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QUANTUM NOISE AND EXCESS NOISE IN OPTICAL
HOMODYNE AND HETERODYNE RECEIVERS

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Abstract

A parallel development of the semiclassical and quantum statistics of multi-spatiotemporal mode direct, homodyne, and heterodyne detection using an ideal (except for its sub-unity quantum efficiency) photon detector is presented. Particular emphasis is placed on the latter two coherent detection configurations. The primary intent is to delineate the semiclassical theory's regime of validity and to show, within this regime of validity, how the quantum theory's signal quantum noise, local oscillator quantum noise, the quantum noise incurred because of sub-unity detector quantum efficiency, plus (for heterodyning only) image band quantum noise produce the quantitative equivalent of the semiclassical theory's local oscillator shot noise. The effects of classical fluctuations on the local oscillator, and the recently suggested dual-detector arrangement for suppressing these fluctuations, are treated. It is shown that previous studies of this arrangement have neglected a potentially significant noise contribution.

I. INTRODUCTION

In coherent optical detection [1] - [3], the optical field to be measured is combined on the surface of a photodetector with the field of a strong local oscillator laser whose center frequency is offset by an amount $\Delta\nu$ from that of the signal field. The detection scheme is referred to as optical homodyning if $\Delta\nu=0$, and optical heterodyning if $\Delta\nu=\nu_{IF}>0$, with ν_{IF} being the intermediate frequency in the latter case. For both schemes, electrical filtering of the photocurrent is used to select the beat frequency components in the vicinity of $\Delta\nu$, yielding an output that contains a frequency translated replica of the signal field components that were coherent in space and time with the local oscillator field. Heterodyne detection is now widely employed in coherent CO_2 laser radars [4], [5], and is being vigorously researched for use with semiconductor injection lasers in fiber optics [6] - [8] and space communications [9], [10]. Performance analyses in these areas routinely employ the semiclassical statistical model for photodetection, which implies that the fluctuations observed in coherent optical detection with signal and local oscillator fields of perfect amplitude and frequency stability comprise an additive white Gaussian noise, representing local oscillator shot noise.

It has long been known [11] that the semiclassical statistics for photodetection are quantum mechanically correct only when the total field illuminating the detector is in a Glauber coherent state or a classically random mixture of such states. Inasmuch as ordinary light sources, including lasers and light emitting diodes, obey this classical state condition, there is no need to abandon the semiclassical approach in the vast majority of photodetection sensitivity calculations. However, non-classical light has been generated via resonance fluorescence, as confirmed by observations of its photon anti-bunching [12] and sub-Poissonian behavior [13] in direct detection. Moreover, there is great theoretical interest in squeezed states (also called two-photon coherent states) [14], [15], which are non-classical states of considerable potential for optical communications [16] - [19] and precision measurements [20] - [23]. For these states, the quantum theory of photodetection is essential, and coherent optical detection schemes are the most interesting.

In [18], Yuen and Shapiro developed the quantum descriptions of single-detector optical homodyne and heterodyne receivers. They employed a quasimonochromatic approximation, and assumed a coherent state local oscillator, corresponding to perfect local oscillator amplitude and frequency stability. Within these limitations, complete statistics for multi-spatiotemporal mode detection are available from [18]. More recently [22], the fact that photodetectors respond to photon flux rather than power [24] has been used to relax somewhat the quasimonochromatic approximation in [18]. Thus, were high power, highly stable local oscillator lasers available at all wavelengths of interest, the quantum photodetection theory of [18] would provide a sufficiently general foundation for all optical homodyne and heterodyne sensitivity calculations. Unfortunately, such is not the case.

Driven by heterodyne-detection problems arising from the excess noise of semiconductor injection lasers, Yuen and Chan [25] proposed a dual-detector arrangement for coherent optical detection, akin to the balanced mixer concept of microwave technology [26], [27]. They gave a direct quantum analysis of single-mode dual-detector homodyning, showing that local oscillator quantum and excess noises can be balanced out, hence alleviating injection laser problems that would have plagued a single-detector system. In subsequent work by Chan and his collaborators, the basic dual-detector excess noise cancellation concept was demonstrated experimentally [28], and a variety of non-ideal device effects (quantum efficiency mismatch, etc.) were analyzed using semiclassical multi-temporal mode techniques [29]. Also, Schumaker [30] has shown that the dual-detector single-mode homodyne arrangement is better than single-detector homodyning for making non-classical squeezed state observations, as a result of its ability to cancel out local oscillator excess noise.

Because the quantum treatments in [25] and [30] are confined to single-mode situations, and the multi-mode results in [28], [29] are in essence semiclassical, there is as yet no fully quantum treatment of multi-mode dual-detector coherent optical reception. This paper will develop such a model by generalizing the results of [18]. Simple explicit representations for all of the relevant output terms in coherent optical detection with a strong but classically random local oscillator field will be derived. It will be seen that the previous dual-detector analyses [25], [28] - [30] neglect excess-noise modulation of the signal and quantum noise terms, and the first of these modulation effects may significantly degrade output signal-to-noise ratio in some circumstances. Moreover, because of the calculational power afforded by [18], our rather general quantum results are more directly comparable with those of the multi-mode semiclassical theory than are the more limited results of [25], [30]. Indeed, that comparison is the primary purpose of this paper.

The paper's core, Section II, is a parallel development of the semiclassical and quantum statistics of multi-spatiotemporal mode direct, homodyne, and heterodyne detection using an ideal (except for its sub-unity quantum efficiency) photon detector. The formulation therein for the coherent optical detection schemes will assume perfectly stable local oscillators in the semiclassical models, and the corresponding coherent state local oscillators in the quantum models. We use Section II to delineate the semiclassical theory's regime of validity, and to show, within this regime, how the combination of the quantum theory's signal quantum noise, local oscillator quantum noise, the quantum noise incurred because of sub-unity detector quantum efficiency, plus (for heterodyning only) image band quantum noise produce the quantitative equivalent of the semiclassical theory's local oscillator shot noise. In Section III we address coherent optical detection with classically random local oscillators. The technique of iterated expectation is used to readily obtain both semiclassical and quantum results for this case. Single-detector and dual-detector systems are considered, and our results are compared, in the case of dual-detector quantum homodyning to those of [25], [30]. Finally, in Section IV we briefly discuss the implications of our work for squeezed state generation experiments, which is the application that motivated our analysis.

II. SEMICLASSICAL VS. QUANTUM PHOTODETECTION

The central element of all the photodetection configurations we will consider is shown in Fig. 1. It is a surface photoemitter with active region $\bar{x} = (x,y) \epsilon A_d$ in the z=0 plane, illuminated by a quasimonochronomatic (center frequency ν_0) paraxial scalar electromagnetic wave from the half space z<0 over an observation time interval $t \epsilon \mathcal{T}$. This detector is assumed to have a constant quantum efficiency η over the frequency band containing the illuminating field. The output of the detector is a scalar current density $J(\bar{x},t)$ for $\bar{x} \epsilon A_d$, $t \epsilon \mathcal{T}$. As will be described below, the field characterization we must employ for the illumination is either classical or quantum mechanical, according to whether semiclassical or quantum photodetection statistics are sought. Although we shall neglect internal time constant and noise effects, which are present in real detectors, our direct detection results will be applicable to photomultiplier tubes (for which the current gain permits internal noise to be overcome) at post-detection bandwidths up to the reciprocal anode response time of the tube. Furthermore, our results will be applicable to coherent optical detection systems using semiconductor photodiodes (for which the mixing gain overcomes the internal noise) up to the post-detection bandwidth of the detector. No particular loss of generality is entailed by the use of scalar rather than vector fields, with the caveat that all the coherent optical detection work herein presumes that the actual signal and local oscillator fields are co-polarized. Finally, by appropriate spatial integrations, we can collapse our current density observation to photocurrent observations for a single detector or a multiple-detector array.

A. Direct Detection

In direct detection, the electromagnetic field to be measured comprises the entire illumination, and the basic observation quantity is the current density $J(\overline{x},t)$.

Semiclassical Model Let $E^{(+)}(\overline{x},t)$ be the positive-frequency complex field (V/m units) associated with the classical scalar electric field incident on the detector, i.e., $E^{(+)}(\overline{x},t)$ is the analytic signal of this electric field. Because of our quasimonochromatic assumption, the Fourier transform [1] of $E^{(+)}$

$$E^{(+)}(\overline{x},\nu) = \int dt\, E^{(+)}(\overline{x},t)\, e^{j2\pi\nu t} \qquad (1)$$

is non-zero only for $|\nu - \nu_0| \leq B$, where the bandwidth B is much less than the center frequency ν_0. Because of our paraxial assumption, the short time average power density falling on the point $\overline{x}$ at time t is

$$I(\overline{x},t) = (c\epsilon_0/2)\;\; E^{(-)}(\overline{x},t)E^{(+)}(\overline{x},t), \qquad (2)$$

where c is the speed of light, ϵ_0 the permittivity of free space, and $E^{(-)} \equiv (E^{(+)})^*$ is the negative-frequency complex field, with * denoting complex conjugate.

The standard semiclassical photodetection model [31], in our notation, presumes that $J(\overline{x},t)$ is a conditional space-time Poisson impulse train with rate function $\mu(\overline{x},t) = RI(\overline{x},t)/e$ where e is the electron charge, and R is the detector's responsivity (A/W units) at the illumination's center frequency ν_0 . This means that:

1) the current density, which is of the form

$$J(\bar{x},t) = \sum_n e\delta(\bar{x}-\bar{x}_n)\delta(t-t_n), \qquad (3)$$

has shot effect noise, i.e., it consists of instantaneous emissions of an electron charge e at the random space-time points $\{(\bar{x}_n,t_n) : \bar{x}_n \epsilon A_d, t_n \epsilon T\}$;

2) conditioned on knowledge of the rate function $\{\mu(\bar{x},t) : \bar{x}\epsilon A_d, t\epsilon T\}$, the number of photoemissions occurring within a spatial region $A' \subseteq A_d$ during a time interval $T' \subseteq T$ is a Poisson random variable with mean value $\int_{A'} d\bar{x}' \int_{T'} dt\ \mu(\bar{x},t)$;

3) conditioned on knowledge of $\{\mu(\bar{x},t) : \bar{x}\epsilon A_d , t\epsilon T\}$, the photoemissions occurring in disjoint spaial regions $A', A'' \subset A_d$ are statistically independent processes,

4) conditioned on knowledge of $\{\mu(\bar{x},t): \bar{x}\epsilon A_d, t\epsilon T\}$, the photoemissions occurring in disjoint time intervals $T', T'' \subset T$ are statistically independent processes.

Even though the semiclassical theory of photodetection employs classical fields, it is customary to recognize in this theory that light of frequency ν is quantized into photons of energy $h\nu$, where h is Planck's constant. Thus, for the quasimonochromatic case at hand, the responsivity is ordinarily written as $R = e\eta/h\nu_0$, in terms of the detector's quantum efficiency η and the photon energy at the field's center frequency, so that $\mu(\bar{x},t) = \eta I(\bar{x},t)/h\nu_0$. In fact, because we are concerned with detectors that, quantum mechanically, respond to photon-flux density rather than power density [24], [22], it is more proper to write

$$\mu(\bar{x},t) = \eta I_{ph}(\bar{x},t), \qquad (4)$$

where I_{ph} is the classical photon-flux density

$$I_{ph}(\overline{x},t) = E^*(\overline{x},t)E(\overline{x},t) \tag{5}$$

obtained from the $(\text{photons/s})^{1/2}/\text{m}$ units positive-frequency complex field

$$E(\overline{x},t) = \int d\nu(c\varepsilon_0/2h\nu)^{1/2}\,\mathcal{E}^{(+)}(\overline{x},\nu)e^{-j2\pi\nu t}. \tag{6}$$

For all practical purposes in the semiclassical theory, with quasimonochromatic light we can use $\mu(\overline{x},t) = \eta I(\overline{x},t)/h\nu_0$ and $\mu(\overline{x},t) = \eta I_{ph}(\overline{x},t)$ interchangeably. This amounts to using $\nu \approx \nu_0$ in the square-root term of (6), an approximation whose validity is guaranteed by (1). In the quantum theory, even with quasimonochromatic light, it is critical to employ the photon-flux density formulation, see [22].

<u>Quantum Model</u> In the quantum photodetection theory, the classical positive-frequency complex field $\mathcal{E}^{(+)}(\overline{x},t)$ is replaced by a positive-frequency field operator $\hat{\mathcal{E}}^{(+)}(\overline{x},t)$, whose quantum state is specified by a density operator ρ. The quasimonochromatic and paraxial conditions of the semiclassical theory become conditions on the density operator, namely, that the excited (non-vacuum state) modes of $\hat{\mathcal{E}}^{(+)}(\overline{x},t)$ lie at frequencies within B of ν_0 and propagate at small angles to the z axis. As in [17], [18], [22], we shall regard the current density $J(\overline{x},t)$ as a classical quantity, corresponding to the macroscopic output[2] of the quantum measurement performed by the detector of Fig. 1 on the field $\hat{\mathcal{E}}^{(+)}(\overline{x},t)$. To provide an explicit representation of this quantum measurement, we must first develop the quantum effective photon-flux density.

Let us convert $\hat{\mathcal{E}}^{(+)}(\overline{x},t)$ to a photon-units field operator by defining (cf. Eqs. (1), (6))

$$\hat{E}^{(+)}(\overline{x},\nu) = \int dt\ \hat{E}^{(+)}(\overline{x},t)\ e^{j2\pi\nu t}\ , \tag{7}$$

and

$$\hat{E}(\overline{x},t) = \int d\nu\ (c\epsilon_o/2h\nu)^{1/2}\ \hat{E}^{(+)}(\overline{x},\nu)e^{-j2\pi\nu t}\ . \tag{8}$$

Equation (8) defines the same basic field operator used in [16] - [18], [22]. We can made a modal expansion

$$\hat{E}(\overline{x},t) = \sum_n \hat{a}_n \xi_n(\overline{x},t),\ \ \overline{x}\epsilon A_d,\ t\epsilon T \tag{9}$$

of this operator, where $\{\hat{a}_n\}$ are modal annihilation operators satisfying the commutation rules

$$[\hat{a}_n,\hat{a}_m] = 0,\ \ [\hat{a}_n,\hat{a}_m^{\dagger}] = \delta_{nm}, \tag{10}$$

and $\{\xi_n\}$ are a complete orthonormal set of classical functions over $\overline{x}\epsilon A_d$, $t\epsilon T$. In Eq. (10), the $\{\hat{a}_n^{\dagger}\}$ are the adjoints of the $\{\hat{a}_n\}$; they are modal creation operators.

For a detector of sub-unity quantum efficiency we must adjoin to (9) a fictitious field

$$\hat{E}_{vac}(\overline{x},t) = \sum_n \hat{c}_n \xi_n(\overline{x},t),\ \ \ \overline{x}\epsilon A_d,\ t\epsilon T, \tag{11}$$

where $\{\hat{c}_n\}$ are modal annihilation operators that commute with $\{\hat{a}_n\}$ and $\{\hat{a}_n^{\dagger}\}$, viz.

$$[\hat{c}_n,\hat{c}_m] = [\hat{c}_n,\hat{a}_m] = [\hat{c}_n,\hat{a}_m^\dagger] = 0 ,$$
$$[\hat{c}_n,\hat{c}_m^\dagger] = \delta_{nm}. \qquad (12)$$

The fields $\hat{E}$ and $\hat{E}_{vac}$ are quantum-mechanically independent, with the latter having all its modes in the vacuum state. In terms of $\hat{E}$ and $\hat{E}_{vac}$, the effective photon-flux density operator for the detector is

$$\hat{I}'_{ph}(\bar{x},t) = \hat{E}'^\dagger(\bar{x},t)\hat{E}'(\bar{x},t) , \qquad (13)$$

with

$$\hat{E}'(\bar{x},t) \equiv \eta^{1/2}\hat{E}(\bar{x},t) + (1-\eta)^{1/2}\hat{E}_{vac}(\bar{x},t) . \qquad (14)$$

The representation theorem of quantum photodetection [18, theorem 1] can now be stated (in our notation) as follows. The classical current density $J(\bar{x},t)$ obtained from photoemissive detection measures the quantum operator

$$\hat{J}(\bar{x},t) = e\hat{I}'_{ph}(\bar{x},t) , \qquad (15)$$

i.e., it is proportional to the effective photon-flux density. In somewhat more detail this means (cf. the semiclassical case):

1) the current density obeys

$$J(\bar{x},t) = \sum_n e\, \delta(\bar{x}-\bar{x}_n)\delta(t-t_n) , \qquad (16)$$

so it is still a collection of instantaneous emissions of an electron charge at random space-time points $\{(\bar{x}_n,t_n)\}$;

2) if $F(J(\overline{x},t))$ is an arbitrary functional of the current density, then the classical average of this random variable $\langle F(J(\overline{x},t))\rangle$ equals the quantum average $tr(\rho' F(\hat{J}(\overline{x},t)))$, where tr denotes trace and $\rho' = \rho \otimes \rho_{vac}$ gives the joint density operator for $\hat{E}$ and $\hat{E}_{vac}$ in terms of the density operator ρ for $\hat{E}$ and the vacuum-state $\rho_{vac} = \bigotimes_{n} |0\rangle\langle 0|$ density operator for $\hat{E}_{vac}$.

Note that we cannot dispense with the vacuum state field $\hat{E}_{vac}$ unless $\eta = 1$, even though its average value obeys $tr(\rho_{vac} \hat{E}_{vac}(\overline{x},t)) = 0$ regardless of the value of η. This is because the zero-point fluctuations (vacuum-state quantum noise) in $\hat{E}_{vac}$ can contribute to $F(J(\overline{x},t))$. Indeed the noise in $J(\overline{x},t)$ has nothing to do with the shot effect associated with the discreteness of the electron charge. Rather, it is the quantum noise in $\hat{E}'$ being observed through measurement of the effective photon-flux density.

<u>Comparison</u> Let us suppose that the density operator for $\hat{E}$ is a classical state[3], i.e.,

$$\rho = \int d^2\underline{\alpha}\; P(\underline{\alpha};\underline{\alpha}^*)\; |\underline{\alpha}\rangle\langle\underline{\alpha}| \tag{17}$$

with

$$\hat{a}_n |\underline{\alpha}\rangle = \alpha_n |\underline{\alpha}\rangle \tag{18}$$

defining the multi-mode Glauber coherent states of the field $\hat{E}$ in terms of the modal expansion (9), and $P(\underline{\alpha};\underline{\alpha}^*)$ being a classical probability density function $P(\underline{\alpha};\underline{\alpha}^*) \geq 0$, $\int d^2\underline{\alpha}\; P(\underline{\alpha};\underline{\alpha}^*) = 1$. It was shown in [18] that this is a necessary and sufficient condition for the semiclassical statistics to be quantitatively correct. In particular, under this condition the quantum model predicts that $J(\bar{x},t)$ is a conditional space-time Poisson impulse train with conditional rate function

$$\mu(\bar{x},t) = \eta E^*(\bar{x},t)E(\bar{x},t) \tag{19}$$

for

$$E(\bar{x},t) \equiv \sum_n \alpha_n \varepsilon_n(\bar{x},t) = \langle \underline{\alpha}|\hat{E}(\bar{x},t)|\underline{\alpha}\rangle , \tag{20}$$

the average field illuminating the detector given the state of $\hat{E}$ is the multi-mode coherent state $|\underline{\alpha}\rangle$.

To illustrate the above behavior, let us examine the statistics of the observed photon count[4]

$$N = e^{-1} \int_{A_d} d\bar{x} \int_{T} dt \; J(\bar{x},t) \tag{21}$$

assuming single-mode illumination, and $\eta = 1$. In the semiclassical theory we shall take

$$E(\bar{x},t) = [\alpha/(A_d T)^{1/2}] e^{-j2\pi\nu_0 t}, \; \bar{x}\epsilon A_d, t\epsilon T , \tag{22}$$

where α is a complex-valued random variable with probability density function $p(\alpha)$, A_d is the area of A_d, and T is the duration of T. We then obtain Mandel's rule [32] for the probability distribution of N

$$\Pr[N = n] = \int d^2\alpha \; p(\alpha) \; (|\alpha|^{2n}/n!) \exp(-|\alpha|^2) , \tag{23}$$

viz. N is a conditionally Poisson random variable. Equation (23) gives the mean and variance of the observed photocount to be

$$\langle N\rangle = \int d^2\alpha \; p(\alpha) \; |\alpha|^2 = \langle|\alpha|^2\rangle \tag{24}$$

and

$$\mathrm{var}(N) = \langle N\rangle + \mathrm{var}\,(|\alpha|^2), \tag{25}$$

respectively, where the first term on the right in (25) represents shot noise and the second term on the right in (25) represents excess noise.

In the quantum theory we let $\xi_1(\overline{x},t) = (A_dT)^{-1/2}e^{-j2\pi\nu_0 t}$ be the only excited mode in (9), so that the density operator for $\hat{E}$ is

$$\rho = \rho_1 \bigotimes_{n>1} |0\rangle\langle 0| \tag{26}$$

for ρ_1 the density operator of mode 1. We then find for the probability distribution of N [11], [17]

$$\Pr[N = n] = \langle n|\rho_1|n\rangle , \tag{27}$$

where

$$\hat{a}_1^{\dagger}\hat{a}_1|n\rangle = n|n\rangle \tag{28}$$

defines the photon number states of the first mode of $\hat{E}$. If ρ_1 is the classical state.

$$\rho_1 = \int d^2\alpha \; p(\alpha) \quad |\alpha\rangle\langle\alpha| \quad , \tag{29}$$

with $p(\alpha)$ being the probability density from the semiclassical theory (cf. Eq. (17)), then (27) reduces to (23) as expected. Thus, in this case the semiclassical theory is quantitatively correct in its prediction of the photon counting probability distribution. It is nevertheless physically incorrect, in that it ascribes the photon counting fluctuations to shot noise, whereas they are actually a manifestation of the illumination field's quantum noise. For example, were $\rho_1 = |k\rangle\langle k|$ where $|k\rangle$ is the k-photon number state (a non-classical state), then we would get

$$\Pr[N=n] = \delta_{nk} \tag{30}$$

from (27), whence

$$\langle N\rangle = k \, , \tag{31}$$

and

$$\mathrm{var}(N) = 0 \, , \tag{32}$$

for the photon count mean and variance. Here the field state is an eigenket of our observation operator, so there is no uncertainty in the measurement outcome. This sub-Poissonian behavior cannot be obtained from the semiclassical theory, because for all $p(\alpha)$ the excess noise term in (25) will be non-negative, forcing $\mathrm{var}(N) \geq \langle N\rangle$ to prevail.

B. Homodyne Detection

The configuration we shall consider for single-detector multi-spatiotemporal mode homodyne detection is shown in Fig. 2. The signal field to be detected is combined, through a lossless beam splitter of intensity transmission ϵ, with a perfectly stable local oscillator field on the surface of the Fig. 1 photodetector. The resulting current density, $J_{hom}(\bar{x},t)$, is our homodyne detection output, whose statistics we shall characterize below. By spatial integration of our results over the detector's active region A_d, we can use our model to describe single-detector homodyning; the extension to dual-detector homodyning will be made in Section III.

Semiclassical Model The total classical photon-units complex field incident on the photodetector is given by[5]

$$E(\bar{x},t) = \epsilon^{1/2}E_S(\bar{x},t) + (1-\epsilon)^{1/2}E_{LO}(\bar{x},t) \quad , \qquad (33)$$

for $\bar{x}\epsilon A_d$, $t\epsilon T$, in terms of a (potentially random) weak signal field $E_S(\bar{x},t)$, and a deterministic strong local oscillator field $E_{LO}(\bar{x},t)$. The latter has a classical photon-flux density

$$I_{phLO}(\bar{x},t) = E_{LO}^*(\bar{x},t)E_{LO}(\bar{x},t) \quad , \qquad (34)$$

that greatly exceeds that of the former

$$I_{phS}(\bar{x},t) = E_S^*(\bar{x},t)E_S(\bar{x},t) \quad , \qquad (35)$$

for $\bar{x}\epsilon A_d, t\epsilon T$. Thus, the rate function driving the photodetector is, from (4), (5), (33)-(35), approximately

$$\mu(\bar{x},t) = \eta[(1-\epsilon)I_{phLO}(\bar{x},t)+2[\epsilon(1-\epsilon)]^{1/2}Re(E_S(\bar{x},t)E_{LO}^*(\bar{x},t))] \ . \qquad (36)$$

It then follows, from the Central Limit Theorem for high density shot noise [33], [34], that at very large values of the local oscillator classical photon number

$$N_{phLO} = \int_{A_d} d\bar{x} \int_T dt \ I_{phLO}(\bar{x},t) \qquad (37)$$

the homodyne detection current density $J_{hom}(\bar{x},t)$ is a conditional Gaussian process. Specifically, conditioned on knowledge of the signal field

$\{E_S(\bar{x},t)\ :\ \bar{x}\epsilon A_d,\ t\epsilon T\}$, $J_{hom}(\bar{x},t)$ is the sum of three current densities:

1) a homodyne-mixing current density signal term $2e\eta[\epsilon(1-\epsilon)]^{1/2}\mathrm{Re}(E_S(\bar{x},t)E_{LO}^*(\bar{x},t))$;
2) a direct-detection local oscillator bias current density $e\eta(1-\epsilon)I_{phLO}(\bar{x},t)$; and
3) a local oscillator shot noise current density, which is a zero-mean spatiotemporal non-stationary white Gaussian noise process $J_{shot}(\bar{x},t)$ with covariance function

$$\langle J_{shot}(\bar{x}_1,t_1)J_{shot}(\bar{x}_2,t_2)\rangle = e^2\eta(1-\epsilon)I_{phLO}(\bar{x}_1,t_1)\delta(\bar{x}_1-\bar{x}_2)\delta(t_1-t_2) \quad . \qquad (38)$$

In order to connect the preceding multi-spatiotemporal mode formulation with more familiar single-detector multi-temporal mode results, let us consider the statistics of the single-detector homodyne photocurrent

$$i_{hom}(t) = \int_{A_d} d\bar{x}\ J_{hom}(\bar{x},t) \quad , \qquad (39)$$

assuming that

$$E_{LO}(\bar{x},t) = (P_{LO}/h\nu_o A_d)^{1/2}\ e^{-j2\pi\nu_o t} \quad , \qquad (40)$$

corresponding to a normally-incident plane wave local oscillator of power P_{LO}. Here we find that, conditioned on knowledge of the signal field, $i_{hom}(t)$ comprises a signal current

$$i_{sig}(t) = 2e\eta[P_{LO}\epsilon(1-\epsilon)/h\nu_o A_d]^{1/2}\mathrm{Re}(\int_{A_d} d\bar{x}\ E_S(\bar{x},t)e^{j2\pi\nu_o t}) \ , \qquad (41)$$

plus a zero-frequency bias current

$$i_{bias} = en(1-\epsilon)P_{LO}/h\nu_0 \quad , \tag{42}$$

plus a zero-mean stationary white Gaussian process shot-noise current $i_{shot}(t)$ with spectral density ei_{bias} (A^2/Hz) .

The signal current is a frequency-translated (to baseband) replica of the normally-incident plane wave component of $E_S(\overline{x},t)$ that is in phase with the local oscillator field. The bias current is the zero-frequency photocurrent produced by the local oscillator field. The noise current is the local-oscillator shot noise, whose spectrum follows the well known Schottky formula [35].

Quantum Model In the quantum model, Eq. (33) becomes an operator-valued expression

$$\hat{E}(\overline{x},t) = \epsilon^{1/2}\hat{E}_S(\overline{x},t) + (1-\epsilon)^{1/2}\hat{E}_{LO}(\overline{x},t), \tag{43}$$

giving the field operator $\hat{E}$ that drives the detector in terms of the signal field operator $\hat{E}_S$ and the local oscillator field operator $\hat{E}_{LO}$. The density operator ρ for $\hat{E}$ is assumed to be

$$\rho = \rho_S \otimes \rho_{LO} \tag{44}$$

where ρ_S is an arbitrary signal field density operator and $\rho_{LO} = |\underline{\alpha}_{LO}\rangle\langle\underline{\alpha}_{LO}|$ is a multi-mode coherent state local oscillator density operator. The latter corresponds to a mean local oscillator field

$$E_{LO}(\overline{x},t) \equiv \langle\underline{\alpha}_{LO}|\hat{E}_{LO}(\overline{x},t)|\underline{\alpha}_{LO}\rangle = \sum_n \alpha_{LOn}\, \xi_n(\overline{x},t) \ , \tag{45}$$

when $\hat{E}_{LO}$ is expanded using the mode set $\{\xi_n\}$ as was done for $\hat{E}$ in Eq. (9).

The strong local oscillator condition of the quantum theory,

$$tr(\rho_S E_S^{\dagger}(\overline{x},t)E_S(\overline{x},t)) << |E_{LO}(\overline{x},t)|^2 \quad , \tag{46}$$

is assumed to prevail (cf. Eqs. (34), (35)), with a very large average local oscillator photon number

$$N_{LO} \equiv \int_{A_d} d\overline{x} \int_T dt |E_{LO}(\overline{x},t)|^2 >> 1, \tag{47}$$

(cf. Eq. (37)).

To obtain the effective photon-flux density operator measured by the detector we adjoin to $\hat{E}$ from (43) a quantum-mechanically independent vacuum-state field operator $\hat{E}_{vac}$,see Eqs. (11) - (14). We can now give a fully quantum characterization of the classical homodyne current density $J_{hom}(\overline{x},t)$,by translating the results of [18, theorem 2] into our notation. The strong local oscillator condition implies that this classical current density measures the quantum operator

$$\hat{J}_{hom}(\overline{x},t) = e\eta(1-\epsilon)\hat{E}_{LO}^{\dagger}(\overline{x},t)\hat{E}_{LO}(\overline{x},t) + 2e[\eta(1-\epsilon)]^{1/2}Re\,\{[(\eta\epsilon)^{1/2}\hat{E}_S(\overline{x},t) + (1-\eta)^{1/2}\hat{E}_{vac}(\overline{x},t)] \cdot \hat{E}_{LO}^{\dagger}(\overline{x},t)\} . \tag{48}$$

Moreover, because $N_{LO}>>1$, the local oscillator direct detection term in (48), $e\eta(1-\epsilon)\hat{E}_{LO}^{\dagger}(\overline{x},t)\hat{E}_{LO}(\overline{x},t)$, yields classical observation values comprising a bias current density $e\eta(1-\epsilon)|E_{LO}(\overline{x},t)|^2$ plus a local oscillator quantum noise current density, which is a zero-mean spatiotemporal non-stationary white Gaussian noise process $J_{LOq}(\overline{x},t)$ with covariance function

$$\langle J_{LOq}(\overline{x}_1,t_1)J_{LOq}(\overline{x}_2,t_2)\rangle = [e\eta(1-\epsilon)]^2|E_{LO}(\overline{x}_1,t_1)|^2\delta(\overline{x}_1-\overline{x}_2)\delta(t_1-t_2) \quad . \tag{49}$$

Furthermore, under this same condition[6], the second term on the right in (48) simplifies to a homodyne-mixing signal operator $2e\eta[\epsilon(1-\epsilon)]^{1/2}\mathrm{Re}(\hat{E}_S(\bar{x},t)E_{LO}^*(\bar{x},t))$ plus a sub-unity quantum efficiency $(\eta<1)$ quantum-noise current density. The latter current density is a zero-mean spatiotemporal non-stationary white Gaussian noise process $J_{vac}(\bar{x},t)$ with covariance function

$$\langle J_{vac}(\bar{x}_1,t_1)J_{vac}(\bar{x}_2,t_2)\rangle =$$

$$e^2\eta(1-\eta)(1-\epsilon)|E_{LO}(\bar{x}_1,t_1)|^2\delta(\bar{x}_1-\bar{x}_2)\delta(t_1-t_2) \; ; \qquad (50)$$

J_{vac} 's statistically independent of J_{LOq}. Thus, the classical homodyne current density $J_{hom}(\bar{x},t)$ measures the operator

$$\hat{J}_{hom}(\bar{x},t) = e\eta(1-\epsilon)|E_{LO}(\bar{x},t)|^2 + J_{LOq}(\bar{x},t)$$

$$+ 2e\eta[\epsilon(1-\epsilon)]^{1/2}\,\mathrm{Re}(\hat{E}_S(\bar{x},t)E_{LO}^*(\bar{x},t)) + J_{vac}(\bar{x},t). \qquad (51)$$

The first term on the right in (51) is the local oscillator bias current density, the second term is the classical representation of the local oscillator's quantum noise contributed by the $e\eta(1-\epsilon)\hat{E}_{LO}^{+}\hat{E}_{LO}$ measurement, and the last term is the classical representation of the $\eta<1$ quantum noise contributed by the $2e[\eta(1-\eta)(1-\epsilon)]^{1/2}\mathrm{Re}(\hat{E}_{vac}(\bar{x},t)E_{LO}^*(\bar{x},t))$ measurement. The signal field contribution to $\hat{J}_{hom}(\bar{x},t)$ cannot be simplified further without knowledge of the density operator ρ_S . In general, this term will contribute signal field quantum noise to the homodyne observation, as will be seen below.

Comparison To facilitate comparison of the semiclassical and the quantum theories of homodyning, we shall restrict our consideration to the single detector case. First, we need the quantum characterization of the homodyne photocurrent (39), which can be obtained by spatial integration of the results just presented. We assume a normally-incident plane wave mean local oscillator field

$$\hat{E}_{LO}(\bar{x},t) = (P_{LO}/h\nu_o A_d)^{1/2} e^{-j2\pi\nu_o t} , \qquad (52)$$

and we find the $i_{hom}(t)$ measures the operator

$$\hat{i}_{hom}(t) = i_{bias} + i_{LOq}(t)$$
$$+ 2e\eta[P_{LO}\epsilon(1-\epsilon)/h\nu_o A_d]^{1/2} \mathrm{Re}\left(\int_{A_d} d\bar{x}\hat{E}_S(\bar{x},t)\, e^{j2\pi\nu_o t}\right) + i_{vac}(t) . \qquad (53)$$

Here, i_{bias} is given by (42), and i_{LOq} and i_{vac} are statistically independent zero-mean stationary white Gaussian noise processes with spectral densities $e\eta(1-\epsilon)i_{bias}$ and $e(1-\eta)i_{bias}$,respectively.

Physically, i_{bias} is the local oscillator bias current, i_{LOq} is the local oscillator quantum-noise current, and i_{vac} is the $\eta<1$ quantum-noise current. Equation (53) differs from the semiclassical description in two respects: the homodyne-mixing signal term involves the quantum field operator $\hat{E}_S$ rather than the classical field E_S ; the noise in the homodyne observation is a combination of local oscillator quantum noise, $\eta<1$ quantum noise, and signal quantum noise, rather than simply being local oscillator shot noise. We know, from the direct detection discussion, that the semiclassical photodetection model is quantitatively correct if the density operator ρ for the field $\hat{E}$ illuminating the detector represents a classical state. This situation occurs, under (44), if and only if ρ_S ,the signal field density operator, is a classical state

$$\rho_S = \int d^2\underline{\alpha}_S P_S(\underline{\alpha}_S;\underline{\alpha}_S^*)|\underline{\alpha}_S\rangle\langle\underline{\alpha}_S| \qquad (54)$$

for $|\underline{\alpha}_S\rangle$ the multi-mode signal field coherent state in modal expansion of $\hat{E}_S$ similar to Eq. (9), with P_S being a classical probability density. When (54) applies, the homodyne-mixing signal term in (53) can be given a classical representation akin to that employed for the $\hat{E}_{vac}$ mixing term in going from

(48) to (51). In particular, for a classical signal field state, the quantum theory of homodyning predicts that

$$i_{hom}(t) = i_{bias} + i_{LOq}(t) + 2e\eta[P_{LO}\epsilon(1-\epsilon)/h\nu_o A_d]^{1/2}\mathrm{Re}\left(\int_{A_d} d\bar{x}E_S(\bar{x},t)\, e^{j2\pi\nu_o t}\right) + i_{Sq}(t) + i_{vac}(t) \quad , \tag{55}$$

where

$$E_S(\bar{x},t) \equiv \langle \underset{\sim}{\alpha}_S|\hat{E}_S(\bar{x},t)|\underset{\sim}{\alpha}_S\rangle \tag{56}$$

is the classical mean signal field when the state of $\hat{E}_S$ is $|\underset{\sim}{\alpha}_S\rangle$, and $i_{Sq}(t)$ is a zero-mean stationary white Gaussian noise current of spectral density $e\eta\epsilon i_{bias}$ that is statistically independent of i_{LOq} and i_{vac}. The classical field E_S is, in general, a random process with probability density $P_S(\underset{\sim}{\alpha}_S;\underset{\sim}{\alpha}_S^*)$ in modal expansion form. The current $i_{Sq}(t)$ is the classical representation of the coherent state signal field quantum noise as observed through the measurement operator (53). Note that

$$i'(t) \equiv i_{LOq}(t) + i_{Sq}(t) + i_{vac}(t) \tag{57}$$

is a zero-mean stationary white Gaussian noise process of spectral density ei_{bias}, in quantitative agreement with the semiclassical $i_{shot}(t)$ result. Of course, the interpretation of the origin of the noise in homodyning is different in these two theories. Local oscillator shot noise is a semiclassical fiction; the noise seen in homodyne detection (with an ideal local oscillator) is local oscillator quantum noise, plus $\eta<1$ quantum noise, plus signal quantum noise. Moreover, in the limit $\eta\epsilon \to 1$ with $\eta(1-\epsilon)N_{LO} >> 1$, the two former contributions disappear, and homodyning gives a direct quantum measurement of the signal field component that is coherent in space and in phase with the local oscillator [18]. It is this characteristic that makes homodyning attractive for squeezed state applications [16] - [19].

C. Heterodyne Detection

The configuration for single-detector multi-spatiotemporal mode heterodyne detection, shown in Fig. 3, mimics that employed for homodyne detection. The only differences are that the signal field is centered at frequency $\nu_0 + \nu_{IF}$, the local oscillator is centered at frequency ν_0, and passband filtering of the current density is used to select beat frequency components in the vicinity of the IF frequency ν_{IF} ($\nu_{IF}T>>1$ will be assumed). The bandwidth B of the signal field will be taken to be much less than ν_{IF}, and we shall concern ourselves with characterizing the statistics of the current density $J_{het}(\bar{x},t)$. The results we need are easily developed by injecting the frequency offset ν_{IF} into the preceding homodyne work.

Semiclassical Model In Eq. (33) let us make the frequency offset of the signal field explicit by writing

$$E_S(\bar{x},t) = F_S(\bar{x},t)e^{-j2\pi(\nu_0+\nu_{IF})t} , \qquad (58)$$

where F_S is a baseband complex signal field of bandwidth B. The results following (37) now provide the semiclassical statistics for heterodyning, namely, conditioned on knowledge of the baseband signal field $\{F_S(\bar{x},t) : \bar{x}\epsilon A_d, t\epsilon T\}$, $J_{het}(\bar{x},t)$ is the sum of three current densities:

1) a heterodyne-mixing current density signal term

$$2e\eta[\epsilon(1-\epsilon)]^{1/2}\mathrm{Re}(F_S(\bar{x},t)e^{-j2\pi(\nu_0+\nu_{IF})t}E_{LO}^*(\bar{x},t)) ;$$

2) a direct-detection local oscillator bias current density $en(1-\epsilon)I_{phLO}(\bar{x},t)$; and

3) a local oscillator shot noise current density $J_{shot}(\bar{x},t)$ characterized by (38).

The single-detector heterodyne photocurrent

$$i_{het}(t) = \int_{A_d} d\bar{x}\, J_{het}(\bar{x},t) \quad , \tag{59}$$

assuming E_{LO} is given by (40), then comprises a signal current

$$i_{sig}(t)=2en[P_{LO}\epsilon(1-\epsilon)/\, h\nu_o A_d]^{1/2}\mathrm{Re}\left(\int_{A_d} d\bar{x} F_S(\bar{x},t)e^{-j2\pi\nu_{IF}t}\right) \quad , \tag{60}$$

plus a bias current i_{bias} from (42), plus a zero-mean stationary white Gaussian process shot-noise current $i_{shot}(t)$ with spectral density ei_{bias}.

The heterodyne current (59) is thus a frequency translated (from $\nu_o + \nu_{IF}$ to ν_{IF}) version of the normally incident plane wave component of E_S plus the usual bias and shot noise terms. Because of the frequency offset ν_{IF} between the signal and the local oscillator fields, both the in-phase and quadrature (relative to the local oscillator) components of the signal field contribute to the output observations.

Quantum Model Here we suppose that the only non-vacuum state modes of the field operator $\hat{E}_S$ lie within a bandwidth B of the frequency $\nu_o+\nu_{IF}$. However, because of zero-point fluctuations, the quantum version of (58) is

$$\hat{E}_S(\bar{x},t) = \hat{F}_S(\bar{x},t)e^{-j2\pi(\nu_o+\nu_{IF})t} + \hat{F}_I(\bar{x},t)e^{-j2\pi(\nu_o-\nu_{IF})t} \quad , \tag{61}$$

where $\hat{F}_S$ and $\hat{F}_I$ are baseband complex signal and image field operators. Physically, the image band, being ν_{IF} Hz below the local oscillator's frequency, contributes quantum noise to J_{het} even when it is unexcited [18], [22]. We shall assume that $\hat{F}_S$ and $\hat{F}_I$ are quantum-mechanically independent, with the latter having all its modes in the vacuum state[7]. We now find, from the quantum homodyning work, that $J_{het}(\bar{x},t)$ measures the operator

$$\hat{J}_{het}(\bar{x},t) = e\eta(1-\epsilon)|E_{LO}(\bar{x},t)|^2 + J_{LOq}(\bar{x},t)$$
$$+ 2e\eta[\epsilon(1-\epsilon)]^{1/2}\mathrm{Re}(\hat{F}_S(\bar{x},t)e^{-j2\pi(\nu_0+\nu_{IF})t}E_{LO}^*(\bar{x},t))$$
$$+2e\eta[\epsilon(1-\epsilon)]^{1/2}\mathrm{Re}(\hat{F}_I(\bar{x},t)e^{-j2\pi(\nu_0-\nu_{IF})t}E_{LO}^*(\bar{x},t)] + J_{vac}(\bar{x},t) , \quad (62)$$

where E_{LO}, J_{LOq}, and J_{vac} are as given in (51). We can use the vacuum-state nature of $\hat{F}_I$ to obtain the classical representation

$$2e\eta[\epsilon(1-\epsilon)]^{1/2}\mathrm{Re}(\hat{F}_I(\bar{x},t)e^{-j2\pi(\nu_0-\nu_{IF})t}E_{LO}^*(\bar{x},t))$$
$$= J_{Iq}(\bar{x},t)/2^{1/2} , \quad (63)$$

where J_{Iq} is a zero-mean non-stationary white Gaussian classical process corresponding to the image-band quantum noise, with covariance function

$$\langle J_{Iq}(\bar{x}_1,t_1)J_{Iq}(\bar{x}_2,t_2)\rangle =$$
$$(e\eta)^2\epsilon(1-\epsilon)|E_{LO}(\bar{x}_1,t_1)|^2\delta(\bar{x}_1-\bar{x}_2)\delta(t_1-t_2) . \quad (64)$$

Thus, the quantum description of the single-detector heterodyne photocurrent $i_{het}(t)$ from (59) is that it measures the operator

$$\hat{i}_{het}(t) = i_{bias} + i_{LOq}(t) + i_{Iq}(t)/2^{1/2} + i_{vac}(t)$$
$$+ 2e\eta[P_{LO}\epsilon(1-\epsilon)/h\nu_0A_d]^{1/2}\mathrm{Re}(\int_{A_d} d\bar{x}\hat{F}_S(\bar{x},t)e^{-j2\pi\nu_{IF}t}) , \quad (65)$$

where i_{bias}, i_{LOq}, and $i_{vac}(t)$ are as in (53) and $i_{Iq}(t)$ is a zero mean white Gaussian noise process (the classical representation of image-band quantum noise) of spectral height $e\eta\epsilon i_{bias}$.

Comparison Suppose the density operator for $\hat{F}_S$ is a classical state, i.e., its density operator ρ_{F_S} obeys

$$\rho_{F_S} = \int d^2\underline{\alpha}_S \, P_{F_S}(\underline{\alpha}_S;\underline{\alpha}_S^*) \; |\underline{\alpha}_S\rangle\langle\underline{\alpha}_S| \,, \tag{66}$$

where P_{F_S} is a classical probability density, and $|\underline{\alpha}_S\rangle$ is the multi-mode Glauber coherent state for the modal expansion

$$\hat{F}_S(\bar{x},t) = \sum_n{}' \hat{a}_{Sn}\xi_n(\bar{x},t)e^{j2\pi(\nu_o+\nu_{IF})t} \,, \tag{67}$$

with $\sum_n{}'$ denoting summation over modes ξ_n lieing within bandwidth B of frequency $\nu_o+\nu_{IF}$. Here we can obtain a classical representation of the $\hat{F}_S$ term in (65) which reduces the quantum description of the heterodyne photocurrent to

$$\begin{aligned} i_{het}(t) &= i_{bias} + i_{LOq}(t) + i_{Iq}(t)/2^{1/2} + i_{vac}(t) \\ &\quad +2e\eta[P_{LO}\epsilon(1-\epsilon)/h\nu_o A_d]^{1/2}\mathrm{Re}\left(\int_{A_d} d\bar{x}\, F_S(\bar{x},t)e^{-j2\pi\nu_{IF}t}\right) \\ &\quad + i_{Sq}(t)/2^{1/2} \,, \end{aligned} \tag{68}$$

where the total noise current, $i_{LOq} + i_{Iq}/2^{1/2} + i_{Sq}/2^{1/2} + i_{vac}$, is a zero-mean white Gaussian process with spectral density ei_{bias}, in quantitative agreement with the semiclassical theory, and

$$F_S(\overline{x},t) \equiv \langle \alpha_S | \hat{F}_S(\overline{x},t) | \alpha_S \rangle \tag{69}$$

is the classical baseband signal field envelope $\hat{F}_S$ associates with the coherent state $|\alpha_S\rangle$. Note that half of the $\hat{E}_S$ quantum noise entering i_{het} comes through the signal field operator $\hat{F}_S$ and the other half comes through the image field operator $\hat{F}_I$.[8]

III. EXCESS NOISE EFFECTS AND DUAL-DETECTOR OPERATION

In this section we shall extend the results of Section II for coherent optical reception to include classical excess noise on the local oscillator field and dual-detector operation. It is convenient to begin with a presentation of dual-detector results in the absence of excess noise.

A. Dual-Detector Coherent Optical Reception

Suppose the homodyne/heterodyne configurations of Figs. 2 and 3 are augmented by the use of another quantum efficiency η detector on the previously unused output port of their beam splitters, see Fig. 4. We take the classical output field for this port to be

$$E(\overline{x},t) = -(1-\varepsilon)^{1/2}E_S(\overline{x},t) + \varepsilon^{1/2}E_{LO}(\overline{x},t) \tag{70}$$

in the semiclassical model, and use the corresponding operator-valued expression in the quantum model. Rather than treat the full multi-spatiotemporal mode situation, we shall restrict our attention to the photocurrents $i_1(t)$ and $i_2(t)$ obtained by spatial integration of the current densities $J_1(\overline{x},t)$ and $J_2(\overline{x},t)$ produced by detectors 1 and 2. We shall assume a perfectly stable (i.e, deterministic) classical local oscillator field

$$E_{LO}(\bar{x},t) = F_{LO}(\bar{x},t)e^{-j2\pi\nu_0 t} \tag{71}$$

with baseband complex envelope F_{LO} in the semiclassical model, and a Glauber coherent state quantum local oscillator with mean field

$$tr(\rho_{LO}\hat{E}_{LO}(\bar{x},t)) = F_{LO}(\bar{x},t)e^{-j2\pi\nu_0 t} \tag{72}$$

with baseband complex envelope F_{LO} in the quantum model. Under these conditions the results of Section II can be used to show that the following statistics apply.

Homodyne Detection In homodyning, the signal field is centered on ν_0, so, because of (71),(72), it is convenient to introduce baseband signal complex envelopes via

$$E_S(\bar{x},t) = F_S(\bar{x},t)e^{-j2\pi\nu_0 t} \tag{73}$$

and

$$\hat{E}_S(\bar{x},t) = \hat{F}_S(\bar{x},t)e^{-j2\pi\nu_0 t} \quad . \tag{74}$$

for the semiclassical and quantum cases, respectively. Now we have, semiclassically, that

$$\begin{aligned} i_1(t) &= e\eta(1-\epsilon)\int_{A_d} d\bar{x}|F_{LO}(\bar{x},t)|^2 \\ &+ 2e\eta[\epsilon(1-\epsilon)]^{1/2}\mathrm{Re}\left(\int_{A_d} d\bar{x}F_S(\bar{x},t)\,F_{LO}^{*}(\bar{x},t)\right) \\ &+ e[\eta(1-\epsilon)\int_{A_d} d\bar{x}|F_{LO}(\bar{x},t)|^2]^{1/2}n_{shot1}(t), \end{aligned} \tag{75}$$

and

$$i_2(t) = en\epsilon\int_{A_d} d\bar{x}|F_{LO}(\bar{x},t)|^2$$
$$-2en[\epsilon(1-\epsilon)]^{1/2}\mathrm{Re}\left(\int_{A_d} d\bar{x}\, F_S(\bar{x},t)F_{LO}^*(\bar{x},t)\right)$$
$$+e[n\epsilon\int_{A_d} d\bar{x}|F_{LO}(\bar{x},t)|^2]^{1/2}n_{shot2}(t) \quad , \qquad (76)$$

for the homodyne photocurrents, where $n_{shot1}(t)$ and $n_{shot2}(t)$ are statistically independent identically distributed zero-mean stationary white Gaussian noise processes of unity spectral density. Equations (75) and (76) have the usual bias plus mixing signal plus local-oscillator shot noise interpretation. Note that the beam splitter phase shift between the output ports forces the mixing signals to be 180° out of phase. Also, the independence of the local-oscillator shot noises follows because they are generated from deterministic illumination of two different detectors.

For the quantum case, we have that $i_1(t)$ and $i_2(t)$ measure the operators

$$\hat{i}_1(t) = en(1-\epsilon)\int_{A_d} d\bar{x}|F_{LO}(\bar{x},t)|^2$$
$$+ 2en[\epsilon(1-\epsilon)]^{1/2}\mathrm{Re}\left(\int_{A_d} d\bar{x}\hat{F}_S(\bar{x},t)F_{LO}^*(\bar{x},t)\right)$$
$$+ en(1-\epsilon)[\int_{A_d} d\bar{x}|F_{LO}(\bar{x},t)|^2]^{1/2}\, n_{LOq}(t)$$
$$+ e[n(1-n)(1-\epsilon)\int_{A_d} d\bar{x}|F_{LO}(\bar{x},t)|^2]^{1/2}n_{vac1}(t) \quad , \qquad (77)$$

and

$$\hat{i}_2(t) = en\epsilon\int_{A_d} d\bar{x}|F_{LO}(x,t)|^2$$
$$-2en[\epsilon(1-\epsilon)]^{1/2}\mathrm{Re}\left(\int_{A_d} d\bar{x}\, \hat{F}_S(\bar{x},t)F_{LO}^*(\bar{x},t)\right)$$

$$+e\eta\epsilon\left[\int_{A_d} d\bar{x}|F_{LO}(\bar{x},t)|^2\right]^{1/2} n_{LOq}(t)$$

$$+e[\eta(1-\eta)\epsilon\int_{A_d} d\bar{x}|F_{LO}(\bar{x},t)|^2]^{1/2} n_{vac2}(t), \qquad (78)$$

where $n_{LOq}(t)$, $n_{vac1}(t)$, $n_{vac2}(t)$ are statistically independent identically distributed zero-mean stationary unity-spectrum Gaussian processes. The familiar bias plus quantum mixing signal plus local-oscillator quantum noise plus $\eta<1$ quantum noise interpretation applies to (77), (78). As in the semiclassical model, the mixing term appears 180^0 out of phase in the two photocurrents. No such phase shift appears on the n_{LOq} term, as this noise arises out of the direct detection of $\hat{E}_{LO}$. Indeed, except for scale factors, the local-oscillator quantum noise contributions to $i_1(t)$ and $i_2(t)$ are completely correlated. The $\eta<1$ quantum noises are, on the other hand, statistically independent because they arise from different detectors. Finally, when the signal field is in a classical state these quantum results can be shown to be in quantitative agreement with the foregoing semiclassical formulas.

Heterodyne Detection For heterodyning we use (58), rather than (73), to introduce a baseband signal complex envelope for the semiclassical analysis. We then find that

$$i_1(t) = e\eta(1-\epsilon)\int_{A_d} d\bar{x}|F_{LO}(\bar{x},t)|^2$$

$$+2e\eta[\epsilon(1-\epsilon)]^{1/2}\mathrm{Re}\left(\int_{A_d} d\bar{x}F_S(\bar{x},t)F_{LO}^*(\bar{x},t)e^{-j2\pi\nu_{IF}t}\right)$$

$$+e[\eta(1-\epsilon)\int_{A_d} d\bar{x}|F_{LO}(\bar{x},t)|^2]^{1/2} n_{shot1}(t) \quad , \qquad (79)$$

and

$$i_2(t) = en\epsilon\int_{A_d} d\overline{x}|F_{LO}(\overline{x},t)|^2$$
$$-2en[\epsilon(1-\epsilon)]^{1/2}\mathrm{Re}\left(\int_{A_d} d\overline{x}F_S(\overline{x},t)F_{LO}^*(\overline{x},t)e^{-j2\pi\nu_{IF}t}\right)$$
$$+e[n\epsilon\int_{A_d} d\overline{x}|F_{LO}(\overline{x},t)|^2]^{1/2}n_{shot2}(t), \qquad (80)$$

with interpretations as given following (75), (76). In the quantum case we use (61) instead of (74) and obtain the measurement operators

$$\hat{i}_1(t) = en(1-\epsilon)\int_{A_d} d\overline{x}|F_{LO}(\overline{x},t)|^2$$
$$+2en[\epsilon(1-\epsilon)]^{1/2}\mathrm{Re}\left(\int_{A_d} d\overline{x}\hat{F}_S(\overline{x},t)F_{LO}^*(\overline{x},t)e^{-j2\pi\nu_{IF}t}\right)$$
$$+en(1-\epsilon)\ [\int_{A_d} d\overline{x}|F_{LO}(\overline{x},t)|^2]^{1/2}\ n_{LOq}(t)$$
$$+e[n(1-n)(1-\epsilon)\int_{A_d} d\overline{x}|F_{LO}(\overline{x},t)|^2]^{1/2}n_{vac1}(t)$$
$$+en[\epsilon(1-\epsilon)2^{-1}\int_{A_d} d\overline{x}|F_{LO}(\overline{x},t)|^2]^{1/2}n_{Iq}(t)\quad , \qquad (81)$$

for detector 1 and

$$\hat{i}_2(t) = en\epsilon\int_{A_d} d\overline{x}|F_{LO}(\overline{x},t)|^2$$
$$-2en[\epsilon(1-\epsilon)]^{1/2}\mathrm{Re}\left(\int_{A_d} d\overline{x}\hat{F}_S(\overline{x},t)F_{LO}^*(\overline{x},t)e^{-j2\pi\nu_{IF}t}\right)$$
$$+en\epsilon[\int_{A_d} d\overline{x}|F_{LO}(\overline{x},t)|^2]^{1/2}n_{LOq}(t)$$
$$+e[n(1-n)\epsilon\int_{A_d} d\overline{x}|F_{LO}(\overline{x},t)|^2]^{1/2}n_{vac2}(t)$$
$$-en[\epsilon(1-\epsilon)2^{-1}\int_{A_d} d\overline{x}|F_{LO}(\overline{x},t)|^2]^{1/2}n_{Iq}(t)\quad , \qquad (82)$$

for detector 2. In Eqs (81), (82) the interpretations and comments following (77), (78) are applicable. The noise $n_{Iq}(t)$, which represents image-band quantum noise, is another zero-mean stationary unity-spectrum white Gaussian process. It is statistically independent of $n_{LOq}(t)$ and $n_{vacj}(t)$ for $j = 1,2$, and appears with a sign reversal in $\hat{i}_1$ and $\hat{i}_2$ because it arises from the mixing term involving $\hat{F}_I$.

B. Local-Oscillator Excess Noise

The extension of the results of Section IIIA to incorporate classical excess noise on the local oscillator is extraordinarily simple, because of the form the preceding results have been cast in. Specifically, for the semiclassical theory we need only make the baseband local oscillator complex envelope F_{LO} in (71) a complex-valued random process with known statistics. Then the homodyne and heterodyne results of the semiclassical theory, namely Eqs. (75), (76) and Eqs. (79), (80), respectively, become conditional statistics assuming F_{LO} is known.[9] Unconditional statistics follow, via iterated expectation [38], from averaging over the local oscillator fluctuations, as will be illustrated below. In a similar manner, classical local-oscillator excess noise can be injected into the quantum model by making ρ_{LO} a classical-state density operator for which F_{LO}, the average baseband local-oscillator complex envelope given the local oscillator is known to be in the multi-mode coherent state $|\underline{\alpha}_{LO}\rangle$, is a complex-valued classical random process. The quantum homodyne and heterodyne results, Eqs. (77), (78), and (81), (82), respectively, are now conditional characterizations given F_{LO}. Unconditional statistics are again obtained by averaging over the local oscillator fluctuations.[10]

To illustrate our excess noise results, and compare them with relevant prior work [25], [28]-[30], we shall consider a single spatial mode/multi-temporal mode local oscillator, for which F_{LO} in the semiclassical theory and $\hat{F}_{LO}$ in the quantum theory are both of the form $(P_{LO}(t)/h\nu_o A_d)^{1/2}\exp(-j\phi_{LO}(t))$, where $P_{LO}(t)$ and $\phi_{LO}(t)$ are classical random power and phase fluctuations. For convenience, we shall assume that these fluctuations are the polar decomposition of a stationary complex-Gaussian random process. We shall also assume that the signal field, in both the semiclassical and quantum pictures, is statistically independent of the local oscillator. Finally, we shall limit our consideration to the differenced output currents $i_1(t) - i_2(t)$.

<u>Homodyne Detection</u> Under the preceding conditions we have the semiclassical result

$$\begin{aligned}
i_1(t) - i_2(t) &= e\eta(1-2\epsilon)P_{LO}(t)/h\nu_o \\
&+4e\eta[\epsilon(1-\epsilon)P_{LO}(t)/h\nu_o A_d]^{1/2}\mathrm{Re}\left(\int_{A_d} d\overline{x}F_S(\overline{x},t)e^{j\phi_{LO}(t)}\right) \\
&+e[\eta(1-\epsilon)P_{LO}(t)/h\nu_o]^{1/2}n_{shot1}(t) \\
&-e[\eta\epsilon P_{LO}(t)/h\nu_o]^{1/2}n_{shot2}(t) ,
\end{aligned} \tag{83}$$

and the quantum result

$$\begin{aligned}
\hat{i}_1(t) - \hat{i}_2(t) &= e\eta(1-2\epsilon)P_{LO}(t)/h\nu_o \\
&+4e\eta[\epsilon(1-\epsilon)P_{LO}(t)/h\nu_o A_d]^{1/2}\mathrm{Re}\left(\int_{A_d} d\overline{x}\hat{F}_S(\overline{x},t)e^{j\phi_{LO}(t)}\right) \\
&+e\eta(1-2\epsilon)(P_{LO}(t)/h\nu_o)^{1/2}n_{LOq}(t) \\
&+e[\eta(1-\eta)(1-\epsilon)P_{LO}(t)/h\nu_o]^{1/2}n_{vac1}(t) \\
&-e[\eta(1-\eta)\epsilon P_{LO}(t)/h\nu_o]^{1/2} n_{vac2}(t) .
\end{aligned} \tag{84}$$

In both (83) and (84), the first term on the right equals a mean bias current $en(1-2\epsilon)\langle P_{LO}(t)\rangle/h\nu_o$ plus a local oscillator power-fluctuation excess noise $en(1-2\epsilon)(P_{LO}(t)-\langle P_{LO}(t)\rangle)/h\nu_o$. Both of these are exactly nulled when the beam splitter is 50/50, i.e., when $\epsilon = 1/2$. The second term on the right in (83) and (84) is the homodyne-mixing signal current; local oscillator randomness both amplitude and phase modulates this term. The remaining terms in the semiclassical result (83) are the shot noises, now modulated by local oscillator power fluctuations. The remaining terms in the quantum result are the local oscillator quantum noise and the $\eta<1$ quantum noises; these too are modulated by the local oscillator power fluctuations. Note that when $\epsilon = 1/2$ the local oscillator quantum noise contribution vanishes.

Let us further specialize the quantum results by supposing that the only excited mode of $\hat{F}_S$ is the monochromatic plane-wave pulse $(A_dT)^{-1/2}$ for $\bar{x}\epsilon A_d$, $t\epsilon T$, and that $\hat{a}_S$ is the annihilation operator for this mode. Matched filtering of the differenced output currents then yields a measurement of

$$\hat{M} \equiv e^{-1}\int_0^T [\hat{i}_1(t) - \hat{i}_2(t)]dt , \qquad (85)$$

where normalization by the electron charge has been used, for convenience, to make the observation values dimensionless. We assume that the mean function and covariance function of the stationary complex-Gaussian local-oscillator random process

$$\underline{v}(t) \equiv (P_{LO}(t)/h\nu_o A_d)^{1/2} \exp(-j\phi_{LO}(t)) \qquad (86)$$

are

$$m_{\underline{v}} = [(1-\gamma)\langle N_{LO}\rangle/A_dT]^{1/2} , \qquad (87)$$

and

$$K_{\underline{yy}}(\tau) = (\gamma\langle N_{LO}\rangle/A_dT)k(\tau) \quad , \tag{88}$$

respectively, where $\langle N_{LO}\rangle$ is the average number of local oscillator photons present over $A_d \times T$, $\gamma^{1/2}$ is the fractional root-mean-square (rms) local oscillator amplitude fluctuation level, and $k(\tau)$ is a real-valued normalized covariance $(k(0) = 1)$. It then follows that

$$\begin{aligned}\langle\hat{M}\rangle &= \eta(1-2\epsilon)\langle N_{LO}\rangle \\ &+ 4\eta[\epsilon(1-\epsilon)(1-\gamma)\langle N_{LO}\rangle]^{1/2} \quad \langle\hat{a}_{S1}\rangle \ ,\end{aligned} \tag{89}$$

and

$$\begin{aligned}\langle\Delta\hat{M}^2\rangle &= \eta(1-\eta)\langle N_{LO}\rangle + \eta^2(1-2\epsilon)^2\langle N_{LO}\rangle + 16\eta^2\epsilon(1-\epsilon)(1-\gamma)\langle N_{LO}\rangle\langle\Delta\hat{a}_{S1}{}^2\rangle \\ &+ [\eta(1-2\epsilon)\langle N_{LO}\rangle/T]^2\int_{-T}^{T} d\tau[\gamma^2k^2(\tau) + 2\gamma(1-\gamma)k(\tau)]\ (T-|\tau|) \\ &+ (2\eta/T)^2(1-2\epsilon)\gamma[\epsilon(1-\epsilon)\langle N_{LO}\rangle^3(1-\gamma)]^{1/2}\, 2\langle\hat{a}_{S1}\rangle\int_{-T}^{T} d\tau k(\tau)(T-|\tau|) \\ &+ (2\eta/T)^2\epsilon(1-\epsilon)\gamma\langle N_{LO}\rangle[T^2+2\langle\hat{a}_S{}^{\dagger}\hat{a}_S\rangle\int_{-T}^{T} d\tau k(\tau)(T-|\tau|)] \quad ,\end{aligned} \tag{90}$$

give the mean and variance of the $\hat{M}$ measurement from which a signal-to-noise ratio

$$SNR_{\hat{M}} \equiv \langle\hat{M}\rangle^2/\langle\Delta\hat{M}^2\rangle \tag{91}$$

may be calculated. In Eq. (89), the first term is the average local oscillator bias current contribution, and the second term is the average signal field mixing term contribution, where $\hat{a}_{S1} = (\hat{a}_S+\hat{a}_S{}^{\dagger})/2$ for $\hat{a}_S$ the annihilation operator of the sole excited $\hat{F}_S$ mode. In Eq. (90), the first term is due to the $\eta<1$ noises n_{vac1} and n_{vac2} , the second term is due to the local oscillator quantum noise n_{LOq} , the third term is the signal field quantum noise, the fourth term is due to the local-oscillator power fluctuations, and the last terms are due to the random modulation of the mixing current by the local oscillator fluctuations.

The previous dual-detector homodyne studies of Yuen and Chan [25] and Schumaker [30] assume $\epsilon = 1/2$, $(1-\gamma)\langle N_{LO}\rangle \gg 1$, and a slowly fluctuating local oscillator (corresponding, in our case, to $k(\tau) = 1$ for $|\tau| \leq T$). In this limit both prior studies find (in our notation)

$$SNR_{\hat{M}} = \frac{\langle \hat{a}_{S1}\rangle^2}{\langle \Delta\hat{a}_{S1}{}^2\rangle + (1-\eta)/4\eta} \quad , \tag{92}$$

whereas we obtain

$$SNR_{\hat{M}} = \frac{\langle \hat{a}_{S1}\rangle^2}{\langle \Delta\hat{a}_{S1}{}^2\rangle + (1-\eta)/4\eta(1-\gamma) + \gamma(1+2\langle \hat{a}_S^{\dagger}\hat{a}_S\rangle)/4(1-\gamma)} \quad . \tag{93}$$

For small fractional rms local-oscillator amplitude fluctuations ($\gamma \ll 1$), (93) differs from (92) because of an additional noise term in the denominator that is approximately $\gamma(1 + 2\langle \hat{a}_S^{\dagger}\hat{a}_S\rangle)/4$. Physically, this term arises from the random local-oscillator modulation of the mixing current, an effect neglected by the earlier studies. In order for this term to be insignificant compared to the signal quantum noise of a coherent state ($\langle \Delta\hat{a}_{S1}{}^2\rangle = 1/4$), we require that

$$\gamma\langle \hat{a}_S^{\dagger}\hat{a}_S\rangle \ll 1 \quad , \tag{94}$$

i.e., the fractional rms local oscillator amplitude fluctuation must be much smaller than the square root of the reciprocal of the average number of signal field photons.[11] This requirement becomes even more stringent if a squeezed state is being probed, for which $\langle \Delta\hat{a}_{S1}{}^2\rangle < 1/4$ prevails.

In addition to exhibiting the potentially significant random modulation of the mixing term, our formulation, (84), shows another effect suppressed in [25] and [30]. This is the random amplitude modulation of the local oscillator and $\eta < 1$ quantum noises by the classical amplitude noise of the local oscillator. Although this modulation does not explicitly enter the signal-to-noise ratio , it does make the last three terms in (84) non-Gaussian random processes, an effect which will modify digital communication error probability calculations somewhat.

Heterodyne Detection The semiclassical description for the differenced output currents in heterodyne detection is

$$
\begin{aligned}
i_1(t) - i_2(t) &= e\eta(1-2\epsilon)P_{LO}(t)/h\nu_o \\
&+4e\eta[\epsilon(1-\epsilon)P_{LO}(t)/h\nu_o A_d]^{1/2}\mathrm{Re}\left(\int_{A_d} d\bar{x}F_S(\bar{x},t)e^{-j(2\pi\nu_{IF}t-\phi_{LO}(t))}\right) \\
&+e[\eta(1-\epsilon)P_{LO}(t)/h\nu_o]^{1/2}\, n_{shot1}(t) \\
&-e[\eta\epsilon P_{LO}(t)/h\nu_o]^{1/2} n_{shot2}(t) \quad ,
\end{aligned}
\qquad (95)
$$

and the quantum description is

$$
\begin{aligned}
\hat{i}_1(t) - \hat{i}_2(t) &= e\eta(1-2\epsilon)P_{LO}(t)/h\nu_o \\
&+4e\eta[\epsilon(1-\epsilon)P_{LO}(t)/h\nu_o A_d]^{1/2}\mathrm{Re}\left(\int_{A_d} d\bar{x}\hat{F}_S(\bar{x},t)e^{-j(2\pi\nu_{IF}t-\phi_{LO}(t))}\right) \\
&+e\eta(1-2\epsilon)(P_{LO}(t)/h\nu_o)^{1/2}n_{LOq}(t) \\
&+e[\eta(1-\eta)(1-\epsilon)P_{LO}(t)/h\nu_o]^{1/2}n_{vac1}(t)
\end{aligned}
$$

$$-e[\eta(1-\eta)\epsilon P_{LO}(t)/h\nu_o]^{1/2} n_{vac2}(t)$$

$$+e\eta[2\epsilon(1-\epsilon)P_{LO}(t)/h\nu_o]^{1/2} n_{Iq}(t) \quad . \qquad (96)$$

These results differ from the corresponding homodyne results, (83) and (84), in only two respects. First, the mixing terms (second terms on the right in (95) and (96)) beat the signal field to an intermediate frequency not baseband, and so they sense both quadratures of the signal field. Second, the quantum result (96) gains a noise contribution from the image band quantum noise through $n_{Iq}(t)$. The local oscillator excess noise (and its cancellation when $\epsilon = 1/2$) and the random modulation of the signal and noise terms by the local oscillator fluctuations thus continue to be present in the heterodyne case, i.e., the interpretations given for the homodyne situation apply here as well. Once again, the relevant previous work on dual detector systems [25], [28], [29] does not include all the effects contained in our treatment; the random local oscillator modulation of the signal and noise terms is absent in the above analyses.

As an illustration of these omissions, let us compare our semiclassical answer (95) assuming a deterministic monochromatic plane-wave pulse signal $F_S(\overline{x},t) = \alpha_S(A_dT)^{-1/2}$ for $\overline{x}\epsilon A_d, t\epsilon T$, with the corresponding $\epsilon=1/2$, equal quantum efficiency result of Abbas and Chan [29]. The latter claim, in our notation, that the differenced output currents consist of a mean current

$$<(i_1(t) - i_2(t))> = 2e\eta T^{-1}<N_{LO}>^{1/2} Re(\alpha_S e^{-j2\pi\nu_{IF}t}) \quad , \qquad (97)$$

embedded in an additive zero-mean white Gaussian noise process with bilateral spectral density

$$S(f) = e^2\eta<N_{LO}>/T \ . \qquad (98)$$

We have, using (86) - (87) in (95), that the differenced output currents consist of a mean current

$$\langle(i_1(t) - i_2(t))\rangle = 2e\eta T^{-1}[(1-\gamma)\langle N_{LO}\rangle]^{1/2}\mathrm{Re}(\alpha_S e^{-j2\pi\nu_{IF}t}) \quad , \qquad (99)$$

plus a conditionally non-stationary zero-mean white Gaussian shot noise process that, given the local oscillator power waveform, has covariance function

$$K(t,s) = e^2\eta(P_{LO}(t)/h\nu_o)\delta(t-s) \quad , \qquad (100)$$

plus a signal dependent zero-mean stationary Gaussian noise process

$$i''(t) = 2e\eta(A_d/T)^{1/2}\mathrm{Re}[\alpha_S(\underline{y}(t) - m_{\underline{y}})^* e^{-j2\pi\nu_{IF}t}] \, , \qquad (101)$$

with covariance function

$$K_{i''i''}(\tau) = 2(e\eta/T)^2\gamma\langle N_{LO}\rangle|\alpha_S|^2 k(\tau)\cos(2\pi\nu_{IF}\tau) \quad . \qquad (102)$$

When Eq.(100) is averaged over the P_{LO} statistics it reduces to a stationary white noise spectrum (98) , however the random P_{LO} fluctuations make the noise non-Gaussian, albeit in a minor way if $\gamma \ll 1$. The noise current $i''(t)$ comes from the random modulation of the mixing term and may present a significant degradation. Consider a high quality ($\gamma \ll 1$), slowly fluctuating local oscillator ($k(\tau) = 1$ for $|\tau| \le T$) and the matched filter processor generating

$$M = e^{-1}\int_0^T (i_1(t) - i_2(t))2^{1/2}\cos[2\pi\nu_{IF}t - \arg(\alpha_S)] \, , \qquad (103)$$

then the Abbas and Chan model gives a signal-to-noise ratio

$$SNR_M = 2\eta|\alpha_S|^2 \quad , \qquad (104)$$

whereas we have that

$$\mathrm{SNR}_M = \frac{2\eta|\alpha_S|^2}{1 + \eta\gamma|\alpha_S|^2} . \tag{105}$$

As in the quantum homodyne example given earlier, at high average detected signal levels there is a very stringent requirement on local oscillator amplitude fluctuations if SNR degradation is to be avoided.

IV. DISCUSSION

At this point, we have clearly established how the quantum theory for coherent optical detection subsumes the familiar semiclassical statistics in a natural way. We have also seen that the quantum approach is essential for studying the photodetection statistics of non-classical field states. There is now considerable interest in a particular class of non-classical states, called the two-photon coherent states [14] or the squeezed states [15]. These states are in essence minimum uncertainty product states for the quadrature components of the photon-units field operator $\hat{E}(\bar{x},t)$. In particular, for a single field mode with annihilation operator $\hat{a}$, the two-photon coherent state $|\beta;\mu,\nu\rangle$ obeys the eigenket relation

$$(\mu\hat{a} + \nu\hat{a}^{+})|\beta;\mu,\nu\rangle = \beta|\beta;\mu,\nu\rangle , \tag{106}$$

where β,μ,ν are complex numbers and μ,ν satisfy $|\mu|^2 - |\nu|^2 = 1$. With $\hat{a}_1 = (\hat{a} + \hat{a}^{+})/2$ and $\hat{a}_2 = (\hat{a} - \hat{a}^{+})/2j$ denoting the quadrature components of $\hat{a}$, we then find that the state $|\beta;\mu,\nu\rangle$ gives

$$\langle\Delta\hat{a}_1{}^2\rangle = |\mu - \nu|^2/4, \tag{107a}$$

and

$$\langle\Delta\hat{a}_2{}^2\rangle = |\mu + \nu|^2/4. \tag{107b}$$

When μ,ν are real valued, (107) implies that $|\beta;\mu,\nu\rangle$ satisfies the Heisenberg relation

$$\langle \Delta\hat{a}_1^2 \rangle \langle \Delta\hat{a}_2^2 \rangle \geq 1/16 \qquad (108)$$

with equality, as does the familiar coherent state $|\alpha\rangle$. Unlike the coherent state, which gives $\langle \Delta\hat{a}_1^2 \rangle = \langle \Delta\hat{a}_2^2 \rangle = 1/4$, (107) shows that there is an asymmetric noise division between the quadratures (a noise squeezing) in the state $|\beta;\mu,\nu\rangle$, with the low-noise quadrature being less noisy than a coherent state. This noise reduction can be used, in principle, to effect important performance improvements in optical communications [16] - [19] and precision measurements [20] - [23].

As yet, there have been no experimental observations of squeezed state light. Theoretical studies, which employ varying degrees of idealization, indicate that such states may be generated by degenerate four-wave mixing (DFWM) [39] - [42], as well as a number of other nonlinear optical processes [14], [15], [43] - [46]. We are presently working on a continuous-wave DFWM experiment using homodyne detection to generate and verify the quadrature noise squeezing. In this experiment, a single frequency-stabilized laser will be used to provide all the input beams to the four-wave mixer, as well as the local oscillators for dual-detector homodyne detection. The results of this paper permit the expected photocurrent statistics for this experiment to be derived, including the effects of the laser's residual amplitude and phase fluctuations. Specifically, an iterated expectation approach is used, as in Section III. The photocurrent statistics are first obtained assuming the laser output to be a particular coherent state. This entails a calculation of the four-wave mixer output state, along the lines of [40], followed by a calculation of the sort performed here in Section IIIA. To average over the input laser fluctuations, we assign to the coherent state value for this laser a classical probability distribution. We can then proceed as in Section IIIB, except that the state of the signal field operator in the homodyne apparatus is now dependent on the coherent-state value of the local-oscillator field in that apparatus, because both fields are derived from the same laser.

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Footnotes

1. The convention we use for this Fourier transform is necessitated by the accepted quantum-optics definition for what constitutes a positive-frequency field.

2. For a photomultiplier tube, the internal current gain amplifies the current we are analyzing by a sufficient amount to warrant its treatment as a classical entity. In the coherent optical detection cases that follow, the mixing gain produced by the strong local oscillator has a similar effect, see [22].

3. A classical state is either a Glauber coherent state or a classically random mixture of such states. In either case, the density operator ρ has a proper P-representation (17). The terminology arises, see below, because a classical state ρ gives rise to the same statistics in quantum photodetection theory as found for a classical field in semiclassical photodetection theory.

4. Because our idealized detector model neglects internal noise sources (dark current shot noise, thermal noise, etc.) N from Eq. (21) corresponds to the output of a pulse-discriminator/counter applied to the output current $\int_{A_d} d\bar{x}\, J(\bar{x},t)$. In other words, Eq. (21) models the output of an ideal (unity quantum efficiency) photomultiplier-tube/pulse-counter setup.

5. Our choice for the beam splitter transformation agrees with that employed in [18], and implies that the field leaving the other port of this optical element is $-(1-\epsilon)^{1/2}E_S(\bar{x},t)+\epsilon^{1/2}E_{LO}(\bar{x},t)$. Other beam splitter relations (see, e.g. [25] , [30]) are equivalent to ours after redefinition of the input and output planes.

6. A critical aspect of the strong local oscillator condition acting through the measurement operator (48) is that the mean local oscillator field and its

quantum noise both contribute to J_{hom} through the direct detection term, but only the mean local oscillator field (not its quantum noise) contributes to J_{hom} through the mixing term.

7. Very interesting noise reductions can accrue when the signal and image bands are quantum-mechanically dependent [22].

8. Because the in-phase and quadrature components of $\hat{F}_S$ are non-commuting observables, the image band noise enters into heterodyning in order to enforce the Heisenberg uncertainty principle on ideal simultaneous observations of these incompatible quantities (see [36], [37]).

9. Implicit in this conditioning statement is the fact that the local oscillator must, with very high probability, remain sufficiently strong to ensure the validity of the Section II theory. Also note that the signal field statistics may depend on the value of the local oscillator field, such as occurs in a laboratory experiment when the same laser is used to obtain both the signal and local oscillator beams (see Section IV).

10. The local oscillator fluctuations must not be such as to invalidate the Section II theory for any state $|\underline{\alpha}_{LO}\rangle$ that occurs with appreciable probability. Also, the signal state (density operator) may depend on the value of the local oscillator field, if, for example, both beams originate from the same laser (see Section IV).

11. For example, to keep this added noise below 10% (in standard deviation) of the coherent-state signal quantum noise when $\langle \hat{a}_S^{\dagger} \hat{a}_S \rangle = 10^4$, we can tolerate no more than 0.3% local oscillator amplitude fluctuation. This limitation may be significant in precision measurement applications for which signal-to-noise ratios far in excess of 40dB are sought.

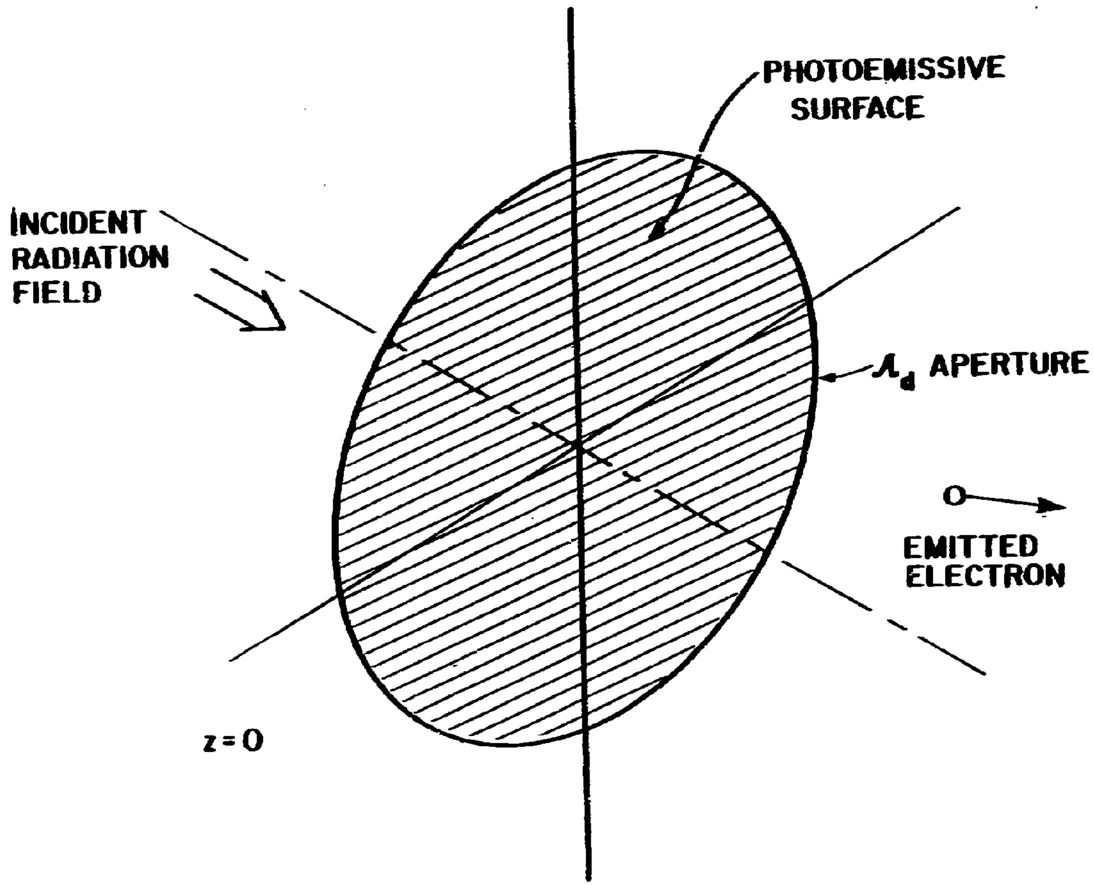
Figure Captions

Fig. 1 Geometry of an idealized surface photoemitter of active region A_d.

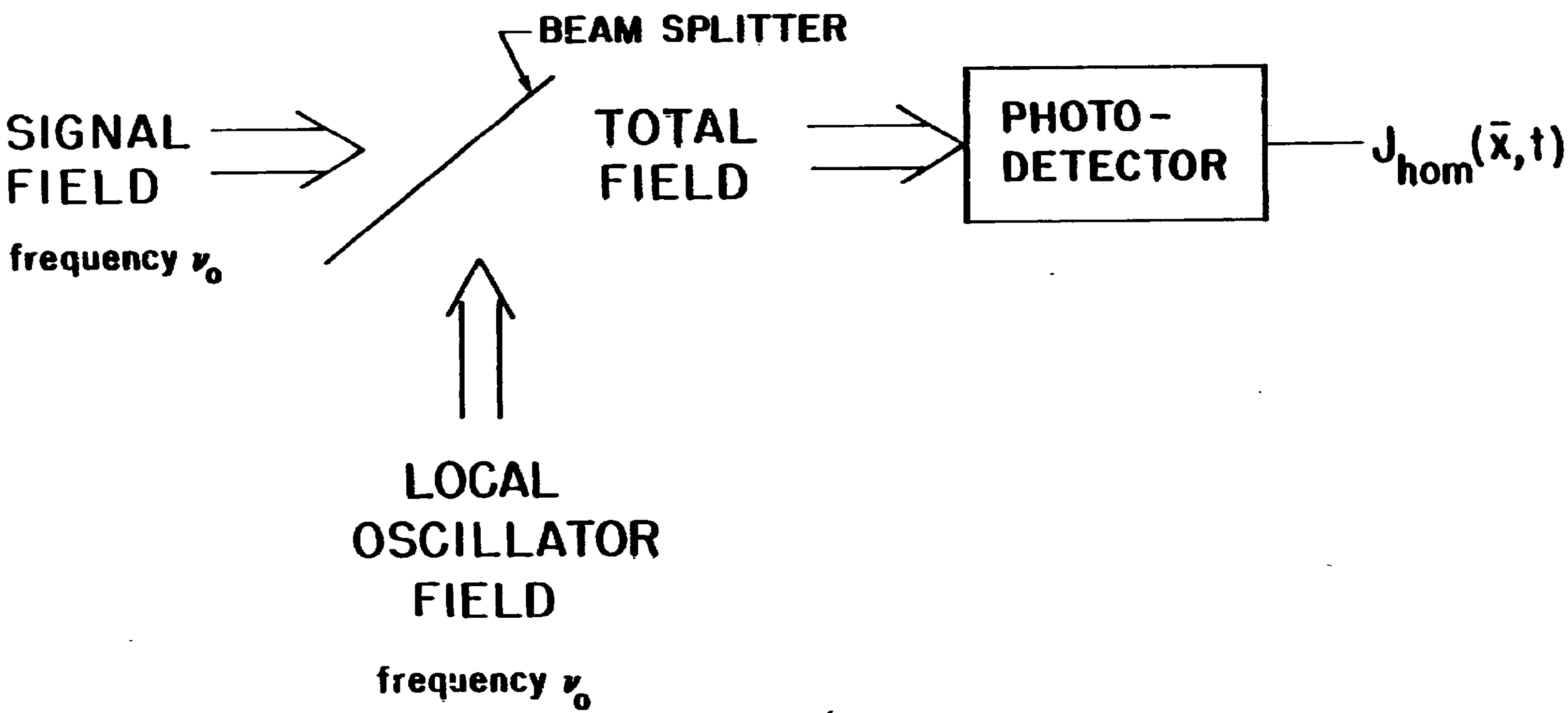
Fig. 2 Configuration for optical homodyne detection.

Fig. 3 Configuration for optical heterodyne detection.

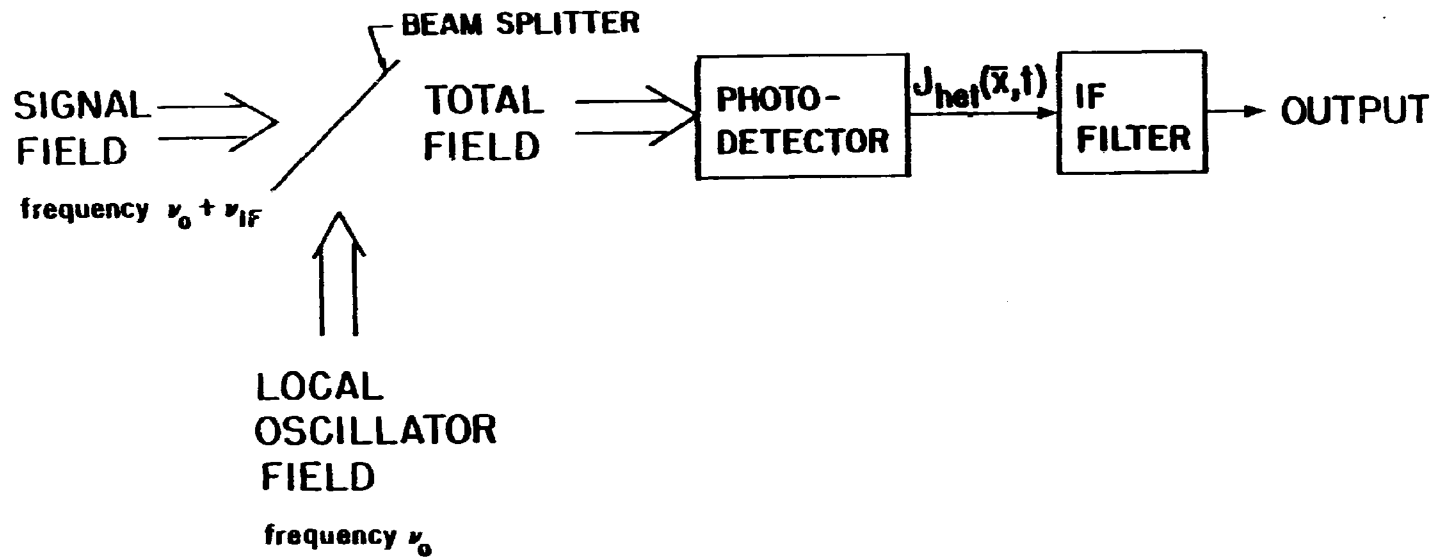
Fig. 4 Configuration for dual-detector coherent optical detection.



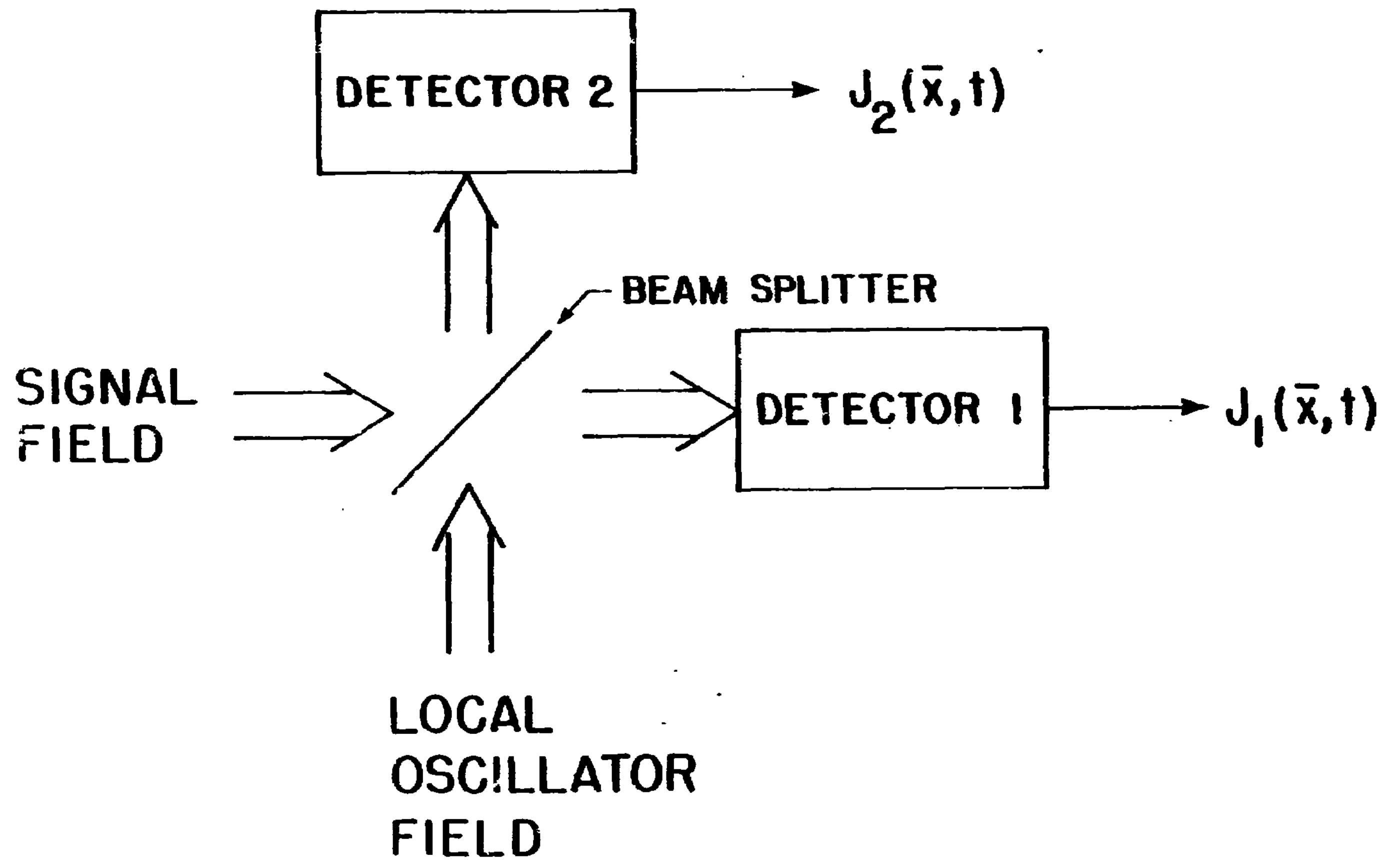
J.H. Shapiro Figure 1



J. H. Shapiro Figure 2



J.H. Shapiro Figure 3



J.H. Shapiro Figure 4