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EVOLUTION OF NONLINEAR WAVE GROUPS ON WATER OF  
SLOWLY-VARYING DEPTH(U) TECHNION - ISRAEL INST OF TECH  
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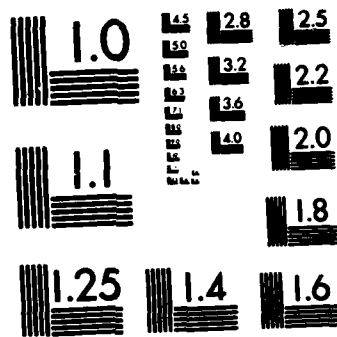
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EVOLUTION OF NONLINEAR WAVE GROUPS ON  
WATER OF SLOWLY-VARYING DEPTH

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FOURTH ORDER EVOLUTION EQUATIONS FOR  
SHOALING WAVE GROUPS

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Third order evolution equation for groups of water-waves moving over an uneven bottom were first derived by Djordjevic' and Redekopp (1978). In the present report we carry their derivation one step further, to fourth order, utilizing the REDUCE 2 algebraic manipulator, see Hearn (1973).

We consider the evolution of a uni-directional progressive gravity wave moving along the x axis on the free surface of a homogeneous liquid with depth  $h = h(x)$  varying in the direction of the propagation. The effect of surface tension is neglected, so the analysis applies to gravity waves only. The fluid motion is irrotational, thus there exists a velocity potential  $\phi(x,z,t)$  satisfying Laplace's equation

$$\phi_{xx} + \phi_{zz} = 0 \quad (1)$$

where  $z$  is the vertical coordinate, and  $z = 0$  is the undisturbed free surface.

The boundary condition on the bottom  $z = -h(x)$  is

$$\phi_z = -h'(x) \phi_x \quad (2)$$

and the boundary conditions on the free surface  $z = \zeta(x,t)$  are the kinematic condition:

$$\phi_z = \zeta_t + \phi_x \zeta_x \quad (3)$$

and the pressure condition:

$$2g\zeta + 2\phi_t + \phi_x^2 + \phi_z^2 = 0 \quad (4)$$

The situation where the depth varies slowly in the direction of propagation is considered, so that properties characterizing the wave will change slowly as well. A small nondimensional parameter  $\epsilon$  that measures the slope of the wavy surface is introduced, and we define the new variables:

$$\tau = \epsilon \left[ \int^x \frac{dx}{C_g(\xi)} - t \right]; \quad \xi = \epsilon^2 x \quad (5)$$

where  $C_g$  is the group velocity.

We suppose that the depth changes on the scale of  $\epsilon^2$  so that  $h = h(\xi)$  with the property  $h'(\xi) = O(1)$ .

The velocity potential and the free surface displacement are expanded as follows:

$$\phi = \phi_0(\xi, \tau, z) + (\phi_1(\xi, \tau, z) e^{i\theta} + \phi_2(\xi, \tau, z) e^{2i\theta} + \dots + c.c) \quad (6)$$

$$\zeta = \zeta_0(\xi, \tau) + (\zeta_1(\xi, \tau) e^{i\theta} + \zeta_2(\xi, \tau) e^{2i\theta} + \dots + c.c) \quad (7)$$

where  $\theta = \int^x k(\xi) dx - \omega t$  (8)

and c.c means complex conjugate.

With  $\epsilon$  chosen to be small, the functions  $\phi_j(\xi, \tau, z)$  and  $\zeta_j(\xi, \tau), j > 1$  are expanded formally in powers of  $\epsilon$  as follows:

$$\phi_j(\xi, \tau, z) = \sum_{m=j}^{\infty} \epsilon^m \phi_{mj}(\xi, \tau, z) \tag{9}$$

$$\zeta_j(\xi, \tau) = \sum_{m=j}^{\infty} \epsilon^m \zeta_{mj}(\xi, \tau) \tag{10}$$

We also assume  $\phi_0(\xi, \tau, z) \leq 0(\epsilon)$  (11)

thus only wave induced currents are considered.

Substituting the Fourier series (6) into the Laplace equation (1) we obtain that the zero order potential satisfies:

$$\nabla^2 \phi_0(\xi, \tau, z) = 0$$

=====

(12)

From (11) and (12) it follows that:

$$\phi_{0,zz}(\xi, \tau, z) \leq 0(\epsilon^3) \tag{13}$$

Expanding the free surface conditions (3) and (4) around the equilibrium position  $z = 0$ , substituting eqs. (6) to (10) and looking for the coefficient of  $\epsilon^0$ , yields respectively:

$$\begin{aligned} & \phi_{0z} + \epsilon \tau_{0\tau} + 2\epsilon^3 \cdot \text{Re}[\zeta_{11} \bar{\phi}_{21} + \zeta_{21} \bar{\phi}_{11} + \frac{ki}{C_g} \zeta_{11} \bar{\phi}_{11} - \\ & - k^2 \phi_{11} \bar{\zeta}_{21} - \frac{ik}{C_g} \zeta_{11} \bar{\phi}_{11} - k^2 \zeta_{11} \bar{\phi}_{21}] + 0(\epsilon^4) = 0 \end{aligned} \quad (14)$$

and

$$\tau_0 = \frac{\epsilon}{g} \phi_{0\tau} + 2\text{Re} \frac{\epsilon^2}{g} [i\omega \phi_{11z} \bar{\zeta}_{11}] - \frac{\epsilon^2}{g} [k^2 \phi_{11} \bar{\phi}_{11} + \phi_{11z} \bar{\phi}_{11z}] + 0(\epsilon^3) \quad (15)$$

where  $\phi_0 = \phi_0(\xi, \tau, 0)$ ,  $\phi_{ij} = \phi_{ij}(\xi, \tau, 0)$ , and the bar denotes the complex conjugate.

From (14) and (15) it follows that the order of  $\tau_0$  is greater or equal to  $\epsilon^2$  and that the order of  $\phi_{0z}$  is greater or equal to  $\epsilon^3$

$$\tau_0 \in O(\epsilon^2) \quad (16)$$

and

$$\phi_{0z} \in O(\epsilon^3) \quad (17)$$

The next step of the derivation is similar to that given by Djordjevic' and Redekopp with the difference that we continue the process to fourth order. Substitution of (6) (8) and (9) into Laplace's equation (1) gives:

$$\phi_{11} = A(\xi, \tau) \frac{\cosh k(z+h)}{\cosh kh} \quad (18)$$

$$\underline{O(\epsilon^2)e^{i\theta}}: \quad \phi_{21} = D(\xi, \tau) \frac{\cosh k(z+h)}{\cosh k h} - \frac{iA_\tau}{C_g} \cdot$$

$$\cdot \frac{(z+h) \sinh k(z+h) - h\sigma \cosh k(z+h)}{\cosh k h} \quad (19)$$

$$\underline{O(\epsilon^3)e^{i\theta}}: \quad \phi_{31} = G(\xi, \tau) \frac{\cosh k(z+h)}{\cosh k h} -$$

$$- \frac{i}{2 \cosh k h} \{2kh'A + (k'A - \frac{i}{C_g^2} A_{\tau\tau})(z+h)\} \cdot$$

$$\cdot (z+h) \cosh k(z+h) + \frac{i}{\cosh k h} \{\sigma(hk)'A -$$

$$- A_\xi - \frac{ih\sigma}{C_g^2} A_{\tau\tau} - \frac{1}{C_g}\} D_\tau \cdot (z+h) \sinh k(z+h) \quad (20)$$

$$\underline{O(\epsilon^4)e^{i\theta}}: \quad \phi_{41} = M(\xi, \tau) \frac{\cosh k(z+h)}{\cosh k h} +$$

$$+ [ \frac{i}{6C_g^3} A_{\tau\tau\tau} - \frac{k'}{2C_g} A_\tau ] [(z+h)^3] \frac{\sinh k(z+h)}{\cosh k h} -$$

$$- \frac{kh'}{C_g} A_\tau (z+h)^2 \frac{\sinh k(z+h)}{\cosh k h} +$$

$$\begin{aligned}
 & + \left\{ \frac{1}{2kC_g} \{ (kh') [3\sigma - 2kh(2\sigma^2 - 1)] + kh'\sigma - hk'\sigma - 2kh\sigma \cdot \frac{C'_g}{C_g} \right. \\
 & + \left. \frac{2\sigma^2}{1-\sigma^2} \frac{k'}{k} \right\} A_\tau + \frac{h\sigma}{C_g} A_{\tau\xi} - \frac{i\sigma^2}{1-\sigma^2} \frac{k'}{k} D - iD_\xi - \frac{iG_\tau}{C_g} \} \cdot \\
 & \cdot (z+h) \frac{\sinh k(z+h)}{\cosh kh} + \left\{ \frac{1}{4kC_g} \left[ \frac{-4\sigma^2}{1-\sigma^2} k' + 2kh\sigma k' + 2k \frac{C'_g}{C_g} \right] A_\tau \right. \\
 & - \left. \frac{i h \sigma}{2C_g^3} A_{\tau\tau\tau} - \frac{A_{\xi\tau}}{C_g} - \frac{D_{\tau\tau}}{2C_g^2} - \frac{ik'}{2} D \right\} (z+h)^2 \frac{\cosh k(z+h)}{\cosh kh} + \\
 & + \left\{ - \frac{h'}{C_g} (1 - kh\sigma) A_\tau - ikh'D \right\} (z+h) \frac{\cosh k(z+h)}{\cosh kh} \tag{21}
 \end{aligned}$$

$$\begin{aligned}
 \underline{\underline{O(\epsilon^2)e^{2i\theta}}}: \quad \phi_{22} = F(\xi, \tau) \frac{\cosh 2k(z+h)}{\cosh 2kh} \tag{22}
 \end{aligned}$$

$$\begin{aligned}
 \underline{\underline{O(\epsilon^3)e^{2i\theta}}}: \quad \phi_{32} = E(\xi, \tau) \frac{\cosh 2k(z+h)}{\cosh 2kh} - \frac{iF_\tau}{C_g} \cdot \\
 \cdot (z+h) \sinh \frac{2k(z+h)}{\cosh 2kh} \tag{23}
 \end{aligned}$$

Substitution of eq (6) to (10) and eqs. (18) to (23) into the expansion around  $z = 0$  of the free surface conditions (3) and (4) gives:

$$\underline{O(\epsilon)e^{i\theta}}: \quad \zeta_{11} = \frac{i\omega}{g} A \quad (24)$$

and

$$\omega^2 = g k \sigma \quad (25)$$

$$\text{where } \sigma = \text{tgh}(kh) \quad (26)$$

$$\underline{O(\epsilon^2)e^{i\theta}}: \quad C_g = \frac{g}{2\omega} [\sigma + kh(1-\sigma^2)] \quad (27)$$

and

$$g\zeta_{21} = i\omega D + A_\tau \quad (28)$$

$$\underline{O(\epsilon^2)e^{2i\theta}}: \quad g\zeta_{22} = -\frac{k^2}{2} \frac{(3-\sigma^2)}{\sigma^2} A^2 \quad (29)$$

and

$$\omega F = \frac{3}{4} ik^2 \frac{(1-\sigma^4)}{\sigma^2} A^2 \quad (30)$$

$$\underline{O(\epsilon^3)e^{i\theta}}: \quad \epsilon^2 \mu_1 A + \epsilon^3 \mu_2 A_\xi + \epsilon^3 \mu_3 A_{\tau\tau} + \epsilon^2 \mu_4 A \phi_{O_\tau} + \epsilon^3 \mu_5 |A|^2 A = 0 \quad (31)$$

where

$$\mu_1 = -i \left\{ \frac{\sigma^2}{1-\sigma^2} \frac{k'}{k} [\sigma + hk(1-\sigma^2)] + (hk)' \right\} \quad (32)$$

$$\mu_2 = - \frac{2\omega i C_g}{g} \quad (33)$$

$$\mu_3 = \frac{1}{g} \left[ 1 - \frac{gh}{C_g^2} (1-kh\sigma)(1-\sigma^2) \right] \quad (34)$$

$$\mu_4 = \frac{k^2}{g} \left[ 2 \frac{C_p}{C_g} + (1-\sigma^2) \right] ; \quad C_p = \omega/k \quad (35)$$

$$\mu_5 = \frac{k^4}{2g} \left[ \frac{9}{\sigma^2} - 12 + 13\sigma^2 - 2\sigma^4 \right] \quad (36)$$

and

$$\begin{aligned} g\zeta_{31} = & i\omega G + \left(1 + \frac{\omega h\sigma}{C_g}\right) D_\tau + \left(\frac{i\omega k\sigma}{g} - \frac{ik}{C_g}\right) \phi_{o_\tau} A + \\ & + \frac{i\omega h^2}{2C_g^2} (2\sigma^2 - 1) A_{\tau\tau} + h\omega\sigma A_\xi - \\ & - \frac{ik^4}{2\omega\sigma^2} \{3 - 9\sigma^2 + 11\sigma^4 - 2\sigma^6\} A^2 \bar{A} + \\ & + \frac{h\omega}{2} [2kh' + k'h - 2\sigma^2(kh)'] A \end{aligned} \quad (37)$$

$$\begin{aligned} \underline{\underline{O(\epsilon^3)e^{2i\theta}}}: \quad & \frac{4\sigma^3}{1+\sigma^2} E = \frac{k}{2\omega C_g} [3kh(3\sigma^4 + 2\sigma^2 - 5) + 9\sigma(1-\sigma^2)] A A_\tau \\ & + \frac{6ik\omega}{g} (1-\sigma^2) AD \end{aligned} \quad (38)$$

and

$$\begin{aligned} g\zeta_{32} = & \frac{ik}{4C_g\sigma^3} [-12kh(1-\sigma^2) - 4\sigma^3 + 12\sigma] A A_\tau \\ & - \frac{k^2}{\sigma^2} (3-\sigma^2) AD \end{aligned} \quad (39)$$

$$\begin{aligned}
 \underline{\underline{0(\epsilon^4)e^{i\theta}}}: & \quad \epsilon^4\mu_1 D + \epsilon^4\mu_2 D_\xi + \epsilon^4\mu_3 D_{\tau\tau} + \epsilon^3\mu_4 D\phi_{O_\tau} + \epsilon^4 \cdot 2\mu_5 |A|^2 D + \\
 & \quad + \epsilon^4\mu_5 A^2 \bar{D} + \epsilon^4\lambda_1 A_\tau + \epsilon^4\lambda_2 A_{\tau\tau\tau} + \epsilon^4\lambda_3 |A|^2 A_\tau + \epsilon^4\lambda_4 A^2 \bar{A}_\tau + \\
 & \quad + \epsilon^4\lambda_5 A_{\xi\tau} + \epsilon^3\lambda_6 A\phi_{O_{\tau\tau}} + \epsilon^3\lambda_7 A_\tau \phi_{O_\tau} + \epsilon^3\lambda_8 A\phi_{O_\xi} + \\
 & \quad + \epsilon\lambda_9 A\phi_{O_z} = 0 \tag{40}
 \end{aligned}$$

where  $\mu_1, \dots, \mu_5$  are given by (32) to (36) and

$$\begin{aligned}
 \lambda_1 = -\frac{h'g^2}{4\omega^2 C_g^3} & \quad (2h^3k^3\sigma^7 - 6h^3k^3\sigma^5 + 6h^3k^3\sigma^3 - 2h^3k^3\sigma - \\
 & \quad 5h^2k^2\sigma^6 + 9h^2k^2\sigma^4 - 5h^2k^2\sigma^2 + h^2k^2 + 4hk\sigma^5 - \\
 & \quad - 4hk\sigma^3 - \sigma^4 + \sigma^2) \tag{41}
 \end{aligned}$$

$$\lambda_2 = \frac{ih^2(1-\sigma^2)}{3C_g^3} [-3\sigma + kh(3\sigma^2-1)] \tag{42}$$

$$\lambda_3 = \frac{ik^3}{2gC_g\sigma^3} \{7\sigma^7 - 47\sigma^5 + 48\sigma^3 - 36\sigma + kh[-7\sigma^8 + 32\sigma^6 - 25\sigma^4 - 18\sigma^2 + 18]\} \tag{43}$$

$$\lambda_4 = \frac{ik^3}{2gC_g} \{-\sigma^4 + 5\sigma^2 + kh\sigma(1-\sigma^2)^2\} \tag{44}$$

$$\lambda_5 = \frac{-2h(1-\sigma^2)}{C_g} (1-kh\sigma) \quad (45)$$

$$\lambda_6 = \frac{i\omega k\sigma}{g} - \frac{2ik}{gC_g} - \frac{i\omega}{gC_g^2} \quad (46)$$

$$\lambda_7 = -\frac{ik}{gC_g} \left\{ \frac{2C_g}{C_g} + 2[1-kh\sigma][1-\sigma^2] + 2 \right\} \quad (47)$$

$$\lambda_8 = \frac{2\omega}{g} k \quad (48)$$

$$\lambda_9 = -\frac{i\omega k\sigma}{g} \quad (49)$$

Combining eqs. (31) and (40) and introducing the new variable

$$A_1 = \epsilon A + \epsilon^2 D \quad (50)$$

one finally obtains the modification to fourth order of the cubic Schrödinger equation

$$\begin{aligned} & \epsilon^2 \mu_1 A_1 + \epsilon^2 \mu_2 A_1 \xi + \epsilon^2 \mu_3 A_1 \tau \tau + \epsilon \mu_4 A_1 \phi_{O\tau} + \mu_5 |A_1|^2 A_1 + \\ & + \epsilon^3 \lambda_1 A_1 \tau + \epsilon^3 \lambda_2 A_1 \tau \tau \tau + \epsilon \lambda_3 |A_1|^2 A_1 \tau + \epsilon \lambda_4 A_1^2 \bar{A}_1 \tau + \\ & + \epsilon^3 \lambda_5 A_1 \xi \tau + \lambda_6 \epsilon^2 A_1 \phi_{O\tau \tau} + \epsilon^2 \lambda_7 A_1 \phi_{O\tau} + \lambda_8 \epsilon^2 A_1 \phi_{O\xi} + \lambda_9 A_1 \phi_{Oz} = 0 \end{aligned} \quad (51)$$

On the other hand substitution of (18) and (24) into (14) and (15) and elimination of  $\zeta_0$ , yields the following equation for  $\phi_0$ :

$$\phi_{0z} + \frac{\epsilon^2 \phi_{0\tau\tau}}{g} = \epsilon \frac{k^2}{g} \left[ \frac{2C}{C_g} + (1-\sigma^2) \right] (|A|^2)_\tau + O(\epsilon^4) \quad (52)$$

which can be written to the same order of accuracy as

$$\phi_{0z} + \frac{\epsilon^2 \phi_{0\tau\tau}}{g} = \epsilon \frac{k^2}{g} \left[ \frac{2C}{C_g} + (1-\sigma^2) \right] (|A_1|^2)_\tau \quad (53)$$

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Laplace's equation (12) and the boundary conditions (2) at  $z = -h$  and eqs. (51), (53) at  $z = 0$  form the system of equations from which  $A_1$  and  $\phi_0$  can be determined.

For the case of infinitely deep water, the terms  $\lambda_6 \cdot \epsilon^2 A_1 \phi_{0\tau\tau}$ ,  $\lambda_7 \epsilon^2 A_{1\tau} \phi_{0\tau}$ , and  $\lambda_8 \epsilon^2 A_1 \phi_{0\xi}$  in (53) are  $O(\epsilon^5)$ , and equations (53) and (51) become respectively:

$$\phi_{0z} = \epsilon \frac{4k^2}{g} (|A_1|^2)_\tau \quad (54)$$

and

$$\begin{aligned} & -\epsilon^2 \cdot \frac{2\omega i C_g}{g} A_{1\xi} + \epsilon^2 \frac{A_1}{g} \tau\tau + \frac{2\omega k}{g C_g} A_1 \phi_{0\tau} + \\ & + \frac{4k^4}{g} |A_1|^2 A_1 + \epsilon \left[ -\frac{14ik^3}{g C_g} |A_1|^2 A_{1\tau} + \frac{2ik^3}{g C_g} A_1^2 \bar{A}_{1\tau} \right] \\ & - \frac{i\omega k}{g} A_1 \phi_{0z} = 0 \end{aligned} \quad (55)$$

The last two equations are identical to those given in K.B. Dysthe (1980).

Note that the following typographical errors were found in Dysthe's paper:

His equations (2.17) and (2.19) should be written as follows:

$$\Gamma = 4k^4 |A|^2 + 8ik^3 (AA_x^* - A^* A_x) - 4ik^3 |A|_x^2 + 2\omega k (\bar{\phi}_x - i\bar{\phi}_z) \quad (\text{Dysthe 2.17})$$

$$2i(A_t + \frac{1}{2}A_x) + \frac{1}{2}A_{yy} - \frac{1}{2}A_{xx} - A|A|^2 = -\frac{1}{8}i(6A_{xyy} - A_{xxx}) + \frac{3}{2}iA(AA_x^* - A^* A_x) - \frac{1}{2}|A|_x^2 + A(\bar{\phi}_x - i\bar{\phi}_z) \quad (\text{Dysthe 2.19})$$

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3. Hearn, A.C. 1973, REDUCE 2 User's Manual, University of Utah.

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