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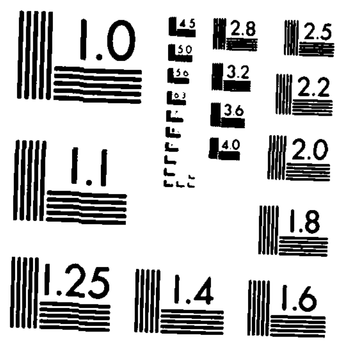

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AN ASYMPTOTIC ANALYSIS OF A TRANSIENT p-n JUNCTION MODEL

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ABSTRACT

In this paper we carry out an asymptotic analysis of the system of differential equations describing the transient behavior of a p-n-junction device (i.e. a diode). We determine the different time-scales present in the equations and investigate which of them actually occur in physical situations. We derive asymptotic expansions of the solution and perform some numerical experiments.

AMS (MOS) Subject Classifications: 35G25, 35M05, 65M15

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SIGNIFICANCE AND EXPLANATION

The present paper is concerned with a mixed elliptic-parabolic singularly perturbed initial boundary value problem describing a p-n-junction device. The purpose of transient modelling is the determination of the response time of a particular device (as opposed to the stationary model where the emphasis lies on the computation of voltage current characteristics). We carry out an asymptotic analysis using a singular perturbation parameter which is proportional to the minimal Debeye length.

Essentially we deal with the involved equations on 2 different time scales: One is proportional to the size of the device. The other one is the so called "dielectric relaxation time" which is several orders of magnitude smaller and essentially a property of the material and the doping concentration. Thus only effects occurring on the "slow" timescale (which is on the order of 10^{-9} sec) can be accelerated by miniaturization of the device whereas the dielectric relaxation time represents a lower bound to this (technological) approach. This fact has been known by engineers for several years. However, this analysis gives a mathematical proof for it. Moreover, it is precisely determined which effects are due to which time scales. We derive asymptotic expansions for the solution in powers of the perturbation parameter and give an existence proof for the solution of the reduced problem. We determine (and prove) the decay rate of the fast time scale solution.

The responsibility for the wording and views expressed in this descriptive summary lies with MRC, and not with the author of this report.

AN ASYMPTOTIC ANALYSIS OF A TRANSIENT p-n JUNCTION MODEL

Christian Ringhofer

0. Introduction

In this paper we present an asymptotic analysis of a singularly perturbed mixed system of parabolic and elliptic equations modelling a p-n-junction device. We consider the following physical situation: A semiconductor (for instance silicone) is doped with donor atoms (positive ions) in the left-hand-side and with acceptor atoms (negative ions) in the right-hand-side of the device. The device possesses 2 ohmic contacts to which a bias is applied. The devive is assumed to have a characteristic length $2\ell (\sim 0.5 * 10^{-3} \text{cm})$. The physics of a p-n junction are explained in Sze [1969], Ashcroft et al. [1976] and R. A. Smith [1978]. The equations governing the potential distribution and the carrier and current densities are in the case of one space dimension (see Van Roosbroeck [1950]):

(0.0) $\psi_{xx} = \frac{q}{\varepsilon}(n - p - C)$ Poisson's equation

(0.1) $n_t = \frac{1}{q} J_{n_x} - R$ continuity equations

(0.2) $p_t = -\frac{1}{q} J_{p_x} - R$ for electrons and holes

(0.3) $J_n = q(D_n n_x - \mu_n n \psi_x)$ current relations

(0.4) $J_p = -q(D_p p_x + \mu_p p \psi_x)$ for electrons and holes .

The system (0.0)-(0.4) is subject to the boundary conditions

(0.5) (a) $\psi(-\ell,t) = -\psi_1(t)$, (b) $\psi(\ell,t) = \psi_1(t)$

(0.6) $n(-\ell,t) = p_{-1} + C(-\ell), \quad p(-\ell,t) = p_{-1}$

(0.7) $n(\ell,t) = p_1 + C(\ell), \quad p(\ell,t) = p_1$

and the initial conditions

(0.8)
$$\psi(x,0) = \psi_I(x)$$
$$n(x,0) = n_I(x)$$
$$p(x,0) = p_I(x) .$$

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The meaning of the dependent variables with units is given in Table 1:

Table 1

ψ	electric potential	(V)
ψ_x	electric field	($V\ cm^{-1}$)
n	electron density	(cm^{-3})
p	hole density	(cm^{-3})
J_n	electron current density	($A\ cm^{-2}$)
J_p	hole current density	($A\ cm^{-2}$)

The parameters $q, \epsilon, \mu_n, \mu_p, D_n, D_p, n_i, U_T$ in (0.0)-(0.4) have the following meaning and approximate value at $T = 300K$ (room temperature).

Table 2

q	elementary charge	10^{-19} As
ϵ	permittivity constant	10^{-12} $AsV^{-1}cm^{-3}$
μ_n	electron mobility	10^3 $cm^2V^{-1}s^{-1}$
μ_p	hole mobility	10^3 $cm^2V^{-1}s^{-1}$
n_i	intrinsic density	10^{10} cm^{-3}
D_n	electron diffusion constant	25 cm^2s^{-1}
D_p	hole diffusion constant	25 cm^2s^{-1}

$C(x)$ in (0.0) is a given function of space and models the doping profile (i.e. the preconcentration of electrons and holes). We assume that $C(x) < 0$ for $x < 0$ (in the p-region) and $C(x) > 0$ for $x > 0$ (in the n-region holds). Thus we locate the p-n junction at $x = 0$. Moreover we assume that $C(x)$ has a jump-discontinuity at $x = 0$ (abrupt junction) and that $C(x)$ is an odd function of x i.e.

(0.9) $$C(-x) = -C(x) \quad \text{for} \quad x \neq 0, \quad C(0) \neq 0$$

holds. (Although the oddness of C does not appear as a special case from a physical point of view it will simplify the analysis considerably). Because of the physical background of the boundary conditions (0.5)-(0.7) the oddness of $C(x)$ results in $p_{-1} = p_1 + C(\ell)$. For the recombination rate R in (0.1), (0.2) we take the so called Shockley Read Hall recombination term:

$$R = \frac{n \cdot p - n_i^2}{\tau_n(n + n_i) + \tau_p(p + n_i)} . \tag{0.10}$$

τ_n and τ_p denote the average lifetime of electrons and holes. Realistic numerical values for τ_n and τ_p are $\tau_n = \tau_p = 10^{-6}(s)$. Other recombination rates can be found in Langer et al. *[1981] and* Schütz et al. [1981]. Generally it can be said that the choice of the recombination rate is not very important for the behavior of the solution of the time dependent problem (0.0)-(0.8) as long as we do not consider impact ionization effects (whereas it is of considerable importance in the steady state case (see Mock [1983])).

For the rest of this paper we assume that the initial functions ψ_I, n_I and p_I in (0.8) are compatible with the boundary conditions (0.5)-(0.7) and the (elliptic) equation (0.0). Thus we request that

$$\psi_{I_{xx}} = \frac{q}{\varepsilon}(n_I - p_I - C) \tag{0.11}$$

$$p(\pm\ell) = p_{\pm 1} \tag{0.12}$$

$$n(\pm\ell) = p_{\pm 1} + C(\pm\ell) \tag{0.13}$$

$$\psi_I(-\ell) = -\psi_1(0), \quad \psi_I(\ell) = \psi_1(\emptyset) \tag{0.14}$$

holds. After an appropriate scaling we carry out a singular perturbation analysis of problem (0.0)-(0.8) where the quantity

$$\lambda = \left(\frac{\varepsilon\, U_T}{\ell^2 q \max_x |C(x)|}\right)^{1/2} \tag{0.15}$$

(which is proportional to the minimal Debeye length) acts as a perturbation parameter. In this analysis we will rely on a similar asymptotic analysis for the steady state problem by Markowich and Ringhofer [1981] and Markowich et al. [1982].

It turns out that in addition to the internal layer behavior already present in the steady state case (see Markowich and Ringhofer [1981] there exist two different time scales in the problem (0.0)-(0.8). One time scale (the "slow" scale) is proportional to ℓ^2/μ and is of the order of magnitude 10^{-9} sec for modern devices. Here $\mu = \mu_n = \mu_p$ denotes the mobility. The other ("fast") time scale is proportional to the so called dielectric relaxation time (see Sze [1969]) which is several orders of magnitude smaller. Thus the asymptotic solution of (0.0)-(0.8) will consist of 4 different parts. Two parts vary on the slow time scale - one of them valid near the p-n junction $(x = 0)$ one of them valid away from $x = 0$. The two other parts - again an outer solution and a p-n-junction layer solution - vary on the fast time scale and are only valid near $t = 0$. From the work of Markowich and Ringhofer [1982] we conclude that the fast time scale is not present if ψ_I, n_I and p_I are solutions of the steady state problem

(0.16) $$\psi_{xx} = \frac{q}{\varepsilon}(n - p - C)$$

(0.17) $$\text{(a)}\quad J_{n_x} = qR, \qquad \text{(b)}\quad J_n = q(D_n n_x - \mu_n n\psi_x)$$

(0.18) $$\text{(a)}\quad J_{p_x} = -qR, \qquad \text{(b)}\quad J_p = -q(D_p p_x + \mu_p p\psi_x)\ .$$

Together with the boundary conditions corresponding to (0.5)-(0.7) in the steady state case.

This paper is organized as it follows: In Section 1 we scale (0.0)-(0.8) and reformulate the problem as a singular perturbation problem. In Section 2 we derive the asymptotic expansion for the slow time scale. We prove the existence of a solution of the reduced problem and give necessary and sufficient conditions for the slow time scale expansion to satisfy the differential equations the boundary and the initial conditions. These conditions are satisfied if the initial data ψ_I, n_I and p_I are solutions of the steady state problem (0.16)-(0.18). In Section 3 we consider the case when these conditions are violated and derive an asymptotic expansion for the solution of (0.0)-(0.8) with general initial data (0.8) which just satisfy the compatibility conditions (0.11)-

(0.14). In Section 4 we present some asymptotic solutions which were obtained by solving the reduced problem and the layer equations numerically by finite difference methods. The proofs of some results stated in Sections 2 and 3 are given in the appendix.

1. Reformulation as Singular Perturbation Problem

First we scale (0.0)-(0.8) appropriately and reformulate it as a singular perturbation problem: We scale the space variable x by the characteristic length ℓ and the time by ℓ^2/D where $D = D_n = D_p$ denotes the diffusion coefficient

(1.0) $$x_s = \frac{x}{\ell}, \quad t_s = t\frac{D}{\ell^2}.$$

For ψ_1, n, p, J_n, J_p we use the following scaling:

(1.1) $$\psi = U_T\psi_s, \quad n = \bar{C}n_s, \quad p = \bar{C}p_s,$$

(1.2) $$J_n = \frac{q\bar{C}D}{\ell}J_{n_s}, \quad J_p = \frac{q\bar{C}D}{\ell}J_{n_s}$$

(1.3) $$\bar{C} := \max_x |C(x)| .$$

(Here the variables with the subscript s denote the scaled variables.)

<u>(1.4) Remark</u>: For the rest of this paper we assume the Einstein relations

(1.5) $$U_T = \frac{D_n}{\mu_n} = \frac{D_p}{\mu_p}.$$

U_T (the thermal voltage) is only a function of the temperature (which is assumed to be constant).

After scaling the system (0.0)-(0.8) assumes the form

(1.6) $$\lambda^2 \frac{\partial^2}{\partial x_s^2}\psi_s = n_s - p_s - C_s, \quad C_s(x_s) = \frac{C(x)}{\max|C(x)|}$$

(1.7) $$\text{(a)} \quad \frac{\partial}{\partial t_s}n_s = \frac{\partial}{\partial x_s}J_{n_s} - R_s, \qquad \text{(b)} \quad J_{n_s} = \frac{\partial}{\partial x_s}n_s - n_s\frac{\partial}{\partial x_s}\psi_s$$

(1.8) $$\text{(a)} \quad \frac{\partial}{\partial t_s}p_s = -\frac{\partial}{\partial x_s}J_{p_s} - R_s, \qquad \text{(b)} \quad J_{p_s} = -\frac{\partial}{\partial x_s}p_s - p_s\frac{\partial}{\partial x_s}\psi_s$$

(1.9) $$R_s = \frac{n_s p_s - \delta^2}{n_s + p_s + 2\delta}, \quad \delta = \frac{n_i}{\bar{C}}$$

boundary conditions:

(1.10) $$\psi_s(-1,t_s) = -\psi_{1_s}(t), \quad \psi_s(1,t_s) = \psi_{1_s}(t)$$

(1.11) $$n_s(-1,t_s) = p_{-1_s} + C(-1), \quad n_s(1,t_s) = p_{1_s} + C(1)$$

(1.12) $$p_s(-1,t_s) = p_{-1_s}, \quad p_s(1,t_s) = p_{1_s}$$

initial conditions

(1.13) $$\psi_s(x_s,0) = \psi_{I_s}(x_s), \; n_s(x_s,0) = n_{I_s}(x_s), \; p_s(x_s,0) = p_{I_s}(x_s) \; .$$

Here $\psi_{\pm 1_s}$, ψ_{I_s}, $p_{\pm 1_s}$, n_{I_s}, p_{I_s} denote $\psi_{\pm 1}$, ψ_I, $p_{\pm 1}$, n_I, p_I scaled in the same way as ψ, n and p. The (dimensionless) singular perturbation parameter λ in (1.6) is of the form

(1.14) $$\lambda = \left(\frac{\varepsilon U_T}{q\ell^2\bar{C}}\right)^{1/2} , \quad \bar{C} = \max_x |C(x)| \; .$$

λ is proportional to the minimal Debeye length (see Sze [1969]). From hereon we will omit the subscript s. Since $C(x)$ is an odd function we can employ the "Ansatz"

(1.15) $$\psi(-x) = -\psi(x), \; n(x) = p(-x), \; J_n(x) = J_p(-x)$$

to obtain a problem posed on $[0,1]$. This enables us to treat the internal layer at the p-n junction (see Markowich and Ringhofer [1981]) as a boundary layer. In order to employ this "Ansatz" we have to assume the corresponding relations for the intial data ψ_I, n_I and p_I.

(1.16) Assumption: The initial data satisfy

(1.17) $$\psi_I(-x) = -\psi_I(x), \; n_I(x) = p_I(-x) \; .$$

After employing (1.15) we obtain (1.6)-(1.13) in its final form:

(1.18) $$\lambda^2\psi_{xx} = n - p - C$$

(1.19) $$\text{(a)} \quad n_t = J_{n_x} - R, \qquad \text{(b)} \quad J_n = n_x - n\psi_x$$

(1.20) $$\text{(a)} \quad p_t = -J_{p_x} - R, \qquad \text{(b)} \quad J_p = -p_x - p\psi_x$$

$$R = \frac{n \cdot p - \delta^2}{n + p + 2\delta} \tag{1.21}$$

boundary conditions:

$$\psi(1,t) = \psi_1(t), \; n(1,t) = p_1 + C(1), \; p(1,t) = p_1 \tag{1.22}$$

$$\psi(0,t) = 0, \; n(0,t) = p(0,t), \; J_n(0,t) = J_p(0,t) \tag{1.23}$$

initial conditions:

$$\psi(x,0) = \psi_I(x), \; n(x,0) = n_I(x), \; p(x,0) = p_I(x) \; . \tag{1.24}$$

(1.25) Notation: For the rest of this paper we denote the solution vector $(\psi,n,p,J_n,J_p)^T$ by

$$w = (\psi,n,p,J_n,J_p)^T \; . \tag{1.26}$$

The boundary conditions (1.23) emerge from the "Ansatz" (1.15). The compatibility conditions now read

$$\lambda^2 \psi_{I_{xx}} = n_I - p_I - C \tag{1.27}$$

$$\psi_I(1) = \psi_1(\emptyset), \; n_I(1) = b, \; p_I(1) = a \tag{1.28}$$

$$\psi_I(0) = 0, \; n_I(0) = p_I(0), \; n_{I_x}(0) = -p_{I_x}(0) \; . \tag{1.29}$$

Again (1.29) is obtained from (1.17) and guarantees (1.23) to hold at $t = \emptyset$. In practice ψ_I, n_I and p_I will be the (scaled) solution of the steady state problem (0.16)-(0.18). From Markowich and Ringhofer [1981] we know that this solution has an asymptotic expansion in powers of λ consisting of an outer solution and layer terms varying in the variable $\xi = x/\lambda$. From here on we will assume that ψ_I, n_I and p_I have an asymptotic expansion of the same form:

$$w_I(x) = w_I(x,\lambda) \sim \sum_{j=0}^{\infty} [\tilde{w}_I^j(x) + \hat{w}_I^j(\tfrac{x}{\lambda})]\lambda^j \tag{1.30}$$

$$w_I = (\psi_I, n_I, p_I)^T$$

where the $\hat{w}_I^j$ satisfy

$$\|\hat{w}_I^j(\xi)\| \le C_1 e^{-C_2 \xi} \quad \text{for some constants } C_1, C_2 \; . \tag{1.31}$$

Moreover, we will assume that the initial data are such that the current densities J_n and J_p stay uniformly bounded at $t = 0$ as λ tends to zero:

(1.32) $$\max_{x \in [0,1]} |J_{n_I}(x,\lambda)|, \quad \max_{x \in [0,1]} |J_{p_I}(x,\lambda)| \le \text{const.} \quad \text{as} \quad \lambda \to 0$$

(1.33) $$J_{n_I} := n_{I_x} - n_I \psi_{I_x}, \quad J_{p_I} = -p_{I_x} - p_I \psi_{I_x} \, .$$

2. Slow Time Scale Expansions

In this section we derive our asymptotic expansion to the solution w of (1.18)-(1.24) of the form

$$w(x,t,\lambda) \sim \sum_{j=0}^{\infty} [\tilde{w}^j(x,t) + \hat{w}^j(\tfrac{x}{\lambda},t)]\lambda^j \; . \tag{2.1}$$

Thus we obtain an approximation of the solution on the "slow" t-time scale. The expansion (2.1) will satisfy the differential equations (1.18)-(1.20) and the boundary conditions (1.22)-(1.23). However, it will not satisfy the initial conditions (1.24) for arbitrary initial data ψ_I, n_I and p_I. We derive a necessary and sufficient condition on (ψ_I,n_I,p_I) such that the expansion (2.1) satisfies (1.24) as well. As it turns out these conditions are satisfied if ψ_I, n_I and p_I in (1.24) are the solution of the steady state problem

$$\lambda^2\psi_{xx} = n - p - C \tag{2.2}$$

$$-R = J_{n_x}, \quad J_n = n_x - n\psi_x \tag{2.3}$$

$$R = J_{p_x}, \quad J_p = -p_x - p\psi_x \tag{2.4}$$

together with the boundary conditions (1.22)-(1.23). Since this will be the case in all "physically meaningful" situations (2.1) actually represents an approximation to the solution of (1.18)-(1.24). Furthermore we give an existence result for the zero'th order term $\tilde{w}^0$ in (2.1).

We start with the equations determining $\tilde{w}^0$: Setting λ equal to zero in (1.18)-(1.24) gives

$$0 = \tilde{n}^0 - \tilde{p}^0 - C \tag{2.5}$$

$$\text{(a)}\quad \tilde{n}^0_t = \tilde{J}^0_{n_x} + \tilde{R}^0, \qquad \text{(b)}\quad \tilde{J}^0_n = \tilde{n}^0_x - \tilde{n}^0\tilde{\psi}^0_x \tag{2.6}$$

$$\text{(a)}\quad \tilde{p}^0_t = -\tilde{J}^0_{p_x} + \tilde{R}^0, \qquad \text{(b)}\quad \tilde{J}^0_p = -\tilde{p}^0_x - \tilde{p}^0\tilde{\psi}^0_x \tag{2.7}$$

$$\tilde{R}^0 := \frac{\tilde{n}^0 \cdot \tilde{p}^0 - \delta^2}{\tilde{n}^0 + \tilde{p}^0 + 2\delta} \; . \tag{2.8}$$

Thus Poisson's equation (1.18) degenerates to the condition of vanishing space charge for $\lambda \to \emptyset$.

Subtracting (2.7) from (2.6) and using (2.5) we can rewrite (2.6)-(2.7) into

(2.9) $$[(2\tilde{p}^0 + C)\tilde{\psi}^0_x]_x = C_{xx}$$

(2.10) $$\text{(a)}\quad \tilde{p}^0_t = -\tilde{J}^0_{p_x} + \tilde{R}^0, \qquad \text{(b)}\quad \tilde{J}^0_p = -\tilde{p}^0_x - \tilde{p}^0\tilde{\psi}^0_x$$

(2.11) $$\tilde{R}^0 = \frac{\tilde{p}^0(\tilde{p}^0 + C)}{2\tilde{p}^0 + C} .$$

(2.9) is an elliptic equation expressing the conservation of the total current $J_n + J_p$ (in the absence of space charge) coupled to the parabolic conservation law (2.10)(a),(b). $\tilde{n}^0$ and $\tilde{J}^0_n$ are then given a posteriori by virtue of (2.5):

(2.12) $$\tilde{n}^0 = \tilde{p}^0 + C$$

(2.13) $$\tilde{J}^0_n = \tilde{p}^0_x + C_x - (\tilde{p}^0 + C)\tilde{\psi}^0_x .$$

Since (2.9)-(2.10) is a system involving two second order differential operators in space we can not expect its solution to satisfy all six boundary conditions (1.22)-(1.23). At the contact $(x = 1)$, however, the boundary conditions have been derived from the assumption of vanishing space charge $(n - p - C = 0)$. Therefore the boundary conditions (1.22)-(1.23) are consistent with (2.5): If we impose the boundary conditions

(2.14) $$\tilde{\psi}^0(1,t) = \psi_1(t),\ \tilde{p}^0(1,t) = p_1$$

$\tilde{n}^0 := \tilde{p}^0 + C$ satisfies automatically the third boundary condition

(2.15) $$\tilde{n}^0(1,t) = p_1 + C(1) .$$

Therefore no boundary layer term is needed at $x = 1$ (the contact). At $x = 0$ (the junction) the condition $n(0,t) = p(0,t)$ contradicts $\tilde{n}^0 - \tilde{p}^0 - C = 0$. Therefore the outer solution $\tilde{w}^0$ has to be supplemented by a boundary layer term there. (In other words the approximation of vanishing space charge can not be valid near the p-n junction.) Thus we supplement $\tilde{w}^0(x,t)$ by the layer term

(2.16) $$\hat{w}^0(\xi,t),\ \xi = \frac{x}{\lambda},\ \hat{w}^0 = (\hat{\psi}^0,\hat{n}^0,\hat{p}^0,\hat{J}^0_n,\hat{J}^0_p)$$

where we request that $\|\hat{w}^0\|$ decays exponentially as $\xi \to \infty$. Inserting

$\tilde{w}^0(\lambda\xi,t) + \hat{w}^0(\xi,t)$ into (1.18)-(1.20) and $\lambda \to 0$ gives (using (2.9)-(2.13)):

$$\hat{\psi}^0_{\xi\xi} = \hat{n}^0 - \hat{p}^0 \tag{2.17}$$

$$\text{(a)}\quad 0 = \hat{J}^0_{n_\xi}, \qquad \text{(b)}\quad 0 = \hat{n}^0_\xi - (\tilde{n}^0(0,t) + \hat{n}^0)\hat{\psi}^0_\xi \tag{2.18}$$

$$\text{(a)}\quad 0 = -\hat{J}^0_{p_\xi}, \qquad \text{(b)}\quad 0 = -\hat{p}^0_\xi - (\tilde{p}^0(0,t) + \hat{p}^0)\hat{\psi}^0_\xi\ . \tag{2.19}$$

Because $\hat{w}^0$ has to vanish at $\xi = \infty$ we can integrate (2.18)-(2.19) and obtain

$$\hat{J}^0_n \equiv 0, \quad \hat{J}^0_p \equiv 0 \tag{2.20}$$

$$\hat{n}^0(\xi,t) = \tilde{n}^0(0,t)[\exp(\hat{\psi}^0(\xi,t)) - 1] \tag{2.21}$$

$$\hat{p}^0(\xi,t) = \tilde{p}^0(0,t)[\exp(-\hat{\psi}^0(\xi,t)) - 1]\ . \tag{2.22}$$

Inserting $\tilde{w}^0 + \hat{w}^0$ into the boundary conditions (1.23) and $\lambda \to \emptyset$ gives

$$\hat{\psi}^0(0,t) = -\tilde{\psi}^0(0,t) \tag{2.24}$$

$$\tilde{n}^0(0,t)\exp(\hat{\psi}^0(0,t)) = \tilde{p}^0(0,t)\exp(-\hat{\psi}^0(0,t)) \tag{2.25}$$

$$\tilde{J}^0_n(0,t) = \tilde{J}^0_p(0,t)\ . \tag{2.26}$$

From (2.24)-(2.26) we obtain two boundary conditions for the system (2.9)-(2.10) at $x = \emptyset$. Thus we have the following

(2.27) Theorem: The zero'th order term $\tilde{w}^0$ of the outer solution **satisfies the reduced** problem:

$$[(2\tilde{p}^0 + C)\tilde{\psi}^0_x]_x = C_{xx} \tag{2.28}$$

$$\text{(a)}\quad \tilde{p}^0_t = -\tilde{J}^0_{p_x} + \tilde{R}^0, \qquad \text{(b)}\quad \tilde{J}^0_p = -\tilde{p}^0_x - \tilde{p}^0\tilde{\psi}^0_x \tag{2.29}$$

$$[\tilde{p}^0(0,t) + C(0)]\exp[-\tilde{\psi}^0(0,t)] = \tilde{p}^0(0,t)\exp[\tilde{\psi}^0(0,t)] \tag{2.30}$$

$$\tilde{J}^0_n(0,t) = \tilde{J}^0_p(0,t) \tag{2.31}$$

$$\tilde{\psi}^0(1,t) = \psi_1(t), \quad \tilde{p}^0(1,t) = p_1 \tag{2.32}$$

$$\tilde{n}^0 = \tilde{p}^0 + C, \quad \tilde{J}^0_n = \tilde{p}^0_x + C_x - (\tilde{p}^0 + C)\tilde{\psi}^0_x \tag{2.33}$$

$$\tilde{p}^0(x,0) = f(x)\ . \tag{2.34}$$

(2.35) Remark: We have not determined the initial function $f(x)$ yet. The appropriate choice of $f(x)$ will be discussed at the end of this section and in Section 3. In any case we can prove an existence result for small f up to a certain value of the bias ψ_1.

(2.36) Theorem: There exist positive constants α_0 and K such that if

$$\alpha = \max|f(x)| < \alpha_0 \quad \text{and} \quad \psi_1 > \ln\frac{K}{\alpha} \tag{2.37}$$

holds then (2.28)-(2.34) has a classical solution which is continuously differentiable with respect to x and t for $x \in [0,1]$ and all t less than some arbitrary but finite time T. The proof of Theorem (2.36) is deferred to the Appendix.

Once $\tilde{w}^0$ is obtained as the solution of problem (2.28)-(2.34) $\hat{w}^0$ can be obtained by solving (2.17)-(2.19) together with the appropriate boundary conditions:

(2.38) Theorem: The layer term $\hat{w}^0 = (\hat{\psi}^0,\hat{n}^0,\hat{p}^0,\hat{J}_n^0,\hat{J}_p^0)$ in (2.1) satisfies

$$\hat{\psi}^0_{\xi\xi} = \tilde{n}^0(0,t)[e^{\hat{\psi}^0} - 1] - \tilde{p}(0,t)[e^{-\hat{\psi}^0} - 1] \tag{2.39}$$

$$\hat{\psi}^0(0,t) = -\tilde{\psi}^0(0,t), \quad \hat{\psi}^0(\infty,t) = 0 \tag{2.40}$$

$$\hat{n}^0(\xi,t) = \tilde{n}^0(0,t)[e^{\hat{\psi}^0(\xi,t)} - 1], \quad \hat{p}^0(\xi,t) = \tilde{p}^0(0,t)[e^{-\hat{\psi}^0(\xi,t)} - 1] \tag{2.41}$$

$$\hat{J}_n^0(\xi,t) = \hat{J}_p^0(\xi,t) = 0 \,. \tag{2.42}$$

We now turn to the question of an appropriate choice for the initial function $f(x)$ in (2.34). As mentioned earlier we can not satisfy the initial conditions (1.24) with our expansion for arbitrary initial functions ψ_I,n_I,p_I which just satisfy the compatibility condition (1.27)-(1.29). From the previous asymptotic analysis we conclude

(2.43) Lemma: If the solution $w(x,t,\lambda)$ of (1.18)-(1.24) possesses an asymptotic expansion of the form (2.17) then the zero'th order term $\tilde{w}_I^0(x)$ in (1.30) has to satisfy

$$[(2\tilde{p}_I^0 + C)\tilde{\psi}^0_{I_x}]_x = C_{xx} \tag{2.44}$$

If (2.44), is satisfied then the zero order term $\tilde{w}^0(x,t) + \hat{w}^0(\xi,t)$ in (2.27), (2.38) satisfies the differential equations the boundary and the initial conditions up to terms of order $O(\lambda)$ if the initial function $f(x)$ in (2.34) is chosen as $\tilde{p}_I^0(x)$.

Proof: Since (2.28) is time independent it obviously has to hold for $t = 0$ if $w(x,t,\lambda)$ has an asymptotic expansion of the form (2.1). Therefore (2.44) is a necessary condition. On the other hand $\tilde{w}^0(x,t) + \hat{w}^0(\xi,t)$ satisfy the differential equations and the boundary conditions up to terms of order $O(\lambda)$. The compatibility condition (2.46) implies

$$0 = \tilde{n}_I^0 - \tilde{p}_I^0 - C \tag{2.47}$$

$$\hat{\psi}^0_{I_{\xi\xi}} = \hat{n}_I^0 - \hat{p}_I^0 \tag{2.48}$$

$$\hat{\psi}_I^0(0) = -\tilde{\psi}_I^0(0) \ . \tag{2.49}$$

If we choose $f(x)$ in (2.34) to be $\tilde{p}_I^0(x)$ then (2.33) and (2.27) imply $\tilde{n}^0(x,0) = \tilde{n}_I^0(x)$. In this case $\tilde{\psi}^0(x,0)$ satisfies

$$[(2\tilde{p}_I^0(x) + C(x))\tilde{\psi}_x^0(x,0)]_x = C_{xx}(x) \tag{2.50}$$

$$[\tilde{p}_I^0(0) + C(0)]e^{-\tilde{\psi}^0(0,0)} = \tilde{p}_I^0(0)e^{\tilde{\psi}^0(0,0)},\ \tilde{\psi}^0(1,0) = \psi_1(0) \ . \tag{2.51}$$

From (1.32) we conclude

$$\hat{J}^0_{n_I} = \hat{n}_{I_\xi} - (\tilde{n}_I(0) + \hat{n}_I)\hat{\psi}_{I_\xi} = 0 \tag{2.52}$$

since $\hat{n}_I$ decays exponentially for $\xi \to \infty$

$$\hat{n}_I(\xi) = \tilde{n}_I(0)[e^{\hat{\psi}_I} - 1] \tag{2.53}$$

and similarly

$$\hat{p}_I(\xi) = \tilde{p}_I(0)[e^{-\hat{\psi}_I} - 1] \tag{2.54}$$

has to hold. Inserting $\tilde{\psi}_I + \hat{\psi}_I$, $\tilde{n}_I + \hat{n}_I$, $\tilde{p}_I + \hat{p}_I$ into (1.29) and $\lambda \to \emptyset$ gives together with (2.47)

(2.55) $$[\tilde{p}_I^0(0) + C(0)]e^{-\tilde{\psi}_I^0(0)} = \tilde{p}_I^0 e^{\tilde{\psi}_I(0)} .$$

Thus $\tilde{\psi}_I$ and $\tilde{\psi}(x,0)$ satisfy the same differential equation (2.50) and the same boundary conditions. Since this problem has a unique solution (which follows from a simple maximum principle) $\tilde{\psi}_I^0(x) \equiv \tilde{\psi}^0(x,0)$ has to hold. Inserting (2.53) and (2.54) into (2.48) gives the same boundary value problem for $\hat{\psi}_I^0$ as $\hat{\psi}^0(\xi,0)$ has to satisfy namely

(2.56) $$\hat{\psi}^0_{I_{\xi\xi}} = \tilde{n}_I^0(0)[e^{\hat{\psi}_I^0} - 1] - \tilde{p}_I^0(0)[e^{-\hat{\psi}_I} - 1]$$

(2.57) $$\hat{\psi}_I^0(0) = -\tilde{\psi}_I(0), \quad \hat{\psi}_I(\infty) = 0 .$$

Since (2.56)-(2.57) again has a unique solution $\hat{\psi}_I^0(\xi) = \hat{\psi}^0(\xi,0)$ and therefore $\hat{n}_I^0(\xi) = \hat{n}^0(\xi,0)$ and $\hat{p}_I^0(\xi) = \hat{p}^0(\xi,0)$ has to hold.

<u>(2.58) Remark</u>: So if the initial functions ψ_I, n_I and p_I satisfy (2.44) in addition to (2.58) we have constructed a zero order asymptotic solution to problem (1.18)-(1.24). Higher order expansions can now be obtained in a straightforward manner. If the corresponding assumptions are made on the higher order terms of ψ_I, n_I and p_I these higher order expansions satisfy (1.18)-(1.24) to any arbitrary order in λ.

We now conclude this section with the following

<u>(2.59) Theorem</u>: If the initial functions ψ_I, n_I and p_I are the solution of the steady state problem

(2.60) $$\lambda^2 \psi_{I_{xx}} = n_I - p_I - C$$

(2.61) $$\text{(a)} \quad J_{n_{I_x}} = R, \qquad \text{(b)} \quad J_{n_I} = n_{I_x} - n_I \psi_{I_x}$$

(2.62) $$\text{(a)} \quad J_{p_{I_x}} = -R, \qquad \text{(b)} \quad J_{p_I} = -p_{I_x} - p_I \psi_{I_x}$$

(2.63) $$\psi_I(0) = 0 , \qquad \psi_I(1) = \psi_1(0)$$
$$n_I(0) = p_I(0) , \qquad n_I(1) = n_1$$

(2.64) $$J_{n_I}(0) = J_{p_I}(0)\ , \qquad p_I(1) = p_1$$

then the conditions (2.44) is satisfied.

For the proof of this statement the reader is referred to Markowich and Ringhofer (1982) where a singular perturbation analysis of the problem (1.18)-(1.24) can be found.

3. Fast Time Scale Expansions

In this section we investigate the asymptotic behavior of the solution of (1.18)-(1.24) if the conditions (2.44)-(2.45) on the initial functions are violated. It turns out that in this case there is an additional "fast" time scale present in the solution. Although this time scale will not be present in the physically meaningful situations, as we saw in the preceding section (since in these cases the initial functions are solutions of the steady state problem (2.17)-(2.19)), it is of some interest to investigate these "fast" solutions.

Any perturbation of the solution will propagate on this time scale. Therefore the understanding of this fast time scale behavior is important for the development of any numerical method for the solution of (1.18)-(1.24) (see Ringhofer [1983]). Thus we derive our asymptotic solution of (1.18)-(1.24) for arbitrary initial functions ψ_I, n_I and p_I which just satisfy (1.27)-(1.29) (and not necessarily (2.44)-(2.45)).

First we determine the possible fast time scale. For this purpose it is convenient to reformulate problem (1.18)-(1.24): Differentiating Poisson's equation with respect to time yields

(3.1) $$\lambda^2\psi_{xxt} = n_t - p_t \ .$$

Inserting for n_t and p_t from (1.19)-(1.20) and replacing n by $p + C + \lambda^2\psi_{xx}$ gives

(3.2) $$\text{(a)}\quad \lambda^2\psi_{xxt} = J_x, \qquad \text{(b)}\quad J(= J_n + J_p) = C_x + \lambda^2\psi_{xxx} - (2p + C + \lambda^2\psi_{xx})\psi_x$$

(3.3) $$\text{(a)}\quad p_t = J_{p_x} - R, \qquad \text{(b)}\quad J_p = -p_x - p\psi_x \ .$$

n and J_n are then given a posteriori by

(3.4) $$n = p + C + \lambda^2\psi_{xx}$$

(3.5) $$J_n = J - J_p = n_x - n\psi_x \ .$$

The boundary conditions (1.22)-(1.23) become

(3.6) $$\psi(0,t) = 0, \quad \lambda^2\psi_{xx}(0,t) = C(0), \quad J(0,t) = 2J_p(0,t)$$

(3.7) $$\psi(1,t) = \psi_1(t), \quad \lambda^2\psi_{xx}(1,t) = 0, \quad p(1,t) = p_1 \ .$$

Obviously (3.2), (3.3), (3.6), (3.7) are equivalent to (1.18)-(1.24) (provided the solutions are sufficiently smooth).

We now try to determine a possible fast time scale:

(3.8) $$\tau = \frac{t}{\lambda^{\alpha}} \, . \ \alpha > 0 \ .$$

Away from the p-n junction at $x = \emptyset$ where we expect the derivatives w.r.t.x to be bounded (3.2)(a)(b) assumes the form

(3.9) $$\lambda^2 \psi_{xxt} = C_{xx} - (2p + C)\psi_{xx} - (2p + C)_x \psi_x + O(\lambda^2) \ .$$

If $2p + C$ is in some sense "almost constant" away from $x = 0$ (which is the case for a constant doping profile and reverse bias) (3.9) behaves like an ordinary differential equation for ψ_{xx} and therefore exhibits the fast time scale t/λ^2. This heuristic argument can be made more rigorous: We supplement the asymptotic expansion (2.1) by an additional term varying on the fast time scale $\tau = t/\lambda^{\alpha}$ where $\alpha > 0$ has to be determined yet:

(3.10)
$$w(x,t,\lambda) \sim \sum_{j=0}^{\infty} [\bar{w}^j(x,t) + \hat{w}^j(\xi,t) + \tilde{z}^j(x,\tau)]$$
$$\xi = \frac{x}{\lambda}, \quad \tau = t\lambda^{-\alpha} \ .$$
$$\tilde{z}^j = (\tilde{\phi}^j, \tilde{u}^j, \tilde{v}^j, \tilde{J}^j_u, \tilde{J}^j_v)^T \ .$$

($\bar{w}^j$ and $\hat{w}^j$ are the terms of the expansion (2.1) derived in the previous chapter.) We assume that $\tilde{z}^j$ decays exponentially as $\tau \to \infty$.

(3.11) $$\|\tilde{z}^j(x,\tau)\| \le C_1 e^{-C_2 \tau} \ .$$

Then we have the following:

<u>(3.12) Lemma</u>: The only value of α for which there exists a non-trivial solution $\tilde{z}^0$ of the corresponding equations is $\alpha = 2$.

The proof of the lemma is trivial but lengthy and is therefor deferred to the appendix.

(3.13) Remark: The time scale $\tau = t/\lambda^2$ has the following physical meaning: In unscaled form it is of the size $\varepsilon U_T(DqC_{max})^{-1}$ sec which is proportional to the dielectric relaxation time (see Sze [1969]).

The equations for $\tilde{z}^0$ are now obtained from (3.2). Inserting the expansion (3.10) into (3.2)-(3.9), fixing x and $\tau = t/\lambda^2$ and $\lambda \to 0$ gives:

(3.14) $$\tilde{\phi}^0_{xx\tau} = -[(2\tilde{p}^0 + 2\tilde{v}^0 + C)\tilde{\phi}^0_x - 2\tilde{v}^0\tilde{\psi}^0_x]_x$$

(3.15) (a) $\tilde{v}^0_\tau = 0$, (b) $\tilde{J}^0_v = -\tilde{v}^0_x - (\tilde{p}^0 + \tilde{v}^0)\tilde{\phi}^0_x - \tilde{v}^0\tilde{\psi}^0_x$

(3.16) $$\tilde{u}^0 = \tilde{v}^0$$

(3.17) $$\tilde{J}^0_u = \tilde{u}^0_x - (\tilde{n}^0 + \tilde{u}^0)\tilde{\phi}^0_x - \tilde{u}^0\tilde{\psi}^0_x .$$

Here $\tilde{\psi}^0$, $\tilde{n}^0$ and $\tilde{p}^0$ have to be taken at $(x,0)$. From (3.15), we conclude $\tilde{u}^0 = \tilde{v}^0 = 0$. So we obtain $\tilde{\phi}^0$ from the equation

(3.18) $$\tilde{\phi}^0_{xx\tau} = -[(2\tilde{p}^0(x,0) + C(x))\tilde{\phi}^0_x]_x .$$

$\tilde{u}^0, \tilde{v}^0, \tilde{J}^0_U, \tilde{J}^0_v$ are then given a posteriori by

(3.19) $$\tilde{u}^0 = \tilde{v}^0 = 0$$

(3.20) (a) $\tilde{J}^0_v(x,\tau) = -\tilde{p}^0(x,0)\tilde{\phi}^0_x(x,\tau)$, (b) $\tilde{J}^0_u(x,\tau) = -\tilde{n}^0(x,0)\tilde{\phi}^0_x(x,\tau)$.

Thus there is (in the zero order approximation) no fast time scale correction to the carrier densities n and p ($\tilde{u}^0 = \tilde{v}^0 = 0$). This is reasonable since we expect the carrier flux to be much slower than the dielectric relaxation time. Correspondingly the fast time scale correction of the currents only involves the drift current. To determine a solution $\tilde{\phi}^0$ of (3.18) it is necessary to impose two boundary conditions at $x = 0$ and $x = 1$ and an initial condition at $\tau = 0$. Since we have already satisfied all boundary conditions with the slow time scale solution $\tilde{w}^0 + \hat{w}^0$ we obtain

(3.21) $$\tilde{\phi}^0(0,\tau) = 0, \quad \tilde{\phi}^0(1,\tau) = 0 .$$

We can prove the following result about the decay of $\tilde{\phi}^0$:

(3.22) Theorem: The solution of the problem

(3.23) $$\tilde{\phi}^0_{xx\tau} = -[(2\tilde{p}^0(x,0) + C(x))\tilde{\phi}^0_x]_x$$

(3.24) $$\tilde{\phi}^0(0,\tau) = 0, \quad \tilde{\phi}^0(1,\tau) = 0, \quad \tilde{\phi}^0(x,0) = g(x)$$

satisfies

(3.25) $$|\tilde{\phi}^0(x,\tau)| < Ce^{-\alpha\tau}$$

for some positive constant C and any α satisfying

(3.26) $$2\tilde{p}^0(x,0) + C(x) > \alpha .$$

The proof is deferred to the appendix.

We now investigate the choice of proper initial functions for the problems (2.28)-(2.34), (3.23)-(3.24) in order to construct an approximation to the solution of (1.18)-(1.24) for general initial data ψ_I, n_I and p_I which just satisfy the conditions (1.27)-(1.32). Since the zero-order-fast-time-scale corrections $\tilde{u}^0, \tilde{v}^0$ vanish (see (3.19)) we have to choose $\tilde{p}_I^0(x)$ as an initial function f for problem (2.28) in any case. This leads to the following procedure to obtain a zero order approximation to the solution of (1.18)-(1.24) for arbitrary initial data:

Step 1: Solve problem (2.28)-(2.34) with $f(x)(= \tilde{p}^0(x,0)) = \tilde{p}_I^0(x)$. Obtain $\tilde{\psi}^0(x,t)$ and $\tilde{n}^0(x,t)$. $\tilde{n}^0(x,0) = \tilde{n}_I^0(x)$ is then satisfied automatically because of (1.27).

Step 2: Solve the slow time-scale-boundary-layer problem (2.39)-(2.40). Obtain $\hat{\psi}^0$, $\hat{n}^0$ and $\hat{p}^0$. Because of conditions (1.27)-(1.33) $\hat{\psi}^0(\xi,0) = \hat{\psi}_I^0(\xi)$, $\hat{n}^0(\xi,0) = \hat{n}_I^0(\xi)$, $\hat{p}^0(\xi,0) = \hat{p}_I^0(\xi)$ holds.

If ψ_I, n_I and p_I are the solution of the steady state problem we are done. If not we have to perform

Step 3: Obtain $\tilde{\phi}^0$ by solving (3.23)-(3.24) with $g(x) = \tilde{\psi}_I^0(x) - \tilde{\psi}^0(x,0)$.

Now the solutions obtained by Steps 1-3 satisfy the differential equations together with the boundary and initial conditions up to terms of order $O(\lambda)$. Higher order approximations can now be obtained in a straightforward manner.

4. Numerical Examples

In this section we present some numerical results for a simple testproblem: We chose a diode of length 1μ and a doping profile C which is constantly $10^{16}cm^{-3}$ on the n-side (and because of symmetry $-10^{16}cm^{-3}$ in the p-region). This corresponds to $\lambda = 0.05$. We varied the applied bias from 0.5V reverse bias to 0.5V forward bias. Figures 1-7 show the solution for $x \in [0,1]$ where the p-n-junction is located at $x = 0$ and the contact at $x = 1$. To obtain a solution on $[-1,1]$ the graphs in Figures 1-7 would have to be continued according to (1.15). For all calculations backward finite differences have been used in time. In the x-direction we used the exponentially fitted finite difference method introduced by Scharfetter and Gummel (see Scharfetter and Gummel [1969] and Markowich et al. [1983b]) with an exponentially graded mesh near $x = 0$ to resolve the junction layer. Figures 1 and 2 show the "full" solutions of (1.18)-(1.24) for p and ψ respectively. As initial data the steady state solution for 0.5V reverse bias has been used. Thus no fast time scale solutions occur near $t = 0$. Figures 3 and 4 show the corresponding solutions of the reduced problem (2.28)-(2.34). In Figures 2 and 4 $\log_{10}(p)$ instead of p is plotted. To examine the fast time scale behavior we held the bias constant at ØV (equilibrium) and perturbed the initial conditions (thus violating (2.44)). The dotted line in Figure 5 is the equilibrium solution for ψ. The other line is the initial condition for ψ. Figure 6 shows the solution for ψ of the full problem (1.18)-(1.24) on the fast time scale $\frac{t}{\lambda^2}$. Figure 7 shows the initial layer term $\tilde{\phi}^0(x,\tau)$ in (3.10).

FIGURE 1

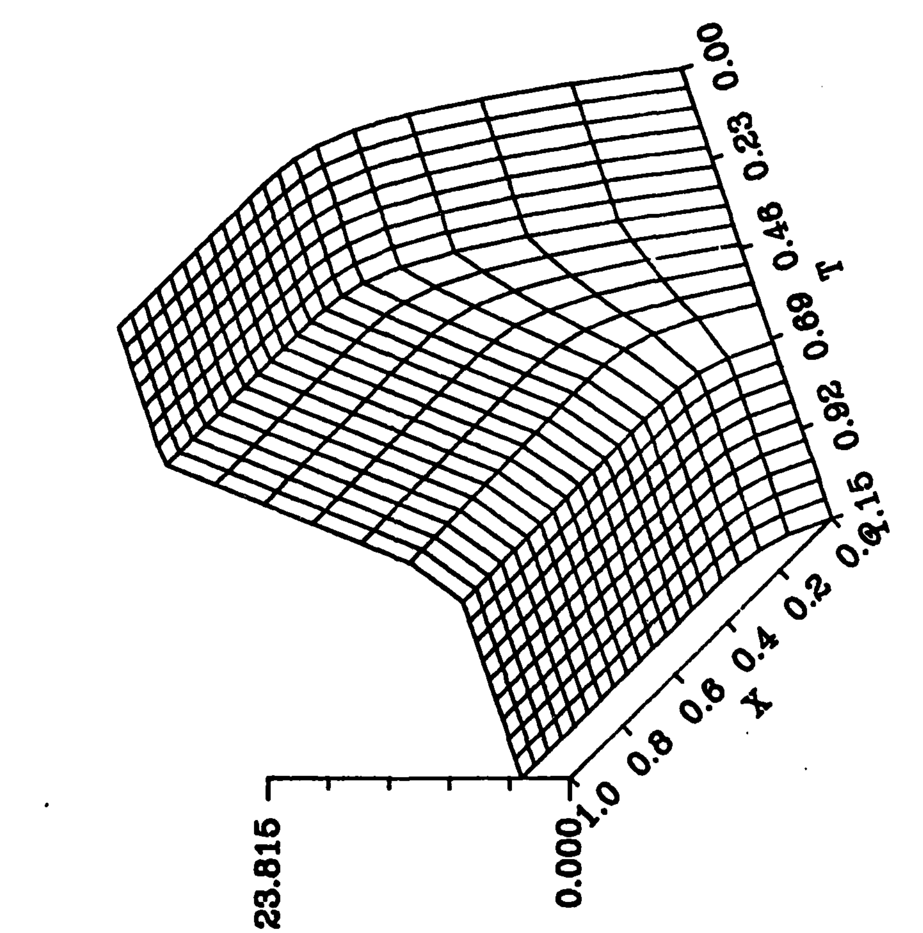


FIGURE 2

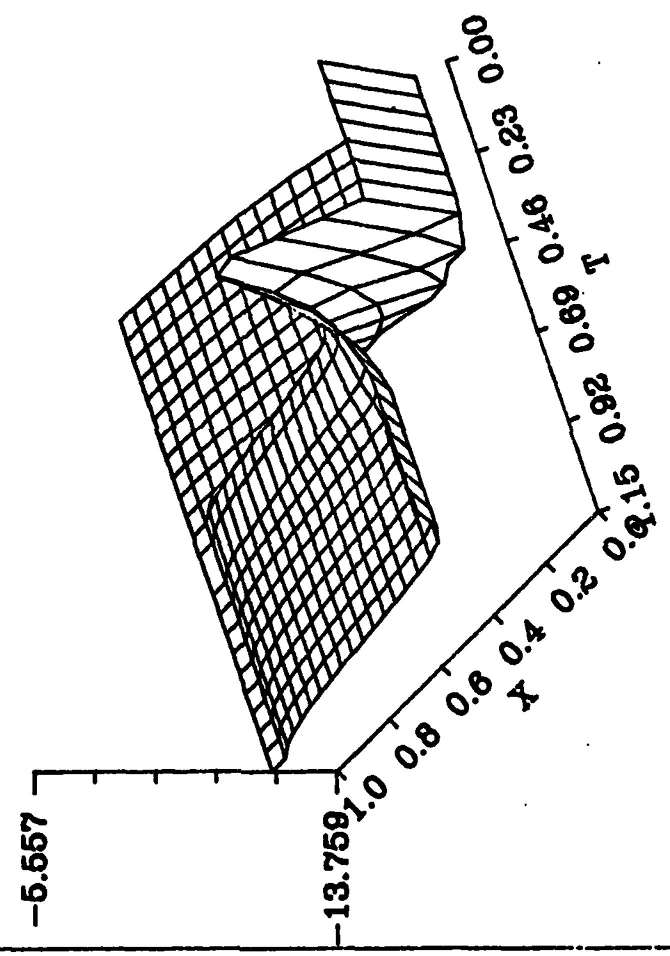
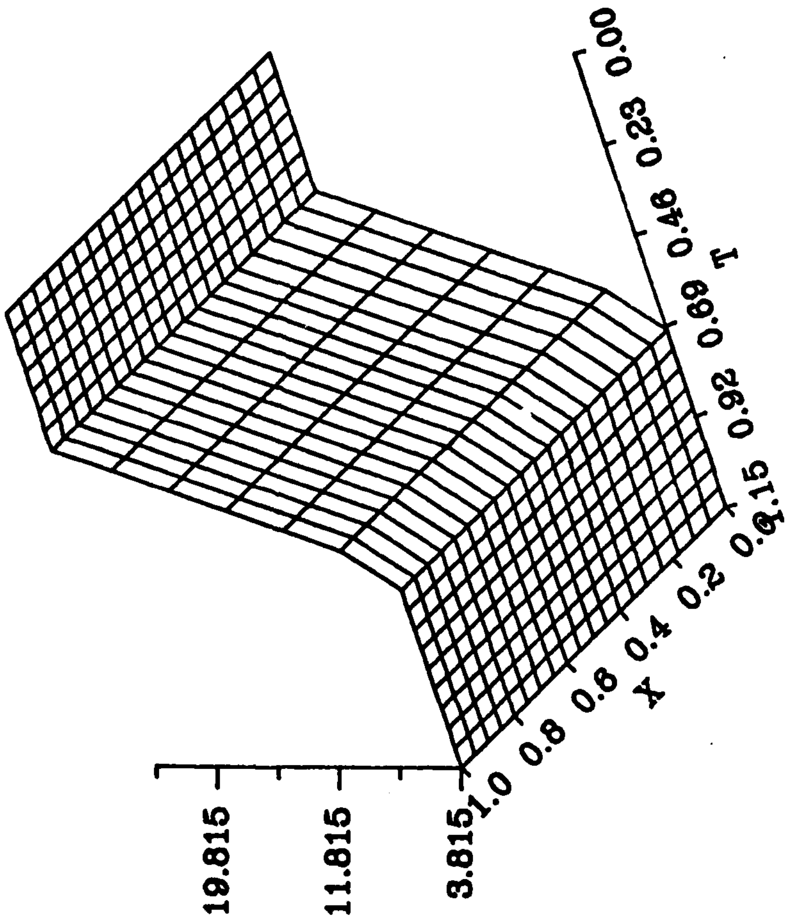


FIGURE 3



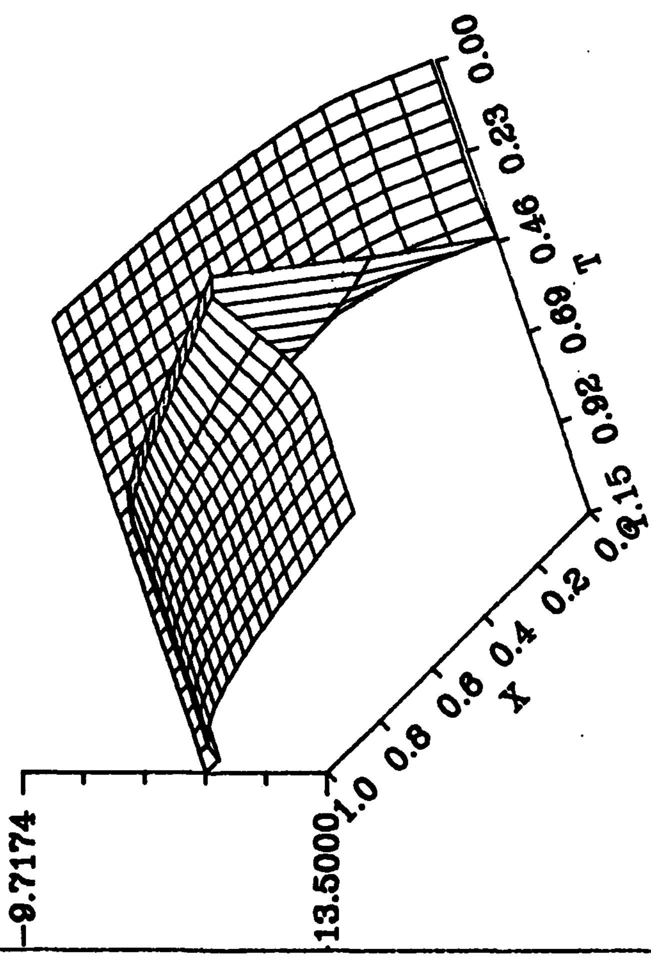


FIGURE 4

FIGURE 5

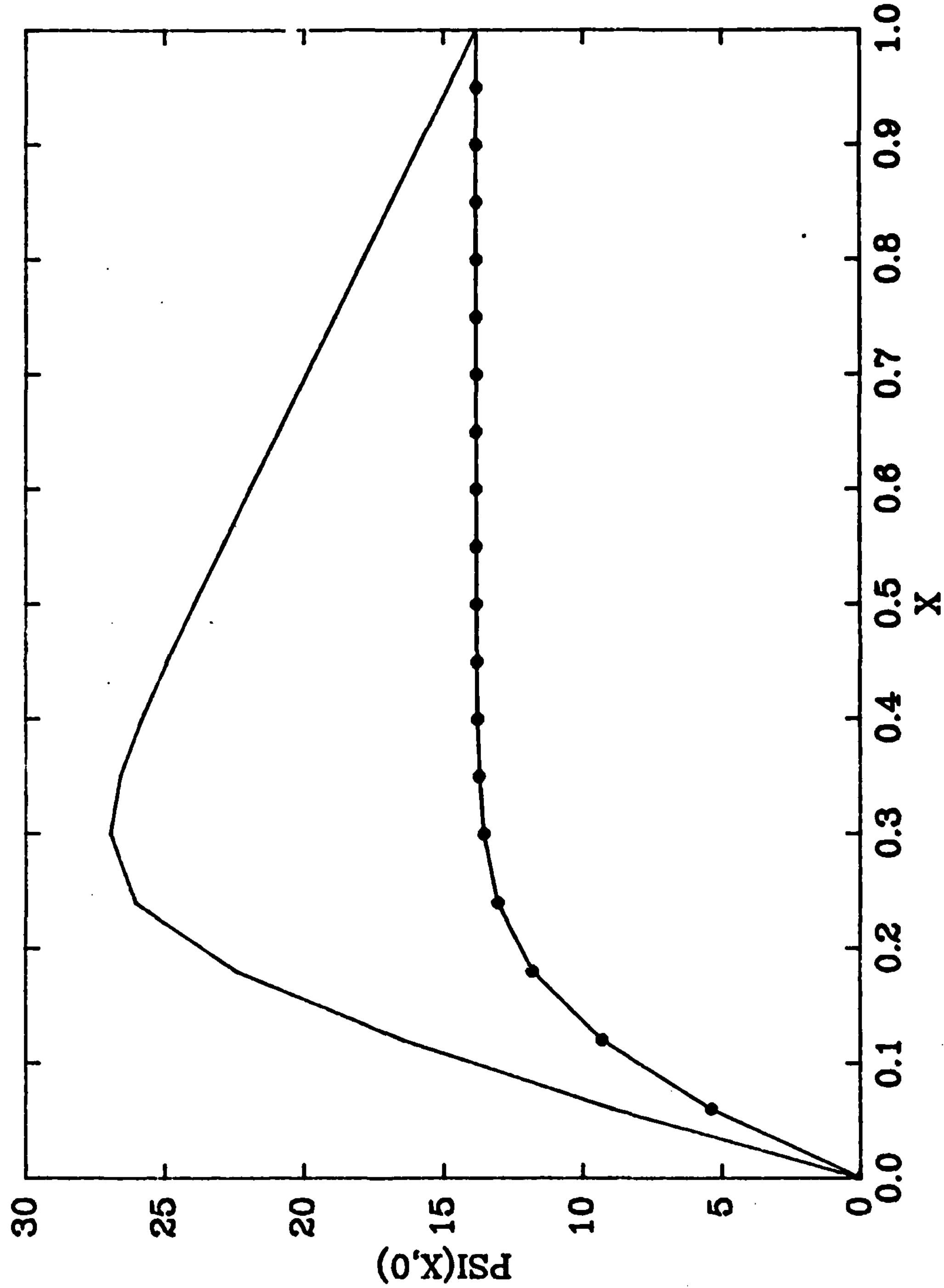


FIGURE 6

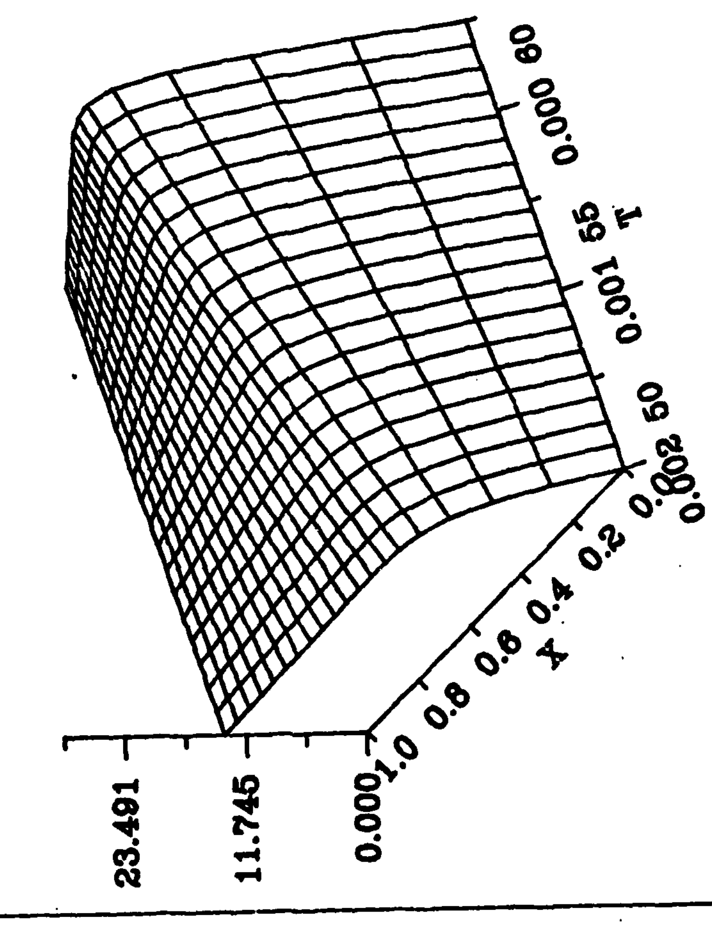
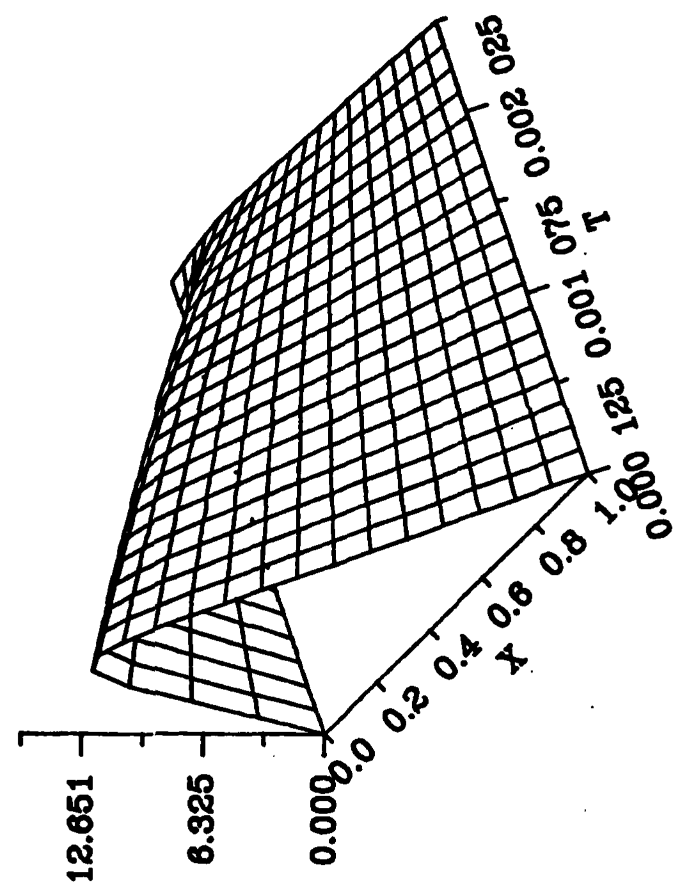


FIGURE 7



5. Appendix

(5.1) Proof of Theorem (2.36): Let α be given by

(5.2) $$\alpha = \max_x |f(x)| .$$

We employ the following transformation:

(5.3) $$p(x,t) = \alpha\bar{p}(x,t), \qquad J_p(x,t) = \alpha\bar{J}_p(x,t)$$

$$\psi(x,t) = \alpha\bar{\psi}(x,t) + \psi_1(t) + \ell n\left(\frac{C(x)}{C(1)}\right)$$

(5.4) (2.28)-(2.34) then becomes

(5.5) $$[(2\alpha\bar{p} + C)\bar{\psi}_x + 2p(\ell n\ C)_x]_x = 0$$

(5.6) (a) $\bar{p}_t = -\bar{J}_{p_x} - \bar{R}$, (b) $\bar{J}_p = -\bar{p}_x - \bar{p}[(\ell n\ C)_x + \alpha\bar{\psi}_x]$

$$\bar{R} = \frac{\bar{p}(\alpha\bar{p} + C)}{2\alpha\bar{p} + C + 2w}$$

(5.7) $$\bar{p}(x,0) = \bar{f}(x), \quad \bar{f}(x) := \frac{f(x)}{\alpha}$$

(5.8) (a) $\bar{p}(0,t) = [\alpha\bar{p}(0,t) + C(0)]e^{-2\alpha\bar{\psi}(0,t)} \cdot \rho(\alpha,t)$

(b) $2\bar{p}_x(0,t) = C(0)\bar{\psi}_x(0,t)$

(5.9) $$\bar{p}(1,t) = \bar{p}_1 = \bar{f}(1), \quad \bar{\psi}(1,t) = \emptyset .$$

$\rho(\alpha,t)$ is dependent on the size of the initial data (α) and the applied bias (ψ_1)

(5.10) $$\rho(\alpha,t) = \frac{C(1)}{C(0)} \cdot e^{-\psi_1(t)} \cdot \frac{1}{\alpha} \qquad (< \alpha_0) .$$

If α is small enough and ψ_1 is such that ρ is bounded (5.5)-(5.9) is up to a small perturbation linear in $\bar{\psi}$ and $\bar{p}$. Thus it can be solved by Picard iteration if the appropriate spaces are chosen. We define

(5.11) $$Y := \{(u,v) \mid u,v \in C^2[0,1] \times [0,T],\ U(1,t) = 0\ ,\ C(0)U_x(0,t) = 2v_x(0,t)\}$$

$$Z := (C[0,1] \times [0,T])^2 \times C^2[0,T] \times C^2[0,1]$$

and the operators

$$A(\psi,p) = ([(C\psi_x) + 2p(\ln C)_x]_x,\ p_t - p_{xx} - [p(\ln C)_x]_x\ ,\ p(0,t),\ p(1,t),\ p(x,0))$$

$$G(\psi,p) = (-2\alpha(p\psi_x)_x,\ \alpha(p\psi_x)_x,\ \rho(\alpha,t)e^{-2\alpha\psi(0,t)}(\alpha p(0,t) + C(0)),\ 0,0)\ .$$

Problem (5.5)-(5.9) can then be written in the form

$$A(\bar{\psi},\bar{p}) - G(\bar{\psi},\bar{p}) = (0,0,0,\bar{f}(1),\bar{f}(x))\ .$$

A is a linear operator from Y onto Z with a bounded inverse A^{-1}. (This can simply be shown by using Fourier expansions.) The Frechet derivative G' of G satisfies

(5.12) $$\|G'(p,\psi)\| \le \alpha[(2C(0) + \alpha|p(0)|)\rho(\alpha,t)e^{2\alpha\|\psi\|}]\ .$$

Thus it can be shown by the usual Picard iteration argument that the sequence defined by

(5.13) $$A(p_{k+1},\psi_{k+1}) = G(\psi_k,p_k) + (0,0,0,\bar{f}(1),\bar{f}(x))$$

converges to a solution $(\bar{p},\bar{\psi})$ of (5.5)-(5.9).

<u>(5.14) Proof of Lemma (3.12):</u> Inserting the expansion (3.10) into (3.2) (which are exponentially small away from $x = 0$) gives

(5.15) $$\lambda^{2-\alpha}\tilde{\phi}^0_{xx\tau} = \lambda^2\tilde{\phi}_{xxxx} - [(2\tilde{p}^0 + C + \lambda^2\tilde{\psi}^0_{xx} + 2\tilde{U}^0 + \lambda^2\tilde{\phi}^0_{xx})\tilde{\phi}^0_x]_x$$
$$- [(2\tilde{U}^0 + \lambda^2\tilde{\phi}^0_{xx})\tilde{\psi}^0_x]_x + \text{h.o.t.}$$

(5.16) $$\tilde{U}^0_\tau = \lambda^\alpha[\tilde{U}^0_x + (\tilde{p}^0 + \tilde{U}^0)\tilde{\phi}^0_x + \tilde{U}^0\tilde{\psi}^0_x]_x + \lambda^\alpha\tilde{R}^0\ .$$

Here $\tilde{\psi}^0$ and $\tilde{p}^0$ have to be evaluated at $(x,\tau\lambda^\alpha)$. (5.16) gives

$$\tilde{U}^0(x,\tau) = \emptyset$$

for $\lambda \to 0$. If $\alpha > 2$ holds then (5.15) becomes

(5.17) $$\tilde{\phi}^0_{xx\tau} = 0$$

for $\lambda \to 0$. Together with the boundary conditions (3.21) this gives

(5.18) $$\tilde{\phi}^0(x,\tau) = 0\ .$$

But in this case $\tilde{z}^0 \equiv \emptyset$ follows immediately. If $\alpha < 2$ holds then (5.15) gives for $\lambda \to 0$

(5.19) $$-[(2\tilde{p}^0 + C)\tilde{\phi}^0_x]_x = \emptyset\ .$$

Again using the boundary conditions (3.21) gives only the trivial solution for $\tilde{\phi}^0$ (and therefore also for $\tilde{z}^0$). Thus $\alpha = 2$ has to hold to get a zero'th order correction of $\tilde{U}^0$.

(5.20) Proof of Theorem (3.33): Let y be defined by

(5.21) $$y(x,\tau) = \tilde{\phi}^0(x,\tau)e^{\alpha\tau}$$

for some α satisfying (3.37). Then y satisfies

(5.22) $$y_{xx\tau} = -[(2\tilde{p}^0(x,0) + C(x) - \alpha)y_x]_x$$

(5.23) $$y(0,\tau) = y(1,\tau) = 0 .$$

Multiplying (5.22) by y and integrating by parts gives

(5.24) $$\frac{1}{2}\frac{\partial}{\partial\tau}\int_0^1 y_x^2 dx = -\int_0^1 (2\tilde{p}(x,0) + C(x)-\alpha)y_x^2 dx .$$

If

$$\min_x |2\tilde{p}(x,0) + C(x) - \alpha| > 0$$

holds then the L_2 norm of y stays bounded and (3.25) holds.

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In this paper we carry out an asymptotic analysis of the system of differential equations describing the transient behavior of a p-n-junction device (i.e. a diode). We determine the different time-scales present in the equations and investigate which of them actually occur in physical situations. We derive asymptotic expansions of the solution and perform some numerical experiments.

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