AD-A148 394 NEAR AXIS SHIP WAKES(U) MITRE CORP MCLEAN VA K M CASE 1/1
AUG 84 JSR-84-203C F19628-84-C-0001

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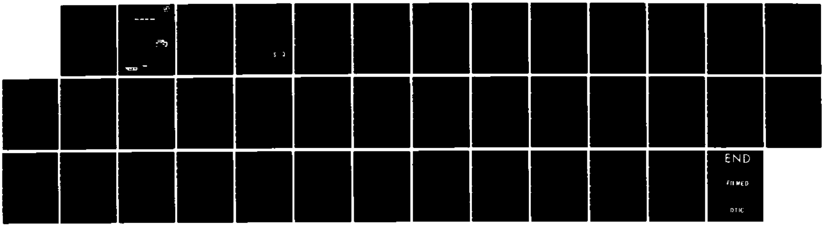

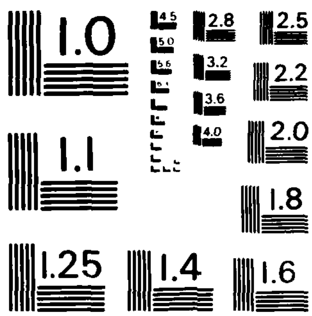

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SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER JSR-84-203C	2. GOVT ACCESSION NO. A148 394	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) Near Axis Ship Wakes		5. TYPE OF REPORT & PERIOD COVERED
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) K. M. Case		8. CONTRACT OR GRANT NUMBER(s) F19628-84-C-0001
9. PERFORMING ORGANIZATION NAME AND ADDRESS The MITRE Corporation 1820 Dolley Madison Blvd. McLean, VA 22102		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
11. CONTROLLING OFFICE NAME AND ADDRESS		12. REPORT DATE August 1984 — 13. NO. OF PAGES 31
14. MONITORING AGENCY NAME & ADDRESS (if diff. from Controlling Office)		15. SECURITY CLASS. (of this report) Unclassified
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this report)

DISTRIBUTION STATEMENT A
Approved for public release;
Distribution Unlimited

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

This report studies the structure of a ship surface wave wake far down stream in the vicinity of the axis. Of particular interest is the effects of finite ship width.

The theoretical work shows that there are singularities in the flow field but they are not on the axis. It is argued that the singularity is not real but is rather due to an injudicious rise of the stationary phase method.

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Near Axis Ship Wakes

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August 1984

JSR-84-203C

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NEAR AXIS SHIP WAKES

I. Introduction

The problem we address is that of the structure of a ship surface wave wake far down stream in the vicinity of the axis, (i.e.: $\frac{L}{X}$, $\frac{Y}{X}$ < < 1, where L is a typical dimension of the ship).

Basic approximations made are:

(i) The linearized theory is used.

(ii) It is assumed that $g\ L/U^2 >> 1$. Further, rather conventional stationary phase methods are employed It is indicated that this is the most suspect part of the calculation. Elucidation is warranted.

We are particularly interested in the effects of finite ship width. Accordingly, we deal with a simplified hull model. It is one of a prolate semi-spheroid. This has the advantage that the underlying protential flow is simple and can be written in closed form.

The main conclusions obtained are:

1) In agreement with other calculations it is found that singularities appear in the flow field. However, the singularity is <u>not</u> on the axis but is displaced therefrom. (In our model the displacement is of the order of the ship width.)

2) It is argued that the singularity is not real, but is rather due to an injudicious rise of the stationary phase method. However, it is probable that the singularity indicates that large disturbances can result far from the Kelvin angle and yet not on the axis.

II. The Basic Equations

1. The Integral Representation

These are as in reference (1). We use the linearized Euler equations and boundary conditions. In a coordinate system at rest with respect to the ship we can write the velocity potential as

$$\Phi = Ux + \phi \tag{1}$$

(U is the velocity of the ship which is moving in the + x direction).

Then ϕ satisfies

$$\nabla^2 \phi = 0, \tag{2}$$

$$\frac{\partial \phi}{\partial n} = -n_x U \text{ (on the ship surface)} \tag{3}$$

and

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{g}{U^2} \frac{\partial \phi}{\partial z} = 0 \text{ (on the free ocean surface } z = 0) \tag{4}$$

If we introduce a Green's function satisfying

$$\nabla'^2 G(\underset{\sim}{r}', \underset{\sim}{r}) = \delta(\underset{\sim}{r}' - \underset{\sim}{r}) \tag{5}$$

and

$$\frac{\partial G}{\partial Z'}(\underset{\sim}{r}', \underset{\sim}{r}) + \frac{U^2}{g} \frac{\partial^2}{\partial X'^2} G(\underset{\sim}{r}', \underset{\sim}{r}) \Big|_{Z' = 0} = 0 \tag{6}$$

then using Green's identity we have the representation:

$$\phi(r) = \int_{S_1} G(\underset{\sim}{r}', r) \frac{\partial}{\partial n'} G(\underset{\sim}{r}', r)\, dS_1$$

$$+ \frac{U^2}{g} \int_{c-} \phi(\underset{\sim}{r}') \frac{\partial}{\partial X'} G(\underset{\sim}{r}', r) - \frac{\partial \phi}{\partial X'} G(\underset{\sim}{r}', \underset{\sim}{r}) \Big|_{\substack{Z'=0 \\ X'=X-(y')}} dy' \tag{7}$$

$$\frac{-U^2}{g} \int_{c+} \phi(\underset{\sim}{r}') \frac{\partial}{\partial X'} G(\underset{\sim}{r}', \underset{\sim}{r}) - \frac{\partial \phi}{\partial X'} G(\underset{\sim}{r}', \underset{\sim}{r}) \Big|_{\substack{Z'=0 \\ X'=X+(y')}} dy'$$

(Here S_ is the hull of the ship and c+ + c- is the waterline.)

The equation (7) plays two roles.

a) When $\underset{\sim}{r}$ is on S_1 this is an inhomogeneous integral for $\phi(\underset{\sim}{r})$.

b) When $\underset{\sim}{r}$ is not on S_1 this gives an integral representation for $\phi(\underset{\sim}{r})$ in terms of its value and normal derivative on S_1.

Our basic approximation is then the following:

The G defined by equation (5) and (6) can be written as

$$G(\underset{\sim}{r}', \underset{\sim}{r}) = G_o(\underset{\sim}{r}', \underset{\sim}{r}) + G_1(\underset{\sim}{r}', \underset{\sim}{r}) \tag{8}$$

where

$$G_o(\underset{\sim}{r}', \underset{\sim}{r}) = \frac{1}{4\pi}\left\{ \frac{1}{|\underset{\sim}{r}'-\underset{\sim}{r}|} + \frac{1}{|\underset{\sim}{r}'-\hat{\underset{\sim}{r}}|} \right\} \tag{9}$$

with $\hat{\underset{\sim}{r}} = (x, y, -z)$,

and

$$G_1(\underset{\sim}{r}', \underset{\sim}{r}) = \frac{-1}{(2\pi)^2} \iint \frac{d^2k \;\; k_x^{\;2}}{k[\frac{gk}{U^2} - k_x^{\;2}]} e^{k(z+z')} \, e^{i[k_x(x'-x)+k_y(y'-y)]} \tag{10}$$

where $k = \sqrt{k_x^{\;2} + ky^2}$.

When $gL/U^2 >> 1$ and $\underset{\sim}{r}$, $\underset{\sim}{r}'$ are on S_1, we expect $G_1 << G_o$. Thus the integral equation (7) for $\phi(\underset{\sim}{r})$ with $\underset{\sim}{r}$ on S_1 can be approximated by putting $G = G_o$. The integral equation which results is then that of a simple potential flow problem--which is readily solved yielding a solution ϕ_o .

To calculate the radiated field (for $\underset{\sim}{r}$ far from S_1) we then replace ϕ in equation (7) by ϕ_o and G by G_1.

More precisely, we use the radiative part of G_1. This is obtained so: Causality tells us that the poles in equation (10) are to be above the contour of integration. The radiative part is just the contribution of the poles. For the part of G_1 which generates the radiation field we then obtain

$$G_1 \tilde{=} \frac{i}{2\pi} \frac{g}{U^2} \int_{-\infty}^{\infty} \frac{d\ell_y \; \ell_x}{1-2\ell} \, e^{\frac{g}{U^2} \ell(z+z')} \, e^{\frac{ig}{U^2}[\ell_x(x'-x)+\ell_y(y'-y)]} \qquad (11)$$

+ complex conjugate

(Here we have introduced dimensionless variables

so that $(k_x, k_y, k) = \frac{g}{U^2} (\ell_x, \ell_y, \ell)$.) (12)

2. Boat Model

We model the hull as half of a prolate spheroid. To be specific, introduce spheroidal coordinates so that

$x = c \cosh \eta \cos \theta$

$$y = c \sinh \eta \sin \theta \cos \omega$$

$$Z = c \sinh \eta \sin \theta \sin \omega$$

Then the hull is the surface

$$\eta = \eta_o, \quad 0 \leq \theta \leq \pi, \quad \pi \leq \omega \leq 2\pi$$

(The major and minor axis are thus $a = c \cosh \eta_o$, $b = c \sinh \eta_o$.)

The scale factors are

$$h_1 = h_2 = c \sqrt{\sinh^2 \eta \cos^2 \theta + \cosh^2 \eta \sin^2 \theta}$$

$$h_3 = c \sinh \eta \sin \theta,$$

or if we introduce $\zeta = \cosh \eta$, $\mu = \cos \theta$

$$h_1 = h_2 = c \sqrt{\zeta^2 - \mu^2}$$

$$h_3 = c \sqrt{(\zeta^2 - 1)(1 - \mu^2)} \; .$$

In reference (1) it is shown that

$$\phi_o = -\mu \; U \; c \; f \; (\zeta) \tag{13}$$

where

$$f(\zeta) = \frac{2 + \zeta \; \ell n \; (\frac{\zeta-1}{\zeta+1})}{\frac{2\zeta_o}{\zeta_o^2-1} + \ell n \; \frac{(\zeta_o-1)}{(\zeta_o+1)}} \tag{14}$$

III. Fundamental Integrals

In our approximation, we then have

$$\phi_{asym} = \sum_{i=1}^{6} I_i \tag{15}$$

where
$$I_1 = \frac{U^2}{g} \int_{c+} \frac{\partial \phi_o}{\partial x'} G(\underset{\sim}{r}', \underset{\sim}{r}) \Big|_{\substack{z'=0 \\ x'=x+(y')}} dy' \tag{16}$$

$$I_2 = - \frac{U^2}{g} \int_{c-} \frac{\partial \phi_o}{\partial x'} G_1 \, dy' \tag{17}$$

$$I_3 = \int_{S_1} \frac{\partial \phi_o}{\partial n'} G_1 (\underset{\sim}{r}', \underset{\sim}{r}) \, dS_1 \tag{18}$$

$$I_4 = \frac{-U^2}{g} \int_{c+} \phi_o(\underset{\sim}{r}') \frac{\partial}{\partial x'} G_1(\underset{\sim}{r}', \underset{\sim}{r}) \, dy' \tag{19}$$

$$I_5 = \frac{U^2}{g} \int_{c-} \phi_o (\underset{\sim}{r}') \frac{\partial}{\partial x'} G_1 (\underset{\sim}{r}', \underset{\sim}{r}) \, dy' \tag{20}$$

$$I_6 = - \int_{S_1} \phi_o \frac{\partial}{\partial n'} G_1 (\underset{\sim}{r}', \underset{\sim}{r}) \, dS_1 \tag{21}$$

Some useful kinematics:

$$\frac{\partial \phi_o}{\partial x'} = \frac{\mu(\zeta^2-1) \frac{\partial \phi_o}{\partial \zeta} + \zeta(1-\mu^2) \frac{\partial \phi_o}{\partial \mu}}{c(\zeta^2 - \mu^2)} \tag{22}$$

$$\frac{\partial \phi_o}{\partial n'} = -\frac{1}{c}\sqrt{\frac{(\zeta_o^2 - 1)}{(\zeta_o^2 - \mu^2)}}\ \frac{\partial \phi_o}{\partial \zeta} \tag{23}$$

$$dS_1 = c^2 \sqrt{\zeta_o^2 - \mu^2}\ \sqrt{\zeta_o^2 - 1}\ \sin\theta\, d\theta\, d\omega\ . \tag{24}$$

Let us see how some typical integrals are to be evaluated.

Consider I_1. This is an integral over c+. c+ is $X' > 0$, $Z' = 0$

$\therefore \quad 0 \leq \theta \leq \frac{\pi}{2}$

For $y' > 0$, we have $y' = b \sin\theta$.

$dy' = b \cos\theta\, d\theta$

As y goes from 0 to b, θ goes from 0 to $\frac{\pi}{2}$.

For $y' < 0$, we have $y' = -b \sin\theta$, $dy' = -b\cos\theta\, d\theta$

As y' goes from $-b$ to 0, θ goes from $\frac{\pi}{2}$ to 0.

$$\therefore \quad I_1 = \int_0^{\frac{\pi}{2}} \cos\theta \, d\theta \; \frac{\partial\phi_o}{\partial x'} \left\{ \begin{array}{l} G_1 \, (a\cos\theta, \, b\sin\theta, \, o \, ; \, \underset{\sim}{r}) \\ + \, G \quad (a\cos\theta, \, -b\sin\theta, \, o \, ; \, \underset{\sim}{r}) \end{array} \right\} \qquad (25)$$

Consider I_3.

Using equations (23) and (24) we have

$$\frac{\partial\phi_o}{\partial n'} \; dS_1 = -c \, (\zeta_o^{\;2} - 1) \, \frac{\partial\phi_o}{\partial\zeta} \sin\theta \, d\theta \, d\omega$$

and thus

$$I_3 = -c \, (\zeta_o^{\;2} - 1) \int_o^{\pi} d\theta \, \sin\theta \, \frac{\partial\phi_o}{\partial\zeta} \int_{\pi}^{2\pi} d\omega \, G_1 \, (\underset{\sim}{r}', \, \underset{\sim}{r}) \; . \qquad (26)$$

Now,

$$\int_{\pi}^{2\pi} d\omega \, G_1 = \frac{-i}{2\pi} \, \frac{g}{U^2} \int_{-\infty}^{\infty} \frac{d\ell_y \ell_x}{1-2\ell} \, e^{-\frac{ig}{U^2}[\ell_x x + \ell_y y]} \; e^{\frac{ig}{U^2} \ell_x \, a\cos\theta} \qquad (27)$$

$$x \, J \; ,$$

$$\text{with } J = \int_{\pi}^{2\pi} d\omega \; e^{\frac{g\ell b}{U^2} \sin\theta \sin\omega} \; e^{i \frac{g}{U^2} \ell_y \, b \sin\theta \cos\omega} \qquad (28)$$

Clearly in our approximation of $\frac{gb}{U^2} >> 1$, the principal contributions come from the vicinity of $\omega = \pi, 2\pi$.

Thus,

$$J \approx e^{\frac{-ig}{U^2} \ell_y b \sin\theta} \int_{\pi}^{3\frac{\pi}{2}} d\omega \; e^{\frac{-g}{U^2} \ell b \sin\theta \sin\omega}$$

$$+ e^{\frac{ig}{U^2} \ell_y b \sin\theta} \int_{3\frac{\pi}{2}}^{2\pi} d\omega \; e^{\frac{-g}{U^2} \ell b \sin\theta \sin\omega}$$

In the first of these integrals, we let

$\omega = \pi + \varepsilon$, then $d\omega = d\varepsilon$

$\sin\omega = \sin(\pi + \varepsilon) = \cos\pi \sin\varepsilon \approx -\varepsilon.$

Then $\int_{\pi}^{3\frac{\pi}{2}} \approx \int_{0}^{\infty} d\varepsilon \; c^{\frac{-g\ell b \sin\theta}{U^2}\varepsilon} = \frac{U^2}{g\ell b \sin\theta}$.

Similarly,

$$\int_{3\frac{\pi}{2}}^{2\pi} \approx \frac{U^2}{g\ell b \sin\theta}$$

and thus

$$J = \frac{U^2}{g\ell\, b \sin\theta} \left\{ e^{i \frac{g}{U^2} \ell\, b \sin\theta} + e^{\frac{-ig}{U^2} \ell\, b \sin\theta} \right\} .$$

Inserting into equations (27) and (26) yields:

$$I_3 = Ub \frac{\partial f}{\partial \zeta} \left(\frac{-i}{2\pi}\right) \int_{-\infty}^{\infty} \frac{d\ell_y\, \ell_x}{\ell\,(1-2\ell)}\, e^{-i \frac{g}{U^2} (\ell_x\, x + \ell_y\, y)}$$

$$\times \int_0^{\pi} \cos\theta\, d\theta \left\{ e^{i \frac{g}{U^2} [\ell_x\, a \cos\theta + \ell_y\, b \sin\theta]} \right. \qquad (29)$$

$$\left. + e^{i \frac{g}{U^2} (\ell_x\, a \cos\theta - \ell_y\, b \sin\theta]} \right\}$$

The remaining I_i are evaluated similarly. The results are:

$$I_1 + I_2 + I_3 = \frac{iUb}{2\pi} \int_{-\infty}^{\infty} \frac{d\ell_y\, \ell_x}{1-2\ell}\, e^{-i \frac{g}{U^2} (\ell_x x + \ell_y\, y]}$$

$$\times \int_0^{\pi} \cos\theta\, d\theta \left\{ \frac{\partial f}{\partial \zeta} \left[\frac{\mu^2(\zeta_o - 1)}{\zeta_o^{\,2} - \mu^2} - \frac{1}{\ell}\right] + \frac{\zeta_o f[1-\mu^2]}{(\zeta_o^{\,2} - \mu^2)} \left\{ e^+ + e^- \right\} . \right. \qquad (30)$$

$$+ \text{ c. c.}$$

and

$$I_4 + I_5 + I_6 = \frac{U}{2\pi} \frac{bg}{U^2}\, cf \int_{-\infty}^{\infty} \frac{d\ell_y \ell_x}{1-2\ell}\, e^{-i \frac{g}{U^2} (\ell_x\, x + \ell_y y)}$$

$$\times \int_0^{\pi} \cos\theta \, d\theta \; \{ \; \ell_x \cos\theta \, (1-\frac{1}{\ell}) \; [e^{+}+e^{-}]$$

$$- \frac{ig}{b} \; (\frac{U^2}{gb}) \, \frac{1}{\ell} \, [e^{+}+e^{-}] \qquad (31)$$

$$- \frac{\ell_y}{\ell} \; \frac{g}{b} \; \sin\theta \, [e^{+}-e^{-}] \; \}$$

$$+ \; c.c.$$

(Here $e^{\pm} = e^{i \frac{g}{U^2} [\ell_x \, a \cos\theta \, \pm \, \ell_y \, b \sin\theta]}$.)

IV. The Stationary Phase Evaluation

So far the approximations made are:

(i) Linearization

(ii) $\phi \to \phi_o$ on S_1. This justified by the assumption $gL/U^2 >> 1$.

(iii) In the integrals over S_1 only the immediate vicinity of the surface contributes significantly. Again this is justified by $gL/U^2 >> 1$.

We are left with generic integrals of the form

$$I_{\pm} = \int_{-\infty}^{\infty} d\ell_y \; g(\ell_y) \; e^{-i\frac{g}{U^2}(\ell_x x + \ell_y y)}$$

$$x \int_0^{\pi} h(\theta) \; e^{i\frac{g}{U^2}[\ell_x a \cos\theta \pm \ell_y b \sin\theta]} d\theta \qquad (32)$$

(Remember: $\ell_x = \sqrt{\ell}$, $\ell = \sqrt{\ell_x^2 + \ell_y^2}$.)

We want to evaluate these integrals in the limits $|y/x| << 1$, $|^b/x| << 1$. Our approach is to use the method of stationary phase twice.

First we do the θ integral by stationary phase. The stationary points θ^{+}_{-} are then functions of l_y. The l_y integral is then done by another application of stationary phase.

The main result is that in the limit of interest there are two classes of stationary points.

(a) $\ell \sim \ell_x \sim 1,\ \ell_y \sim 0, \qquad \cos^2 \theta_o \approx 1$

(b) $\ell \sim \ell_y >> \ell_x >> 1, \qquad \cos^2 \theta_o \approx 0$

In the integrals I^{+}_{-}, we first encounter

$$\int_o^\pi h(\theta)\, e^{i\,\phi_{\pm}}$$

where $\phi^{+}_{-} = \frac{g}{U^2}\,[\ell_x\, a \cos\theta \pm \ell_y b \sin\theta]$

The stationary phase condition $\frac{\partial \phi_{\pm}}{\partial\, \theta} = 0$

$$\text{yields } \tan\ \theta_{\pm} = \pm \frac{\ell_y}{\ell_x}\frac{b}{a} \tag{33}$$

Then

$$\int_0^{\pi} h(\theta)\, e^{i\,\phi_\pm} = e^{i\,\phi_\pm(\theta_\pm)}\, h(\theta_\pm) \int_{-\infty}^{\infty} e^{\frac{-i}{2}(\theta-\theta_\pm)^2 \phi_\pm''(\theta_\pm)}\, d\theta \, .$$

Then

$$I_\pm = \int_{-\infty}^{\infty} g(\ell_y)\, h(\theta_\pm)\, \left[\int_{-\infty}^{\infty} e^{\frac{-i}{2}(\theta-\theta_\pm)^2 \phi_\pm''(\theta_\pm)}\, d\theta \right] e^{-i\Phi_\pm} \quad (34)$$

where

$$\Phi_\pm = \frac{g}{U^2}\, [\ell_x(x - a\cos\theta_\pm) \mp \ell_y\, (y \mp b\sin\theta_\pm)] \quad (35)$$

The stationary phase condition $\frac{\partial\Phi}{\partial\ell_y} = 0$

yields

$$\frac{\partial\ell_x}{\partial\ell_y} = \frac{\pm\, b\sin\theta_\pm - y}{x - a\cos\theta_\pm} + \left[\frac{\pm\,\ell_y\, b\cos\theta_\pm - \ell_x\, a\sin\theta_\pm}{x - a\cos\theta_\pm}\right] \frac{\partial\theta_\pm}{\partial\ell_y} \, . \quad (36)$$

We are interested only in cases where $|x| >> a$. Therefore, we are always justified in taking as our basic equation

$$\frac{\partial\ell_x}{\partial\ell_y} = \frac{\pm b\sin\theta_\pm - y}{x} + \left[\frac{\pm\ell_y\, b\cos\theta_\pm - \ell_x\, a\sin\theta_\pm}{x}\right] \frac{\partial\theta_\pm}{\partial\ell_y} \quad (37)$$

Since $\theta_\pm$ are implicit functions of ℓ_y via equation (33) this is in principle a very difficult equation to solve for ℓ_y .

However, in our limit it is readily solved by a perturbation theory. We assume the second term on the right is small compared to the first. Calculate ℓ_y using only the first term and then verify that the second term is indeed small.

Thus, in first approximation our equation is

$$\frac{\partial \ell_x}{\partial \ell_y} = \Theta_{\pm} \quad , \tag{38}$$

where $\Theta_{\pm} = \dfrac{\pm b \sin \theta_o - y}{x}$, and by assumption

$$| \Theta_{\pm} | << 1 .$$

Since $\dfrac{\partial \ell_x}{\partial \ell_y} = \dfrac{\ell_y}{\ell_x(2\ell-1)}$

Our basic equation (38) becomes

$$\frac{\ell_y}{\ell_x(2\ell-1)} = \Theta_{\pm} \tag{39}$$

Squaring gives

$$\frac{\ell_y^{\,2}}{\ell_x^{\,2}\ (2\ell-1)^2} = \Theta_{\pm}^2 \quad . \tag{40}$$

If we note that $\ell_y^{\ 2} = \ell(\ell-1)$, $\ell_x^{\ 2} = \ell$ and set $2\ell - 1 = A$, we obtain a quadratic equation for A which can be solved to give

$$A = \frac{1 \pm \sqrt{1 - 8\ \Theta_{\pm}^2}}{4\ \Theta_{\pm}^2} \ . \qquad (41)$$

Clearly, the two solutions correspond to the wave trains whose caustics give the classical Kelvin 19 $1/2^{o}$ cone.

First, we look at the - in equation (41). This gives us the simpliest (and least interesting) of the two wave sets. For small Θ this gives

$$A \approx 1-2\ \Theta^2 \ .$$

Then $$\ell = \frac{A+1}{2} \approx 1 - \Theta^2 \qquad (42)$$

and $$\ell_x = \sqrt{\ell} \approx 1 - \frac{\Theta^2}{2} \ , \qquad (43)$$

From equation (39) we then see that

$$\ell_y \mathrel{\hat{=}} \Theta_{\pm} \quad (44)$$

What does this imply for θ_o ? The equation (33) tells us that

$$\tan \theta_{\pm} \approx \pm \Theta \frac{b}{a} \tag{45}$$

Thus $\theta_{\pm}$ is close to either 0 or π depending on the sign of the right side of equation (45).

Then $\Theta_{\pm} \approx -y/x$. (Here $x < 0$ and we will always look at $y > 0$.)

Then $\theta_{+} = \Theta = -y/x$

$$\theta_{-} = \pi - \varepsilon$$

and solving for ε we obtain

$$\varepsilon = \Theta = -y/x \text{ , i.e. } \theta_{-} = \pi - \Theta$$

Then $\phi_{\pm}(\theta_{\pm}) = \pm \frac{ga}{U^2}$.

From this it follows that

$$\int_{-\infty}^{\infty} e^{\frac{-i}{2}(\theta-\theta_{\pm})^2 \phi_{\pm}(\theta_{\pm})} d\theta = \sqrt{\frac{2\pi U^2}{ga}} e^{\pm i \pi/4}$$

$$I_{\pm} = \sqrt{\frac{2\pi U^2}{ga}}\, e^{\mp i \frac{\pi}{4}}\, h(\theta\pm)\; g(\Theta)\; e^{-i\Phi_{\pm}(\Theta,\theta_{\pm})}$$

$$x \int_{-\infty}^{\infty} e^{-i(\ell_y - \ell_y^0)^2 \frac{1}{2} \frac{\partial^2 \Phi}{\partial \ell_y^2}}\, d\ell_y \;.$$

In our approximation

$$\frac{\Theta^2 \Phi}{\Theta \ell_y} \approx \frac{g}{U^2}\, x\, \frac{2^2 \ell_y}{2\ell_y^2} = \frac{gx}{U^2} \;.$$

Thus, $$I_{\pm} = \sqrt{\frac{2\pi U^2}{ga}} \sqrt{\frac{2\pi U^2}{g(-x)}}\; e^{i\frac{\pi}{4}}\; e^{\pm i\frac{\pi}{4}}\; g(\Theta) h(\theta_{\pm}) e^{-i\Phi_{\pm}} \;. \qquad (46)$$

Here $$\Phi_{\pm}(\Theta,\ \theta_{\pm}) = \frac{g}{U^2}\,[x \mp a - \frac{y^2}{x}]$$

Now let us turn to the solution of equation (41) taking the + sign. Thus,

$$A = \frac{1 + \sqrt{1 - 8\,\Theta_{\pm}^2}}{4\,\Theta_{\pm}^2} \approx \frac{1}{2\Theta_{\pm}^2}$$

Then $$\ell = \frac{A+1}{2} \approx \frac{1}{4\Theta_{\pm}^2}$$

$$\ell_x = \frac{1}{2|\Theta_{\pm}|}$$

$$\ell_y = \Theta_{\pm}\, \ell_x (2\ell - 1) = \frac{\text{sgn}\ \Theta}{4\Theta^2}$$

The equation for the $\theta_{\pm}$ are

$$\tan \theta_{\pm} = \pm \frac{b}{a} \frac{1}{2\Theta_{\pm}} \quad .$$

To be definite, let us discuss the case

$y > b$. Then $\Theta_{\pm} > 0$.

θ_+ (θ_-) is then just less than (greater than) $\pi/2$.

Explicitly,

$$\theta_+ = \frac{\pi}{2} - \frac{2\Theta_+ a}{b}$$

$$\theta_- = \frac{\pi}{2} + \frac{2\Theta_- a}{b}$$

Substituting in the expressions for $\phi_{\pm}(\theta_{\pm})$ yields

$$\phi_{\pm}(\theta_{\pm}) = \pm \frac{g}{U^2} \frac{b}{4\Theta_{\pm}^{\ 2}}$$

and then

$$\int_{-\infty}^{\infty} e^{\frac{1}{2}(\theta-\theta\pm)^2 \phi\pm(\theta\pm)} d\theta$$

$$= \sqrt{\frac{\infty\pi U^2 \Theta_\pm^2}{gb}} \; e^{\pm i\pi/4} .$$

$$I\pm = \sqrt{\frac{\infty\pi U^2 \Theta_\pm^2}{gb}} \; e^{\pm i\pi/4} \; e^{-i\Phi_\pm(\Theta_\pm, \theta_\pm)}$$

$$\int_{-\infty}^{\infty} e^{-\frac{i}{2}(\ell_y-\ell_y)^2} \frac{\partial^2\Phi}{\partial \ell_y^2} \, d\ell_y .$$

Here $\Phi\pm = \frac{g}{U^2}\left[\ell_x x - \ell_y (\pm b \sin\theta \pm - y)\right]$

Using the expressions for ℓ_x, ℓ_y, $\theta\pm$ in terms of $\Theta_\pm$ simplifies this to

$$\Phi_\pm = \frac{g}{4U^2} \frac{x}{\Theta_\pm}$$

To lowest order (in Θ) we have

$$\frac{\partial^2\Phi}{\partial \ell_y^2} \simeq \frac{gx}{U^2} \frac{\partial^2 \ell_x}{\partial \ell_y^2} .$$

But $$\frac{\partial^2 \ell_x}{\partial \ell_y^2} \approx -2\,\Theta_\pm^3 .$$

$$\therefore \quad \int_{-\infty}^{\infty} e^{-\frac{i}{2}(\ell_y-\ell_y^0)^2} \frac{\partial^2\Phi}{\partial \ell_y^2} \, d\ell_y = \sqrt{\frac{\pi U^2}{g(-x)\Theta_\pm^3}} \; e^{-i\pi/4}$$

The results for $I_{\pm}$ are then

$$I_{\pm} = \frac{2\pi U^2}{g} e^{\pm i\frac{\pi}{4}} e^{-i\frac{\pi}{4}} \sqrt{\frac{2}{b(-x)\ \Theta_{\pm}}} \tag{47}$$

$$x\ e^{\frac{-igx}{4U^2\Theta_{\pm}}}\ g\ (\Theta_{\pm})\ h\ (\theta_{\pm})$$

To recapitulate:

$$\Theta_{\pm} = \frac{\pm b - y}{x}$$

$$\theta_{+} = \frac{\pi}{2} - \frac{2\ \Theta_{+} a}{b}$$

$$\theta_{-} = \frac{\pi}{2} + \frac{2\ \Theta_{-} a}{b}$$

$$\ell = \frac{1}{4\Theta_{\pm}} = \approx \ell_y\ ,\ \ell_x = \frac{1}{2\Theta_{\pm}}$$

(Remember we have assumed $y \geq b$!)

V. Explicit Forms

A. The Contributions From The Stationary Point Where $\ell \sim \ell_x \sim 1,\ \ell_y \sim \theta$

Here $\Theta = -y/x$.

and $\theta_+ = \Theta\ ,\ \theta_- = \pi - \Theta$.

From the results of Section IV, it is readily seen that all of $I_1 + I_2 + I_3$ vanish at least as Θ^2 .

The only term in $I_4 + I_5 + I_6$ which does not vanish similarly is

$$I = \frac{U}{2\pi}\, c\ f\ i\, \frac{a}{b} \int_{-\infty}^{\infty} \frac{d\ell_y\ \ell_x}{\ell(1-2\ell)}\ e^{-i \frac{g}{U^2}(\ell_x x + \ell_y y)}$$

$$x \int_0^{\pi} \cos\theta\, d\theta\, [e^+ + e^-]\ , \qquad + \text{c.c.}$$

Using equation (46) we then find

$$I = -U\ c\ f\ i\ \frac{U^2}{gb} \sqrt{\frac{a}{(-x)}}\ e^{+i\frac{\pi}{4}} \tag{48}$$

$$\left\{ e^{-i\frac{\pi}{4}} e^{\frac{-ig}{U^2}\left(x - a - \frac{y^2}{x}\right)} - e^{+i\frac{\pi}{4}} e^{-i\frac{g}{U^2}\left[(x+a) - \frac{y^2}{x}\right]} \right\}$$

B. Contributions From the Far Stationary Point

Here $\ell \approx \dfrac{1}{4\Theta_{\pm}^2}$, $\ell_y \approx \dfrac{1}{4\Theta_{\pm}^2}$, $\ell_x \approx \dfrac{1}{2\Theta_{\pm}}$

$$\theta_+ = \frac{\pi}{2} - 2\,\frac{\Theta + a}{b} \quad , \quad \theta_- = \frac{\pi}{2} + 2\,\frac{\Theta\ a}{b}$$

$$\Theta_{\pm} = \frac{\pm b - y}{x} \ .$$

Then

$$\sin\Theta_{\pm} = 1 + 0\,(\Theta_{\pm}^2)$$

$$\cos\theta_{\pm} = \pm 2\,\frac{\Theta \pm a}{b} + 0\,(\Theta_{\pm}^3)$$

One might suspect that because of the overall factor of $(\Theta_{\pm})^{-1/2}$ in equation (47) that there may be singularities in some derivatives of our potential. However, it can be shown that almost all the terms in equations (30) and (31) are at worst of order $\Theta^{7/2}$

We consider two examples:

(a). The term in equation (30) proportional to

$$\int_o^{\pi} \cos\theta \, d\theta \, \frac{\partial f}{\partial \zeta} \left[\frac{\mu^2(\zeta_o^{\,2}-1)}{\zeta_o^{\,2} - \mu^2} - \frac{1}{\ell}\right] \tag{49}$$

By our rules μ^2 and $\frac{1}{\ell}$ are of order Θ^2. The multiplicative cos θ makes the expression $\sim \Theta^3$. The factor $\frac{\ell_x}{1-2\ell}$ gives another Θ. Finally the overall $\Theta^{-1/2}$ shows that the contribution to ϕ is $\sim \Theta^{+7/2}$.

(b) Somewhat more subtle is the evaluation of the terms in equation (31) proportional to

$$\int_o^{\pi} \cos\theta \, d\theta \left\{ \ell_x \cos\theta \, [e^+ + e^-] - \frac{\ell_y}{\ell} \frac{a}{b} \sin\theta \, [e^+ - e^-] \right\} \tag{50}$$

The terms $\sim [e^+ + e^-]$ and $[e^+ - e^-]$ separately are of order Θ. However, there is a cancellation such that the overall expression is $\sim \Theta^3$. Thus consider

$$\int_o^{\pi} \cos\theta \, d\theta \left\{ \ell_x \cos\theta - \frac{\ell_y}{\ell} \frac{a}{b} \sin\theta \right\} e^+$$

$$\approx \cos\theta_+ \left\{ \frac{\cos\theta_+}{2\Theta_+} - \frac{a}{b} \sin\theta_+ \right\} e^+$$

$$= \frac{2\Theta_+ a}{b} \left\{ \frac{2\Theta_+ \frac{a}{b}}{2\Theta_+} - \frac{a}{b} \right\} e^+ = 0 \text{ to } 0\ (\Theta^3)\ !$$

As a consequence we see that the dominant contriibution for $\Theta << 1$ is:

$$\phi = \frac{iUb}{2\pi} \int_{-\infty}^{\infty} \frac{d\ell_y \ell_x}{1-2\ell} e^{\frac{-ig}{U^2}[\ell_x x + \ell_y y]}$$

$$x \int_0^{\pi} \cos\theta\, d\theta\, \zeta_o\, f(\zeta_o) \frac{[1-u^2]}{(\zeta_o^2-\mu^2)} [e^+ + e^-] \qquad (51)$$

$$+ \text{ c.c.}$$

Using our rules for integration this becomes

$$\phi = -2Ua \frac{U^2}{g} \sqrt{\frac{2}{b(-x)}} \frac{f(\zeta_o)}{\zeta_o} \qquad (52)$$

$$x \left\{ \Theta_+^{\frac{3}{2}} e^{\frac{-igx}{4U^2\Theta_+}} + i\, \Theta_-^{\frac{3}{2}} e^{\frac{-igx}{4U^2\Theta_-}} \right\}$$

$$+ \text{ c.c.}$$

VI. Discussion

We conclude that with our approximations the dominant term for the potential near the axis is given by equation (52). This has a singularity at $y = b$ not at $y = 0$.

The potential and displacement at the singularity are both zero. But the slopes are infinite.

Where does the singularity come from? We think not from our use of the approximation $U^2/gL << 1$. Indeed tracing back to the origin of the term we see it arises from $\partial\phi/\partial x$ on the waterline. Our procedure guarantees that $\partial\phi/\partial x$ *there is represented* exactly.

Rather the origin would seem to result from an inappropriate use of the stationary phase method when evaluating equation (51). (This does not seem to matter for all other terms of equations (30) and (31). They are multiplied by a high powers of the vanishing quantity.) The problem with evaluating equation (51) by stationary phase is that $\frac{\partial^2 \Phi}{\partial \ell_y^2} \to 0$ as $\Theta_+ \to 0$.

Normally one would handle such a situation by proceeding to the next term in the Taylor series of Φ around the stationary point. Thus instead of the Fresnel type integral one would have an Airy integral to describe the result in the vicinity of the bogus singular point. Here, however, one readily shows that not only does $\frac{\partial^2 \Phi}{\partial \ell_y^2} \to 0$ but so does $\frac{\partial^3 \Phi}{\partial \ell_y^3}$. Indeed all higher derivatives go to zero as ${}^y\Theta_+ \to 0$. (They even go to zero faster and faster as the derivative increases.)

We conclude:

(1) The result of equation (52) would seem to indicate that there are large slopes in the wake near but not on the axis.

(2) A definitive conclusion (of the linear theory) awaits a better evaluation of the integrals of equation (51).

(3) Since we have used a very specific hull model it is not immediately obvious as to what the shape dependence really is. However, the procedure outlined here strongly suggests that large slopes should be produced in the wake at distances from the axis comparable to some measure of the ship width.

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