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A subset selection procedure R_B for binomial populations is considered for the problem of selecting the best of k vendors whose manufacturing processes have the probabilities $p_1,\ldots,p_k$ of turning out an item which conforms to spefications. Let $X_1,\ldots,X_k$ denote the number of conforming items from samples of size n from the k processes. Then the rule R_B is of the form: Select π_i if and only if $X_i \geq \max_{1\leq j\leq k} X_j - d$, where d is a nonnegative integer. The operating characteristics (over)

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SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered) (i.e. selection probabilities and expected size of the selected subset) of this rule are related to the underlying p_i's, the common sample size n, and d. Formulae (both exact and asymptotic) are given for these quantities for slippage as well as equi-spaced parametric configurations. Tables and graphs relating these quantities are presented for three specific slippage configurations. Numerical illustrations are given to show the use of the tables in determining the sample size n and the constant d to be used in the rule R_B. Also, a rule R_{BC} is mentioned for selecting vendors who are better than (that is, having higher success-probability) a given (control) vendor.

A STATISTICAL APPROACH TO VENDOR SELECTION

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A Statistical Approach to Vendor Selection

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Introduction

A common problem that arises in practice is the comparison of several Bernoulli processes (or populations) with unknown parameters $p_1,\ldots,p_k$, respectively, where the p_i's denote the success probabilities. A particular realization of this problem is the critical issue of vendor selection. Deming (1982) notes the importance of vendor selection in a company's efforts to achieve high quality and productivity. In his 14 points, Deming's point 4 suggests the reduction of the number of suppliers to a subset of vendors who can furnish statistical evidence of dependable quality.

Vendor selection involves a consideration of many aspects -- cost, service, reliability, and quality. Pettit (1984) described the approach that 3M Corporation uses in the evaluation of prospective suppliers. It consists of evaluating potential vendors in four areas: quality, price, performance, and facility capabilities. While quality is explicitly considered in this approach, it is not evaluated in a statistical sense. It is the intent of this (present) article to indicate how statistics can be utilized as one objective evaluation tool in this decision setting.

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Short Title: VENDOR SELECTION

Abstract:

A subset selection rule R_B for binomial populations is considered for selecting the best of k vendors whose manufacturing processes have probabilities $p_1,\ldots,p_k$ of turning out an item conforming to specifications. Let X_i denote the number of conforming items in a sample of size n from the i-th vendor (success probability p_i), $i = 1,\ldots,k$. Rule R_B selects the i-th vendor if and only if $X_i \geq \max_{1\leq j\leq k} X_j - d$, where d is a nonnegative integer. Operating characteristics of R_B are studied for slippage and equi-spaced parametric configurations. Tables and graphs relating to selection probabilities and expected subset size are presented as well as examples for illustrating use of these. Also, a rule R_{BC} is discussed for selecting vendors who are better than a given (control) vendor.

Key Words: Binomial model, subset selection rules, operating characteristics, comparisons with control, tables, graphs, numerical illustrations, applications to vendor selection.

To formalize the above problem consider k Bernoulli processes, which may represent the manufacturing processes of k vendors. Let p_i denote the probability that items manufactured by the ith vendor will conform to specifications. The ith vendor we'll denote simply by π_i. Let $p_{[1]} \leq \cdots \leq p_{[k]}$ denote the ordered parameters. It is assumed that there is no prior knowledge regarding the correct pairings of the ordered and the unordered p_i's. The vendors (or processes) are ranked according to the values of p_i's. The vendor associated with $p_{[k]}$, the largest p_i, is called the best.

Let $X_1, X_2, \ldots, X_k$ denote the number of conforming items from these vendors based on a random sample of n items from each. Our interest is to define a statistical procedure based on $X_1, \ldots, X_k$ to select a nonempty subset of the k vendors with a guarantee of minimum probability P* that the best vendor is included in the selected subset. Selection of any subset which includes the best is called a correct selection (CS). Thus the probability of a correct selection using a rule R $P(CS|R)$, should satisfy the condition that

$$P(CS|R) \geq P^* \tag{1}$$

whatever be the unknown values of the p_i's. This condition is generally referred to as the P*-condition. Obviously, for a meaningful problem, $1/k < P^* < 1$.

Any procedure R that satisfies (1) is a valid procedure. To distinguish between valid procedures we need to evaluate criteria that characterize effectively procedure performance. One such criterion is the expected value of S, the number of populations included in the selected subset. S is known as the subset size and it is a positive integer-valued random variable. One may also consider the related quantity E(S'), where S' denotes the number of non-best populations included in the selected subset. Let α_i denote the probability of selecting the process associated with $p_{[i]}$, $i = 1,\dots,k$. Obviously, α_k = PCS. It is also easy to see that

$$E(S) = \alpha_1 + \dots + \alpha_k$$
$$E(S') = \alpha_1 + \dots + \alpha_{k-1}. \tag{2}$$

The α_i's are called the individual selection probabilities. One may also consider a criterion which combines E(S) and PCS. Such a criterion, namely, E(S)/PCS has been considered in the literature. All these criteria that are used to evaluate a valid procedure are called the operating characteristics of the procedure. In our present study, we use the expected subset size and the individual selection probabilities.

The Gupta-Sobel Rule

Gupta and Sobel (1960) proposed and studied a rule R_B defined as follows.

R_B: Select π_i if and only if $X_i \geq \max_{1 \leq j \leq k} X_j - d$,

where $d = d(k,n,P^*)$ is the smallest nonnegative integer satisfying

$$\inf_{\Omega} P(CS|R_B) \geq P^*, \tag{3}$$

where $\Omega = \{\underline{p} | \underline{p} = (p_1,\dots,p_k),\ 0 \leq p_i \leq 1,\ i = 1,\dots,k\}$ is the parameter space. Gupta and Sobel (1960) have shown that the infimum on the left-hand side of (3) is attained when $p_1 = \dots = p_k$. Thus, we evaluate $P(CS|R_B)$ for $p_1 = \dots = p_k = p$ (say) and rewrite (3) as

$$\inf_{0 \leq p \leq 1} \sum_{j=0}^{n} \binom{n}{j} p^j (1-p)^{n-j} \left\{ \sum_{y=0}^{d+j} \binom{n}{y} p^y (1-p)^{n-y} \right\}^{k-1} \geq P^*. \tag{4}$$

There is no known result regarding the value of p for which the infimum in (4) is attained except in the special case of k = 2. When k = 2, the infimum is attained for p = 0.5.

When n is large enough to justify normal approximation, then equation (4) can be approximated by

$$\inf_{0\le p\le 1} \int_{-\infty}^{\infty} \Phi^{k-1}[x + (d + .5)/(npq)^{\frac{1}{2}}]\varphi(x)dx = P^* ,$$

where $q = 1-p$. The infimum of the expression on the left hand side above occurs at $p = \frac{1}{2}$ which gives the approximation for d as the solution to

$$\int_{-\infty}^{\infty} \Phi^{k-1}[x + (2\hat{d}+1)/(n^{\frac{1}{2}})]\varphi(x)dx = P^*, \tag{5}$$

where Φ and φ denote the cdf and density of a standard normal variable. Since $\hat{d}$ is not necessarily an integer, to implement the procedure we simply replace $\hat{d}$ by the smallest integer greater than or equal to $\hat{d}$. These values have been tabulated by Gupta and Sobel (1960), for $k = 2(1)20(5)50$ and $n = 1(1)20(5)50(10)100(25)200(50)500$. Tables 1 and 2, extracted from Gupta and Sobel (1960), provide the values of $\hat{d}$ for $P^* = .90$ and $.95$, respectively, for $k = 2,5(5)30(10)50$, and $n = 5(5)50(10)100, 250, 500$.

Operating Characteristics

Let us assume without loss of generality that $p_1 \le \ldots \le p_k$. As we pointed out earlier, we consider the rule: Select π_i if and only if $X_i \ge \max_{1\le j\le k} X_j - d$, where $0 \le d \le n$. The operating characteristics studied are the expected subset size and the individual selection probabilities. We consider two types of parametric configurations, namely, (1) the <u>slippage configuration</u> defined by $p=p_1=\ldots=p_{k-1} = p_k-\delta$, $0 < \delta < 1-p$, and (2) the <u>equi-spaced parametric configuration</u> defined by $p_{i+1}-p_i = \delta$, $i = 1,\ldots,k-1$, $0 < \delta < (1-p)/(k-1)$. For convenience, let

$$\begin{aligned} b(x;\, n,p) &= \binom{n}{x}p^x(1-p)^{n-x}, \quad x = 0,1,\ldots,n \\ B(t;\, n,p) &= \sum_{x=0}^{t} b(x;\, n,p), \quad t = 0,1,\ldots,n. \end{aligned} \tag{6}$$

Slippage Configurations

For the configuration $(p,p,\ldots,p,p+\delta)$, $0 < \delta < 1-p$, we get

$$PCS = \alpha_k = \sum_{x=0}^{n} b(x;\ n,p+\delta)[B(x+d;\ n,p)]^{k-1},$$

$$\alpha_i = \sum_{x=0}^{n} b(x;\ n,p)B(x+d;\ n,p+\delta)[B(x+d;\ n,p)]^{k-2}, \tag{7}$$

$$i = 1,\ldots,k-1.$$

Any specified non-best population has the same probability of being selected and we denote this by P(NCS). Also, $E(S) = (k-1)\alpha_1 + PCS$.

We present tables and graphs for the operating characteristics in the case of three slippage configurations. These are given by the following pairs of p and δ values:

(I) $p = .50$, $\delta = .10$, (II) $p = .75$, $\delta = .05$, (III) $p = .90$, $\delta = .03$.

Tables 3 through 5 give the values of PCS, P(NCS), and E(S) for $k = 3,5,10,15$; $d = 2,3,4,5$; and $n = 5(5)50\ (10)100,250,500$ in the case of the three configurations I - III. Figure 1 shows the graph of E(S) as a function of n for the rule with $d = 2$ for $k = 3,5,10$ when the slippage configuration is given by $p = .90$ and $\delta = .03$. This figure also shows for $n = 10(10)50$, the value of PCS when $\delta = 0$, that is, when all the parameters are equal to .90. Figures 2 and 3 are graphs of E(S) as a function of n for $d = 2,3,4,5$, and for $k = 3,5$, and 10. Figure 2 is for the slippage configuration with $p = .75$ and $\delta = .05$ and Figure 3 is for the configuration with $p = .90$ and $\delta = .03$. These results and examples are discussed in the next section.

For sufficiently large n, one can use the normal approximation and obtain

$$PCS \approx \int_{-\infty}^{\infty} \Phi^{k-1}\left[x\sqrt{\frac{(p+\delta)(q-\delta)}{pq}} + \frac{d+n\delta+1/2}{\sqrt{npq}}\right]\varphi(x)dx,$$

$$\alpha_i \approx \int_{-\infty}^{\infty} \Phi^{k-2}\left[x + \frac{d+1/2}{\sqrt{npq}}\right]\Phi\left[\sqrt{\frac{pq}{(p+\delta)(q-\delta)}}\, x + \frac{1/2+d-n\delta}{\sqrt{n(p+\delta)(q-\delta)}}\right]\varphi(x)dx, \quad (8)$$

$$i = 1,\ldots,k-1.$$

Equi-spaced Parametric Configuration

For the configuration $(p,p+\delta,\ldots,p+(k-1)\delta)$, $0 < \delta < (1-p)/(k-1)$, we have

$$\alpha_i = \sum_{x=0}^{n} b(x;\, n,p+(i-1)\delta) \prod_{j \neq i} B(x+d;\, n,p+(j-1)\delta),\ i = 1,\ldots,k. \quad (9)$$

We note that α_i is the probability of including the non-best population with parameter $p + (i-1)\delta$, $i = 1,\ldots,k-1$, and α_k is the PCS. For large n, the normal approximation yields

$$\alpha_i \approx \int_{-\infty}^{\infty} \prod_{j \neq i} \Phi\left[\sqrt{\frac{\theta_i(1-\theta_i)}{\theta_j(1-\theta_j)}}\, x + \frac{d+1/2+(i-j)n\delta}{\sqrt{n\theta_j(1-\theta_j)}}\right]\varphi(x)dx, i = 1,\ldots,k, \quad (10)$$

where $\theta_i = p+(i-1)\delta$, $i = 1,\ldots,k$.

A Modified Procedure R_B'

Suppose, the experimenter has the a priori information that for all vendors the unknown probabilities p_i's are at least as large as p_0 where p_0 is some specified number and which in many situations can be assumed to be greater than $\frac{1}{2}$. Then, intuitively speaking, one should be able to use this information to reduce d-value, for fixed values of P* and n. This can be shown as follows:

In the least favorable case, i.e. when $p_1 = p_2 = \ldots = p_k = p$, and n is large, we have

$$P(CS) = \int_{-\infty}^{\infty} \Phi^{k-1}\left(x + \frac{d+1/2}{\sqrt{npq}}\right)\varphi(x)\,dx,$$

so that as $n \to \infty$, the infimum of the P(CS) takes place as $p \to \frac{1}{2}$. Since the P(CS) given above decreases with p for values of $p > \frac{1}{2}$, it follows that for $p_0 > \frac{1}{2}$,

$$\inf_{0 \le p \le 1} P(CS) = \int_{-\infty}^{\infty} \Phi^{k-1}\left(x + \frac{2d+1}{\sqrt{n}}\right)\varphi(x)\,dx$$

$$< \int_{-\infty}^{\infty} \Phi^{k-1}\left(x + \frac{d+1/2}{\sqrt{np_0q_0}}\right)\varphi(x)\,dx, \text{ where } q_0 = 1-p_0.$$

Equating the two integrals above to P* and relabelling the d-value in the second integral as d*, we have

$$d^* = (2d+1)\sqrt{p_0q_0} - 1/2 < d.$$

Thus, for fixed n and P*, the a priori constraint on p_i's leads one to use the following modified procedure,

R_B': Select the ith vendor if and only if

$$X_i \ge \max_{1 \le j \le k} X_j - d^*.$$

The modified procedure R_B' will result in a smaller value of the expected size, E(S), keeping n and P* fixed. If one is willing to give up the saving in the value of E(S), one can, for a fixed P*, find a smaller n corresponding to this smaller value d* of d. This can be done by interpolation in Tables 1 and 2.

Comparison with a Control

In some situations, one may want to compare several competing vendors with a specific vendor who serves as the control. The goal is to select all vendors who are better than (that is, having higher success probability) the control vendor. Based on random samples of n items, let $X_1,\ldots,X_m$ denote the numbers of conforming items from m competing vendors and let X_0 denote the number for the control vendor. This problem was studied separately by Gupta and Sobel (1958). Their rule is

R_{BC}: Select the vendor with X_i success if and only if $X_i \geq X_0 - D$,

where $D = D(m,n,P^*)$ is the smallest nonnegative integer such that with specified probability P^* the selected subset will include all vendors who are better than the control vendor. For selected values of m, n, and P^*, the value of D can be obtained from Tables 1 and 2 by setting $m = k-1$.

Examples

For the purpose of illustrating our rule and the use of the tables, let's assume that we have five potential vendors for an item. Our goal is to identify a subset of these in such a manner that the best is contained in the subset with a high probability. Having identified this subset, we'll then proceed to investigate other nonstatistical criteria (such as facility capability, price, etc.) upon which to base a final decision on vendor selection. We note that this approach is applicable only if test samples of the item can be obtained. For the five candidates, let x_i denote the realized value of X_i based on random samples of size $n = 30$. (We'll say more about the sample size choice later). Suppose that

$$X_1 = 27,\ X_2 = 25,\ X_3 = 24,\ X_4 = 22, \text{ and } X_5 = 28.$$

In simple terms, vendor 1 supplied 30 test items (chosen at random from its production process) and 27 of the 30 items conformed satisfactorily to all specifications.

Now we use the statistical selection procedure R_B with d = 2 to select a subset of these vendors. (We'll say more about the choice of d later.) The rule can now be simply stated as: choose all vendors for which $X_i \geq \max X_j - d = 28-2 = 26$. This results in the selection of vendors 1 and 5. How good is this procedure? What probabilistic guarantees do we have with its use? That's where our tables and figures are helpful as we'll now illustrate.

In the event that four of the vendors could produce 90% conforming items (i.e., p = .90) and one could produce 93% conforming items, the selection rule R_B as we used it (n = 30, d = 2, k = 5) would select the best vendor with probability 0.86 and would retain a nonbest vendor with probability 0.66 (see Table 5). The expected size of the selected subset can be read from either Table 5 or Figure 1 and is 4(.66) + 0.86 = 3.5. Also from Figure 1 we find the probability of making a correct selection (i.e., choosing the best vendor to be in the selected subset) decreases to 0.702 as the process of the best vendor decreases to 90% conformance -- the same as the other four vendors.

If these operating characteristics are not satisfactory from the decision maker's perspective then alternative choices for n and/or d should be made. Note, however, that all of the probabilities given in the preceding paragraph were obtainable <u>before</u> any data was obtained from the vendors. The operating characteristics of the selection procedure are determined prior to the actual data analysis. Let's look at how alternative choices of n and d can be generated so as to meet a decision maker's requirements or preferences. This search and specification is usually conditioned on some statement about the parameter configuration over which the probabilistic statements should be applicable.

For example, if we now focus our concern on parameters in a slippage configuration with $p = .75$ and $\delta = .05$ we can look for a pair (n,d) for which PCS is at least a specified number -- say 0.90. Since this criterion will yield more than one (n,d) choice we might then choose the pair which has the smallest $E(S)$. Consulting Table 4 we generate the options listed below:

n	d	PCS	E(S)
5	2	.96	4.65
10	3	.96	4.56
15	3	.92	4.13
20	4	.95	4.32
25	4	.93	4.06
30	4	.91	3.82
35	4	.90	3.62
40	5	.93	3.91
45	5	.93	3.74
50	5	.92	3.60

It should be noted that because of the discrete nature of the distribution involved, an increase in n does not produce necessarily a better option. In this illustration the best option would be $n = 50$ and $d = 5$. That is, ask for a random sample of 50 items from each vendor and select those for which $X_i \geq \max_{1 \leq j \leq k} X_j - 5$.

Alternatively, one may want to set an upper bound for $E(S)/k$, the expected proportion of populations selected. If we set this bound as .80, then we look for pairs (n,d) for which $E(S) \leq 5 \times .80 = 4$. If there are more than one such pair with same n, we take the pair for which the PCS is maximum. Consulting Table 4 again, we have the following options -- the best being $n = 45$ and $d = 5$.

n	d	E(S)	PCS
10	2	3.89	.87
15	2	3.37	.81
20	3	3.77	.88
25	3	3.48	.86
30	4	3.82	.91
35	4	3.62	.90
40	5	3.91	.93
45	5	3.74	.93
50	5	3.60	.92

It is possible to use other criteria for choosing the pair (n,d). If we feel that the true parametric configuration can in some sense be described by one of two possible slippage configurations given by, say, $p = .75$, $\delta = .05$ and $p = .90$, $\delta = .03$, then we can choose the pair (n,d) that controls the PCS or E(S) at given levels for both configurations.

Summary and Concluding Remarks

In this paper we have presented two statistical selection rules applicable to the important problem of vendor (or process) selection. The first rule is appropriate for the selection of a subset to contain the best vendor with a preassigned probabilistic guarantee. The second rule is directed towards selection of a subset to contain all vendors better than a standard -- again with a specified probabilistic guarantee. Additionally we've indicated how prior knowledge on vendor quality level can be explicitly incorporated in the form of inequality constraints on the binomial probability parameters. Such incorporation, where applicable, can reduce substantially the expected subset size while preserving the stated minimum probability of making a correct selection.

Implementation of such procedures requires several choices by the analyst. That is, in a sense, similar to consideration involved in statistical hypothesis testing. In the latter case the analyst determines a critical region (or rejection region) and sample size by examining operating characteristics (e.g., Type I and Type II errors) and choosing combinations appropriate for the application. With respect to the selection procedures herein discussed the analyst must choose the constant d to be used with the rule R_B and the sample size for each vendor (or process).

Once the number of vendors (k) is specified the choice of d and n depends in turn on operating characteristics of the selection procedure. (For rule R_{BC} the choice is D and n). We recommend the analyst first specify a P* value which is the minimum probability of a correct selection (the analog of Type I error). This specification can generate many (d,n) combinations. At this point the analyst should specify an upper bound on the expected subset size for a parametric configuration meaningful for the application (the analog of Type II error). Then referring to the figures and tables given here, determine a (d,n) choice which achieves the requirements on both the probability of a correct selection <u>and</u> the expected subset size. In situations where these tables and figures are not sufficient to represent an application, the reader is referred to the additional references. New calculations may be required using the formulae given.

Once the d value and sample size n have been determined the data analysis proceeds by random sampling and testing of n items from each vendor (process) and then selecting a subset according to the rule R_B with d as the constant. The resultant subset of vendors, chosen on the basis of a statistical comparison of quality, can then be examined further on other important aspects such as price, facilities, delivery, etc.

Statistical methods can play a significant role in vendor selection. Those described here are applicable only to those situations where vendors are currently producing the product of interest. Since the rules are data dependent, they would not be applicable for decision situations involving new products currently not being produced.

We've illustrated here techniques applicable to attribute data represented by the binomial model. Similar procedures have been developed for continuous measurement data emanating from a wide variety of statistical distributions such as normal, gamma, and exponential. A good discussion of these many rules can be found in the book by Gupta and Panchapakesan (1979). Distribution-free (nonparametric) rules have also been developed and can be applied when only ordinal information is obtained about the vendors (or processes). A review of such procedures can be found in Gupta and McDonald (1982).

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Table 1. Values of d for implementing the rule R_B for selecting the best of k binomial populations or the rule R_{BC} for selecting from k-1 binomial populations that are better than an unknown control.

P* = .90

n \ k	2	5	10	15	20	25	30	40	50
5	2	3	3	3	4	4	4	4	4
10	3	4	5	5	5	5	5	6	6
15	4	5	6	6	6	7	7	7	7
20	4	6	7	7	7	8	8	8	8
25	5	6	7	8	8	8	9	9	9
30	5	7	8	9	9	9	9	10	10
35	5	8	9	9	10	10	10	11	11
40	6	8	9	10	10	11	11	11	12
45	6	9	10	11	11	11	12	12	12
50	6	9	11	11	12	12	12	13	13
60	7	10	12	12	13	13	13	14	14
70	8	10	12	13	14	14	14	15	15
80	8	12	13	14	15	15	15	16	16
90	9	12	14	15	16	16	16	17	17
100	9	13	15	16	16	17	17	18	18
250	14	21	24	25	26	27	27	28	29
500	20	29	33	35	37	38	39	40	41

The above values of d were computed by using the normal approximation as given in equation (5).

Table 2. Values of d for implementing the rule R_B for selecting the best of k binomial populations or the rule R_{BC} for selecting from k-1 binomial populations that are better than an unknown control.

$P^* = .95$

n \ k	2	5	10	15	20	25	30	40	50
5	3	3	4	4	4	4	4	4	5
10	4	5	5	6	6	6	6	6	6
15	5	6	7	7	7	7	8	8	8
20	5	7	8	8	8	8	9	9	9
25	6	8	8	9	9	9	10	10	10
30	6	8	9	10	10	10	11	11	11
35	7	9	10	11	11	11	11	12	12
40	7	10	11	11	12	12	12	13	13
45	8	10	11	12	12	13	13	13	14
50	8	11	12	13	13	13	14	14	14
60	9	12	13	14	14	15	15	15	16
70	10	13	14	15	16	16	16	17	17
80	10	14	15	16	17	17	17	18	18
90	11	14	16	17	18	18	18	19	19
100	12	15	17	18	19	19	19	20	20
250	18	24	27	28	29	30	31	32	32
500	25	34	38	40	42	43	43	45	46

The above values of d were computed by using the normal approximation as given in equation (5).

Table 3. PCS (top line), P(NCS) (middle line) and E(S) (bottom line)

Slippage Configuration: p = 0.50 delta = 0.10

	k= 3				5				10				15			
n	d=2	3	4	5	2	3	4	5	2	3	4	5	2	3	4	5
5	0.95 0.87 2.69	0.99 0.97 2.93	1.00 1.00 2.99	1.00 1.00 3.00	0.92 0.82 4.19	0.99 0.95 4.80	1.00 0.99 4.98	1.00 1.00 5.00	0.87 0.74 7.56	0.97 0.93 9.33	1.00 0.99 9.92	1.00 1.00 10.00	0.83 0.70 10.60	0.96 0.91 13.69	1.00 0.99 14.83	1.00 1.00 15.00
10	0.90 0.69 2.29	0.96 0.84 2.64	0.99 0.93 2.85	1.00 0.98 2.95	0.84 0.62 3.30	0.93 0.79 4.10	0.98 0.91 4.61	0.99 0.97 4.87	0.75 0.51 5.33	0.89 0.71 7.32	0.96 0.87 8.77	0.99 0.95 9.56	0.69 0.45 6.98	0.86 0.67 10.17	0.95 0.84 12.68	0.99 0.94 14.14
15	0.88 0.58 2.04	0.94 0.73 2.39	0.98 0.84 2.66	0.99 0.92 2.83	0.81 0.50 2.81	0.90 0.66 3.56	0.96 0.80 4.16	0.98 0.90 4.57	0.70 0.40 4.26	0.84 0.57 5.97	0.92 0.73 7.51	0.97 0.86 8.66	0.64 0.34 5.37	0.79 0.51 7.99	0.90 0.69 10.51	0.96 0.83 12.51
20	0.87 0.50 1.88	0.93 0.64 2.20	0.97 0.75 2.47	0.98 0.85 2.68	0.80 0.43 2.50	0.88 0.57 3.16	0.94 0.70 3.76	0.97 0.81 4.23	0.69 0.33 3.63	0.81 0.47 5.07	0.89 0.62 6.51	0.95 0.76 7.76	0.62 0.27 4.47	0.76 0.42 6.62	0.86 0.57 8.88	0.93 0.72 10.96
25	0.87 0.44 1.75	0.92 0.56 2.05	0.96 0.68 2.32	0.98 0.78 2.54	0.79 0.37 2.28	0.87 0.50 2.87	0.93 0.62 3.43	0.96 0.74 3.92	0.68 0.28 3.21	0.79 0.41 4.45	0.88 0.54 5.74	0.93 0.67 6.96	0.62 0.23 3.88	0.74 0.35 5.69	0.84 0.49 7.68	0.91 0.62 9.66
30	0.87 0.40 1.66	0.92 0.50 1.93	0.95 0.61 2.18	0.98 0.71 2.40	0.79 0.33 2.12	0.87 0.44 2.64	0.92 0.56 3.16	0.96 0.67 3.63	0.68 0.25 2.91	0.79 0.36 3.98	0.86 0.48 5.14	0.92 0.60 6.30	0.62 0.20 3.47	0.73 0.31 5.01	0.83 0.42 6.77	0.90 0.55 8.60
35	0.87 0.36 1.59	0.92 0.46 1.83	0.95 0.56 2.07	0.97 0.65 2.28	0.80 0.30 1.99	0.87 0.40 2.45	0.92 0.50 2.93	0.95 0.61 3.39	0.69 0.22 2.68	0.78 0.32 3.62	0.86 0.42 4.66	0.91 0.54 5.73	0.62 0.18 3.16	0.73 0.27 4.50	0.82 0.37 6.06	0.88 0.49 7.73
40	0.88 0.32 1.52	0.92 0.41 1.75	0.95 0.51 1.97	0.97 0.60 2.17	0.80 0.27 1.88	0.87 0.36 2.30	0.91 0.46 2.74	0.95 0.56 3.17	0.70 0.20 2.49	0.78 0.28 3.33	0.85 0.38 4.27	0.91 0.48 5.26	0.63 0.16 2.92	0.73 0.24 4.10	0.81 0.33 5.49	0.88 0.44 7.01
45	0.88 0.30 1.47	0.92 0.38 1.68	0.95 0.46 1.88	0.97 0.55 2.08	0.81 0.25 1.79	0.87 0.33 2.18	0.91 0.42 2.58	0.95 0.51 2.98	0.71 0.18 2.34	0.79 0.26 3.09	0.85 0.34 3.94	0.90 0.44 4.85	0.64 0.15 2.72	0.73 0.22 3.77	0.81 0.30 5.02	0.87 0.39 6.40
50	0.89 0.27 1.43	0.92 0.35 1.61	0.95 0.43 1.80	0.97 0.51 1.99	0.82 0.23 1.72	0.87 0.30 2.07	0.91 0.38 2.44	0.94 0.47 2.81	0.72 0.17 2.22	0.79 0.23 2.89	0.85 0.31 3.67	0.90 0.40 4.51	0.65 0.14 2.56	0.74 0.20 3.50	0.81 0.27 4.63	0.87 0.36 5.80

Table 3 (Continued). PCS (top line), P(NCS) (middle line) and E(S) (bottom line)

Slippage Configuration: p = 0.50 delta = 0.10

	k= 3				5				10				15			
	d=2	3	4	5	2	3	4	5	2	3	4	5	2	3	4	5
n																
	0.90	0.93	0.95	0.97	0.83	0.88	0.92	0.94	0.73	0.80	0.86	0.90	0.67	0.75	0.82	0.87
60	0.23	0.29	0.36	0.44	0.19	0.25	0.32	0.40	0.14	0.20	0.26	0.34	0.12	0.17	0.23	0.30
	1.36	1.51	1.68	1.84	1.60	1.89	2.21	2.54	2.01	2.57	3.22	3.93	2.30	3.07	3.99	5.05
	0.91	0.93	0.95	0.97	0.85	0.89	0.92	0.95	0.76	0.81	0.86	0.90	0.70	0.76	0.82	0.87
70	0.20	0.25	0.31	0.38	0.17	0.22	0.28	0.34	0.12	0.17	0.22	0.29	0.10	0.14	0.19	0.25
	1.30	1.44	1.58	1.72	1.51	1.76	2.02	2.31	1.86	2.33	2.88	3.49	2.11	2.75	3.53	4.43
	0.92	0.94	0.96	0.97	0.86	0.90	0.93	0.95	0.78	0.83	0.87	0.91	0.72	0.78	0.83	0.88
80	0.17	0.22	0.27	0.32	0.14	0.19	0.24	0.29	0.11	0.15	0.19	0.25	0.09	0.12	0.17	0.22
	1.26	1.37	1.49	0.62	1.44	1.65	1.88	2.13	1.74	2.14	2.61	3.13	1.95	2.50	3.16	3.93
	0.93	0.95	0.96	0.97	0.88	0.91	0.93	0.95	0.79	0.84	0.88	0.91	0.74	0.80	0.85	0.88
90	0.15	0.19	0.23	0.28	0.12	0.16	0.21	0.26	0.09	0.13	0.17	0.21	0.08	0.11	0.14	0.19
	1.22	1.32	1.42	1.54	1.38	1.56	1.76	1.97	1.64	1.99	2.39	2.84	1.83	2.30	2.87	3.53
	0.93	0.95	0.96	0.98	0.89	0.92	0.94	0.96	0.81	0.86	0.89	0.92	0.76	0.81	0.86	0.89
100	0.13	0.16	0.20	0.25	0.11	0.14	0.18	0.22	0.08	0.11	0.15	0.19	0.07	0.09	0.13	0.16
	1.19	1.28	1.37	1.47	1.33	1.48	1.65	1.85	1.56	1.86	2.21	2.60	1.73	2.14	2.63	3.20
	0.99	0.99	0.99	0.99	0.98	0.98	0.99	0.99	0.96	0.97	0.97	0.98	0.94	0.95	0.96	0.97
250	0.02	0.02	0.03	0.04	0.02	0.02	0.03	0.04	0.02	0.02	0.03	0.03	0.01	0.02	0.02	0.03
	1.03	1.04	1.05	1.07	1.05	1.07	1.10	1.13	1.10	1.14	1.20	1.26	1.13	1.20	1.28	1.37
	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
500	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	1.00	1.00	1.00	1.00	1.00	1.00	1.01	1.01	1.01	1.01	1.01	1.02	1.01	1.01	1.02	1.02

For values of $n \geq 60$, the values in the above table were computed by using the normal approximations given in (8).

Table 4. PCS (top line), P(NCS) (middle line) and E(S) (bottom line)

Slippage Configuration: p = 0.75 delta = 0.05

	k=	3			5				10				15			
	d=2	3	4	5	2	3	4	5	2	3	4	5	2	3	4	5
n																
5	0.97 0.94 2.86	1.00 0.99 2.98	1.00 1.00 3.00	1.00 1.00 3.00	0.96 0.92 4.65	1.00 0.99 4.95	1.00 1.00 5.00	1.00 1.00 5.00	0.95 0.90 9.08	0.99 0.99 9.86	1.00 1.00 9.99	1.00 1.00 10.00	0.94 0.90 13.52	0.99 0.98 14.78	1.00 1.00 14.99	1.00 1.00 15.00
10	0.91 0.81 2.54	0.97 0.93 2.83	0.99 0.98 2.95	1.00 1.00 2.99	0.87 0.75 3.89	0.96 0.90 4.56	0.99 0.97 4.87	1.00 0.99 4.97	0.80 0.68 6.89	0.93 0.86 8.69	0.98 0.96 9.59	1.00 0.99 9.90	0.77 0.63 9.63	0.92 0.84 12.65	0.98 0.95 14.23	1.00 0.99 14.82
15	0.87 0.72 2.31	0.95 0.85 2.65	0.98 0.93 2.85	0.99 0.98 2.95	0.81 0.64 3.37	0.92 0.80 4.13	0.97 0.91 4.61	0.99 0.97 4.85	0.72 0.54 5.57	0.87 0.73 7.47	0.95 0.87 8.79	0.98 0.95 9.52	0.67 0.48 7.45	0.84 0.69 10.52	0.93 0.85 12.79	0.98 0.94 14.10
20	0.85 0.65 2.15	0.93 0.78 2.49	0.97 0.88 2.73	0.99 0.94 2.88	0.77 0.56 3.02	0.88 0.72 3.77	0.95 0.84 4.32	0.98 0.92 4.68	0.67 0.45 4.75	0.82 0.63 6.52	0.91 0.79 7.98	0.96 0.89 8.98	0.61 0.40 6.16	0.77 0.58 8.93	0.89 0.75 11.36	0.95 0.87 13.10
25	0.83 0.59 2.02	0.91 0.73 2.36	0.96 0.83 2.62	0.98 0.91 2.79	0.75 0.51 2.77	0.86 0.66 3.48	0.93 0.78 4.06	0.97 0.88 4.47	0.63 0.40 4.19	0.78 0.56 5.80	0.88 0.71 7.26	0.94 0.83 8.39	0.57 0.34 5.31	0.73 0.50 7.77	0.85 0.66 10.14	0.93 0.80 12.07
30	0.82 0.55 1.92	0.90 0.68 2.25	0.95 0.78 2.51	0.97 0.87 2.71	0.73 0.46 2.58	0.84 0.60 3.25	0.91 0.73 3.82	0.95 0.83 4.27	0.61 0.35 3.79	0.75 0.50 5.25	0.85 0.64 6.65	0.92 0.77 7.84	0.55 0.30 4.72	0.70 0.44 6.91	0.82 0.59 9.13	0.90 0.73 11.11
35	0.82 0.51 1.85	0.89 0.63 2.16	0.94 0.74 2.42	0.97 0.83 2.63	0.72 0.43 2.43	0.82 0.56 3.05	0.90 0.68 3.62	0.94 0.78 4.08	0.60 0.32 3.49	0.73 0.45 4.82	0.83 0.59 6.15	0.90 0.71 7.34	0.53 0.27 4.28	0.67 0.40 6.24	0.79 0.54 8.31	0.88 0.67 10.26
40	0.81 0.48 1.78	0.88 0.60 2.08	0.93 0.70 2.33	0.96 0.79 2.55	0.72 0.40 2.31	0.81 0.52 2.89	0.89 0.64 3.44	0.93 0.74 3.91	0.59 0.30 3.25	0.71 0.42 4.47	0.81 0.54 5.72	0.89 0.67 6.89	0.52 0.24 3.94	0.66 0.36 5.71	0.77 0.49 7.63	0.86 0.62 9.51
45	0.81 0.46 1.73	0.88 0.56 2.01	0.92 0.67 2.26	0.96 0.76 2.47	0.72 0.37 2.21	0.81 0.49 2.76	0.88 0.60 3.28	0.93 0.70 3.74	0.59 0.27 3.06	0.70 0.39 4.18	0.80 0.51 5.35	0.88 0.62 6.49	0.52 0.23 3.67	0.64 0.33 5.28	0.75 0.45 7.07	0.84 0.57 8.87
50	0.81 0.43 1.68	0.87 0.54 1.94	0.92 0.64 2.19	0.95 0.73 2.40	0.71 0.35 2.13	0.80 0.46 2.64	0.87 0.57 3.14	0.92 0.67 3.60	0.58 0.26 2.90	0.70 0.36 3.93	0.79 0.47 5.04	0.86 0.59 6.13	0.51 0.21 3.45	0.63 0.31 4.93	0.74 0.42 6.59	0.83 0.53 8.30

Table 4 (Continued). PCS (top line), P(NCS) (middle line) and E(S) (bottom line)

Slippage Configuration: p = 0.75 delta = 0.05

	k= 3				5				10				15			
	d=2	3	4	5	2	3	4	5	2	3	4	5	2	3	4	5
n																
	0.81	0.87	0.91	0.94	0.70	0.78	0.85	0.90	0.56	0.66	0.75	0.83	0.48	0.59	0.69	0.78
60	0.39	0.48	0.58	0.66	0.31	0.41	0.50	0.60	0.22	0.30	0.40	0.50	0.18	0.25	0.34	0.44
	1.59	1.83	2.06	2.27	1.96	2.41	2.86	3.30	2.55	3.41	4.35	5.34	2.94	4.11	5.48	6.97
	0.81	0.87	0.91	0.94	0.71	0.78	0.85	0.89	0.57	0.66	0.75	0.82	0.49	0.59	0.68	0.77
70	0.36	0.44	0.53	0.61	0.29	0.37	0.46	0.55	0.20	0.28	0.36	0.45	0.16	0.23	0.31	0.39
	1.53	1.75	1.96	2.16	1.86	2.26	2.68	3.08	2.38	3.14	3.99	4.88	2.72	3.75	4.96	6.30
	0.82	0.87	0.91	0.94	0.71	0.78	0.84	0.89	0.57	0.66	0.74	0.81	0.49	0.59	0.68	0.76
80	0.33	0.41	0.49	0.57	0.27	0.34	0.42	0.50	0.19	0.25	0.33	0.41	0.15	0.21	0.28	0.36
	1.48	1.68	1.88	2.07	1.78	2.15	2.52	2.90	2.25	2.93	3.69	4.51	2.55	3.47	4.55	5.75
	0.82	0.87	0.90	0.93	0.72	0.79	0.84	0.89	0.58	0.67	0.74	0.81	0.50	0.59	0.68	0.75
90	0.31	0.38	0.45	0.53	0.25	0.31	0.39	0.47	0.17	0.23	0.30	0.38	0.14	0.19	0.25	0.33
	1.44	1.63	1.81	1.99	1.71	2.05	2.40	2.75	2.14	2.75	3.45	4.20	2.41	3.24	4.21	5.30
	0.83	0.37	0.91	0.93	0.73	0.79	0.84	0.88	0.59	0.67	0.74	0.80	0.51	0.60	0.68	0.75
100	0.29	0.35	0.42	0.49	0.23	0.29	0.36	0.43	0.16	0.22	0.28	0.35	0.13	0.18	0.23	0.30
	1.41	1.58	1.75	1.92	1.66	1.96	2.28	2.62	2.05	2.61	3.24	3.93	2.30	3.05	3.93	4.93
	0.90	0.92	0.94	0.95	0.84	0.87	0.89	0.91	0.74	0.78	0.82	0.85	0.68	0.72	0.77	0.81
250	0.13	0.15	0.18	0.21	0.11	0.13	0.15	0.18	0.08	0.10	0.12	0.15	0.06	0.08	0.10	0.12
	1.16	1.22	1.29	1.37	1.26	1.38	1.51	1.65	1.44	1.65	1.90	2.16	1.56	1.84	2.17	2.55
	0.97	0.97	0.98	0.98	0.94	0.95	0.96	0.96	0.89	0.91	0.92	0.93	0.85	0.87	0.89	0.91
500	0.04	0.05	0.06	0.07	0.04	0.04	0.05	0.06	0.03	0.04	0.04	0.05	0.02	0.03	0.04	0.04
	1.05	1.07	1.09	1.11	1.08	1.12	1.16	1.20	1.15	1.22	1.30	1.39	1.20	1.30	1.41	1.53

For values of $n \geqq 60$, the values in the above table were computed by using the normal approximations given in (8).

Table 5. PCS (top line), P(NCS) (middle line) and E(S) (bottom line)

Slippage Configuration: p = 0.90 delta = 0.03

n	k = 3				5				10				15			
	a=2	3	4	5	2	3	4	5	2	3	4	5	2	3	4	5
5	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	0.99	1.00	1.00	1.00	0.99	1.00	1.00	1.00	0.99	1.00	1.00	1.00	0.99	1.00	1.00	1.00
	2.98	3.00	3.00	3.00	4.96	5.00	5.00	5.00	9.92	10.00	10.00	10.00	14.88	14.99	15.00	15.00
10	0.98	1.00	1.00	1.00	0.98	1.00	1.00	1.00	0.97	1.00	1.00	1.00	0.97	1.00	1.00	1.00
	0.95	0.99	1.00	1.00	0.94	0.99	1.00	1.00	0.93	0.99	1.00	1.00	0.93	0.99	1.00	1.00
	2.88	2.98	3.00	3.00	4.73	4.95	4.99	5.00	9.35	9.86	9.99	10.00	13.99	14.82	14.98	15.00
15	0.96	0.99	1.00	1.00	0.94	0.99	1.00	1.00	0.93	0.98	1.00	1.00	0.92	0.98	1.00	1.00
	0.89	0.97	0.99	1.00	0.86	0.96	0.99	1.00	0.83	0.95	0.99	1.00	0.82	0.95	0.99	1.00
	2.74	2.93	2.98	3.00	4.38	4.82	4.96	4.99	8.39	9.52	9.89	9.98	12.40	14.23	14.82	14.97
20	0.94	0.98	1.00	1.00	0.91	0.98	0.99	1.00	0.87	0.96	0.99	1.00	0.86	0.96	0.99	1.00
	0.83	0.94	0.98	1.00	0.78	0.92	0.97	0.99	0.73	0.89	0.97	0.99	0.70	0.88	0.96	0.99
	2.60	2.86	2.96	2.99	4.05	4.64	4.89	4.97	7.44	8.99	9.68	9.92	10.71	13.28	14.45	14.86
25	0.92	0.98	0.99	1.00	0.88	0.96	0.99	1.00	0.83	0.94	0.98	1.00	0.80	0.93	0.98	1.00
	0.77	0.90	0.96	0.99	0.72	0.87	0.95	0.98	0.65	0.83	0.93	0.98	0.61	0.81	0.92	0.97
	2.47	2.77	2.92	2.98	3.76	4.44	4.79	4.93	6.66	8.41	9.38	9.80	9.35	12.24	13.90	14.64
30	0.91	0.97	0.99	1.00	0.86	0.95	0.98	1.00	0.80	0.92	0.97	0.99	0.76	0.90	0.97	0.99
	0.73	0.86	0.94	0.98	0.66	0.82	0.92	0.97	0.58	0.77	0.89	0.96	0.54	0.74	0.88	0.95
	2.36	2.69	2.87	2.95	3.52	4.24	4.67	4.88	6.04	7.86	9.02	9.62	8.30	11.28	13.26	14.31
35	0.90	0.96	0.99	1.00	0.85	0.93	0.98	0.99	0.77	0.90	0.96	0.99	0.73	0.88	0.95	0.98
	0.68	0.82	0.91	0.96	0.62	0.78	0.89	0.95	0.53	0.72	0.85	0.94	0.48	0.68	0.83	0.92
	2.27	2.61	2.82	2.92	3.31	4.06	4.54	4.80	5.53	7.36	8.65	9.40	7.47	10.42	12.60	13.92
40	0.89	0.95	0.98	0.99	0.83	0.92	0.97	0.99	0.75	0.88	0.95	0.98	0.70	0.85	0.94	0.98
	0.65	0.79	0.89	0.95	0.58	0.74	0.86	0.93	0.49	0.67	0.81	0.91	0.44	0.63	0.79	0.89
	2.19	2.53	2.76	2.89	3.14	3.88	4.40	4.72	5.12	6.90	8.28	9.16	6.81	9.66	11.95	13.47
45	0.88	0.95	0.98	0.99	0.82	0.91	0.96	0.99	0.73	0.86	0.94	0.98	0.68	0.83	0.92	0.97
	0.62	0.76	0.86	0.93	0.54	0.70	0.83	0.91	0.45	0.63	0.77	0.88	0.40	0.58	0.74	0.86
	2.11	2.46	2.70	2.85	2.99	3.72	4.27	4.63	4.77	6.50	7.91	8.89	6.27	8.99	11.32	13.00
50	0.88	0.94	0.97	0.99	0.81	0.90	0.96	0.98	0.72	0.85	0.93	0.97	0.67	0.81	0.91	0.96
	0.59	0.73	0.83	0.91	0.51	0.67	0.80	0.89	0.42	0.59	0.74	0.85	0.37	0.54	0.70	0.83
	2.05	2.39	2.64	2.81	2.86	3.58	4.14	4.53	4.48	6.14	7.57	8.62	5.82	8.41	10.74	12.53

Table 5. (Continued). PCS (top line), P(NCS) (middle line) and E(S) (bottom line)

Slippage Configuration: p = 0.90 delta = 0.03

	k= 3				5				10				15			
	d=2	3	4	5	2	3	4	5	2	3	4	5	2	3	4	5
n																
	0.86	0.93	0.96	0.98	0.77	0.87	0.93	0.97	0.64	0.78	0.88	0.94	0.56	0.72	0.84	0.92
60	0.53	0.66	0.77	0.86	0.44	0.58	0.71	0.82	0.33	0.47	0.62	0.75	0.27	0.41	0.56	0.70
	1.91	2.24	2.51	2.71	2.54	3.20	3.79	4.25	3.61	5.04	6.46	7.69	4.37	6.47	8.69	10.74
	0.86	0.92	0.96	0.98	0.77	0.86	0.93	0.96	0.64	0.77	0.86	0.93	0.56	0.70	0.82	0.90
70	0.49	0.61	0.73	0.82	0.41	0.54	0.66	0.77	0.30	0.43	0.57	0.69	0.25	0.37	0.51	0.64
	1.83	2.14	2.41	2.62	2.39	3.01	3.58	4.06	3.34	4.63	5.96	7.18	3.99	5.86	7.91	0.90
	0.86	0.92	0.95	0.98	0.77	0.86	0.92	0.96	0.63	0.76	0.85	0.92	0.55	0.69	0.81	0.89
80	0.45	0.57	0.68	0.78	0.38	0.50	0.62	0.73	0.28	0.39	0.52	0.65	0.22	0.34	0.46	0.59
	1.76	2.06	2.32	2.53	2.27	2.84	3.39	3.87	3.12	4.30	5.54	6.73	3.70	5.38	7.26	9.16
	0.86	0.91	0.95	0.97	0.77	0.85	0.91	0.95	0.63	0.75	0.84	0.91	0.55	0.68	0.79	0.88
90	0.42	0.53	0.64	0.74	0.35	0.46	0.58	0.69	0.26	0.36	0.48	0.60	0.21	0.31	0.42	0.55
	1.70	1.98	2.23	2.45	2.17	2.70	3.22	3.70	2.94	4.02	5.18	6.32	3.46	4.99	6.73	8.52
	0.86	0.91	0.95	0.97	0.77	0.85	0.91	0.95	0.64	0.75	0.84	0.90	0.56	0.68	0.79	0.87
100	0.40	0.50	0.61	0.70	0.33	0.43	0.54	0.65	0.24	0.34	0.45	0.56	0.19	0.28	0.39	0.51
	1.65	1.91	2.16	2.38	2.08	2.58	3.08	3.54	2.79	3.78	4.87	5.96	3.27	4.66	6.27	7.97
	0.90	0.93	0.95	0.97	0.84	0.88	0.91	0.94	0.73	0.79	0.84	0.89	0.66	0.73	0.79	0.85
250	0.19	0.24	0.29	0.35	0.16	0.21	0.26	0.32	0.12	0.16	0.21	0.26	0.10	0.13	0.18	0.23
	1.28	1.40	1.53	1.67	1.47	1.70	1.94	2.20	1.80	2.22	2.70	3.24	2.02	2.59	3.26	4.04
	0.96	0.97	0.97	0.98	0.92	0.94	0.95	0.97	0.86	0.89	0.91	0.93	0.81	0.85	0.88	0.91
500	0.07	0.09	0.11	0.13	0.06	0.08	0.10	0.12	0.05	0.07	0.08	0.10	0.04	0.06	0.07	0.09
	1.10	1.15	1.19	1.25	1.18	1.26	1.35	1.45	1.32	1.48	1.66	1.87	1.43	1.65	1.90	2.20

For values of $n \geq 60$, the values in the above table were computed by using the normal approximations given in (8).

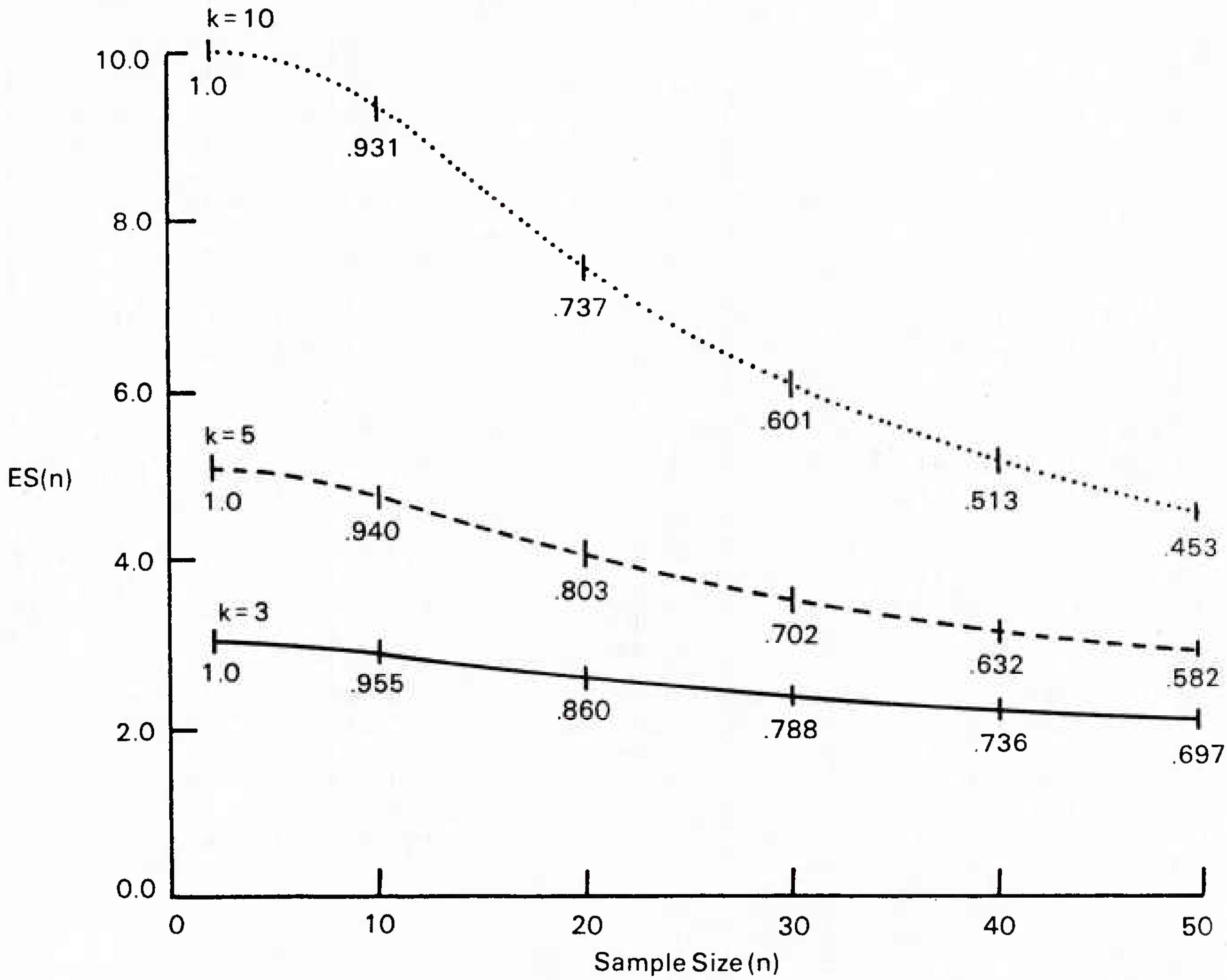

Figure 1. Expected size of selected subset for p = .90, δ = .03, d = 2, and k = 3,5,10. Inserted numbers are probability of a correct selection with δ = 0 and p = .90.

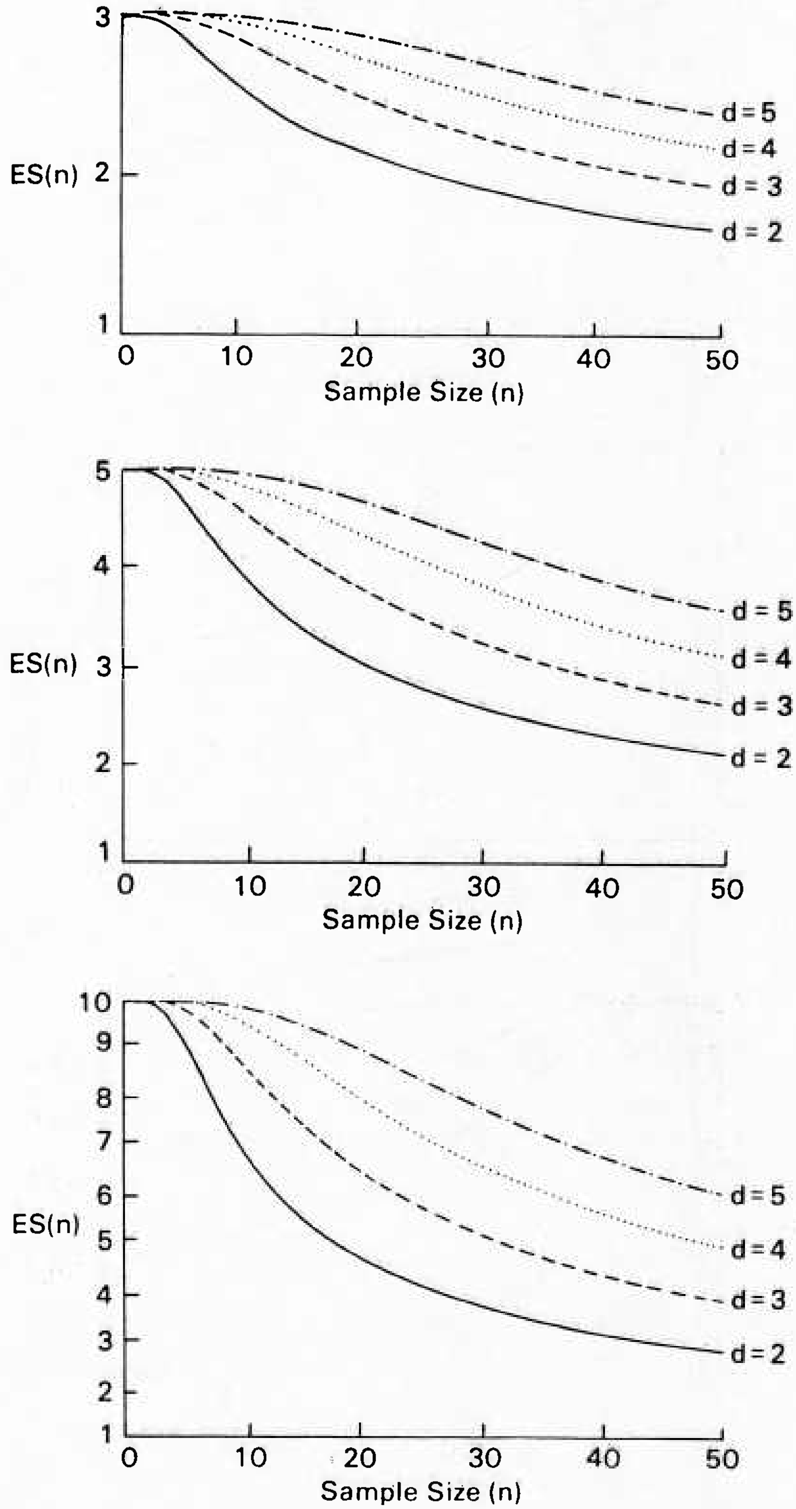

Figure 2. Expected size of selected subset for p = .75, δ = .05 and k = 3 (top), k = 5 (middle) and k = 10 (bottom).

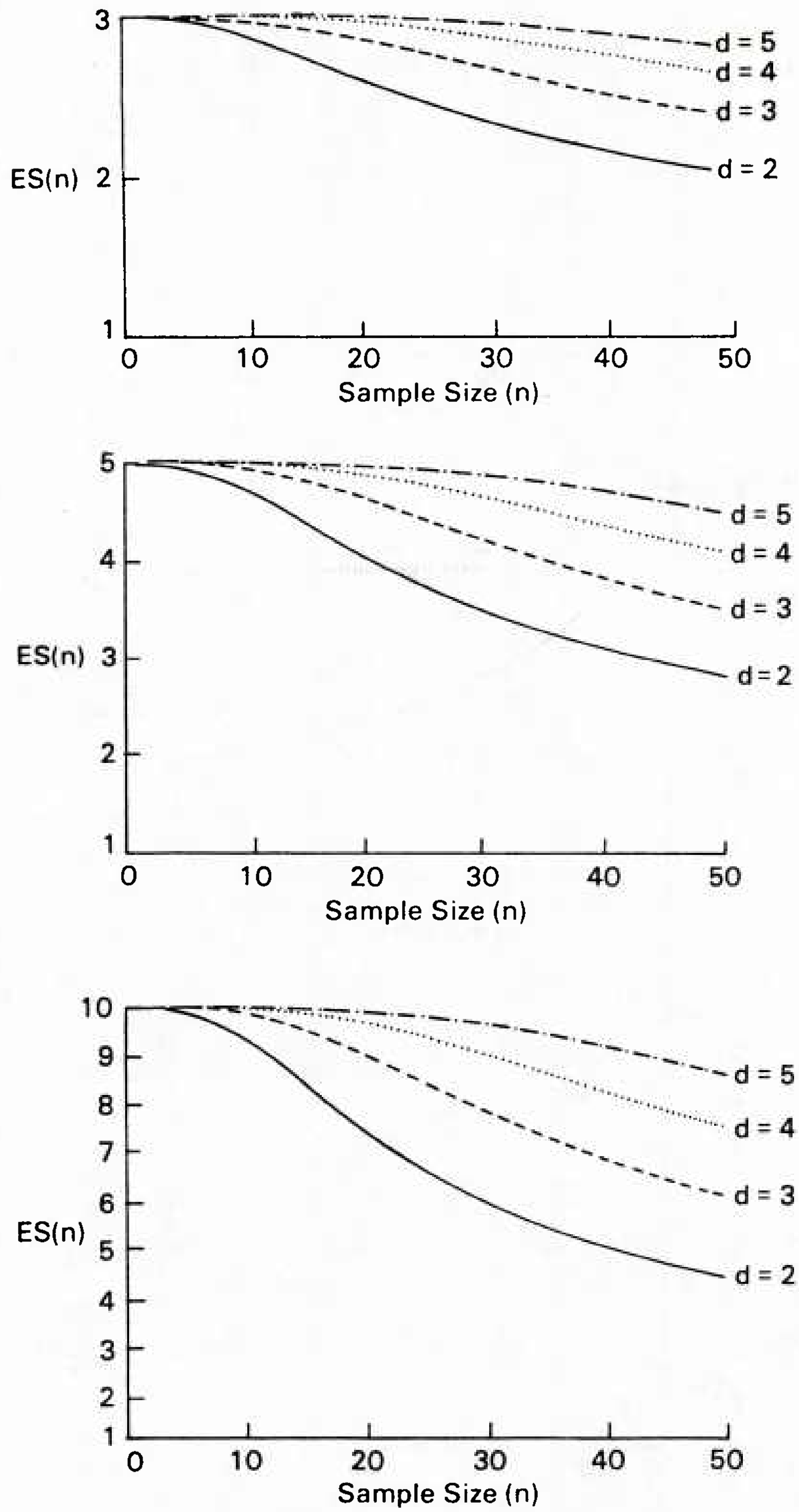

Figure 3. Expected size of selected subset for p = .90, δ = .03 and k = 3 (top), k = 5 (middle, and k = 10 (bottom).