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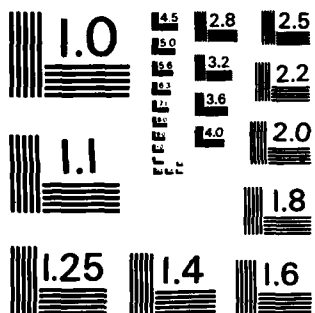
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PRODUCTION RATE VARIATIONS COST MODELS

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14 August 1984

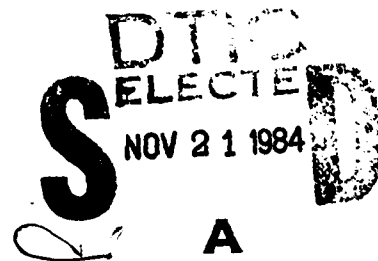
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19. ABSTRACT (Continue on reverse if necessary and identify by block number) This research examines a model that may be used to estimate the cost impact of production rate changes on a timely and real world basis. Part I analyzes current models that are being implemented by Air Force Systems Command, and demonstrates that these models are deficient in their theoretical definition and empirical construction. Part II presents current status of the development of a more realistic rate variations model for the F-16 aircraft program.					
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I. INTRODUCTION

There has been much interest in recent years in the relationships among learning, production rate, and program costs. These relationships are of particular interest in the military acquisition of made-to-order equipment. At the outset of a weapon system program, a tentative monthly production schedule for the life of the program is negotiated between the contracting parties. This planning schedule covers the life of program, but formal contractual agreements between the Department of Defense and manufacturers usually cover only annual delivery requirements. Since annual funding allocations are characterized by political uncertainties, there is often a need to deviate from the planned production rate during the production phase of the program. Coincident with these rate changes, new cost estimates are required to support contract negotiations and additional funding requests.

There are many proposed methodologies for assessing the cost impact of a production rate change. A recent group of models constructed for Air Force Systems command is based on application of the Alchian^{1,2} cost function. Even within this framework there is very little agreement about the relationships among learning, production rate, and program cost. While some studies, for example, Womer¹⁴ and Womer and Gullledge¹⁵ make assumptions concerning the cost impact of the above factors in developing models of optimal contractor behavior, others [e.g., Smith¹³, Large, et. al.¹², Bemis^{4,5}, Cox and Gansler⁸, Crouch⁹, Cox, et. al.⁷, Bohn and Kratz⁶ address the problem directly by attempting to statistically estimate the above influences. In these latter studies contractor behavior is not a part of the modeling effort. The purpose of this research is to interface the two types of studies and show how estimates of the influences of learning



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and production rate, which are often statistically unreliable, may be inconsistent with optimal contractor behavior. In addition, an alternative method for estimating rate variations is proposed.

II. OBJECTIVES

The specific objectives related to the more general introduction are as follows:

1. to review and criticize the current Air Force Systems Command production rate model,
2. to propose and begin testing of a new production rate model that is more sensitive to the needs of Aeronautical Systems Division (ASD/ACCR).

It was understood from the beginning that it would be impossible to complete this project. That is, the present Air Force Systems Command model was developed by a team of researchers over a one year period with a \$150,000 budget. It would be impossible to review the existing system and devise a new system in a ten week research period. The agreed objective was to extend the work as far as possible.

III. ESTIMATING COST IMPACTS

The learning curve, first formulated by Wright¹⁶, is an empirically specified relationship that yields declining units costs with increases in cumulative output. In recent years the more commonly used terminology has been "improvement curve." The improvement curve allows for reductions in cost that are due to factors other than repetition (learning). Gold¹⁰ includes changes in product design, product mix, technology, facilities, etc. in this listing of other factors. Both the learning and improvement curves are described mathematically as

$$Z = \beta_0 X_1^{\beta_1} \quad (1)$$

where

Z = the unit cost of the X_1 th unit,

β_0 = a constant, commonly called the first unit cost,

β_1 = a parameter describing the slope of the quantity/cost curve,

X_1 = cumulative quantity produced.

Studies attempting to ascertain the relationships among production rate, learning, and program costs generally use the following augmented model:

$$Z = \beta_0 X_1^{\beta_1} X_2^{\beta_2} \quad (2)$$

where

X_2 = some measure (usually a proxy) for production rate,

β_2 = a parameter describing the slope of the rate/cost curve.

Some researchers (e.g., Bohn and Kratz⁶ call equation (2) the "rate analysis curve model."

The parameters in equation (2) are estimated from the log-linear form of the relationship using the linear regression or directly from (2) using nonlinear regression. Unfortunately both of these techniques often are plagued with statistical problems due to the collinearity between the independent variables, X_1 and X_2 . The source of this collinearity may be reasoned as follows. Often made-to-order production programs are characterized by initial production at a low rate with a gradual buildup in production rate throughout the program. In fact, given a learning curve, if the resource use rate does not decline, production rate must increase during the program. As a result cumulative quantity is highly correlated with production rate.

There are additional problems with the formulation described by equation (2). Cox and Gansler⁸ and Bohn and Kratz⁶ use lot size as a proxy

for production rate. However, the time required to produce a lot often changes over the life of the program. This is true in much of the data that these authors have analyzed, namely the C141 airframe program, the F102 airframe program, the F4 airframe program, and the Army's Black Hawk helicopter program. For example, lot sizes of 15 and 20 are not good proxies for production rate as the time horizons for the two lots are 1 and 4/3 years respectively.

Still, the main problem encountered is collinearity between output rate and cumulative output. Large, et. al.¹² concluded that the influence of production rate could not be estimated with confidence. Many later studies have also been unable to significantly measure the influence of production rate. Both positive and negative estimates of β_2 , the slope of the rate/cost curve, have been obtained. Assuming that an increase in rate requires an increase in resources, a positive slope for the rate/cost curve implies decreasing returns, that is, an increase in production rate causes an increase in required resources (and hence cost). A negative slope implies increasing returns since an increase in rate requiring an increase in resources decreases unit cost.

As discussed by Cox and Gansler⁸, different signs for β_2 , even if statistically significant, are not necessarily contradicting. In the short-run, both increasing and decreasing returns can exist. Even if the data indicate falling unit cost as rate increases, this does not necessarily imply increasing returns to the variable factors. The firm could be producing in the region of diminishing returns on the short-run cost surface, but the dominating learning (cumulative quantity) effect could be causing unit costs to decline.

After considering the above cost impacts, this research demonstrates that estimates obtained from production data using equation (2) are often

inconsistent with optimal contractor behavior. In addition, the assertion that the production on all major Air Force programs since the F-100 have been characterized by increasing returns (lower than optimal production rates) is also examined. On the surface this assertion seems illogical if the contract is written so as to induce cost minimizing behavior and the contractor is interested in making a profit. If the assertion were true, the contractor would certainly have incentives to increase production rate.

IV. CONTRACTOR BEHAVIOR

The model presented by Womer¹⁴ is used to demonstrate that negative slopes for rate/cost ($\beta_2 < 0$) result in optimal behavior which is inconsistent with observed and logical contractor behavior. Consider the following defining notation:

C = total discounted program cost,

$q(t)$ = production rate at time t ,

$Q(t)$ = cumulative production at time, i.e., $Q(t) = \int_0^t q(\tau) d\tau$,

$x(t)$ = the use rate of a variable composite resource at time t ,

γ = a parameter describing the returns to the variable resource,

δ = a learning parameter,

ρ = the discount rate,

A = a constant,

V = the total planned units to be produced,

T = the planned time horizon for the program.

The following production function is specified:

$$q(t) = Ax^{1/\gamma(t)}Q^\delta(t). \quad (3)$$

For the moment no assumption is made about the sign of γ , but it is assumed that $A > 0$ and $0 < \delta < 1$. Notice that solving equation (3) for $x(t)$ yields an

improvement curve if production rate is assumed constant, i.e.,

$$\frac{x(t)}{q(t)} = A^{-\gamma} q^{\gamma-1}(t) Q^{-\delta\gamma}(t). \quad (4)$$

Also, notice the equivalence with equation (2) when production rate is allowed to vary. Note that $\beta_1 = -\delta\gamma$ and $\beta_2 = \gamma-1$. The relationship in equation (4) is combined with a behavioral assumption to construct a simple model of optimal firm behavior.

There may be some discussion about the appropriate assumption that governs the firm's behavior. The assumption here is that the contract is structured so as to induce cost minimizing behavior on the part of the contractor. This could be in the form of a fixed price contract, but most likely as a cost-plus incentive or award fee contract.

If cost is measured in units of the variable resource, the firm's objective may be stated as

$$\text{Minimize } C = \int_0^T x(t) e^{-\rho t} dt \quad (5)$$

subject to:

$$q(t) = A x^{1/\gamma}(t) Q^{\delta}(t),$$

$$Q(0) = 0,$$

$$Q(T) = V,$$

$$x(t) \geq 0.$$

This is a problem in optimal control theory, but if it is assumed that the last constraint $[x(t) \geq 0]$ is satisfied, it is possible to use classical variational techniques to solve the problem. The complete solution to the problem presented in (5) is not needed to demonstrate the hypothesized result, however a transformation simplifies the required analysis somewhat. Let $Z(t) = Q^{1-\delta}(t)/(1-\delta)$. This implies $z(t) = dZ/dt = Q^{-\delta}(t)q(t)$. The optimization problem may now be restated as

$$\text{Minimize } C = \int_0^T A^{-\gamma} z^{\gamma}(t) e^{-\rho t} dt \quad (6)$$

subject to:

$$z(0) = 0$$

$$z(T) = v^{1-\delta}/(1-\delta).$$

A sketch of the calculus of variations solution is as follows. Let

$$I = \int_0^T [z_0(t) + \epsilon h(t)]^{\gamma} e^{-\rho t} dt \quad (7)$$

where $h(t)$ is a function that gives the difference between the assumed optimal path, $z_0(t)$, and any other path, and ϵ is an arbitrary constant (see Kamien and Schwartz¹¹ for a comprehensive derivation). If equation (7) is treated as function of ϵ , the optimum must occur when $\epsilon=0$. The derivative is

$$I'(\epsilon) = \int_0^T \gamma [z_0(t) + \epsilon h(t)]^{\gamma-1} h(t) e^{-\rho t} dt, \quad (8)$$

and after equating with zero the following is obtained:

$$I'(0) = 0 = \int_0^T \gamma z_0^{\gamma-1}(t) h(t) e^{-\rho t} dt. \quad (9)$$

After integrating equation (9) by parts, it is possible to obtain the Euler equation of the calculus of variations. Womer¹⁴ has derived the extremals for production rate, resource use rate, and discounted cost by solving the Euler equation.

Additional insight is gained by examining the second variational. This is stated as

$$I''(\epsilon) = \int_0^T \gamma(\gamma-1) [z_0(t) + \epsilon h(t)]^{\gamma-2} h^2(t) e^{-\rho t} dt. \quad (10)$$

After evaluating at zero, the following expression is obtained:

$$I''(0) = \int_0^T \gamma(\gamma-1) z_0^{\gamma-2}(t) h^2(t) e^{-\rho t} dt. \quad (11)$$

For the problem in (6) to have a minimum, it is necessary that (11) be

nonnegative. As long as $\gamma > 1$, equation (11) is nonnegative and the solution is a minimum. From an economic point of view, it is obvious what type of behavior is implied when $\gamma < 1$. The contractor has incentive to delay all production to the end of the program because of the combined efforts of increasing returns and discounting. It is easy to show mathematically that you can make the integral in (6) approach zero by letting $z(t)$ be zero until the last instance of time. In short, the solution does not make sense. It implies contractor behavior that is inconsistent with observed contractor behavior.

The author does not deny that increasing returns ($\gamma < 1$) may exist, particularly during the start-up period of production. However, economic theory suggests that the contractor will add resources if the contract is written to induce such cost minimizing behavior. It is highly unlikely that increasing returns to the variable resources exist throughout the production program. This type of irrational contractor behavior has not been noticed in previous research. Certainly the contractor would not plan (as shown by the model) to be in such a situation after start-up.

There are many applications where estimates of the parameters in equation (2) are provided. For example, Bemis⁴ provides a table of estimates for many defense items. The estimated values for the quantity slope (β_1) and the rate slope (β_2) are transformed to the corresponding γ and δ values as presented in equation (4) and are presented in Table 1. Notice that all of the estimated values for γ are less than one. The estimates for tactical missile programs presented by Cox and Gansler⁸ are transformed and presented in Table 2. While the Bullpup and Tow estimates seem reasonable, the Sparrow and Sidewinder estimates have $\gamma < 1$.

<u>System</u>	<u>γ</u>	<u>δ</u>
Aircraft A	.9635	.4692
Aircraft C	.6690	.5580
Aircraft D	.4436	.4084
Aircraft E	.4265	.3377
Aircraft F	.1966	1.0560
Aircraft G	.7031	.3578
Aircraft H	.8703	.4902
Helicopter	.8367	.3047
Jet Engine A	.8797	.4718
Jet Engine B	.8400	.5786
Missile G&C	.2485	.4904
Ordnance Item A	.9561	.1912
Radar Set A	.8286	.1245
Radar Set B	.8734	.0183

Note: Six additional items were included in Bemis' data summary. These were not included because in four cases the rate slope was not provided, and in two cases the quantity slope was not provided.

Table 1. Values of γ and δ implied by Bemis' data summary.

<u>System</u>	<u>γ</u>	<u>δ</u>
Sparrow (1st source)	.9782	.2467
Sparrow (2nd source)	.8844	.2197
Bullpup	1.0058	.2794
Tow	1.0101	.0129
Sidewinder	.7119	-.0931

Table 2. Estimates of γ and δ for missile programs as presented by Cox and Gansler

V. DISCUSSION

The combined influence of improvement and production rate on cost is still a topic that requires much additional research. Most previous modeling attempts must be interpreted with extreme care because they suffer from severe statistical problems. If there were no data problems (e.g., engineering change orders) and production rate could be measured accurately, the regression equation may be a valid tool for prediction purposes. However, any attempt to make any statement about the estimates from equation (2) is futile.

In terms of production planning, a contractor certainly would not plan to operate at a less than optimal rate if cost minimization is induced by the contract. In many production programs planned production has exceeded actual production. Only the inability of the contractor to deliver on cost and on schedule has resulted in decreased production. In view of the results of this research, it would seem that the above phenomena could imply that contractors are producing at greater than optimal rates given fixed facilities, so that diminishing returns to the variable resources exist throughout the program after some start-up period. Again it must be noted that this does not imply the average cost per unit would have to rise as output rate is increased. The improvement effect could dominate the rate effect and average cost per unit could decline.

This research suggests that regression models as presented in equation (2) are not the answer to the problem. These results are noted and studied since Air Force Systems Command has developed and most likely will use equations such as those analyzed in this paper.

VI. AN ALTERNATIVE RATE VARIATIONS MODEL

After discussing this problem with Aeronautical Systems Division (ASD/ACCR) personnel, it became clear that a pure statistical model was not the proper methodology for solving this problem. In particular, the data was extremely deficient. On some programs (e.g., B1-B), the data on lengthy production runs is just not available. On other programs significant engineering changes caused the data to be unreliable. Also, conversations with Col. L. L. Smith shed additional insight on the problem. Smith noted

the following. The models constructed for Air Force Systems Command (these models use the Bohn and Kratz methodology) are used to predict total flyaway cost for a particular program. The modeling methodology was never intended to examine total flyaway cost. The total flyaway cost problem is a much more difficult problem, a cost accounting problem. Smith calls this problem the "budget" problem. The methodology developed by Smith was meant to be applied in the production phase of an airframe program. It was not designed or deemed capable of predicting total flyaway cost.

The above comments were confirmed by Aeronautical Systems Division (ASD/ACCR). In fact, the cost information that is needed as output from the rate variations model is a proforma cost budget for each program. This budget provides annual cost estimates for various categories for 10 years into the future. For example, the cost breakdowns for the F-15 weapon system program are presented in Table 3.

Airframe	Nonrecurring	Gross Weapon System
Propulsion	Flyaway (total)	Gross Weapon System (U)
Electronics	Flyaway (unit)	Advance Buy Credit
Armament	Training	Net Weapon System
Other	PGSE	Advance Buy
ECO	Data	Total Requirements
Recurring Flyaway (total)	SE Conformal Full Tanks	Initial Spares
Recurring Flyaway (unit)	Peculiar Support Total	Total Procurement

Table 3. Cost Categories required for the F-15 Weapon System program for a ten year time horizon.

Many of the items in Table 3 may be estimated without a model by Air Force personnel with a high level of confidence. But the point is, a single equation statistical model will never be able to achieve the level of disaggregation required by Table 3.

The initial strategy of this research effort was to identify the important factors that impact cost through production rate changes and then

to try to integrate these factors into some type of prediction model. Two things became clear immediately: (1) some of the important cost drivers are qualitative and should be included subjectively, and (2) the redistribution of contractor overhead costs was by far the most important cost driver. In fact, Aeronautical System Division personnel stated that if they could determine the redistribution of overhead costs. they could construct a reasonable consistent estimated cost budget.

Therefore, to reduce the scope of the research effort, the decision was made to construct a model that examines overhead redistribution after production rate changes. Also, because of the size of the project and the severe time limitations, it was decided that only the F-16 program should be examined. This decision was partly for convenience since the F-16 System Program Office is located at the research location. At the time the project was initiated, it was understood that there was no chance of completing the project during the ten week research period. This effort is one component of an ongoing research project.

VII. OVERHEAD REDISTRIBUTION MODELS

After examining the relevant literature, one methodology appeared to be clearly superior considering the type of data that is available on weapon systems programs. Balut³ presents a model for redistributing overhead costs. The model was developed for the Office of the Assistant Secretary of Defense (Program Analysis and Evaluation). The model is not described in detail in this report because of space limitations. The interested reader should consult Balut's³ original paper. The technique presented by Balut is a two step procedure. The first step requires the construction of a program price curve, at the recurring flyaway level, and repricing according to the

given quantity change. The second step is an adjustment for changes in production rate.

The methodology is extremely data intensive since contractor overhead data is required for each program, however the data is available through the Contractor Cost Data Reporting System. Balut stated in personal conversations that the data analysis is an absolute requirement, but it is worth the effort. He has had extremely good luck with the model on a variety of major weapon systems programs. Still, the model was approached with cautious optimism since the F-16 program is quite different from many other programs. The overhead costs are affected by the fact that General Dynamics does not own the building or the land where the aircrafts are produced. This leads to a situation where the short-run fixed overhead percentage is much lower than the aerospace industry average. In short, some modifications on Balut's methodology were anticipated.

VIII. THE DATA

The data was taken from two sources:

1. The overhead data was taken from the annually produced plant wide data summaries (DD Form 1921-3).
2. The quantity data was provided by the F-16 System Program Office.

The compilation of the data into the proper form was a major part of the research effort. In fact, half of the research period was spent collecting and adjusting data. In the last week of the research period, a complete and consistent data base for the F-16 program was finally obtained. The data is not reproduced here since it was submitted in confidence to the government. It is for official use only, and it cannot be disclosed without prior written permission from General Dynamics, Fort Worth Division.

IX. PRELIMINARY FINDINGS

The initial results from the modeling effort indicate that the F-16 model will be able to predict overhead redistributions as rate changes. It must be stressed that these results are tentative. Time limitations and software limitations at the research location prevented a complete analysis of the model. The final model that will be presented to the Business Research Management Center will be completed early in the Fall 1984.

X. RECOMMENDATIONS

The recommendations are somewhat dependent on the final outcome of the empirical work, however it appears that the following can be stated:

1. The production rate variations problem at the total flyaway level is a cost accounting problem, not a simple statistical problem. It should be recognized as such.
2. The problem is solvable if it is possible to predict overhead cost redistribution after production rate changes. Therefore Air Force Systems Command should supplement its current statistical formulations with models that provide the information that is needed by Aeronautical Systems Division.

It should be noted that the construction of appropriate models will be a major effort. However, a considerable amount of work has been completed. For example, the data is available at the Office of the Assistant Secretary of Defense. The major effort will involve analyzing and organizing the data in the proper form for model construction, but irregardless of the research hours required, the project should be initiated.

REFERENCES

1. A. A. Alchian. 1959. Costs and outputs. In the Allocation of Economic Resources (M. Abramovitz, et. al., eds.) Stanford: Stanford University Press, 23-40.
2. A. A. Alchian. 1963. Reliability of progress curves in airframe production. *Econometrica*, 31 (October), 679-693.
3. S. J. Balut. 1981. Redistributing fixed overhead costs. *Concepts*, 4, 63-76.
4. J. C. Bemis. 1983. A model for examining the cost implications of production rate. Paper presented at 50th meeting of the Military Operations Research Society, Annapolis, Maryland.
5. J. C. Bemis. 1981. A model for examining the cost implications of production rate. *Concepts*, 4 (Spring), 84-94.
6. M. Bohn and L. A. Kratz. 1984. The impact of production rate on unit costs. (Paper presented at the ORSA/TIMS joint national meeting) San Francisco, California.
7. L. Cox, B. Dembroski, D. Elam, and P. Martin. 1981. Analyses to support evaluation of the potential benefits of competition in cruise missile production. Arlington, Virginia: The Analytic Sciences Corporation.
8. L. W. Cox and J. S. Gansler. 1981. Evaluating the impact of quantity, rate, and competition. *Concepts*, 4 (Autumn), 29-53.
9. R. Crouch, 1980. Avoiding bias in progress functions. *Defense Management Journal*, 16 (Third Quarter), 40-45.
10. B. Gold. 1981. Changing perspectives on size, scale, and returns. *Journal of Economic Literature*, 19 (March), 5-33.
11. M. I. Kamien and N. L. Schwartz. 1981. *Dynamic Optimization: The Calculus of Variations and Optimal Control in Economics and Management*. New York: North-Holland.
12. J. P. Large, K. Hoffman, and F. Kontrovich. 1976. *Production Rate and Production Costs*, R-1609-PA&E Santa Monica: The Rand Corporation.
13. L. L. Smith. 1976. *An Investigation of Changes in Direct Labor Requirements Resulting from Changes in Airframe Production Rate*, PhD. Thesis. Eugene: The University of Oregon.
14. N. K. Womer. 1979. Learning curves, production rate, and program costs. *Management Science*, 25 (April), 312-319.

15. N. K. Womer and T. R. Gullledge, Jr. 1983. A dynamic cost function for an airframe production program. Engineering Costs and Production Economics (September), 213-27.
16. T. P. Wright. 1936. Factors affecting the cost of airplanes. Journal of Aeronautical Sciences, 3 (February), 122-128.

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