

AD-A146 739

A METHOD FOR SOLVING THE INVERSE PROBLEM OF TRANSONIC
CASCADE FLOW(U) FOREIGN TECHNOLOGY DIV WRIGHT-PATTERSON
AFB OH C ZENGJING ET AL. 28 SEP 84 FTD-ID(RS)T-1118-84

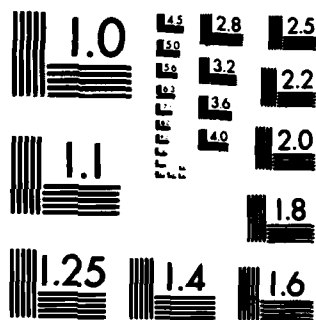
171

UNCLASSIFIED

F/G 20/4

NL





OPY RESOLUTION TEST CHART

AD-A146 739

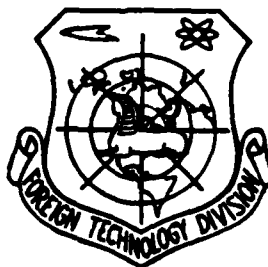
FOREIGN TECHNOLOGY DIVISION



A METHOD FOR SOLVING THE INVERSE PROBLEM OF
TRANSONIC CASCADE FLOW

by

C. Zengjing, Z. Zixiang



DTIC
ELECTE
S OCT 26 1984
A

DTIC FILE COPY

Approved for public release;
distribution unlimited.

84 10 22 083

EDITED TRANSLATION

FTD-ID(RS)T-1118-84

28 September 1984

MICROFICHE NR: FTD-84-C-000951

A METHOD FOR SOLVING THE INVERSE PROBLEM OF
TRANSONIC CASCADE FLOW

By: C. Zengjing, Z. Zixiang

English pages: 12

Source: Shuzhi Jisuan yu Jisuanji Yingyong,
Vol. 4, Nr. 4, 1983, pp. 241-246

Country of origin: China

Translated by: LEO KANNER ASSOCIATES
F33657-81-D-0264

Requester: FTD/TQTA

Approved for public release; distribution unlimited.

THIS TRANSLATION IS A RENDITION OF THE ORIGINAL FOREIGN TEXT WITHOUT ANY ANALYTICAL OR EDITORIAL COMMENT. STATEMENTS OR THEORIES ADVOCATED OR IMPLIED ARE THOSE OF THE SOURCE AND DO NOT NECESSARILY REFLECT THE POSITION OR OPINION OF THE FOREIGN TECHNOLOGY DIVISION.

PREPARED BY:

TRANSLATION DIVISION
FOREIGN TECHNOLOGY DIVISION
WP.AFB, OHIO.

GRAPHICS DISCLAIMER

All figures, graphics, tables, equations, etc. merged into this translation were extracted from the best quality copy available.

Accession For

NTIS GRA&I	☒
DTIC TAB	☐
Unannounced	☐
Justification	

By _____

Distribution/ _____

Availability Codes

Dist	Avail and/or Special
------	-------------------------

1A1

A METHOD FOR SOLVING THE INVERSE PROBLEM OF TRANSONIC CASCADE FLOW

Cong Zengjing

Computing Center, Academia Sinica

Zou Zixiang

Institute of Engineering Thermophysics, Academia Sinica

Abstract

This paper presents a method for solving the inverse problem of transonic cascade flow in (ϕ, φ) coordinate system. Given the upstream and downstream conditions and the velocity distribution on cascade, the flow field is obtained by solving the full potential equation with velocity as the unknown with the line relaxation method. Subsequently, the field is transformed back into the physical plane and we obtain the profile coordinates. Thickness and deflection are not restricted, since the full potential equation is used. The numerical result of the inverse problem for the Hobson cascade flow by the above method shows the applicability and fast convergence of our method.

I. Preface

In the last ten years, many numerical methods have appeared for the analysis of transonic cascade flow such as those by Denton [1,8], MacCormack [2], FLIC [3] etc. It is necessary to obtain the velocity distribution on the blade surface for given cascade configuration and inlet and outlet conditions. However, the calculation time of this method is too long and moreover, when there are small blade surface configuration corrections near the adjacent velocity line, this can cause tremendous changes of the blade surface velocity. Therefore, to obtain a cascade with rational velocity distribution, it is necessary to carry out a great deal of molding and calculations.

One suitable method for designing a cascade with low loss [4] begins with the rationality of the velocity distribution and as regards the configuration of the cascade, for the velocity distribution of the cascade, the experiences have been voluminous (mainly research on the boundary layer) and for the velocity distribution and the inlet and outlet conditions of a given cascade, they find the configuration of the blade surface. This then composes the so-called inverse problem of cascade flow.

It is well known that when solving the mixed potential flow equation using the potential function as the unknown quantity, there are two problems worthy of attention. One is finding the hyperbolic point and the elliptic point positions in the solution range which are unknown and irregular. If the calculation mode used is not suitable this will produce calculation instability. The other problem worthy of attention is the use of the physical boundary condition $\phi_n=0$ is not convenient especially the problem of the substance surface's form being too complex. The reason is the grid lines included in the substance surface are often nonorthogonal.

In 1971, Murman and Cole [5] first pointed out that the "type correlation form" caused research on the transonic potential flow problem to make considerable progress yet it is only limited to small perturbation equations. When the flow direction and grid line direction are different, the negative artificial viscosity possibly produced in the supersonic region causes the calculations to be unstable. In 1974, Jameson [6] pointed out that the "rotation finite difference form" eliminated the limitation of the small perturbation. However, the problem of how to correctly use the substance surface's boundary condition $\phi_n=0$ is to date still difficult to process. When the wing or cascade thickness is very thin, $\phi_n=0$ and we can approximately substitute with $\phi_y=0$. In reality, this further adds to the limitation of the small perturbation.

This paper uses velocity q as the unknown quantity, finds solution in the (ψ, φ) coordinate system (similar to [4]) and readily solves the above two problems. Because the upper and lower sides of the grid here are flow lines themselves, it does not require structural rotation form. Moreover, because the grid is rectangular on the (ψ, φ) calculation plane, this is advantageous for structuring the implicit form advanced along the flow line. At the same time, the orthogonality of the explicit grid naturally satisfies the impermeable conditions of the solid wall boundary.

Symbols

a_{Lj} : critical velocity
 s : arc length along flow line
 n : arc length along potential line
 ψ : flow function
 β : inlet and outlet angle
 r : specific heat ratio
 Γ : circulation
 w : velocity
 q : w/a_{Lj}
 ρ : density
 φ : potential function
 θ : local flow angle
 t : cascade distance

Lower Symbols

$"1,2"$: inlet and outlet parameters
 $"0"$: stagnation parameter

II. Fundamental Equations

Assuming the flow satisfies the continuity conditions, flow

function Ψ can be defined as

$$d\psi = \rho w dn$$

Assuming the flow satisfies the non-rotational conditions, potential function φ can be defined as

$$d\varphi = w ds$$

Because the dn along the flow line is zero, when the definition of Ψ is known, Ψ is constant. The ds along the velocity potential line (vertical to the flow line) is zero and when the definition of φ is known, φ is constant.

Naturally, the following relational formulas exist:

$$\begin{aligned} dx &= \cos\theta ds = (\cos\theta/w) d\varphi, \\ dy &= \sin\theta ds = (\sin\theta/w) d\varphi, \end{aligned} \quad (2.1)$$

$$\begin{aligned} dx &= -\sin\theta dn = (-\sin\theta/\rho w) d\psi, \\ dy &= \cos\theta dn = (\cos\theta/\rho w) d\psi. \end{aligned} \quad (2.2)$$

Considering the two-dimensional, non-rotational and constant compressible flows

$$\frac{\partial}{\partial s} (\rho w \delta_n) = 0, \quad (2.3)$$

$$\frac{\partial}{\partial n} (w \delta_s) = 0. \quad (2.4)$$

Further, based on considerations of geometric attributes (see Fig. 1), we have

$$\frac{1}{\delta_n} \frac{\partial(\delta_n)}{\partial s} = \frac{\partial\theta}{\partial n}, \quad (2.5)$$

$$\frac{1}{\delta_s} \frac{\partial(\delta_s)}{\partial n} = -\frac{\partial\theta}{\partial s}. \quad (2.6)$$

Based on formulas (2.3), (2.4), (2.5) and (2.6), we obtain the following set of differential equations:

$$\frac{1}{\rho^2 w} (\rho w)_\varphi + \theta_\varphi = 0, \quad (2.7)$$

$$\frac{\rho}{w} w_\varphi - \theta_\varphi = 0. \quad (2.8)$$

From $\frac{\partial}{\partial \varphi}$ (2.7) and $\frac{\partial}{\partial \gamma}$ (2.8), we obtain the logarithmic differential equation

$$(\ln \rho)_{\varphi\varphi} + (\ln w)_{\varphi\varphi} - ((\ln \rho)_\varphi)^2 - (\ln \rho)_\varphi (\ln w)_\varphi + \rho^2 (\ln \rho)_\varphi (\ln w)_\varphi + \rho^2 (\ln w)_{\varphi\varphi} = 0. \quad (2.9)$$

We introduce the isentropic relationship

$$(\rho/\rho_0) = \left(1 - \frac{r-1}{r+1} q^2\right)^{1/(r-1)}. \quad (2.10)$$

Dimensionless factors ρ_0 , a_{Lj} and t are used to make formulas (2.9) and (2.10) dimensionless. The entire potential equation on the calculation plane ($\gamma - \varphi$ coordinate system) can be written as

$$c_1 (\ln q)_{\varphi\varphi} + c_2 (\ln q)_{\varphi\varphi} + c_3 [(\ln q)_\varphi]^2 + c_4 [(\ln q)_\varphi]^2 = 0. \quad (2.11)$$

Formula (2.10) changes into

$$\rho = \left(1 - \frac{r-1}{r+1} q^2\right)^{1/(r-1)}, \quad (2.12)$$

In the formula, $c_1 = (1-q^2)^2 / k^{(r+1)(r-1)}$, $c_2 = 1$,

$$c_3 = k_2(1+q^2)/k^{2/(r-1)}, \quad c_4 = k_3/k_2,$$

$$k_2 = 1 - \frac{r-1}{r+1} q^2, \quad k_3 = -\frac{2}{r+1} q^2.$$

We can see that the local subsonic and supersonic regions of the flow separately correspond to the elliptical and hyperbolic regions.

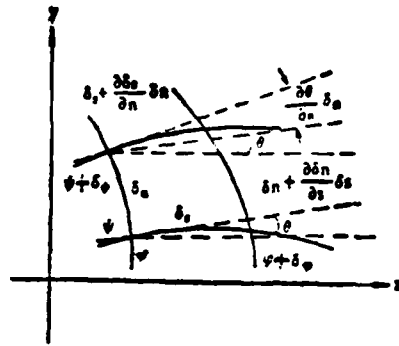


Fig. 1 Geometric relationship under $(\psi - \varphi)$ grid.

III. Boundary Conditions

The boundary conditions of the solution region shown in Fig. 2 are given as follows:

Inlet: q_1, β_1 ; outlet: $q_2, (\beta_2)$; blade surface: q_p, q_s .
In this, (β_2) is the reference value.

Letting $Q = (\rho_2 q_2 \sin \beta_2) / (\rho_1 q_1 \sin \beta_1)$, we consider that the boundary layer causes the main flow region of the wake flow section and can select $Q > 1$ so that the wake flow section is not sealed.

From the continuous equations and isentropic conditions, we can calculate

$$\beta_2 = \arcsin \left(\left(Q q_1 \sin \beta_1 \right. \right. \\ \times \left[\left(1 - \frac{r-1}{r+1} q_1^2 \right) \right]^{1/(r-1)} \\ \left. \left. / \left(q_2 \left[\left(1 - \frac{r-1}{r+1} q_2^2 \right) \right]^{1/(r-1)} \right) \right) \right)$$

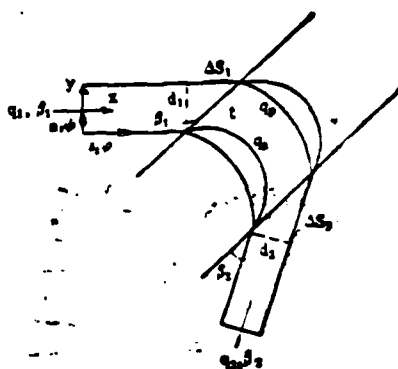


Fig. 2 Solution region and boundary conditions.

Naturally, when Q gives β_2 , we can calculate from q_1 , β_1 and q_2 .

Δs_1 and Δs_2 on the pressure boundary flow line are calculated as follows:

$$\Delta s_1 = \frac{d_1}{\lg \beta_1} \quad \Delta s_2 = \frac{d_2}{\lg \beta_2}$$

$$d_1 = t \sin \beta_1 \quad d_2 = t \sin \beta_2$$

Circulation Γ along the cascade profile is calculated based on strict periodic conditions

$$\Gamma = t(q_2 \cos \beta_2 - q_1 \cos \beta_1).$$

If under design conditions, it is permissible to have a supersonic region. The necessity to thus select the velocity distribution causes it to further compress the isentropy. The final shock wave strength will be weakened and thus avoid the separation of the boundary layer.

It is only necessary that the boundary layer not have separation to be able to use the cascade velocity distribution with the largest pressure gradient.

The velocity distribution in this paper is given along the flow line isopotential.

IV. The Discete Technique

When there is discretization, there is central difference with second-order precision along the ψ direction. However, along the φ direction, we then use the "type correlation" form.

The elliptic point:

$$(\ln q)_{\varphi\varphi}|_{ij} = \frac{1}{(\Delta\varphi)^2} [\ln q_{i+1,j} - 2\ln q_{ij} + \ln q_{i-1,j}]$$

The hyperbolic point: the finite difference forms of first-order and second-order precision are separately:

$$(\ln q)_{\varphi\varphi}|_{ij} = \frac{1}{(\Delta\varphi)^2} [\ln q_{ij} - 2\ln q_{i-1,j} + \ln q_{i-2,j}]$$

$$(\ln q)_{\varphi\varphi}|_{ij} = \frac{1}{(\Delta\varphi)^2} [2\ln q_{ij} - 5\ln q_{i-1,j} + 4\ln q_{i-2,j} - \ln q_{i-3,j}].$$

In order to avoid processing the set of nonlinear equations, the value of the (n+1) layer only appears in the i line, on the unknown quantity $(\ln q)$ obtained after using the second-order difference quotient.

Finally, we obtain a set of finite difference equations which assume the form of three opposite angles

$$A_{ij}^{(n)}(\ln q)_{i+1,j}^{(n+1)} + B_{ij}^{(n)}(\ln q)_{i,j}^{(n+1)} + C_{ij}(\ln q)_{i,j-1}^{(n+1)} = D_{ij}^{(n)} \quad (4.1)$$

Coefficient $A_{ij}^{(n)}$ etc. are calculated as follows

$$\begin{aligned}
A_{ij}^{(n)} &= c_j^{(n)} / (\Delta\phi)^2 = c_{ij}^{(n)} \\
B_{ij}^{(n)} &= -2c_j^{(n)} / (\Delta\phi)^2 + H[-2c_i^{(n)} / (\Delta\phi)^2] + (1+H)[H_1 \\
&\quad + 2(1-H_1)]c_i^{(n)} / (\Delta\phi)^2, \\
D_{ij}^{(n)} &= -c_j^{(n)}[(\ln q_{i+1,j} - \ln q_{i-1,j}) / (2\Delta\phi)]^2 \\
&\quad - c_i^{(n)}[(\ln q_{i,j+1} - \ln q_{i,j-1}) / (2\Delta\phi)]^2 \\
&\quad - H[\ln q_{i+1,j} + \ln q_{i-1,j}]c_i^{(n)} / (\Delta\phi)^2 \\
&\quad + (1-H)\{H_1[(2\ln q_{i-1,j} - \ln q_{i-2,j})c_i^{(n)} / (\Delta\phi)^2] \\
&\quad + (1-H_1)[(5\ln q_{i-1,j} - 4\ln q_{i-2,j} + \ln q_{i-3,j})]\}, \\
H &= \begin{cases} 1, & (1) \text{ when } q < 1, \\ 0, & q > 1. \end{cases}
\end{aligned}$$

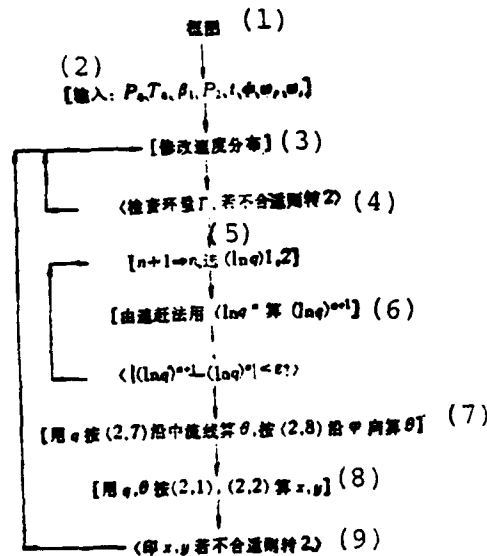
Key: (1) When.

When $H_1=1$, formula (1) has first-order precision (the q direction) and when $H_1=0$, formula (4) has second-order precision.

We use the method of pursuit to solve the set of finite difference equations based on the following steps:

For a fixed i , we select the number from 2 to $I-1$ and carry out

1. $E_j^{(n)} = -A_j^{(n)} / (B_j^{(n)} + c_j^{(n)} E_{j-1}^{(n)})$,
 $F_j^{(n)} = (D_j^{(n)} - c_j^{(n)} F_{j-1}^{(n)}) / (B_j^{(n)} + c_j^{(n)} E_{j-1}^{(n)}), \quad j = 2 \cdots k-1$
2. $(\ln q)_{ij}^{n+1} = E_j^{(n)} (\ln q)_{ij}^{n+1} + F_j^{(n)}, \quad j = k-1 \cdots 2.$



Key: (1) Flow chart; (2) Input; (3) Revised velocity distribution; (4) Investigated circulation Γ , if not suitable then change to 2; (5) Send; (6) From method of pursuit use $\ln q^n$ to calculate $(\ln q)^{n+1}$; (7) Use q to calculate θ along middle flow line based on formula (2.7) and calculate θ along ϕ direction based on formula (2.8); (8) Use q, θ to calculate x, y based on formulas (2.1) and (2.2); (9) Print x, y and if not suitable then change to 2.

After the calculation of the entire field, we then further relax

$$(\ln q)_{ij}^{n+1} = \omega (\ln q)_{ij}^{(n)} + (1 - \omega) (\ln q)_{ij}^{n+1},$$

In this, relaxation factor $\omega \in [0, 2]$.

The total calculation procedure is shown in the flow chart.

V. Conclusion

Firstly, based on the positive problem solved in Reference [7], we obtained the velocity distribution on the Hobson cascade blade surface and afterwards used the method proposed in this paper to solve the inverse problem. After 23 iterations, the maximum value of the entire field's absolute value of error

$\delta(\ln q)$ decreased to 0.0002 and we obtained the profile shown in Fig. 3. The symmetry accords quite well.

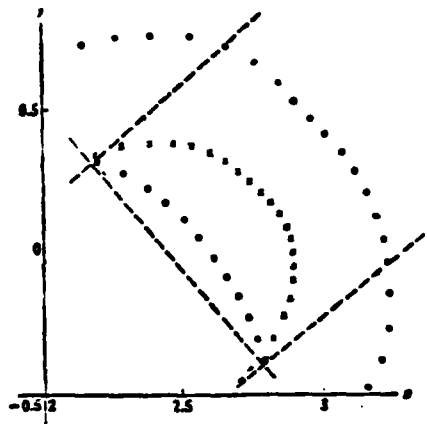


Fig. 3 Calculate Hobson profile.

References

- [1] J.D. Denton, A Time Marching Method For Two and Three-Dimensional Blade to Blade Flows, ARC RM No. 3775, 1975.
- [2] R.W. MacCormack, A.J. Paullay, Computational Efficiency Achieved by Time Splitting of Finite Difference Operators, AIAA Paper No. 72-154, 1972.
- [3] N. Ukeguchi, H. Sakata, T. Adachi, On the Numerical Analysis of Compressible Flow Problems by the "Modified FLIC Method", Computers and Fluids, Vol. 8, 1980.
- [4] E. Schmidt, Computation of Supercritical Compressor and Turbine Cascade With a Design Method for Transonic Flows, ASME Paper No. 79-GT-30, 1979.
- [5] E.D. Murman, J.D. Cole, Calculation of Plane Steady Transonic Flows, AIAA. J. Vol. 9, 1971.

- [6] Antony. Jameson, Iterative Solution of Transonic Flows Over Airfoils and Wings Including Flow at Mach 1, Comm. Pure Appl. Math., Vol. 27, No. 3, 1974.
- [7] Shen Mengyu, Gong Zengjin and Zhang Yaoke, A new Method For Calculating the Transonic Flow Around Plane Cascades, Journal Of Engineering Thermophysics, 1:3 (1980).
- [8] Zhang Yaoke, Shen Mengyu and Gong Zengjin, Numerical Tests on the Transonic Flow Around Plane Cascades, Computation Mathematics, 4(1978).

END

FILMED

DTIC