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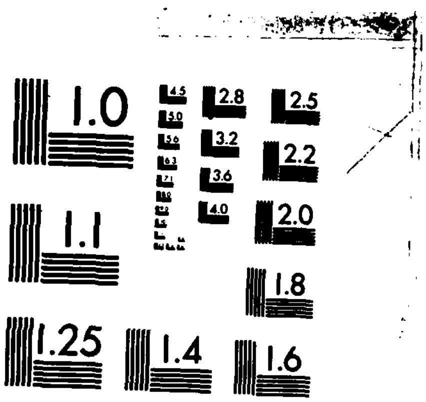
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MRC Technical Summary Report #2730

EXISTENCE OF SLOW STEADY FLOWS OF VISCOELASTIC FLUIDS WITH DIFFERENTIAL CONSTITUTIVE EQUATIONS

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ABSTRACT

We consider a viscoelastic fluid filling a bounded domain in $\mathbb{R}^3$ under the influence of a small body force. The fluid is described by certain differential constitutive equations. We use an iterative method to prove the existence of steady flows.

AMS (MOS) Subject Classifications: 35M05, 35Q99, 76A10

Key Words: Viscoelastic fluids, steady flows

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SIGNIFICANCE AND EXPLANATION

Questions of existence and uniqueness of steady flows of viscoelastic fluids have thus far not been understood, even for slow flows perturbing rest. This paper provides an existence result for slow flows with no in- and outflow boundaries. The fluid is assumed to be described by constitutive equations of a differential nature. The method used to prove existence is constructive and in fact very close to procedures used in numerical calculations.

The responsibility for the wording and views expressed in this descriptive summary lies with MRC, and not with the author of this report.

EXISTENCE OF SLOW STEADY FLOWS OF VISCOELASTIC FLUIDS
WITH DIFFERENTIAL CONSTITUTIVE EQUATIONS

Michael Renardy

1. INTRODUCTION

Although existence theorems for low Reynolds number steady flows of Newtonian fluids are well known (see e.g. [5]), no such theorems have been established for viscoelastic fluids. There are formal perturbation expansions which are used for slow flows (see e.g. [10]), but the justification of these expansions leads to difficult problems in singular perturbation theory, which have not been solved. Niggemann [8] has given a convergence proof for expansions of this nature in a one-dimensional model problem, which has certain features in common with equations in viscoelasticity.

In this paper, we prove the existence of slow steady flows of certain viscoelastic fluids by using an iterative method. The basic idea is very similar to existence proofs for initial value problems in hyperbolic partial differential equations. We first show that all iterates are bounded and small in a certain norm, and we then show that the iteration converges in a weaker norm. The iteration we use is similar to procedures employed in numerical calculations (for a review, see [1]), and the ideas used here should therefore be useful in proving convergence of numerical schemes. We shall also use our results to justify the formal perturbation methods as asymptotic expansions, but we do not prove their convergence.

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We study steady flows of a viscoelastic liquid in a bounded domain $\Omega \subset \mathbb{R}^3$. The fluid satisfied the no-slip condition on the wall and moves under the influence of a given body force. (Problems arising in applications usually have inflow boundaries, which require additional boundary conditions. Such problems are more difficult than the one studied here). The equations for steady flow are as follows

$$(1.1)\qquad \begin{aligned} \rho(\underline{u}\cdot\nabla)\underline{u} - \operatorname{div}\underline{\underline{T}} + \nabla p - \underline{f} &= \underline{0} \\ \operatorname{div}\underline{u} &= 0 \end{aligned} \quad \text{in } \Omega \subset \mathbb{R}^3$$

$$\underline{u} = \underline{0} \quad \text{on } \partial\Omega \ .$$

Here $\underline{u} = (u_1, u_2, u_3)$ is the velocity vector, p the pressure and $\underline{\underline{T}}$ the extra stress. $\underline{f}$ is the given body force and ρ is the density. Throughout the paper, Ω is assumed to be a bounded domain with a smooth (for simplicity, say C^∞-) boundary.

The extra stress is related to the velocity field by a constitutive equation. Here we deal with differential constitutive equations. For simplicity, we adopt a particular constitutive law, which exemplifies the typical structure. This model, the "rubberlike liquid" [3], [7], is given by

$$(1.2)\qquad \begin{aligned} &\underline{\underline{T}} = 2\eta_0\underline{\underline{D}} + \sum_{k=1}^{N} \underline{\underline{T}}_k \ , \\ &(\underline{u}\cdot\nabla)\underline{\underline{T}}_k - (\nabla\underline{u})\underline{\underline{T}}_k - \underline{\underline{T}}_k(\nabla\underline{u})^T + \lambda_k\underline{\underline{T}}_k = 2\eta_k\lambda_k\underline{\underline{D}} \ , \end{aligned}$$

where $\nabla\underline{u}$ is the velocity gradient : $(\nabla\underline{u})_{ij} = \frac{\partial u_i}{\partial x_j}$, and $\underline{\underline{D}} = \frac{1}{2}(\nabla\underline{u} + (\nabla\underline{u})^T)$. N is an arbitrary positive integer and η_k, λ_k are positive constants (η_0 may be zero).

The essential difficulty for the analysis arises from the terms $(\underline{u}\cdot\nabla)\underline{\underline{T}}_k$. The particular form of the terms $(\nabla\underline{u})\underline{\underline{T}}_k$ and $\underline{\underline{T}}_k(\nabla\underline{u})^T$ is unimportant, and we could replace them by other nonlinear combinations of $(\nabla\underline{u})$ and $\underline{\underline{T}}_k$. Our analysis can thus be extended to fluids with other differential constitutive equations, such as those of Oldroyd [9], Leonov [6] and Giesekus [2].

2. THE CASE OF THE UPPER CONVECTED MAXWELL FLUID

In this case, we set $\eta_0 = 0$ and $N = 1$, i.e. the constitutive equation takes the form

$$(2.1)\qquad (\underline{u}\cdot\nabla)\underline{\underline{T}} - (\nabla\underline{u})\underline{\underline{T}} - \underline{\underline{T}}(\nabla\underline{u})^T + \lambda\underline{\underline{T}} = 2\eta\lambda\underline{\underline{D}} .$$

By applying the divergence operator, we obtain

$$(2.2)\qquad (\underline{u}\cdot\nabla)\mathrm{div}\underline{\underline{T}} - (\nabla\underline{u})\mathrm{div}\,\underline{\underline{T}} + \lambda\mathrm{div}\underline{\underline{T}} = \underline{\underline{T}} : \partial^2\underline{u} + \eta\lambda\Delta\underline{u} .$$

Here we use the notation $\underline{\underline{T}} : \partial^2 = \sum_{j,k} T_{jk}\dfrac{\partial^2}{\partial x_j \partial x_k}$. Next, we substitute $\mathrm{div}\,\underline{\underline{T}} = \rho(\underline{u}\cdot\nabla)\underline{u} + \nabla p - \underline{f}$ from (1.1), and obtain

$$(2.3)\qquad \begin{aligned} &\nabla[(\underline{u}\cdot\nabla)p + \lambda p] - [(\nabla\underline{u}) + (\nabla\underline{u})^T]\nabla p - (\underline{u}\cdot\nabla)\underline{f} \\ &+ (\nabla\underline{u})\underline{f} - \lambda\underline{f} = \underline{\underline{T}} : \partial^2\underline{u} + \eta\lambda\Delta\underline{u} - \rho(\underline{u}\cdot\nabla)(\underline{u}\cdot\nabla)\underline{u} \\ &+ \rho(\nabla\underline{u})(\underline{u}\cdot\nabla)\underline{u} - \lambda\rho(\underline{u}\cdot\nabla)\underline{u} . \end{aligned}$$

In the following, we regard (2.3), with the condition $\mathrm{div}\,\underline{u} = 0$ and the no-slip condition, as a perturbation of the Stokes problem. This equation contains a "modified pressure" $q = (\underline{u}\cdot\nabla)p + \lambda p$. Solutions are found by the following iteration scheme.

$$(2.4)\qquad \underline{u}_0 = \underline{0}\ ,\ p^0 = q^0 = 0\ ,\ \underline{\underline{T}}^0 = \underline{\underline{0}}\ ,$$

$$(2.5)\qquad \begin{aligned} &\underline{\underline{T}}^n : \partial^2\underline{u}^{n+1} + \eta\lambda\Delta\underline{u}^{n+1} - \rho(\underline{u}^n\cdot\nabla)(\underline{u}^n\cdot\nabla)\underline{u}^{n+1} - \nabla q^{n+1} \\ &= -[\nabla\underline{u}^n + (\nabla\underline{u}^n)^T]\nabla p^n - (\underline{u}^n\cdot\nabla)\underline{f} + (\nabla\underline{u}^n)\underline{f} - \lambda\underline{f} \\ &- \rho(\nabla\underline{u}^n)(\underline{u}^n\cdot\nabla)\underline{u}^n + \lambda\rho(\underline{u}^n\cdot\nabla)\underline{u}^n , \\ &\mathrm{div}\,\underline{u}^{n+1} = 0\ ,\ \underline{u}^{n+1} = \underline{0}\ \text{on}\ \partial\Omega\ ,\ \iiint_\Omega q^{n+1} = 0\ , \end{aligned}$$

$$(2.6) \qquad (\underline{u}^{n+1}\cdot\nabla)p^{n+1} + \lambda p^{n+1} = q^{n+1} ,$$

$$(2.7) \qquad (\underline{u}^{n+1}\cdot\nabla)\underline{\underline{T}}^{n+1} - (\nabla\underline{u}^{n+1})\underline{\underline{T}}^{n+1} - \underline{\underline{T}}^{n+1}(\nabla\underline{u}^{n+1})^T$$

$$+ \lambda\underline{\underline{T}}^{n+1} = \eta\lambda[\nabla\underline{u}^{n+1} + (\nabla\underline{u}^{n+1})^T] .$$

We denote by $H^s(\Omega)$ the usual Sobolev spaces and by $\|\cdot\|_s$ the norm in $H^s(\Omega)$. The following lemma is immediate from the invertibility of the Stokes operator [5] and elementary perturbation theory [4].

LEMMA 2.1: *Let* s *be an integer* ≥ 1. *Then there are positive constants* ε_1 *and* ε_2 *such that the following holds: If* $\|\underline{\underline{T}}^n\|_{s+1} \leq \varepsilon_1$ *and* $\|\underline{u}^n\|_{s+1} \leq \varepsilon_2$, *then equation* (2.5) *has a unique solution* $\underline{u}^{n+1}$, q^{n+1}. *This solution obeys an estimate of the form*

$$\|\underline{u}^{n+1}\|_{s+2} + \|q^{n+1}\|_{s+1} \leq C_1[\|p^n\|_{s+1}\|\underline{u}^n\|_{s+2} + \|\underline{u}^n\|_{s+1}\|\underline{f}\|_{s+1}$$

$$+ \|\underline{f}\|_s + \|\underline{u}^n\|^2_{s+2}\|\underline{u}^n\|_s + \|\underline{u}^n\|^2_{s+1}] .$$

The following lemma concerns the solvability of equation (2.6).

LEMMA 2.2: *There is some* $\varepsilon_3 > 0$ *such that, for* $\|\underline{u}^{n+1}\|_{s+2} \leq \varepsilon_3$, $\operatorname{div} \underline{u}^{n+1} = 0$, $\underline{u}^{n+1}|_{\partial\Omega} = \underline{0}$, *the unique solution* p^{n+1} *of* (2.6) *satisfies an estimate of the form*

$$\|p^{n+1}\|_{s+1} \leq C_2\|q^{n+1}\|_{s+1} .$$

Sketch of the proof: Equation (2.6) is easily solved by the method of characteristics, and existence and uniqueness follow immediately. Moreover, the operator $(\underline{u}^{n+1}\cdot\nabla)$ is skew-adjoint in $L^2(\Omega)$, whence

$$\|p^{n+1}\|_0 \leq \frac{1}{\lambda}\|q^{n+1}\|_0 .$$

Estimates for derivatives are obtained by differentiating the equation. (Such estimates are formal and the calculations involve derivatives not a priori known to exist. However, it is easy to see that p is smooth if $\underline{u}$ and q are, and we can thus construct approximating sequences satisfying uniform estimates.)

Since equation (2.7) can be regarded as a perturbation of (2.6), we have

LEMMA 2.3: *There is a constant* ε_4 *such that, for* $\|\underline{u}^{n+1}\|_{s+2} \leq \varepsilon_4$, *the unique solution of (2.7) satisfies an estimate of the form*

$$\|\underline{\underline{T}}^{n+1}\|_{s+1} \leq C_3 \|\underline{u}^{n+1}\|_{s+2} .$$

By combining the estimates contained in lemmas 2.1 - 2.3, it can be shown that all iterates remain bounded if $\underline{f}$ is small.

LEMMA 2.4: *If* $\|\underline{f}\|_{s+1}$ *is sufficiently small, then* $\|\underline{u}^n\|_{s+2}$, $\|p^n\|_{s+1}$, $\|q^n\|_{s+1}$ *and* $\|\underline{\underline{T}}^n\|_{s+1}$ *have (small) bounds independent of* n .

Next we show that the iteration generates a convergent sequence in a weaker norm.

LEMMA 2.5: *Let* $\|\underline{f}\|_{s+1}$ *be sufficiently small. Then there is a constant* $\gamma < 1$ *such that*

$$\|\underline{u}^{n+1} - \underline{u}^n\|_{s+1} + \|q^{n+1} - q^n\|_s \leq \gamma [\|\underline{u}^n - \underline{u}^{n-1}\|_{s+1} + \|q^n - q^{n-1}\|_s] .$$

Sketch of the proof: From (2.6), we obtain

$$(\underline{u}^n \cdot \nabla) p^n - (\underline{u}^{n-1} \cdot \nabla) p^{n-1} + \lambda (p^n - p^{n-1}) = q^n - q^{n-1} .$$

This is equivalent to

$$((\underline{u}^n - \underline{u}^{n-1})\cdot\nabla)p^n + (\underline{u}^{n-1}\cdot\nabla)(p^n - p^{n-1}) + \lambda(p^n - p^{n-1})$$

$$= q^n - q^{n-1} .$$

From this and the bounds already established by lemma 2.4, we conclude that, for some constant C_4, we have

$$\|p^n - p^{n-1}\|_s \le C_4[\|q^n - q^{n-1}\|_s + \|\underline{u}^n - \underline{u}^{n-1}\|_{s+1}] .$$

Similarly, we find from (2.7)

$$\|\underline{\underline{T}}^n - \underline{\underline{T}}^{n-1}\|_s \le C_5\|\underline{u}^n - \underline{u}^{n-1}\|_{s+1} .$$

Next, we subtract $(2.5)_n$ and $(2.5)_{n-1}$. By using lemma 2.4 and the already established estimates for $p^n - p^{n-1}$ and $\underline{\underline{T}}^n - \underline{\underline{T}}^{n-1}$, we easily obtain the lemma. We omit the details of the calculation.

Thus we have proved

THEOREM 2.6: *Let* s *be an integer* ≥ 1 *and let* $\|\underline{f}\|_{s+1}$ *be sufficiently small. Then there exists a solution* $\underline{u} \in H^{s+2}$, $p \in H^{s+1}$, $\underline{\underline{T}} \in H^{s+1}$ *for equations* (1.1), (2.1), *obtainable by the iteration procedure* (2.4) - (2.7).

Remark:

Let us replace $\underline{f}$ by $\epsilon\underline{f}$. If s is chosen large enough, then we can, by following similar procedures as above, obtain estimates for difference quotients of the solution with respect to ϵ. This shows that the solution depends smoothly on ϵ and therefore establishes the asymptotic validity of Rivlin-Ericksen expansions. In problems with inflow boundaries, we should expect the situation to be quite different. Rivlin-Ericksen expansions are uniquely determined by prescribing velocities alone on the boundary. However, these boundary conditions are clearly not enough to uniquely define flows of a Maxwell fluid.

3. THE CASE OF SEVERAL RELAXATION MODES

We apply the divergence operator to (1.2), and obtain as before

$$(3.1)\qquad (\underline{u}\cdot\nabla)\operatorname{div}\underline{\underline{T}}_k - \nabla\underline{u}\operatorname{div}\underline{\underline{T}}_k + \lambda_k\operatorname{div}\underline{\underline{T}}_k$$
$$= \eta_k\lambda_k\Delta\underline{u} + \underline{\underline{T}}_k : \partial^2\underline{u} \ .$$

We write this as

$$(3.2)\qquad \operatorname{div}\underline{\underline{T}}_k = ((\underline{u}\cdot\nabla) + \lambda_k)^{-1}[\eta_k\lambda_k\Delta\underline{u} + \underline{\underline{T}}_k : \partial^2\underline{u} + \nabla\underline{u}\operatorname{div}\underline{\underline{T}}_k] \ .$$

By inserting this into (1.1), we find

$$(3.3)\qquad \rho(\underline{u}\cdot\nabla)\underline{u} = \eta_0\Delta\underline{u} + \sum_k ((\underline{u}\cdot\nabla) + \lambda_k)^{-1}$$
$$[\eta_k\lambda_k\Delta\underline{u} + \underline{\underline{T}}_k : \partial^2\underline{u} + \nabla\underline{u}\operatorname{div}\underline{\underline{T}}_k] - \nabla p + \underline{f} \ .$$

The cases $\eta_0 \neq 0$ and $\eta_0 = 0$ are treated in different ways. We begin with $\eta_0 \neq 0$. In this case, we set

$$L[\underline{u}] = \eta_0 + \sum_k \eta_k\lambda_k((\underline{u}\cdot\nabla) + \lambda_k)^{-1} \ .$$

LEMMA 3.1: *The operator* $L[\underline{u}]$ *is a bijection* $L^2(\Omega) \to L^2(\Omega)$.

Proof: Since $(\underline{u}\cdot\nabla)$ is skew-adjoint, we have $(((\underline{u}\cdot\nabla) + \lambda_k)^{-1}x,x) \geq 0$ for $x \in L^2(\Omega)$. Hence $(L[\underline{u}]x,x) \geq \eta_0(x,x)$, and the invertibility follows from the Lax-Milgram theorem.

By differentiating the equation $L[\underline{u}]x = y$, it is not difficult to show that, if $\underline{u}$ and its derivatives are small, then $L[\underline{u}]$ also maps higher Sobolev spaces bijectively into themselves.

We set $q = L[\underline{u}]^{-1}p$ and apply the operator $L[\underline{u}]^{-1}$ to (3.3). We thus obtain

$$\rho L[\underline{u}]^{-1}(\underline{u}\cdot\nabla)\underline{u} = \Delta\underline{u} - \nabla q + L[\underline{u}]^{-1}\{\sum_k ((\underline{u}\cdot\nabla) + \lambda_k)^{-1}$$

$$[\underline{\underline{T}}_k : \partial^2\underline{u} + \nabla u \operatorname{div} \underline{\underline{T}}_k]\} + L[\underline{u}]^{-1}\underline{f} \tag{3.4}$$

$$+ L[\underline{u}]^{-1} \sum_k \eta_k\lambda_k((\underline{u}\cdot\nabla) + \lambda_k)^{-1}(\nabla\underline{u})^T\nabla[((\underline{u}\cdot\nabla) + \lambda_k)^{-1}q] .$$

We use the following iteration scheme

$$\underline{u}^0 = \underline{0} , \; q^0 = 0 , \; \underline{\underline{T}}_k^0 = 0 \tag{3.5}$$

$$\Delta\underline{u}^{n+1} - \nabla q^{n+1} = \rho L[\underline{u}^n]^{-1}(\underline{u}^n\cdot\nabla)\underline{u}^n \tag{3.6}$$

$$- L[\underline{u}^n]^{-1} \sum_k ((\underline{u}^n\cdot\nabla) + \lambda_k)^{-1}[\underline{\underline{T}}_k^n : \partial^2\underline{u}^n + \nabla\underline{u}^n \operatorname{div} \underline{\underline{T}}_k^n]$$

$$- L[\underline{u}^n]^{-1}\underline{f} - L[\underline{u}^n]^{-1}\sum_k \eta_k\lambda_k((\underline{u}^n\cdot\nabla) + \lambda_k)^{-1}(\nabla\underline{u}^n)^T$$

$$\nabla[((\underline{u}^n\cdot\nabla) + \lambda_k)^{-1}q^n]$$

$$\operatorname{div} \underline{u}^{n+1} = 0 , \; \underline{u}^{n+1}|_{\partial\Omega} = \underline{0} , \; \iiint_\Omega q^{n+1} = 0 .$$

$$(\underline{u}^{n+1}\cdot\nabla)\underline{\underline{T}}_k^{n+1} - (\nabla\underline{u})^{n+1}\underline{\underline{T}}_k^{n+1} - \underline{\underline{T}}_k^{n+1}(\nabla\underline{u}^{n+1})^T \tag{3.7}$$

$$+ \lambda_k\underline{\underline{T}}_k^{n+1} = \eta_k\lambda_k[\nabla\underline{u}^{n+1} + (\nabla\underline{u}^{n+1})^T] .$$

We can now proceed in precisely the same manner as in section 2 to establish convergence of the iteration scheme.

If $\eta_0 = 0$, the operator $L[\underline{u}]$ as defined above is not coercive and lemma 3.1 does not hold. In this case, we adopt a different procedure. Let $\tilde{\lambda}$ be any positive real number. We apply the operator $(\underline{u}\cdot\nabla) + \tilde{\lambda}$ to

(3.3) and obtain

$$\rho((\underset{\sim}{u}\cdot\nabla) + \tilde{\lambda})(\underset{\sim}{u}\cdot\nabla)\underset{\sim}{u} = ((\underset{\sim}{u}\cdot\nabla) + \tilde{\lambda})\{\sum_k ((\underset{\sim}{u}\cdot\nabla) + \lambda_k)^{-1}$$
$$[\eta_k\lambda_k\Delta\underset{\sim}{u} + \underset{\approx}{T}_k : \partial^2\underset{\sim}{u} + \nabla\underset{\sim}{u}\ \mathrm{div}\ \underset{\approx}{T}_k] - \nabla p + \underset{\sim}{f}\} . \tag{3.8}$$

We now set $L[\underset{\sim}{u}] = ((\underset{\sim}{u}\cdot\nabla) + \tilde{\lambda}) \sum_k \eta_k\lambda_k\ ((\underset{\sim}{u}\cdot\nabla) + \lambda_k)^{-1}$

$= \sum_k \eta_k\lambda_k\{1 + (\tilde{\lambda}-\lambda_k)((\underset{\sim}{u}\cdot\nabla) + \lambda_k)^{-1}\}$.

$L[\underset{\sim}{u}]$ is a coercive operator in $L^2(\Omega)$ and $L[\underset{\sim}{u}]^{-1}$ exists. We can now apply the operator $L[\underset{\sim}{u}]^{-1}$ to (3.8), define $q = L[\underset{\sim}{u}]^{-1}((\underset{\sim}{u}\cdot\nabla) + \tilde{\lambda})p$ and set up an iteration scheme in an analogous fashion as before.

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We consider a viscoelastic fluid filling a bounded domain in $\mathbb{R}^3$ under the influence of a small body force. The fluid is described by certain differential constitutive equations. We use an iterative method to prove the existence of steady flows.

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