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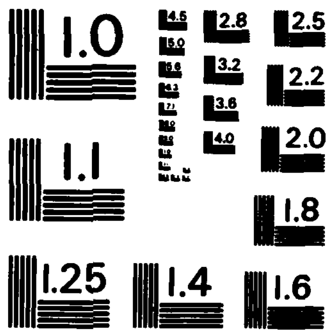

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AD-A146 446

A-10 ALTIMETRY SYSTEM ERROR ANALYSIS

Paul E. Hundley, 1st Lt., USAF

Instruments Branch
Information Engineering Division
Directorate of Avionics Engineering

April 1981

Final Report for Period September 1979 - September 1980

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DIRECTORATE OF AVIONICS ENGINEERING
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Paul E Hundley

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FOREWORD

This work was accomplished by the author as a member of the Instruments Branch, Information Engineering Division, Directorate of Avionics Engineering, Aeronautical Systems Division, Wright-Patterson Air Force Base, Ohio. The work was accomplished from September 1979 to September 1980 and submitted in part to the University of Dayton, Dayton, Ohio, in partial fulfillment of the requirements for the Degree of Master of Science.

The author wishes to express his gratitude to Dr. John Fraker, University of Dayton, for his guidance during the course of the work. The author also expresses his deepest appreciation to Mrs. Patricia Ennever for her assistance.

This report was submitted by the author during August 1980.

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A study was made to identify and validate field-reported A-10 altimetry system bias. Preliminary evaluation indicated a positive bias (i.e., the aircraft's actual height was lower than the displayed altitude) at the positive extreme of the root sum square of the altimetry system component tolerances. As a result, data taken on actual flights were gathered and statistically analyzed. The results indicate a mean positive bias of 60 feet @ 99.5 percent confidence level. However, in further analysis interactions were present. Since the interactions could not be made nonsignificant, no direct conclusions concerning

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the main effects of altitude or aircraft could be made. The altimetry system component analysis demonstrated a positive bias trend, but it was not large enough to explain the flight test results. Ground effects appear to increase the positive bias significantly (i.e., below 100 feet altitude). As a result, it is concluded that there is definitely a positive bias in the system but that a better designed, follow-on flight test is required to pinpoint this bias or main effects that could be further studied.

TABLE OF CONTENTS

TABLE OF CONTENTS (Continued)

TABLE OF CONTENTS (Continued)

Page

LIST OF TABLES

LIST OF ILLUSTRATIONS

SECTION I

INTRODUCTION

1. PROBLEM STATEMENT

The purpose of this report is to determine if the A-10 aircraft vehicle is consistently experiencing altitude errors of 200 feet or more at high airspeeds and low altitudes. This is a recent problem due to changes in the tactics of the A-10 in its role as a close air support aircraft. See #1473

If it is true that errors of 200 feet or greater are being consistently experienced, the A-10's capability, flexibility, and mission effectiveness will be severely handicapped. No other aircraft flies as low for as long as the A-10. In terms of human factors, this type of flying introduces an extremely heavy pilot workload. If the altimetry system is reporting large altitude errors and the pilot reads an incorrect display and believes it, the consequences can be extremely grave. A lost life is irreplaceable, and the loss of an aircraft is very costly. The key player and pilot-identified problem area is the barometric altimetry system (Figure 1).

Due to the nature of the A-10's close air support role, altimetry system reporting is critical with gross errors intolerable. This report will determine, with the data available, whether or not the A-10 is consistently experiencing large altimetry errors.

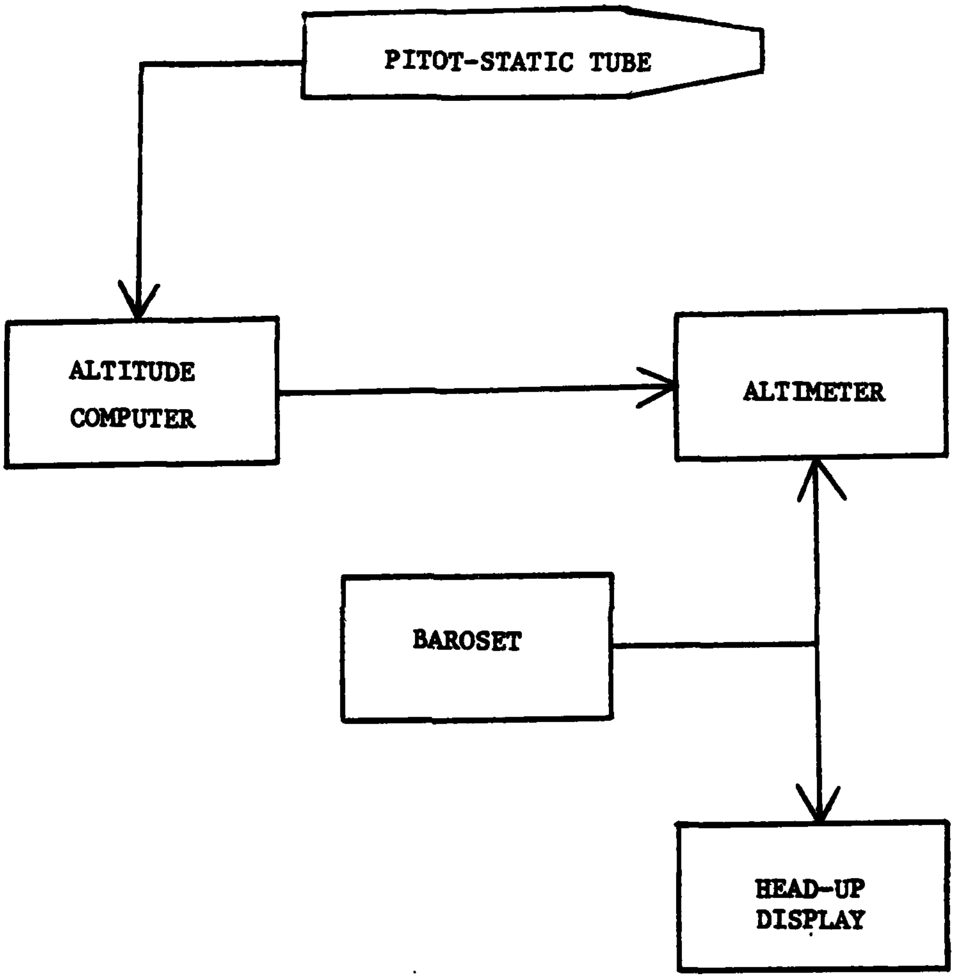


Figure 1
Altimetry System

2. BACKGROUND

To place the problem in perspective requires some knowledge of the A-10's physical and functional characteristics.

The A-10 can fly low to the ground at moderate airspeeds while remaining extremely manueverable. The A-10 can carry a multitude of ordnance, such as unguided and guided bombs, both single-load and multiload missiles, and the 30 mm/cannon. The cannon alone can destroy a tank. This enables the A-10 to be used as an offensive, as well as a defensive, weapon for conventional as well as nuclear warfare. Suffice it to say, the A-10 is an impressive close ground support aircraft. However, the A-10 does have its limitations. In terms of flight performance compared to interceptor fighters, the A-10 is inferior. Its maximum altitude is 35,000 feet, and its maximum airspeed is 450 knots. Close to the ground in manuevers, the aircraft rarely exceeds 350 knots. These performance limitations, as well as the aircraft size, severely limit the aircraft's survivability in combat.

Initially, the aircraft was a low-cost aircraft with unsophisticated avionics. The altitude ground level designed for the aircraft was 500 feet. Due to the aircraft's manueverability and its projected survivability, the fighter tactics changed. The operating altitude ground level was lowered to 300 feet and more recently to 100 feet. At the 300-foot level, no altimetry system problems were noted. However, at the 100-foot level, pilots reported experiencing large altimetry errors as a common occurrence.

The first reported case was in November of 1978. During an exercise in Florida, a flight of A-10s was disqualified for flying below

100 feet. Subsequent review of gun camera film demonstrated the aircraft to have an indicated altitude greater than 300 feet. Further telephone conversations with pilots at different Air Force bases revealed that the A-10 altimetry system errors were viewed as commonplace.

The prime contractor, the Fairchild Republic Company, was notified. Fairchild dispatched a team to one of the A-10 bases for further investigation. An A-10 that exhibited an error of 300 feet was selected by the pilots and flown at the gunnery range. The A-10 flew over a surveyed point on the range and was viewed through the range tower window. The tower window is imprinted with a graph allowing a determination of the height of the aircraft to be made. The aircraft's altimetry system exhibited a 70-foot positive error, i.e., indicated altitude 70 feet higher than actual altitude. The pitot-static tube was removed from the aircraft and was sent to Fairchild's contractor for analysis. The tube was tested against the standard and exhibited a negative position error of 10 feet. After the tube was cleaned and retested, it agreed exactly with the standard. A key point here is that the indicated altitudes found thus far were higher than actual aircraft altitude.

A preliminary evaluation of the altimetry system components was performed by the Instruments Branch of the Aeronautical Systems Division (ASD/ENAID) at Wright-Patterson Air Force Base, Ohio. Based upon flight test data from Edwards Air Force Base, California, and the specifications (References 7, 8, 9) for the individual components, the cumulative system tolerance limits are from minus 77 feet to positive 113 feet. The root sum square tolerance limits are from minus 42 feet to positive 77 feet. This is approximately a three standard deviation range (3 sigma) within which approximately 99 percent of the aircraft's altimetry system readings should fall. A 2-sigma range is from minus 22 feet to positive 68 feet and should account for approximately 97 percent of the A-10 aircraft. A 1-sigma range is from positive 2 feet to positive 36 feet and should account for approximately 67 percent of the A-10 aircraft. However, these ranges of altitude errors are not consistent with field-reported discrepancies.

3. SUMMARY OF RESULTS

The results of this study indicate a positive bias in the A-10 altimetry system in the neighborhood of 60 feet above actual aircraft altitude. This can have a detrimental effect for close air-support aircraft, such as the A-10.

Independent analysis of the A-10 altimetry system, component by component, indicates a small positive bias. However, the component error does not explain the total error seen from the flight test data. Hence, other aircraft effects, such as pitch attitude, local aircraft barometric conditions, and bank angle, probably play a small but contributing part to the total altimetry error.

The ground proximity error appears to significantly increase the positive bias in altimetry system error. This is probably due to aircraft compressibility effects on the aerodynamically compensated pitot tube. However, this area bears further investigation.

The results of this analysis should be confirmed with a follow-on flight test. Section VI presents a flight test designed from the results of this analysis. The follow-on flight test will allow the suspected error contributors to be pinpointed and some investigation into the area of ground effects.

SECTION II

INITIAL FLIGHT TEST DESIGN

After reviewing the previously performed investigations, analysis, and current literature, nothing specific could be stated concerning the A-10 altimetry system other than that 99 percent of the A-10 fleet's displayed altitude should be in the range of minus 42 feet to positive 77 feet. To validate the expected or substantiate the reported altimetry system errors, further investigation was required.

Since the A-10 production program was well underway, there were no funds available for a controlled flight test. The Air Force Tactical Air Command (TAC) was requested to gather some data for the Avionics Division of the A-10 System Program Office (ASD/YXEA) in June 1979. The Tactical Air Command replied that they could, as long as specific, dedicated flights were not required. A simple flight test was designed to allow the necessary data to be gathered during training missions. The following requested test allowed all variables normally encountered in flight to affect the displayed altitude:

1. Data base was to include five different aircraft and pilots.

2. A ground visual inspection of the Pitot-static probe and an altimeter and airspeed ground check were to be performed by qualified maintenance personnel in accordance with the applicable technical order. All readings were to be recorded.

3. A field elevation check of the altimetry system prior to the flight was to be performed and the readings recorded.

4. An in-flight altitude check was to be accomplished by flying the aircraft straight and level at an altitude above ground level of 100 feet past the base or range tower at an airspeed of 300 knots. The aircraft was to make four passes. At a point in time called out by an operations person, the pilot was to read his head-up display altitude and altimeter altitude while trained aircraft observers were to ascertain the aircraft's altitude. All readings were to be recorded.

5. After the flight, another field elevation check was to be performed.

6. The tail number of the aircraft was to be recorded.

The test was assigned to an A-10 training base. Telephone conversations in July 1979 with the assigned project officer revealed that the range or base tower and observers requested were unavailable because the base was only a training base and flights at 100 feet were prohibited. As an alternative, it was suggested to use a flight of four aircraft and match their altitudes against an A-7 aircraft with a radar altimeter at an altitude of 500 feet. This suggestion was agreed upon as an acceptable method with the addition of a 200-foot altitude test point and an assurance that aircraft vortices would not cross-interfere with the pitot-static probes. The redesigned test was accomplished as follows:

1. The aircraft selected by the project officer were selected at random with the exception that two aircraft had AAU-19/A-type designated altimeters and the other two aircraft had AAU-34/A-type designated altimeters. The AAU-34/A is an updated version of the AAU-19/A.

2. The maintenance checks by maintenance personnel were performed, but the data were not recorded. However, the altimetry systems did meet the technical order tolerance requirement of ±45 feet.

3. The field elevation check was performed and was recorded.

4. Calibration of the A-7 aircraft radar altimeter was checked and was found to be within tolerance.

5. Four passes of the aircraft were made—two at 500 feet and two at 200 feet.

6. The test formation flown was with two A-10 aircraft maintaining wings-level flight with each other while the A-7 came up wings level. When all aircraft were wings level, the altitude readings from the A-10 head-up display and altimeter, along with the A-7's radar altimeter readings, were noted and were recorded.

7. The tail numbers of the aircraft were recorded.

The test was completed in August 1979, and the data were forwarded and received by ASD/YXEA in September 1979. The test described allowed all variables to affect the system. But, some extraneous variables, such as accuracy of formation flying and the radar altimeter, were ignored. The author feels that the preciseness of the data, but not the accuracy, may have been affected (Reference 11).

SECTION III

METHODS AND PROCEDURES FOR ANALYSIS

The methods used for analysis of the data depend to some extent on the format of the data. For example, Table 1 is the data format for one aircraft (see Appendix A for all data).

Table 1

DATA FORMAT FOR ONE AIRCRAFT

Nominal Altitude	A-10 True Altitude	HUD	HUD Error	Altimeter	Altimeter Error
500	550	565	+15	565	+15
500	490	505	+15	510	+20
200	210	295	+85	295	+85
200	210	295	+85	295	+85

The primary instrument for reading the altitude is the altimeter, and the normal mode of operation is the electrical mode. The head-up-displayed altitude is a cross-check to the altimeter and is used when the pilot is under a heavy workload and cannot afford to look inside the cockpit. But, the primary interest is the altimeter deviation from the true altitude as established by the A-7's radar altimeter. Therefore, it is the errors of the altimeter and then the head-up display that will be analyzed.

The analysis will seek to achieve the following objectives:

1. Confirm the possibility of 200- to 300-foot positive errors reported from the A-10 bases.

2. Quantify, if reasonable, the errors.

3. Analyze the system and determine if:

   a. Differences between aircraft exist

   b. Differences due to altitudes exist

   c. System component effects exist

4. Determine differences between the head-up display and the altimeter.

5. Determine effects of ground proximity.

The determinations and any inferences proposed will be as a result of the statistical analysis of the data available. The specific techniques that will be used are analysis of variance and regression. Since the aircraft were picked at random but the altitudes were predetermined, a mixed effects model will be used for the analysis of variance as follows:

$$Y_{ijk} = \mu_{..} + \alpha_i + \beta_j + (\alpha\beta)_{ij} + \epsilon_{ijk}$$

Y_{ijk} = error

$\mu_{..}$ = mean of the errors

α_i = altitude effects, fixed

β_j = aircraft (altimetry system) effects, random

$(\alpha\beta)_{ij}$ = interaction

ϵ_{ijk} = remaining random error

In the following section, the discussion will be organized according to the previous objectives. The error exhibited will be viewed as strictly random error. A confidence interval and a prediction interval will be stated in relation to the field-reported errors. This also will allow statements to be made regarding the expected value of error for the fleet. Analysis of variance techniques will be applied to assign and refine the previous random error and will allow new statements to be introduced. Each system component will be evaluated with statistical techniques as applicable. The head-up display errors will be considered and compared to the altimeter errors to ascertain if a significant difference between the two display modes exists. Finally, ground effects will be considered. This area will be difficult to examine due to compressibility effects by the aircraft and Mach number. Only very general statements will be made in the latter two areas since the scope of investigation touches upon it but does not delve into it. However, it is reemphasized that the primary purpose of this report is to substantiate the reported errors.

SECTION IV

ANALYSIS AND DISCUSSION

To begin this section, the objectives are briefly restated as follows:

1. Substantiate the 200- to 300-foot reported errors
2. Quantify the errors
3. Determine if:
   a. Differences between aircraft exist
   b. Differences between altitudes exist
   c. System component effects exist
4. Determine altitude readout differences between head-up display and altimeter
5. Investigate ground proximity effects

The sections that deal with the previous objectives are as follows:

1. Total Random Effects--Objectives 1 and 2
2. Analysis of Variance--Objectives 3.a. and 3.b.
3. System Component Effects--Objective 3.c.
4. Head-up display analysis--Objective 4
5. Ground effects analysis--Objective 5

Before delving into the analysis, the following assumptions are stated for simplification:

1. Radar altimeter readings from the A-7 aircraft are without error.
2. The formation flying was accomplished without error.

3. The data come from a normal population.

1. TOTAL RANDOM ERROR

This section of analysis is predicated on the assumption that all error is totally random. Therefore, the data are looked upon as one sample, size 16. The following statistics are calculated (Data in Appendix A):

$$\text{Mean, } \bar{Y} = \frac{\Sigma Y}{16} = 61.5626 \text{ feet}$$

$$\text{Standard deviation } Sy = \frac{\Sigma(Y-\bar{Y})^2}{n-1} = 26.56 \text{ feet}$$

The significance level (α) established is $\alpha = 0.01$. Thus, a 99 percent confidence interval for the mean is computed as follows:

$$\bar{Y} - t_t(99.5,15)S\bar{y} \leq \mu \leq \bar{Y} + t_t(99.5,15)S\bar{y}$$

where: $t_t(0.99.5,15) = 2.942$ (19).

$$S\bar{y} = \frac{Sy}{\sqrt{n}} = 6.64 \text{ feet}$$

The resulting interval is:

$$42 \leq \mu \leq 82 - \text{feet}$$

(NOTE: numbers are rounded)

The above corresponds to where one would most likely expect to find the mean error value of the total fleet. However, to state where one would expect to find the error value of an individual aircraft requires a prediction interval. Choosing again a 99 percent confidence level ($\alpha = 0.01$), the following interval is obtained:

$$Y_h - t_t(99.5,15)S(Y_{h[new]}) \le Y_{h(new)} \le Y_h + t_t(99.5,5)S(Y_{h[new]})$$

where: $S(Y_{h[new]}) = (\frac{17}{16}\ Sy^2)^{1/2} = 27.38$

Y_h = estimated mean of the distribution

Then a 99 percent prediction interval is:

$$-20 \le Y_{h(new)} \le 143 \text{ feet}$$

Based on the data and the computed intervals, one can say that it is not a common occurrence for an A-10 aircraft to yield an altimetry error of 200 to 300 feet. If one were to quantify a likely range for the mean error for the A-10 fleet, it would appear to be from approximately 40 feet to 80 feet. This falls into the previous 3-sigma range stated in the background section, but the results definitely demonstrate a positive bias.

2. ANALYSIS OF VARIANCE

To determine the effects of aircraft and altitude, a computer routine (Reference 3) was used. The previous assumptions apply with the addition of the assumption of constant variance. Applying the previously stated mixed-effects model, the computed F test results are shown in Table 2. To determine significance, the standard F test was used with significance levels of 0.01 and 0.001. The complete Analysis of Variance Table is shown in Appendix B.

With $\alpha = 0.001$, it appears that none of the factors is significant. However, if the confidence level is dropped to 99 percent (remaining consistent with the previous section), the interaction factor appears to be significant. The interaction could be masking the true factor effects of aircraft and altitude, especially altitude, since a mixed-effects model divides the mean squares (MS) of altitude by the interaction mean squares and not the random error. The first step is to validate the assumptions of normalcy and constant variance.

Table 2

F TEST RESULTS

Factor	F calculated	F test 99%	F test 99.9%
α - altitude	1.6571	34.1	167
β - aircraft	2.874	7.59	15.8
$\alpha\beta$ - interaction	11.744	7.59	15.8

The residual terms are plotted (Figure 2) and demonstrate normalcy. To examine the variances, the Hartley Test is used (Appendix C). The differences in variance were tested within aircraft and within altitudes, and the results are shown in Table 3.

Table 3

HARTLY VALUES FOR VARIANCE

$H = \frac{\max(S^2_{ij})}{\min(S^2_{ij})}$	H @ 95%	H @ 99%
36	≥ 403	≥ 2063

e	f	Cum f	Z
-15.0	1	0.0625	-1.23
-12.5	1	0.125	-1.03
-10.0	1	0.1875	-0.82
-7.5	1	0.25	-0.61
-5.0	2	0.375	-0.41
-2.5	1	0.9375	-0.205
0.0	2	0.5625	0.00
2.5	1	0.625	0.205
5.0	2	0.75	0.41
7.5	1	0.8125	0.61
10.0	1	0.875	0.82
12.5	1	0.9375	1.03
15.0	1	1.000	1.23

$(\sigma = \sqrt{MSE} = 12.18)$

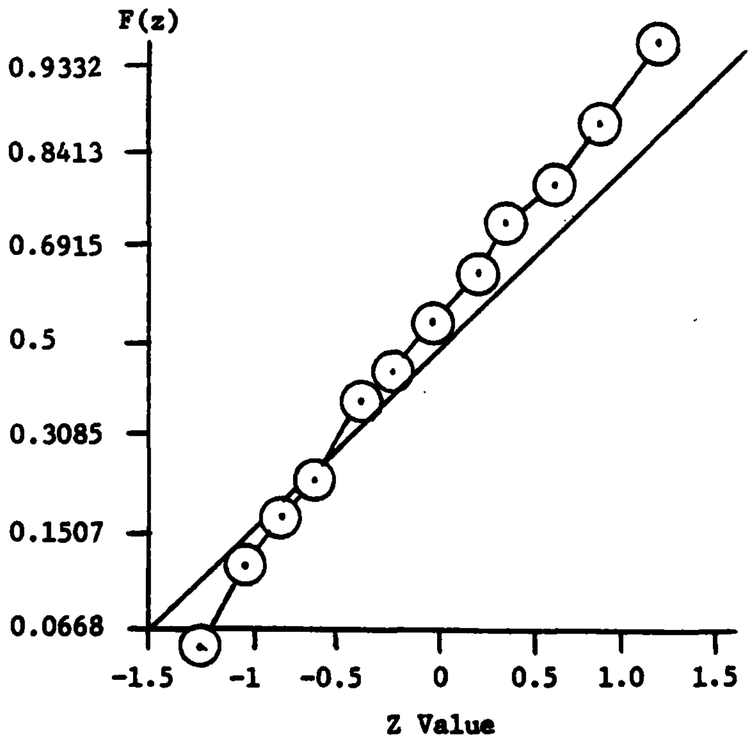


Figure 2

Normal Error Plot

This indicates that the null hypotheses, all variances being equal, cannot be rejected. Thus, it is concluded that the variance is constant.

This implies that the model chosen is adequate and that any transformation used to reduce the effect of interactions would destroy the normality of the data. Several transformations were attempted (Appendix D), and interactions were reduced by the transformation $1/Y^2$. However, Figure 3 demonstrates the loss of normality. Therefore, transformation of the data is not acceptable.

The specific affects for the factor levels were analyzed. As can be seen in Tables 4 and 5, a real difference to aircraft and altitude response is evident.

Table 4

AIRCRAFT SPECIFIC EFFECTS

Altitude Level	AAU-19 Aircraft 1	AAU-19 Aircraft 2	AAU-34 Aircraft 3	AAU-34 Aircraft 4
500	-30.625	-18.125	16.875	31.875
200	10.0	5.0	7.5	-22.5
Aircraft Main Effects	-10.3125	-6.5625	12.1875	4.6875

e	f	Cum f	Z
-1.975	1	0.0625	-1.786
-0.97	1	0.125	-0.88
-0.1942	1	0.1875	-0.175
-0.074	1	0.25	-0.067
-0.0197	2	0.4375	-0.0178
0.0	2	0.5626	0.0
+0.0197	2	0.6875	0.0178
+0.0272	1	0.75	0.025
+0.074	1	0.8125	0.067
+0.1942	1	0.875	0.175
+0.97	1	0.9375	0.88
+1.975	1	1.0000	1.786

($\sigma = \sqrt{MSE} = 1.106$)

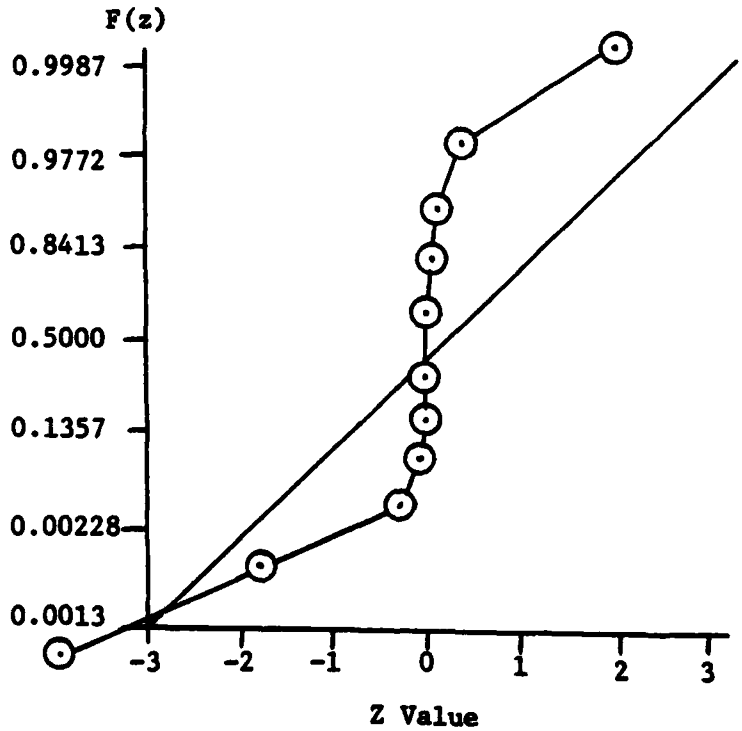


Figure 3

$^1/_{Y^2}$ Transformation Normal Plot

Table 5

ALTITUDE SPECIFIC EFFECTS

Altitude Level	AAU-19 Aircraft 1	AAU-19 Aircraft 2	AAU-34 Aircraft 3	AAU-34 Aircraft 4	Altitude Main Effects
500	-33.75	-25	-8.75	13.75	-13.4375
200	33.75	25	8.75	-13.75	13.4375

Plotting the cell means for the data (Figure 4) clearly demonstrates interaction. However, the figure does appear to show some differences between the types of altimeters within the altimetry systems.

In general, the main effects of Aircraft 1 and Aircraft 2 (the AAU-19-equipped aircraft) are below the mean response, and Aircraft 3 and Aircraft 4 (the AAU-34-equipped aircraft) are above the mean response. The altitude main effects are below the mean response at 500 feet and above the mean response at 200 feet altitude. But, the specific effects for aircraft demonstrate that the AAU-19-equipped aircraft read below the mean response at 500 feet and above the mean response at 200 feet.

The above analysis indicates that it may be more appropriate to group the aircraft by altimeter types rather than considering them as individuals. This was accomplished, and the aircraft system mean data plot (Figure 5) demonstrates interaction. The analysis of variance results (Table 6) at 99.9 percent demonstrate no significant main effects but significant interactions. Again, if the confidence level

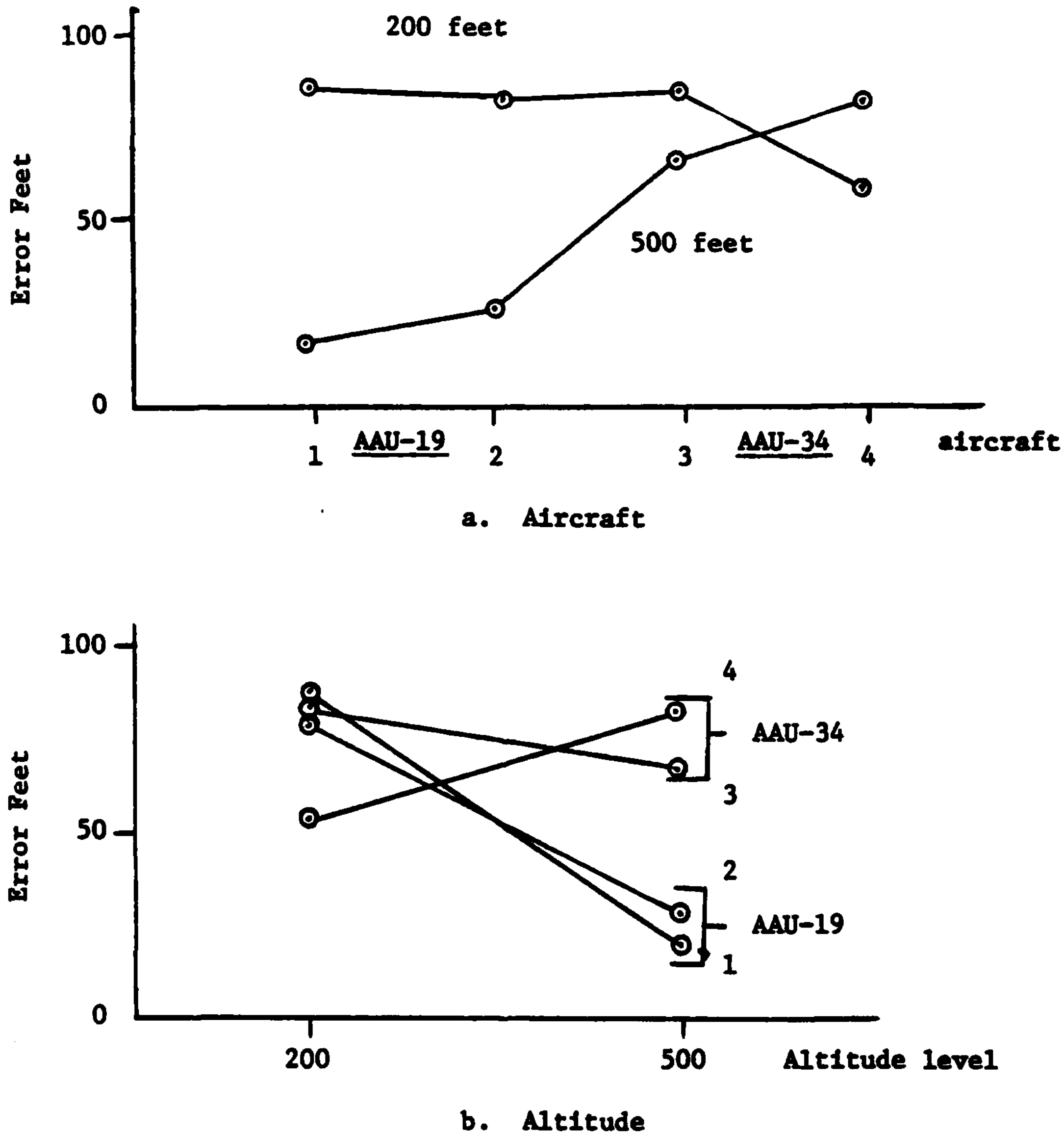
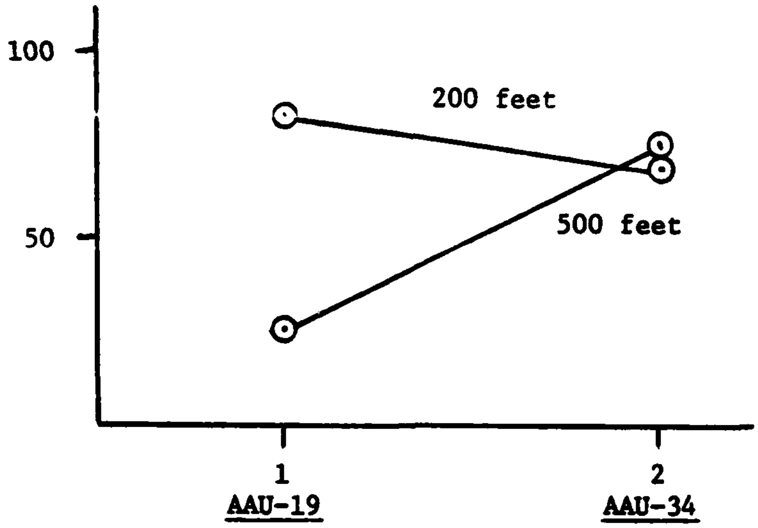
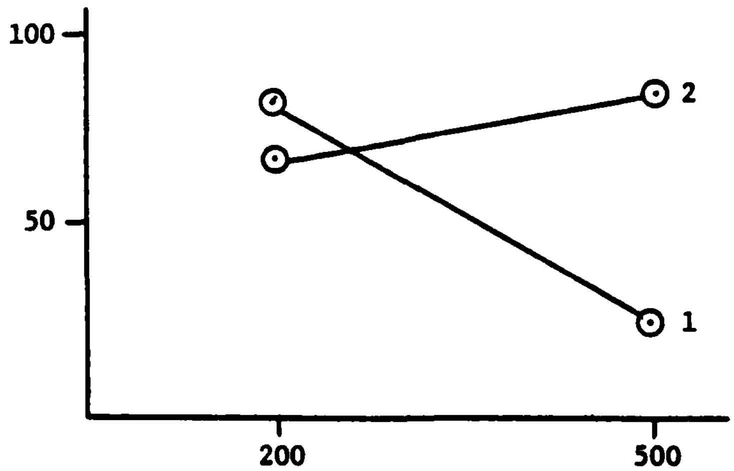


Figure 4

Cell Mean Data Plots



a. Aircraft



b. Altitude

Figure 5

Aircraft System Mean Data Plots

is dropped to 99 percent to remain consistent with the previous section, the same information is evident--significant interactions and insignificant factor effects (Appendix E for ANOVA table).

Looking at specific effects in the regrouped data (Tables 7 and 8), the same relationship is evident. The AAU-19-equipped aircraft read below the mean response or near it for both levels of altitude, and the AAU-34-equipped aircraft read above or near the mean response.

Table 6

ALTIMETER F TEST

Factor	F Value	F Test 99%	99.9%
α - Altitude	0.71	Not Significant	
β - Systems	5.48	9.33	18.6
αβ - Interaction	19.556	9.33	18.6

Table 7

AIRCRAFT SPECIFIC EFFECTS

Altitude Level	Aircraft AAU-19s	Aircraft AAU-34s
500	-24.375	24.375
200	7.5	- 7.5
Main Aircraft Effects	-8.4375	8.4375

Table 8

ALTITUDE SPECIFIC EFFECTS

Altitude Level	Aircraft AAU-19s	Aircraft AAU-34s	Main Altitude Effects
500	-29.375	2.5	-13.4375
200	29.375	-2.5	13.4375

Since transformation of the data is not feasible and interaction appears to be important, it must be concluded that there is a definite possibility of interaction between altimetry systems and altitude. This is an opposite effect from the one that is expected, especially at such small changes in altitude. Only two reasonable explanations for the interaction term are available, either the interactions are due to the changes in the local pressure field (barometric) conditions about the aircraft or the test method used for obtaining the data allows too much variation from absolute levels.

Since the interaction term cannot be discounted but can be questioned as to significance, the author judges the interactions to be significant. This implies that the effects of the factors involved cannot be discussed in terms of the factor level means. However, based on the F tests, it is concluded that there is not a significant difference between altimetry systems and performance of systems at different altitudes. It is realized that to substantiate this claim would require similar results from repeated experiments.

3. SYSTEM COMPONENT EFFECTS

The altimetry system is broken down into its components (sensor, transducer, and display) as shown in Figure 1. These components will be discussed and analyzed individually in the following paragraphs.

a. Sensor

The sensor is an aerodynamically-compensated Pitot-static tube. The tube senses rampressure through the Pitot opening and static (ambient) atmospheric conditions through the static ports. The data analyzed come from the development flight test performed at Edwards Air Force Base, California, on the production wing tip boom Pitot-static tube (Appendix F). The data (39 points) are represented in terms of the following parameters:

M = Mach number

$\frac{\Delta P}{qc}$ = pressure ratio

where: $\Delta P = Psi \quad Ps$

Psi = indicated static pressure

Ps = true static pressure as given in standard atmospheric tables

qc = true dynamic pressure

The statistical technique applied was regression analysis (Reference 12) fitting pressure ratios to Mach number. The significance level is $\alpha = 0.05$. The results demonstrate no significant regression relationship (Appendix G). This implies that the mean of the data is an adequate predictor of the pressure ratio for any Mach number. Thus, the mean and standard deviation are $(\frac{\Delta P}{qc} = x)$:

$$\bar{x} = -0.00185$$

$$Sx = 0.00548$$

(NOTE: Data are presented as a correction to be added to the altitude displayed, i.e., a minus implies the altitude displayed will be higher than the aircraft actually is.)

The airspeeds of interest are from 275 knots to 330 knots. To calculate the error in feet requires use of the equation:

$$\Delta H = \frac{\Delta P}{k}$$

where: $\Delta P = \bar{x}\ qc;\ Sx\ qc$

ΔH is error in feet

k is the conversion constant and equals 0.00108 in Hg/ft. (Reference 5)

qc values are obtained as a function of airspeed from the tables in Reference 5.

Table 9 gives some examples of the error expected for selected airspeeds using the computed mean and standard deviation. Thus, selecting 300 knots as a representative airspeed, the error expected from the specification (Reference 9) at 300 knots is from 65 feet to a minus 80 feet negative correction. However, the error demonstrated by the data (Appendix F) is biased in the negative direction with a mean of -7.77 feet and a standard deviation of 23 feet. A 99 percent confidence interval for the mean of the Pitot-static tube at 300 knots is:

μ_1: -7.77 ± 9.98 feet

(Note: Computed by same method as used in Random Effect section, numbers rounded) (Appendix H).

Table 9

PITOT-STATIC ERROR

Airspeed	Mach Number	ΔH Feet	Sx Feet
280	0.427	-6.72	19.9
290	0.441	-7.23	21.4
300	0.457	-7.77	23.0
310	0.471	-8.32	24.1
320	0.488	-8.9	26.4
330	0.51	-9.45	28.1

b. Transducer

The altitude computer receives the input from the sensor and applies the standard air data equations to compute an output altitude. The data analyzed are production acceptance test data for down-scale error of 22 computers. Only the points for 500 and 0 feet were used (Appendix I). The average error is a positive correction of approximately 6 feet with an approximate standard deviation of 9 feet. A 99 percent confidence interval for the mean of the altitude computers is computed as the above interval for the sensor:

μ_2: -6.25 ± 3.66 feet

c. Display

The altimeter receives its input from the transducer and displays this information by a dial, pointer, and counter to the pilot. The data analyzed are acceptance test production data for down-scale error of 30 altimeters. Only points for 300 feet and below were used since, again, this is the area of interest. An additional limitation of using only the 25°C test data was used since the cockpit is environmentally maintained. The computed mean is 5.6 feet negative correction with a standard deviation of 11 feet. A 99 percent confidence interval for the mean of the altimeters is:

$$\mu_3\text{:}\ \ 5.6 \pm 2.71 \text{ feet}$$

An additional error to account for is baroset. This is a mechanically-induced error which accounts for the local barometric conditions. Since no data were available and all altimeters must pass the acceptance test, the assumed confidence interval is the specification tolerance of ±15 feet. Telephone conversations with the vendors indicate the error to be only 5 feet to 10 feet, but the tolerance of ±15 feet will be used, to be conservative. Thus, the confidence interval for the mean of the baroset is:

$$\mu_4\text{:}\ \ 0 \pm 15 \text{ feet}$$

d. System

The intent of the analysis in this section is to determine an aggregate confidence interval for the mean of the systems in the A-10 fleet. This interval will be compared to the mean of the flight test data.

Due to the large standard deviations, it is realized that the components will vary to the extremes of the individual specification tolerances. However, the assumption of randomness for the A-10 fleet will permit the use of the computed confidence intervals to determine if the bias previously noted is internal or external. Since each component is the input to the next component, arriving at an overall confidence interval requires an additive process as follows:

Pitot-static interval: μ_1: -2.77 ± 9.98 feet

Altitude Computer Interval: μ_2: -6.2 ± 3.66 feet

Altimeter interval: μ_3: 5.6 ± 2.71 feet

Baroset interval: μ_4: 0 ± 15 feet

Altimetry system interval:

$\mu_o = \mu_1 + \mu_2 + \mu_3 + \mu_4$:

μ_o: -8.42 ± 18.58

where the $\pm$ value was computed by the root sum square technique:

$$3\sigma = \sqrt{(9.98)^2 + (3.66)^2 + (2.71)^2 + (15)^2}$$

Therefore, a 99 percent confidence interval for the mean of the altimetry system is:

$$-27 \leq \mu_o \leq 10.16 \text{ correction to be added}$$

Switching the signs of the interval to obtain an interval showing the error expected instead of correction to be added as presented in the first section, the following interval results:

$$-10 < \mu_o < 27 \text{ - altimetry system}$$

(NOTE: Numbers are rounded)

As the interval demonstrates, there is a positive, higher-than-actual altitude bias in the altimetry system. However, the confidence interval for the altimetry system components does not include the mean (62 feet) of the flight test data. This leads one to speculate that the system components do not account for all the error demonstrated by the flight test data, that additional variables may need to be analyzed, or that the test method of formation flying may be too variable in nature.

One final point should be noted. Based on knowledge of the current state of the art in barometric air data systems, the individual components are the state of the art, and the tolerances cannot be improved without substantial redevelopment and cost.

4. HEAD-UP DISPLAY

Since the head-up display is an optical display in the forward part of the canopy and uses the same sensor and transducer, the only analysis made will be a comparison to determine if the head-up display is displaying altitude information significantly greater in error than the altimeter. The statistics of interest, the the mean and standard deviation, are computed as follows:

$$\bar{z} = 43.75 \text{ feet}$$

$$SX = 34.286 \text{ feet}$$

(NOTE: Last set of aircraft data, Appendix A, is deleted as it is suspect for the head-up display only.)

The test conducted at a confidence level of $\alpha = 0.01$ (99%) is as follows:

Base Hypothesis H_o: $\mu \leq 62$ feet

Alternate Hypothesis H_1: $\mu < 62$ feet

Where: 62 feet is the mean of the altimeter data.

The computed t value is:

$$t = \frac{z-\mu}{Sz}\sqrt{n} = \frac{43.75-62}{34.286}\sqrt{16}$$

$$t = -2.129$$

The test value is: $t_t(99.5,11) = 2.718$ (Reference 19)

Since the absolute value of $t < t_t$, it is concluded that the head-up display mean altitude is not significantly greater in error than the altimeter.

5. GROUND PROXIMITY ERROR

Exactly at what altitude ground proximity affects the altimetry system is not known, but current practice indicates 1 1/2 to 2 wingspans or in this case, 87 feet to 116 feet altitude above ground level. The only data available on the A-10 are development flight test data (Figure 6) (Reference 1). As observed, there are relatively few data points and none for the airspeed range of interest. The author extrapolated the curve as shown. It is realized that this is extremely crude, but it does indicate that the error increases as airspeed increases. Assuming the data and extrapolation to reveal somewhat the true nature of the altimetry

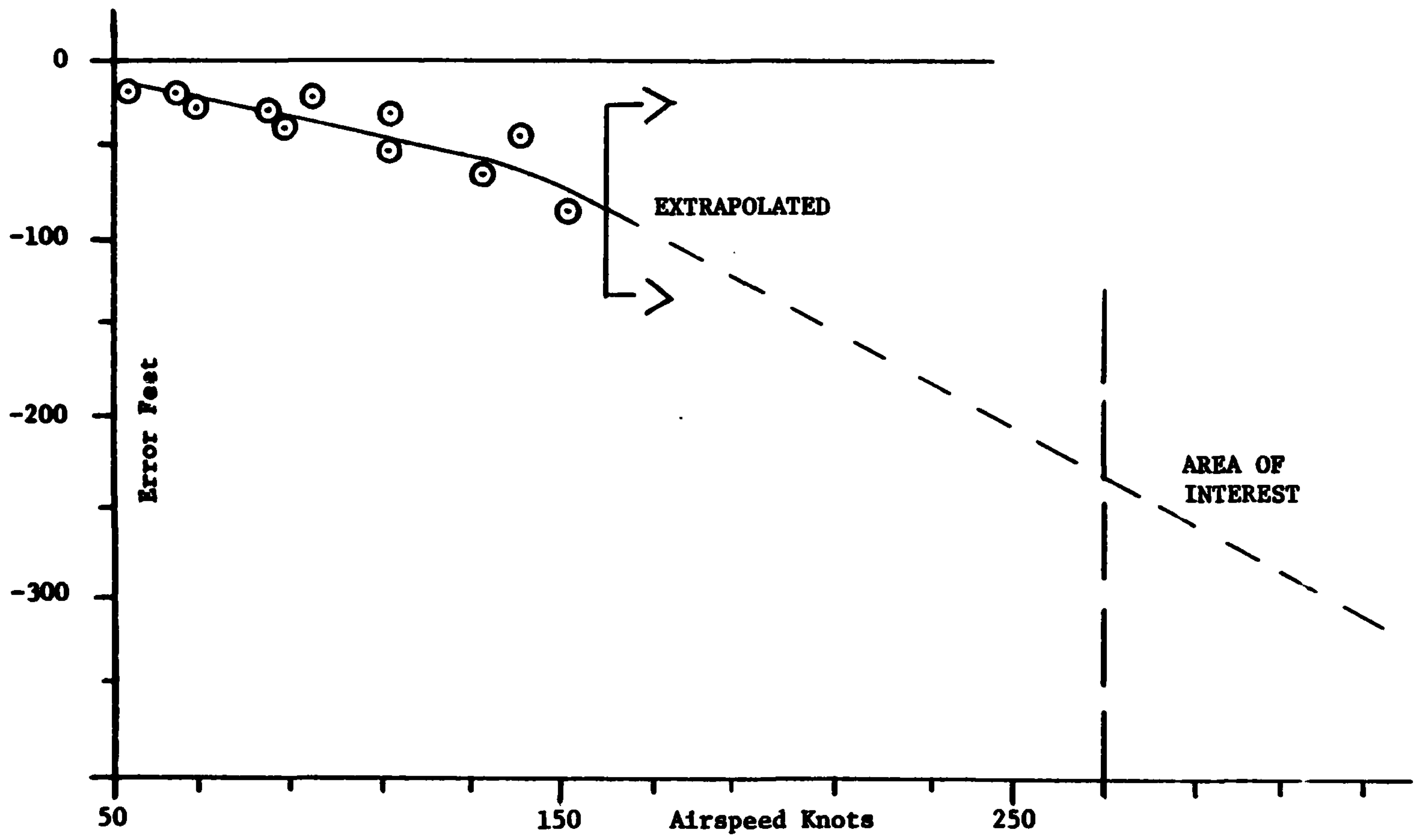


Figure 6

A-10 USAF S/N 73-01644
Position Error Ground Proximity

systems at low altitude, one can hypothesize a plausible explanation for the large errors being reported. The systems are reporting altitude with an average error of approximately a positive 60 feet. This error becomes an increasingly small percent of altitude as altitude increases. However, at low altitudes this is an extremely large error (i.e., at 100 feet altitude ground level, it is 60 percent). It seems reasonable that a pilot could read 160 feet on his altimeter but actually be near 100 feet, dip lower to what he perceives as 100 feet altitude ground level, and then observe large altimetry system errors due to ground and Mach compressibility effects.

SECTION V

CONCLUSIONS AND RECOMMENDATIONS

1. CONCLUSIONS

Based upon the analysis, the following conclusions are drawn:

1. It is possible, but extremely unlikely, that the A-10 altimetry systems are reporting errors greater than 300 feet positive without external influence.

2. The errors demonstrated do indicate a positive bias, and the mean error appears to be in the range of 42 feet to 82 feet for the A-10 fleet.

3. There is no significant difference between altimetry systems or altitude. There is a very strong probability of interaction between systems and altitude.

4. System components do indicate, as an aggregate, some positive bias but do not account for the bias observed in the data. The errors present cannot be reduced without costly development.

5. The head-up display is not significantly greater in error than the altimeter.

6. Extrapolation of the ground proximity data indicates that large errors (100 feet to 400 feet) could be induced into the altimetry system if the aircraft is in ground effects.

7. Although the field-reported errors could not be substantiated, the results have shown the altimetry system to be inadequate

for reporting accurate (±20 feet) altitude for the A-10's mission of close ground support.

2. RECOMMENDATIONS

Based upon the conclusions drawn from the results of the analysis, the following recommendations are offered:

1. Accomplish a follow-on test or study to accurately define and quantify the altimetry system error and publish this information in the applicable technical order.

2. Define a pull-up altitude and provide a radar altimeter. If the minimum altitude is broken, immediate recovery of the aircraft is required.

3. Provide an altitude alerting system that, for a preset altitude and various combinations of airspeed, dive angle, and roll angle, will alert the pilot to pull up in a definite positive manner.

4. Provide either 2 or 3 above to train pilots only.

The author recommends either of the following. The choice will depend on the user's requirements and available funds.

1. Accomplish a follow-on test in accordance with the test designed in Section VI. If the error cannot be quantified to predict the error within ±20 feet of actual altitude at 99 percent confidence, the next alternative should be considered. Automatic aural and visual warnings also should be provided.

2. The altitude alerting system described in 3 above should be accomplished.

SECTION VI

FOLLOW-ON FLIGHT TEST

If the alternative to quantify the altitude error is selected, another flight test is required. In order to minimize the risk of obtaining unwanted interactions, the conditions assumed to contribute to interaction must be controlled. The barometric conditions must be accurately obtained, and the variability associated with wings-level flight must be tightly controlled. Several considerations for the test must be made to insure that meaningful data are obtained.

The goal of the flight test is to estimate the altimetry system bias for the A-10 fleet for altitudes below 500 feet with reasonable precision and to determine if there is interaction between A-10 altimetry systems and altitude levels. The precision required to be a useful measure to the pilot is ±20 feet about the grand mean over all aircraft altimetry systems and altitude levels. However, if it appears that there are different biases associated with different altitude levels, then the bias at each level must have a precision of ±20 feet over all aircraft at that level. Of additional consideration are the effects of airspeed, ground proximity, and possible interaction.

The test design will be the analysis of variance mixed-effects model. The aircraft (Factor B) will be selected at random, but the altitude levels (Factor A) and airspeeds (Factor C) will be preselected. Preselecting altitude levels will allow one to choose "easy (physically able)-to-fly" altitudes above ground level. Due to consistency in local barometric conditions from 0 to 500 feet, if the variation in the altitude mean holds to ±20 feet, it will be safe to assume that the altitude levels in between the preselected altitude levels will react similarly. The upper altitude ground level limit is fixed at 500 feet because measurement accuracy of actual aircraft height in feet above ground degrades as altitude increases. Reasonable measurement accuracy is ±2 percent of altitude. It is believed that airspeed will have little or no affect upon the altitude bias. However, to confirm this and thus increase the applicability of the obtained bias over a range of airspeeds, this variable is introduced. Again, preselecting airspeeds will allow one to choose "easy-to-fly" airspeeds. The upper and lower limits will be selected based upon the most commonly flown airspeeds at low altitude levels. The model to be employed is in Appendix J.

1. ESTIMATION OF PARAMETERS

The first parameters to be identified are the preselected ones (Table 10) of altitude levels and airspeeds. The following levels are selected because they will be easy to read on the instruments and, hopefully, easy to maintain for a period of time. The 50-foot point is added to obtain information concerning ground proximity and to correlate the previous conclusions concerning ground proximity. It may be, in the final analysis, that the data for the 50-foot level may have to be deleted.

Table 10

PRESELECTED VALUES

Altitude Levels (Feet)	Airspeeds (Knots)
50	200
100	250
200	300
300	350
400	
500	

With the above parameters selected, one needs to estimate the number of aircraft altimetry systems (Factor B) and the number of replications (n) at each airspeed and altitude level to obtain the necessary precision. Also, the possibility of interaction must be considered. Specifically, different ways to arrive at the above numbers will be employed. The method that affords the best set of parameters will be used. The confidence coefficient established is 99 percent.

1. The first method looks at the precision about the grand mean of the data. Ideally, it is hoped that the grand mean can be obtained with a precision of ±20 feet. An estimate of the variance associated with the grand mean is the mean squared error (Appendix B) from the previous flight test. The estimation is a 99 percent confidence interval as follows:

$$\bar{Y}_{....} - 20 < \mu_{...} < \bar{Y}_{....} + 20 \text{ feet}$$

$$\bar{Y}_{....} - t\sigma_1 < \mu_{...} < \bar{Y}_{....} + t\sigma_1$$

where: $\sigma^2_1 = \frac{MSE}{abcn} = \frac{148.44}{(6)b(4)n} = \frac{6.185}{bn}$

$$t = t(99.5,[n-1]abc)$$

$$t = t(99.5,[n-1]24b)$$

NOTE: This method assumes that there are no significant main effects or interactions and that the data come from a normal population.

Let b = n as a first estimate:

$$t\sigma_1 = t\left(\frac{6.185}{n^2}\right)^{1/2} = 20$$

$$n = \frac{t}{20}(6.185)^{1/2}$$

$$n = 0.124(t)$$

After several trials: $n \approx 2$

Let b = 5 as a second estimate:

$$\sigma^2_1 = \frac{6.185}{5n} = \frac{1.237}{n}$$

$$t = t(99.5,[n-1]120)$$

$$n = 1.237\left(\frac{t}{20}\right)^{1/2}$$

Again, $n \approx 2$

It appears that for almost any selection of b, $n \approx 2$. Therefore, for economics in test costs, let b = 2, n = 2.

2. The second method looks at the precision required (±20 feet) for altitude levels if altitude effects are significant. A simultaneous

comparison using the Bonferroni approach will be made for each altitude level. Since the altitude levels are fixed, the estimation is as follows with the variance estimated by MSAB (Appendices B and J):

$$[Y_{i\ldots} - B\sigma_2 \le \mu_{i..} \le Y_{i\ldots} + B\sigma_2]_{i=1}^{6}$$

$$\sigma_2^{\,2} \approx \frac{MSAB}{bcn} = \frac{1743.23}{4bn} = \frac{435.81}{bn}$$

$$B = t(1 - \alpha/2s;\ [a-1][b-1])$$

$$B = t(1 - 0.01/2[6];\ 5(b-1)$$

$$B = t(0.999;\ 5(b-1)$$

Then:

$$B\sigma_2 = 20$$

$$t(\frac{435.81}{bn})^{1/2} = 20$$

$$bn = t^2\ \frac{435.81}{400} = t^2\ 1.09$$

Let $b = n$; $n = t(1.02)^{1/2} = 1.044t$

This leads to $n \approx 4$

This is a pretty good estimate. Therefore, for the second method, $b = n = 4$.

3. The third approach involves estimation and detection of interactions. Specifically, the question is for what values of b and n will the experiment be able to detect interactions at a high probability with a confidence coefficient of 95 percent.

Of particular interest is the interaction between the A-10 altimetry systems and the altitude levels--Factors A and B. Also, to verify that airspeed has no adverse effects on A-10 altimetry system bias, attention will be paid to this interaction--Factors B and C. Of additional interest for adverse effects on altimetry system factor is the three-way interaction of airspeed, altitude levels, and altimetry systems--Factors A, B, and C. The interaction of airspeed and altitude levels is, for practical purposes, assumed to be insignificant, a priori.

The power approach suggested by Duncan (Reference 4) will be employed. This approach estimates the noncentrality parameter phi (ϕ) with β fixed at 0.1 and an α of 0.05. These values were previously established as follows:

For the AB interaction:

$$\phi_1^{\ 2} = \frac{\sigma^2 + nc\sigma^2_{\alpha\beta}}{\sigma^2}$$

For the BC interaction:

$$\phi_2^{\ 2} = \frac{\phi^2 + na\sigma^2_{\beta\gamma}}{\sigma^2}$$

For the ABC interaction:

$$\phi_3^{\ 2} = \frac{\sigma^2 + n\sigma^2_{\alpha\beta\gamma}}{\sigma^2}$$

The experiment is to be designed to detect the above interactions for an F ratio equal to or greater than four. Thus, estimating the above variance components by the appropriate mean squares, the approximate phi's are as follows:

$$\phi_1^{\ 2} \approx \frac{MSAB}{MSE}$$

$$\phi_2^{\ 2} \approx \frac{MSBC}{MSE}$$

$$\phi_3^{\ 2} \approx \frac{MSABC}{MSE}$$

Substituting the F value of four for detection leads to:

$$\phi_1^{\ 2} = \phi_2^{\ 2} = \phi_3^{\ 2} = 4$$

For the interactions of interest, Duncan's Table L (Reference 8) reveals the following values for b and n where these values are obtained from the degrees of freedom as follows:

For a first estimate let:

$$\nu_1 = 15$$

$$\nu_2 = 20$$

For the AB interaction:

$$\nu_1 = (a-1)(b-1)$$

$$15 = (6-1)(b-1)$$

$$b = 3$$

$$\nu_2 = (n-1)abc$$

$$20 = (n-1)(6)(3)(4)$$

Therefore: $n \approx 1$

For the BC interaction:

$$\nu_1 = (b-1)(c-1)$$

$$15 = (B-1)(4-1)$$

$$b = 6$$

Again, $n \approx 1$

For the ABC interaction:

$$\nu_1 = (a-1)(b-1)(c-1)$$

$$15 = (6-1)(b-1)(4-1)$$

$$b = 2$$

Again $n \approx 1$

It is readily apparent that the driving interaction for the choice of b is the BC interaction.

For a second estimation let:

$$\nu_1 = 10$$

$$\nu_2 = 120$$

For the BC interaction:

$$b = \frac{10}{3} + 1 \approx 4$$

$$n = \frac{120}{96} + 1 \approx 2$$

Thus, for detection of the interactions, AB, BC, and ABC for $\beta \leq 0.1$ and $\alpha = 0.05$, the values of $b = 4$ and $n = 2$ are sufficient.

4. From the previous flight test, no altimetry system effects appeared to be prominent and airspeed effects are prejudged to be minimal. Therefore, the numbers for b and n to provide adequate precision come from Approach 2 and are: $b = 4$, $n = 4$.

2. PARAMETERS

With the mixed-effects model selected and the number of aircraft, altitude levels, and replications determined, the following parameters are established:

1. Number of aircraft is 4.
2. Number of altitude levels is 6.
3. Number of replications (passes) per altitude level is 4.
4. Number of airspeeds selected is 4.
5. The altitude levels are: (in feet)
   - A. 500
   - B. 400
   - C. 300
   - D. 200
   - E. 100
   - F. 50

6. The airspeeds are: (in knots)

A. 200

B. 250

C. 300

D. 350

7. The tail numbers for the four aircraft should be selected randomly. A suggested method is to employ a random number table.

3. PROCEDURES

Although the tail numbers will be selected randomly, the order of experimentation (i.e., which aircraft flies first, second, etc.) should also be randomized to protect against systematic error.

The aircraft's height-above-ground determination should be accomplished by the tower fly-by method. Both radar tracking and the phototheadolite methods should be used to determine the aircraft's actual height. If only one method can be used or if large discrepancies between the two methods occur, the phototheadolite method is preferred.

Barometric readings should be taken prior to each aircraft pass. This information should be radioed to the pilot, and the pilot should enter this in the altimeter's baroset. This will minimize errors due to fluctuating barometric conditions.

A precheck of each aircraft by maintenance personnel should be performed to insure that the altimetry system meets the overall tolerance of ±45 feet for low altitude settings in the servo mode.

A field elevation check at a known surveyed point should be taken and recorded just prior to takeoff.

Both the head-up display and the altimeter should be read and recorded at the same time that the actual height is being read and recorded.

The aircraft must fly at the altitude level at a wings-level attitude for at least two seconds before any readings are taken. This is to eliminate altimetry lag.

4. ANALYSIS OF THE DATA

After the data are collected, an analysis of variance using the mixed-effects model in Appendix J should be performed (Figure 7).

		$\underline{C_1}$	$\underline{C_2}$	$\underline{C_4}$
		B_1 B_2 B_3 B_4	- - - -	- - - -
A_1	N_1	- - -		
	N_2	- - -		
	N_3	- - -		
	N_4	- - -		
A_6	N_1	- - -		
	N_2	- - -		
	N_3	- - -		
	N_4	- - -		

Figure 7

Analysis of Variance Design

If there are no significant interactions, altimetry system or altitude effects, and the analysis falls within the predetermined confidence interval for the grand mean, the grand mean should be published as the altimetry system bias for altitudes below 500 feet for airspeeds between 200 and 350 knots.

If there are significant altitude effects but no significant altimetry system effects, each altitude bias should be determined and published separately for airspeeds of 200 to 350 knots at intervals of 0 to 100, 100 to 200, 300 to 400, and 400 to 500 feet. It may be preferable to make a chart rather than a table for this instance.

If there are significant altimetry system and interaction effects, significant interaction effects, or the data scatter is large, i.e., more than 25 percent of the data points exceed the ±20 feet confidence interval established for the means, it will be impossible to generalize for the fleet. In these instances, it is recommended that the second preferred alternative as defined in Section V be pursued.

SECTION VII

APPLICATION TO OTHER AIRCRAFT

1. INTRODUCTION

The applicability of performing an analysis to define a bias in the aircraft's altimetry system is dependent on the mission. If the mission is a noncombat one, the value of an analysis of this type is probably not cost effective. However, if the mission is combative, especially bombing or low-level air support, the knowledge of exactly what the bias is could mean the difference between a successful and a nonsuccessful mission.

2. REQUIRED PARAMETER ESTIMATION

To develop a flight test to aid in pinpointing areas (i.e., Pitot-static tubes, air data computer, altimeters, etc.) for improvement, one needs to estimate the mean altimetry system bias and the standard deviation of this bias. An analysis of variance at this point, considering altitude, airspeeds, and aircraft as main factors, can be made to determine the mean squares of the standard deviation and interactions. These estimations can then be used to estimate the parameters for a follow-on flight test at some chosen confidence level.

3. ESTIMATION OF PARAMETERS

Once the required estimation of a mean, standard deviation, and interaction mean squares has been obtained, the parameters for the follow-on flight test at some chosen confidence level can be estimated. Here, also, time and flight test cost considerations will play a part. One should attempt the estimation by looking at the precision required in the particular case about the grand mean, the altitude levels, and the interactions (Section VI.1). The different altitude levels and airspeeds should be chosen in a manner that will span the critical altitudes and airspeeds in which the investigator is interested. The parameters can be chosen for convenience, as was done in Section VI.

4. ANALYSIS

With the parameters estimated and the flight test completed, an analysis of variance considering the parameters of interest should be performed. Ideally, one hopes for main effects so that efforts in those areas can be made to reduce the error. However, if there are significant interactions, there is a dependence in those particular factors that may require a test redesign or reanalysis to make interactions nonsignificant. The importance of having nonsignificant interactions is to eliminate any masking effects that interactions may have on the main effects.

5. VALUE OF INFORMATION

Once one is able to determine the main effects, in this case airspeed, altitude, and aircraft, specific areas can be observed in more detail.

For instance, if there are aircraft effects, the production methods used may require reevaluation. There may be a discrepancy in the installation of Pitot-static booms or the routing of Pitot-static lines. If there are altitude eff cts, one may wish to investigate the design of the static ports on the Pitot tube, the nature of error in the central air data computer, or the altimeter. If airspeed is a factor, the Pitot port on the Pitot tube may be at fault or the Pitot lines may be distorting the pressure with resonant pressure pulses in areas of changing volume.

There also may be some grand average effect that would allow the publishing of an altitude or airspeed bias to offset a true bias. Also, one should check his results after an aircraft has been in the field for some time. This will enable one to determine any trendable age effects.

As demonstrated, the statistical technique of analysis of variance is a powerful tool to use in actual design or in designing a test. It helps pinpoint problem areas with some level of confidence and is limited in use only by the user.

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APPENDIX A

Flight Test Data

	Nominal ALTITUDE	A-10 True ALTITUDE	A-10 Indicated ALT HUD	HUD Indication DEV	A-10 Indicated ALT CKPIT	CKPT ALT Indication DEV
A-10	500'	550'	565'	+15'	565'	+15'
75-0288	500'	490'	505'	+15'	510'	+20'
AAU-19/A	200'	210'	295'	+85'	295'	+85'
Altimeter	200'	210'	295'	+85'	295'	+85'
	Field Elevation Check		HUD Indication Error +15' CKPT Altimeter Error +15'			
A-10	500'	550'	595'	+45'	595'	+45'
75-0268	500'	490'	495'	+05'	505'	+15'
AAU-19/A	200'	210'	295'	+85'	285'	+75'
Altimeter	200'	210'	305'	+95'	295'	+85'
	Field Elevation Check		HUD Indication error +15' CKPT Altimeter error +15'			

APPENDIX A (Cont'd)

	Nominal ALTITUDE	A-10 True ALTITUDE	A-10 Indicated ALT HUD	HUD Indication DEV	A-10 Indicated ALT CKPIT	CKPT ALT Indication DEV
A-10	500'	400'	395'	-05'	475'	+75'
77-0225	500'	420'	435'	+15'	475'	+55'
AAU-34/A	200'	105'	145'	+40'	155'	+90'
Altimeter	200'	130'	175'	+45'	205'	+75'
	Field Elevation Check		HUD Indication error -10' CKPT Altimeter error +15'			
A-10	500'	400'	295'	-105'	485'	+85'
77-0223	500'	420'	345'	-75'	495'	+75'
AAU-34/A	200'	105'	45'	-60'	145'	+40'
Altimeter	200'	130'	95'	-35'	195'	+65'

Pneumatic Set on Altimeter

Field Elevation Check

HUD Indication error -105'
CKPT Altimeter error +35'

ALT – $\bar{x} = 61.5625'$, $Sx = 26.5600'$ all data (error) as sample

HUD – $\bar{z} = 15.6250$, $Sz = 60.3566$ all data (error) as sample

ALT – $\bar{x} = 60$, $Sx = 27.914$

HUD – $\bar{z} = 43.75$, $Sz = 34.286$

Last aircraft, 77-0223, omitted as the HUD data, which is large in the negative direction, is suspect.

APPENDIX B

ANOVA Table

BMD02V, Analysis of Variance for Factorial Design. Revised February 3, 1972, Health Sciences Computing Facility, UCLA, California.

ANOVA Table (Excerpt from Computer Run)

Source of Variation	Degrees of Freedom	Sums of Squares	Mean Squares	F Ratio Corrected	F Test 99%	F Test 99.9%
α - altitude	1	2889.0625	2889.0625	1.6573		
β - systems	3	1279.6875	426.5625	2.87368	34.10	167.
αβ - interaction	3	5229.6875	1743.22917	11.74386	7.59	15.8
Within Replicates	8	1187.5	148.4375			
TOTAL	15	10,585.9375				

Table of Means

Altitude	Aircraft 1	Aircraft 2	Aircraft 3	Aircraft 4	Row Average
1	17.5	30.0	65.0	80.0	48.125
2	65.0	80.0	82.5	52.5	75.00
Column Average	51.25	55.0	73.75	66.25	61.5625 Grand Averages

APPENDIX C

Hartley Test

	Variances	Aircraft		
Altitude	$S^2_{11} = 25$	$S^2_{12} = 900$	$S^2_{13} = 400$	$S^2_{14} = 100$
	$S^2_{11} = 0$	$S^2_{22} = 100$	$S^2_{23} = 225$	$S^2_{24} = 400$

<u>Aircraft and Altitude</u>

$$H = \frac{\max(S^2_{ij})}{\min(S^2_{ij})} = \frac{900}{25} = 36 \qquad H(0.99,8,2) \geq 2063$$
$$H(0.95,8,2) \geq 403$$

Therefore: you cannot say variances are nonconstant.

APPENDIX D

Data Transformations

$\sqrt{Y}$	$\frac{1}{Y}$	Log Y	$\frac{1}{Y^2}$
3.870	0.067	1.176	0.004444444
6.708	0.022	1.653	0.000493827
8.666	0.013	1.875	0.000177777
9.219	0.012	1.929	0.000138408
8.666	0.013	1.875	0.000177777
9.487	0.011	1.954	0.000123456
6.324	0.025	1.602	0.000625
4.472	0.05	1.301	0.0025
3.87	0.067	1.176	0.004444444
7.416	0.018	1.74	0.000330578
8.666	0.013	1.875	0.000177777
9.219	0.012	1.929	0.000138403
9.219	0.012	1.929	0.000138408
8.666	0.013	1.875	0.000177777
8.062	0.015	1.813	0.000235585

ANOVA Table

Sources of Variation	Degrees of Freedom	Sums of Squares	Mean Squares	F Ratio Corrected	F Test 90%
α - altitude	1	7.49573	7.49573	2.4855	5.54
β - system	3	7.37571	2.45857	2.01024	2.92
$\alpha\beta$ - interaction	3	7.04462	8.01487	2.46509	2.92
Within Replicates	8	9.78421	1.22303		
Total	15	33.70023			

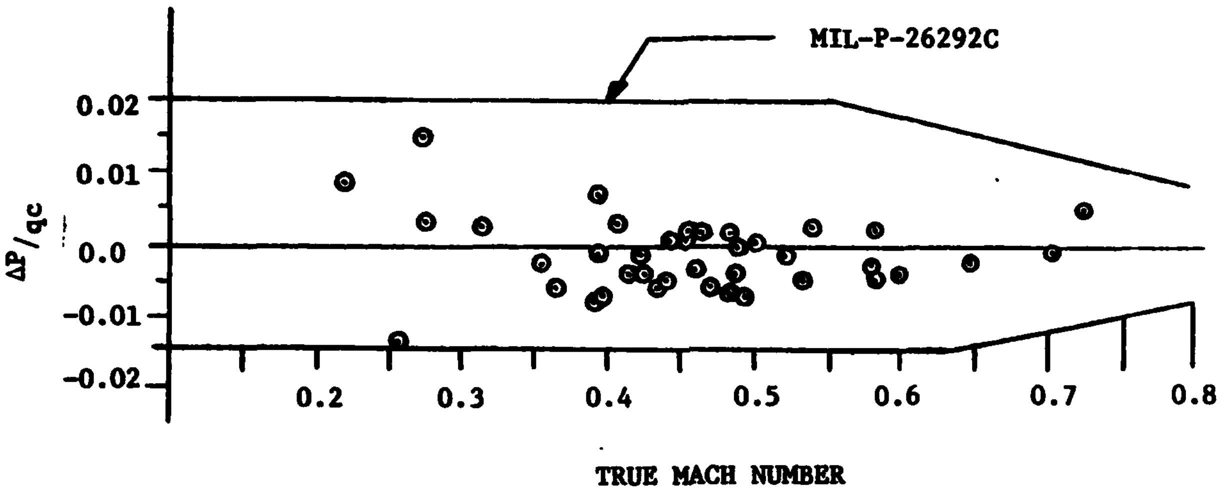
APPENDIX E

ANOVA Table

Altimeter Grouped Data

Source of Variation	Degrees of Freedom	Sums of Squares	Mean Squares	F Ratio Corrected	F Test 99%	F Test 99.9%
α - altitude	1	2889.0625	2889.0625	13.90226	405,280	4052
β - systems	1	1139.0625	1139.0625	5.4812	9.33	18.6
αβ - interaction	1	4064.0625	4064.0625	19.55639	9.33	18.6
Within Replicates	12	2493.75	207.8125			
TOTAL	15	10,585.9375				

APPENDIX F



A-10A USAF S/N 73-01664
Production Wing Boom Static Pressure
Error (Modified Head P/N 855DU-2)
Pacer and Nose Boom Referenced Data
Cruise Configuration

APPENDIX G

Regression Analysis Wing Boom

BMD02R, Stepwise Regression, Revised, November 27, 1972, Health Sciences Computing Facility, UCLA, California (Excerpts)

Variables: $\frac{\Delta P}{qc}$ - pressure ratio, dependent; M - Mach number, independent

Step 1: $\frac{\Delta P}{qc} = 0.0023 - 0.0091M$

ANOVA Table

(Data transformed by 10^{-3} for $\frac{\Delta P}{qc}$, 10^{-1} for M)

Sources of Variation	Degrees of Freedom	Sums of Squares	Mean Squares	F Ratio	F Test 95%
M	1	40.296	40.296	1.357	4.13
Error	37	1099.121	29.706		
Total	38	1138.417			

Step 2: $\frac{\Delta P}{qc} = 0.021 - 0.093M + 0.009M^2$

ANOVA Table
(Same transformation as Step 1)

Sources of Variation	Degrees of Freedom	Sums of Squares	Mean Squares	F Ratio	F Test 95%
M, M^2	2	143.452	71.726	2.593	3.28
Error	36	995.965	27.666		
Total	38	1138.417			

Conclusion: Inclusion of M, M^2, or M^3 is not significant at the 95 percent level; therefore, the mean is an adequate predictor for the data. The statistics are ($\frac{\Delta P}{qc} = x$): $\bar{x} = 0.00185$; Sx - 0.00548

APPENDIX H

Confidence Interval Computation $\alpha = 0.01$ (99%)

1. Sensor at 300 knots: $\bar{x}_1 = -7.77$, $Sx_1 = 23$ feet (Table 9)

$$-7.77 - t(99.5,38)S\bar{x}_1 \leq \mu_1 \leq -7.77 + t(99.5,38)S\bar{x}_1$$

$$-7.77 - 2.71\left(\frac{23.0}{\sqrt{39}}\right) \leq \mu_1 \leq -7.77 + 2.71\left(\frac{23.0}{\sqrt{39}}\right)$$

$-17.75 \leq \mu_1 \leq 2.21$ correction to be added; or μ_1: -7.77 ± 9.98

2. Transducer, 500 feet to ground: $\bar{x}_2 = -6.25$, $Sx_2 = 8.99$ feet (Appendix I)

$$-6.25 - t(99.5,43)S\bar{x}_2 \leq \mu_2 \leq -6.25 + t(99.5,43)S\bar{x}_2$$

$$-6.25 - 2.7\left(\frac{8.99}{\sqrt{44}}\right) \leq \mu_2 \leq -6.25 + 2.7\left(\frac{8.99}{\sqrt{44}}\right)$$

$-9.91 \leq \mu_2 \leq -2.59$ correction to be added; or μ_2: -6.25 ± 3.66

3. Display, 500 feet to ground: $\bar{x}_3 = 5.6$, $Sx_3 = 7.89$ feet (Appendix I)

$$5.6 - t(99.5,59)S\bar{x}_3 \leq \mu_3 \leq 5.6 + t(99.5,59)S\bar{x}_3$$

$$5.6 - 2.66\left(\frac{7.89}{\sqrt{60}}\right) \leq \mu_3 \leq 5.6 + 2.66\left(\frac{7.89}{\sqrt{60}}\right)$$

$2.89 \leq \mu_3 \leq 8.31$ correction to be added; or μ_3: 5.6 ± 2.71

APPENDIX I

Altimeter and Computer Production Data

Altitude Computer Scale Error Down

Altitude Feet	Scale Error in Feet																	
0	-8	14	-2	-6	-21	-6	-6	-12	-16	-10	-3	-20	6	-20	-12	-12	-8	-6
500	5	14	-14	2	-8	1	1	-6	-5	-10	0	16	10	-18	-2	-14	-18	-10
	-20	-18	-2	-10														
	-8	-5	-4	-4														

Altitude	$\bar{x}$	Sx	0----0.5 n=44
0	-9	8.7	$\bar{x}_2 = -6.25$
500	-3.5	9.27	$Sx_2 = 8.99$

NOTE: Negative scale error is equivalent to a positive correction to altitude displayed.

Altimeter Data, AAU-34/A Down-Scale Only

Altitude Feet	Scale Error in Feet																			
0	0	-5	0	0	5	10	10	-10	-5	20	10	10	20	0	25	5	10	-5	5	0
500	0	5	10	0	10	5	10	-10	5	10	20	10	20	5	25	5	5	-5	10	5
	5	-5	5	5	0	0	5	10	0	20										
	5	-10	0	5	0	5	5	10	5	10										

Altitude	$\bar{x}$	Sx	0----0.5 n=60
0	5.33	8.193	$\bar{x}_3 = 5.6$
500	6.00	7.588	$Sx_3 = 7.89$ feet

APPENDIX J

Follow-on Flight Test Model

$$Y_{ijk} = \mu_{...} + \alpha_i + \beta_j + \gamma_k + (\alpha\beta)_{ij} + (\alpha\gamma)_{ik} + (\beta\gamma)_{jk} + (\alpha\beta\gamma)_{ijk} + \varepsilon_{ijkm}$$

ANOVA Table

Variable	Mean Squares	Degrees of Freedom	Expected Mean Square	F Ratio
Altitude	MSA	a-1	$\sigma^2 + nbc \frac{\Sigma\alpha_i^2}{a-1} + nc\sigma^2\alpha\beta$	MSA/MSAB
Aircraft	MSB	b-1	$\sigma^2 + nac\sigma^2_\beta$	MSB/MSE
Airspeed	MSC	c-1	$\sigma^2 + nab \frac{\Sigma\gamma_k^2}{c-1} + na\sigma^2_{\beta\gamma}$	MSC/MSBC
Inter-actions	MSAB	(a-1)(b-1)	$\sigma^2 + nc\sigma^2_{\alpha\beta}$	MSAB/MSE
	MSAC	(a-1)(c-1)	$\sigma^2 + nb \frac{\Sigma\Sigma(\alpha\gamma)^2_{ik}}{(a-1)(c-1)} + n\sigma^2\alpha\beta\gamma$	MSAC/MSABC
	MSBC	(b-1)(c-1)	$\sigma^2 + na\sigma^2\beta\gamma$	MSBC/MSE
	MSABC	(a-1)(b-1)(c-1)	$\sigma^2 + n\sigma^2_{\alpha\beta\gamma}$	MSABC/MSE
Error	MSE	(n-1)abc	σ^2	

EN