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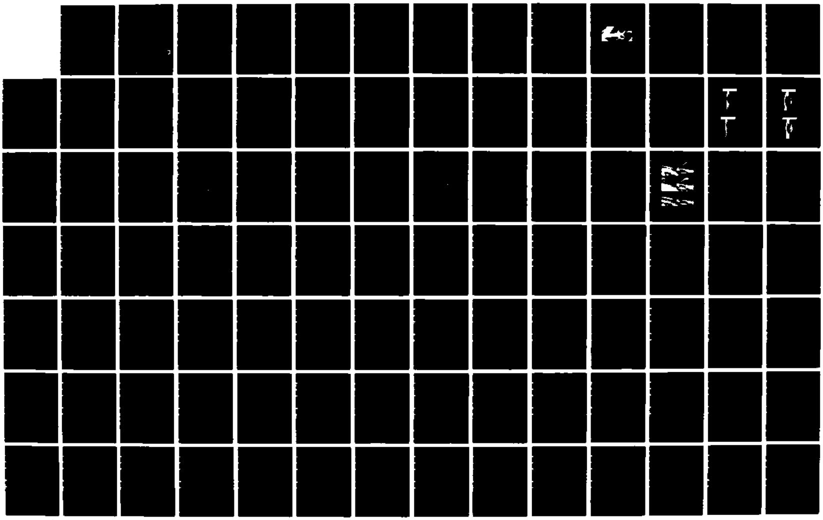

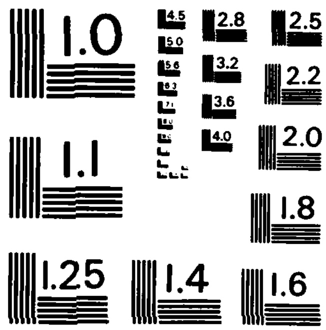

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MODELING OF PROPULSIVE JET PLUMES--EXTENSION OF MODELING CAPABILITIES BY UTILIZING WALL CURVATURE EFFECTS

by
S. E. Doerr

June 1984

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

Modeling of aerodynamic interference effects of propulsive jet plumes, by using inert gases as substitute propellants, introduces design limits. To extend the range of modeling capabilities, nozzle wall curvature effects may be utilized. Numerical calculations, using the Method of Characteristics, were made and experimental data were taken to evaluate the merits of the theoretical predictions. A bibliography, listing articles that led to the present report, is included.

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NOMENCLATURE

A	area (in^2)
D	forebody diameter (in.)
K	specific heat ratio (-)
L	boattail length (in.)
M	Mach number (-)
P	pressure (psi)
r	radius (in.)
R	radius of curvature (in.)
R_e	Reynolds number (-)
S	separation distance measured from end of boattail (in.)
T	absolute temperature (°R)
U_I^*	measure of the local rate of acceleration near the nozzle lip [12]
x	intrinsic coordinate system (in.)
α	angle-of-attack (deg)
β	boattail angle (deg)
δ	control surface deflection angle (deg) or velocity boundary layer thickness (in.)
δ^*	displacement thickness of the boundary layer (in.)
δ^{**}	momentum thickness of the boundary layer (in.)
γ	specific heat ratio (-)
ρ	density (lb_m/in^3)
θ	conical divergence angle (deg) or momentum thickness of the boundary layer (in.)
ω	Prandtl-Meyer angle (deg)

Subscripts

b	back or base conditions
e	exit nozzle plane
E	external
F	free jet surface conditions
L	conditions at nozzle lip
m	model
o	stagnation state
p	prototype
x	flow conditions before normal shock
y	flow conditions after normal shock
*	throat conditions
∞	free stream conditions

*See APPENDIX for nomenclature for a particular program.

1. INTRODUCTION

High acceleration rates required for tactical missiles and the resulting jet-slipstream interaction can give rise to undesirable aerodynamic performance. Degradation in performance is due to introducing severe drag penalties through lower than ambient base pressures or, as the ratio of jet stagnation-to-ambient pressure increases, by leading to plume induced separation [1,2]†, see Fig. 1. Separation, in addition to increasing drag, can adversely affect missile stability and control surface effectiveness [3].

The flow component method of Korst, et al. [4], has led to a basic understanding of the problem, including the establishment of quantitative relations accounting for the influence of all the pertinent variables and parameters. But confidence in the method to predict actual separation locations, even for the simplest of vehicle geometries [5], leaves much to be desired.

For actual missile or vehicle configurations, especially when large angles-of-attack, canted fins and mixed subsonic, transonic and supersonic regions are present, the resulting flowfields are very complex, see Fig. 2. The usual support of predictive and corrective prototype evaluations will, therefore, be strongly dependent on wind tunnel modeling.

Even then difficult problems arise in dealing with the aerodynamic and thermodynamic effects of propulsive jet plumes. Because it is inconvenient and often impossible to carry out wind tunnel experiments

†Numbers in brackets refer to entries in the BIBLIOGRAPHY.

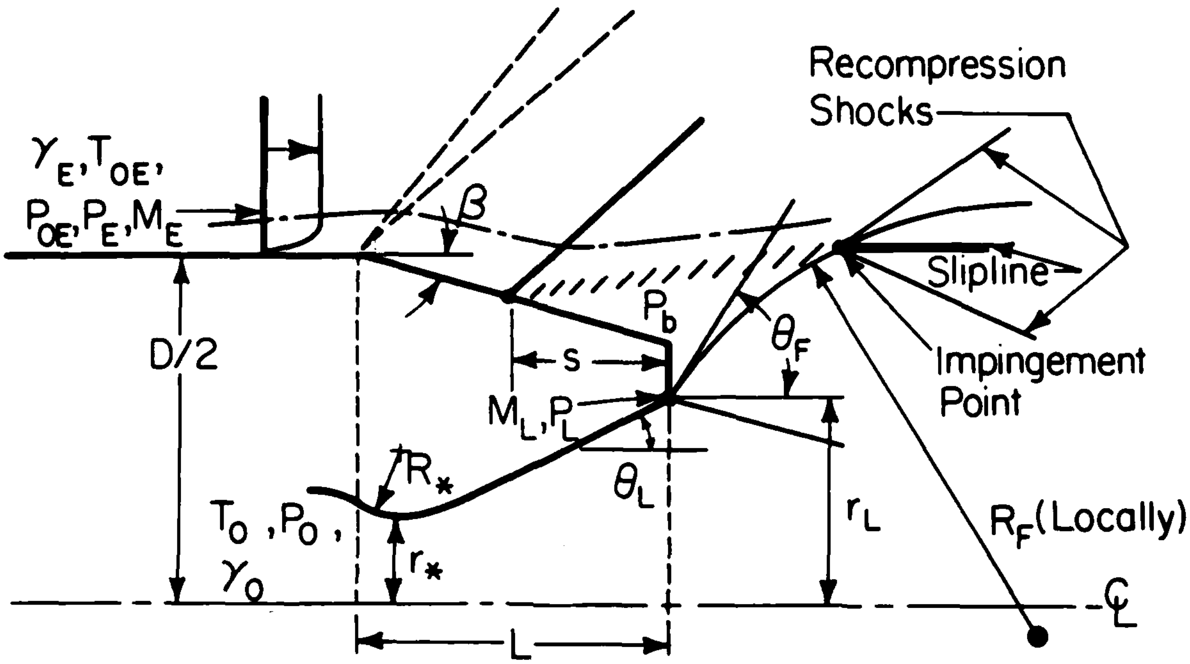


Figure 1 Flow Configuration for Plume Induced Separation from a Conical Afterbody and Identification of Geometric and Operational Parameters

Figure 2 Finned Missile Afterbody,
$M_\infty = 2$, $\alpha = -6°$, $\delta = +10°$

with the actual hot propellants, one must revert to appropriate methods of plume simulation.

Jet plumes have been generated in wind tunnels using a variety of methods including the use of cold or heated air through geometrically modeled nozzles, small rocket motors, radial injection and solid bodies which simulate the plume shape. Deficiencies inherent in these methods can be traced to failure to account for such factors as specific heat ratio influence, mass entrainment, viscous effects, wake closure, temperature and plume deflection. Accounting for all of the pertinent parameters in wind tunnel tests, simultaneously, is impossible.

Requirements for modeling interactions include, in particular:

1. Geometrically congruent inviscid jet contours,
2. Correct pressure rise-jet boundary deflection (plume stiffness) and
3. Mass entrainment characteristics along the wake boundary.

Realizing that the predominant dynamic interactions depend strongly on the specific heat ratio of the propellant and to a lesser degree on its temperature, several investigators [6,7] have performed cold gas tests using gases with specific heat ratios similiar to those of the actual propellant at its elevated temperature. However, the large amounts required and the relative high costs of running with such gases makes this type of testing often impractical.

A second approach to the problem is to design test nozzles so that the dynamics of the actual plume can be simulated when the nozzles are run with unheated air. Investigators [1,8,9] taking this approach have used nozzles which meet the simple simulation requirements proposed by

Goethert and Barnes [10]. These include duplicating the initial plume angle as the jet exits from the nozzle and matching the ratio of jet-to-ambient pressure between the model and the actual (prototype) flow.

However, Love and Lee [11] have shown that duplicating the initial plume angle is not a stringent enough requirement to cope with plume induced separation where the curvature of the jet boundary is of importance. An analysis of inviscid jet boundaries near centers of expansion by Johannesen and Meyer [12] provided a means for introducing the initial plume radius of curvature. Korst [13] then interpreted their work as a device for geometric plume modeling.

The assumption of conical source flow as an approach condition to the nozzle exit, while greatly simplifying the analysis, becomes inaccurate for nozzles where the transonic throat region produces nozzle flowfields which are significantly different from those of ideal conical source flow. Thus, the supersonic flowfield in axisymmetric nozzles as affected by the throat wall curvature requires careful consideration [14,15,16].

In addition, replacing a prototype propellant by an (inert) gas of higher specific heat ratio, will generally lead to model nozzles with smaller exit Mach numbers and exit wall angles. This, in effect, imposes a lower limit on the range of geometrical modeling [17], see Fig. 3. The main problems are a relatively thick boundary layer [18,19,20], shock formations [21], and manufacturing nozzle contours with small conical divergence angles.

In order to avoid these effects and to extend the useful range of model testing, the possibility of utilizing model nozzles with con-

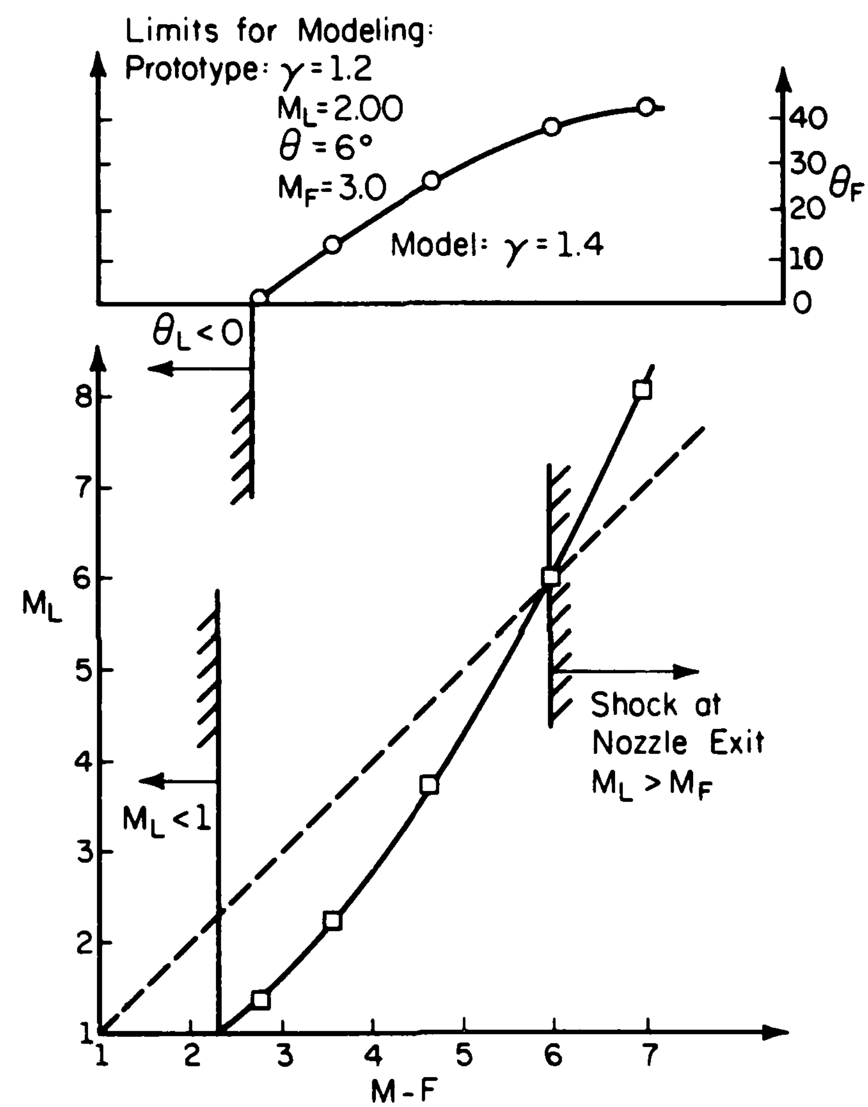


Figure 3 Lower Limit on Range of Conical Geometrical Modeling

verging-diverging-converging (CDC) cross sections (including the conventional transonic near throat geometry), see Fig. 4, is here investigated by analytical and experimental methods.

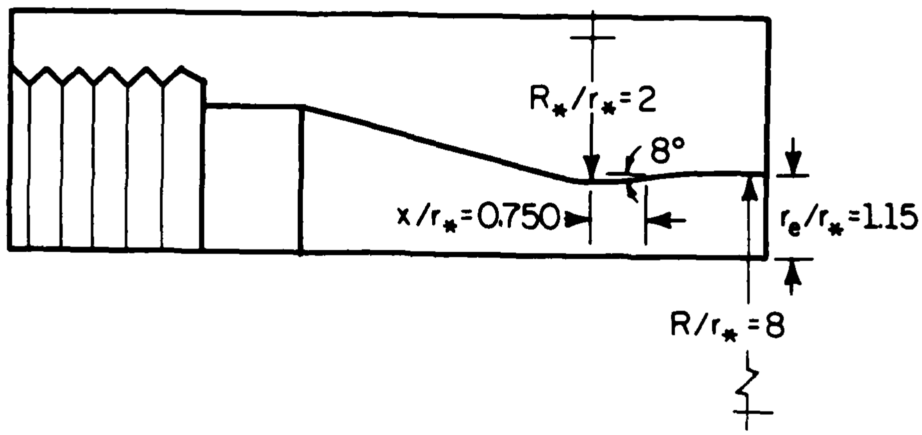


Figure 4a CDC Model (Nozzle A)

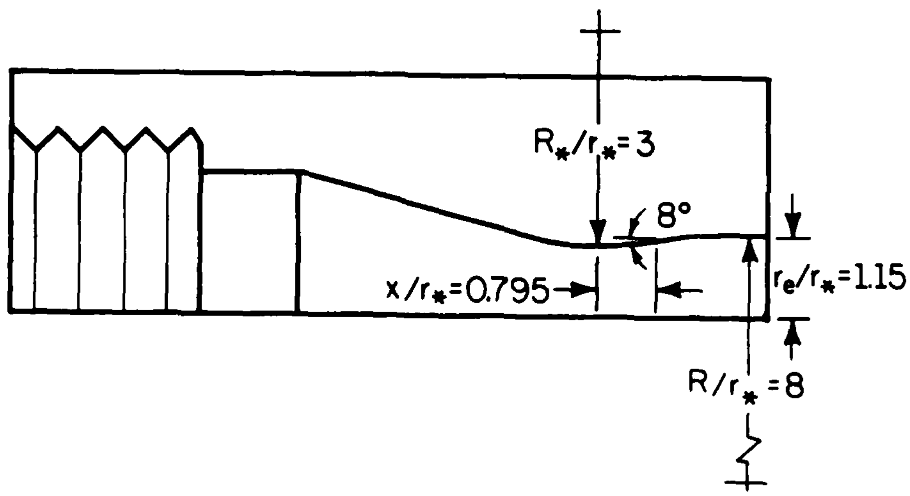


Figure 4b CDC Model (Nozzle B)

2. PLUME SIMULATION REQUIREMENTS

Plume simulation is the attempt to achieve geometrically congruent jet contours between the model and prototype flows with the most pertinent dynamic characteristics satisfied.

2.1 Geometric Simulation

Previous techniques have been inadequate in that they only duplicate the initial plume angle. The current method matches, in addition, the initial radius of curvature needed to adequately define the plume shape bordering the near wake. With reference to Fig. 5, this can be expressed as

$$\theta_{Fm} = \theta_{Fp} \quad \text{and} \quad R_{Fm} = R_{Fp} \qquad (1,2)$$

where m denotes the model flow and p the prototype flow. It is assumed that the following parameters are given: γ_{Fp}, M_{Lp}, θ_{Lp} and M_{Fp}.

The geometric simulation parameters, θ_F and R_F, are related to flow conditions at the nozzle lip by a second-order series solution of the axisymmetric compressible potential flow equations near centers of expansion [12]. As a result of such an analysis, a model nozzle configuration can be determined (for either ideal or non-ideal conical source flow) [17]. It is important to note that geometrical plume modeling accomplished by this analysis, while establishing proper values for initial expansion angle and plume radius of curvature, can be interpreted to be a second order initial condition for a more accurate determination of plume contours by the Method of Characteristics (MOC). In

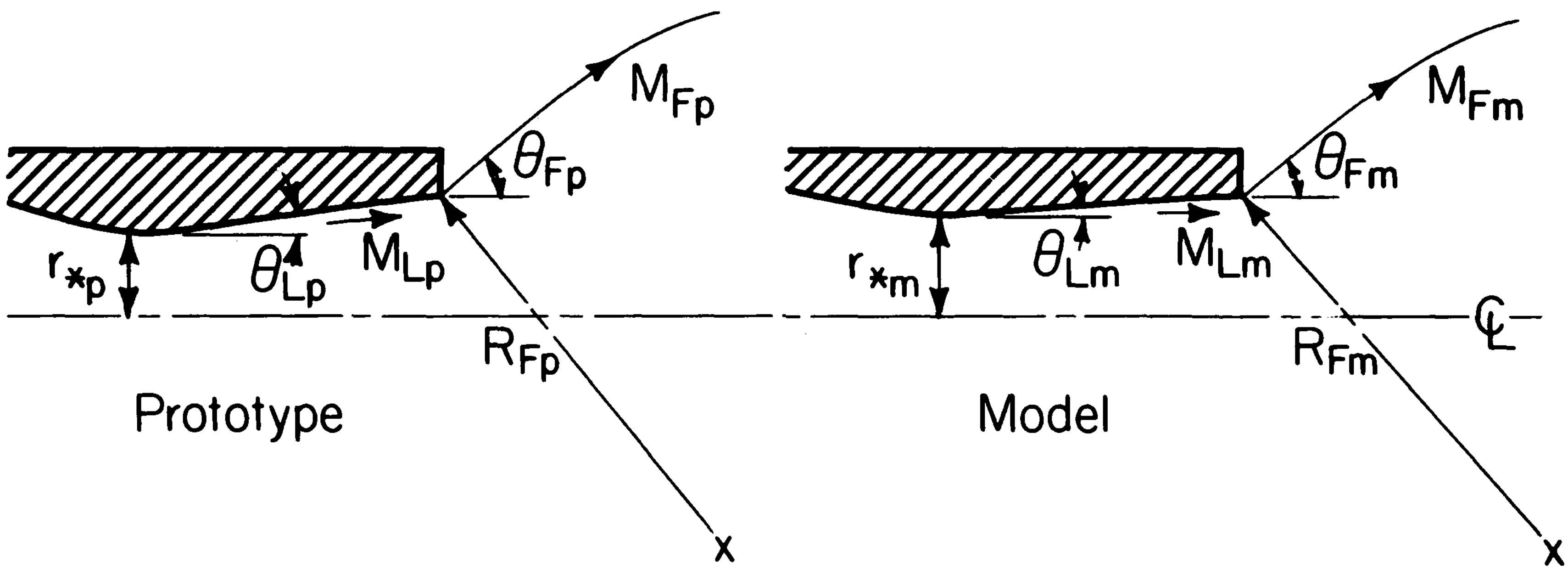


Figure 5 Plume Simulation Parameters

addition, it is significant that satisfying the geometrical congruence of the plume contour still leaves one free parameter available for achieving dynamic simulation.

2.2 Dynamic Simulation

Ideally the dynamic specifying condition should account for both inviscid and viscid aspects of the plume. This apparently requires two closure conditions to be satisfied--the recompression ratio at the end of the wake and conservation of mass within the wake. Due to geometrical requirements, only one parameter is, however, available.

For studying plume induced separation, it is recommended [22] that plume surface pliability be modeled to satisfy both the inviscid and viscid interaction mechanisms of plume-slipstream confluence at the end of the wake. The selection of inviscid pressure rise-deflection relations along the plume boundary was chosen because the jet plume essentially contributes to the wake closure through its inviscid jet boundary geometries before and after impingement. The viscid aspects of jet mixing, attributed predominately to the separated slipstream, are diminished by the relaminarization of the nozzle boundary layer during its rapid expansion after the nozzle exit.

For relatively small angular deflections (beyond the linearization limits), the requirement for plume pliability (jet surface Mach number) is found from Weak Shock Modeling (Program WSHOCK)†.

†Program listings are found in the APPENDIX.

2.3 Calculation Procedure

One intends to model a prototype plume that results from γ_p, M_{Lp}, θ_{Lp} and M_{Fp}, yielding an initial slope θ_{Fp} and an initial radius of curvature R_{Fp}. For a given model gas γ_m, using either Weak or Strong Shock Modeling will determine M_{Fm}.

For a selected trial value of M_{Lm}, the Prandtl-Meyer relation for θ_{Lm} and the analysis of Johanessen and Meyer [12] will yield the radius of curvature R_{Fm}. If a solution exists, it will be found by satisfying $R_{Fm} = R_{Fp}$.

Nozzle geometries, nozzle Mach numbers, and jet-to-ambient pressure ratios can differ widely from model and prototype. These parametric variations do not seem to impair the accuracy of matching the plume surface geometry.

Due to the complexity of the iterative calculation procedures, one must revert to computational solutions.

3. FLOW MODEL

Although geometric simulation requirements involve only inviscid flow mechanisms, the inherent dependence of the problem on viscous mass interchange requires viscid-inviscid analysis for adequate closure.

3.1 Nozzle Flow

3.1.1 Inviscid Flowfield

The inviscid analysis of the flow in the nozzle consists of a series solution of the transonic flow in the vicinity of the throat and an inviscid MOC solution for the supersonic flow downstream of the throat region.

The disturbing effects of throat wall curvature on the transonic flow region require an accurate initial condition to properly calculate the supersonic flowfield. Transonic flow analyses by Sauer [15], Kliegel and Levine [16], or Dutton and Addy [23] can be used to provide the noncharacteristic initial condition for the MOC nozzle solution.

The MOC (Program CDCSECT) is then used to calculate the supersonic flowfield. Nozzle wall geometry and model propellant properties serve as input. Characteristics are calculated throughout the nozzle's region of influence on the developing plume.

The exit conditions of the inviscid nozzle flow serve as input to the MOC plume solution, see Section 3.2.1, and are also used in determining the second-order modeling scheme for non-ideal conical source flow.

3.1.2 Viscous Boundary Layer Effects

The viscid analysis of the nozzle flow is an extension of established methods for compressible laminar-transitional-turbulent boundary layer development [19,24] (Program GRTRAN1).

Beginning with the Gruschwitz [25] method for a laminar compressible boundary layer, the transition to turbulent flow is based on a criterion related to the local Reynolds number based on an equivalent displacement thickness as a function of the local second-shape factor [19]. The turbulent portion of the boundary layer was then calculated using a scheme based on the method of Culick and Hill [24]. No distinction has here been made between the point of instability and the actual onset of turbulence.

Since it was found that the nozzle boundary layer was predominately turbulent (transition occurred before the throat), comparison with the fully turbulent results based on the Bartz method [18] (Program BARTZ) was attractive. Close agreement between the two can be observed, see Fig. 6.

Both methods utilize the approximate integral momentum and energy boundary layer equations which are solved for local velocity, displacement, momentum and energy thicknesses. As part of the output, the BARTZ program calculates a Reynolds number (based on the velocity boundary layer thickness) that is used in the calculation of the free shear layer development, see Section 3.2.2, and in the calculation of plume impingement, see Section 5.3.

The boundary layer displacement thicknesses were small enough so that an iterative procedure for obtaining improved inviscid nozzle flow field solutions was not deemed necessary.

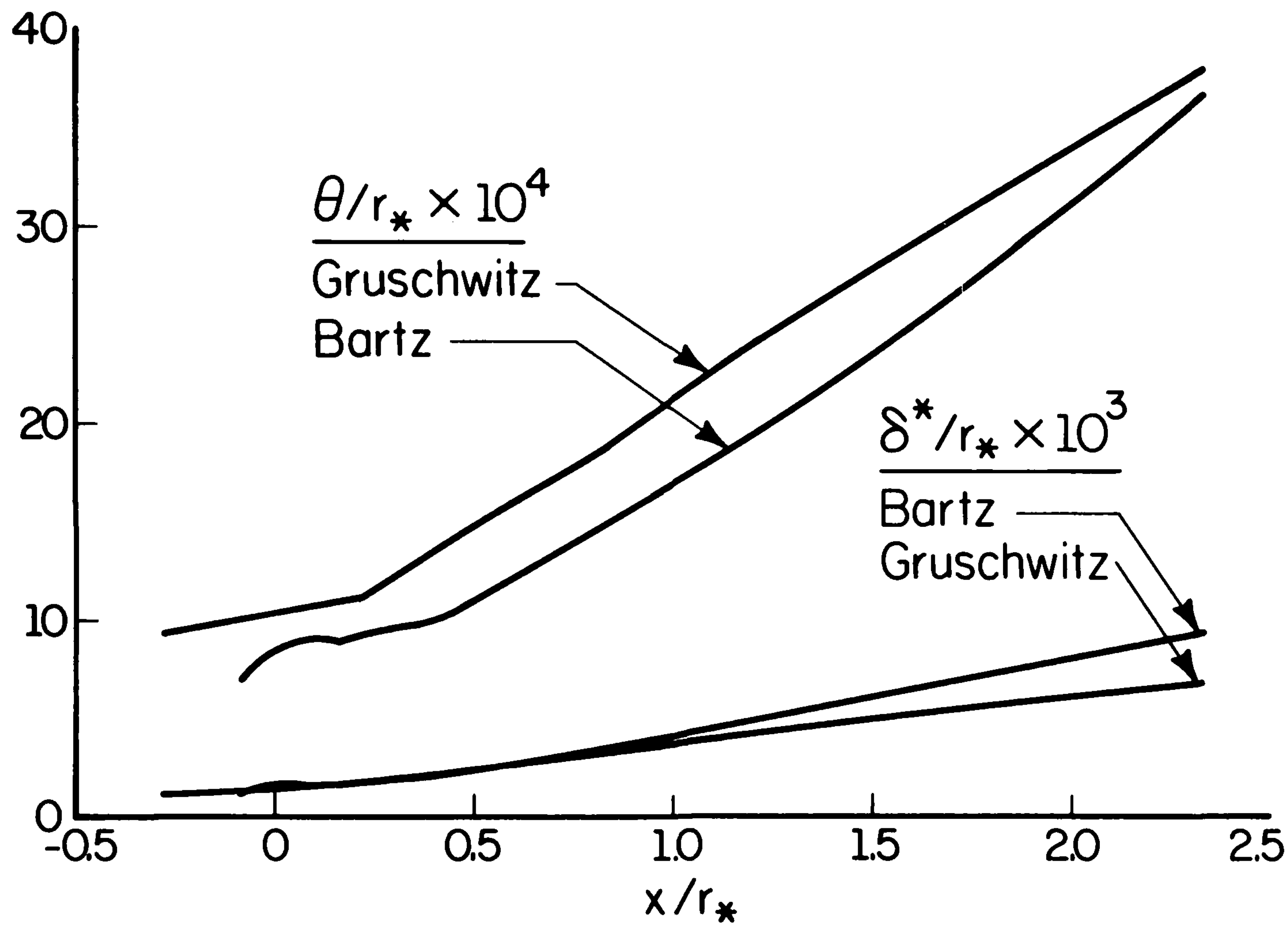


Figure 6 Nozzle Boundary Layers

3.2 Plume Development

3.2.1 Inviscid Plume Boundary

The MOC (Program JETC) is used to calculate the expanded inviscid plume boundary. The final characteristic line calculated by the MOC nozzle solution, see Section 3.1.1, is used as a starting condition for the expansion centered at the nozzle exit. Jet surface Mach number is specified to account for plume pliability, see Section 2.2. Circular arcs are then fitted between the characteristic boundary points to account for angular variation in the plume surface.

A second-order modeling scheme for non-ideal conical source flow (Program NOCON) is used as a comparison, see Fig. 7. Flow parameters at the nozzle exit (M_L, θ_L, U_1^*) calculated by the MOC nozzle solution, serve as input.

3.2.2 Viscous Mass Entrainment

While it can be expected in cases involving large regions of plume induced separation that the mass and energy balance in the wake is predominantly controlled by the viscous mechanisms of the slipstream (external flow), it is nevertheless of interest to investigate the viscous mechanisms along the jet boundary. In absence of a confluent slipstream, plume impingement of solid walls will indeed be entirely controlled by the viscid-inviscid interactions of the plume alone [26]. Mass bleed may be necessary for the proper modeling of such impingement flows [27,28].

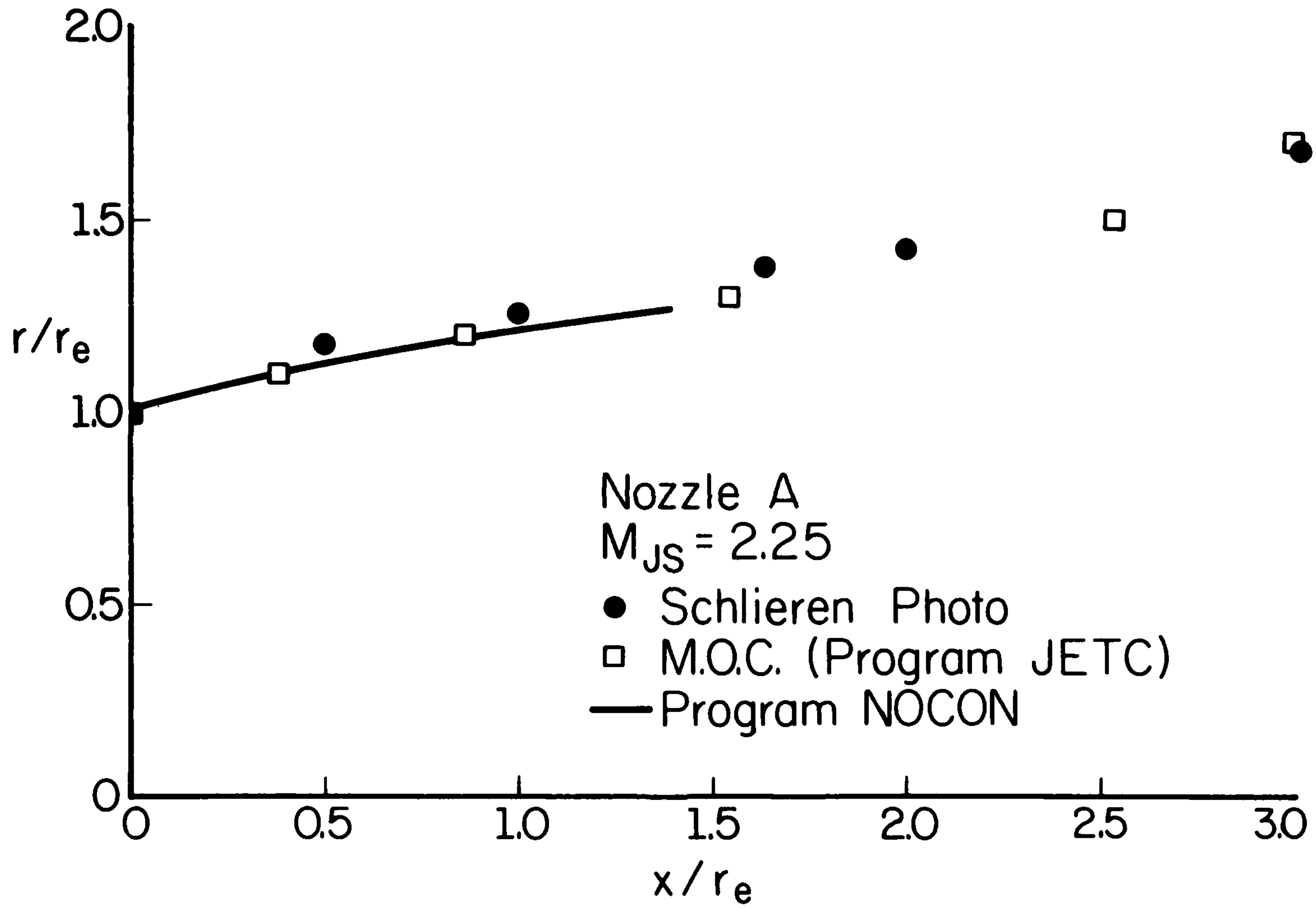


Figure 7 Plume Shape Comparison

A computer program (Program FREES) is available for determining the development of the free shear layer in the form of a two-layer compressible laminar-transitional-turbulent model based on local similarity concepts [29], see Fig. 8. While it is not possible to make accurate measurements from the Schlieren photograghs, Figs. 9, there is general agreement between theory and experiments.

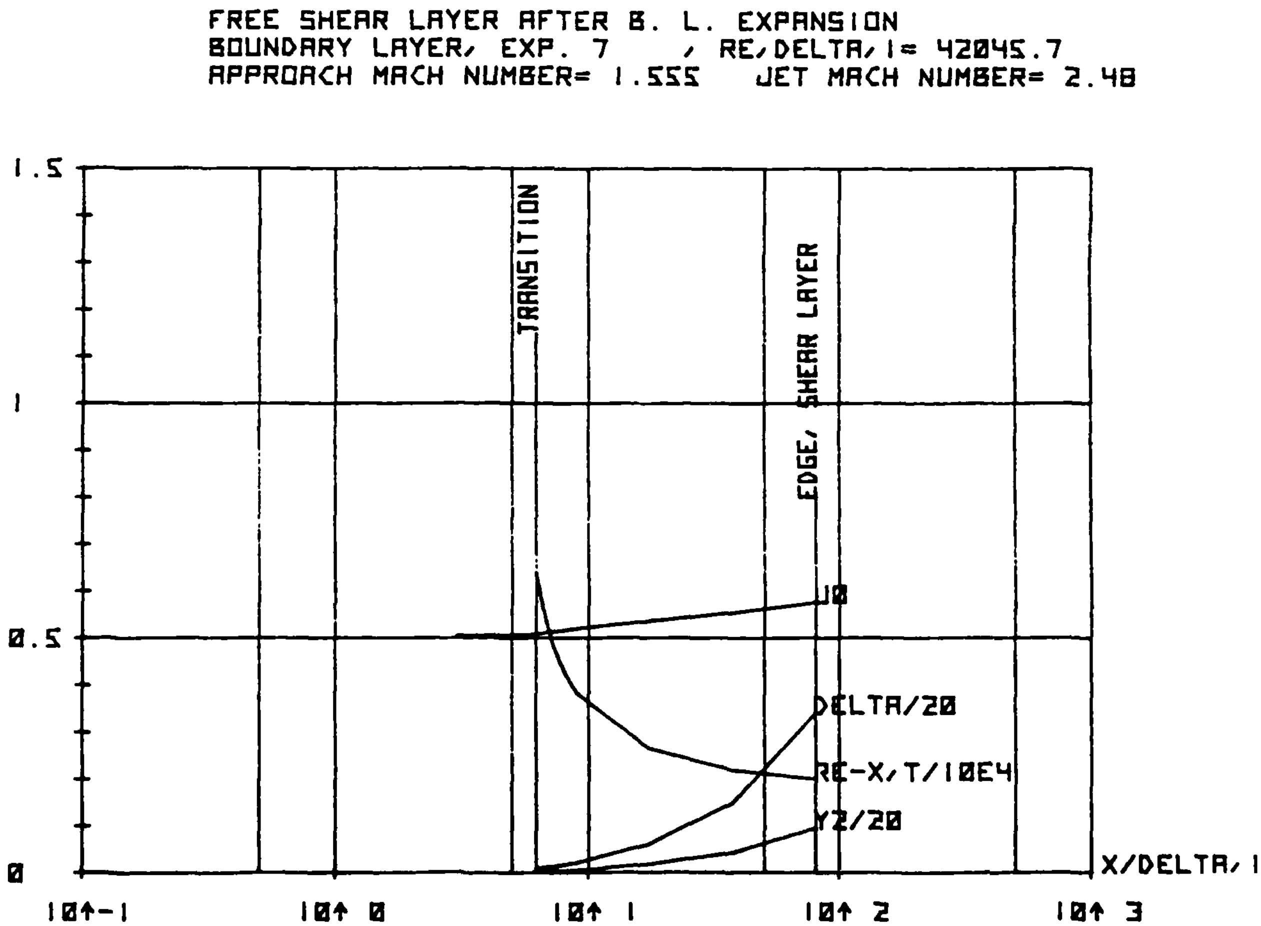


Figure 8 Free Shear Layer Development

(a) $M_{JS} = 1.73$ (b) $M_{JS} = 1.98$

Figure 9 Schlieren Photographs--Nozzle A

(c) $M_{JS} = 2.22$

(d) $M_{JS} = 2.41$

Figure 9 Schlieren Photographs--Nozzle A

4. EXPERIMENTAL INVESTIGATION

4.1 Objectives

The objective of this thesis is to determine the feasibility of designing and operating a CDC nozzle to extend the range of plume simulation. To do so, the following were designated as specific objectives:

1. Establish limits for conical-to-conical nozzle modeling,
2. Design a nozzle based on theoretical analysis and explore the extension of these limits by use of CDC nozzles,
3. Experimentally verify the theoretical calculations including starting characteristics and flowfield peculiarities (coalescence of disturbance lines, etc.), and
4. Determine the potential and limitations of using such nozzles as modeling tools.

4.2 Apparatus and Instrumentation

Two specific nozzle configurations were selected from a variety of candidate configurations. The reason for the selection was that the two produced the intended plume shape and had an almost constant Mach number wall distribution after the conical supersonic expansion section. This allowed changing the exit lip angle, θ_L, without a change in exit lip Mach number. The CDC configuration was derived from the idea of replacing the conventional conical supersonic expansion section and free jet boundary by an appropriate combination of conical and circular arc wall sections.

The two nozzles, see Fig. 4, were fabricated to study the effect of throat wall curvature. This effect is evident in Fig. 10. As seen in

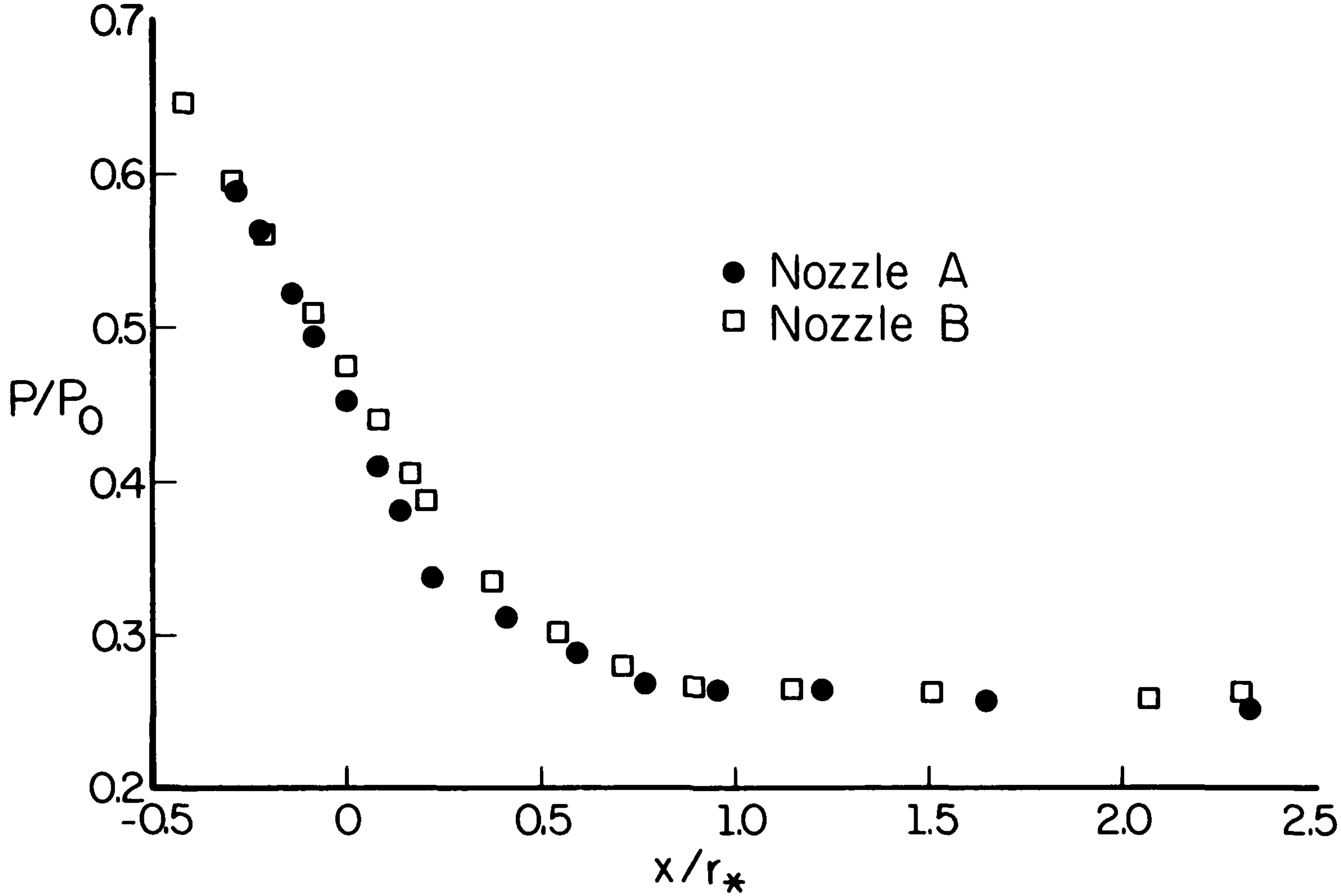


Figure 10 Effect of Throat Wall Curvature on Pressure Distribution

Figs. 11 and 12, application of the Kliegel and Levine [16] throat solution successfully predicts the transonic flowfield.

Since the analytical design process was done non-dimensionally, the size of the throat determined all other dimensions. An analysis of the compressor/storage system led to a throat radius of 0.375 inches.

The wall pressure data and Schlieren photographs were taken with the nozzles exhausting into the ambient. Pressure measurements were taken with a conventional transducer-scanivalve-amplifier setup. An Apple II-e computer was employed as a data reduction/recording device.

For the impingement and stagnation pressure data, an enclosure was fabricated. The enclosure served several uses, it positioned the exhaust tube that the impingement data was taken from, it secured the pitot probe, it allowed evacuation of the test section (lowering the back pressure) and lastly it significantly decreased the noise level during tests. If mass bleed was necessary, a metering valve could easily be connected to the enclosure.

Pressure taps in the exhaust tube provided a means for collecting impingement data. The inner radius of the tube was 1.4 times larger than the nozzle exit radius. This allowed plume expansion with the assurance of impingement.

All testing was done at the Gas Dynamics Laboratory, Department of Mechanical and Industrial Engineering, University of Illinois at Urbana-Champaign.

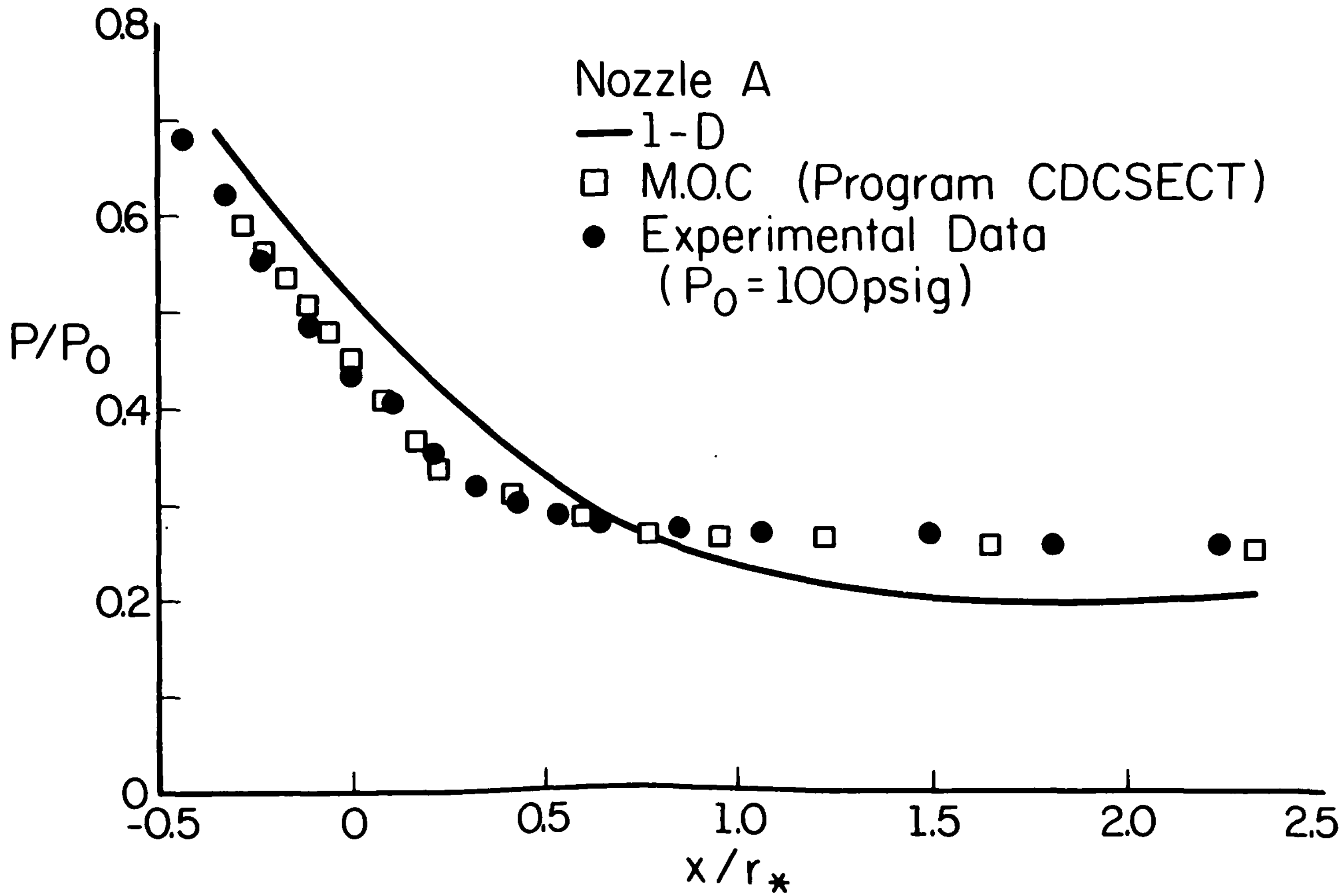


Figure 11 Wall Pressure Distribution for Nozzle A

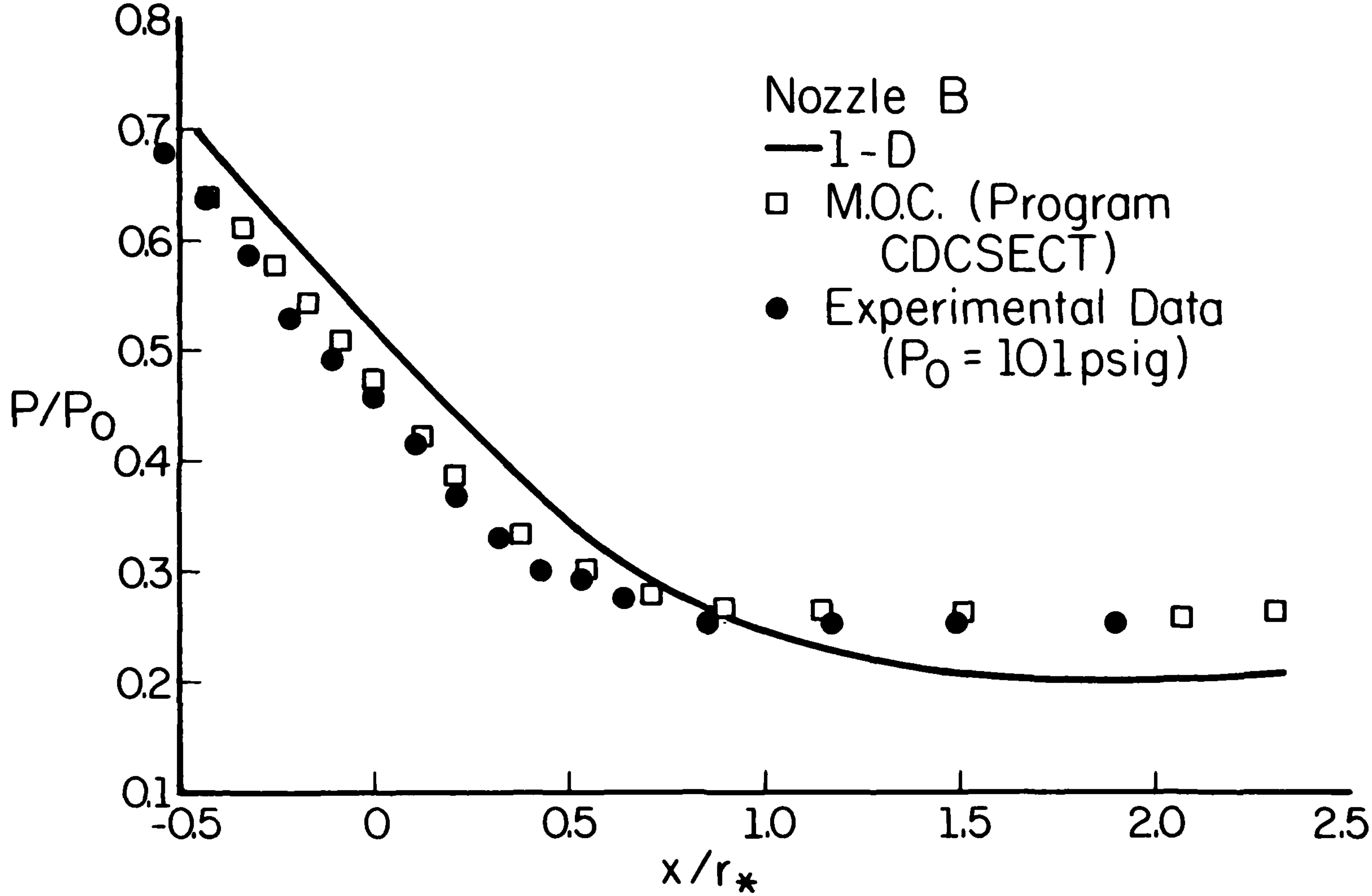


Figure 12 Wall Pressure Distribution for Nozzle B

5. EXPERIMENTAL RESULTS

5.1 Nozzle Wall Pressure Distribution

Nozzle wall pressure data taken from the models are shown in Figs. 11 and 12 compared to the MOC (Program CDCSECT) and one-dimensional solutions. Note the constancy of the Mach number after the conical divergence section.

5.2 Plume Development

5.2.1 Plume Shape Comparison

The Schlieren photograph, Fig. 9c, was used to compare the inviscid solutions of the MOC (Program JETC) and of the second-order, non-ideal conical source flow approximation (Program NOCON). The comparison is shown in Fig. 7.

In Fig. 9, note the disturbance lines emerging from inside the nozzle. These were predicted and can be shown graphically on a plotting version of the MOC nozzle solution (Program CDCSECT). The disturbance lines are focused by the concave wall curvature of the nozzle and eventually coalesce into oblique shock waves. Neither program, JETC or NOCON, takes these disturbance lines into account in calculating plume shapes. Even so, agreement between calculated and observed plume boundaries is satisfactory, see Fig. 7.

Even though the coalescence of compressive disturbance lines cause discontinuities in plume slope, this is detrimental only if the coalescence occurs upstream of the confluence with the external slipstream or if it interferes with the plume boundary in the wake closure region.

5.2.2 Mach Number Survey

The survey was done by measuring the Rayleigh-pitot stagnation pressure using a 0.050 in. diameter pitot probe and using normal shock relations to determine the Mach number M_x. Figure 13 shows the comparison of experimental data with the inviscid solutions (Programs CDCSECT and SECTP).

Non-parallel outflow, shock wave interference, and mechanical inaccuracy are some of the reasons why the theory does not better predict the Mach number distribution.

5.3 Jet Impingement on Solid Cylindrical Surface

The impingement study was done to further characterize the modeling capabilities (or limitations) of the CDC configurations. The theoretical calculations (Program ASUEXP) have been previously determined to be reliable [30] for predicting viscid and inviscid impingement aspects of conventional converging-diverying nozzles discharging into sudden enlargements in cross section.

In the present study, predicted and measured pressure distributions, see Fig. 14, were analyzed. While in agreement for base pressure, they exhibit distinctive differences near and after plume impingement. This may be attributable to the proximity of the coalescent compression waves, which are presently unaccounted for by the inviscid jet plume boundary calculations. The calculated length of the viscid interaction region, $(x_{inviscid}-x_{viscid})/r_e$, can be considered as a measure for the pressure feedback domain through the impingement wall boundary layer. This serves as an illustration for the limitations inherent in the use of CDC nozzles alluded to in Section 5.2.1.

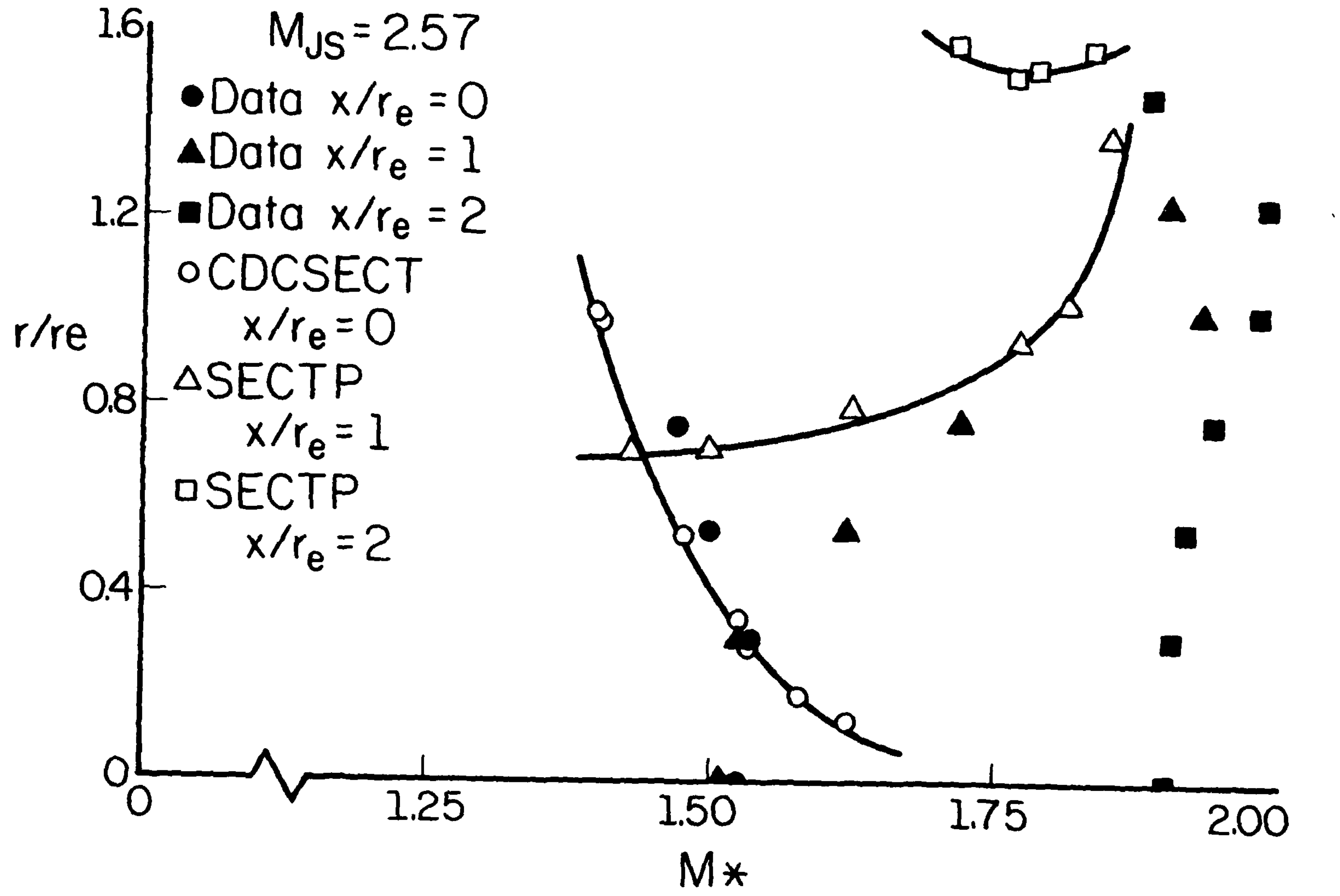


Figure 13 Mach Number Distribution through Plume

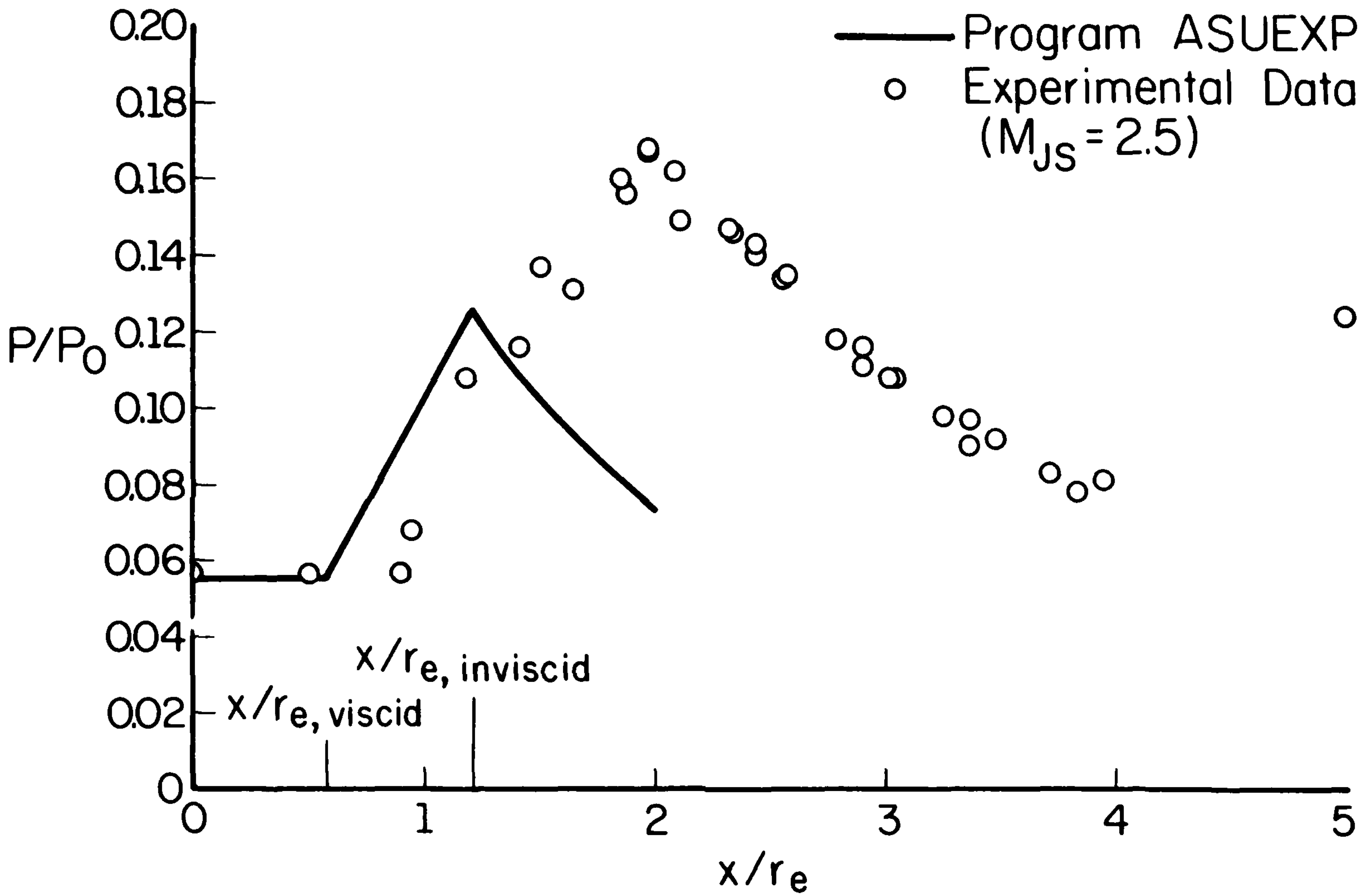
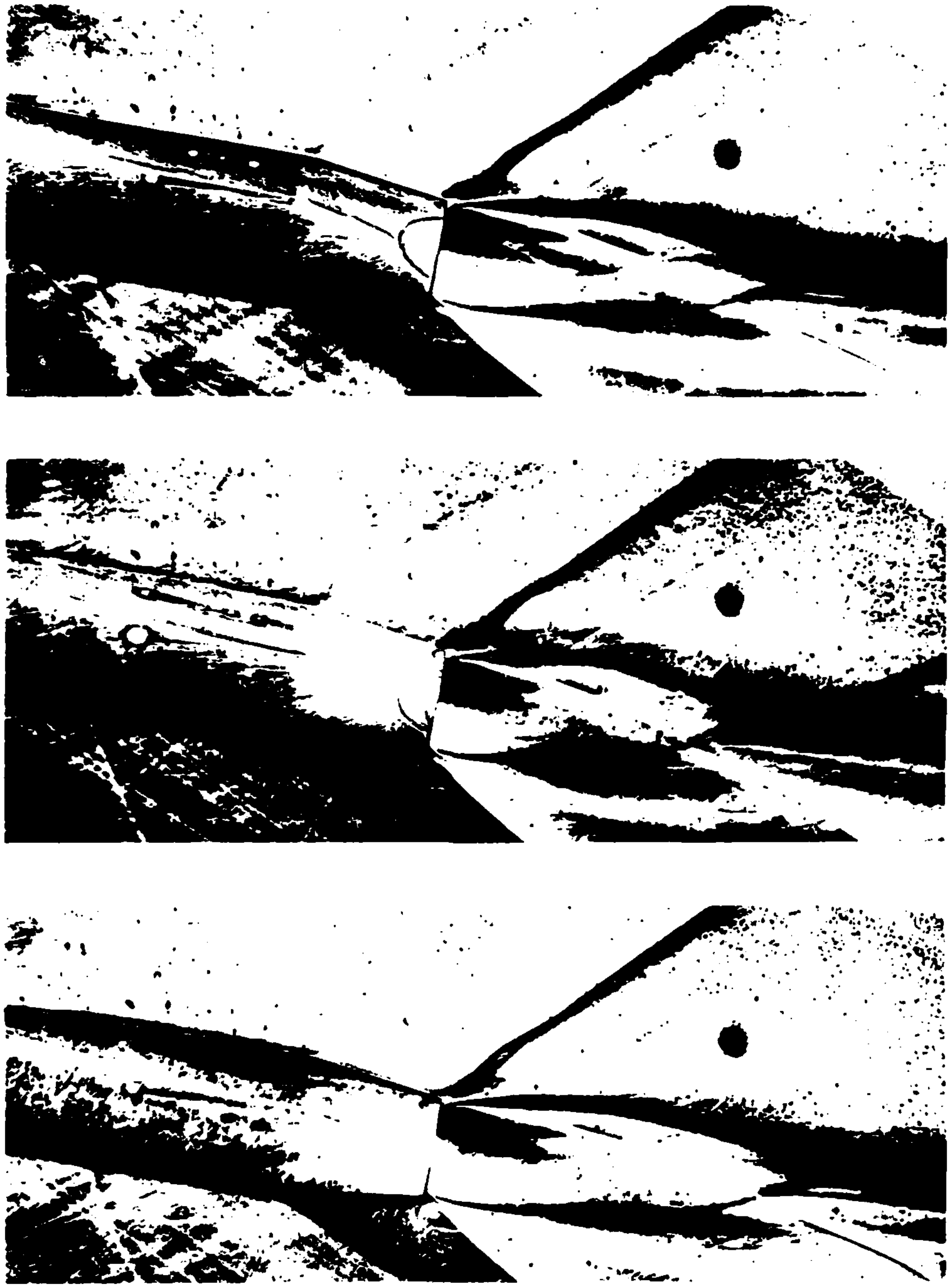


Figure 14 Impingement Pressure Distribution

6. CONCLUSIONS

1. Success of the general modeling scheme has been established in as much as the method allows simulation of aerodynamic plume interaction effects of great detail even under complex operational conditions (large angles-of-attack, boattail, etc.), see Fig. 15.
2. The CDC nozzle configurations do extend the range of geometrical modeling, see Fig. 16, allowing the modeling of prototype nozzles with lower exit lip Mach numbers using model propellants with higher specific heat ratios.
3. The wall curvature of the end section of the CDC nozzle produces convergence of compressive lines which leads to plume boundary deflections downstream of the nozzle, this limits the modeling range unless large radii of curvature, $R/r^*>8$, are used or these effects are downstream or small in the region of interest.
4. Agreement between predicted and experimentally determined flow fields (inviscid and viscid), within the limits stated above, has been observed.
5. While plume-slipstream interactions have not been carried out in this investigation, it can be expected that the successful extension of the modeling methodology can be established by experimentation in appropriate wind tunnel facilities [31].



(a)

(b)

(c)

Figure 15 Oil Flow and Schlieren Photographs at $\alpha = -10°$ and P_o = 1.00 MPa for the Freon (Prototype) and Air (Model) Nozzles

(a) Freon M_L = 2.6
(b) Air M_L = 1.41
(c) Air M_L = 2.03

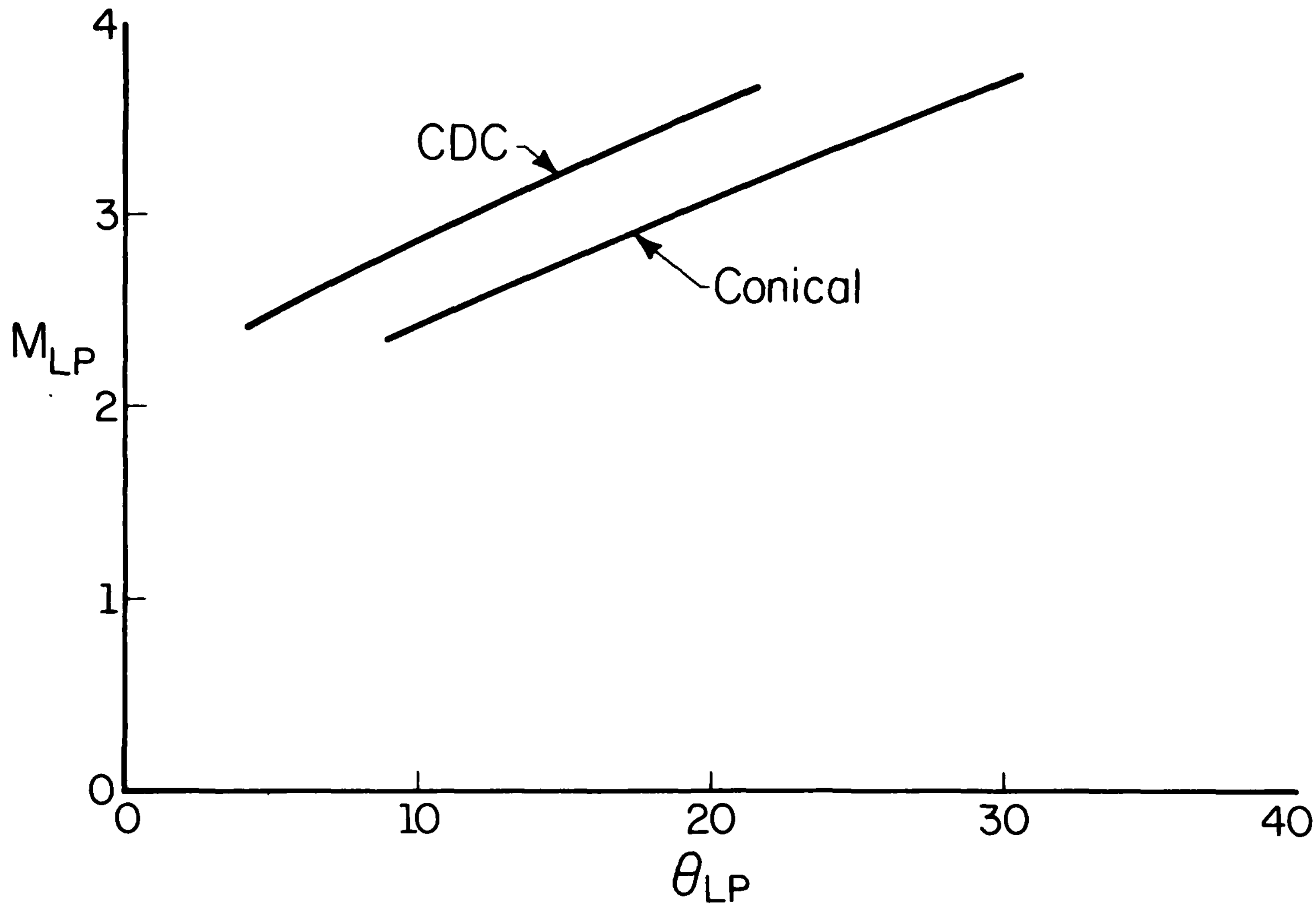


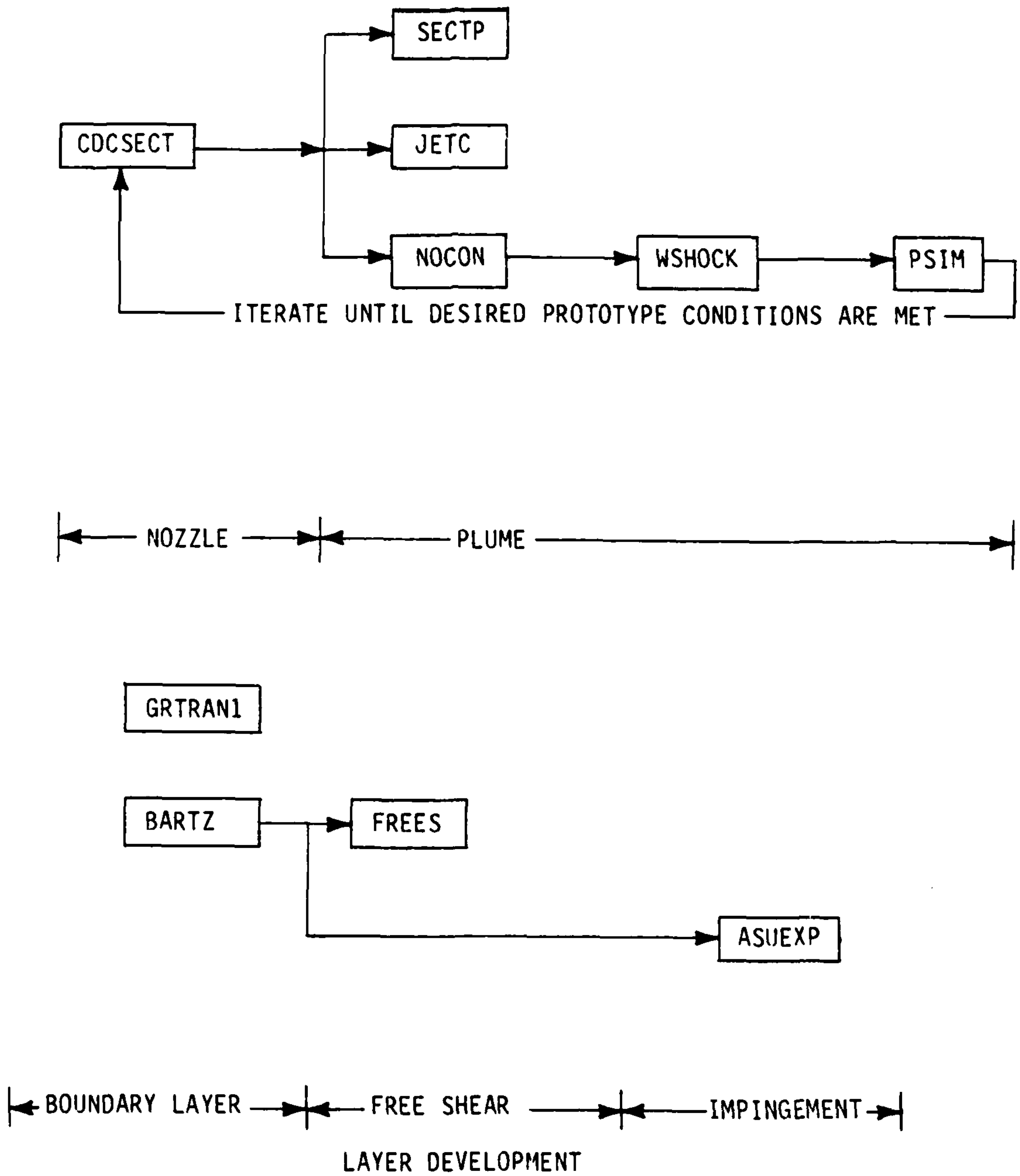
Figure 16 Extension of Geometrical Modeling Range (M_{LP} and θ_{LP}, $\gamma = 1.16$) through the Use of CDC Model Nozzles ($\gamma = 1.4$)

APPENDIX

This APPENDIX is a compilation of all of the computer programs that were used in this thesis. Almost all of the theoretical work done for the thesis has been incorporated into these programs which were developed in the Department of Mechanical and Industrial Engineering, University of Illinois at Urbana-Champaign, Urbana, Illinois. Because of the number and complexity of the programs, each is listed with a brief discussion of what the program does, the input and output and pertinent nomenclature.

The listing includes the following programs:

All programs, unless otherwise stated, are written in BASIC and were run on the CYBER 175. As an aid in understanding what each program does and its relationship to the thesis and other programs, a flowchart is included on the following page.


SECTP
CDCSECT
JETC
NOCON
WSHOCK
PSIM
ITERATE UNTIL DESIRED PROTOTYPE CONDITIONS ARE MET
NOZZLE
PLUME
GRTRAN1
BARTZ
FREES
ASUEXP
BOUNDARY LAYER
FREE SHEAR
IMPINGEMENT
LAYER DEVELOPMENT

Program CDCSECT

Program CDCSECT calculates the inviscid supersonic nozzle flowfield by the MOC after the transonic flow region has been calculated by a method based on the analysis of Kliegel and Levine [16]. Individual characteristics are printed and the last characteristic line is stored in file "SONIC" to be used as input for Programs JETC and SECTP. In addition, a routine has been added to allow the calculation of flow properties at a given cross section.

input quantity	symbol	explanation
γ	GO	specific heat ratio
r_e/r^*	OO	nozzle exit radius†
R^*/r^*	CO	throat radius of curvature, (see Fig. 4)
θ	TO	conical wall divergence angle (deg)
	k	number of points in the MOC net
	X4	x location of the end of the conical divergence section, measured from the throat
R/r^*	UO	radius of curvature for the circular arc
	LO	x location, from the nozzle exit, for flow profile determination

†All nozzle dimensions are non-dimensionalized by the throat radius.

<u>output</u>

x/r^*, r/r^*, M^* and θ (deg) at distinct points throughout the flowfield

M_L, θ_L and U_1^* at the nozzle lip

```
[illegible]
[illegible]
[illegible]
[illegible]
70 FILE #1="SONIC"
80 DIM A$(3),B$(3),C$(3)
90 B$="YES"
100 GOTO 5140
110 IF X3>E AND P<0 THEN 2130
120 Y3=X2
130 Y4=X2+(U0*(SQR(1-((Z2-Y3)/U0)^2)-COS(T0))+Z3-R2)/T9
140 IF ABS(Y4-Y3)<0.0001 THEN 170
150 Y3=Y4
160 GOTO 130
170 X3=Y4
180 R3=R2+(X3-X2)*T9
190 T3=FNZ((Z2-X3)/U0)
200 GOTO 2160
210 DIM X(9,11),R(9,11),M(9,11),T(9,11),D(3,14),E(3,14),F(3,14)
220 DIM G(3,35),K(7),H(7),I(7),J(7),V(4,20)
230 DIM P(1,11),Q(1,11),B(1,11),C(1,11),A(15,11)
240 DEF FNB(X)=ATN(SQR(1-X*X)/X)
250 DEF FNA(X)=ATN(1/SQR(X*X-1))
260 DEF FNS(X)=SQR(2)*X/SQR(G0+1-(G0-1)*X*X)
270 DEF FNQ(A)=1/(M*TAN(A))
280 DEF FNF(A)=SIN(T)*SIN(A)/SIN(T+A)
290 DEF FNG(A)=SIN(T)*SIN(A)/SIN(T-A)
300 DEF FNZ(X)=ATN(X/SQR(1-X*X))
310 PRINT "CONICAL NOZZLE SECTION STARTS AT X="X7;"R="R7
320 PRINT "FOR CDC NOZZLE INPUT X4>X7>0=";
330 INPUT X4
340 IF X4=0 THEN 420
350 Z3=R7+(X4-X7)*TAN(T0)
360 PRINT "ENTER RADIUS OF CURV.,CDC SECTION=";
370 INPUT U0
380 PRINT "CURVED WALL STARTS AT X="X4"AT RADIUS="Z3"RADIUS OF CURV.="U0
390 Z2=X4+U0*SIN(T0)
400 PRINT "MAX. RADIUS OF NOZZLE="Z3+U0*(1-COS(T0))"AT X="Z2
410 GOTO 6060
420 I=S=P=W=1
430 PRINT "NOZZLE EXIT AT X="E
440 Q=N=Z0=0
450 Z=+1
460 PRINT "INITIAL POINTS ALONG NON-CHARACTERISTIC CURVE"
470 PRINT "I=1"
480 FOR J=1 TO K
490 PRINT J,X(I,J),R(I,J),M(I,J),T(I,J)
500 NEXT J
```

```
510 PRINT
520 PRINT
530 PRINT "FLOWFIELD COMPUTED, ALL POINTS PRINTED"
540 PRINT
550 GOTO 740
560 FOR J=1 TO K-S
570 X1=X(I,J)
580 X2=X(I,J+1)
590 R1=R(I,J)
600 R2=R(I,J+1)
610 M1=M(I,J)
620 M2=M(I,J+1)
630 T1=T(I,J)
640 T2=T(I,J+1)
650 GOSUB 930
660 X(I+1,J)=X3
670 R(I+1,J)=R3
680 M(I+1,J)=M3
690 T(I+1,J)=T3
695 IF X3>LO AND X1<LO THEN 697
696 GOTO 705
697 GOSUB 6675
705 IF X3>LO AND X2<LO THEN 707
706 GOTO 720
707 GOSUB 6730
720 NEXT J
730 RETURN
740 GOSUB 560
750 IF I=1 THEN 810
760 IF Z>0 THEN 810
770 IF P<0 THEN 6190
780 GOSUB 1350
790 GOSUB 2210
800 GOSUB 1850
810 GOSUB 2030
860 I=I+1
870 S=S+1
880 W=W+1
890 Z=-Z
900 IF I<3 THEN 920
910 GOSUB 5910
920 GOTO 740
930 M9=M1
940 A1=FNA(FNS(M1))
950 A2=FNA(FNS(M2))
960 A4=A1
970 A5=A2
980 T4=T1
990 T5=T2
1000 M4=M1
1010 M5=M2
```

```
1020 R4=R1
1030 R5=R2
1040 X3=(R2-R1+X1*TAN(T4-A4)-X2*TAN(T5+A5))/(TAN(T4-A4)-TAN(T5+A5))
1050 R3=R1+(X3-X1)*TAN(T4-A4)
1060 T=T4
1070 A=A4
1080 M=M4
1090 Q1=FNQ(A)
1100 G1=FNG(A)
1110 T=T5
1120 A=A5
1130 M=M5
1140 Q2=FNQ(A)
1150 IF R2=0 THEN 2180
1160 F2=FNF(A)
1170 M3=((T1-T2)+G1*(R3-R1)/R4+F2*(R3-R2)/R5+Q1*M1+Q2*M2)/(Q1+Q2)
1180 IF M3>1 THEN 1200
1190 M3=M1
1200 T7=Q2*(M3-M2)-F2*(R3-R2)/R5
1210 T3=T2+T7
1220 IF ABS(M9-M3)<0.0001 THEN 1340
1230 M9=M3
1240 T4=(T1+T3)/2
1250 T5=(T2+T3)/2
1260 M4=(M1+M3)/2
1270 M5=(M2+M3)/2
1280 A3=FNA(FNS(M3))
1290 A4=(A1+A3)/2
1300 A5=(A2+A3)/2
1310 R5=(R3+R2)/2
1320 R4=(R3+R1)/2
1330 GOTO 1040
1340 RETURN
1350 X2=X(I,1)
1360 R2=R(I,1)
1370 M2=M(I,1)
1380 T2=T(I,1)
1390 A2=FNA(FNS(M2))
1400 M9=M2
1410 A5=A2
1420 T5=T2
1430 M5=M2
1440 R5=R2
1450 T9=TAN(T5+A5)
1460 IF X2>X7 THEN 2120
1470 IF X3>X7 THEN 2120
1480 IF B$="YES" THEN 1600
1490 X0=((X2*T9-R2)*T9)/(T9*T9-C0)
1500 X9=(R2*R2+X2*X2*T9*T9-2*R2*X2*T9-1)/(T9*T9-C0)
1510 X3=+X0+SQR(X0*X0-X9)
1520 IF X3>X7 THEN 2120
```

```
1530 R3=SQR(1+C0*X3*X3)
1540 T3=C0*X3/SQR(1+C0*X3*X3)
1550 T3=ATN(T3)
1560 Q2=FNQ(A5)
1570 F2=FNF(A5)
1580 M3=M2+(T3-T2+F2*(R3-R2)/R5)/Q2
1590 IF B$<>"YES" THEN 1700
1600 X0=C0+1-R2+T9*X2
1610 X9=T9*X0/(1+T9*T9)
1620 X3=X9-SQR(X9*X9+(C0*C0-X0*X0)/(T9*T9+1))
1630 IF X3>X7 THEN 2120
1640 R3=1+C0*(1-SQR(1-X3*X3/(C0*C0)))
1650 T3=ATN(1/SQR(C0*C0/(X3*X3)-1))
1660 PRINT X3,R3,T3*57.2958
1670 Q2=FNQ(A5)
1680 F2=FNF(A5)
1690 M3=M2+(T3-T2+F2*(R3-R2)/R5)/Q2
1700 IF ABS(M9-M3)<0.0001 THEN 1780
1710 M9=M3
1720 T5=(T2+T3)/2
1730 M5=(M2+M3)/2
1740 A3=FNA(FNS(M3))
1750 A5=(A2+A3)/2
1760 R5=(R3+R2)/2
1770 GOTO 1450
1780 X8=X3
1790 R8=R3
1800 M8=M3
1810 T8=T3
1820 IF X3>LO AND X2<LO THEN 1822
1821 GOTO 1840
1822 GOSUB 6730
1840 RETURN
1850 FOR J=K-[illegible]+1 TO 1 STEP (-1)
1860 X(I+1,J+1)=X(I+1,J)
1870 R(I+1,J+1)=R(I+1,J)
1880 M(I+1,J+1)=M(I+1,J)
1890 T(I+1,J+1)=T(I+1,J)
1900 NEXT J
1910 J=1
1920 X(I+1,1)=X8
1930 R(I+1,1)=R8
1940 M(I+1,1)=M8
1950 T(I+1,1)=T8
1960 IF X(I+1,1)<X7 THEN 1990
1970 IF P<0 THEN 1990
1980 GOSUB 2570
1990 K=K+2
2000 IF P<0 THEN 2020
2010 IF X(I+1,1)>E THEN 2740
2020 RETURN
```

```
2030 IF I=2 AND P<0 THEN 3020
2040 PRINT "ROW #W="W
2050 FOR J=1 TO K-S
2060 IF J<Q THEN 2100
2070 IF J=Q AND P<0 THEN 3040
2080 PRINT J,X(I+1,J),R(I+1,J),M(I+1,J),T(I+1,J)*57.2958
2100 NEXT J
2110 RETURN
2120 IF X3>X4 AND X4>0 THEN 110
2130 X3=(R2-R7+X7*TAN(T0)-X2*T9)/(TAN(T0)-T9)
2140 R3=R7+(X3-X7)*TAN(T0)
2150 T3=T0
2160 IF B$="YES" THEN 1670
2170 GOTO 1560
2180 M3=(Q2*M2/2+T1+Q1*M1+G1*(R3-R1)/R4)/(Q2/2+Q1)
2190 T7=Q2*(M3-M2)/2
2200 GOTO 1210
2210 X1=X(I,K-S+1)
2220 R1=R(I,K-S+1)
2230 M1=M(I,K-S+1)
2240 T1=T(I,K-S+1)
2250 A1=FNA(FNS(M1))
2260 M9=1.01
2270 A4=A1
2280 T4=T1/2
2290 M4=M1
2300 R4=R1/2
2310 T9=TAN(A4-T4)
2320 X3=X1+R1/T9
2330 G1=FNG(A4)
2340 Q1=FNQ(A4)
2350 M3=M1+(T1-G1)/Q1
2360 PRINT "M3,AXIS="M3
2370 IF ABS(M3-M9)<0.0001 THEN 2430
2380 M9=M3
2390 M4=(M1+M3)/2
2400 A3=FNA(FNS(M3))
2410 A4=(A1+A3)/2
2420 GOTO 2310
2430 X(I+1,K-S+1)=X3
2440 R(I+1,K-S+1)=0
2450 M(I+1,K-S+1)=M3
2460 T(I+1,K-S+1)=0
2470 IF X3>L0 AND X1<L0 THEN 2472
2471 GOTO 2490
2472 GOSUB 6675
2490 RETURN
2500 FOR J=1 TO K-S
2510 X(1,J)=X(3,J)
2520 R(1,J)=R(3,J)
2530 M(1,J)=M(3,J)
```

```
2540 T(1,J)=T(3,J)
2550 NEXT J
2560 GOTO 2020
2570 D1=FNA(FNS(M(I-1,1)))
2580 D2=FNA(FNS(M(I,1)))
2590 D3=-COS(D1)+COS(D2)*M(I,1)/M(I-1,1)
2600 D4=SQR((X(I-1,1)-X(I,1))^2+(R(I-1,1)-R(I,1))^2)/R(I-1,1)
2610 U1=D3*M(I-1,1)/D4
2620 PRINT "U1*="U1
2630 D5=D3/D4+SIN(T0)*(SIN(D2))^2
2640 D6=D5/(2*(COS(D2))^2)
2650 PRINT "ACCELERATION TERM="D6
2660 PRINT "AT WALL POINT WHERE R/R*="R(I-1,1)"AND X/R*="X(I-1,1)
2670 PRINT
2680 PRINT
2690 RETURN
2700 PRINT "EXIT CONDITIONS FOR M#="FNS(M8)
2710 PRINT "AT X="X8;"R="R8;"WHERE THETA="T8*57.2958"DEG"
2711 B6=X8
2712 B7=R8
2713 B8=M8
2714 B9=T8
2716 PRINT
2735 GOTO 3000
2740 W1=0
2750 GOSUB 2950
2760 W0=(E-X(I-1,1))/(X(I+1,1)-X(I-1,1))
2770 PRINT W0
2780 H1=X(I-1,1)+(X(I,1)-X(I-1,1))*W0
2790 H2=R(I-1,1)+(R(I,1)-R(I-1,1))*W0
2800 H3=M(I-1,1)+(M(I,1)-M(I-1,1))*W0
2810 E4=T(I-1,1)+(T(I,1)-T(I-1,1))*W0
2820 X(I,1)=H1
2830 R(I,1)=H2
2840 T(I,1)=E4
2850 M(I,1)=H3
2860 GOSUB 1350
2870 IF ABS(W0-W1)<0.00001 THEN 2700
2880 R(I+1,1)=R8
2890 X(I+1,1)=X8
2900 M(I+1,1)=M8
2910 T(I+1,1)=T8
2920 W1=W0
2930 GOTO 2760
2940 GOTO 2700
2950 V1=X(I,1)
2960 V2=R(I,1)
2970 V3=T(I,1)
2980 V4=M(I,1)
2981 A6=V1
2982 A7=V2
```

```
2983 A8=V3
2984 A9=V4
2990 RETURN
3000 P=-1
3010 GOTO 2020
3020 Q=Q+1
3030 GOTO 2040
3040 N=N+1
3050 D(1,N)=X(I+1,J)
3060 E(1,N)=R(I+1,J)
3070 F(1,N)=M(I+1,J)
3080 G(1,N)=T(I+1,J)
3090 GOTO 2080
5130 PRINT "TRANSONIC THROAT FLOW AND SUPERSONIC CONICAL NOZZLE, 1/12/78"
5140 PRINT "KLIEGEL THROAT SOLUTION"
5150 PRINT "USE OF KLIEGEL'S THIRD ORDER THROAT FLOW SOLUTION "
5160 REM: CONVERGENCE EXPECTED FOR R=1
5170 PRINT "ENTER GAMMA=";
5180 INPUT G0
5190 PRINT "GAMMA="G0
5200 PRINT
5210 A$="NO"
5220 PRINT "ENTER EXIT RADIUS=";
5230 INPUT O0
5240 PRINT "EXIT RADIUS="O0
5242 PRINT "ENTER X/R,E FOR FLOW PROFILE DETERMINATION";
5243 INPUT L0
5244 PRINT "FLOW PROFILE DETERMINED AT X/R,E="L0
5250 GOTO 5290
5260 PRINT "ENTER EXIT MACH #=";
5270 INPUT M0
5280 PRINT "EXIT MACH #="M0
5290 PRINT
5300 PRINT "ENTER THROAT RADIUS OF CURVATURE=";
5310 INPUT C0
5320 PRINT "THROAT RADIUS OF CURVATURE="C0
5330 PRINT "ENTER CONICAL WALL DIVERGENCE ANGLE (DEG)=";
5340 INPUT T0
5350 PRINT "WALL DIVERGENCE ANGLE="T0"(DEG)"
5360 PRINT "ENTER # OF POINTS FOR NET=";
5370 INPUT K
5380 PRINT "NUMBER OF INPUT POINTS=";K
5390 T0=T0/57.2958
5400 X7=C0*SIN(T0)
5410 R7=1+C0*(1-SQR(1-(X7/C0)^2))
5420 PRINT
5430 GOTO 5440
5440 PRINT "NUMBER OF INITIAL POINTS="K
5450 FOR X=0 TO X7 STEP (X7/10)-1E-06
5460 PRINT
5470 Y0=1+C0*(1-SQR(1-(X/C0)^2))
```

```
5480 PRINT "X="X
5490 J=1
5500 FOR Y=0 TO Y0 STEP (Y0/(K-1))
5510 GOSUB 5660
5520 P(1,J)=X
5530 Q(1,J)=Y
5540 J=J+1
5550 NEXT Y
5560 IF B(1,1)>1.025 THEN 5590
5570 NEXT X
5580 GOTO 5440
5590 FOR J=K TO 1 STEP (-1)
5600 X(1,K+1-J)=P(1,J)
5610 R(1,K+1-J)=Q(1,J)
5620 M(1,K+1-J)=B(1,J)
5630 T(1,K+1-J)=C(1,J)
5640 NEXT J
5650 GOTO 310
5660 Z0=X+SQR(2*C0/(G0+1))
5670 L1=Y*Y/2-0.25+Z0
5680 L2=((2*G0+9)*Y^4-(4*G0+15)*Y*Y)/24
5690 L2=L2+(10*G0+57)/288+Z0*(Y*Y-0.625)-(2*G0-3)*Z0^3/6
5700 L3=(556*G0^2+1899*G0+3231)*Y^6/10368-(388*G0^2+1233*G0+1953)*Y^4/2304
5710 L3=L3+(304*G0^2+858*G0+1269)*Y*Y/1728-(2708*G0^2+7839*G0+14211)/82944
5720 L4=(52*G0^2+99*G0-375)*Y^4/384-(52*G0^2+99*G0+303)*Y*Y/192
5730 L4=L4+(92*G0^2+180*G0+639)/1152
5740 L5=((13*G0-27)/48-(5*G0-5)*Y*Y/8)*Z0*Z0+Z0*Z0*Z0*((4*G0^2-57*G0+27)/144)
5750 L3=L3+Z0*L4+L5
5760 B(1,J)=1+L1/(C0+1)+(L1+L2)/(C0+1)^2+(L1+2*L2+L3)/(C0+1)^3
5770 N1=Y*Y*Y/4-Y/4+Y*Z0
5780 N2=(8*G0+15)*Y^5/72-(20*G0+45)*Y^3/96+(28*G0+75)*Y/288
5790 N2=N2+Z0*((4*G0+9)*Y^3/12-(4*G0+9)*Y/12)
5800 N3=(6836*G0^2+16695*G0+14211)*Y^7/82944-(3380*G0^2+8703*G0+7875)*Y^5/13824
5810 N3=N3+(3424*G0^2+9183*G0+8964)*Y^3/13824
5820 N3=N3-([illegible]*G0^2+19575*G0+20745)*Y/82944
5830 N4=(556*G0^2+1113*G0+981)*Y^5/1728-(388*G0^2+801*G0+693)*Y^3/576
5840 N4=N4+(304*G0^2+645*G0+549)*Y/864
5850 N5=Z0*Z0*((52*G0^2+3*G0-33)*Y^3/192-(52*G0^2+27*G0-9)*Y/192)
5860 N6=Z0^3*(G0+1)*Y/4
5870 N3=N3+Z0*N4+N5-N6
5880 Z9=SQR((G0+1)/(2*(C0+1)))
5890 C(1,J)=Z9*(N1/(C0+1)+(1.5*N1+N2)/(C0+1)^2+(15*N1/8+2.5*N2+N3)/(C0+1)^3)
5900 RETURN
5910 FOR J=1 TO K-S+1
5920 X(1,J)=X(3,J)
5930 R(1,J)=R(3,J)
5940 M(1,J)=M(3,J)
5950 T(1,J)=T(3,J)
5960 NEXT J
5970 I=I-2
5980 RETURN
```

```
6060 B=(OO+UO*COS(TO)-Z3)/UO
6070 PRINT "IS CDC (NOT CD) NOZZLE WANTED?";
6080 INPUT C$
6090 IF C$<>"YES" THEN 6120
6100 B=FNB(B)
6110 GOTO 6170
6120 B=-FNB(B)
6170 E=X4+UO*(SIN(TO)+SIN(B))
6175 LO=E+LO*OO
6180 GOTO 420
6190 GOSUB 2570
6200 GOSUB 560
6210 FOR J=1 TO K-S
6220 PRINT I,J,X(I,J),R(I,J)
6230 NEXT J
6240 I=I+1
6250 S=S+1
6260 IF K-S<0 THEN 6280
6270 GOTO 6200
6280 X6=X(1,1)
6290 R6=R(1,1)
6300 PRINT "NOZZLE EXIT CHARACTERISTICS NORMALIZED AND STORED IN FILE <SONIC>"
6310 PRINT "     N      X(1,N)      R(1,N)       M(1,N)       T(1,N)<RAD>"
6311 FOR N=I-1 TO 2 STEP (-1)
6312 X(N+1,1)=X(N,1)
6313 R(N+1,1)=R(N,1)
6314 M(N+1,1)=M(N,1)
6315 T(N+1,1)=T(N,1)
6316 NEXT N
6317 GOTO 6500
6320 FOR N=1 TO I
6330 X(1,N)=(X(N,1)-X6)/R6
6340 R(1,N)=R(N,1)/R6
6350 M(1,N)=M(N,1)
6360 T(1,N)=T(N,1)
6370 PRINT USING 6380,N,X(1,N),R(1,N),M(1,N),T(1,N)
6380 :   ##.     ##.####     ##.####      ##.####      ##.####
6385 PRINT #1,X(1,N),R(1,N),M(1,N),T(1,N)
6390 NEXT N
6395 CLOSE #1
6490 STOP
6500 X2=A6
6510 R2=A7
6520 M2=A9
6530 T2=A8
6540 X1=B6
6550 R1=B7
6560 M1=B8
6570 T1=B9
6580 GOSUB 930
6630 X(2,1)=X3
```

```
6640 R(2,1)=R3
6650 T(2,1)=T3
6660 M(2,1)=M3
6670 GOTO 6320
6675 PRINT "AT CROSS SECTION X/R,E="L0
6680 B=B+1
6685 B1=(L0-X1)/(X3-X1)
6690 V(1,B)=L0
6695 V(2,B)=R1+(R3-R1)*B1
6700 V(3,B)=M1+(M3-M1)*B1
6705 V(4,B)=T1+(T3-T1)*B1
6710 PRINT USING 6715, B,L0,V(2,B),V(3,B),V(4,B)*57.2958
6715:#######.    ##.####    ##.####    ##.####    ##.####
6720 RETURN
6730 PRINT "AT CROSS SECTION X/R,E="L0
6735 B=B+1
6740 B2=(L0-X2)/(X3-X2)
6745 V(1,B)=L0
6750 V(2,B)=R2+(R3-R2)*B2
6755 V(3,B)=M2+(M3-M2)*B2
6760 V(4,B)=T2+(T3-T2)*B1
6765 PRINT USING 6715,B,L0,V(2,B),V(3,B),V(4,B)*57.2958
6770 RETURN
6780 END
READY.
```

Program JETC

Program JETC calculates the inviscid plume shape using the MOC. The last characteristic line, calculated and stored in file "SONIC" by program CDCSECT, serves as a starting condition.

input

quantity	symbol	explanation
γ	G0	specific heat ratio
M_L	M0	exit lip Mach number
r_e/r^*	00	exit radius
M_{JS}	M6	jet surface Mach number
	K	number of points in MOC net

output

x/r_e†, r/r_e and θ (deg) for the plume

†All plume dimensions are non-dimensionalized by the exit radius.

```
00005 PRINT"PROGRAM JETC"
00010 PRINT "JET BOUNDARY FOR GIVEN NOZZLE EXIT CONDITIONS,HHK, 1/31/78"
00015 PRINT "FLOW THROUGH NON IDEAL C_D NOZZLE"
00020 DIM A$(3),B$(3)
00025 PRINT "ENTER GAMMA=";
00030 INPUT G0
00035 PRINT "GAMMA="G0
00040 PRINT
00045 PRINT "IS EXIT MACH # GIVEN?";
00050 INPUT A$
00055 IF A$="YES" THEN 80
00060 PRINT "ENTER EXIT RADIUS=";
00065 INPUT O0
00070 PRINT "EXIT RADIUS="O0
00075 GOTO 110
00080 PRINT "ENTER EXIT MACH #=";
00085 INPUT M0
00090 PRINT "EXIT MACH #="M0
00095 PRINT "ENTER JET SURFACE MACH #=";
00100 INPUT M6
00105 PRINT "JET SURFACE MACH #="M6
00110 PRINT
00115 PRINT "ENTER # OF POINTS FOR NET=";
00120 INPUT K
00125 K0=K
00130 PRINT "NUMBER OF INPUT POINTS=";K
00135 PRINT "INPUT FROM SONIC FILE?";
00136 INPUT B$
00137 IF B$="YES" THEN 1390
00140 DIM X(21,11),R(21,11),M(21,11),T(21,11)
00145 DEF FNA(X)=ATN(1/SQR(X*X-1))
00150 DEF FNS(X)=SQR(2)*X/SQR(G0+1-(G0-1)*X*X)
00155 DEF FN[illegible]
00160 DEF FNF(A)=SIN(T)*SIN(A)/SIN(T+A)
00165 DEF FNG(A)=SIN(T)*SIN(A)/SIN(T-A)
00170 PRINT "INITIAL POINTS ALONG CHARACTERISTIC CURVE"
00180 I=1
00185 PRINT "I=1"
00190 FOR J=1 TO K
00195 INPUT X(1,J),R(1,J),M(1,J),T(1,J)
00200 PRINT J,X(I,J);R(I,J);M(I,J);T(I,J)
00205 NEXT J
00210 T0=T(1,1)
00215 PRINT
00220 PRINT
00225 PRINT "FLOWFIELD COMPUTED, ALL POINTS PRINTED"
00230 PRINT"          J        X        R        M*        THETA"
00235 PRINT
```

```
00240 PRINT
00245 S=1
00255 IF I>1 THEN 270
00260 GOTO 850
00265 I=1
00270 GOSUB 285
00280 GOTO 365
00285 FOR J=1 TO K-1
00290 X1=X(I+1,J)
00295 X2=X(I,J+1)
00300 R1=R(I+1,J)
00305 R2=R(I,J+1)
00310 M1=M(I+1,J)
00315 M2=M(I,J+1)
00320 T1=T(I+1,J)
00325 T2=T(I,J+1)
00330 GOSUB 395
00335 X(I+1,J+1)=X3
00340 R(I+1,J+1)=R3
00345 M(I+1,J+1)=M3
00350 T(I+1,J+1)=T3
00355 NEXT J
00360 RETURN
00365 GOSUB 635
00370 PRINT
00375 I=I+1
00380 S=S+1
00385 IF I>(N-1) THEN 1210
00390 GOTO 255
00395 M9=1.01
00400 A1=FNA(FNS(M1))
00405 A2=FNA(FNS(M2))
00410 A4=A1
00415 A5=A2
00420 T4=T1
00425 T5=T2
00430 M4=M1
00435 M5=M2
00440 R4=R1
00445 R5=R2
00450 X3=(R2-R1+X1*TAN(T4-A4)-X2*TAN(T5+A5))/(TAN(T4-A4)-TAN(T5+A5))
00455 R3=R1+(X3-X1)*TAN(T4-A4)
00460 T=T4
00465 A=A4
00470 M=M4
00475 Q1=FNQ(A)
00480 [illegible]=FN[illegible](A)
00485 T=T5
00490 A=A5
00495 M=M5
00500 Q2=FNQ(A)
```

```
00505 IF R2=0 THEN 665
00510 F2=FNF(A)
00515 M3=((T1-T2)+G1*(R3-R1)/R4+F2*(R3-R2)/R5+Q1*M1+Q2*M2)/(Q1+Q2)
00520 T7=Q2*(M3-M2)-F2*(R3-R2)/R5
00525 T3=T2+T7
00530 IF ABS(M9-M3)<0.0001 THEN 580
00535 M9=M3
00540 T4=(T1+T3)/2
00545 T5=(T2+T3)/2
00550 M4=(M1+M3)/2
00555 M5=(M2+M3)/2
00560 A3=FNA(FNS(M3))
00565 A4=(A1+A3)/2
00570 A5=(A2+A3)/2
00572 R5=(R2+R3)/2
00573 R4=(R1+R3)/2
00575 GOTO 450
00580 RETURN
00585 FOR J=K-S+1 TO 1 STEP (-1)
00590 X(I+1,J+1)=X(I+1,J)
00595 R(I+1,J+1)=R(I+1,J)
00600 M(I+1,J+1)=M(I+1,J)
00605 T(I+1,J+1)=T(I+1,J)
00610 NEXT J
00615 [illegible]=K+1
00620 GOTO [illegible]
00625 IF I=[illegible] THEN [illegible]
00630 GOTO 365
00635 PRINT "ROW #="I+1
00640 FOR J=1 TO K
00645 PRINT USING 650,J,X(I+1,J),R(I+1,J),M(I+1,J),T(I+1,J)*180/3.14159
00650 : J=#.          X=##.####     R=##.####  M=##.####  T=##.####
00655 NEXT J
00660 RETURN
00665 M3=(Q2*M2/2+T1+Q1*M1+G1*(R3-R1)/R4)/(Q2/2+Q1)
00670 T7=Q2*(M3-M2)/2
00675 GOTO 525
00680 X1=X(I,K-S+1)
00685 R1=R(I,K-S+1)
00690 M1=M(I,K-S+1)
00695 T1=T(I,K-S+1)
00700 A1=FNA(FNS(M1))
00705 M9=1.01
00710 A4=A1
00715 T4=T1/2
00720 M4=M1
00725 R4=R1/2
00730 T9=TAN(A4-T4)
00735 X3=X1+R1/T9
00740 G1=FNG(A)
00745 Q1=FNQ(A)
```

```
00750 M3=M1+(T1*3.14159/180-O1)/O1
00755 IF ABS(M3-M9)<0.0001 THEN 785
00760 M9=M3
00765 M4=(M1+M3)/2
00770 A3=FNA(FNS(M3))
00775 A4=(A1+A3)/2
00780 GOTO 730
00785 X(I+1,K-S+1)=X3
00790 R(I+1,K-S+1)=0
00795 M(I+1,K-S+1)=M3
00800 T(I+1,K-S+1)=0
00805 RETURN
00810 FOR J=1 TO K-S
00815 X(1,J)=X(3,J)
00820 R(1,J)=R(3,J)
00825 M(1,J)=M(3,J)
00830 T(1,J)=T(3,J)
00835 NEXT J
00840 I=I-2
00845 GOTO 630
00850 LO=SQR((GO+1)/(GO-1))
00855 PRINT
00860 PRINT "PRANDTL-MEYER EXPANSION"
00865 PRINT
00870 DEF FNB(X)=X*SQR((GO+1)/(2+(GO-1)*X*X))
00875 DEF FNC(X)=SQR((X*X-1)/((GO+1)/(GO-1)-X*X))
00880 DEF FNO(X)=LO*ATN(FNC(X))-ATN(LO*FNC(X))
00885 L9=(FNO(FNB(M6))-FNO(FNB(M0))+T0)*180/3.14159
00890 PRINT
00895 PRINT "INITIAL SLOPE (DEG)="L9
00900 I=1
00905 P=2
00910 H(1)=0
00915 I(1)=1
00920 J(1)=L9*3.14159/180
00925 J=1
00930 PRINT "ENTER # STEPS=";
00935 INPUT N
00940 NO=N
00945 PRINT "# OF STEPS="N
00950 FOR M=MO TO M6 STEP (M6-MO)/(N-1)
00955 M1=FNB(M)
00960 O=FNO(M1)
00965 PRINT "M="M;"M*="M1;"OMEGA="O;"DEG="O*180/3.14159
00970 X(I,J)=0
00975 R(I,J)=1
00980 M(I,J)=M1
00985 T(I,J)=O-FNO(FNB(MO))+TO
00990 I=I+1
00995 NEXT M
01000 GOTO 265
```

```
01005 J=1
01010 X2=X(N,J+1)
01015 R2=R(N,J+1)
01020 M2=M(N,J+1)
01025 T2=T(N,J+1)
01030 A2=FNA(FNS(M2))
01035 A3=FNA(FNS(M(N,J)))
01040 R1=R(N,J)
01045 X1=X(N,J)
01050 M1=M(N,J)
01055 T1=T(N,J)
01060 A5=A2
01065 M3=FNB(M6)
01070 A3=FNA(FNS(M3))
01075 T5=T2
01080 M5=M2
01085 R5=R2
01090 T9=T1
01095 X3=X2
01100 X3=(R1-R2+X2*TAN(T5+A5)-X1*TAN(T9))/(TAN(T5+A5)-TAN(T9))
01105 R3=R2+(X3-X2)*TAN(T5+A5)
01110 Q2=1/(M5*TAN(A5))
01115 F2=SIN(A5)*SIN(T5)/SIN(A5+T5)
01120 T3=T2+Q2*(M1-M2)-F2*(R3-R2)/R5
01125 T9=(T1+T3)/2
01130 X7=(R1-R2+X2*TAN(T5+A5)-X1*TAN(T9))/(TAN(T5+A5)-TAN(T9))
01135 IF ABS(X3-X7)<0.001 THEN 1170
01140 X3=X7
01145 T5=(T2+T3)/2
01150 M5=(M2+M3)/2
01155 A5=(A2+A3)/2
01160 R5=(R2+R3)/2
01165 GOTO 1105
01170 R0=(R1-R3)/(COS(T1)-COS(T3))
01175 PRINT "R0="R0;"X3="X7;"R3="R3;"T3(DEG)="T3*180/3.14159;"M3="M6
01180 X(N+1,1)=H(P)=X3
01185 K(P)=R0
01190 R(N+1,1)=I(P)=R3
01195 M(N+1,1)=M3
01200 T(N+1,1)=J(P)=T3
01205 RETURN
01210 J=1
01215 GOSUB 1005
01220 GOSUB 1250
01225 N=N+1
01230 P=P+1
01235 IF N>N0+K0-2 THEN 1330
01240 K=K-1
01245 GOTO 1215
01250 FOR J=1 TO K-2
01255 X1=X(N+1,J)
```

```
01260 X2=X(N,J+2)
01265 R1=R(N+1,J)
01270 R2=R(N,J+2)
01275 M1=M(N+1,J)
01280 M2=M(N,J+2)
01285 T1=T(N+1,J)
01290 T2=T(N,J+2)
01295 GOSUB 395
01300 X(N+1,J+1)=X3
01305 R(N+1,J+1)=R3
01310 M(N+1,J+1)=M3
01315 T(N+1,J+1)=T3
01320 NEXT J
01325 RETURN
01330 PRINT "ENTER SEGMENT #=";
01335 INPUT P
01340 P=P+1
01345 PRINT "P="P
01350 PRINT "INPUT RADIUS=";
01355 INPUT Q
01360 U=J(P-1)
01365 U1=COS(U)
01370 U2=U1+(Q-I(P-1))/K(P)
01375 U2=ATN(SQR(1-U2*U2)/U2)
01380 PRINT "AT RADIUS="Q"THETA="U2*180/3.14159;"X="H(P-1)+K(P)*(SIN(U)-SIN(U2))
01385 GOTO 1330
01390 I=1
01400 FILE #1="SONIC"
01405 FOR N=1 TO K
01410 INPUT #1,D(1,N),E(1,N),F(1,N),G(1,N)
01415 NEXT N
01420 FOR J=1 TO K
01425 X(1,J)=D(1,J)
01430 R(1,J)=E(1,J)
01435 M(1,J)=F(1,J)
01440 T(1,J)=G(1,J)
01445 NEXT J
01450 GOTO [illegible]
01455 END
READY.
```

Program NOCON

Program NOCON uses a second-order, non-ideal conical source flow approximation to determine a plume shape. The reason for running this program, besides as a comparison to Program JETC, is that as an output the initial deflection angle, θ_F, and the initial radius of curvature, R_F, are given (convenience). These parameters serve as input to Program PSIM.

input

quantity	symbol	explanation
γ	A	specific heat ratio
M_L	B	exit lip Mach number
θ_L	C	nozzle exit angle (deg)
M_{JS}	D	jet surface Mach number
U_1^*	CO	measure of the local rate of acceleration near the nozzle lip [12]

output

θ_F, R_F, x/r_e, r/r_e, and θ (deg) for the plume

```
[illegible] PRINT"PROGRAM NICON"
10 PRINT"CALCULATION OF A[illegible]
15 PRINT "FOR IDEAL AND NON-IDEAL CONICAL [illegible]
20 DIM A$(3)
30 DEF FNA(X)=ATN(X/SQR(1-X*X+1E-99))
35 DEF FNB(X)=ATN(SQR(1-X*X)/X)
40 DEF FNR(M)=((2+(A-1)*M*M)/(A+1))^((A+1)/(2*(A-1)))/M
45 Z=0
50 PRINT "PROTOTYPE FLOW SPECIFIED"
55 PRINT "ENTER GAMMA-P";
60 INPUT A
65 PRINT "GAMMA="A
70 PRINT "ENTER NOZZLE EXIT MACH";
75 INPUT B
80 PRINT "ML-P="B
85 PRINT "ENTER NOZZLE EXIT ANGLE";
90 INPUT C
95 PRINT "THETA-L-P="C
98 C=C/57.296
100 PRINT "IS NOZZLE IDEAL-CONICAL?";
105 INPUT A$
110 PRINT "ENTER JET SURFACE MACH";
115 INPUT D
120 PRINT "MFP="D
125 PRINT
130 F=((A-1)/(A+1))^0.5
135 G=FNA(1/B)
140 H=C-G+1.5708
145 E1=ATN(F/TAN(G))
150 B0=(E1)/F-H
155 DEF FNC(X)=(((A+1)/2)*X^2)/(1+((A-1)*X^2)/2)
160 I=FNC(B)
165 J=FNC(D)
170 X=I
175 GOSUB 205
180 O1=L
185 X=J
190 GOSUB 205
195 O2=L
200 GOTO 220
205 K=((1-X)/(X-1/F^2))^0.5
210 L=ATN(K)/F-ATN(K/F)
215 RETURN
220 C2=-2*F/(COS(E1)^(((3*A)-1)/(2*(A-1)))*SIN(E1)^0.5)
225 IF A$="YES" THEN 255
230 PRINT "NOZZLE IS NOT IDEAL-CONICAL"
235 PRINT "ENTER EXIT U1*=";
240 INPUT C0
245 PRINT "EXIT VALUE OF U1*="C0
```

```
250 GOTO 260
255 C0=3*SIN(C)*SQR(FNC(B))/(B*B)
260 C1=C0*C2
265 E2=E1+F*(O2-O1-FNA(1/D)+FNA(1/B))
270 D1=(E2-E1)/10
275 S1=S2=S3=S4=0
280 FOR N=1 TO 10 STEP 1
285 E3=E1+D1*(N-1/2)
290 S1=COS(E3/F)*SIN(E3)^(-0.5)*COS(E3)^(-1/(2*F^2))*D1+S1
295 S2=SIN(E3/F)*SIN(E3)^(-0.5)*COS(E3)^(-1/(2*F^2))*D1+S2
300 S3=SIN(E3/F)*SIN(E3)^(-1.5)*COS(E3)^((A-3)/(2*(A-1)))*D1+S3
305 S4=COS(E3/F)*SIN(E3)^(-1.5)*COS(E3)^((A-3)/(2*(A-1)))*D1+S4
310 NEXT N
315 U0=-((COS(E2)^((3*A-1)/(2*(A-1))))*SIN(E2)^0.5)/(2*F)
320 U1=U0*((S1-F*S3)*COS(B0)+(S2+F*S4)*SIN(B0)+C1)
325 G2=FNA(1/D)
330 T2=C+O2-O1
335 R0=-((U1/SQR(J)+SIN(T2)*SIN(G2)^2)/(SIN(2*G2)))
340 R1=-1/R0
345 PRINT
350 PRINT "SOLUTION"
352 T2=T2*57.296
355 PRINT "INITIAL SLOPE OF PLUME THETA-F="T2
360 PRINT "(INITIAL) RADIUS OF CURVATURE R1="R1
365 PRINT
370 PRINT
375 PRINT "PLUME SHAPE APPROXIMATED BY CIRCULAR ARC"
380 T2=T2/57.296
390 FOR S=0 TO 1.5 STEP 0.1
395 T=FNA(SIN(T2)-(S/R1))
400 S=R1*(SIN(T2)-SIN(T))
405 R=1+R1*(COS(T)-COS(T2))
410 PRINT S;R;T*57.296
420 NEXT S
425 PRINT
430 PRINT
435 END
READY.
```

Program WSHOCK

Program WSHOCK uses Weak Shock Modeling to calculate an equivalent jet surface Mach number to model a prototype gas with a different specific heat ratio having the same pressure rise-deflection angle relationship.

<u>input</u>

<u>quantity</u>	<u>symbol</u>	<u>explanation</u>
γ_p	K1	specific heat ratio of the prototype gas
M_{Fp}	M1	jet surface Mach number for the prototype gas
γ_m	K2	specific heat ratio of the model gas

<u>output</u>

M_{Fm}	M2	jet surface Mach number for the model gas

```
5 PRINT "WEAK SHOCK MODELING, JET SURFACE MACH NUMBERS, HHK,1,15,80"
10 PRINT "ENTER GAMMA,PROT. K1 AND GAMMA MODEL, K2=";
20 INPUT K1,K2
30 PRINT "GAMMA, PROTOTYPE="K1;"GAMMA, MODEL="K2
40 PRINT "ENTER M,F JET MACH #, PROTOTYPE=";
50 INPUT M1
60 PRINT
70 A=M1^4/(M1*M1-1)
80 B=A*K1*K1/(2*K2*K2)
90 X=B+SQR(B*B-A*K1*K1/(K2*K2))
100 M2=SQR(X)
110 PRINT "M,F-PROTOTYPE="M1;"M,F-MODEL="M2
120 GOTO 40
130 END
READY.
```

Program SECTP

Program SECTP calculates flow properties at a given cross section within the plume using the MOC.

input

quantity	symbol	explanation
γ	GO	specific heat ratio
M_{JS}	M6	jet surface Mach number
	K	number of points in the MOC net
	ZO	cross section at which flow properties are to be calculated
x/r_e, r/r_e, M^* and θ (deg)		characteristics stored in file "SONIC"

output

x/r_e, r/r_e, M^* and θ (deg) at the prescribed cross section

```
5 PRINT "PROGRAM SECTR"
15 PRINT "FLOW THROUGH C-D NOZZLE, EFFECT OF THROAT CURVATURE, (AXISYMMETRIC)"
20 PRINT "PLUME FLOW CALCULATED FOR GIVEN Z,CROSS SECTION"
22 FILE #1="SONIC"
25 DIM A$(3),B$(3),C$(3)
27 Q0=3.141592
30 PRINT "ENTER GAMMA=";
35 INPUT G0
40 PRINT "GAMMA="G0
45 DEF FNE(X)=ATN(X/SQR(1-X*X+1E-99))
50 PRINT
105 PRINT "ENTER JET SURFACE MACH #=";
110 INPUT M6
115 PRINT "JET SURFACE MACH #="M6
120 Y=0
125 PRINT
130 PRINT "ENTER # OF POINTS FOR NET=";
135 INPUT K
140 K0=K
145 PRINT "NUMBER OF INPUT POINTS=";K
155 DIM X(11,11),R(11,11),M(11,11),T(11,11),A(4,21),V(4,11)
160 PRINT "ENTER Z0>0 IF FLOW CROSS SECTION TO BE CALCULATED AT Z0";
165 INPUT Z0
170 PRINT "FLOW CROSS SECTION DETERMINED FOR Z0="Z0
175 DEF FNA(X)=ATN(1/SQR(X*X-1))
180 DEF FNS(X)=SQR(2)*X/SQR(G0+1-(G0-1)*X*X)
185 DEF FNQ(A)=1/(M*TAN(A))
190 DEF FNF(A)=SIN(T)*SIN(A)/SIN(T+A)
195 DEF FNG(A)=SIN(T)*SIN(A)/SIN(T-A)
200 PRINT "INITIAL POINTS ALONG CHARACTERISTIC CURVE"
210 I=1
215 PRINT "I=1"
220 PRINT "DATA FROM FILE=0,FROM KEY=1";
225 INPUT L
230 IF L=0 THEN 1965
235 FOR J=1 TO K
237 PRINT"ENTER X/RE, R/RE, M*, THETA(RAD) FOR POINT # "J
240 INPUT X(1,J),R(1,J),M(1,J),T(1,J)
245 PRINTJ,X(I,J),R(I,J),M(I,J),T(I,J)
250 A(1,J)=X(1,J)
255 A(2,J)=R(1,J)
260 A(3,J)=M(1,J)
265 A(4,J)=T(1,J)
270 NEXT J
275 PRINT "SHALL DATA BE STORED IN FILE 1(SONIC)";
280 INPUT B$
285 IF B$<>"YES" THEN 295
288 FOR J=1 TO K
```

```
289 PRINT #1, X(1,J),R(1,J),M(1,J),T(1,J)
290 NEXT J
292 CLOSE #1
295 T0=T(1,1)
300 PRINT
302 M0=FNS(A(3,1))
303 PRINT"EXIT MACH # ="M0
305 PRINT
310 PRINT "FLOWFIELD COMPUTED, ALL POINTS PRINTED"
315 PRINT" J              X/RE=                 R/RE=              M*=              THETA="
320 PRINT
325 PRINT
330 S=1
340 IF I>1 THEN 355
345 GOTO 965
350 I=1
355 GOSUB 365
360 GOTO 475
365 FOR J=1 TO K-1
370 X1=X(I+1,J)
375 X2=X(I,J+1)
380 R1=R(I+1,J)
385 R2=R(I,J+1)
390 M1=M(I+1,J)
395 M2=M(I,J+1)
400 T1=T(I+1,J)
405 T2=T(I,J+1)
410 GOSUB 505
415 X(I+1,J+1)=X3
420 R(I+1,J+1)=R3
425 M(I+1,J+1)=M3
430 T(I+1,J+1)=T3
435 IF X3>T0 AND X2<T0 THEN 445
440 GOTO 450
445 GOSUB 1640
450 IF X3>Z0 AND X2<Z0 THEN 460
455 GOTO 465
460 GOSUB 1685
465 NEXT J
470 RETURN
475 GOSUB 745
480 PRINT
485 I=I+1
490 S=S+1
495 IF I>(N-1) THEN 1360
500 GOTO 340
505 M9=1.01
510 A1=FNA(FNS(M1))
515 A2=FNA(FNS(M2))
520 A4=A1
525 A5=A2
```

```
530 T4=T1
535 T5=T2
540 M4=M1
545 M5=M2
550 R4=R1
555 R5=R2
560 X3=(R2-R1+X1*TAN(T4-A4)-X2*TAN(T5+A5))/(TAN(T4-A4)-TAN(T5+A5))
565 R3=R1+(X3-X1)*TAN(T4-A4)
570 T=T4
575 A=A4
580 M=M4
585 Q1=FNQ(A)
590 G1=FNG(A)
595 T=T5
600 A=A5
605 M=M5
610 Q2=FNQ(A)
615 IF R2=0 THEN 780
620 F2=FNF(A)
625 M3=((T1-T2)*Q0/180+G1*(R3-R1)/R4+F2*(R3-R2)/R5+Q1*M1+Q2*M2)/(Q1+Q2)
630 T7=Q2*(M3-M2)-F2*(R3-R2)/R5
635 T3=T2+T7
640 IF ABS(M9-M3)<0.0001 THEN 690
645 M9=M3
650 T4=(T1+T3)/2
655 T5=(T2+T3)/2
660 M4=(M1+M3)/2
665 M5=(M2+M3)/2
670 A3=FNA(FNS(M3))
675 A4=(A1+A3)/2
680 A5=(A2+A3)/2
685 GOTO 560
690 RETURN
695 FOR J=K-S+1 TO 1 STEP (-1)
700 X(I+1,J+1)=X(I+1,J)
705 R(I+1,J+1)=R(I+1,J)
710 M(I+1,J+1)=M(I+1,J)
715 T(I+1,J+1)=T(I+1,J)
720 NEXT J
725 K=K+2
730 GOTO 735
735 IF I=2 THEN 925
740 GOTO 475
745 PRINT "RIGHT RUNNING CHARACTERISTIC #"I+1
755 FOR J=1 TO K
760 PRINTJ,X(I+1,J),R(I+1,J),M(I+1,J),T(I+1,J)*180/Q0
770 NEXT J
775 RETURN
780 M3=(Q2*M2/2+T1*Q0/180+Q1*M1+G1*(R3-R1)/R4)/(Q2/2+Q1)
785 T7=Q2*(M3-M2)/2
790 GOTO 635
```

```
795 X1=X(I,K-S+1)
800 R1=R(I,K-S+1)
805 M1=M(I,K-S+1)
810 T1=T(I,K-S+1)
815 A1=FNA(FNS(M1))
820 M9=1.01
825 A4=A1
830 T4=T1/2
835 M4=M1
840 R4=R1/2
845 T9=TAN(A4-T4)
850 X3=X1+R1/T9
855 G1=FNG(A4)
860 Q1=FNQ(A4)
865 M3=M1+(T1*Q0/180-G1)/Q1
870 IF ABS(M3-M9)<0.0001 THEN 900
875 M9=M3
880 M4=(M1+M3)/2
885 A3=FNA(FNS(M3))
890 A4=(A1+A3)/2
895 GOTO 845
900 X(I+1,K-S+1)=X3
905 R(I+1,K-S+1)=0
910 M(I+1,K-S+1)=M3
915 T(I+1,K-S+1)=0
920 RETURN
925 FOR J=1 TO K-S
930 X(1,J)=X(3,J)
935 R(1,J)=R(3,J)
940 M(1,J)=M(3,J)
945 T(1,J)=T(3,J)
950 NEXT J
955 I=I-2
960 GOTO 740
965 L0=SQR((G0+1)/(G0-1))
970 PRINT
975 PRINT "PRANDTL-MEYER EXPANSION"
980 PRINT
985 DEF FNB(X)=X*SQR((G0+1)/(2+(G0-1)*X*X))
990 DEF FNC(X)=SQR((X*X-1)/((G0+1)/(G0-1)-X*X))
995 DEF FNO(X)=L0*ATN(FNC(X))-ATN(L0*FNC(X))
1000 L9=(FNO(FNB(M6))-FNO(FNB(M0))+T0)*180/Q0
1005 PRINT
1010 PRINT "INITIAL SLOPE (DEG)="L9
1015 I=1
1020 P=2
1025 H(1)=0
1030 I(1)=1
1035 J(1)=L9*Q0/180
1040 J=1
1045 PRINT "ENTER # STEPS=";
```

```
1050 INPUT N
1055 NO=N
1060 PRINT "# OF STEPS="N
1065 FOR M=MO TO M6+1E-05 STEP (M6-MO)/(N-1)
1070 M1=FNB(M)
1075 O=FNO(M1)
1080 PRINT "M="M;"M*="M1;"OMEGA="O;"DEG="O*180/Q0
1085 X(I,J)=0
1090 R(I,J)=1
1095 M(I,J)=M1
1100 T(I,J)=O-FNO(FNB(MO))+TO
1105 I=I+1
1110 NEXT M
1115 GOTO 350
1120 J=1
1125 X2=X(N,J+1)
1130 R2=R(N,J+1)
1135 M2=M(N,J+1)
1140 T2=T(N,J+1)
1145 A2=FNA(FNS(M2))
1150 A3=FNA(FNS(M(N,J)))
1155 R1=R(N,J)
1160 X1=X(N,J)
1165 M1=M(N,J)
1170 T1=T(N,J)
1175 A5=A2
1180 M3=FNB(M6)
1185 A3=FNA(FNS(M3))
1190 T5=T2
1195 M5=M2
1200 R5=R2
1205 T9=T1
1210 X3=X2
1215 X3=(R1-R2+X2*TAN(T5+A5)-X1*TAN(T9))/(TAN(T5+A5)-TAN(T9))
1220 R3=R2+(X3-X2)*TAN(T5+A5)
1225 Q2=1/(M5*TAN(A5))
1230 F2=SIN(A5)*SIN(T5)/SIN(A5+T5)
1235 T3=T2+Q2*(M1-M2)-F2*(R3-R2)/R5
1240 T9=(T1+T3)/2
1245 X7=(R1-R2+X2*TAN(T5+A5)-X1*TAN(T9))/(TAN(T5+A5)-TAN(T9))
1250 IF ABS(X3-X7)<0.001 THEN 1285
1255 X3=X7
1260 T5=(T2+T3)/2
1265 M5=(M2+M3)/2
1270 A5=(A2+A3)/2
1275 R5=(R2+R3)/2
1280 GOTO 1220
1285 R0=(R1-R3)/(COS(T1)-COS(T3))
1290 PRINT
1295 PRINT "RIGHT RUNNING CHARACTERISTIC #"N+1
1300 PRINT "R0="R0
```

```
1305 PRINT1,X3,R3,M3,T3*180/Q0
1310 IF X3>Z0 AND X2<Z0 THEN 1320
1315 GOTO 1325
1320 GOSUB 1730
1325 X(N+1,1)=H(P)=X3
1330 K(P)=R3
1335 R(N+1,1)=I(P)=R3
1340 M(N+1,1)=M3
1345 T(N+1,1)=J(P)=T3
1350 IF H(P)>Z0 AND H(P-1)<Z0 THEN 1320
1355 RETURN
1360 J=1
1365 GOSUB 1120
1370 GOSUB 1400
1375 N=N+1
1380 P=P+1
1385 IF N>N0+M0-2 THEN 1515
1390 K=K-1
1395 GOTO 1365
1400 FOR J=1 TO K-2
1405 X1=X(N+1,J)
1410 X2=X(N,J+2)
1415 R1=R(N+1,J)
1420 R2=R(N,J+2)
1425 M1=M(N+1,J)
1430 M2=M(N,J+2)
1435 T1=T(N+1,J)
1440 T2=T(N,J+2)
1445 GOSUB 505
1450 X(N+1,J+1)=X3
1455 R(N+1,J+1)=R3
1460 M(N+1,J+1)=M3
1465 T(N+1,J+1)=T3
1470 PRINTJ+1,X3,R3,M3,T3*180/Q0
1475 IF X3>Z0 AND X1<Z0 THEN 1485
1480 GOTO 1490
1485 GOSUB 1640
1490 IF X3>Z0 AND X2<Z0 THEN 1500
1495 GOTO 1505
1500 GOSUB 1730
1505 NEXT J
1510 RETURN
1515 PRINT "ENTER SEGMENT #=";
1520 INPUT P
1525 IF P=0 THEN 1850
1530 P=P+1
1535 PRINT "P="P-1
1540 PRINT "INPUT RADIUS=";
1545 INPUT Q
1550 U=J(P-1)
1555 U1=COS(U)
```

```
1560 U2=U1+(Q-I(P-1))/K(P)
1565 U2=ATN(SQR(1-U2*U2)/U2)
1570 PRINT "AT RADIUS="Q"THETA="U2*180/Q0;"X="H(P-1)+K(P)*(SIN(U)-SIN(U2))
1575 GOTO 1515
1585 FOR J=1 TO K
1590 X(1,J)=A(1,J)
1595 R(1,J)=A(2,J)
1600 M(1,J)=A(3,J)
1605 T(1,J)=A(4,J)
1610 PRINTJ,X(1,J),R(1,J),M(1,J),T(1,J)
1615 NEXT J
1620 FOR J=2 TO K
1625 IF A(1,J)>Z0 AND A(1,J-1)<Z0 THEN 1775
1630 NEXT J
1635 GOTO 295
1640 PRINT "AT CROSS SECTION X="Z0
1645 Y=Y+1
1650 V1=(Z0-X1)/(X3-X1)
1655 V(1,Y)=Z0
1660 V(2,Y)=R1+(R3-R1)*V1
1665 V(3,Y)=M1+(M3-M1)*V1
1670 V(4,Y)=(T3-T1)*V1+T1
1675 PRINTY,V(1,Y),V(2,Y),V(3,Y),V(4,Y)*180/Q0
1680 RETURN
1685 PRINT "AT CROSS SECTION X="Z0
1690 Y=Y+1
1695 V2=(Z0-X2)/(X3-X2)
1700 V(1,Y)=Z0
1705 V(2,Y)=R2+(R3-R2)*V2
1710 V(3,Y)=M2+(M3-M2)*V2
1715 V(4,Y)=(T3-T2)*V2+T2
1720 PRINTY,V(1,Y),V(2,Y),V(3,Y),V(4,Y)*180/Q0
1725 RETURN
1730 PRINT "AT CROSS SECTION X="Z0
1735 Y=Y+1
1740 V2=(Z0-X2)/(X3-X2)
1745 V(1,Y)=Z0
1750 V(2,Y)=R2+(R3-R2)*V2
1755 V(3,Y)=M2+(M3-M2)*V2
1760 V(4,Y)=(T3-T2)*V2+T2
1765 PRINTY,V(1,Y),V(2,Y),V(3,Y),V(4,Y)*180/Q0
1770 RETURN
1775 PRINT "AT CROSS SECTION X="Z0
1780 Y=Y+1
1785 V3=(Z0-A(1,J-1))/(A(1,J)-A(1,J-1))
1790 V(1,Y)=Z0
1795 V(2,Y)=A(2,J-1)+(A(2,J)-A(2,J-1))*V3
1800 V(3,Y)=A(3,J-1)+(A(3,J)-A(3,J-1))*V3
1805 V(4,Y)=A(4,J-1)+(A(4,J)-A(4,J-1))*V3
1810 PRINTY,V(1,Y),V(2,Y),V(3,Y),V(4,Y)
1815 GOTO 1630
```

```
1820 Y=Y+1
1825 V(4,Y)=FNE((H(P)+K(P)*SIN(J(P))-Z0)/K(P))
1830 V(2,Y)=I(P)+K(P)*(COS(V(4,Y))-COS(J(P)))
1835 PRINT "CROSS SECTION AT Y="Z0
1840 PRINTY,Z0,V(2,Y),M3,V(4,Y)*180/Q0
1845 GOTO 1355
1850 PRINT "SHALL NEW PLUME BE CALCULATED?=";
1855 INPUT C$
1860 IF C$="YES" THEN 105
1861 IF C$<>"YES" THEN 1950
1865 PRINT
1870 PRINT" X/RE, R/RE, M* AND THETA(RAD) ARE BEING READ FROM 'SONIC'"
1875 FOR J=1 TO K
1880 INPUT #1,D(1,J),E(1,J),F(1,J),G(1,J)
1885 NEXT J
1890 FOR J=1 TO K
1895 A(1,J)=X(1,J)=D(1,J)
1900 A(2,J)=R(1,J)=E(1,J)
1905 A(3,J)=M(1,J)=F(1,J)
1910 A(4,J)=T(1,J)=G(1,J)
1911 PRINT A(1,J),A(2,J),A(3,J),A(4,J)
1915 NEXT J
1920 GOTO 295
1950 END
READY.
```

Program PSIM

Program PSIM calculates what flow parameters are required to produce an equivalent plume shape when a different specific heat ratio is used.

input

quantity	symbol	explanation
θ_F	T2	initial plume deflection angle (deg)
R_F	R1	initial plume radius of curvature
γ_m	A	specific heat ratio of the model gas
M_{Fm}	D	jet surface Mach number for the model gas
M_{Lm-1}	B	trial value of exit lip Mach number for the model gas

output

M_{Lm} and θ_{Lm}-final values for the model gas at the exit

P-final pressure ratio,

P_b/P_0, x/r_e, r/r_e and θ (deg) of the model plume

```
00005 PRINT " PROGRAM PSI*-REVISED 3/21/77"
00020 DEF FNA(X)=ATN(X/SQR(1-X*X+1E-99))
00025  DEF FNB(X)=ATN(SQR(1-X*X)/X)
00027 DEF FNR(M)=((2+(A-1)*M*M)/(A+1))^((A+1)/(2*(A-1)))/M
00035 PRINT "PLUME SHAPE GIVEN ?YES0,NO 1";
00040 INPUT K1
00045 IF K1=0 THEN 710
00050 Z=0
00055 PRINT "PROTOTYPE FLOW SPECIFIED"
00060 PRINT "ENTER GAMMA-P";
00065 INPUT A
00070 PRINT "GAMMA="A
00075 PRINT "ENTER NOZZLE EXIT MACH";
00080 INPUT B
00085 PRINT "ML-P="B
00090 PRINT "ENTER NOZZLE EXIT ANGLE";
00095 INPUT C
00100 PRINT "THETA-L-P="C
00102 C=C*3.14159/180
00105 PRINT "ENTER JET SURFACE MACH";
00110 INPUT D
00115 PRINT "MFP="D
00120 E=1
00135 PRINT
00140 F=((A-1)/(A+1))^0.5
00145 G=FNA(1/B)
00150 H=C-G+3.14159/2
00155 E1=ATN(F/TAN(G))
00160 B0=(E1)/F-H
00165 DEF FNC(X)=(((A+1)/2)*X^2)/(1+((A-1)*X^2)/2)
00170 I=FNC(B)
00175 J=FNC(D)
00180 X=I
00185 GOSUB 215
00190 01=L
00195 X=J
00200 GOSUB 215
00205 02=L
00210 GOTO 230
00215 K=((1-X)/(X-1/F^2))^0.5
00220 L=ATN(K)/F-ATN(K/F)
00225 RETURN
00230 C0=3*SIN(C)/(B*B)
00235 C2=-2*F*SQR(I)/(COS(E1)^(((3*A)-1)/(2*(A-1)))*SIN(E1)^.5)
00240 C1=C0*C2
00245 E2=E1+F*(02-01-FNA(1/D)+FNA(1/B))
00250 D1=(E2-E1)/10
00255 S1=S2=S3=S4=0
```

```
00260 FOR N=1 TO 10 STEP 1
00265 E3=E1+D1*(N-1/2)
00270 S1=COS(E3/F)*SIN(E3)^(-0.5)*COS(E3)^(-1/(2*F^2))*D1+S1
00275 S2=SIN(E3/F)*SIN(E3)^(-0.5)*COS(E3)^(-1/(2*F^2))*D1+S2
00280 S3=SIN(E3/F)*SIN(E3)^(-1.5)*COS(E3)^((A-3)/(2*(A-1)))*D1+S3
00285 S4=COS(E3/F)*SIN(E3)^(-1.5)*COS(E3)^((A-3)/(2*(A-1)))*D1+S4
00290 NEXT N
00295 U0=-((COS(E2)^((3*A-1)/(2*(A-1))))*SIN(E2)^0.5)/(2*F)
00300 U1=U0*((S1-F*S3)*COS(B0)+(S2+F*S4)*SIN(B0)+C1)
00305 G2=FNA(1/D)
00310 T2=C+02-01
00315 R0=-((U1/SQR(J)+SIN(T2)*SIN(G2)^2)/(SIN(2*G2)))
00320 R1=-1/R0
00325 PRINT
00330 PRINT "SOLUTION"
00335 PRINT "INITIAL SLOPE OF PLUME THETA F="T2*57.2958
00340 PRINT "(INITIAL) RADIUS OF CURVATURE R1="R1
00345 PRINT
00350 IF E<1 THEN 505
00355 IF E>1 THEN 375
00360 IF Z=1 THEN 375
00365 GOSUB 765
00370 GOSUB 510
00375 IF E >= 2 THEN 560
00380 PRINT "ENTER GAMMA-M";
00385 INPUT A
00390 PRINT "ENTER MF-M";
00395 INPUT D
00400 PRINT
00405 PRINT "INPUT FOR MODEL PLUME"
00410 PRINT "GAMMA-M="A,"MF-M="D
00415 F=((A-1)/(A+1))^0.5
00420 Q=T2
00425 R2=R1
00430 PRINT "FIRST TRIAL VALUE ML-M-1"
00435 PRINT "ENTER ML-M-1";
00440 INPUT B
00445 I=FNC(B)
00450 L=FNC(D)
00455 K=I
00460 GOSUB 215
00465 03=L
00470 X=J
00475 GOSUB 215
00480 04=L
00485 C=Q-04+03
00490 PRINT "ML-M-1="B,"THETA-L-M-1="C*57.2958
00495 E=E+1
00500 GOTO 140
00505 STOP
00510 PRINT
```

```
00515 PRINT "PLUME SHAPE CALCULATED"
00520 PRINT "X","R","THETA"
00525 FOR S=0 TO 1.2 STEP 0.2
00530 T=FNA(SIN(T2)-(S/R1))
00535 S=R1*(SIN(T2)-SIN(T))
00540 R=1+R1*(COS(T)-COS(T2))
00545 PRINT S,R,T*57.2958
00550 NEXT S
00555 RETURN
00560 IF E>2 THEN 635
00565 B5=B
00570 R5=R1
00575 PRINT "ENTER ML-M-2";
00580 INPUT B
00585 I=FNC(B)
00590 X=I
00595 GOSUB 215
00600 O3=L
00605 C=Q-O4+O3
00610 PRINT "SECOND TRIAL VALUE ML-M-2"
00615 PRINT "ML-M-2="B,"THETA-L-M-2="C*57.2958
00620 E=E+1
00625 B6=B
00630 GOTO 140
00635 B6=B
00640 R6=R1
00645 B=B6+(R6-R2)*(B6-B5)/(R5-R6)
00650 I=FNC(B)
00655 X=I
00660 GOSUB 215
00665 O3=L
00670 C=Q-O4+O3
00675 PRINT
00680 PRINT
00685 PRINT "ML-M="B,"THETA-L-M="C*57.2958
00690 IF ABS(B-B6)<0.001 THEN 750
00695 B5=B6
00700 R5=R6
00705 GOTO 140
00710 PRINT "INPUT THETA-F,R1";
00715 INPUT T2,R1
717 T2=T2/57.296
00720 Z=E=1
00725 PRINT "PLUME GEOMETRY SPECIFIED"
00730 PRINT "THETA-F="T2*57.2958,"R1="R1
00735 PRINT
00740 GOTO 350
00745 Z=0
00750 GOSUB 765
00755 GOSUB 510
00760 GOTO 790
```

```
00765 P=(1-J*F^2)^(A/(A-1))
00770 PRINT
00775 PRINT "PRESSURE RATIO P="P
00780 PRINT
00785 RETURN
00790 PRINT "THIS IS THE FINAL RESULT"
00795 PRINT
00800 END
READY.
```

Program GRTRAN1

GRTRAN1 is a laminar-transitional-turbulent boundary layer program based on the methods of Gruschwitz [25] and Culick and Hill [24].

input

quantity	symbol	explanation
γ	GO	specific heat ratio
Pr	P	Prandtl number
M_{ref}	MO	reference Mach number
P_0	PO	stagnation pressure (psia)
T_0	TO	stagnation temperature (R)
x/r* vs. M		Mach number distribution in the nozzle

output

X/r*, M, θ, HC, δ^*, RI along the nozzle wall

$R_{e,\delta}$ (test section), δ, δ^* and θ at the nozzle exit.

where

HC - compressible shape factor δ^*/θ

RI - equivalent incompressible boundary layer Reynolds number (based on displacement thickness)

```
5 PRINT "PROGRAM GRTRAN1.BAS. HHK 11/20/1983"
10 PRINT "TRANSITIONAL BOUNDARY LAYER, GRUSCHWITZ-CULICK/HILL-HHK,9/21/83"
15 PRINT "MERGED PROGRAMS GRUT AND BLT2. HHK 10/28/83"
20 Q0=3.141592
25 DIM X(50),M(50),U(50),D(50),S(50),K(50),H(50),B(3),A(50,[illegible])
30 DIM R(50),V(50),L(50),F(50),G(50),N(50)
35 PRINT "LAMINAR COMPRESSIBLE BOUNDARY LAYER (GRUSCHWITZ)"
40 DEF FNA(G)=2*FNH(G)*(1-G/15+G^2/240)
45 DEF FNB(G)=FNI(G)-P*(12+G)^2/2160
50 DEF FNC(G)=1-(G0-1)*FNB(G)/(2*FNH(G))
55 DEF FND(G)=(G0-1)*(1-FNB(G)/(2*FNH(G)))
60 DEF FNE(G)=(G0-1)*(1-(FNI(G)-(12+G)^2/2160)/(2*FNH(G)))
65 DEF FNT(M)=1+(G0-1)*M*M/2
70 DEF FNK(Z)=Z*(37/315-Z/945-Z*Z/9072)^2
75 DEF FNM(F)=SQR(F^2*M0*M0/(1+(G0-1)*M0*M0*(1-F^2)/2))
80 DEF FNN(M)=(FNT(M0)/FNT(M))^(G0-1/(G0-1))
85 DEF FNG(Y)=Y/(37/315-Y/945-Y [illegible]
90 DEF FNH(G)=37/315-G/945-G^2/9072
95 DEF FNI(G)=(7 [illegible] 48 -465 [illegible] 3 [illegible] 4320
100 DEF FNJ(G)=(1+(G0-1 [illegible] *FND(G))/(1-M*M [illegible]
105 DEF FNL(M)=SQR(M*M*(1+(G0-1)*M0*M0/2)/(1+M0*M0*(1+(G0-1)*M*M/2)))
110 PRINT "ENTER GAMMA=";
115 INPUT G0
120 PRINT "ENTER PRANDTL #=";
125 INPUT P
130 PRINT "ENTER M0 (=REF. MACH #)";
135 INPUT M0
140 PRINT"IS RE00 GIVEN OR COMPUTED [illegible]";
145 INPUT A$
150 IF A$="Y" THEN 225
155 PRINT "ENTER RE00= [illegible],0*L/NU,0) OR 1 IF COMPUTED [illegible] STAG. STATE=";
160 L=1
165 INPUT R1
170 PRINT "RE0=";R1;" (REF [illegible]=";
175 IF R1<>0 THEN 210
180 PRINT "ENT[illegible] P0(PSIA),T0(DEG.F0)=";
185 INPUT P0,T0
190 PRINT "ENTER REFERENCE LENGTH [illegible]=";
195 INPUT L
200 PRINT "P0 (PSIA)=";P0;"T0(DEG.F)=";T0;"L-REF(FT)=";L
205 R1=1.26175*P0*P0*L [illegible]
210 PRINT "R1=";R1
215 R0=R1*M0 [illegible] 0-1 [illegible] M0 [illegible]
220 GOTO 235
225 PRINT "ENTER R0 (=U,REF,*L/NU,REF)";
230 INPUT R0
235 Q=.76
240 PRINT "GAMMA=";G0;" PRANDTL #=";P;" OMEGA=";Q
245 PRINT "M0=";M0;" R0 (=U,INF,*L/NU,INF)=";R0
```

```
250 PRINT "INPUT MACH# (YES),OR U/UO(NO)=";
255 INPUT B$
260 PRINT "ENTER 0 FOR STAGN. POINT, 1 FOR SHARP L.E.";
265 INPUT Z
270 PRINT "INPUT  # OF POINT PAIRS=";
275 INPUT NO
280 PRINT "NUMBER OF POINT PAIRS X,M (OR U/U)="NO
281 PRINT "INPUT FROM FILE GDATA (Y/N)=";
282 INPUT C$
283 IF C$<>"N" THEN 1441
285 FOR I=1 TO NO
286 IF B$<>"YES" THEN 297
287 PRINT "I="I;"ENTER X,M";
290 INPUT X(I),M(I)
292 A(I,1)=X(I)
295 A(I,2)=M(I)
296 GOTO 300
297 PRINT "POINT "I" ENTER X(I),U/U(I)=";
298 INPUT X(I),U(I)
299 M(I)=FNM(U(I))
300 A(I,1)=X(I)
301 A(I,2)=M(I)
304 PRINT "I="I;"X="X(I);"M="A(I,2)
305 NEXT I
306 PRINT "SHALL DATA A(1,I),A(2,I) BE STORED IN FILE GDATA (Y/N)?";
307 INPUT D$
308 IF D$="Y" THEN 1470
310 IF M(1)<>0 THEN 580
315 IF Z<>0 THEN 580
320 X(1)=  M(1)=  U(1)=  F1=0
330 PRINT "NEAR STAGNATION POINT"
335 FOR I=1 TO 3
340 PRINT "I="I,"X="X(I);"M="M(I)
345 NEXT I
350 F2=FNL(M(2))
355 K(1)=7.700001E-02
360 K=K(1)
365 T0=7.700001E-02*(X(2)-X(1))*FNN(M(1))/(R0*F2)
370 D(1)=SQR(T0)
375 GOSUB 620
380 D1=FNH(G)
385 D2=FNI(G)
390 PRINT "THETA-0="SQR(T0);"K="K;"STAGNATION POINT"
395 X3=X(3)
400 X2=X(2)
405 F3=FNL(M(3))
410 D(2)=D(1)*(1-.424*((F3/F2-1)/(X(3)/X(2)-1)-1))
415 PRINT "THETA-2="D(2);"AT X/L="X(2)
420 M=FNM(F2)
425 K2=FNJ(G)*(D(2))^2*R0*(F3-F2)/(FNN(M(2))*(X(3)-X(2)))
430 K(2)=K2
```

```
435 I=3
440 GOSUB 715
445 I=4
446 PRINT "   X        M        THETA/L        HC        DELTA1/L        RI"
450 GOTO 875
455 D1=FNH(G)
460 F2=FNL(M2)
465 H=(M*M*(G0-1)*FNB(G)/2+FNJ(G)*(.3-G/60))/FNH(G)
470 V(I)=1+(G0-1)*A(I,2)^2/2
475 H(I)=H
480 F(I)=(H(I)+1)/V(I)-1
485 G(I)=D(I)/V(I)^3
490 PRINT
492 M2=FNM(F2)
493 D2=SQR(T2)
494 D9=D2/D1
496 H2=H
525 L(I)=R1*G(I)^2*(A(I,2)-A(I-1,2))*V(I)^.5/(A(I,1)-A(I-1,1))
530 IF L(I)>=.02 THEN 545
535 N(I)=10^(1.194*SIN((L(I)-.02)*4.84*Q0))*1550/F(I)
540 GOTO 550
545 N(I)=10^(1.036*SIN((L(I)-.02)*6.06*Q0))*1550/F(I)
550 R(I)=R1*G(I)*A(I,2)
551 T1=R(I)*V(I)^(-3.5)
555 IF R(I)*V(I)^(-3.5)>N(I) THEN 985
560 PRINT "T,W/T,0="FNJ(G)*(1+(G0-1)*M0^2/2)/(1+(G0-1)*M^2/2)
561 PRINT X2;M2;D2;H2;D9;T1
565 IF G<-12 THEN 595
570 PRINT
575 RETURN
580 PRINT "SHARP LEADING EDGE"
585 PRINT "M1="M0;"AT X1="0
590 GOTO 860
595 PRINT "LAMINAR SEPARATION"
596 PRINT "TURBULENT REATTACHMENT CONSIDERED"
597 R(I)=R0*G(I)*A(I,2)
598 T=I
599 GOTO 1010
600 STOP
605 DIM Z(200)
610 PRINT "ENTER K=";
615 INPUT K
620 IF K<9.479999E-02 THEN 635
630 K=9.469999E-02
635 IF K>-.157 THEN 645
640 K=-.157
645 Z0=.000001
650 Y0=FNX(Z0)-K
655 IF K<0 THEN 670
660 Z1=12
665 GOTO 675
```

```
670 Z1=-12
675 Y1=FNX(Z1)-K
680 Z2=Z0-Y0*(Z1-Z0)/(Y1-Y0)
685 IF ABS(Z2-Z1)<.01 THEN 700
690 Z1=Z2
695 GOTO 675
700 G=Z2
705 RETURN
710 STOP
715 X1=X(I-1)
720 X2=X(I)
725 T1=(D(I-1))^2
730 M1=M(I-1)
735 M2=M(I)
740 U1=FNL(M1)
745 U2=FNL(M2)
750 U9=(U1+U2)/2
755 M9=(M1+M2)/2
760 K1=K(I-1)
765 B1=1
770 K=K1
775 GOSUB 620
780 T2=T1+2*(X2-X1)*FNN(M9)*(FNA(G)-K*(2-M9*M9*FNC(G))/B1)/(R0*FNL(M9))
785 K2=B1*R0*(T1+T2)*(U2-U1)/(2*(X2-X1)*FNN(M9))
790 K=K2
795 IF K2<-.1567 THEN 595
800 GOSUB 620
805 M=M9
810 B2=FNJ(G)
815 IF ABS(K2-K1)<.0001 THEN 835
820 B1=B2
825 K1=(K2+K1)/2
830 GOTO 770
835 D(I)=SQR(T2)
840 K(I)=K2
845 PRINT
850 RETURN
855 STOP
860 I=2
865 D(1)=  K(1)=0
867 X(1)=A(1,1)
870 M(1)=M0
875 PRINT
885 X(I)=A(I,1)
886 M(I)=A(I,2)
910 GOSUB 715
915 D(I-1)=D(I)
920 GOSUB 455
925 K(I-1)=K(I)
930 I=I+1
935 IF I>N0 THEN 980
```

```
940 GOTO 875
980 STOP
985 PRINT"POINT OF INSTABILITY, TRANSITION STARTS"I
990 T=I
995 D8=1.43-1/(.02086*(LOG(R(T)))^2.685)
1000 H8=F(T)-D8
1005 L8=(H8-.045)^(-5.556)-.185
1010 T(6)=A(T,2)
1015 T(7)=A(T,1)
1020 T(8)=D(T)*L
1024 T(1)=I
1025 T(2)=N0
1026 T(3)=R1
1027 T(4)=H8
1028 T(5)=L8
1029 T(9)=L
1030 PRINT "TURBULENT COMPRESSIBLE BOUNDARY LAYER, AIR, HHK, 8/24/81"
1035 PRINT
1038 N=4
1040 R0=T(3)
1045 D=T(9)
1050 M0=T(6)
1055 X0=T(7)
1060 N0=T(2)
1065 D0=T(8)
1070 IF D0<>0 THEN 1080
1075 D0= 9.999999E-10
1080 PRINT "EXPONENT OF SHEAR LAW, N=(TENTATIVELY)="N
1090 C0=(N+1)*.1266*N^(-2.6815)/(R0^(1/N))
1095 S0=((D0/D)*(M0/(1+.2*M0*M0))^3)^(1+1/N)
1100 PRINT "TRANSITION CONDITIONS X0(FT)="X0;"M0="M0;"D0="D0
1105 PRINT "SHEAR LAW EXPONENT N="N;"REY-0,D="R0
1107 PRINT "    X       M       THETA/L      HC       DISP/L       RI"
1110 L0=T(5)
1115 PRINT
1120 X1=X0
1125 M1=M0
1130 D1=D0
1135 J1=0
1140 I1=0
1145 Q0=(((D1*(1+.2*M0*M0)^(-3)/D)^(1+1/N))/C0)^4
1150 Q1=Q0
1155 Z1=0
1160 FOR O=T(1)+1 TO N0
1165 X2=A(O,1)
1170 M2=A(O,2)
1175 IF X2=0 AND M2=0 THEN 1290
1180 M9=(M1+M2)/2
1185 I2=(X2-X1)*(M9^(3+2/N)*(1+.2*M9*M9)^((1/N)-4))
1190 I1=I1+I2
1195 D2=D*(1+.2*M2*M2)^3*((C0*I1+S0)*M2^(-3-3/N))^(N/(N+1))
```

```
1200 D9=(D1+D2)/2
1205 Q2=(I1/M0^(-3*(1+1/N))+Q0^(1/4))^4
1210 R2=R0*D9*M9/(((1+.2*M9*M9)^3)*D)
1215 B=.0304*LOG(R2)-.23
1220 J2=(B-LOG(M9/M0))*(Q2-Q1)
1225 L2=Q0*L0/Q2+LOG(M2/M0)+(1/Q2)*(J2+J1)
1230 IF L2<-.15 THEN 1400
1235 IF L2<-.185 THEN 1410
1240 K2=.045+(L2+.185)^(-.18)
1245 H2=(K2+1)*(1+.2*M2*M2)-1
1250 T1=R0*D2*M2/(((1+.2*M2*M2)^3)*D)
1255 D9=H2*D2
1260 PRINT X2;M2;D2;H2;D9;T1
1265 X1=X2
1270 M1=M2
1275 J1=J1+J2
1280 Q1=Q2
1285 NEXT Q
1290 T1=R0*D2*M2/(((1+.2*M2*M2)^3)*D)
1295 PRINT "R,T= TEST REYNOLDS #="T1
1300 PRINT "SELECTION OF SHEAR LAW EXPONENT (HERE USED N="N")"
1305 PRINT "FOR 100<R,T<5E3, N=4"
1310 PRINT "FOR 500<R,T<5E4, N=5"
1315 PRINT "FOR 3E3<R,T<6E5, N=6"
1320 PRINT "RECALCULATE WITH PROPER N IF NEEDED"
1325 N1=2*N-1
1330 C2=M2*M2/(5+M2*M2)
1335 Y=1
1340 Y0=1/(N1+1)-1/(N1+2)
1345 Y=Y+1
1350 Y2=C2^(Y-1)/(N1-1+2*Y)-C2^(Y-1)/(N1+2*Y)
1355 Y0=Y0+Y2
1360 IF Y2<.0001 THEN 1370
1365 GOTO 1345
1370 D3=(1-C2)*N1*Y0
1375 PRINT "DELTA/THETA="1/D3
1380 PRINT "BOUNDARY LAYER THICKNESS (FT)="D2/D3
1385 PRINT "BOUNDARY LAYER MOMENTUM THICKNESS (FT)="D2
1390 PRINT "BOUNDARY LAYER DISPLACEMENT THICKNESS (FT)="D2*H2
1395 GOTO 1415
1400 PRINT "SEPARATION IS POSSIBLE"
1405 GOTO 1235
1410 PRINT "SEPARATION MUST HAVE OCCURRED"
1412 GOTO1325
1415 PRINT "IS NEW SHEAR LAW EXPONENT DESIRED FOR RECALCULATION?(Y/N)";
1420 INPUT B$
1425 IF B$<>"Y" THEN 1540
1430 PRINT "ENTER NEW SHEAR LAW EXPONENT";
1435 INPUT N
1440 GOTO 1040
1441 FILE#1="GDATA"
```

```
1442 FOR I=1 TO NO
1443 INPUT #1, A(I,1),A(I,2)
1444 X(I)=A(I,1)
1445 M(I)=A(I,2)
1446 NEXT I
1447 CLOSE #1
1448 FOR I=1 TO NO
1449 PRINT "I="I;"X="A(I,1);"M="A(I,2)
1450 X(I)=A(I,1)
1451 NEXT I
1452 GOTO 310
1470 FILE#1="GDATA"
1480 FOR I=1 TO NO
1490 PRINT #1, USING"##.####";A(I,1),A(I,2)
1510 NEXT I
1520 CLOSE #1
1530 GOTO 310
1540 END
READY.
```

Program BARTZ

BARTZ is a fully turbulent boundary layer program applying the Bartz method [18].

input

quantity	symbol	explanation
γ	K	specific heat ratio
R	R8	gas constant (ft lb_f/lb_m R)
P_0	P0	stagnation pressure (psia)
T_0	T0	stagnation temperature (R)
μ	M0	stagnation dynamic viscosity (lb_m/sec in.)
r*	R0	throat radius (-)
R*	K0	throat radius of curvature (-)
Cp	C1	specific heat at constant pressure (Btu/lb_m R)
k	K1	wall conductivity (Btu/in. sec R)
	L	wall thickness (in.)
h	H1	film coefficient-coolant side
	T9	coolant temperature (R)
x/r* vs. r/r*		nozzle wall contour

output

x/r*, r/r*, δ**/r*, δ/r*, δ*/r*, DELT/δ, TW/TO, HG along the nozzle wall

$R_{e,\delta}$ at the nozzle exit

```
5 PRINT "PROGRAM BARTZ-FULLY TURBULENT BOUNDARY LAYER"
10 DIM X(25,3),A$(3),B$(3),A(25,7)
20: ##.### ##.### ##.### ##.### ##.### ##.### ##.### ##.###
30 DEF FNX(M)=(2/(K+1)+(K-1)*M*M/(K+1))^((K+1)/(2*(K-1)))/M
40 DEF FNY(M)=(K+1)*M/(M*M*(K-1)+2)-1/M
50 DEF FNA(M)=T*(1+(K-1)*M*M/2)
60 DEF FNB(M)=(1/T-1)*V^(-1/7)
70 DEF FNC(M)=(K-1)*M*M/((2+(K-1)*M*M)*T)
80 DEF FND(M)=FNC(M)*V^(2/7)
90 DEF FNS(M)=1/((0.5*T*(1+(K-1)*M*M/2)+0.5)^(0.6)*(1+(K-1)*M*M/2)^(0.15))
100 PRINT "TURBULENT BOUNDARY LAYER IN COOLED C-D NOZZLE"
110 PRINT "BARTZ METHOD, MODIFIED BY HHK 7/1/76"
120 PRINT "ALL BL. THICKNESSES DIMENSIONLESS IN TERMS OF R-THROAT"
130 REM: NOTE THAT INPUT DATA ARE IN UNITS CONFORMING WITH BARTZ
150 PRINT "ENTER GAMMA,GAS CONST,P0,T0,M0,R0,K0";
160 INPUT K,R8,P0,T0,M0,R0,K0
170 PRINT "ENTER C-P,K1,L,H1,T9";
180 INPUT C1,K1,L,H1,T9
190 PRINT "ENTER NOZZLE GEOMETRY X/R0,R/R0"
200 PRINT "NOTE THAT THROAT (R/R0=1) MUST BE ONE OF LOCATIONS"
210 :##.  #######.###    ######.###
220 PRINT "ENTER # OF POINTS (<25)";
230 INPUT N
240 PRINT "NUMBER OF POINTS="N
250 PRINT "POINT# X/R0           R/R0"
260 PRINT "GEOMETRY FROM FILE 3?";
270 INPUT A$
280 IF A$="YES" THEN 2250
290 FOR I=1 TO N
310 PRINT "ENTER X("I"),1),X("I",2)";
320 X(I,3)=I
340 INPUT X(I,1),X(I,2)
350 PRINT USING 20,I,X(I,1),X(I,2)
360 NEXT I
370 PRINT "SHALL DATA BE STORED IN FILE 3?";
380 INPUT B$
390 IF B$="YES" THEN 2300
400 PRINT "GAMMA="K;"GAS CONST.="R8
410 PRINT "STAGN. PRESS.="P0;"STAGN. TEMP="T0;"STAGN. VISC.="M0
420 PRINT "THROAT RADIUS="R0;"THROAT CURV.="K0
430 PRINT "C-P="C1;"K="K1;"L="L;"H-E="H1;"T-E="T9
440 U0=(2/(K+1))^((K+1)/(2*(K-1)))*P0*SQR(K*32.174/(R8*T0))
450 R9=R0*U0/M0
460 P=4*K/(9*K-5)
470 N1=C1*M0/(R0*H1)
480 N2=L*C1*M0/(R0*K1)
490 N3=0.0228*R9^(3/4)/P^(0.46)
500 PRINT "REF. REYNOLDS #="R9;"PRANDTL #="P
510 V1=P^(-0.36)
```

```
520 GOTO 1130
530 A0=R^2
540 IF C=0 THEN 660
550 M8=1.4
560 GOSUB 580
570 GOTO 630
580 A1=FNX(M8)
590 IF ABS(A0-A1)<0.0001 THEN 630
600 M9=M8+(A0-A1)/(A0*FNY(M8))
610 M8=M9
620 GOTO 580
630 M=M9
640 RETURN
650 STOP
660 M8=0.01
670 GOSUB 580
680 GOTO 640
690 I1=0
700 Z1=0
710 D0=0.05
720 Z2=Z1+D0
730 Z9=Z1+D0/2
740 IF Z9>1 THEN 780
750 I1=I1+Z9^7*(1-Z9)*D0/(1+B0*Z9-C0*Z9*Z9)
760 Z1=Z2
770 GOTO 720
780 RETURN
790 I2=0
800 Z1=0
810 D0=0.05
820 Z2=Z1+D0
830 Z9=Z1+D0/2
840 IF Z9>1 THEN 880
850 I2=I2+Z9^7*D0/(1+B0*Z9-C0*Z9*Z9)
860 Z1=Z2
870 GOTO 820
880 RETURN
890 IF M1<1 THEN 930
900 F=1-1.25*(1+S1)*(X2-X1)/SQR((K+1)*K0/2)
910 N0=I
920 GOTO 940
930 F=1-5*(M1*M1-2*S1-3)*LOG(R2/R1)/(4*(1-M1*M1))
940 F0=0.0285*FNS(M1)*(W1*R1*R1/R9)^(1/4)*(X2-X1)
950 F2=(F1^1.25*F+F0)^(0.8)
960 RETURN
970 B0=FNB(M)
980 C0=FNC(M)
990 RETURN
1000 B0=1/T-1
1010 C0=FND(M)
1020 RETURN
```

```
1030 G1=F1*K8*(1-T7)/(I8*R1)
1040 G2=F2*K9*(1-T)/(I9*R2)
1050 G0=1+9*(LOG(G1)-LOG(G2))/8
1060 G3=0.02565*FNS(M1)*I8*(W1/R9)^0.25*SQR(R1)*(X2-X1)/(K8*F1^1.25*P^(0.46))
1070 G4=G8^(9/7)*G0+G3
1080 G9=(ABS(G4))^(7/9)
1090 N4=N3*FNS(M2)*R2^(-3/2)*W2^(1/4)*G9^(-1/7)/F2^(1/4)
1100 T2=1-(1-T9/T0)/(1+N4*(N1+N2))
1110 PRINT
1120 RETURN
1130 Q=1
1140 C=0
1142 Q=Q
1150 FOR I=2 TO N
1160 X2=X(I,1)
1170 R2=X(I,2)
1180 R=R2
1190 IF Q<2 THEN 1920
1200 T=T7
1210 T1=T7
1220 GOSUB 1240
1230 GOTO 1540
1240 R=R2
1250 IF R=1 THEN 1290
1260 GOSUB 530
1270 M2=M
1280 GOTO 1300
1290 M2=1
1300 V=G8
1310 GOSUB 970
1320 GOSUB 690
1330 I9=I1
1340 GOSUB 790
1350 J9=I2
1360 GOSUB 1000
1370 GOSUB 690
1380 K9=I1
1390 GOSUB 790
1400 L9=I2
1410 W2=7*I9/FNA(M2)
1420 S2=(FNA(M2)/7-J9)/I9
1430 GOSUB 890
1440 IF Q=2 THEN 1460
1450 GOSUB 1030
1460 IF Q<3 THEN 2180
1470 RETURN
1480 GOSUB 1730
1490 IF R2>1.00001 THEN 1520
1500 C=1
1510 Q=Q
1520 NEXT I
```

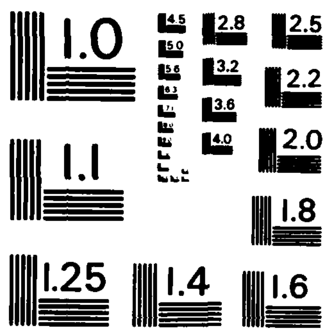
AD-A146 262 MODELING OF PROPULSIVE JET PLUMES--EXTENSION OF MODELING CAPABILITIES BY..(U) ILLINOIS UNIV AT URBANA DEPT OF MECHANICAL AND INDUSTRIAL ENG.. S E DOERR 2/2

UNCLASSIFIED JUN 84 UILU-ENG-84-4005 ARO-19823.3-EG F/G 20/4 NL

END

FILMED

DTIC


1.0
1.1
1.25
1.4
1.6
1.8
2.0
2.2
2.5
2.8
3.2
3.6
4.0
4.5
5.0
5.6

```
1530 GOTO 2220
1540 IF Q<3 THEN 1710
1550 IF ABS(T2-T1)>0.01 THEN 1880
1560 A(I,1)=F2
1570 A(I,2)=F2*FNA(M2)/(7*I9)
1580 A(I,3)=S2*F2
1590 A(I,4)=G9
1600 A(I,5)=T2
1610 A(I,6)=N4*C1*M0/R0
1620 A(I,7)=M2
1630 PRINT " X/R0      R/R0      DEL**/R0 DEL/R0    DEL*/R0  DELT/DEL TW/T0     H-G"
1640 PRINT  X2;R2;F2;F2*FNA(M2)/(7*I9);S2*F2;G9;T2;N4*C1*M0/R0
1660 PRINT "COMPLETES ITERATION FOR POINT"Q
1680 PRINT "WHERE R/R0="R2;"AND MACH#="M2
1690 T8=T2
1700 GOTO 1720
1710 T8=T2=T7
1720 GOTO 1460
1730 X1=X2
1740 R1=R2
1750 Q=Q+1
1760 F1=F2
1770 G8=G9
1780 M1=M2
1790 I8=I9
1800 J8=J9
1810 K8=K9
1820 L8=L9
1830 IF Q<3 THEN 2160
1840 T7=T8
1850 S1=S2
1860 W1=W2
1870 RETURN
1880 T1=2*T1*T2/(T1+T2)
1890 T=T1
1900 GOSUB 1240
1910 GOTO 1540
1920 F1=0
1930 V=V1
1935 X2=X(1,1)
1936 R2=X(1,2)
1940 GOSUB 530
1950 M1=M
1960 G8=V1
1970 T=T7=0.25
1980 GOSUB 970
1990 GOSUB 690
2000 I8=I1
2010 GOSUB 790
2020 J8=I2
2030 GOSUB 1000
```

```
2040 GOSUB 690
2050 K8=I1
2060 GOSUB 790
2070 L8=I2
2080 W1=7*I8/FNA(M1)
2090 S1=(FNA(M1)/7-J8)/I8
2100 Q=Q+1
2110 X1=X2
2120 R1=R2
2130 GOTO 1150
2140 G9=V'
2150 GOTO 1090
2160 T7=T7
2170 GOTO 1830
2180 G9=G8
2190 N4=0
2200 PRINT "POINTS 1&2 GENERATE INPUT, EXEPT FOR DEL**/R0 AT POINT 2"
2210 GOTO 1470
2220 R8=R9*F2*FNA(M2)*(1+(K-1)*M2*M2/2)^0.6/(7*I9*R2*R2)
2230 PRINT "AT LAST STATION, REYNOLDS# RE,FREE STREAM,DELTA="R8
2240 GOTO 2340
2250 FILE #1="BDATA"
2260 FOR I=1 TO N
2270 INPUT #1,X(I,3),X(I,1),X(I,2)
2280 NEXT I
2290 GOTO 400
2300 FILE#1="BDATA"
2305 FOR I=1 TO N
2307 PRINT #1,X(I,3),X(I,1),X(I,2)
2308 NEXT I
2309 CLOSE #1
2310 PRINT "NOZZLE COORDINATES STORED IN FILE BDATA"
2320 GOTO 400
2340 END
READY.
```

Program FREES

Program FREES calculates the free shear layer development.

<u>input</u>

quantity	symbol	explanation
γ	G	specific heat ratio
n	N	flow profile exponent
M_L	M1	exit lip Mach number
M_{JS}	M2	jet surface Mach number
$R_{e,\delta}$	R1	Reynolds number based on velocity boundary layer thickness

<u>output</u>

Shear stress and eddy viscosity functions and spread rate parameters along the plume boundary.

```
5 PRINT "PROGRAM FREES-DEVELOPMENT OF FREE SHEAR LAYER"
10 DIM A(5,100)
15 P0=3.141592
20 PRINT "THIS VERSION WITH PLOT,HHK 8/30/80"
30 PRINT "LINEAR MIXING PROFILE IN EXPANDED 1/N PROFILE B.L."
40 PRINT "THIS VERSION OF 8/28/80 CAN HANDLE THIN APPR. B.L."
50 DEF FNZ(F)=F^N/SQR(1-(1-C1^2*F^2)*P^((G-1)/G))
60 DEF FNC(M)=SQR(M^2/(2/(G-1)+M^2))
70 DEF FNM(F)=F^N*FND(F)/(1-F*F*C1*C1)
80 DEF FND(F)=(1-(1-C1^2*F^2)*P^((G-1)/G))^0.5
90 DEF FNA(F)=F^N/(1-F^2*C1^2)
100 DEF FNY(F)=D2*(-F3+LOG((1+C0)/(1-C0))/(2*C3))
120 PRINT "GAMMA=";
130 INPUT G
140 PRINT "PROFILE EXPONENT N=";
150 INPUT N
160 PRINT "M1=";
170 INPUT M1
180 PRINT "M2=";
190 INPUT M2
200 PRINT "# OF INTEGRATION STEPS=";
210 INPUT D
220 PRINT "ENTER REYNOLDS #(1-A-DELTA1)";
230 INPUT R1
240 K=12+2.76*M2
250 Z0=Y0=F8=J=1
260 R9=10^(4.754+0.039*M2+0.053*M2^2)
270 P=((1+(G-1)*M1^2/2)/(1+(G-1)*M2^2/2))^(G/(G-1))
280 PRINT "GAMMA="G;"UPSTREAM B.L. PROFILE EXP.="N
290 PRINT "M1="M1;"M2="M2;"P2/P1="P
300 PRINT "REYNOLDS #,(1-A-DELTA1)="R1;"SIGMA (CONST.)="K
310 PRINT "# OF INTEGRATION STEPS="D;"REYNOLDS #, TRANS.="R9
320 C1=FNC(M1)
330 C2=FNC(M2)
340 J3=SQR(1-(1-C2^2)*EXP(LOG((1+C2)/(1-C2))*C2-2))/C2
350 PRINT "SIMILARITY VALUE FOR J0="J3
360 Z1=P*R1*(1-C1^2)^2/C1
370 F=Y1=I1=J1=0
380 X1=Z3=0
390 Z5=15+10
400 F1=F
410 IF ABS(F1-F8)>.0001 THEN 440
420 D=D/10
440 F2=F1+1/D
450 F9=(F1+F2)/2
460 IF F9>1 THEN 1020
470 A=FND(F9)
480 I2=I1+FNM(F9)*(F2-F1)
490 Y2=Y1+P^(-1/G)*N*C1*FNZ(F9)*(F2-F1)
```

```
500 Y3=Y2
510 D1=I1*P^(-1)*N*C1/(-1+LOG((1+FND(F1))/(1-FND(F1)))/(2*FND(F1)))
520 D2=I2*P^(-1)*N*C1/(-1+LOG((1+FND(F2))/(1-FND(F2)))/(2*FND(F2)))
530 IF D1=0 THEN 550
540 Z7=2*P^(-1)*C1*FND(F1)*N/D1
550 Z=2*P^(-1)*C1*FND(F2)*N/D2
560 J2=J1+FNA(F9)*(F2-F1)
570 IF D1=0 THEN 590
580 Z8=J1*Z7
590 IF D1=0 THEN 660
600 Q0=SQR(1-(1-(FND(F1))^2)*EXP(Z8))
610 Y0=Q0/C2
620 J2=J1+FNA(F9)*(F2-F1)
630 Q3=FND(F1)
640 F4=Q0/Q3
650 Z9=D1*(-F4+LOG((1+Q0)/(1-Q0))/(2*Q3))
660 I0=J2*Z
670 IF D1<>0 THEN 690
680 Z9=0
690 C0=SQR(1-(1-(FND(F2))^2)*EXP(I0))
700 J0=C0/C2
710 IF Y0=1 THEN 790
720 PRINT
730 PRINT "PHI-1(I)="F1;"PHI-1(I+1)="F2
740 PRINT "PHI-2(I)="FND(F1)/C2;"PHI-2(I+1)="FND(F2)/C2
750 PRINT "Y2/Y1="Y0;"DELTA/DELTA1="D2;D1
760 PRINT "J0="J0;"YL/DELTA1="Y2-D2
770 PRINT "J0-RATIO="J0/J3
780 J3=FND(F2)
790 IF Z0=0 THEN 1450
800 Z6=C3*(1-J0^2*C2^2)/D2
810 F3=C0/C3
820 Z2=(Z5+Z6)/2
830 Z4=D2*(-F3-LOG((1+C0)/(1-C0))/(2*C3))
840 T1=SQR(P(1)*((C2^2-1)*(Z4-Z9)/(D2-D1)))
850 E0=T1*(1-C0^2)*C2/(A*(1-C2*C2))
851 IF Z0=1 THEN 870
860 PRINT "SHEAR STRESS FUNCTION="T1;"EDDY-VISCOSITY FUNCTION="E0
870 X2=X1+(Z4-Z3)*Z1/Z2
880 GOSUB 1790
890 IF Y0=1 THEN 910
900 PRINT "X/DELTA1, LAM.="X2
910 R2=(C2*R1*(2*P*((1-C1^2)/(1-C2^2))^2))/C1
920 PRINT "REYNOLDS # R/2,X ="R2
930 IF R2>R9 THEN 1330
940 Z3=Z4
950 Z5=Z6
960 X1=X2
970 I1=I2
980 J1=J2
990 Y1=Y2
```

```
1000 F1=F2
1010 GOTO 410
1020 Y3=Y2
1025 I=J
1030 PRINT
1040 X1=X2
1050 PRINT "EDGE OF SHEAR LAYER REACHED"
1055 W0=J
1060 PRINT
1070 S0=-1+LOG((1+C2)/(1-C2))/(2*C2)
1080 S1=N*C1*I2/P-Y2*C2^2/(1-C2^2)
1090 T1=C1*N*J2/P-Y2*C2/(1-C2^2)
1100 S2=Y3*C2^2/(1-C2^2)
1110 D2=(S1+S2)/S0
1120 PRINT "Y3/DELTA1="Y3;"DELTA/DELTA1="D2
1130 T2=Y3*C2/(1-C2^2)
1140 L=2*C2*(T1-T2)/D2
1150 J0=SQR(1-(1-C2^2)*EXP(L))/C2
1160 PRINT "J0="J0
1170 PRINT "J0-RATIO="J0/J3
1180 IF R2>R9 THEN 1600
1190 Z6=C2*(1-J0)/J0*C2/C0)/D2
1200 F3=C0/C2
1210 Z2=(Z5+Z6)/2
1220 Z4=D2*(-F3+LOG((1+C0)/(1-C0))/(2*C2))
1230 X2=X1+(Z4-Z3)*Z1/Z2
1240 R2=(C2*R1*X2*P*((1-C1^2)/(1-C2^2))^2)^.1
1250 PRINT "X/DELTAI,LAM.="X2;"REYNOLDS# R/2,X="R2
1260 PRINT
1270 X1=X2
1280 Y3=Y3+2
1290 IF R2>2E+06 THEN 1900
1300 GOSUB 1790
1310 GOTO 1100
1320 PRINT
1330 IF Y0=0 THEN 1400
1340 F8=F3
1350 D=10*D
1360 W6=J
1370 Y0=0
1380 J0=1
1390 GOTO 410
1400 PRINT "TRANSITION HAS OCCURED"
1405 I=J
1410 Z0=0
1420 Q8=X2
1430 Q7=J
1440 X1=X2
1450 GOSUB 1050
1460 PRINT "X/DELTA1, TURB.="X2
1470 T1=SQR(P0*(1/C2^2-1)*(J0-J9)/(Q9-D))
```

```
1480 E0=T1*(1-C0^2)*C2/(A*(1-C2*C2))
1490 PRINT "SHEAR STRESS FUNCTION="T1;"EDDY-VISCOSITY FUNCTION="E0
1500 GOSUB 1790
1510 GOTO 960
1520 GOSUB 1550
1530 GOSUB 1790
1540 GOTO 1460
1550 F3=C0/C3
1560 Z3=Z4
1570 Z4=FNY(F3)
1580 X2=X1+(Z4-Z3)*K*D2/(FNY(F3)*SQR(P0))
1590 RETURN
1600 S2=Y3*C2^2/(1-C2^2)
1610 D2=(S1+S2)/S0
1620 PRINT "Y2/DELTA1="Y3;"DELTA/DELTA1="D2
1630 T2=Y3*C2/(1-C2^2)
1640 L=2*C2*(T1+T2)/D2
1650 J0=SQR(1-(1-C2^2)*(BP/L)^2)
1660 PRINT "J0="J0
1670 PRINT "J0-RATIO="J0/C2
1680 F3=C0/C2
1690 Z3=Z4
1700 Z4=FNY(F3)
1710 X2=X1+(Z4-Z3)*K*D2/(FNY(F3)*SQR(P0))
1720 R2=(D2*R1*X2*P*((1-C1^2)/(1-C0^2))^(1/2))/C1
1730 PRINT "X/DELTA1, TURB.="X2;"REYNOLDS# R/2,X="R2
1740 GOSUB 1790
1750 Y3=Y3+0.5
1760 X1=X2
1770 IF R2>1E+06 THEN 1920
1780 GOTO 1600
1790 A(1,J)=Y2
1800 A(2,J)=D2
1810 A(3,J)=J0
1820 A(4,J)=K*X2/(E)^2
1830 A(5,J)=Y3
1840 PRINT
1850 PRINT "J="J
1860 PRINT
1880 D9=J
1890 J=J+1
1900 RETURN
1910 GOTO 1600
1920 END
READY.
```

Program ASUEXP†

Program ASUEXP calculates the sudden expansion and impingement of a plume on a solid cylindrical wall.

input

quantity	symbol	explanation
γ	A	specific heat ratio
r_e/r^*	R8	nozzle exit radius
θ_L	C	nozzle exit lip angle (deg)
M_L	B	nozzle exit Mach number
U_1^*	CO	measure of the local rate of acceleration near the nozzle lip [12]
r_{imping}/r_e	O	radius of the impingement wall
$R_{e,\delta}$	C(5)	Reynolds number based on velocity boundary layer thickness
δ/r^*	D	velocity boundary layer thickness at the nozzle lip

output

Peak pressure ratio, location of pressure peak, inviscid impingement angle, base pressure ratio and location of the viscid pressure rise.

†This program was written in HP-BASIC and run on a HP 9830A.

```
1 COM C[25]
10 PRINT "PLUME IMPACT ON CYLINDRICAL WALL, ZERO BLOW-BY, THEORY AND EXP.11 7 80
710 PRINT "PLUME IMPINGEMENT ON A CYLINDRICAL WALL, HHK 8 31 79"
715 PRINT "ITERATIVE SOLUTION FOR FINDING ZERO BLOW-BY CASE, HHK 9 5 79
720 PRINT "CALCULATION OF APPROXIMATE PLUME SHAPE"
725 PRINT "FOR IDEAL OR NON-IDEAL CONICAL C-D NOZZLES"
730 DIM D$[3],I[8,3]
735 DEG
740 DEF FNA(X)=ATN(X/SQR(1-X*X+1E-99))
745 DEF FNB(X)=ATN(SQR(1-X*X)/(X+1E-99))+2*ATN1E+99*(X<0)
750 DEF FNR(M)=((2+(A-1)*M*M)/(A+1))↑((A+1)/(2*(A-1)))/M
755 Z=1
760 Y=1
765 PRINT "NOZZLE FLOW SPECIFIED"
770 DISP "ENTER GAMMA=";
775 INPUT A
780 PRINT "GAMMA="A
785 DISP "IS NOZZLE IDEAL CONICAL?";
790 INPUT D$
795 DISP "ENTER NOZZLE EXIT/THROAT RADIUS RATIO=";
800 INPUT R8
805 DISP "ENTER NOZZLE EXIT ANGLE=";
810 INPUT C
815 PRINT "R-E R*="R8;"THETA(L-R)(DEG)="C
820 W3=C
825 GOTO 915
830 PRINT
835 F=((A-1)/(A+1))↑0.5
840 G=FNA(1/W1)
845 H=W3-G+90
850 W4=ATN(F*TAN(G))
855 W5=(W4)/F-H
860 DEF FNC(X)=((A+1)/2*X↑2/(1+(A-1)*X*X/2))
865 I0=FNC(W1)
870 X=I0
875 GOSUB 890
880 Q1=L
885 GOTO 905
890 K=((1-X)/(X-1/F↑2))↑0.5
895 L=ATN(K)/F-ATN(K/F)
900 RETURN
905 W6=-2*F*(COS(W4)↑((3*A)-1)/(2*(A-1)))*SIN(W4)↑0.5)
910 GOTO 1030
915 IF D$="YES" THEN 990
920 PRINT "NOZZLE IS NOT IDEAL CONICAL"
925 DISP "ENTER NOZZLE LIP MACH #=";
930 INPUT B
935 W1=B
940 PRINT "NOZZLE LIP MACH NUMBER="B
945 DISP "ENTER EXIT VALUE OF U1*=";
950 INPUT C0
955 W2=C0
960 PRINT "EXIT VALUE OF U1* (FROM M.O.C.)="C0
965 DISP "ENTER RADIUS OF IMPINGEMENT, WALL=";
970 INPUT Q
975 PRINT "RADIUS RATIO, IMPINGEMENT WALL="Q
978 C[10]=Q
980 GOTO 830
985 GOTO 1030
990 PRINT "IDEAL CONICAL SOURCE FLOW NOZZLE"
995 B=W1=FNO(R8*R8*2/(1+COS(W3)))
1000 PRINT "NOZZLE EXIT MACH NUMBER="W1
1005 C0=W2=3*SIN(W3)*SQR(FNC(W1))/(W1*W1)
1010 DISP "ENTER RADIUS OF IMPINGEMENT, WALL=";
1015 INPUT Q
1020 PRINT "RADIUS RATIO, IMPINGEMENT WALL="Q
1022 C[10]=Q
1025 GOTO 830
1030 W7=W2+W6
1035 C[1]=A
1045 C[2]=W1
1055 C[3]=C
```

```
1066 DISP "ENTER RE-DELTA1=";
1067 INPUT C[5]
1068 PRINT "RE-DELTA1="C[5]
1069 DISP "ENTER DELTA1/R*=";
1070 INPUT D
1071 PRINT "DELTA1/R*="D
1072 DISP "ENTER B.L. EXP. N=";
1073 INPUT N
1074 C[15]=N
1075 DISP "ENTER FIRST ESTIMATE J0=";
1085 INPUT J0
1095 PRINT "FIRST GUESS, J0-FACTOR="J0
1105 C[11]=J0
1110 GOTO 1165
1115 DEF FNO(X)
1120 M1=1.4
1125 M2=FNJ(M1)
1130 IF ABS(X-M2)<0.0001 THEN 1155
1135 M4=(A+1)*M1/(M1*M1*(A-1)+2)-1/M1
1140 M3=M1+(X-M2)/(X*M4)
1145 M1=M3
1150 GOTO 1125
1155 RETURN M3
1160 DEF FNJ(X)=((2/(A+1))*(1+(A-1)*X*X/2))^((A+1)/(2*(A-1)))/X
1165 C[12]=P8
1167 PRINT "B.L. EXP. N="N
1168 C[6]=D/P8
1170 LOAD #5,3
1500 END
```

```
1 COM C[25]
10 PRINT "JET IMPINGEMENT ON CYLINDRICAL WALL, HHK, 3/25/80"
15 DIM A$[3],B$[3],C$[3]
20 DISP "DATA ENTERED BY DIGITIZER=";
25 INPUT B$
30 IF B$#"YES" THEN 290
35 PRINT "DIGITIZATION OF STRIP-CHART DATA"
40 PRINT
45 PRINT "DIGITIZER COORDINATE ORIENTATION"
50 PRINT
55 ENTER (9,*)X,Y
60 T0=ATN(Y/X)
65 PRINT "ANGLE THETA="T0*180/PI"DEGREES"
70 DIM B[100,2],A[100,2],D[100,2]
75 DISP "ENTER ORIGIN COORDINATES OF GRAPH, X8,Y8=";
80 INPUT X8,Y8
85 PRINT "ORIGIN OF GRAPH AT LOCATION X="X8;"Y="Y8
90 DISP "ENTER X9, REF. VALUE X AXIS=";
95 INPUT X9
100 PRINT "REF. VALUE, X AXIS="X9
105 WRITE (9,*)
110 ENTER (9,*)X0,Y0
115 X9=X9-X8
116 PRINT X9
120 DISP "ENTER Y9, REF. VALUE Y AXIS=";
125 INPUT Y9
130 Y9=Y9-Y8
135 PRINT "REF. VALUE Y1, Y AXIS="Y9
140 WRITE (9,*)
145 ENTER (9,*)X1,Y1
150 DISP "ENTER # OF POINTS";
155 INPUT N0
160 PRINT "NUMBER OF POINTS="N0
165 FOR I=1 TO N0
170 WRITE (9,*)
175 ENTER (9,*)D[I,1],D[I,2]
180 WAIT 200
185 NEXT I
190 FOR I=1 TO N0
195 B[I,1]=D[I,1]*COS T0+D[I,2]*SIN T0
```

```
200 B[I,2]=D[I,2]*COST0-D[I,1]*SINT0
205 NEXT I
210 FOR I=1 TO N0
215 A[I,1]=X9*B[I,1]/SQR(X0*X0+Y0*Y0)+X8
220 A[I,2]=Y9*B[I,2]/SQR(X1*X1+Y1*Y1)+Y8
225 NEXT I
230 FOR I=1 TO N0
235 PRINT "I="I;A[I,1];A[I,2]
240 NEXT I
245 DISP "SHALL DATA BE STORED=";
250 INPUT A$
255 IF A$#"YES" THEN 390
260 STORE  DATA #5,4,A
265 PRINT "DATA STORED IN FILE 4,ON FLOPPY DISC"
270 PRINT "NOTE DIM(100,2)"
275 PRINT "IF POINTS TO BE PLOTTED CONT 345";
280 PRINT
285 STOP
290 DISP "POINTS FROM KEYBOARD=";
295 INPUT C$
300 IF C$#"YES" THEN 345
305 DISP "ENTER # POINTS=";
310 INPUT N
315 FOR I=1 TO N
320 DISP "ENTER X("I"),Y("I")=";
325 INPUT A[I,1],A[I,2]
330 PRINT "="I;"X="A[I,1];"Y="A[I,2]
335 NEXT I
340 GOTO 345
345 PRINT "DATA LOADED FROM DISK FILE #4"
350 LOAD  DATA #5,4,A
355 DISP "NUMBER OF POINTS IN FILE"=";
360 INPUT N
365 GOTO 375
370 N=N0
375 FOR I=1 TO N
380 PRINT "I="I,"X="A[I,1];"Y="A[I,2]
385 NEXT I
390 PRINT "SELECT MAX AND MIN VALUES SUCH THAT DIFFERENCES ARE POWERS OF 10"
395 PRINT "OR 2 OR 5 TIMES 10↑N"
400 PRINT
405 DISP "ENTER X-MIN,X-MAX,Y-MIN,Y-MAX";
410 INPUT P1,P2,Q1,Q2
415 SCALE -0.1*P2+1.1*P1,1.1*P2-0.1*P1,1.1*Q1-0.1*Q2,1.1*Q2-0.1*Q1
420 XAXIS Q1,(P2-P1)/10,P1,P2
425 FOR X=P1 TO P2 STEP (P2-P1)/10
430 PLOT X,0,1
435 CPLOT -4,-2
440 LABEL (485)X
445 NEXT X
450 YAXIS P1,(Q2-Q1)/10,Q1,Q2
455 FOR Y=Q1 TO Q2 STEP (Q2-Q1)/10
460 PLOT P1,Y,1
465 CPLOT -7,-0.3
470 LABEL (485)Y
475 NEXT Y
480 LABEL (*,1.5,1.7,0,8/10)
485 FORMAT F6.3
490 DISP "ENTER # POINT PAIRS=";
495 INPUT N
500 FOR I=1 TO N
505 PRINT "I="I;"X="A[I,1];"Y="A[I,2]
510 PLOT A[I,1],A[I,2]
515 CPLOT -0.3,-0.3
520 LABEL (*)"O"
525 IPLOT 0,0
530 PEN
535 NEXT I
540 STOP
545 P=C[10]
555 PRINT "R,W/R,E="R
```

```
560 K=C[1]
570 PRINT "K="K
575 DEF FNA(Y)=((K+1)-(K-1)*Y*Y)/(2*Y*Y)
580 Y=C[16]
590 M1=Y
595 PRINT "M*,0="Y
600 P9=FNP(Y)
605 B=C[17]
615 PRINT "DM*/DR, REATTACHMENT="B
620 A=1-FNA(Y)
625 PRINT "U1*="-2*A*B
630 C=-2*A*B
635 DISP "ENTER INTEGRATION STEP DELTA-X/R,N=";
640 INPUT D
645 PRINT "INTEGRATION STEP DELTA X/R,N="D
650 X0=C[18]
660 PLOT X0,P9
665 FOR I=1 TO 10
670 M2=M1-(C/(2*(1-FNA(M1))))*D
675 P2=FNP(M2)
680 PRINT "X/R,N="I*R*D;"P/P0,N="P2;"M*="M2
685 X=X0+I*R*D
690 PLOT X,P2
695 M1=M2
700 NEXT I
705 PEN
710 X1=C[7]
720 P1=C[19]
730 PRINT "X/R,E, VISC.="X1;"X/R,E-INVISC.="X0
735 PRINT "P,B/P0,N="P1;"P,PEAK/P0,N="P9
740 DEF FNP(X)=(1-(K-1)*X*X/(K+1))↑(K/(K-1))
745 PLOT 0,P1
750 PLOT X1,P1
755 PLOT X0,P9
760 PEN
765 END
```

```
1 COM C[35]
10 DIM A[7,75],B[2,20],K[2,75]
20 PRINT "THIS VERSION WITH PLOT,HHK 8/30/80"
30 PRINT "LINEAR MIXING PROFILE IN EXPANDED 1/N PROFILE B.L."
40 PRINT "THIS VERSION OF 8/28/80 CAN HANDLE THIN APPR. B.L."
50 DEF FNZ(F)=F↑N/SQR(1-(1-C1↑2*F↑2)*P↑((G-1)/G))
60 DEF FNC(M)=SQR(M↑2/(2/(G-1)+M↑2))
70 DEF FNM(F)=F↑N*FNDF/(1-F↑2*C1↑2)
80 DEF FND(F)=(1-(1-C1↑2*F↑2)*P↑((G-1)/G))↑0.5
90 DEF FNA(F)=F↑N/(1-F↑2*C1↑2)
100 DEF FNY(F)=D2*(-F3+LOG((1+C0)/(1-C0))/(2*C3))
110 FLOAT 4
120 G=C[1]
125 N=C[15]
130 M1=C[2]
135 M2=C[14]
140 D=10
145 P1=C[5]
150 I=C[7]/C[6]
234 PRINT "X/DELTA, FINAL="I
240 K=12+2.76*M2
250 Z0=Y0=F8=J=1
260 R9=10↑(4.754+0.039*M2+0.053*M2↑2)
270 P=((1+(G-1)*M1↑2/2)/(1+(G-1)*M2↑2/2))↑(G/(G-1))
280 PRINT "GAMMA="G;"UPSTREAM B.L. PROFILE EXP.="N
290 PRINT "M1="M1;"M2="M2;"P2/P1="P
300 PRINT "REYNOLDS #,(1-A-DELTA1)="R1;"SIGMA (CONST.)="K
310 PRINT "# OF INTEGRATION STEPS="D;"REYNOLDS #, TRANS.="R9
320 C1=FNCM1
330 C2=FNCM2
340 J3=SQR(1-(1-C2↑2)*EXP(LOG((1+C2)/(1-C2))/C2-2))/C2
350 PRINT "SIMILARITY VALUE FOR J0="J3
360 Z1=P*P1*(1-C1↑2)↑2/C1
```

```
370 F=Y1=I1=J1=O=0
380 X1=Z3=0
390 Z5=1E+10
400 F1=F
410 IF F1#F8 THEN 440
420 D=D/10
430 O6=J
440 F2=F1+1/D
445 GOSUB 450
446 GOSUB 940
447 GOTO 410
450 F9=(F1+F2)/2
460 IF F9>1 THEN 1020
470 A=FNDF9
480 I2=I1+FNMF9*(F2-F1)
490 Y2=Y1+P↑(-1/G)*N*C1*FNZF9*(F2-F1)
500 Y3=Y2
510 D1=I1+P↑(-1)*N*C1*(-1+LOG((1+FNDF1)/(1-FNDF1))/(2*FNDF1))
520 D2=I2+P↑(-1)*N*C1*(-1+LOG((1+FNDF2)/(1-FNDF2))/(2*FNDF2))
530 IF D1=0 THEN 550
540 Z7=2*P↑(-1)*C1*FNDF1*N/D1
550 Z=2*P↑(-1)*C1*FNDF2*N/D2
560 J2=J1+FNAF9*(F2-F1)
570 IF D1=0 THEN 590
580 Z8=J1*Z7
590 IF D1=0 THEN 660
600 O0=SQR(1-(1-(FNDF1)↑2)*EXPZ8)
610 K0=O0/C2
620 J2=J1+FNAF9*(F2-F1)
630 O3=FNDF1
640 F4=O0/O3
650 Z9=D1*(-F4+LOG((1+O0)/(1-O0))/(2*O3))
660 I0=J2*Z
670 IF D1#0 THEN 690
680 Z9=0
690 C0=SQR(1-(1-(FNDF2)↑2)*EXPI0)
700 J0=C0/C2
710 IF Y0=1 THEN 780
720 PRINT
730 PRINT "PHI-1(I)="F1;"PHI-1(I+1)="F2
740 PRINT "PHI-2(I)="FNDF1/C2;"PHI-2(I+1)="FNDF2/C2
750 PRINT "Y2/Y1="Y2;"DELTA/DELTA1="D2;D1
760 PRINT "J0="J0;"YL/DELTA1="Y2-D2
770 PRINT "J0-RATIO="J0/J3
771 PRINT I1;I2;J1;J2;F1;F2;X1;X2
780 C3=FNDF2
790 IF Z0=0 THEN 1450
800 Z6=C3*(1-J0↑2*C2↑2)/D2
810 F3=C0/C3
820 Z2=(Z5+Z6)/2
830 Z4=D2*(-F3+LOG((1+C0)/(1-C0))/(2*C3))
840 T1=SQRPI*(1/C2↑2-1)*(Z4-Z9)/(D2-D1)
850 E0=T1*(1-C0↑2)*C2/(A*(1-C2*C2))
851 IF Z0=1 THEN 870
860 PRINT "SHEAR STRESS FUNCTION="T1;"EDDY-VISCOSITY FUNCTION="E0
870 X2=X1+(Z4-Z3)*Z1/Z2
880 GOSUB 1790
890 IF Y0=1 THEN 910
900 PRINT "X/DELTA1, LAM.="X2
910 R2=(C2*R1*X2*P*((1-C1↑2)/(1-C2↑2))↑2)/C1
920 PRINT "REYNOLDS # R/2,X ="R2
930 IF R2>R9 THEN 1330
935 RETURN
940 Z3=Z4
950 Z5=Z6
960 X1=X2
970 I1=I2
980 J1=J2
990 Y1=Y2
1000 F1=F2
1010 RETURN
1020 Y3=Y2
```

```
1030 PRINT
1040 X1=X2
1050 PRINT "EDGE OF SHEAR LAYER REACHED"
1052 Q=J
1060 PRINT
1070 S0=-1+LOG((1+C2)/(1-C2))/(2*C2)
1080 S1=N*C1*I2/P-Y2*C2↑2/(1-C2↑2)
1090 T1=C1+N*J2/P-Y2*C2/(1-C2↑2)
1100 S2=Y3*C2↑2/(1-C2↑2)
1110 D2=(S1+S2)/S0
1120 PRINT "Y2/DELTA1="Y3;"DELTA DELTA1="D2
1130 T2=Y3*C2/(1-C2↑2)
1140 L=2*C2*(T1+T2)/D2
1150 J0=SQR(1-(1-C2↑2)*EXPL)/C2
1160 PRINT "J0="J0
1170 PRINT "J0-RATIO="J0/J3
1180 IF R2>R9 THEN 1680
1190 Z6=C2*(1-J0*J0+C2*C2)/D2
1200 F3=C0/C2
1210 Z3=(Z5+Z6)/2
1220 Z4=D2*(-F3+LOG((1+C0)/(1-C0))/(2*C2))
1230 X2=X1+(Z4-Z3)*Z1/Z2
1240 R2=(C2*R1*X2*P*((1-C1↑2)/(1-C2↑2))↑2)/C1
1250 PRINT "X/DELTA1,LAM.="X2;"REYNOLDS# R(2,X)="R2
1260 PRINT
1270 X1=X2
1280 Y3=Y3+2
1290 IF R2>2E+06 THEN 1920
1300 GOSUB 1790
1310 GOTO 1100
1320 PRINT
1330 IF Y0=0 THEN 1400
1340 F8=F2
1350 D=10*D
1360 O6=J
1370 Y0=0
1380 Z0=1
1390 GOTO 410
1400 PRINT "TRANSITION HAS OCCURED"
1410 Z0=O=0
1420 O8=X2
1430 O7=J
1440 X1=X2
1450 GOSUB 1550
1460 PRINT "X/DELTA1, TURB.="X2
1470 T1=SQRPI*(1/C2↑2-1)*(Z4-Z9)/(D2-D1)
1480 E0=T1*(1-C0↑2)*C2/(A*(1-C2*C2))
1490 PRINT "SHEAR STRESS FUNCTION="T1;"EDDY-VISCOSITY FUNCTION="E0
1491 PRINT I1;I2;J1;J2;F1;F2;X1;X2
1500 GOSUB 1790
1505 IF O#0 THEN 1550
1510 GOSUB 940
1512 GOTO 410
1520 GOSUB 1550
1530 GOSUB 1790
1540 GOTO 1460
1550 F3=C0/C3
1551 IF O#1 THEN 1560
1552 Z3=U4
1553 GOTO 1570
1560 Z3=Z4
1570 Z4=FNYF3
1580 X2=X1+(Z4-Z3)*K*D2/(FNYF3*SQRPI)
1581 PRINT X1;X2,F1;F2
1582 PRINT "Z3="Z3;"Z4="Z4;"F3="F3;"C0="C0;"C3="C3
1590 RETURN
1600 S2=Y3*C2↑2/(1-C2↑2)
1610 D2=(S1+S2)/S0
1620 PRINT "Y2/DELTA1="Y3;"DELTA/DELTA1="D2
1630 T2=Y3*C2/(1-C2↑2)
1640 L=2*C2*(T1+T2)/D2
1650 J0=SQR(1-(1-C2↑2)*EXPL)/C2
```

```
1660 PRINT "J0="J0
1670 PRINT "J0-RATIO="J0/J3
1680 F3=C0/C2
1690 Z3=Z4
1700 Z4=FNYF3
1710 X2=X1+(Z4-Z3)*K*D2/(FNYF3*SQRPI)
1712 K[1,J]=X2
1713 K[2,J]=J0
1720 R2=(C2*R1*X2*R*((1-C1↑2)/(1-C2↑2))↑2)/C1
1730 PRINT "X/DELTA1, TURB.="X2;"REYNOLDS# R 2,X="R2
1735 IF X2>I THEN 2965
1740 GOSUB 1790
1750 Y3=Y3+0.5
1760 X1=X2
1770 IF R2>2E+07 THEN 1920
1780 GOTO 1600
1790 K[1,J]=A[1,J]=X2
1800 A[2,J]=D2
1810 K[2,J]=A[3,J]=J0
1820 A[4,J]=K*X2/(E0*D2)
1830 A[5,J]=Z4
1831 A[6,J]=I2
1832 A[7,J]=J2
1840 PRINT
1850 PRINT "J="J
1852 IF X2>I THEN 2710
1860 PRINT
1880 Q9=J
1890 J=J+1
1900 RETURN
1910 GOTO 1600
1920 STOP
1930 PRINT
1940 FORMAT 3X,"J",3X,"X DELTA,1",2X,"DELTA DELTA,1",2X,"PHI-J",3X,"RE-T"
1950 WRITE (15,1940)
1960 FOR J=1 TO Q9
1961 IF J = Q7 THEN 1970
1962 A[4,J]=0
1970 WRITE (15,1990)J,A[1,J],A[2,J],A[3,J],A[4,J]
1980 NEXT J
1990 FORMAT 1F4.0,4E12.2
1995 PRINT "NOTE SCALE LGT.01,LGT5000,-.2,2"
2000 GOTO 2210
2010 FOR J=Q7 TO Q9
2020 PLOT LGT(A[1,J]),A[3,J]
2030 NEXT J
2035 LABEL (*)"J0"
2040 PEN
2050 STOP
2060 FOR J=Q7 TO Q9
2070 PLOT LGT(A[1,J]),A[4,J]/10000
2080 NEXT J
2085 LABEL (*)"RE-X,T/10E4"
2090 PEN
2100 STOP
2110 FOR J=Q7 TO Q9
2120 PLOT LGT(A[1,J]),A[5,J]/20
2130 NEXT J
2135 LABEL (*)"Y2/20"
2140 PEN
2150 STOP
2160 FOR J=Q7 TO Q9
2170 PLOT LGT(A[1,J]),A[2,J]/20
2180 NEXT J
2185 LABEL (*)"DELTA/20"
2190 PEN
2200 GOTO 2670
2210 DISP "ENTER LOWER LEFT=";
2220 STOP
2230 DISP "ENTER UPPER RIGHT=";
2240 STOP
2250 SCALE LGT0.01,LGT5000,-0.2,2
```

```
2260 XAXIS 0,LGT10,LGT0.1,LGT1000
2270 YAXIS LGT0.1,0.1,0,1.5
2280 A=0.1
2290 PLOT LGTA,0
2300 PLOT LGTA,1.5
2310 PEN
2320 A=10*A
2330 IF A 1000 THEN 2350
2340 GOTO 2290
2350 FOR B=0 TO 1.5 STEP 0.5
2360 PLOT LGT0.1,B
2370 PLOT LGT1000,B
2380 PEN
2390 NEXT B
2400 A=0.5
2410 PLOT LGTA,0
2420 PLOT LGTA,1.5
2430 PEN
2440 A=10*A
2450 IF A 1000 THEN 2470
2460 GOTO 2410
2470 STOP
2480 STANDARD
2490 FOR X=-1 TO 3
2500 X=INTX
2510 PLOT X,-0.1,1
2520 CPLOT -2,0
2530 LABEL (*)"10↑"INTX
2540 NEXT X
2550 FOR Y=0 TO 1.5 STEP 0.5
2560 PLOT -1.5,Y,1
2570 CPLOT 2,-0.3
2580 LABEL (*)Y
2590 NEXT Y
2600 FORMAT F8.2
2610 PLOT -0.5,1.8,1
2620 LABEL (*)"FREE SHEAR LAYER AFTER B. L. EXPANSION"
2630 LABEL (*)"BOUNDARY LAYER, EXP."N", RE,DELTA,1="R1
2640 LABEL (*)"APPROACH MACH #="M1"JET MACH #="M2
2641 LABEL (*)"GAMMA="G"SIGMA="K
2642 PLOT 3.05,0,1
2644 LABEL (*)"X DELTA,1"
2650 GOTO 3010
2660 END
2670 PLOT LGT(A[1,07]),0
2680 PLOT LGT(A[1,07]),1.15
2690 PEN
2700 LABEL (*,1.5,1.7,PI/2,8/10)"TRANSITION"
2701 PLOT LGT(I),0
2702 PLOT LGT(I),0.85
2703 PEN
2704 LABEL (*)"RECOMPRESSION START"
2705 GOTO 3020
2710 IF O#0 THEN 2820
2715 IF F1>0.8 THEN 2965
2720 W1=X1
2730 W2=X2
2740 V2=F2
2750 V1=F1
2760 U1=I1
2770 U2=Y1
2780 U3=J1
2785 U4=A[5,J-1]
2790 W0=A[1,J]-I
2795 U5=C3
2800 H=O=1
2810 B[2,1]=A[1,J-1]-I
2815 B[1,1]=F1
2820 B[1,H+1]=V2-W0*(B[1,H]-V2)/(B[2,H]-W0)
2822 PRINT "H="H;B[1,H+1];B[1,H];B[2,H]
2830 F1=V1
2840 I1=U1
```

```
2850 V1=U2
2860 J1=U3
2870 X1=W1
2875 C3=U5
2880 F2=B[1,H+1]
2890 GOSUB 450
2930 IF ABS(B[2,H])<1E-03 THEN 2950
2935 H=H+1
2938 B[2,H]=X2-I
2940 GOTO 2820
2950 PRINT "SOLUTION, X2="B[2,H]+I;"J0="J0;"J0-RATIO="J0/J3
2952 IF ABS(C[11]-J0/J3)<0.01 THEN 2960
2955 C[11]=J0/J3
2956 LOAD #5,3
2960 PRINT
2961 PRINT "THIS IS THE FINAL RESULT FOR THE SUDDEN ENLARGEMENT PROBLEM"
2962 GOTO 2980
2965 J0=K[2,J-1]+(K[2,J]-K[2,J-1])*(I-K[1,J-1])/(K[1,J]-K[1,J-1])
2970 PRINT "J0="J0"AT X/DELTA1="I
2975 C[11]=J0/J3
2978 GOTO 2952
2980 DISP "IF PLOTTING DESIRED, ENTER 0";
2990 INPUT W9
3000 IF W9#0 THEN 3030
3010 GOTO 1990
3020 LOAD #5,1
3030 END
```

```
1 COM C[25]
10 PRINT "PLUME IMPACT ON CYLINDRICAL WALL, ZERO BLOW-BY, THEORY AND EXP.11 7 80
710 PRINT "PLUME IMPINGEMENT ON A CYLINDRICAL WALL, HHK 8 31 79"
715 PRINT "ITERATIVE SOLUTION FOR FINDING ZERO BLOW-BY CASE, HHK 9 5 79"
720 PRINT "CALCULATION OF APPROXIMATE PLUME SHAPE"
725 PRINT "FOR IDEAL OR NON-IDEAL CONICAL C-D NOZZLES"
730 DIM D$[3],I[8,3]
735 DEG
740 DEF FNA(X)=ATN(X/SQR(1-X*X+1E-99))
745 DEF FNB(X)=ATN(SQR(1-X*X)/(X+1E-99))+2*ATN1E+99*(X<0)
750 DEF FNR(M)=((2*(A-1)*M*M)/(A+1))↑((A+1)/(2*(A-1)))/M
755 Z=1
760 Y=1
765 PRINT "NOZZLE FLOW SPECIFIED"
770 A=C[1]
780 PRINT "GAMMA="A
785 W1=B=C[2]
790 C=C[3]
795 C0=C[4]
800 F8=C[12]
820 W3=C
825 GOTO 935
830 PRINT
835 F=((A-1)/(A+1))↑0.5
840 G=FNA(1/W1)
845 H=W3-G+90
850 W4=ATN(F/TAN(G))
855 W5=(W4)/F-H
860 DEF FNC(X)=(((A+1)/2)*X↑2)/(1+((A-1)*X*X)/2)
865 I0=FNC(W1)
870 X=I0
875 GOSUB 890
880 O1=L
885 GOTO 905
890 K=((1-X)/(X-1/F↑2))↑0.5
895 L=ATN(K)/F-ATN(K/F)
900 RETURN
905 W6=-2*F/(COS(W4)↑(((3*A)-1)/(2*(A-1)))*SIN(W4)↑0.5)
910 GOTO 1030
935 W1=B
940 PRINT "NOZZLE LIP MACH NUMBER="B
950 C0=C[4]
```

```
955 W2=C0
970 Q=C[10]
975 PRINT "RADIUS RATIO, IMPINGEMENT WALL="Q
980 GOTO 830
985 GOTO 1030
995 B=W1=FNO(R8*R8*2/(1+COS(W3)))
1000 PRINT "NOZZLE EXIT MACH NUMBER="W1
1005 C0=W2=3*SIN(W3)*SQR(FNC(W1))/(W1*W1)
1010 DISP "ENTER RADIUS OF IMPINGEMENT, WALL=";
1015 Q=C[10]
1020 PRINT "RADIUS RATIO, IMPINGEMENT WALL="Q
1025 GOTO 830
1030 W7=W2*W6
1040 D=1.2*W1
1045 PRINT "FIRST TRIAL JET MACH NUMBER"
1055 J0=C[11]
1060 PRINT "J0-FACTOR="J0
1065 I[1,1]=D
1070 PRINT "JET SURFACE MACH NUMBER="D
1075 J=FNC(D)
1080 X=J
1085 GOSUB 890
1090 O2=L
1095 E2=W4+F*(O2-O1-FNA(1/D)+FNA(1/B))
1100 D1=(E2-W4)/10
1105 B0=W5
1110 E1=W4
1115 C1=W7
1120 GOSUB 1130
1125 GOTO 1190
1130 S1=S2=S3=S4=0
1135 FOR N=1 TO 10
1140 E3=E1+D1*(N-1/2)
1145 S1=COS(E3/F)*SIN(E3)↑(-0.5)*COS(E3)↑(-1/(2*F↑2))*D1+S1
1150 S2=SIN(E3/F)*SIN(E3)↑(-0.5)*COS(E3)↑(-1/(2*F↑2))*D1+S2
1155 S3=SIN(E3/F)*SIN(E3)↑(-1.5)*COS(E3)↑((A-3)/(2*(A-1)))*D1+S3
1160 S4=COS(E3/F)*SIN(E3)↑(-1.5)*COS(E3)↑((A-3)/(2*(A-1)))*D1+S4
1165 NEXT N
1170 U0=-((COS(E2)↑((3*A-1)/(2*(A-1))))*SIN(E2)↑0.5)/(2*F)
1175 U1=U0*((S1-F*S3)*COS(B0)+(S2+F*S4)*SIN(B0)+C1*180/PI)*PI/180
1180 PRINT "U1*(F)="U1
1185 RETURN
1190 G2=FNA(1/D)
1195 T2=C+O2-O1
1200 R0=-((U1/SQR(J)+SIN(T2)*SIN(G2)↑2)/(SIN(2*G2)))
1205 R1=-1/R0
1210 PRINT
1215 PRINT "SOLUTION"
1220 PRINT "INITIAL SLOPE OF PLUME, THETA-F="T2
1225 PRINT "(INITIAL) RADIUS OF CURVATURE, PLUME="R1
1230 IF R1*(1-COS(T2))+1>Q THEN 1235
1231 D=D+0.2
1232 PRINT "NO PLUME IMPINGEMENT"
1233 GOTO 1065
1235 PRINT
1240 PRINT "PLUME SHAPE APPROXIMATED BY CIRCULAR ARC"
1245 WRITE (15,1250)
1250 FORMAT 5X,"X(-)",6X,"R(-)",5X,"THETA(DEG)"
1255 FOR S=0 TO 1.2 STEP 0.1
1260 T=FNA(SIN(T2)-(S/R1))
1265 S=R1*(SIN(T2)-SIN(T))
1270 R=1+R1*(COS(T)-COS(T2))
1275 WRITE (15,1280)S,R,T
1280 FORMAT 3F10.3
1285 NEXT S
1290 PRINT
1295 PRINT
1300 G=FNA(1/D)
1305 C=FNB(COS(T2)+(Q-1)/R1)
1310 B=D
1315 E1=ATN(F/TAN(G))
```

```
1325 GOSUB 890
1330 Q2=L-C
1335 X4=FNC(B)
1340 X=X4
1345 GOSUB 890
1350 L1=L
1355 X=X2=3.5
1360 GOSUB 890
1365 L2=L
1370 X3=(Q2-L1)*(X2-X4)/(L2-L1)+X4
1375 X=X3
1380 GOSUB 890
1385 L3=L
1390 IF ABS(X3-X2)<0.001 THEN 1420
1395 X4=X2
1400 X2=X3
1405 L1=L2
1410 L2=L3
1415 GOTO 1370
1420 PRINT
1425 D=SQR(2*X3/(A+1-(A-1)*X3))
1430 IF Z<0 THEN 1585
1435 M8=SQR(FNC(D))
1437 C[16]=M8
1440 PRINT "M, ATT.="D;"M*, ATT.="SQR(FNC(D))
1445 C0=SQR(FNC(B))*SIN(G)*(2*COS(G)*Q/R1-SIN(G)*SIN(C))
1450 C2=-2*F/(COS(E1)↑((3*A-1)/(2*(A-1)))*SIN(E1)↑0.5)
1455 C1=C0*C2
1460 B0=ATN(F/TAN(FNA(1/B)))/F-90+FNA(1/B)
1465 E2=F*(90-FNA(1/D)+B0)
1470 D1=(E2-E1)/10
1475 GOSUB 1130
1480 Q8=M8*U1/(2*COS(FNA(1/D))↑2)
1485 PRINT
1490 PRINT "INVISCID REATTACHMENT, WALL PRESSURES"
1495 P8=(1+(A-1)*M8*M8/(A+1))↑(A/(A-1))
1500 PRINT "DM*/DR="Q8;"PEAK PRESSURE RATIO="P8
1504 C[20]=P8
1505 Q4=R1*(SIN(T2)-SIN(C))
1506 C[17]=Q8
1510 PRINT "LOCATION OF PRESSURE PEAK, X/R R/NOZZLE="Q4
1512 C[18]=Q4
1515 PRINT "INVISCID IMPINGEMENT ANGLE="C
1520 P9=-Q8*P8*2*A*M8/(D*((A+1)-(A-1)*M8*M8))
1525 PRINT "INITIAL SLOPE, D(P/PO)/D(X/R,N)="P9
1530 C3=B*B/(2/(A-1)+B*B)
1535 C6=SQR(C3)
1540 E9=SQR(PI)*(1-(1/C3-1)*(-1+LOG((1+C6)/(1-C6))/(2*C6)))
1545 E8=((T2-C)*PI/180)*E9/(12+2.76*B)
1550 C8=FNA(SIN(C)+E8/SIN(C))
1555 PRINT
1560 Z=-Z
1565 PRINT
1570 PRINT "BASE PRESSURE, VISCID SOLUTION (PAGE, MODIFIED)"
1575 C=C8+0.59
1580 GOTO 1315
1585 C4=D*D/(2/(A-1)+D*D)
1590 C5=SQR((C3-C4)/(1-C4))/C6
1595 C7=J0*SQR(1-(1-C3)*EXP(LOG((1+C6)/(1-C6))/C6-2))/C6
1600 PRINT "PHI,D="C5;"PHI,J="C7
1605 D8=(T2-C8)*SQRPI*C5/((12+2.56*B)*180)
1610 C9=FNB((COS(C8)-D8)/(1+(T2-C8)*PI*E9/(180*(12+2.56*B))))
1615 Q9=R1*(SIN(C9)*(1+D8)-SIN(C8))
1620 I[V,2]=C5
1625 I[V,3]=C7
1630 M0=R1*(T2-C8)*PI*(FNI(C5)-FNI(C7))/(180*(12+2.76*B))
1635 PRINT
1640 Q6=(1-C3)↑(A/(A-1))
1645 PRINT "BASE PRESSURE RATIO="Q6
1647 C[19]=Q6
1650 Q7=R1*(SIN(T2)-SIN(C8))-Q9
1655 PRINT "LOCATION OF VISCID PRESSURE RISE X/V/R,N="Q7
```

```
1660 DEF FNI(X)=(C3-1)*SQR(PI)*LOG(1-C3*X*X)↑2*C3)
1665 PRINT "M0="M0
1670 DEF FNJ(X)=((2/(A+1))*(1+(A-1)*X*X/2))↑((A+1)/(2*(A-1)))/X
1675 PRINT "MASS RATIO="2*D*M0*R8*R8/FNJ(B)
1680 V=V+1
1685 IF V>2 THEN 1785
1690 I[2,1]=I[1,1]+0.2
1695 D=I[2,1]
1700 B=W1
1705 C0=W2
1710 C=W3
1715 PRINT
1720 PRINT "SECOND TRIAL JET MACH NUMBER"
1725 PRINT
1730 Z=-Z
1735 GOTO 1070
1740 DEF FNO(X)
1745 M1=1.4
1750 M2=FNJ(M1)
1755 IF ABS(X-M2)<0.0001 THEN 1780
1760 M4=(A+1)*M1/(M1*M1*(A-1)+2)-1/M1
1765 M3=M1+(X-M2)/(X*M4)
1770 M1=M3
1775 GOTO 1750
1780 RETURN M3
1785 I[V,1]=(I[V-1,1]-I[V-2,1])*(I[V-2,2]-I[V-2,3])
1790 I[V,1]=I[V-2,1]+I[V,1]/(I[V-1,3]-I[V-2,3]-I[V-1,2]+I[V-2,2])
1795 IF ABS(I[V,1]-I[V-1,1])<0.001 THEN 1845
1800 D=I[V,1]
1805 B=W1
1810 C0=W2
1815 C=W3
1820 Z=-Z
1825 PRINT
1830 PRINT "NEW JET MACH NUMBER, BY INTERPOLATION, ITERATION #="V
1835 PRINT
1840 GOTO 1070
1845 PRINT
1850 PRINT "THIS IS THE SOLUTION FOR ZERO BLOW-BY"
1855 C[7]=Q7
1856 C[14]=B
1858 LOAD #5,2
1860 END
```

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