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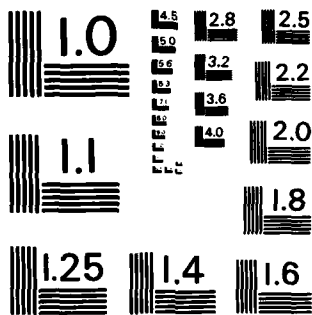
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Modulational instability of lower hybrid waves

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**Abstract.** We investigate the excitation of a purely growing modulational mode together with two lower hybrid sidebands by a lower hybrid pump. The condition,  $|k_{oz}/k_{ol}| < \omega_{pi}/\Omega_e$ , needs to be satisfied for the instability, where  $|k_{oz}/k_{ol}|$  is the ratio of the perpendicular to the parallel scale lengths of the lower hybrid pump. The growth rate of the instability is found to be quite large for the power densities employed in the lower hybrid heating experiments in large tokamaks.

## 1. Introduction

The need of supplementary plasma heating in magnetic fusion devices has been generally recognized for achieving the operational temperatures for fusion power production. Among various RF heating schemes, lower hybrid waves have received much attention not only for plasma heating (e.g., Porklab, 1977) but also for plasma confinement (e.g., Fisch, 1978) purposes. The effectiveness of these processes relies on the penetration of RF wave energy into the interior of the plasmas. Hence, the self-modulational effects on the lower hybrid wave propagation are believed to be important on this aspect (e.g., Morales and Lee, 1975). In this paper, we perform a linear stability analyses of the modulational instability whereby a purely growing modulational mode together with two lower hybrid sidebands are excited by a lower hybrid pump wave.

## 2. Coupled mode equations

We consider the propagation of a lower hybrid pump wave  $\phi = \tilde{\phi}_0 \exp[i(\mathbf{k}_0 \cdot \mathbf{r} - \omega_0 t)]$  in a uniform, collisionless plasma embedded in a constant magnetic field  $\mathbf{B}_0 = \hat{z}B_0$ . Here,  $\phi_0$  is the lower hybrid wave field potential;  $\mathbf{k}_0 (= \hat{y}k_{ol} + \hat{z}k_{oz})$  and  $\omega_0$  are respectively, the wave vector and the angular wave frequency of the lower hybrid pump that satisfies the dispersion relation:  $\omega_0 = \omega_{LH}[1 + (M/m)(k_{oz}/k_{ol})^2]^{1/2}$ , where  $\omega_{LH} = \omega_{pi}/(1 + \omega_{pe}^2/\Omega_e^2)^{1/2}$  is the lower hybrid resonance frequency.

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The parameters,  $M/m$ ,  $\omega_{pi}/\omega_{pe}$ , and  $\Omega_e$  are the ratio of ion to electron masses, the ion(electron) plasma frequency, and the electron gyro-frequency, respectively.

The process under consideration is a modulational instability whereby the lower hybrid pump wave ( $\omega_0, k_0$ ) excites two lower hybrid sidebands ( $\omega_{\pm}, k_{\pm}$ ) and a purely growing modulational mode ( $\omega_s, k_s$ ). This process can be described by the following frequency and wave vector matching relations:  $\omega_+ - \omega_s = \omega_0 = \omega_- + \omega_s^*$  and  $k_{\pm} = k_0 \pm k_s$ , where  $k_s \parallel k_0$ . The lower hybrid sidebands are excited through the beating current density driven by the lower hybrid pump field on the density perturbations,  $n_s = \tilde{n}_s \exp[i(k_s \cdot r - \omega_s t)]$ , of the zero-frequency modulational mode.

From the Poisson's equation,  $k_{\pm}^2 \phi_{\pm} = 4\pi e(\delta n_{i\pm} - \delta n_{e\pm})$ , where the ion and electron density perturbations ( $\delta n_{i\pm}$  and  $\delta n_{e\pm}$ ) associated with the lower hybrid sidebands are given, respectively, by  $\delta n_{i\pm} = (n_0 e / M \omega_0^2) k_{\pm}^2 [\phi_{\pm} + (k_{01}/k_{\pm 1})(\omega_0/\omega_{\pm})\phi_0(n_{s\pm}/n_0)]$  and  $\delta n_{e\pm} = (n_0 e / m \Omega_e^2)(1 - k_{02}^2 \Omega_e^2 / k_{01}^2 \omega_0^2) k_{\pm 1} [\phi_{\pm} + (k_{01}/k_{\pm 1})(\omega_0/\omega_{\pm})\phi_0(n_{s\pm}/n_0)]$ , the coupled mode equation for the lower hybrid sidebands is obtained as

$$\phi_{\pm} = (k_0/k_{\pm})(\omega_{\pm}/\omega_0)(k_{\pm 1}^2/k_{\pm}^2)[\omega_{LH}^2 \omega_0^2 / \omega_{pi}^2 \omega_{s\pm}(\omega_{s\pm} \pm 2\omega_0)]\phi_0(n_{s\pm}/n_0) \quad (1)$$

where the relations  $\omega_{s+} = \omega_s = \omega_s^*$  have been used.

The nonlinear effects responsible for the excitation of the purely growing modulational mode include the nonlinear beating current and the pondermotive force appearing, respectively, in the continuity equation and the momentum equation. Under the assumption of quasi-neutrality, the coupled mode equation for the purely growing mode is derived from the continuity and the momentum equations of electrons and ions and has the following form:

$$\begin{aligned} [-\omega_s^2(1 + k_{is}^2 c_s^2 / \Omega_e \Omega_i) + k_{zs}^2 c_s^2](n_s/n_0) = & -\omega_s k_{is} (\delta v_{ey}^{NL} + \frac{m}{M} \delta v_{iy}^{NL}) \\ & - k_{zs} (\delta v_{ez}^{NL} + \frac{m}{M} \delta v_{iz}^{NL}) + k_{is} \frac{\omega_s}{\Omega_e} \left( \frac{F_{ix}}{M} - \frac{F_{ex}}{m} \right) + i k_{is} \frac{\omega_s^2}{m \Omega_e^2} (F_{iy} + F_{ey}) - i \frac{k_z}{M} (F_{iz} + F_{ez}) \end{aligned} \quad (2)$$

where  $c_s = [(T_e + T_i)/M]^{1/2}$  is the ion acoustic velocity. The nonlinear beating current density  $n_0 \delta v_{e,i}^{NL}$  and the pondermotive force  $F_{e,i}$  are given, respectively, by

$$n_o \delta v_{e,i}^{NL} = \delta n_{e,i0} \delta v_{e,i-}^* + \delta n_{e,i0}^* \delta v_{e,i+} + \delta n_{e,i-}^* \delta v_{e,i0} + \delta n_{e,i+} \delta v_{e,i0}^* \\ \cdot \delta v_{e,i0}^*$$

$$\text{and } F_{e,i} = -im_{e,i} \{ k_o \cdot \delta v_{e,i-}^* \delta v_{e,i0} - k_- \cdot \delta v_{e,i0} \delta v_{e,i-}^* + k_+ \delta v_{e,i0}^* \delta v_{e,i+} \\ \cdot \delta v_{e,i+} - k_o \cdot \delta v_{e,i+} \delta v_{e,i0}^* \}$$

$$\text{where } \delta v_{e,\alpha} = -ik_{\alpha 1} [ \hat{x} + i\hat{y}(\omega_{\alpha}/\Omega_e) - i\hat{z}(k_{ox}\Omega_e/k_{o1}\omega_{\alpha}) ] (e\phi_{\alpha}/m\Omega_e),$$

$$\alpha = 0 \text{ or } \pm \quad (3)$$

$$\delta v_{i0} = ik_{o1} [ \hat{x}(\Omega_i/\Omega_o) - i\hat{y} - i\hat{z}(k_{oz}/k_{o1}) ] (e\phi_o/M\omega_o), \quad (4)$$

$$\delta v_{i\pm} = ik_{\pm 1} [ \hat{x}(\Omega_i^2/\omega_{\pm}^2) - i\hat{y}(\Omega_i/\omega_{\pm}) - i\hat{z}(k_{oz}\Omega_i/k_{o1}\omega_{\pm}) ] (e\phi_{\pm}/m\Omega_e), \quad (5)$$

$$\delta n_{eo} = n_o k_{o1}^2 (1 - k_{oz}^2 \Omega_e^2 / k_{o1}^2 \omega_o^2) (e\phi_o / m\Omega_e^2) \quad (6)$$

$$\delta n_{io} = n_o k_o^2 (e\phi_o / M\omega_o^2). \quad (7)$$

With the aid of (3) - (7), two things have been noted from the RHS of (2): Firstly, the ion-nonlinearity terms are relatively insignificant compared to the electron-nonlinearity terms; Secondly, the nonlinear beating current effect is partially cancelled by the pondermotive force effect. Therefore, the electron-nonlinearity term that remains on the RHS of (2) is  $-\omega_s k_z \delta v_{ez}^{NL}$ , contributed from the induced nonlinear beating current flowing along the imposed magnetic field.

Eliminating  $n_s$  and  $\phi_{\pm}$  from Equations (1) and (2) leads to the dispersion relation

$$\omega_s^2 = k_z^2 \{ C_s^2 - [(1 - k_{oz}^2 \Omega_e^2 / \omega_{pi}^2 k_{o1}^2) \Omega_e^2 / (1 + Mk_{oz}^2 / mk_{o1}^2) (\Omega_e^2 + \omega_{pe}^2)] \\ \cdot |v_{\phi}|^2 \} / (1 + k_1^2 C_s^2 / \Omega_e \Omega_i) \quad (8)$$

where  $v_{\phi} = k_o e\phi_o / m\Omega_e$ . For a purely growing mode (i.e.,  $\omega_s = i\gamma$ ), the growth rate obtained from (8) is given by

$$\gamma = k_z \{ [ (1 - k_{oz}^2 \Omega_e^2 / \omega_{pi}^2 k_{o1}^2) \Omega_e^2 / (1 + Mk_{oz}^2 / mk_{oz}^2) (\Omega_e^2 + \omega_{pe}^2)] |v_{\phi}|^2 - C_s \}^{1/2} / (1 + k_1^2 C_s^2 / \Omega_e \Omega_i)^{1/2}. \quad (9)$$

Since  $\gamma$  has to be positive for instability to occur, it is clear from (9) that (I)  $|k_{oz}/k_{o1}| < \omega_{pi} \Omega_e$ , (10)  
namely, the ratio of the perpendicular to the parallel scale lengths of the lower hybrid pump wave has to be less than the ratio of the ion plasma frequency to the electron gyrofrequency, (II)  $|\tilde{\phi}_o| > |\tilde{\phi}_{oth}| = ($

$$m/e)(\Omega_e^2 + \omega_{pe}^2)^{1/2}(C_s/k_o)(1 + Mk_{oz}^2/mk_{ol}^2)^{1/2}/(1 - \Omega_e^2 k_{oz}^2/\omega_{pi}^2 k_{ol}^2)^{1/2} \quad (11)$$

namely, the pump field intensity represented by  $|\tilde{\phi}_0|$  has to be greater than a threshold defined by  $|\tilde{\phi}_{oth}|$ .

The growth rate has a minimum when  $k_{oz}^2 \Omega_e^2/\omega_{pi}^2 k_{ol}^2 = (1 - |\tilde{\phi}_{oth}/\tilde{\phi}_0|^2)/(2 - |\tilde{\phi}_{oth}/\tilde{\phi}_0|^2)$ . In this optimum case and for a strong lower hybrid pump wave, (9) may be expressed as

$$\gamma_m \sim k_z |v_\phi| \Omega_e^2/(2\Omega_e^2 + \omega_{pe}^2)^{1/2}(\Omega_e^2 + \omega_{pe}^2)^{1/2}(1 + k_i^2 C_s^2/\Omega_e \Omega_i)^{1/2} \quad (12)$$

which is proportional to the wave length of the modulational instability.

#### 4. Summary and conclusions

The condition and the threshold power have been derived for the modulational instability of lower hybrid waves as shown, respectively, in (10) and (11). The nonlinearity for the excitation of purely growing modes is provided by the nonoscillatory beating current in the direction of the imposed magnetic field. For the power densities commonly used in the lower hybrid heating experiments in large tokamaks, the growth rate given in (12) is found to be quite large and it covers a broad spectrum. However, it should be stressed that the spatial modulational on the lower hybrid pump wave as discussed here only occurs in the resonance region where the condition,  $|k_{oz}/k_{ol}| < \omega_{pi}/\Omega_e$ , can be satisfied.

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