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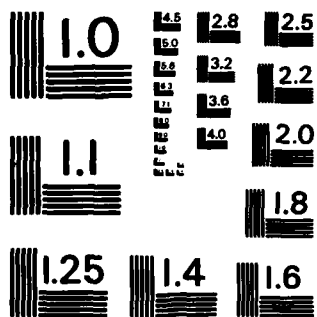
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18 ABSTRACT (Continue on reverse if necessary and identify by block number)

During this period the investigator worked on the development of parallel algorithms to be used in the following linear algebra areas --- stability and inertia problems, controllability and observability problems, pole assignment problems, and matrix equations problems (Sylvester, Lyapunov, Riccati, etc). In particular, algorithms have been developed which require $O(n \log n)$ steps for solution on $O(n^2)$ processors. Several presentations on these results were given, including a talk at the SIAM Fall Meeting. Three papers have been accepted for publication and several more are in preparation.

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Interim Report on
"Sequential and Parallel Matrix Computations"

by
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DeKalb, Illinois 60115



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The system of differential equations

$$\dot{x}(t) = Ax(t) + Bu(t) \quad (1.1)$$

and its discrete counter part

$$\dot{x}_{k+1}(t) = Ax_k(t) + Bu(t) \quad (1.2)$$

Where A and B are constant matrices of appropriate dimensions and x and u are time dependent vectors, arise in a wide variety of practical situations. These include design and analyses of industrial complexes such as chemical plants, electro-mechanical machines, like motor cars, aircraft, spaceships, economics structures of countries, etc. Associated with these systems are many interesting Linear Algebra problems. The very well-knowns are

STABILITY AND INERTIA PROBLEMS

CONTROLLABILITY AND OBSERVABILITY PROBLEMS

POLE ASSIGNMENT PROBLEMS

MATRIX EQUATIONS PROBLEMS (SYLVESTER, LYAPUNOV, RICCATI, etc.)

In addition, there are many other subproblems such as (1) evaluating e^{At} (2) determining Relative Primeness of two polynomials or matrices (3) finding the Cauchy index of rational functions, etc.

Computer solutions of these problems are of utmost importance. Unfortunately, only very few viable sequential methods are available for solutions of these problems. Moreover, FAST ($O(n)$ or $O(n \log_2 n)$ steps) PARALLEL algorithms for these problems do not exist at all (as far as these investigators are aware).

The major objectives of this project are to develop fast computational algorithms, both sequential and parallel, for the problems mentioned above and, to study parallel arithmetic complexities of these problems, i.e. how far these problems can be solved assuming that sufficient number of processors are available [8, 43].

Up to date, the following accomplishments have been made:

(A) We have developed FAST parallel algorithms ($O(n \log n)$ steps - $O(n^2)$ processors algorithms) for (i) determining controllability of the pair (A,B) where A is a companion matrix and B is full (ii) single-input pole assignment problem (iii) stability problems for polynomials and (iv) problems for determining relative primeness of two polynomials and the Cauchy index of a rational function. These algorithms have parallel efficiency of $O(\frac{1}{\log n})$ in most cases.

A desirable feature of these algorithms is that they use only linear algebraic operations for which parallel algorithms already do exist.

(B) We have also been able to show (sometimes reconstructing the proposed $O(n \log n)$ steps - $O(n^2)$ processors algorithms, sometimes using entirely different techniques) that parallel arithmetic complexities of all the control problems mentioned above are upper bounded by $O(\log^2 n)$ parallel steps.

(C) We have developed a FAST sequential algorithm (two to three times faster than the best known methods) for solving the symmetric positive semidefinite Lyapunov matrix equation $XA + A^T X = -C^T C$.

(D) A FAST direct sequential method for finding the eigenvalue distribution of a matrix inside and outside various regions of the complex plane including half planes, shifted half planes, hyperbolas, sectors, quadrants, imaginary axis, regions contained within two straight lines that pass through the origin etc.

(E) We have just developed a parallel algorithm for computing the zeros of a polynomial whose zeros are all real and distinct. This algorithm requires

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