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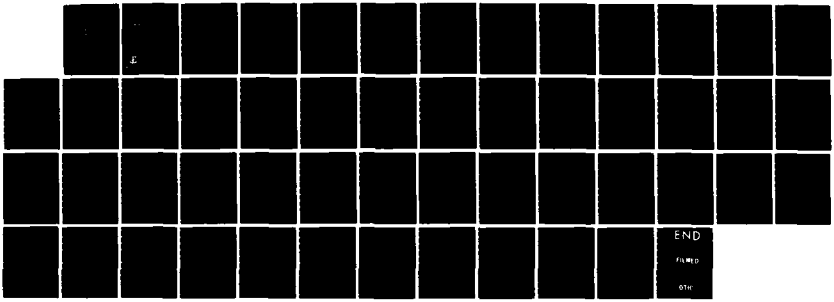

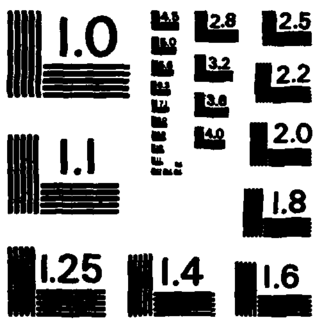

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ESTIMATE OF SOUND ABSORPTION DUE TO MAGNETOELASTIC INTERACTIONS

BY KURT P. SCHARNHORST

RESEARCH AND TECHNOLOGY DEPARTMENT

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Estimates of sound absorption by means of magnetoelastic induction of conduction currents are given. Two examples are analyzed: (a) Induction of eddy currents by means of magnetoelastic coupling between a viscoelastic absorber and a conducting ground plane, and (b) Induction of eddy currents in an electrically conducting viscoelastic material in a uniform magnetic field. We conclude that significant absorption occurs only if the average magnetic field energy density and the mechanical kinetic energy density given by the product

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20. Abstract (Cont.)

of the mass density and the square of the speed of sound in the material are of comparable magnitude.

FOREWORD

The present work arose out of a need to consider sound absorption processes in viscoelastic materials other than naturally occurring dissipation by molecular relaxation. The latter becomes especially weak at low frequencies and should be supplemented by alternative mechanisms. Of several possible electromechanical and magnetomechanical dissipation processes we choose to investigate absorption due to mechanically induced eddy currents.

The work was carried out in the Nonmetallic Materials Branch of the Materials Division of the Research and Technology Department under an IR grant.

Approved by:

J. R. Dixon

JACK R. DIXON, Head
Materials Division

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ILLUSTRATIONS

ILLUSTRATIONS (Cont.)

CHAPTER 1

ACOUSTIC REFLECTION FROM A UNIFORMLY MAGNETIZED VISCOELASTIC PLATE ON A CONDUCTING GROUND PLANE

The purpose of this report is to assess the performance of an acoustic attenuator in the form of a viscoelastic sheet attached to an electrically conducting backing, the sheet consisting of either alternating magnetized and unmagnetized strips of material or a distribution of magnetized disks in an unmagnetized lattice. In the present paper we consider the situation where the magnetized regions, assumed to be electrically nonconducting, are well separated, thus focussing attention on absorption due to induced eddy currents in the backing or ground plane while neglecting interactions between magnetized regions. We also treat the magnetized regions as if they were infinite in extent as far as the acoustic response is concerned, thus neglecting edge effects due to differences in the elastic responses of magnetized and unmagnetized regions. These elastic responses may be different not only because of the magnetic moment but also because of basic differences in the elastic moduli and densities of the materials.

The two simplest magnetic structures in this category are uniformly magnetized, either in the plane of the sheet or perpendicular to it. In the present paper we focus on uniform, perpendicular magnetization.

In order to come up with a readily calculable problem, we consider only one-dimensional geometries. We realize at the outset however, that finite uniformly magnetized plates do not generate uniform magnetic fields. We therefore have to make a number of simplifying assumptions to reduce the magnetoelastic interaction problem to a one-dimensional one. Consequently, the resulting theoretical developments will be highly approximate. In addition, we have to make certain mathematical approximations. Nevertheless it is hoped that the results will convey at least the order of magnitude of the expected attenuation.

Consider a uniformly magnetized plate with the magnetization vector perpendicular to a conducting ground plane and bonded laterally to an unmagnetized viscoelastic material, as shown in Figure 1. Let the shape of the plate be either a circular disk or a long, rectilinear slab. We will argue that we may replace the magnetization by an equivalent surface current distribution, as shown in Figure 1. We may then solve for time varying fields above and below the ground plane as functions of time varying surface currents, $I_o(z,t)$ (ampere-turn/cm), where the time variation of I_o is ultimately due to acoustic vibrations in the plate.

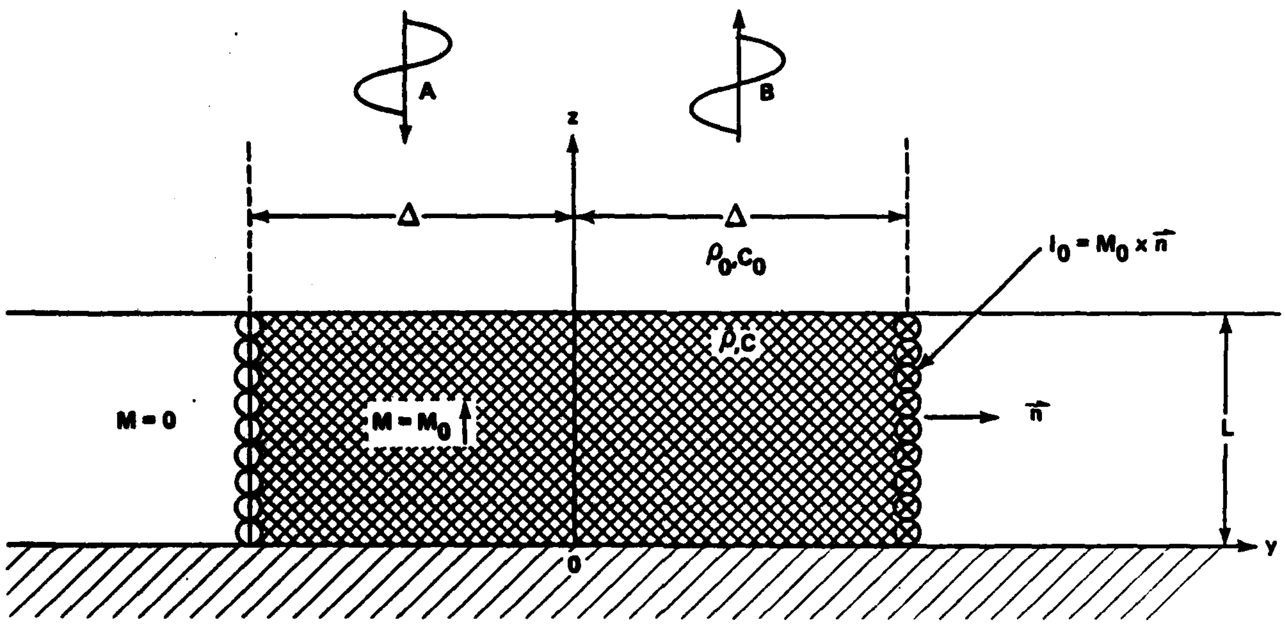

FIGURE 1. CROSS SECTION OF A MAGNETOVISCOELASTIC SLAB IN A VISCOELASTIC MATRIX WITH EQUIVALENT SURFACE CURRENTS.

We assume that the local magnetic dipole moment is frozen into the viscoelastic material. Hence, it does not change because of the relative displacement of two neighboring points of the material. The only change in the magnetization envisioned here is that due to changes in the local density of dipole moments which occurs as a consequence of such local relative displacements of the material. The magnetization, M_o, is the magnetic moment per unit volume:

$$M_o = \frac{dm}{dV} \tag{1}$$

In line with our goal of considering a one dimensional problem, we assume that the motion of the plate in response to a normally incident acoustic wave is strictly in the z-direction as shown in Figure 1. This makes M_o and I_o functions of z only. Furthermore, in this case $\nabla \times M_o = 0$ and since the most general source function for the vector potential, a_o, is:

$$a_o = \frac{\mu_o}{4\pi} \left\{ \int_V \frac{\nabla \times M_o}{r} \, dV + \int_S \frac{M_o \times \vec{n}}{r} \, da \right\} \tag{2}$$

we find that it is a function of the surface current $I_o = M_o \times \hat{n}$ only - even when a compressional wave exists in the slab. Throughout the motion of the slab we may therefore calculate the vector potential on the basis of the current source distribution shown in Figure 1. The source potential becomes:

$$a_o = \frac{\mu_o}{4\pi} \int_S \frac{\vec{I}(z_o,t)}{r} \, da \tag{3}$$

where $I\ (z_o,t)$ is a function of the local displacement $u(z_o,t)$. Thus coupling between the magnetic and the viscoelastic problem occurs because of the presence of $u\ (z_o,t)$ in a_o.

The dependence of I on u may be calculated as follows. Suppose we are given a distribution $f(z)$ on $a \leq z \leq b$. Consider the new distribution $h_t(z')$ on $a'_t \leq z' \leq b'_t$, $a'_t \neq a$, $b'_t \neq b$, which is obtained from the coordinate transformation $z'_t = g(z, t)$, where we indicate that the transformation may be a function of a parameter t, such as time. Since:

$$\int_a^b f(z)\, dz = \int_{a'_t}^{b'_t} h(z')\, dz' = \int_a^b h[g(z,t)] \frac{\partial g}{\partial z}\, dz \tag{4}$$

we have

$$h[g(z,t)]\,\frac{\partial g}{\partial z} = f(z) \tag{5}$$

In the present problem we are working with the transformation:

$$z'(t) = z + u(z,t) \equiv g(z,t) \tag{6}$$

and the distribution $f(z) = I_o$ = constant on the interval $0 \leq z \leq L$, which exists when no acoustic disturbance is present in the plate. Identifying $h(z'(t))$ with $I(z,t)$ at any given time, we find:

$$I[z'(t)] = \left[\frac{I_o}{1 + \frac{\partial u(z,t)}{\partial z}}\right] \tag{7}$$

Hence, in the presence of a compressional wave, the current at $z' = z + u(z,t)$ is modified by $\partial u(z,t)/\partial z$ at z. For the long wavelength limit, which is the one we are interested in here $\partial u/\partial z$ is small and we therefore disregard this dependence and replace $I(z')$ by I_o.

In order to solve for the acoustic response, we need the magnetic volume force exerted by the total magnetic field at any point inside the slab. We are going to assume that this force may be calculated in a certain average sense from the same equivalent surface current, I_o, that is being used to calculate the magnetic field. Suppose we have solved the magnetodynamic boundary value problem for the current $I_o(z,t)$ and have found the vector potential at $z \geq 0$, $A(z,t)$. Let $A(z,\omega)$ be the Fourier transform. The resulting Lorentz force on an equivalent current loop will generally not be normal to the plane of the slab. But in keeping with our aim of simplifying this problem to the point where only vertical forces and motions remain, we consider only the y-component of the total magnetic field. We also consider only the case of a rectilinear strip of material of width 2Δ and thickness L, since this geometry is of primary interest to us. The case of the circular disk can be analyzed in exactly the same way. We give the corresponding vector potential below. Hence the Fourier transform of the Lorentz force per unit height and unit length of slab on e.g., a right hand surface current element of strength $I(z,\omega)$ at z is:

$$F(z,\omega) = I(z,\omega) \times B_y(z,\omega)$$

$$= -I(z,\omega) \times \frac{\partial A(z,\omega)}{\partial z} \tag{8}$$

In order to obtain an expression for the total magnetic volume force (force/unit volume) normal to the plane of this current, we first add the force on the left hand current filament. This force is in the same direction, even though the field is in the opposite direction, because the current is reversed. Dividing by the width of the strip, 2Δ, we obtain the average magnetic volume force or, more accurately, we obtain a simple analytical quantity which we will use in place of the true average force inside the material:

$$\overline{P_m}(z,\omega) = -\frac{1}{\Delta}\left[\vec{I}(z,\omega) \times \frac{\partial A(z,\omega)}{\partial z}\right] \tag{9}$$

From this force we have to substract the equilibrium magnetic force which exists when no acoustic field is present because the displacement, $u(z,t)$ in the solution of the acoustic problem represents the displacement from position coordinates reached not only under the influence of static, mechanical forces but also under the influence of magnetic forces due to the static magnetization, M_o. Hence the resultant volume force is:

$$\begin{aligned} P_m(z,\omega) &= -\frac{1}{\Delta}\left\{\left[I(z,\omega) \times \frac{\partial A(z,\omega)}{\partial z}\right] - \left[I_o \times \frac{\partial A_o(z)}{\partial z}\right]\right\} \\ &\cong -\frac{I_o}{\Delta} \times \left[\frac{\partial A(z,\omega)}{\partial z} - \frac{\partial A_o(z)}{\partial z}\right] \end{aligned} \tag{10}$$

This is the Fourier transform of the local magnetic force due to all sources; internal, due to other magnetic moments, as well as external, due to induced eddy currents in the ground plane. The latter force will enter the dynamic equations explicitly, whereas the magnetic force due to other internal moments will be taken care of by means of a phenomenological modification of Hooke's Law. However, for the moment we will not distinguish between these various sources and calculate the total vector potential resulting from all time varying internal (equivalent surface) and external (eddy) currents. The part of the resultant potential that is due to internal sources will be discarded later.

Let the applied vector potential for unit stationary surface current at $z = z_o$, be $a_o(z,z_o)$. This yields the slab's static magnetization which is unmodified by the presence of the ground plane if we assume that the magnetic permeability of the ground plane is that of free space, which we do. Then the applied potential due to a suface current element to I_o at $z = z_o + u(z_o,t)$ at time t is $a_o(z,z_o + u(z_o,t))$. Expanding this to first order with respect to z_o, or more accurately, taking the Frechet differential with respect to $u(z_o,t)$ at $z = z_o$, we write:

$$\begin{aligned} a_o(z,z_o + u(z_o,t)) &= a_o(z,z_o) + u(z_o,t)\left.\frac{\partial a_o}{\partial z_o}\right|_{u=o} \\ &= a_o(z,z_o) + \Delta a_o(z,z_o) \end{aligned} \tag{11}$$

Now let a and Δa be the Fourier transforms of the solutions for the total vector potentials corresponding to a_o and Δa_o, respectively. Then:

$$A_o(z) = \int_o^L a(z,z_o)\, dz_o \tag{12}$$

and:

$$A(z,\omega) = \int_o^L [a(z,z_o) + \Delta a(z,z_o,\omega)]\, dz_o \tag{13}$$

Hence, the total local magnetic pressure is:

$$P_m(z,\omega) = -\frac{I_o}{\Delta}\frac{\partial}{\partial z}\int_o^L \Delta a(z,z_o,\omega)\, dz_o$$

$$= -\frac{I_o}{\Delta}\frac{\partial}{\partial z}\int_o^L u(z_o,\omega)\,\frac{\partial a(z,z_o,\omega)}{\partial z_o}\, dz_o \tag{14}$$

where we recall that the vector potential in Equation (14) is understood to be only that part of the total potential that is due to induced eddy currents in the ground plane.

This pressure has to be accounted for in the dynamic equations. When $P_m = 0$ these are:

Hooke's Law:

$$\rho[c]^2 \frac{\partial^2 u}{\partial z\,\partial t} = -\frac{\partial P}{\partial t} \tag{15a}$$

and:

Newton's 2nd Law:

$$\rho \frac{\partial^2 u}{\partial t^2} = -\frac{\partial P}{\partial z} \tag{15b}$$

where P is the mechanical pressure, ρ the mass density and $[c]$ the speed of sound in the absence of magnetic forces. In order to obtain the corresponding magnetoviscoelastodynamic equations we simply add the magnetic volume force on the righthand side of Equation (15b) and modify Hooke's Law implicitly by changing the wave speed in the material from [c] to $c=c(M_o)$. In other words, the internal magnetic interactions change the elastic properties of the material, conceivably even its viscoelastic properties, by moving the equilibrium state of the dynamic response to a different point on a nonlinear response characteristic. To account for this change, we simply modify Hooke's Law in the way indicated above. The resulting equations are:

$$\rho c^2 \frac{\partial u}{\partial z \partial t} = - \frac{\partial P}{\partial t} \tag{16a}$$

$$\rho \frac{\partial^2 u}{\partial t^2} = - \frac{\partial P}{\partial z} + P_m \tag{16b}$$

We regard c as an emperically determined quantity. It's dependence on M_o should be kept in mind throughout the remainder of this paper. When the change in performance of the magnetic slab is investigated as a function of the interaction with the ground plane, via P_m, it must be kept in mind that as P_m (specifically I_o) changes, so does $c(M_o)$, unless we assume that the longitudinal modulus of the material in which the magnetic moments are embedded is adjusted to conpensate for the effects of internal magnetic interactions. Conversely, if we change c while keeping P_m fixed, we will ultimately encounter incompatible combinations of the two and may obtain unphysical results.

Since u(z,t) appears under the integral sign on the right hand side of Equation (16b) this is an integro-partial differential equation. It can be solved readily by means of perturbation techniques and we indicate below how this is done. A quicker way is by resorting to the following approximation. We replace $u(z_o,t)$ in P_m by its average value on $o \leq z_o \leq L$:

$$u(z_o,\omega) \rightarrow \frac{1}{L} \int_o^L u(z_o,\omega)\, dz_o \equiv \overline{u(z_o,\omega)} \equiv \alpha(\omega) \tag{17}$$

and obtain:

$$P_m(z,\omega) = \frac{I_o \alpha}{\Delta} \frac{\partial}{\partial z} \Big[a(z,L,\omega) - a(z,0,\omega) \Big]$$

$$= \frac{I_o \alpha}{\Delta} \Big[b_y(z,L,\omega) - b_y(z,0,\omega) \Big] \tag{18}$$

In this form the equations may be integrated readily. We are still taking account of the coupling between the acoustic and electromagnetic fields, but are sacrificing the point by point amplitude and phase relationship that exists between the local displacement and the y-component of the magnetic field. In view of other approximations that were made earlier, this seems to be a reasonable step at this point. The solution for $u(z_o,t)$ will be a function of α and later on we will have to solve for α using Equation (17). Since $\alpha(\omega)$ will occur linearly on both sides of this equation, this step is trivial.

For the reflection coefficient we need the steady state solution of Equation (16) with P_m as defined by Equation (18). Like $\alpha(\omega)$, the magnetic fields in Equation (18) remain unspecified for the moment. Introducing the time dependence $e^{-i\omega t}$ and setting $v = \partial u/\partial t$ we obtain:

$$\rho c^2 \frac{dv}{dz} = i\omega P \qquad (19a)$$

$$-i\omega\rho v = -\frac{dP}{dz} + P_m \qquad (19b)$$

Substituting Equation (19a) into Equation (19b) one finds:

$$\frac{d^2v}{dz^2} + k^2 v = \frac{ik}{\rho c} P_m \ ; \ k \equiv \frac{\omega}{c} \qquad (20)$$

This inhomogeneous, second order, ordinary differential equation must now be solved subject to certain boundary conditions. Introducing the incident and reflected waves:

$$P_o = A_1\rho^{-ik_o(z-L)} + B_1\rho^{ik_o(z-L)}$$

$$v_o = -\frac{1}{\rho_o c_o}\left[A_1\rho^{-ik_o(z-L)} - B_1\rho^{ik_o(z-L)}\right]$$

and requiring that both v and P be continuous across the boundaries, we find:

$$z = L \ ; \ \rho_o c_o v(L) = B_1 - A_1 = \rho_o c_o v_o \qquad (21)$$

and:

$$z = L \ ; \ -\frac{i\rho c}{k}\frac{dv}{dz} = A_1 + B_1 = P_o \qquad (22)$$

Defining the known input admittance of the backing as $G(0)=-v(0)/P(0)$, one has:

$$z = 0 \quad ; \quad G(0)\left(\frac{i\rho c}{k}\right)\frac{dv}{dz} - v(0) = 0 \tag{23}$$

Combining Equations (21) and (22) we obtain the boundary condition on v at z=L:

$$z = L \quad ; \quad \frac{2ikA_1}{\rho c} = \frac{dv}{dz} - \left(\frac{i\rho_o c_o k}{\rho c}\right) v \tag{24}$$

and from Equation (23) at z=0 we have:

$$0 = \frac{dv}{dz} + \left(\frac{ik}{\rho c\ G(0)}\right) v \tag{25}$$

Suppose the solution of Equation (20) has been found. The final step is to calculate the reflection coefficient, $\beta^2 = |B_1/A_1|^2$. Using Equations (21) and (22) one finds that:

$$-\frac{\rho_o c_o v(L)}{P(L)} = \frac{1-\beta}{1+\beta} = -ik\left(\frac{\rho_o c_o}{\rho c}\right)\left[\frac{v(L)}{\left.\frac{dv}{dz}\right|_{z=L}}\right] = \rho_o c_o\ G(L) \tag{26}$$

where, by definition, G(L) is the input admittance of the magnetic layer. Equation (26) will therefore yield the answer to the problem once v(z) has been found.

The most compact expression for v(z) is obtained with the aid of the Green's function, g, of this problem. This function must satisfy mixed homogeneous boundary conditions according to Equations (24) and (25), i.e., for g we must have:

$$z = 0 \quad ; \quad D_1(g) \equiv \nabla g + \frac{ik}{\rho c\ G(0)}\, g = 0 \tag{27}$$

$$z = L \quad ; \quad D_2(g) \equiv \nabla g - \frac{ik\rho_o c_o}{\rho c}\, g = 0 \tag{28}$$

Referring to the general solution of this type of problem in three dimensions as given in standard texts, the solution of the present problem may be written:

$$v(z,\omega) = \left[g(z,z_o,\omega)\,\frac{dv(z,\omega)}{dz} - v(z,\omega)\,\frac{dg(z,z_o,\omega)}{dz} \right] \Bigg|_{z_o = 0}^{z_o = L}$$

$$- \frac{ik}{\rho c} \int_0^L P_m(z_o,\omega)\, g(z,z_o,\omega)\, dz_o \;;\; 0 \leq z \leq L \tag{29}$$

where g is the solution of:

$$\frac{dg^2(z,z_o,\omega)}{dz^2} + k^2 g(z,z_o,\omega) = -\,\delta(z-z_o) \tag{30}$$

In Equation (29) the usual surface integral has been reduced to function evaluations at the two boundary points and the volume integral has become a line integral. Using the boundary conditions, Equations (24), (25), (27), and (28), one finds:

$$v(z,\omega) = \psi_1 + \psi_2 \tag{31}$$

where:

$$\psi_1 = \frac{2ikA_1}{\rho c}\, g(z,L,\omega) \tag{32}$$

and:

$$\psi_2 = -\,\frac{ik}{\rho c} \int_0^L P_m(z_o,\omega)\, g(z,z_o,\omega)\, dz_o \tag{33}$$

The function ψ_1 therefore yields the reflection coefficient of the nonmagnetic layers, whereas ψ_2 yields the effects due to magnetization.

Had we not used the average displacement, $\alpha(\omega)$ in Equation (18), we would now have a double integral on the right hand side of Equation (33) with v appearing in the integrand. Equation (31) could, therefore, be used as the starting equation for a perturbation expansion of the solution. For instance, with the aid of Equations (14), (31), (32), and (33) one finds that to first order, the solution for v is:

$$v(z,\omega) = \frac{2ikA_1}{\rho c} \left\{ g(z,L,1) - \left(\frac{I_o}{\rho c^2 \Delta}\right) \int_o^L \int_o^L g(z_o,L,\omega) \right.$$

$$\left. * \frac{\partial a(z',z_o,\omega)}{\partial z' \; \partial z_o} g(z,z',\omega) \; dz' \; dz_o \right\} \tag{34}$$

with remainder:

$$\Delta v(z,\omega) = \left(\frac{I_o}{\rho c^2 \Delta}\right)^2 \int_o^L \int_o^L \int_o^L \int_o^L v(t_o,\omega) \frac{\partial a(t',t_o,\omega)}{\partial t' \; \partial t_o}$$

$$* \; g(z_o,t',\omega) \frac{\partial a(z',z_o,\omega)}{\partial z' \; \partial z_o} g(z,z',\omega) \; dz' \; dz_o \; dt' \; dt_o \tag{35}$$

and Δ follows from Equation (26). Since a (z,z_o,ω) is proportional to $\mu_o I_o$ and $g(z,z_o,\omega)$ to $1/k$, it is clear that the expansion parameter in this perturbation series is:

$$\lambda = \left(\frac{1}{\Delta k}\right)\left(\frac{\mu_o I_o{}^2}{\rho c^2}\right) \tag{36}$$

The ratio of the magnetic field energy density, $\mu_o I_o{}^2$ and the mechanical kinetic energy density ρc^2 in Equation (36) is a small quantity in all real magnetoelastic materials. Furthermore, it may be assumed that $|\Delta k|$ is of order unity or larger at all frequencies. Hence, λ is a small number and the expansion converges rapidly. On the other hand, the smallness of this parameter also tells us that the effects we are going to observe will be weak since the solution with finite P_m will differ little from that for a nonmagnetic slab.

In order to find v explicitly, we need g. By definition:

$$g = \frac{-1}{\Delta_o(y_1,y_2)} \begin{cases} y_1(z) \; y_2(z_o) & z \leq z_o \\ y_2(z) \; y_1(z_o) & z \geq z_o \end{cases} \tag{37}$$

where y_1, and y_2 are two linerarly independent solutions of:

$$\frac{d^2y}{dz^2} + k^2 y = 0 \tag{38}$$

and $\Delta_o,(y_1y_2)$ is the Wronskian:

$$\Delta_o(y_1,y_2) = \left(y_1 \frac{dy_2}{dz} - y_2 \frac{dy_1}{dz}\right) \tag{39}$$

Evidently, g will satisfy its boundary conditions if y_1 and y_2 are chosen such that:

$$\begin{aligned} z = 0 \quad &; \qquad D_1(y_1) = 0 \\ z = L \quad &; \qquad D_2(y_2) = 0 \end{aligned} \tag{40}$$

We use the two linearly independent functions:

$$y_1 = \sin(kz - \theta_1) \tag{41}$$

and

$$y_2 = \sin[k(z-L) - \theta_2] \tag{42}$$

with Wronskian:

$$\Delta_o(y_1,y_2) = -k \sin(kL - \theta_1 - \theta_2) \tag{43}$$

Applying the boundary conditions yields:

$$\tan[\theta_1] = -i\rho c\ G(0) \tag{44}$$

and:

$$\tan[\theta_2] = -\frac{i\rho c}{\rho_o c_o} \tag{45}$$

for the two integration constants in Equation (31) and the problem is solved.

For $g(z,z_o,\omega)$ one finds:

$$g(z,z_o,\omega) = \left\{ \begin{array}{ll} \left[\dfrac{\sin[kz-\theta_1]\ \sin[k(L-z_o) - \theta_2]}{k \sin(kL - \theta_1 - \theta_2)}\right] & z \leq z_o \\ \left[\qquad z \longleftrightarrow z_o \qquad\right] & z \geq z_o \end{array} \right. \tag{46}$$

Removing the singularity in the slope of the Green's function from the integral in Equation (33) leads to a more explicit form of the solution:

$$v(z) = c_1\ y_1(z) - \frac{ik}{\rho c} \int_0^z \frac{[y_1(z)\ y_2(z_o) - y_2(z)\ y_1(z_o)]\ P_m(z_o)}{\Delta} dz_o \tag{47}$$

where:

$$c_1 = c_2 + \frac{ik}{\rho c} \int_0^L \frac{P_m(z_o)\ y_2(z_o)\ dz_o}{\Delta} \tag{48}$$

and:

$$c_2 = - \frac{2ikA_1}{\rho c} \left(\frac{y_2(L)}{\Delta}\right) \tag{49}$$

Observing that the Wronskian is independent of z, we note that:

$$\Delta_o(y_1,y_2)\Big|_L = (y_1 y_2' - y_2 y_1')\Big|_L = -y_2\left(y_1' - i\,\frac{\rho_o c_o k}{\rho c}\,y_1\right)\Big|_L$$

$$= -y_2(L)\ D_2[y_1(L)]$$

Substituting into c_2, we find that:

$$c_2 = \frac{2ikA_1}{\rho c\ D_2[y_1(L)]} \tag{50}$$

Before writing down the reflection coefficient, we calculate $\alpha(\omega)$. Writing Equation (31) in the form:

$$v(z) = A_1 f_1(z) + \alpha(\omega)\, f_2(z) \tag{51}$$

where:

$$f_1(z) = \frac{2ik}{\rho c}\, g(z,L,\omega) \tag{52}$$

and:

$$f_2(z) = -\frac{ik}{\rho c} \int_0^L \left[\frac{P_m(z_o,\omega)}{\alpha(\omega)}\right] g(z,z_o,\omega)\, dz_o \tag{53}$$

and using Equation (17) one finds:

$$\alpha(\omega) = \frac{A_1 \phi_1}{1 - \phi_2} \tag{54}$$

where:

$$\phi_1 = \frac{i}{\omega L} \int_0^L f_1\, dz \tag{55}$$

and:

$$\phi_2 = \frac{i}{\omega L} \int_0^L f_2\, dz \tag{56}$$

For the purpose of estimating the performance difference between magnetized and unmagnetized states we write β in the form $\beta = \beta_o + \Delta\beta$, where β_o is the reflection coefficient in the absence of magnetization and $\Delta\beta$ is the part depending on M_o. A measure of the effectiveness of the magnetization for two otherwise similar materials is given by:

$$R = [|\beta|^2 - |\beta_o|^2] = 2 \text{ Real } [\Delta\beta \; \beta_o^*] + |\Delta\beta|^2 \tag{57}$$

We therefore want to calculate both β and β_o. With the aid of Equations (24), (26), (51) and (54) we find:

$$\frac{1-\beta}{1+\beta} = \rho_o c_o \; G(L) = -\rho_o c_o \left\{ \frac{f_1(L)\;(1-\phi_2) + \phi_1 \; f_2(L)}{[2 + \rho_o c_o \; f_1(L)]\;(1-\phi_2) + \rho_o c_o \; \phi_1 \; f_2(L)} \right\} \tag{58}$$

Setting $\phi_2 = 0 = P_m$ we obtain:

$$\frac{1-\beta_o}{1+\beta_o} = \rho_o c_o \; G(L)_{P_m = 0} \equiv \rho_o c_o \; G(L)_o = \left[\frac{-\rho_o c_o \; f_1(L)}{2 + \rho_o c_o \; f_1(L)} \right] \tag{59}$$

where $G(L)_o$, is the input admittance of a nonmagnetic layer. An explicit expression for this function may be obtained by observing that according to Equations (46) and (52):

$$f_1(L) = -\frac{2i}{\rho c} \left\{ \frac{\sin\,(kL - \theta_1)\,\sin(\theta_2)}{\sin\,(kL - \theta_1 - \theta_2)} \right\} \tag{60}$$

Substituting in Equation (59) and using Equations (44) and (45), we obtain the well-known expresssion:

$$G(L)_o = \left\{ \frac{G(0) - \frac{i}{\rho c}\tan\,(kL)}{1 - i\rho c \; G(0) \tan\,(kL)} \right\}$$

With the aid of Equations (58) and (59) we now find the desired expressions for β, β_o, and $\Delta\beta$:

$$\beta = \left[\frac{1 - \rho_o c_o \; G(L)}{1 + \rho_o c_o \; G(L)} \right] = \beta_o + \left\{ \frac{q}{\frac{1}{\rho_o c_o} + G(L)_o} \right\} \tag{61}$$

where:

$$\beta_o = \left[\frac{1 - \rho_o c_o \; G(L)_o}{1 + \rho_o c_o \; G(L)_o} \right] \tag{62}$$

from Equation (59) and:

$$q = \frac{\phi_1}{1 - \phi_2}\left[\frac{f_2(L)}{2 + \rho_o c_o\ f_1(L)}\right] \tag{63}$$

Hence from Equations (59), (61) and (63):

$$\Delta B = \frac{\rho_o c_o}{2}\left[\frac{\phi_1}{1 - \phi_2}\right] f_2 \tag{64}$$

Combining Equations (59) and (63) we also obtain:

$$\frac{\Delta B}{B_o} = \left\{\frac{\rho_o c_o\ \phi_1\ f_2(L)}{2(1 + \rho_o c_o\ f_1(L))\ (1 - \phi_2)}\right\} \tag{65}$$

At this point we have an explicit expression for R, the change of the reflection coefficient due to the magnetization M_o, in terms of the Fourier transform of the vector potential in Equation 14. Hence, as a final step we calculate the quantity $\Delta a(z,z_o,\omega)/\alpha(\omega) = \partial a(z,z_o,\omega)/\partial z_o$ from the gradient of the applied potential in Equation (11), i.e., from $\Delta a_o(z,z_o,\omega) = \alpha(\omega) \cdot \partial a_o(z,z_o,\omega)/\partial z_o|u=0$. Because of our approximation of $u(z_o,\omega)$ it is not necessary to perform the differentiation on z_o, and we calculate the resultant potential $a(z,z_o,\omega)$ from the applied potential $a_o(z,z_o)e^{-i\omega t}$. Clearly, if we did not want to use the average of u in the dynamic equations we could obtain the integrand in the expression for the magnetic pressure by simply differentiating the expression for $a(z,z_o,\omega)$, as calculated below, with respect to z_o, and multiplying by $u(z_o,\omega)$. Conceptually the potential $a(z,z_o,\omega)$ is the result of applying a bifilar current at $z=z_o$ with unit amplitude and frequency ω. Physically we are dealing with the motion of a constant current at $z = z_o + u(z_o,t)$. This current is changing its flux linkage with the ground plane and hence is inducing eddy currents. If the motion $\partial u(z_o,t)/\partial t$ were uniform we could have derived the form of $\Delta a_o(z,z_o,t)$ from the classical theory of electromagnetism in moving media. For if the applied flux is denoted by ϕ we have for any arbitrary current filament in the plane of the boundary with area element da and line element ds:

$$\frac{d\phi_o}{dt} = \frac{d}{dt}\int \Delta a_o \cdot ds = -\int E_o \cdot ds$$

$$= -\int \nabla \times (v \times b_o) \cdot \vec{n}\ da = -\int (v \times b_o) \cdot ds\ , \tag{66}$$

i.e.:

$$\frac{d\Delta a_o}{dt} = - \frac{dv}{dt} X\ b_o(z,z_o) \tag{67}$$

or:

$$\Delta a_o = - u\ X\ b_o = -\vec{u}\ X\ (b_{oz}\ \vec{k} + b_{oy}\ \vec{j}) = \vec{i}u\ b_{oy}$$

$$= - \vec{i}\ u \frac{\partial a_o}{\partial z} = \vec{i}\ u \frac{\partial a_o}{\partial z_o} \tag{68}$$

since a_o, is a function of $(z-z_o)$ and the subscript zero on the electric field E_o, the magnetic field b_o refers to "applied" quantities. Equation (68) was obtained previously, Equation (11), in spite of the fact that in the present case motion was assumed to be uniform. This agreement is evidently due to the approximation we made in the derivation of Equation (11). Had we gone to second order, we would have picked up a term of the form $[u(z_o,t)]^2\ \partial^2 a_o/\partial z_o^2 |u=0$ which would have introduced the curvature of the potential as well as the first harmonic, with frequency 2ω.

The differential equations for $a(z,z_o,\omega)$ are:

$$z \geq 0;\ \nabla^2 a = -\mu_o I_o \delta(z - z_o)\ [\delta(y - \Delta) - \delta(y + \Delta)] \tag{69}$$

and:

$$z \leq 0;\ \nabla^2 a = \mu_o \sigma \frac{da}{dt} = -i\omega\sigma\mu_o a \tag{70}$$

where the time dependence $e^{-i\omega t}$ has been cancelled on both sides. The Fourier transform of the source potential is logarithmic and may be represented by the following integral:

$$a_o = \frac{\mu_o I_o}{\pi} \int_o^\infty e^{-k|z-z_o|} \frac{\sin(k\Delta)\ \sin(ky)}{k} dk \tag{71}$$

Solving the boundary value problem by requiring the continuity of $a(z,z_o,\omega)$ and the parallel component of the magnetic field, i.e., of the gradient $\partial a(z,z_o,\omega)/\partial z$ at $z = o$ leads to the Fourier transform of the total potential $a(z,z_o,\omega)$. When $z \geq o$:

$$a(z,z_o,\omega) = a(z,z_o) + \frac{\mu_o I_o}{\pi} \int_0^\infty e^{-k(z+z_o)}$$

$$\frac{\sin(k\Delta)\ \sin(ky)}{k} \left(\frac{k-q}{k+q}\right) dk \ , \tag{72}$$

where $q = (k^2 - i\omega\sigma\mu_o)^{1/2}$. In passing, we note that the corresponding solution for the case of a circular disk of radius b_1 is:

$$a(z,z_o,\omega) = -\frac{I_o\mu_o b_1}{2} \int_0^\infty \left\{ e^{-k|z-z_o|} + \left(\frac{k-q}{k+q}\right) e^{-k(z+z_o)} \right\}$$

$$* \ J_1(kb_1)\ J_1(kr)\ dk \tag{73}$$

where $J_1(x)$ is the cylindrical Bessel function of order one which is regular at $x = 0$.

Note that the absolute magnitudes of the integrands decrease exponentially with increasing k. The important contributions to the values of the integrals therefore occur in the neighborhood of $k = 0$. The intergals are readily evaluated if in the neighborhood of $k = 0$ the expression $((k-q)/(k+q))$ is approximated by an exponential function with an exponent linear in k. We therefore write:

$$f(k) = \frac{k-q}{k+q} = \exp[g(k)] \tag{74}$$

and expand $g(k) = \log\ [f(k)]$ in a Taylor series near $k=0$ to first order in k. We obtain:

$$g(0) = \log[f(0)] = 0$$

and:

$$\left.\frac{dg}{dk}\right|_{k=0} = \left.\frac{1}{f(k)}\frac{df}{dk}\right|_{k=0} = \left.\frac{-2}{\sqrt{k^2 - i\omega\sigma\mu_o}}\right|_{k=0} = \frac{-2}{\sqrt{-i\omega\sigma\mu_o}}$$

Hence:

$$f(k) \cong \exp[-k(1 + i)\delta] \ , \ \delta^2 = \frac{2}{\omega\sigma\mu_o} \tag{75}$$

and δ is the electromagnetic "skin depth." Substituting this into $a(z,z_o,\omega)$ and integrating one finds:

$$a(z,z_o,\omega) = \frac{\mu_o I_o}{\pi} \log \left\{ \left[\frac{(z - z_o)^2 + (y + \Delta)^2}{(z - z_o)^2 + (y - \Delta)^2} \right] * \right.$$

$$\left. \left[\frac{(z + z_o + x)^2 + (y + \Delta)^2}{(z + z_o + x)^2 + (y - \Delta)^2} \right] \right\}_{x = (1 + i)\delta} \tag{76}$$

Evidently, the part of $a(z,z_o,\omega)$ in Equation (76) that is independent of δ is due to internal sources. The only reason for carrying it to this point in the calculation was that it was needed as the source-term for the eddy current problem. Its effect on the dynamic response of the plate has already been accounted for in Hooke's Law, Equation (16a). We will therefore discard it now.

Substituting the remaining term in Equation (76) into Equation (18) and setting $y = + \Delta$ ($y = -\Delta$ was accounted for when the volume force was calculated) one finds:

$$P_m(z,\omega) = - \frac{2 I_o^2 \alpha \mu_o}{\Delta\pi} \left\{ \frac{z + L + x}{(z + L + x)^2 + (2\Delta)^2} - \frac{1}{z + L + x} \right.$$

$$\left. - \frac{(z + x)}{(z + x)^2 + (2\Delta)^2} + \frac{1}{z + x} \right\}_{x = (1 + i)\delta} \tag{77}$$

where $\mu_o I_o^2 = B^2 = (\text{gauss})/4\pi$ is the magnetic field energy density per cm^3 in cgs units.

The expression for $P_m(z,\omega)$ may now be substituted into Equation (61) to yield the reflection coefficient for a normally incident pressure wave and into R, Equation (57), to yield the change in performance.

We have evaluated R for several different combinations of ρ, c, B and δ, see Figures 2 through 9. The plate was assumed to be 10 cm thick and 20 cm wide, i.e., both L and Δ in Figure 1 were 10 cm. The XFIL integration routine of the NSWC/WOL computer library was used to calculate the various integrals in Equation (64).

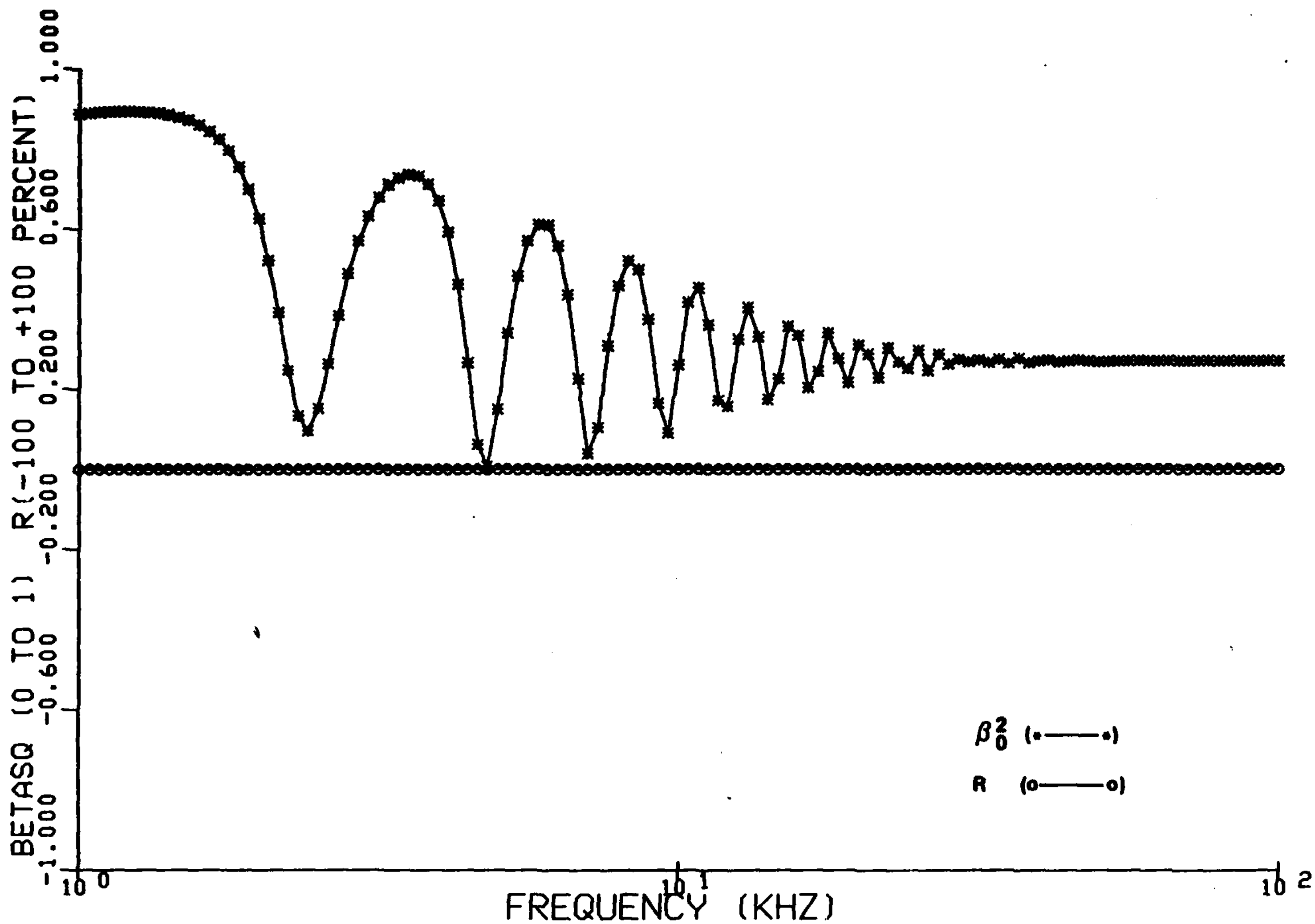

FIGURE 2. ACOUSTIC REFLECTION FROM VISCOELASTIC AND MAGNETOVISCOELASTIC PLATES; β_0^2 AND R ($B_0 = 10^5$ GAUSS, $\sigma = 1.0$ MHO/M, $\rho c^2/\rho_0 c_0^2 = 0.1*(1.-i0.1)$, $\rho/\rho_0 = 1.0$)

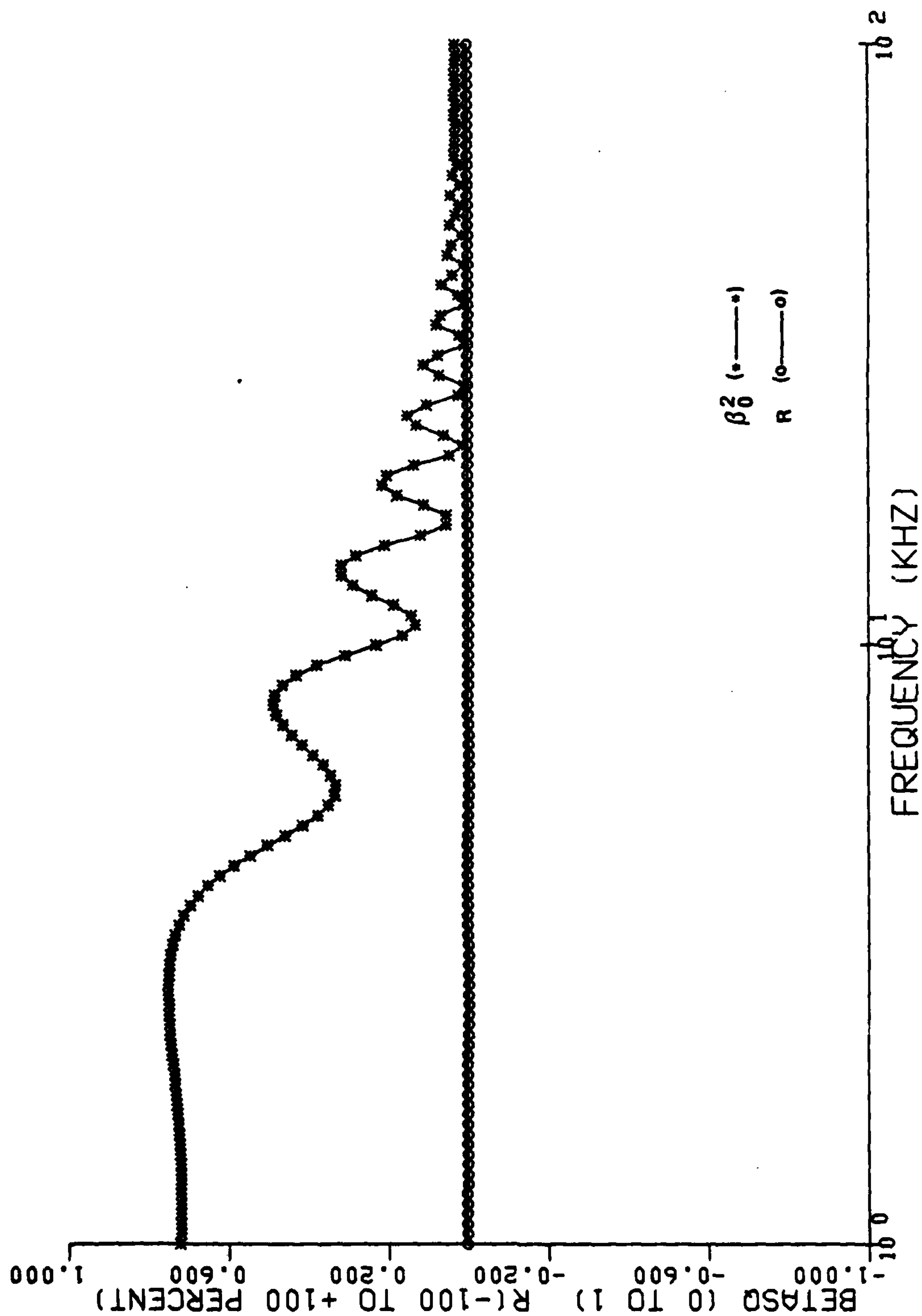

FIGURE 3. ACOUSTIC REFLECTION FROM VISCOELASTIC AND MAGNETOVISCOELASTIC PLATES; β_0^2 AND R ($B_0 = 10^5$ GAUSS, $\sigma = 10^7$ MHO/M, $\rho c^2/\rho_0 c_0^2 = 0.5*(1.-i0.1)$, $\rho/\rho_0 = 1.0$)

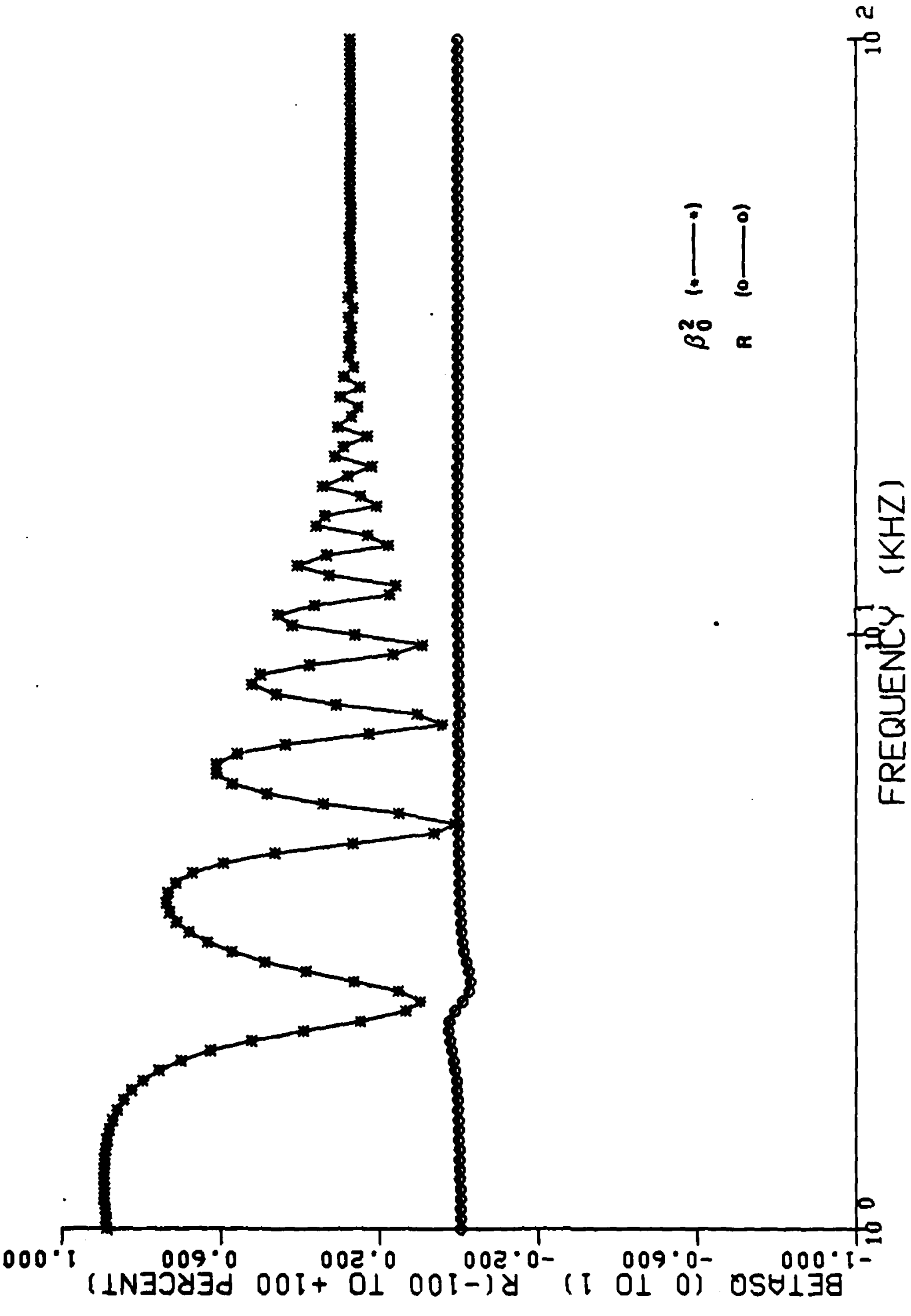

FIGURE 4. ACOUSTIC REFLECTION FROM VISCOELASTIC AND MAGNETOVISCOELASTIC PLATES; β_0^2 AND R ($B_0 = 10^5$ GAUSS, $\sigma = 10^7$ MHO/M, $\rho c^2/\rho_0 c_0^2 = 0.1*(1.-i0.1)$, $\rho/\rho_0 = 1.0$)

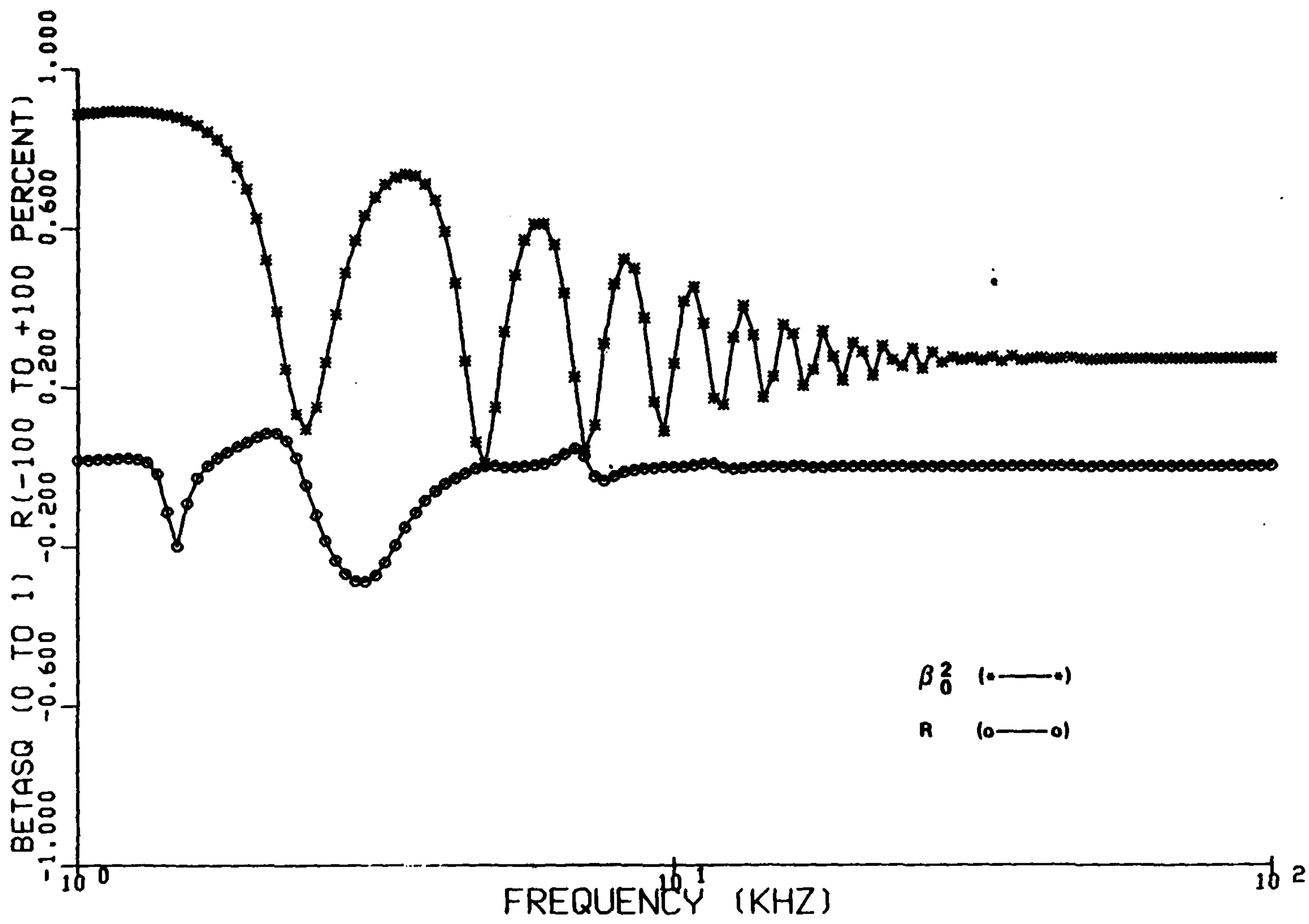

FIGURE 5. ACOUSTIC REFLECTION FROM VISCOELASTIC AND MAGNETOVISCOELASTIC PLATES; β_0^2 AND R ($B_0 = 5 \times 10^5$ GAUSS, $\sigma = 10^7$ MHO/M, $\rho c^2/\rho_0 c_0^2 = 0.1*(1.-i0.1)$, $\rho/\rho_0 = 1.0$)

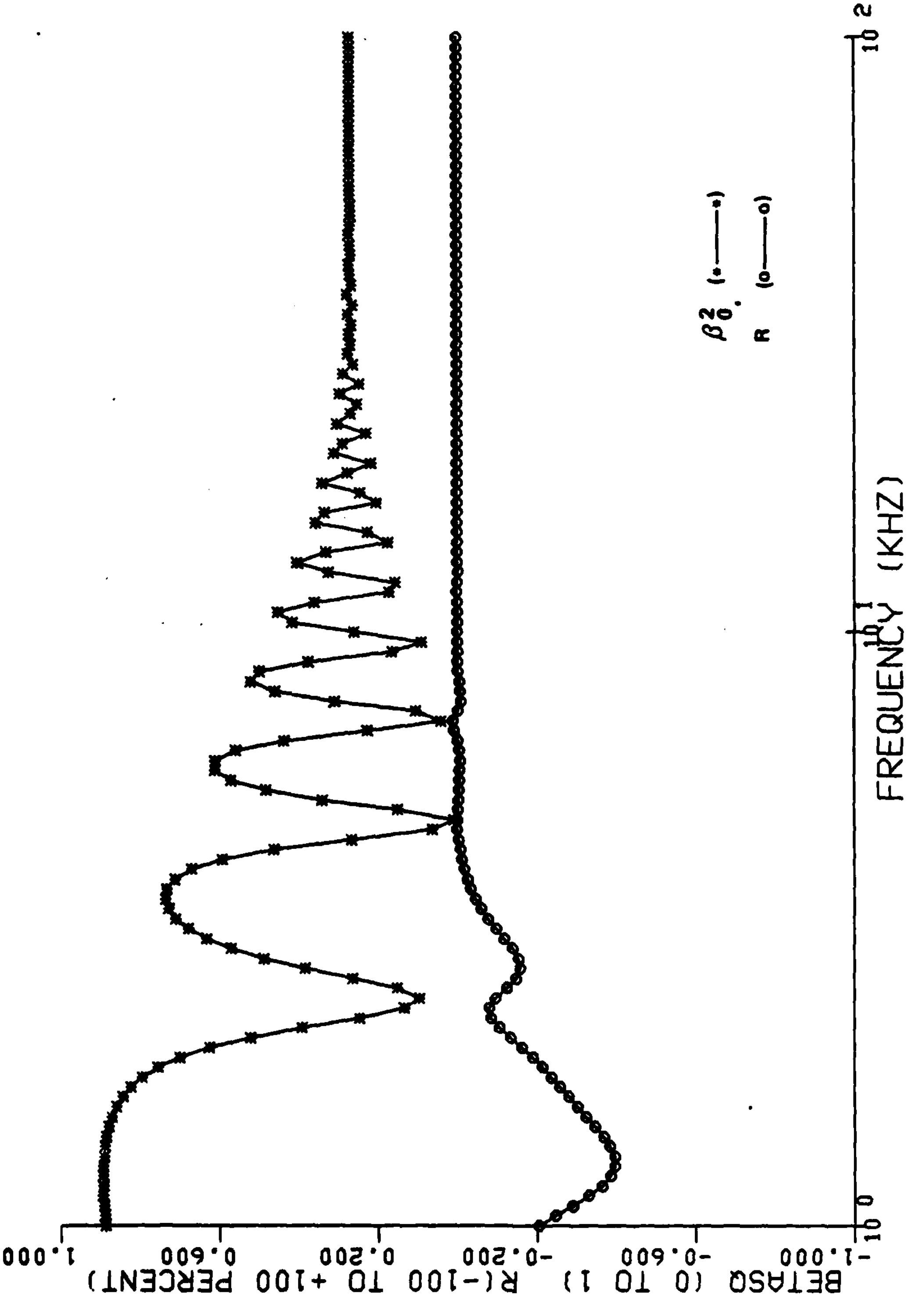

FIGURE 6. ACOUSTIC REFLECTION FROM VISCOELASTIC AND MAGNETOVISCOELASTIC PLATES; β_0^2 AND R ($B_0 = 10^6$ GAUSS, $\sigma = 10^4$ MHO/M, $\rho c^2/\rho_0 c_0^2 = 0.1*(1.-i0.1)$, $\rho/\rho_0 = 1.0$)

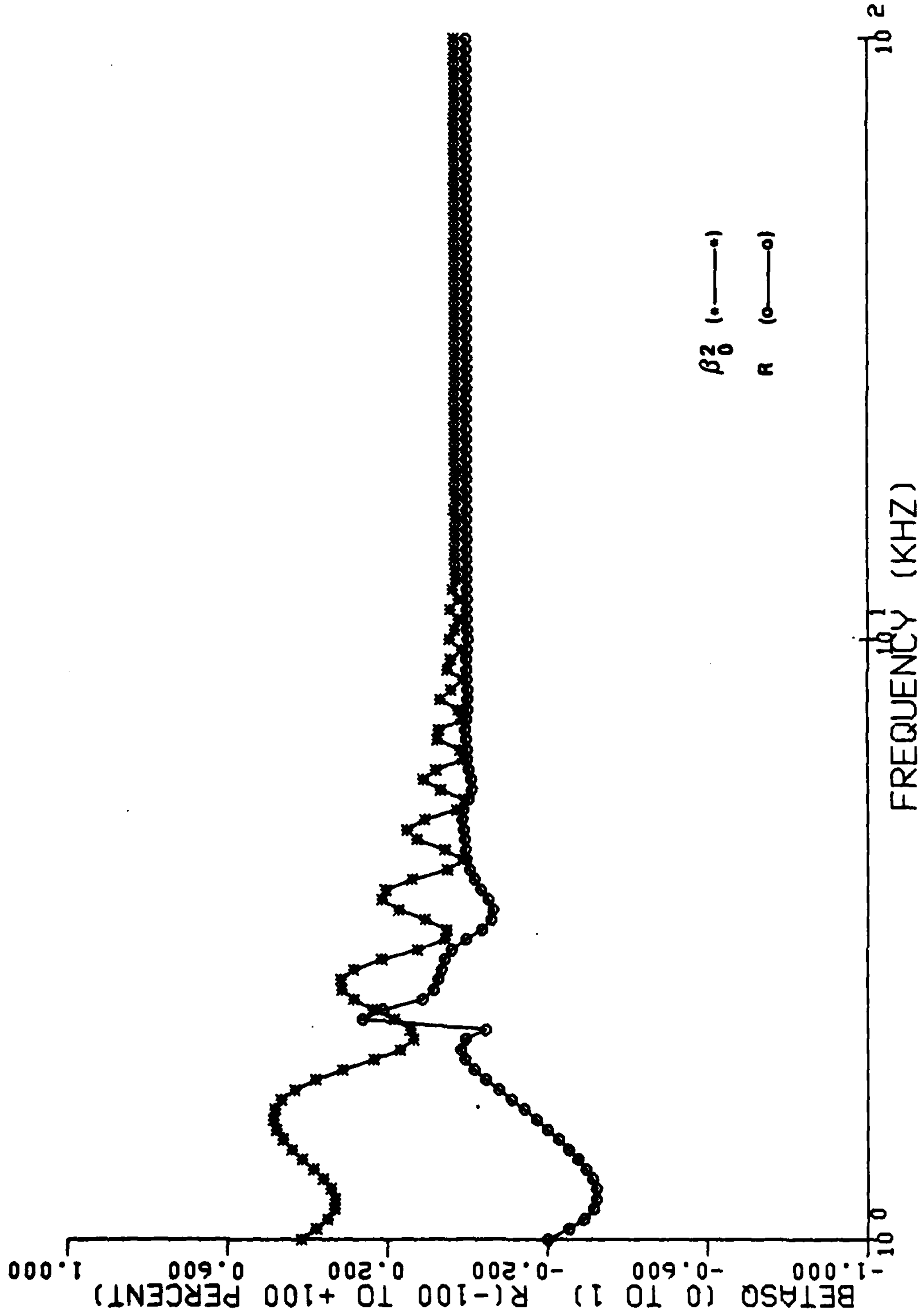

FIGURE 7. ACOUSTIC REFLECTION FROM VISCOELASTIC AND MAGNETOVISCOELASTIC PLATES; β_0^2 AND R ($B_0 = 10^6$ GAUSS, $\sigma = 10^7$ MHO/M, $\rho c^2/\rho_0 c_0^2 = 0.1*(1.-i0.1)$, $\rho/\rho_0 = 5.0$)

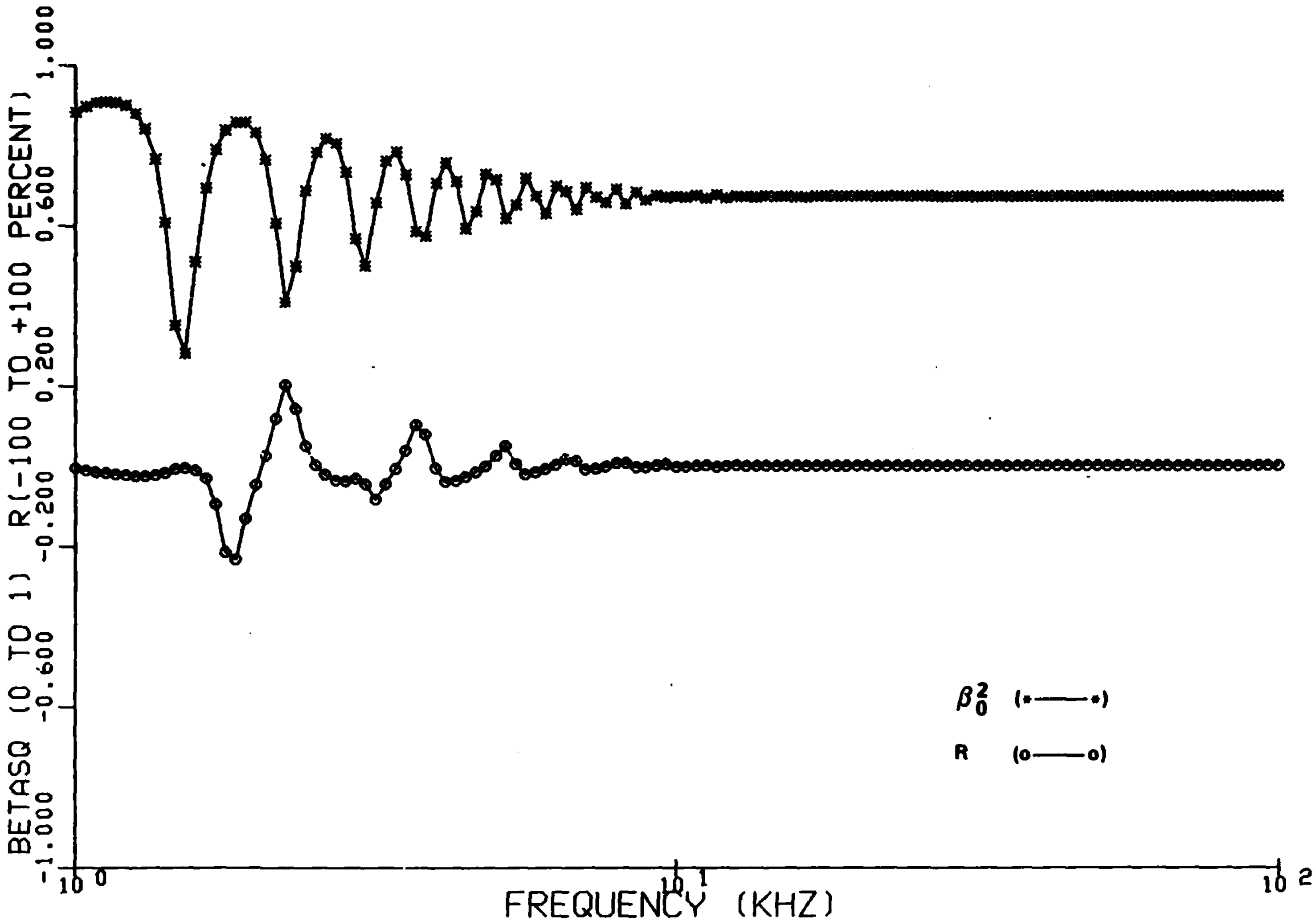

FIGURE 8. ACOUSTIC REFLECTION FROM VISCOELASTIC AND MAGNETOVISCOELASTIC PLATES; β_0^2 AND R ($B_0 = 10^6$ GAUSS, $\sigma = 10^7$ MHO/M, $\rho c^2/\rho_0 c_0^2 = 0.01*(1.-i0.1)$, $\rho/\rho_0 = 1.0$)

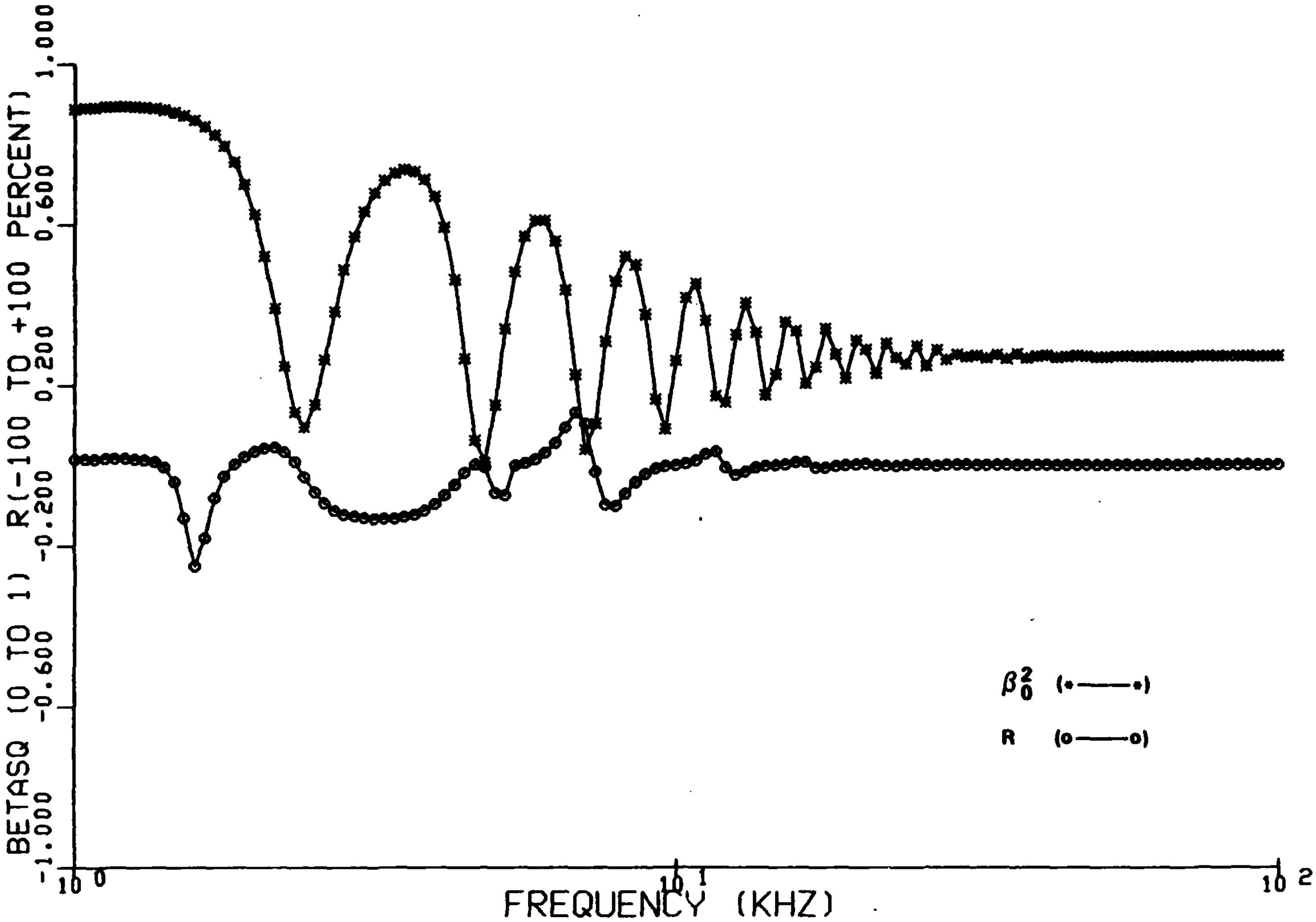

FIGURE 9. ACOUSTIC REFLECTION FROM VISCOELASTIC AND MAGNETOVISCOELASTIC PLATES; β_0^2 AND R ($B_0 = 10^6$ GAUSS, $\sigma = 10^7$ MHO/M, $\rho c^2/\rho_0 c_0^2 = 0.1*(1.-i0.1)$, $\rho/\rho_0 = 1.0$)

As indicated in the text we expected to have to use rather large internal fields in order to obtain observable effects. The results bear this out; unless we apply at least several hundred thousand gauss, unless we use an M_o that produces a B of this order of magnitude, there are no discernible differences in the responses of magnetic and nonmagnetic plates. We find that the energy ratio in Equation (36) is indeed the controlling parameter in this situation. The conductivity of the substrate is of relatively minor importance.

In closing we might add that although we believe the order of magnitude of the effect was calculated correctly, we could have done better had we used the perturbation expansion of the solution in our numerical calculations. In retrospect we feel that the phase information that was suppressed when the local displacement in the magnetic volume force was averaged, might have had a significant influence on the details of the frequency dependence of β^2. On the other hand, in view of the many conceptual simplifications we have made in the course of solving this problem, this refinement would have been academic.

CHAPTER 2

ACOUSTIC ENERGY ABSORPTION IN AN ELECTRICALLY CONDUCTING, VISCOELASTIC MATERIAL IN A UNIFORM MAGNETIC FIELD

In Chapter 1 the sources of the time varying magnetic field on the one hand and the electromagnetic absorber on the other were spatially separate. If the magnetoviscoelastic material was made electrically conducting both functions could be combined in the same volume. Two apparently different mechanisms would now give rise to conduction currents. In addition to the mechanism discussed in Chapter 1, which we think of as a $\partial B/\partial t$ effect, there would now be the (v X B) effect, which is due to convective transport of charge. Both effects are due to the relative motion of "sources" and "observers." In the first case the "sources" move, hence $\partial B/\partial t$ at the observer was picked up as (v X B) in Chapter 1, where v referred to the motion of the source (the magnetic moment). In the second case the "observer" moves, i.e., v refers to the motion of the observer (the mobile charge carrier). Note that in the former case relative motion of source and observer is detectable only because B is nonuniform.

The method of analysis used in Chapter 1 is too approximate for a situation in which the magnetic material discussed in that section simply becomes conducting. The direction of the magnetic field inside the magnetized slab would also be wrong, since it is roughly in the same direction as the local displacement and therefore eliminates the (v X B) effect. If on the other hand the uniform magnetization is assumed to be parallel to the ground plane instead of perpendicular to it, then the internal induction will also be roughly uniform and parallel to the ground plane and effects due to finite conductivity are maximized. These effects may then be analyzed readily if at the same time we neglect the magnetic field variations that were the source of dissipation in Chapter 1, namely those due to local displacements of magnetic moments under the influence of dilatational waves and focus only on convective charge transport arising from the translational motion of the molecules; i.e., on the (v X B) effect. The constant magnetization M_o may then be replaced by the constant magnetic induction, B_o, where $B_o = \mu_o M_o$, or equivalently we may consider an unmagnetized material in a constant parallel field with a wave travelling at right angles to the field direction. In fact, since we really only need the attenuation constant, or the imaginary part of the propagation vector, k, in order to estimate changes in power dissipation, we will only solve for a wave in an infinite medium rather than for reflections from a layer. Hence, we are looking for the complex dispersion relation $k(\omega,\sigma,B)$.

Under the action of a pressure wave in the material we think of the mobile charge as being "carried along" by local displacements of molecules, i.e., there is no charge separation. The (v X B) electric field creates a current, J, and this current locally exerts a Lorentz force, J X B on the material. As in Chapter 1 the equations of motion are:

$$\rho \frac{\partial v}{\partial t} = - \frac{\partial P}{\partial z} + \mu[J \times H] \tag{78}$$

and:

$$\rho c^2 \frac{\partial v}{\partial z} = - \frac{\partial P}{\partial t} \tag{79}$$

As in the theory of magnetohydrodynamics, the equation for the magnetic field is obtained from:

$$\nabla \times E = - \frac{\partial B}{\partial t} \tag{80}$$

$$\nabla \times B = \mu J \tag{81}$$

and:

$$J = \sigma[E + (v \times B)] \tag{82}$$

in mks units, where the last equation is stricly true only for a uniform velocity, v. Eliminating J and E from Equations (80), (81), and (82) yields:

$$\frac{1}{\mu\sigma} \nabla^2 B + \nabla \times (v \times B) = \frac{\partial B}{\partial t} \tag{83}$$

for the magnetic induction. We assume that B consists of a strong, stationary background field, B_o, and a relatively weak response field, b; $B = B_o + b$. Furthermore, if B_o is assumed to point in the x-direction and v in the z-direction, then J is in the y-direction and the response field, b is parallel to B_o.

Assuming the time dependence $e^{-i\omega t}$ in Equations (78), (79), and (83) yields:

$$i\omega(\nabla \times H) \times B - \rho c^2 \frac{dv}{dz} = \rho\omega^2 v \tag{84}$$

and:

$$\frac{1}{\mu\sigma} \nabla^2 B + \nabla \times (v \times B) = -i\omega b \quad (85)$$

Taking account of the fact that the only coordinate dependence is on z, we find:

$$\frac{-i\omega}{2\mu\rho c^2} \frac{d(B^2)}{dz} = \frac{d^2 v}{dz^2} + \frac{\omega^2}{c^2} v \quad (86)$$

and:

$$\mu\sigma \frac{d(vB)}{dz} = \frac{d^2 B}{dz^2} + i\omega\mu\sigma b \quad (87)$$

Using the fact that B_o, is constant, $|b/B_o| << 1$ and neglecting products involving v and b compared with products involving B_o, in combination with v or b, we obtain:

$$\frac{d^2 b}{dz^2} + i\omega\mu\sigma b = \mu\sigma B_o \left(\frac{dv}{dz}\right) \quad (88)$$

$$\frac{d^2 v}{dz^2} + \frac{\omega^2}{c^2} v = - \frac{i\omega B_o}{\mu\rho c^2} \left(\frac{db}{dz}\right) \quad (89)$$

The desired dispersion relation for the propagation of dilatational waves in the material is now obtained if we substitute the solution,

$$v = v_o \, e^{ikz} \; , \; b = b_o \, e^{ikz} \quad (90)$$

into Equations (88) and (89):

$$\left[k^2 - \left(\frac{\omega}{c}\right)^2\right] \left[k^2 - i\omega\mu\sigma\right] - \frac{i\sigma\omega k^2 B_o{}^2}{\rho c^2} = 0 \quad (91)$$

or:

$$k = \pm \frac{1}{\sqrt{2}} \frac{\omega}{c} \left\{(1 + i\,x(1 + y)) + [(1 + i\,x(1 + y))^2 - 4ix]^{1/2}\right\}^{1/2} \quad (92)$$

where:

$$x = \frac{\mu\sigma c^2}{\omega}$$

and:

$$y = \frac{H_o B_o}{\rho c^2}$$

The expression in the curly bracket of Equation (92) therefore modifies the propagation constant $k_o = \omega/c$ which holds in the absence of a background field or when $\sigma = o$. Note that the parameter x may also be written:

$$x = \frac{2}{k_o^2 \delta^2} \quad (93)$$

where:

$$\delta = \sqrt{2/\mu\sigma\omega} \quad (94)$$

is the electromagnetic "skin depth," or:

$$x = 5 \times 10^{-2} \left(\frac{\lambda_o}{\delta}\right)^2 \quad (95)$$

where λ_o is the wavelength in the material in the absence of magnetic effects. The two parameters, y and δ are seen to be the controlling variables of this problem, as they were in the reflectivity problem of section (A). Let:

$$\mu = \mu_o = \frac{4\pi}{10^7} \frac{\text{Henry}}{\text{m}}$$

$$\sigma = 10^2 \frac{\text{mho}}{\text{m}}$$

$$\omega = 2\pi \times 10^2 \text{ Hz}$$

$$c = 1.5 \times 10^3 \frac{\text{m}}{\text{sec}}$$

$$B_o = 10^3 \text{ gauss} = 10^{-1} \frac{\text{Weber}}{m^2}$$

$$\rho = 1 \frac{gm}{cm^3} = 10^3 \frac{kg}{m^3}$$

Then:

$$x = .45$$

and:

$$y = 3.54 \times 10^{-6}$$

Hence, with these orders of magnitudes for the parameters, the ratio of magnetostatic to kinetic energy density is very small; $y=o$ and from Equation (92) it folows that the dispersion relation is the same as in the case of $\sigma = o$.

$$k \approx \frac{\omega}{c}$$

The induction inside the material has to rise to the order of magnitude of several hundred thousand gauss before the dispersion relation, and hence the absorption mechanisms are significantly altered. Note also from Equation (88) that:

$$b_o = -\left[\frac{ixB_o v_o}{1 - ix}\right] \tag{96}$$

Since v_o in this type of situation is of the order of magnitude of 10^{-5} to 10^{-6} m/sec, the assumption that $|b/B_o| << 1$ is satisfied. Evidently, the inequality remains valid even when $x << 1$.

In Figures 10 through 18 we have plotted the real and imaginary parts of k, Equation (92), over the real and imaginary parts of (ω/c) respectively as functions of frequency.

As in Chapter 1 we find that large magnetic fields are needed for strong effects.

Summarizing the results of both chapters we conclude that unless the ratio of the characteristic magnetic field energy density associated with a given situation to the mechanical energy density, ρc^2, of the material is at least of order unity, neither the electrical conductivity of a magnetoviscoelastic material nor induced eddy currents in neighboring structures should be expected to lead to significant electromagnetic absorption of elastic waves or dissipation of acoustic energy.

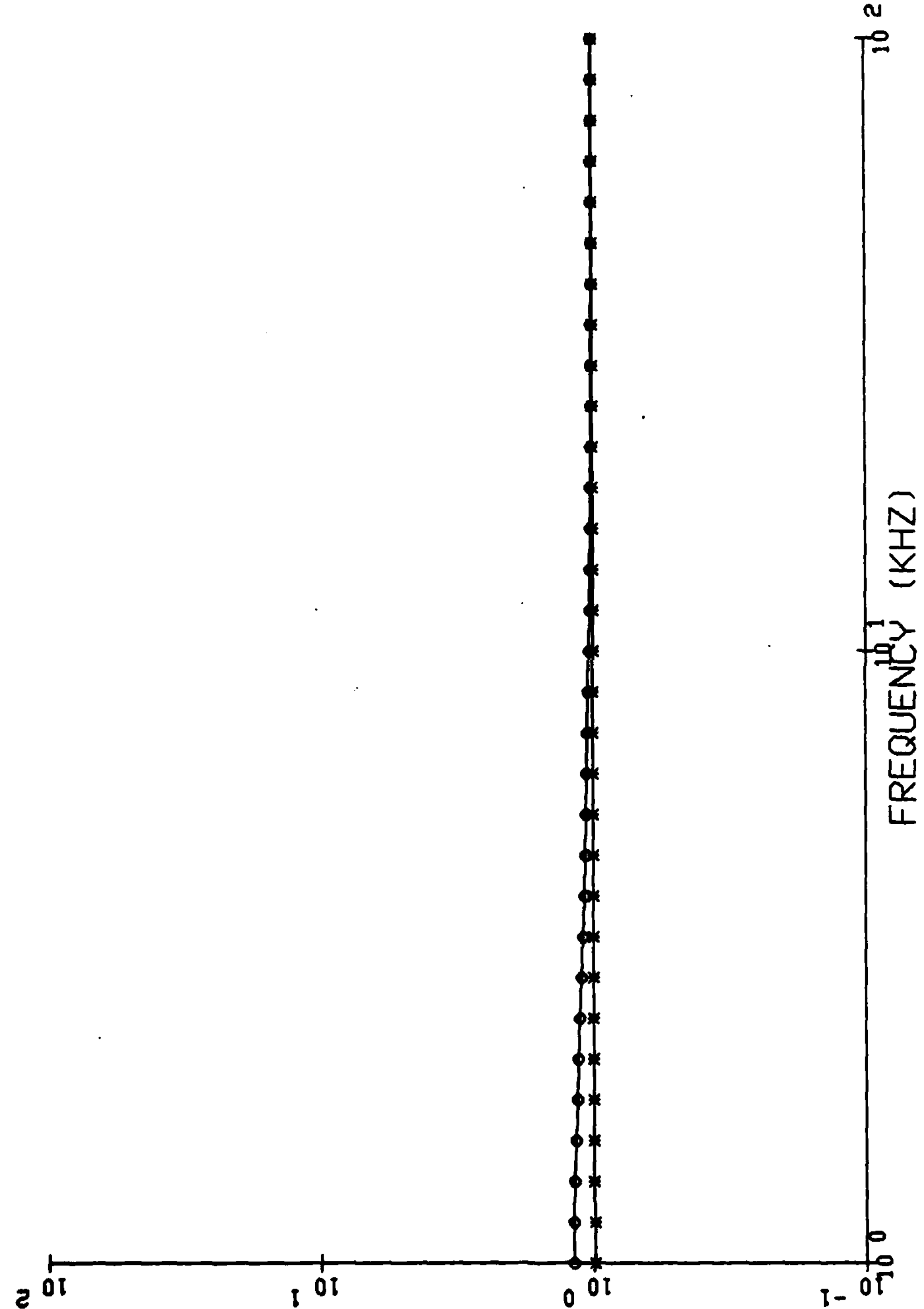

FIGURE 10. DISPERSION RELATION FOR SOUND WAVES IN A VISCOELASTIC ELECTRICAL CONDUCTOR ($B_0 = 10^5$ GAUSS, $\sigma = 10^4$ MHO/M, $\rho c^2/\rho_0 c_0^2 = 1.0*(1.-i0.1)$, $\rho/\rho_0 = 5.0$)

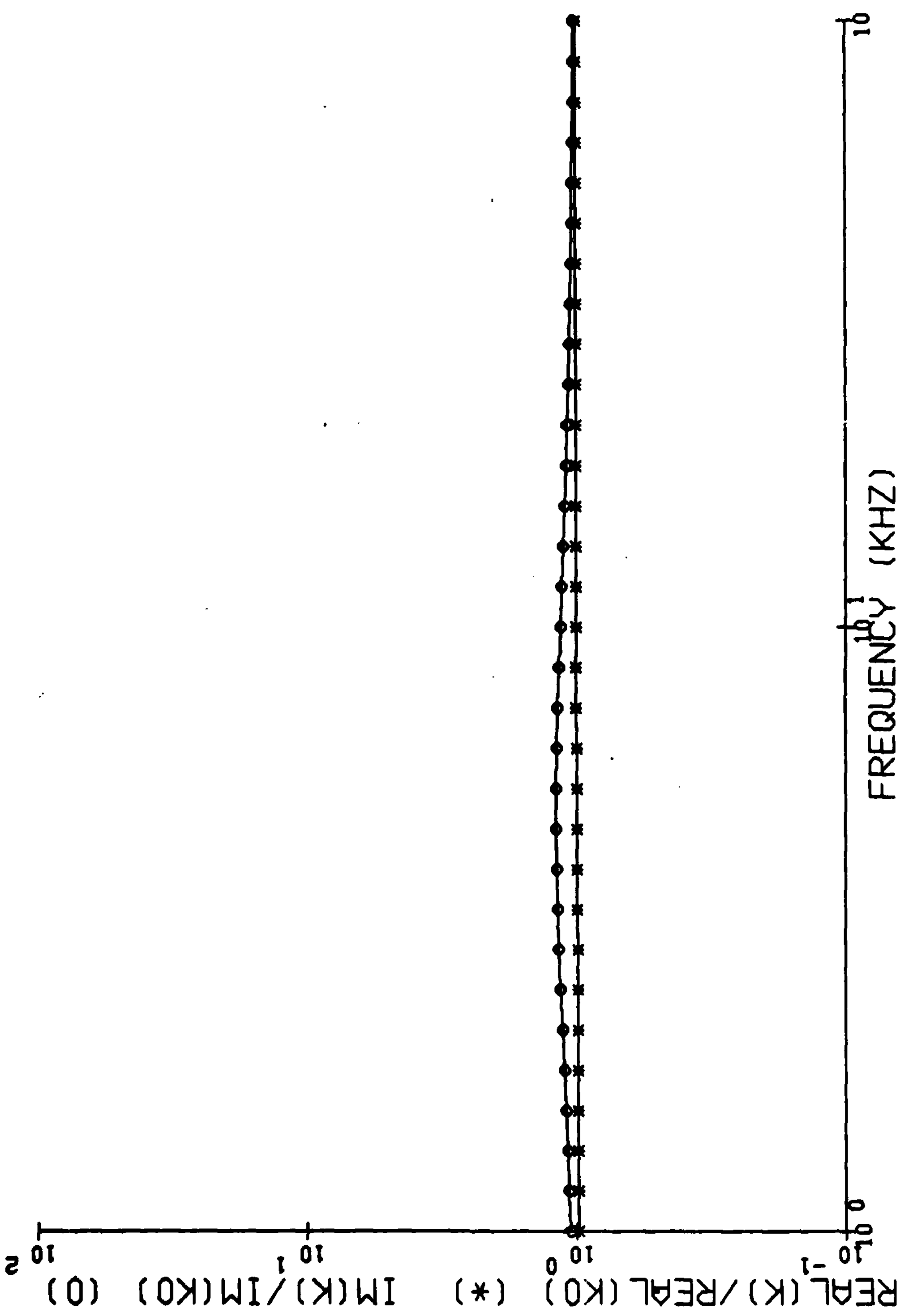

FIGURE 11. DISPERSION RELATION FOR SOUND WAVES IN A VISCOELASTIC ELECTRICAL CONDUCTOR ($B_0 = 10^5$ GAUSS, $\sigma = 10^4$ MHO/M, $\rho c^2/\rho_0 c_0^2 = 1.0*(1.-i0.1)$, $\rho/\rho_0 = 1.0$)

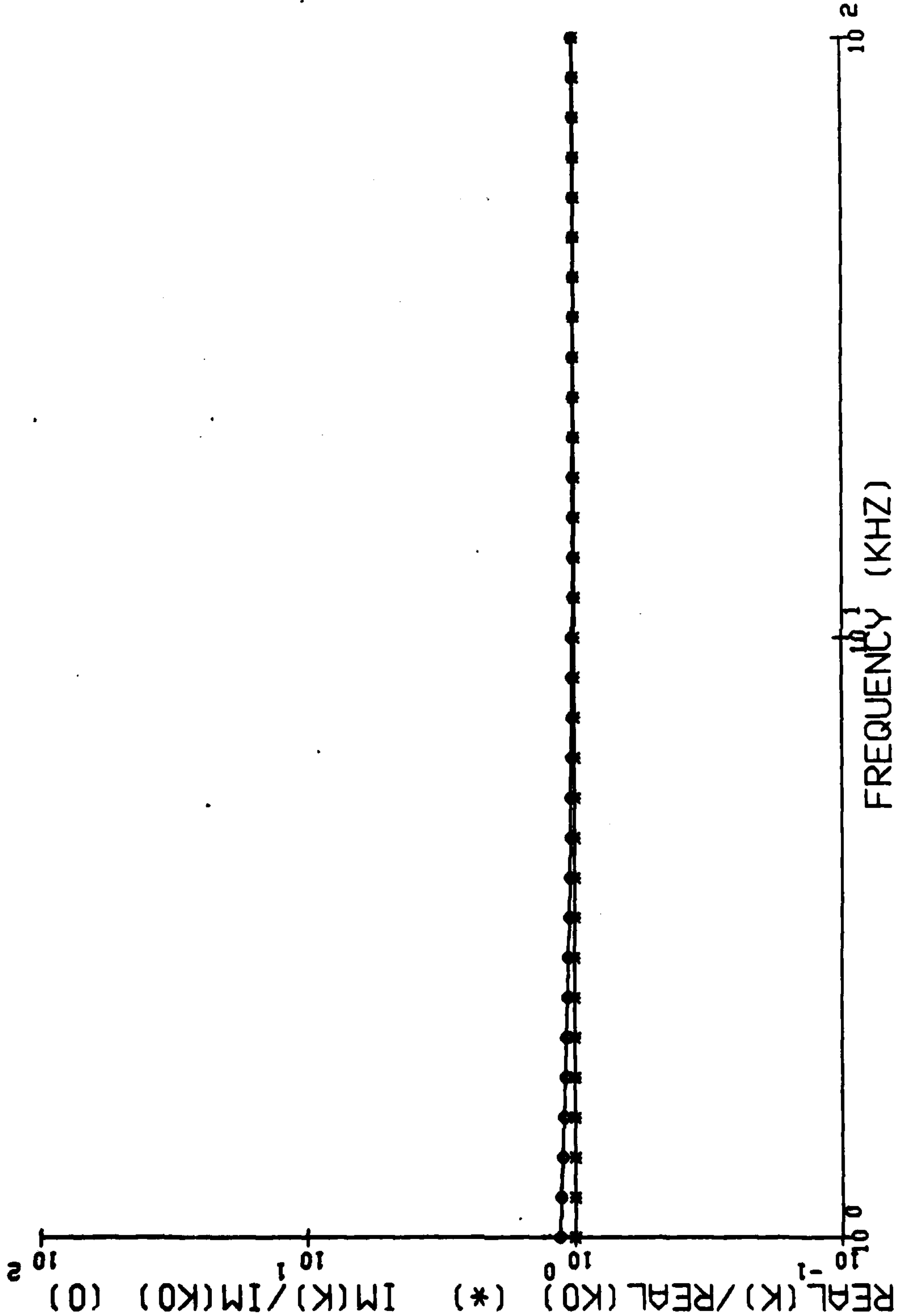

FIGURE 12. DISPERSION RELATION FOR SOUND WAVES IN A VISCOELASTIC ELECTRICAL CONDUCTOR ($B_0 = 10^5$ GAUSS, $\sigma = 10^4$ MHO/M, $\rho c^2/\rho_0 c_0^2 = 0.1*(1.-i0.1)$, $\rho/\rho_0 = 1.0$)

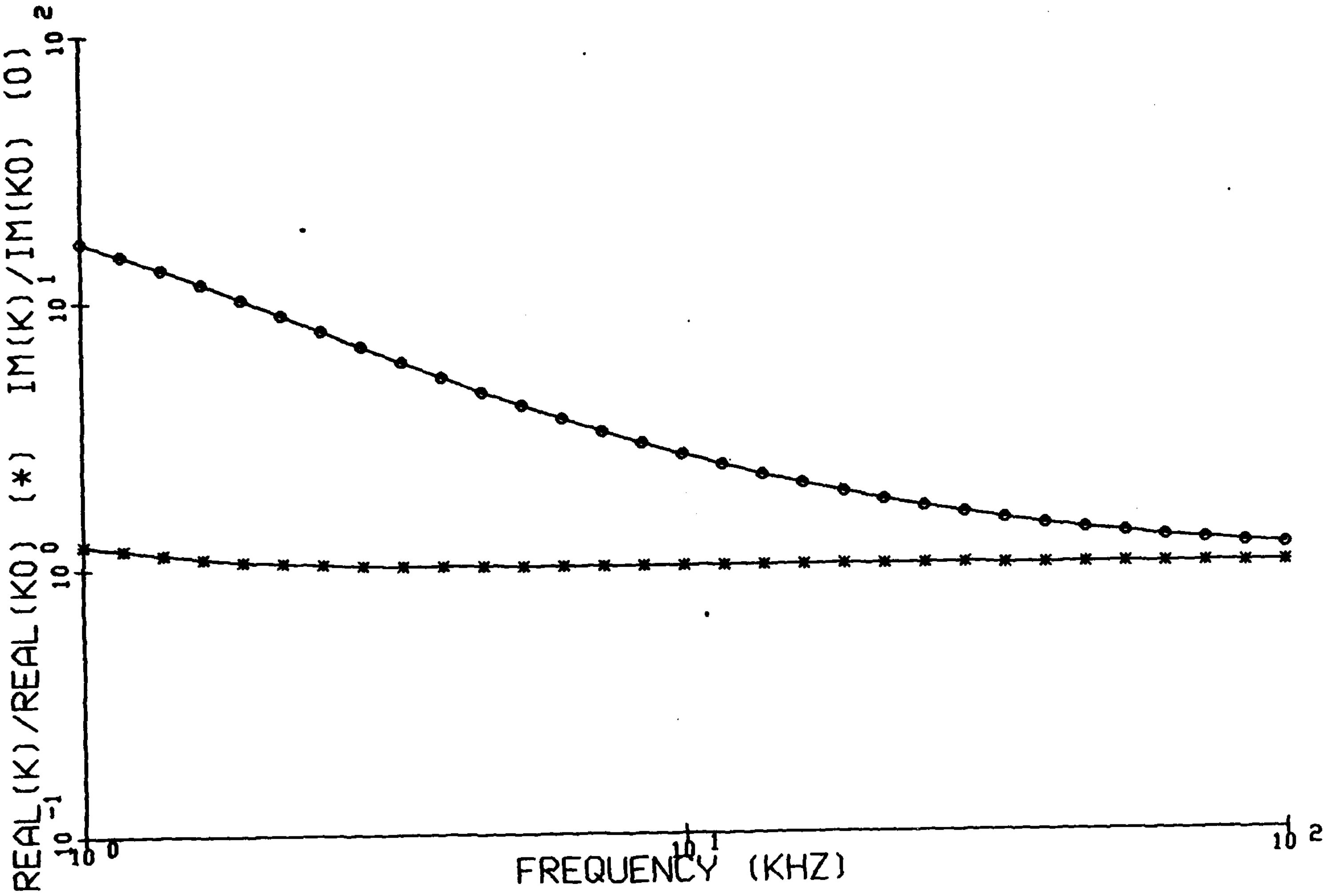

FIGURE 13. DISPERSION RELATION FOR SOUND WAVES IN A VISCOELASTIC ELECTRICAL CONDUCTOR (B_0 = 10^6 GAUSS, $\sigma = 10^4$ MHO/M, $\rho c^2/\rho_0 c_0^2 = 0.1*(1.-i0.1)$, $\rho/\rho_0 = 1.0$)

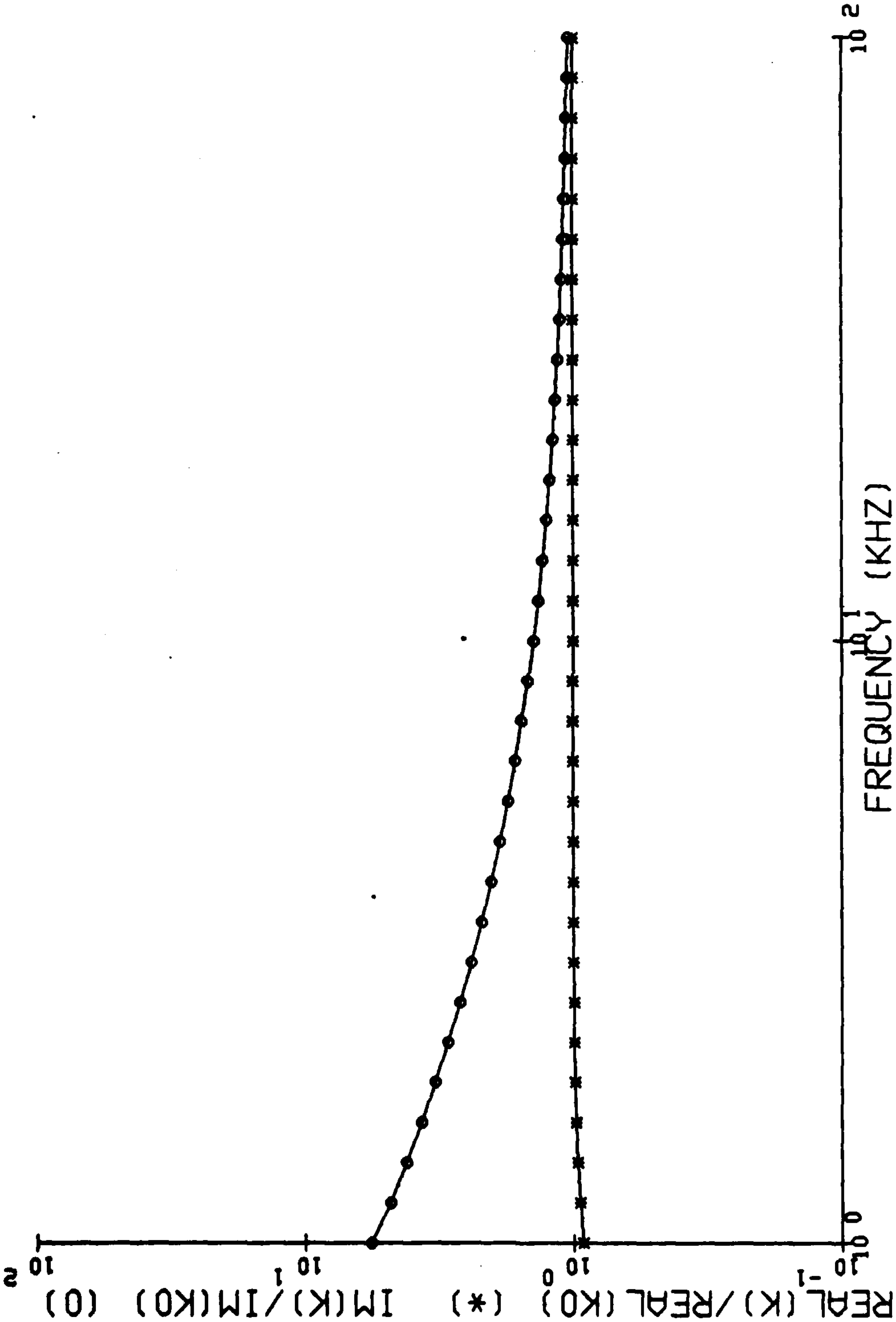

FIGURE 14. DISPERSION RELATIONS FOR SOUND WAVES IN A VISCOELASTIC ELECTRICAL CONDUCTOR (B_0 = 5 x 10^5 GAUSS, σ = 10^4 MHO/M, $\rho c^2/\rho_0 c_0^2$ = 0.1*(1.-i0.1), ρ/ρ_0 = 1.0)

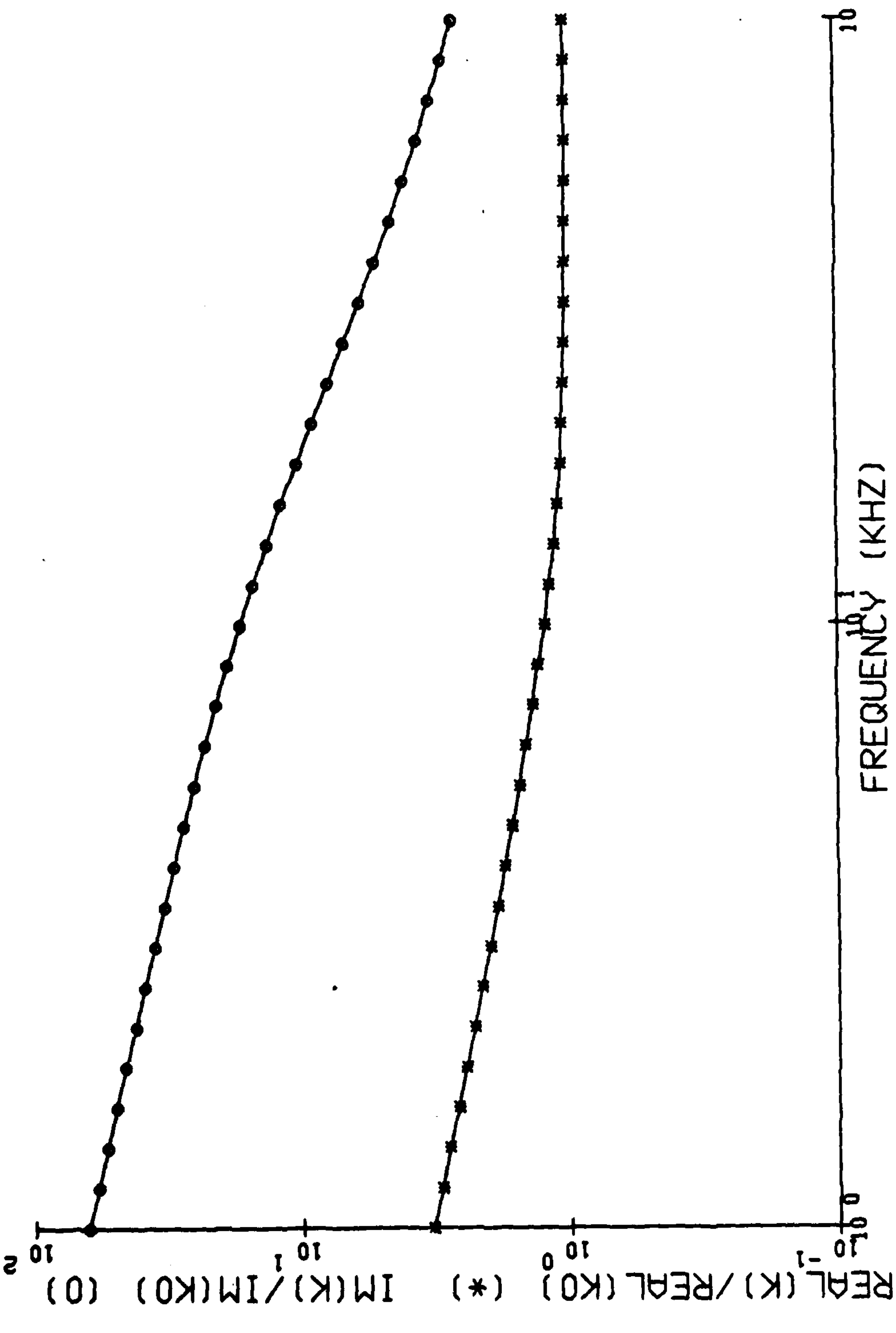

FIGURE 15. DISPERSION RELATIONS FOR SOUND WAVES IN A VISCOELASTIC ELECTRICAL CONDUCTOR ($B_0 = 10^6$ GAUSS, $\sigma = 10^5$ MHO/M, $\rho c^2/\rho_0 c_0^2 = 0.1*(1.-i0.1)$, $\rho/\rho_0 = 1.0$)

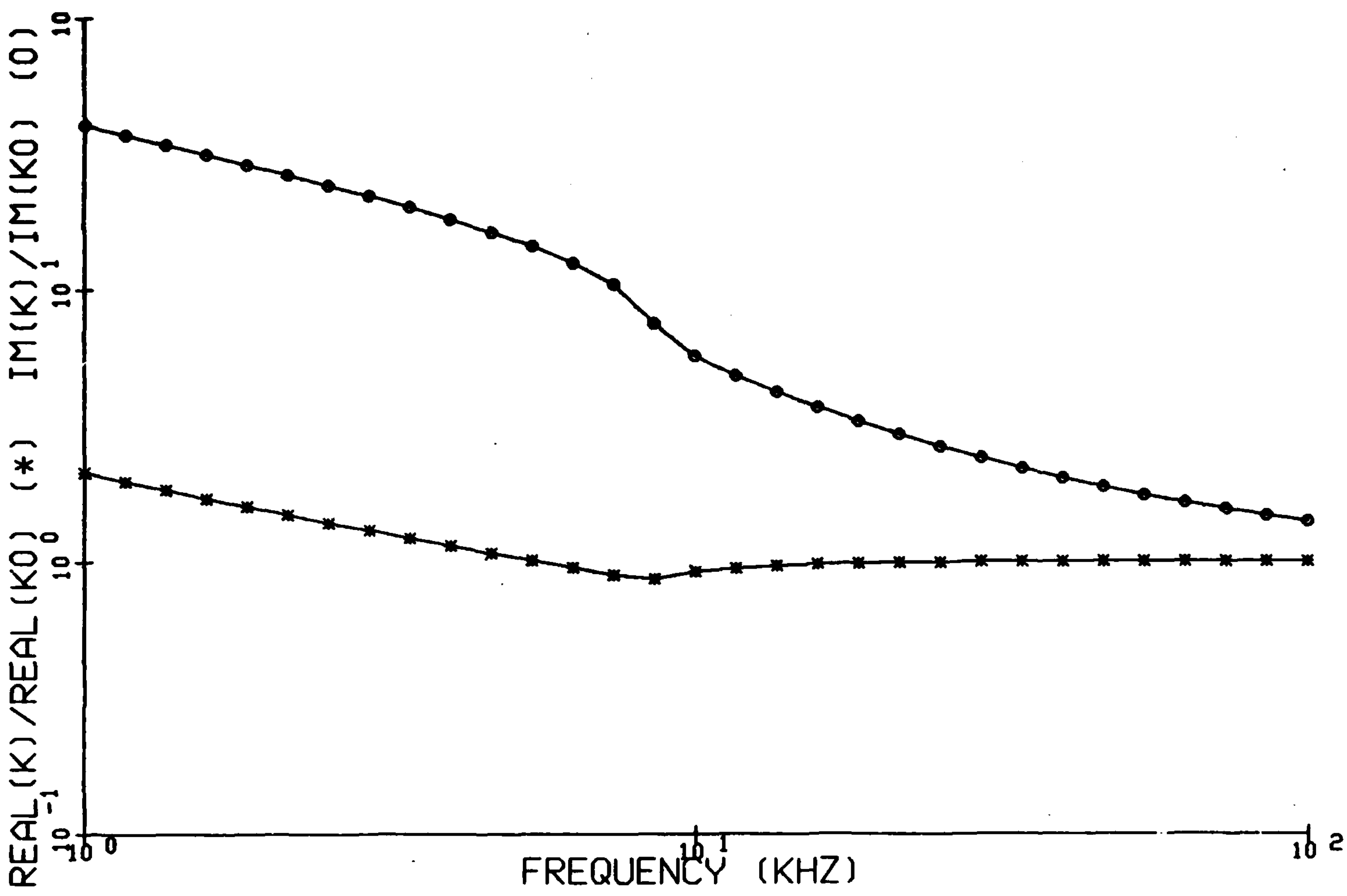

FIGURE 16. DISPERSION RELATIONS FOR SOUND WAVES IN A VISCOELASTIC ELECTRICAL CONDUCTOR ($B_0 = 5 \times 10^5$ GAUSS, $\sigma = 10^5$ MHO/M, $\rho c^2/\rho_0 c_0^2 = 0.1*(1.-i0.1)$, $\rho/\rho_0 = 1.0$)

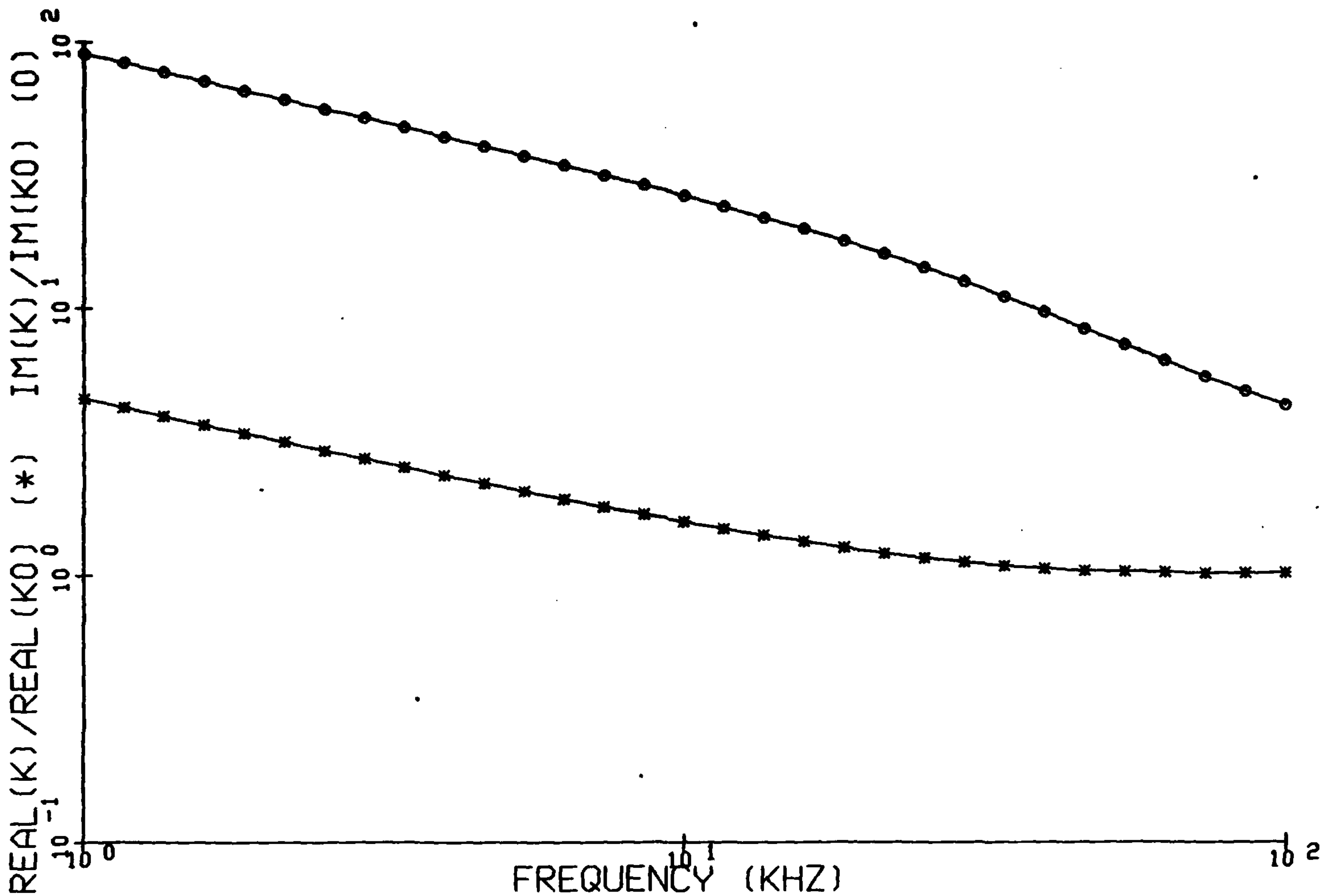

FIGURE 17. DISPERSION RELATIONS FOR SOUND WAVES IN A VISCOELASTIC ELECTRICAL CONDUCTOR (B_0 = 10^6 GAUSS, $\sigma = 10^5$ MHO/M, $\rho c^2/\rho_0 c_0^2 = 0.2*(1.-i0.1)$, $\rho/\rho_0 = 1.0$)

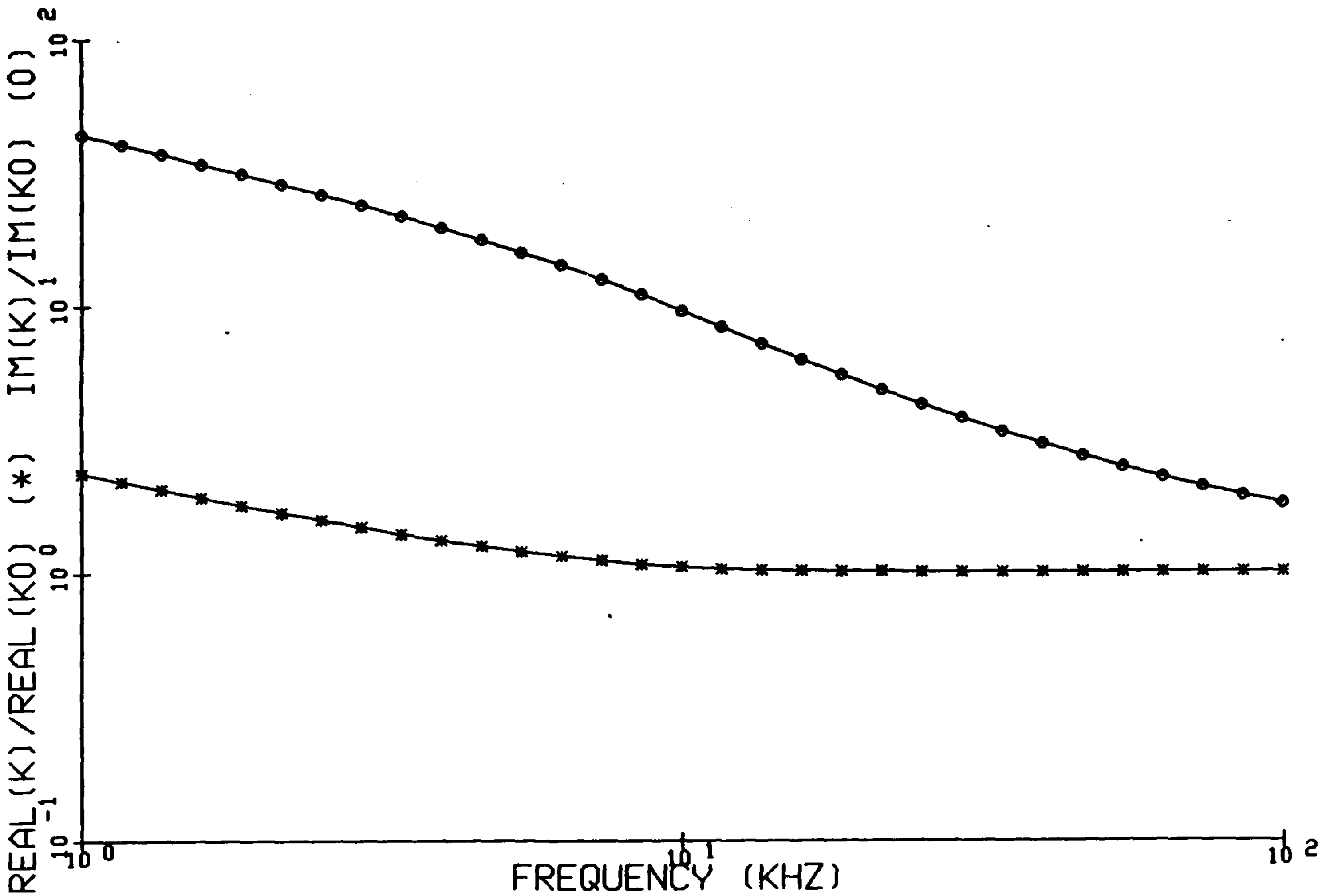

FIGURE 18. DISPERSION RELATIONS FOR SOUND WAVES IN A VISCOELASTIC ELECTRICAL CONDUCTOR ($B_0 = 10^6$ GAUSS, $\sigma = 10^5$ MHO/M, $\rho c^2/\rho_0 c_0^2 = 0.05*(1.-i0.1)$, $\rho/\rho_0 = 1.0$)

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