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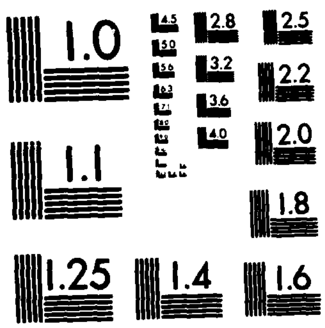


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Interim REPORT FOR GRANT AFOSR-83-0296

"CODING FOR SPREAD-SPECTRUM CHANNELS

IN THE PRESENCE OF JAMMING"

The long-term goals of this project are: a) a thorough analysis of the problems involved in communicating reliably in the presence of hostile jamming, and b) the design of effective "anti-jam" (A/J) countermeasures at the systems level. In the short-term, our research is presently focused on the detailed mathematical analyses of several specific A/J modulation and coding strategies of our own design, which are applicable to modern spread-spectrum communication systems.

Our approach to this research has been, from the beginning, based on a combination of Shannon's Information Theory and Von Neumann's Game Theory. The "game" involved is, of course, the conflict between the Communicator and the Jammer; an important feature of our approach is the use of Shannon's Channel Capacity as the payoff function in the game. Since Channel Capacity is convex-concave in just the right way, Von Neumann's saddlepoint theorems often allow us to calculate strategies for the Communicator and the Jammer which are simultaneously optimal for both players. Channel Capacity, according to Shannon, measures the maximum possible data rate in a communication system fully protected with error-control coding (ECC); this explains the title of our study. However, Shannon's Theorems about Channel Capacity are all nonconstructive existence proofs, and so much of our research is based on modern Algebraic Coding Theory, which might be called a "constructive approach to Shannon's theorems."

Recently, our research has been focused on the A/J problem for non-coherently modulated spread-spectrum frequency-hopped (SS/FH) systems. In particular, we have been studying the use of what we call pseudo-random ratio-threshold techniques (PRT) to combat jamming. Using techniques borrowed from the calculus of variations, we have been able to identify the worst-case jamming threat vs. PRT, and show that PRT's performance is better by several dB than conventional SS/FH systems. (This technique is descended from a technique introduced by Viterbi in 1982, but our analyses show it to be markedly superior in many applications.) This work will be reported in our paper "A Study of Viterbi's Ratio-Threshold AJ Technique," which has been accepted by and will appear in the proceedings of MILCOM'84 (copy attached).

This general subject area is very active; the 1982 and 1983 MILCOM (Military Communications Conference) Conference Proceedings are both filled with papers in this same general area. Many of these papers are devoted to the analysis of the performance of known A/J stategies vs. known jamming threats. Our own work (as befits university research, perhaps) focuses on more fundamental issues and innovative A/J strategies. However, we are following the DOD and industrial research closely in order to maintain contact with the latest technological advances. (For example: Viterbi's 1982 MILCOM paper led to our PRT technique, and the VHSIC program's Reed-Solomon decoder has influenced our thinking about practical ECC in A/J systems.)

# A Study of Viterbi's Ratio-Threshold AJ Technique

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**Abstract**

In this paper we study the performance of several AJ systems based on Viterbi's ratio-threshold technique for FH/SS communications. Innovative features of our work include the use of channel capacity as the figure of merit, and the use of randomly varying thresholds.

## 1. Introduction

Recently Viterbi [1] introduced a new technique, which he called a *ratio threshold* (R/T) technique, for combatting partial-band and tone jamming in an FH/SS environment. This technique, which we will describe in detail below, differs from most other AJ methods in that it is able to provide its own 'side information' about the current severity of the jamming threat. Roughly speaking, it does this by setting a threshold and declaring the jammer to be present unless the received signal exceeds the threshold. When the jammer is thus detected, less weight is attached to the received symbols. Viterbi showed that by using this technique, several dB of signal power could be saved over a conventional 'hard decision' MFSK receiver. In this paper, we shall further analyse the R/T technique, using channel capacity to optimise the selection of threshold parameter vs. various jamming threats.

In the remainder of this section, we will describe the mathematical model which we use to analyse the ratio-threshold technique, and give a game-theoretic formulation of our main problem. In Section 2, we we will analyse the performance of the ratio- threshold technique vs. the worst-case one-dimensional jammer. There we will see that the best choice of the threshold varies with the signal-to-noise ratio, and that for sufficiently small SNR, the worst-case jammer is not one-dimensional, a fact apparently not noticed by Viterbi. In section 3, we introduce an innovation of our own — a *randomly varying* threshold. We show that a random threshold is 1–2 dB superior to a fixed threshold vs. a worst-case one-dimensional jammer. The random threshold has the additional advantage of being more robust than the fixed threshold, in that the AJ strategy does not depend on the SNR. In section 4, we evaluate the performance of the R/T technique vs. the worst case partial band jammer. Our calculations show that the partial-band jammer is almost as effective vs. the R/T technique as is the less realistic worst-case one-dimensional jammer discussed earlier.

### 1. 1 An Abstract Model

In this subsection we introduce a model for noncoherent binary FSK modulation much like the one in [2]. The *transmitted signal* is a two-dimensional vector : $X = (0, \sqrt{\lambda})$ or $(\sqrt{\lambda}, 0)$ with probability 1/2 each. Without loss of generality, we assume $X = (\sqrt{\lambda}, 0)$ represents '0', and $X = (0, \sqrt{\lambda})$ represents '1'. The parameter $\lambda$ represents the signal power, and also the signal-noise ratio. since we assume by convention that the jamming power is 1.

The *jamming noise* is a two-dimensional random vector $Z = (\sqrt{Z_1}, \sqrt{Z_2})$ independent of $X$ , and $Z_1$, $Z_2$ are non-negative random varibles. We denote the two-dimensional distribution function of $Z$ by $F(z_1, z_2)$, and assume the distribution function F is symmetric in $z_1, z_2$. The jammer is assumed to have average power 1. This assumption can be stated mathematically as follows:

$$E(Z_1 + Z_2) = 2. \tag{1.1}$$

The *received signal* in our abstract model is a two-dimensional random vector $R = (R_1, R_2)$, where $R_i = |X_i + Z_i e^{j\theta_i}|^2$, $X_i$ and $Z_i$ are the components of $X$ and $Z$, and $\theta_i$ are independent random phase angles, uniformly distributed on $[0, 2\pi]$ for $i = 1, 2$. $R_i$ is supposed to represent the output of the i-th energy detector of a noncoherent binary FSK receiver. A *hard-decision* receiver chooses the largest $R_i$ and output the bit which is represented by the largest $R_i$.

We now introduce Viterbi's *threshold*, a real number $\geq 1$, which we denote by $\theta$. The receiver tries to decide whether '0' or '1' was transmitted, based on the received vector $(R_1, R_2)$, as follows:

If $R_1 \geq \theta R_2$: Decision : '0'
If $R_2 \geq \theta R_1$: Decision : '1'
Otherwise : Decision : '?'

where '?' is an *erasure symbol*. With this rule, our communication system becomes a *binary errors and erasures channel*, as depicted in Fig. 1. A simple calculation shows that the capacity of this channel is given by

$$C = P_c \log(2P_c/(P_e + P_c)) + P_e \log(2P_e/(P_e + P_c)). \quad (1.2)$$

This quantity depends on the signal-noise ratio $\lambda$, the threshold $\theta$, and the distribution of jammer's power.

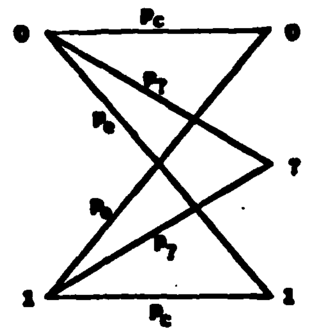


Fig. 1. The errors-and-erasures channel

## 1. 2 Problem Statement

We view this problem as a game with two players. The first player, the *Communicator*, tries to maximise the capacity $C$. He does this by selecting the threshold $\theta$. The second player, the *Jammer*, tries to minimise $C$, by carefully selecting his energy distribution $F$. When stated this way, the Communicator vs. Jammer problem becomes a problem in game-theory, and there are naturally two quantities of interest :

(a) The Communicator's Value of Capacity:

$$C_1 = \max_{\theta} \min_{F} C(\theta; F; \lambda)$$

(b) The Jammer's Value of Capacity:

$$C_2 = \min_{F} \max_{\theta} C(\theta; F; \lambda)$$

It is easy to see that $C_1 \leq C_2$, but in general the two values are unequal. In the remainder of this paper we will confine our attention to the problem of calculating $C_1$, which is the largest capacity that the Communicator can guarantee himself against any jammer subject to (1.1). In the next section we will discuss the problem of selecting the best threshold $\theta$ vs. the restricted class of one-dimensional jammers. As a corollary of our results, we will see that one-dimensional strategies are not optimal for the jammer for small values of $\lambda$, and we will exhibit a class of two-dimensional jammers which are superior to all one-dimensional jammers for sufficiently small SNR's. In section 3 we will see that improved (and more robust) performance is possible if the threshold $\theta$ is allowed to be a random variable.

## 2. Performance of a Fixed Threshold

In this section and section 3, we will discuss the performance of the R/T technique when the adversary is a one-dimensional jammer. In this section we will assume that the threshold $\theta$ is fixed and known to the jammer. In this case it is clear that the jammer needs only expend an energy of $(\lambda/\theta)^+$ to cause an erasure, and an energy of $(\lambda\theta)^+$ to cause an error, provided that the jammer's energy appears in the component of $R$ opposite to that in which the signal appears. Hence in order to calculate the minimimum over $F$ that appears in the definition of $C_1$, it is sufficient to consider jammers that assume only the three values 0, $(\lambda/\theta)^+$, and $(\lambda\theta)^+$. It is a relatively simple matter to perform this minimisation, using techniques of calculus. If we denote this minimum by $C(\theta; \lambda)$, then the value $C_1$ is given by

$$C_1 = \max_{\theta \geq 1} C(\theta; \lambda).$$

This maximisation apparently can not be done in closed form, but can easily be calculated numerically. We have graphed $C_1$ as a function of $\lambda$ (curve (a)) in Fig. 2; curve (b) in Fig. 2 is the channel capacity with $\theta = 1$ (i.e., a hard decision receiver.)

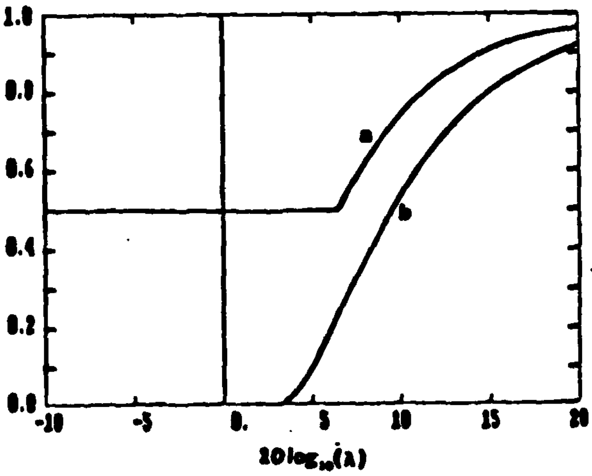


Fig. 2. Performance of one dim. jammer vs. (a) fixed $\theta$ (b) $\theta = 1$

For small values of $\lambda$, we see an interesting phenomenon. The channel capacity is 1/2, not the expected zero. This is disturbing, because it says that the jammer cannot reduce the capacity below 1/2, no matter how much of a power advantage he enjoys. On reflection, it can be seen that this is a result of the one-dimensional nature of the jammer's strategy, since there *will* always be a probability of 1/2 that the jammer will *reinforce* the transmitted signal. Hence if the signaller's threshold is set to $\theta = \infty$, the channel of Fig. 1 becomes the binary erasure channel of Fig. 3, which has capacity 1/2 bits. Indeed we find numerically that for all $\lambda \leq 6.12$ dB, the optimal threshold vs. a worst case one-dimensional jammer is $\theta = \infty$. Of course what this shows is that for small SNR's the worst case jammer is certainly not one-dimensional! In section 4, for example, we will see that in the presence of the worst-case partial band jammer, the channel capacity of the R/T technique does approach 0 as $\lambda$ approaches 0.

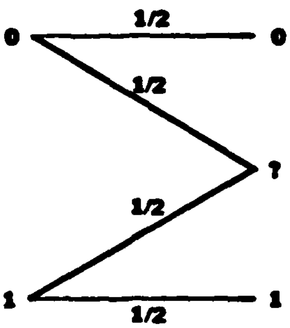


Fig. 3. With $\theta = \infty$, capacity equals 1/2 for all $\lambda$

## 3. Performance of a Random Ratio-Threshold

In this section we will see what happens when the threshold $\theta$ is varied randomly by the Communicator. This possibility enlarges the set of strategies allowed to the Communicator. When dealing with varible thresholds, it is more convinient to deal with $1/\theta$ than with $\theta$ directly, and so we introduce the notation $\varphi = 1/\theta$.

Ideally we would like to identify the best possible distribution of $\varphi$, vs. a jammer restricted only by (1.1). Unfortunately, we have been unable to do this. However, we have identified a class of $\varphi$ distributions which perform very well, compared to the fixed-threshold performance discussed in section 2, above. As in section 2, however, our analysis has so far been restricted to the class of one-dimensional jammers. Our main result here shows that for a wide class of random ratio-thresholds, the worst case one-dimensional jammer assumes at most two distinct values.

**Theorem 1.** *Let the signal-noise ratio $\lambda$ be fixed. Suppose the distribution function $G(\varphi)$ of the reciprocal threshold $\varphi$ is convex $\cup$ on [0,1]. Then the jamming strategy which satisfies (1.1) and minimizes $C$ assumes at most two distinct values. There will be a critical value of $\lambda$, say $\lambda^*$, such that for $\lambda \leq \lambda^*$, the optimal jammer assumes only one value, viz. $Z = 1$. For $\lambda \geq \lambda^*$, the optimal jammer assumes only two values, one of which is zero.*

The class of thresholds considered in Theorem 1 is very broad, and we have only investigated a small subclass of them in detail, viz. the distributions of the form $G(\varphi) = \varphi^n$ for some real number $n \geq 1$. It is possible to identify the best of these distributions.

**Theorem 2.** *Among all threshold distributions of the form $G(\varphi) = \varphi^n$, the distribution with $n = 1$ is uniformly the best, for all $n \geq 1$.*

In view of Theorem 2, it is worthwhile to investigate the performance of the $n = 1$ random threshold vs. the corresponding worst case jammer (which can be calculated with the help of Theorem 1.). We plot this in Fig. 4, together with the two curves from Figure 3.

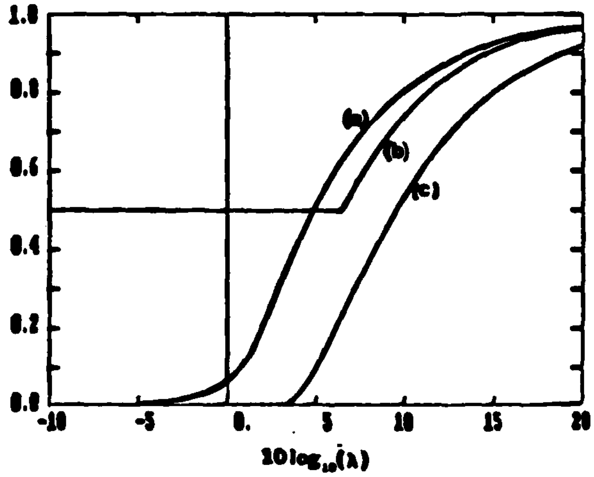


Fig. 4. Performance of one dim. jammer vs. (a) $G(\varphi) = \varphi$; (b) Best fixed $\theta$; (c) $\theta = 1$

In Fig. 4, we see that this one particular distribution of threshold performs better than the best fixed threshold scheme for all sufficiently large $\lambda$. Moreover, the random threshold strategy is the same for all values of $\lambda$, whereas the optimal fixed threshold varies with $\lambda$. We find this to be a very attractive feature of this AJ strategy.

## 4. Performance of Partial Band Jammer

In this section we describe the performance of the R/T technique against a *partial band noise jammer*. That is, the jammer is a white stationary gaussian process with zero mean and two-sided spectral density $N_0/2$. It is well-known [3], that for a hard-decision receiver, if the jammer uniformly distributes his power over the entire spread bandwidth, then the resulting bit error probability is :

$$P_e = \frac{1}{2}e^{-\lambda/2} \tag{4.1}$$

Houston has shown [4] that the jammer can do much better than this by distributing his power uniformly over a fraction $\rho$, $0 \leq \rho \leq 1$, of the total spread bandwidth (the so-called partial band noise jammer with duty factor $\rho$). Here

$$P_e = \frac{\rho}{2}e^{-\lambda\rho/2} \tag{4.2}$$

and the jammer will choose $\rho$ to minimize $P_e$. Letting $P_e^*$ denote the minimum of (4.2), we have:

$$P_e^* = \begin{cases} \frac{1}{2}e^{-\lambda/2} & \text{if } \lambda \leq 2 \quad (\rho = 1) \\ 1/e\lambda & \text{if } \lambda \geq 2 \quad (\rho = 2/\lambda). \end{cases} \tag{4.3}$$

In the R/T model, if the threshold $\theta$ is fixed and known to the partial band jammer, we have the following:

$$P_e = \frac{\rho}{\theta+1}e^{-\lambda\rho\theta/(\theta+1)} \tag{4.4}$$

$$P_c = 1 - \frac{\rho\theta}{\theta+1}e^{-\lambda\rho/(\theta+1)} \tag{4.5}$$

when $\theta = 1$, then equation (4.4) is the same as equation (4.2), and the channel capacity is given by:

$$C_1(\lambda) = \max_{\theta\geq 1} \min_{0\leq\rho\leq 1} C(\theta;\rho;\lambda) \tag{4.6}$$

The minisation over $\rho$ in equation (4.6) can be obtained by setting

$$\partial C(\theta;\rho;\lambda)/\partial\rho = 0 \tag{4.7}$$

It is not possible to find a closed form for the solution to equation (4.7) except for the case $\theta = 1$ (which is equation(4.3)), but as before it can be calculated numerically. Let us denote the minimum in (4.6) by $C(\theta;\lambda)$. In Fig. 5 we have graphed $C(1;\lambda)$ ,$C(2;\lambda)$ and $C_1(\lambda)$. We see that in this case the R/T technique offers little improvement.

As $\lambda$ approches zero, we have the following interesting result, which shows that curves (a) and (b) in Figure 5 are identical for small values of $\lambda$:

**Theorem 3.** *If the signal-noise ratio $\lambda$ is sufficiently small, and if we allow the threshold to be a random varible, then among all possible distribution function of $\theta$ against partial band noise jammer, $\theta = 2$ attains maximum channel capacity.*

Theorem 3 shows that $\theta = 2$ is uniformly the best for sufficiently small $\lambda$. For larger values of $\lambda$, the value of $\theta$ that attains the maximum in equation (4.6) depends on $\lambda$.

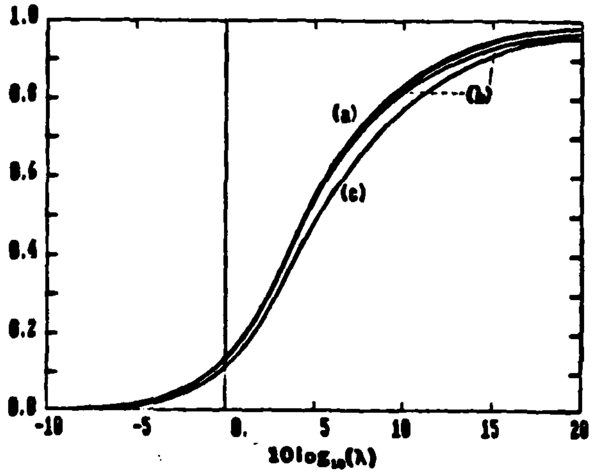


Fig. 5. Performance of worst-case partial band jammer vs. (a) best fixed $\theta$ (b) $\theta = 2$ (c) $\theta = 3$

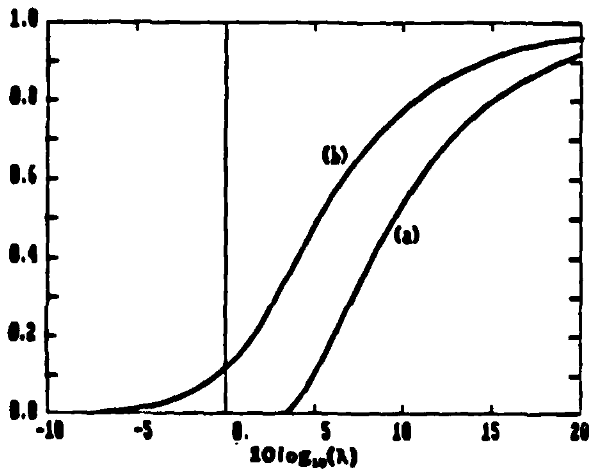


Fig. 6. Performance of worst-case (a) one dim. jammer (b) partial band jammer, vs. $\theta = 1$

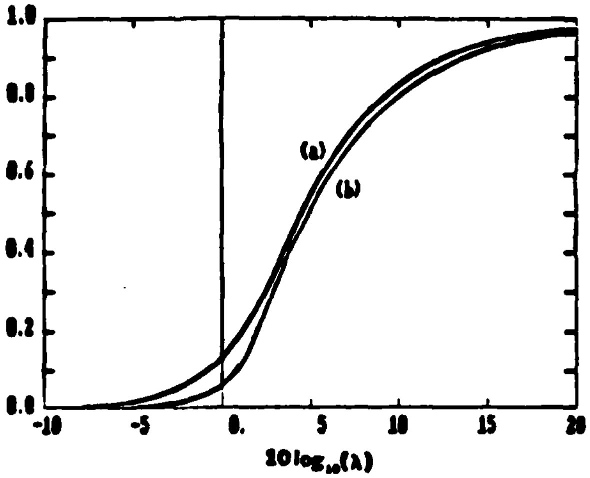


Fig. 7. Performance of worst-case (a) partial band jammer vs. best fixed $\theta$ (b)one-dim. jammer vs. $G(\varphi) = \varphi$

In Fig. 6 we plot the maximum channel capacity vs. one-dimensional jammer (curve (a)) and worst case partial band jammer (curve(b)) against $\theta = 1$ (hard decisions). Apparently the worst-case one-dimensional jammer is 4-5 dB more powerful than the worst-case partial band jammer. This problem is considerably improved by our AJ system, as can be seen in Fig 7, where we plot the channel capacity vs. a worst-case partial band jammer against optimal fixed threshold (curve(a)) and a worst-case one-dimensional jammer against $1/\theta$ uniformly distributed (curve(b)). Now the one-dimensional jammer is only about 0.5 dB less favorable than the worst-case partial band jammer.

## 5. Acknowledgement

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During this period research progressed in these areas: (1) a thorough analysis of the problems involved in communicating reliably in the presence of hostile jamming, and (2) the design of effective "anti-jam" (A/J) countermeasures at the systems level. In the short-term, research is presently focused on the detailed mathematical analyses of several specific A/J modulation and coding strategies of the investigators' own design, which are applicable to modern spread-spectrum communication systems.


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