

AD-A150 067 ON PAGE

1a. REPORT SECURITY CLASSIFICATION UNCLASSIFIED		1b. RESTRICTIVE MARKINGS													
2a. SECURITY CLASSIFICATION AUTHORITY		3. DISTRIBUTION/AVAILABILITY OF REPORT Approved for public release; Distribution unlimited													
2b. DECLASSIFICATION/DOWNGRADING SCHEDULE		5. MONITORING ORGANIZATION REPORT NUMBER(S) AFOSR-TR- 84-1140													
4. PERFORMING ORGANIZATION REPORT NUMBER(S)	7a. NAME OF MONITORING ORGANIZATION AFOSR/NP														
6a. NAME OF PERFORMING ORGANIZATION University of Miami	6b. OFFICE SYMBOL (If applicable)	7b. ADDRESS (City, State and ZIP Code) Building 410 Bolling AFB DC 20332													
6c. ADDRESS (City, State and ZIP Code) Department of Physics Coral Gables, FL 33124	9. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER AFOSR-83-0338														
8a. NAME OF FUNDING/SPONSORING ORGANIZATION AFOSR	8b. OFFICE SYMBOL (If applicable) NP	10. SOURCE OF FUNDING NOS. <table border="1"><tr><td>PROGRAM ELEMENT NO. 61102F</td><td>PROJECT NO. 2301</td><td>TASK NO. D9</td><td>WORK UNIT NO.</td></tr></table>		PROGRAM ELEMENT NO. 61102F	PROJECT NO. 2301	TASK NO. D9	WORK UNIT NO.								
PROGRAM ELEMENT NO. 61102F	PROJECT NO. 2301	TASK NO. D9	WORK UNIT NO.												
8c. ADDRESS (City, State and ZIP Code) Building 410 Bolling AB DC 20332		11. TITLE (Include Security Classification) "ELECTROMAGNETIC RAIL LAUNCHER AS A CIRCUIT ELEMENT"													
12. PERSONAL AUTHOR(S) Dr. Manuel A. Huerta															
13a. TYPE OF REPORT FINAL	13b. TIME COVERED FROM 1 Sep 83 TO 31 Aug 84	14. DATE OF REPORT (Yr., Mo., Day) 25 Oct 84	15. PAGE COUNT 23												
16. SUPPLEMENTARY NOTATION															
17. COSATI CODES <table border="1"><tr><th>FIELD</th><th>GROUP</th><th>SUB GR</th></tr><tr><td> </td><td> </td><td> </td></tr><tr><td> </td><td> </td><td> </td></tr><tr><td> </td><td> </td><td> </td></tr></table>		FIELD	GROUP	SUB GR										18. SUBJECT TERMS (Continue on reverse if necessary and identify by block number)	
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20. DISTRIBUTION/AVAILABILITY OF ABSTRACT UNCLASSIFIED/UNLIMITED ☒ SAME AS RPT. ☐ DTIC USERS ☐		21. ABSTRACT SECURITY CLASSIFICATION UNCLASSIFIED													
22a. NAME OF RESPONSIBLE INDIVIDUAL Dr Robert J. Barker	22b. TELEPHONE NUMBER (Include Area Code) 202/767-5011	22c. OFFICE SYMBOL NP													

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Dr. Robert J. Barker
AFOSR
Attention: Physics
Bolling AFB
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October 25, 1984

Reference: Grant AFOSR 83-0338 , Final Scientific Report
'The Electromagnetic Rail Launcher
as a Circuit Element'
P. I. Dr. Manuel A. Huerta
University of Miami, Department of Physics.

Attached please find the Final Report for the referenced Grant. The grant had a budget of \$11,998 and supported about 1.5 months of work. The effort was concentrated on numerical and analytical solutions of the circuit equations for an inductor driven rail launcher. A simple analytical expression that agrees with the full numerical solution to a few percent was found.

I would like to express my appreciation for the support I received under this grant.

Sincerely

Manuel A. Huerta

Manuel A. Huerta
Professor of Physics

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FINAL SCIENTIFIC REPORT

AIR FORCE OFFICE OF SCIENTIFIC RESEARCH

GRANT NUMBER AF AFOSR 83-0338

'The Electromagnetic Launcher as a Circuit Element'

Principal Investigator: Dr. Manuel A. Huerta
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ABSTRACT

The circuit equations for the inductor driven rail launcher are integrated numerically using a fourth order Runge-Kutta method. Their solution is also obtained as a perturbation expansion for small resistance. The EMACK parameters are used to study the results. For high current shots the full numerical solution agrees with the first order approximation to 3%. For low current shots they agree to 5%. A very simple zeroth order approximation agrees with the full numerical solution to 5% in the high current case and to 3% in the low current case.

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INTRODUCTION

To avoid repetition the initial proposal for this grant is attached as an Appendix to this Final Report. The work in the grant was to study the circuit equations for the inductor driven rail launcher. The basic equations are Eq. (15) of the proposal, which comes from the circuit equation after converting from time to distance as the independent variable,

$$(L_0 + L'x)v \frac{dv}{dx} + \frac{L'}{2} v^2 + 2(R_0 + R_p) v + 2R' \int x dv = b' ,$$

and the force equation

$$m v \frac{dv}{dx} = L' i^2 / 2 . \quad (1)$$

This is an integro-differential equation for the speed v as a function of the distance x . It is in a form suitable for a regular perturbation expansion for small resistance.

We introduce dimensionless variables in terms of the characteristic parameters of the problem as follows. Let s be a dimensionless length defined as

$$x = gs , \quad (2)$$

where

$$g = \frac{L_0}{L'} . \quad (3)$$

We introduce a velocity scale v_f by

$$mv_f^2 = L_0 i_0^2 , \quad (4)$$

where i_0 is the initial current in the inductor. We define a dimensionless speed u by

$$v = v_f u ,$$

and a dimensionless current I by

$$I = \frac{i}{i_0} . \quad (5)$$

The inductance and resistance have a time constant given by

$$t_1 = \frac{L_0}{R_0 + R_p} , \quad (6)$$

in terms of which we define an expansion parameter ε

$$\varepsilon = \frac{2g}{v_f t_1} . \quad (7)$$

Let k be the resistance ratio

$$k = \frac{R'_g}{R_0 + R_p} , \quad (8)$$

and q be the dimensionless initial speed

$$q = qv_0/v_f , \quad (9)$$

where v_0 is the initial speed of the projectile.

In terms of the above variables, Eq. (1) is rewritten as

$$(1 + s) u \frac{du}{ds} + \frac{u^2}{2} + \varepsilon u + \varepsilon k \int_q^u s \, du = b ,$$

and

$$\frac{d}{ds}(u^2) = I^2 , \quad (10)$$

where b is given by

$$b = \frac{(1 + q^2)}{2} + \varepsilon q . \quad (11)$$

We integrate the full Eq. (1) using a fourth order Runge-Kutta method and take the result as the standard for comparison with the approximation scheme

developed below. We expand the solution $u(s)$ in an asymptotic series

$$u = u_0(s) + \varepsilon u_1(s) + \varepsilon^2 u_2(s) + \dots, \quad (12)$$

and substitute the series into Eq. (10). The lowest order equation that results is

$$(1 + s) \frac{d}{ds}(u_0^2) + u_0^2 = 1 + q^2. \quad (13)$$

This equation is integrated easily to yield

$$u_0^2 = 1 + q^2 - \frac{1}{1 + s},$$

and

$$I = \frac{1}{1 + s}. \quad (14)$$

The lowest order approximation u_0 amounts to neglecting the resistance completely. Then the kinetic energy gained by the projectile is given by the drop in the magnetic energy stored in the system inductance $L_0 + L'x$. Despite its simplicity, the speed of Eq. (14) agrees rather well with the Runge-Kutta solution. It overestimates the speed, but the error is not much greater than other errors that are inherent in determining what are the correct parameter values to use to simulate the experimental situation.

The error is reduced if we calculate the first order correction u_1 from the equation

$$\frac{d}{ds} [(1 + s)u_0 u_1] = -u_0 + q - k \int_q^{u_0} s \, du_0. \quad (15)$$

After some straightforward integrations we obtain

$$u_1 = \frac{(k-1)(u_0 - q)}{u_0} + \frac{[k + 2(1+q^2)(1-k-k(1+s))]}{2(1+q^2)^{3/2} u_0 (s+1)} \ln \left[\frac{(r+u_0)(r-q)}{(r-u_0)(r+q)} \right] + \frac{k[u_0(1+s) - q]}{2(1+q^2)^{3/2} u_0 (1+s)}.$$

with the first order speed given by

$$v = v_f (u + u_1) \quad (16)$$

Conclusion

Below we present the numerical results of the full Runge-Kutta integration, spd R-K, compared with the first approximation $\text{spd first} = v_f u_0$, and the second approximation $\text{spd second} = v_f (u_0 + \epsilon u_1)$. The results are presented in the tables that follow for the EMACK rail gun¹ shots 1-5. The parameters L_0 , L' , $R_0 + R_p$, R' , m , i_c , etc. are given above each table.

Table I contains the results for shot 1. It is the least accurate case because of the low current which leads to a relatively large value of ϵ . In this case spd first is 8% above spd R-K while spd second is 4% above spd R-K . For the other shots the current is larger and the errors are reduced. The value of spd second is off from spd R-K by only 3%. It is remarkable that spd first , which from u_0 and neglects the resistance entirely is off by only 5%.

References

1. Deis, D. W., and Scherbarth, D. W., 'Emack Electromagnetic Launcher Commissioning', IEEE Trans. Magnetics, Vol.Mag-20, 1984, p.245.

EMACK shot number 1

LO = 0.500E-05H,

RO+Rp = 0.271E-04cha,

mass = 0.195E+00Kg,

io = 0.500E+06A,

epsilon = .0369936

L1 = 0.585E-06H,

R1 = 0.450E-04cha,

barrel length = 5.00m

v0 = 0.000E+00m/s,

g = -8.55m

t1 = 0.185E+00s

vf = 2531.85m/s

time	position	spd R-V	spd second	spd first
0.00000E+00	0.00000E+00	0.00000E+00	0.00000E+00	0.00000E+00
0.50637E-02	0.53375E-02	0.63212E+02	0.63210E+02	0.63252E+02
0.10127E-02	0.21324E-01	0.12615E+03	0.12615E+03	0.12630E+03
0.15191E-02	0.47899E-01	0.18866E+03	0.18866E+03	0.18901E+03
0.20255E-02	0.84977E-01	0.25058E+03	0.25058E+03	0.25121E+03
0.25319E-02	0.13245E+00	0.31176E+03	0.31177E+03	0.31276E+03
0.30383E-02	0.19017E+00	0.37206E+03	0.37209E+03	0.37353E+03
0.35446E-02	0.25799E+00	0.43134E+03	0.43141E+03	0.43339E+03
0.40510E-02	0.33572E+00	0.48946E+03	0.48960E+03	0.49221E+03
0.45573E-02	0.42315E+00	0.54631E+03	0.54655E+03	0.54990E+03
0.50636E-02	0.52006E+00	0.60177E+03	0.60217E+03	0.60636E+03
0.55700E-02	0.62621E+00	0.65575E+03	0.65637E+03	0.66151E+03
0.60763E-02	0.74134E+00	0.70815E+03	0.70907E+03	0.71528E+03
0.65826E-02	0.86518E+00	0.75890E+03	0.76022E+03	0.76762E+03
0.70890E-02	0.99743E+00	0.80793E+03	0.80977E+03	0.81847E+03
0.75953E-02	0.11378E+01	0.85519E+03	0.85769E+03	0.86782E+03
0.81016E-02	0.12860E+01	0.90064E+03	0.90395E+03	0.91563E+03
0.86080E-02	0.14418E+01	0.94426E+03	0.94854E+03	0.96189E+03
0.91143E-02	0.16047E+01	0.98601E+03	0.99146E+03	0.10066E+04
0.96206E-02	0.17745E+01	0.10259E+04	0.10327E+04	0.10498E+04
0.10127E-01	0.19509E+01	0.10639E+04	0.10723E+04	0.10915E+04
0.10633E-01	0.21336E+01	0.11001E+04	0.11103E+04	0.11316E+04
0.11140E-01	0.23222E+01	0.11345E+04	0.11467E+04	0.11703E+04
0.11646E-01	0.25164E+01	0.11671E+04	0.11815E+04	0.12075E+04
0.12152E-01	0.27161E+01	0.11979E+04	0.12148E+04	0.12433E+04
0.12659E-01	0.29207E+01	0.12270E+04	0.12466E+04	0.12777E+04
0.13165E-01	0.31302E+01	0.12544E+04	0.12770E+04	0.13108E+04
0.13671E-01	0.33441E+01	0.12802E+04	0.13059E+04	0.13427E+04
0.14178E-01	0.35622E+01	0.13044E+04	0.13336E+04	0.13732E+04
0.14685E-01	0.37843E+01	0.13271E+04	0.13599E+04	0.14026E+04
0.15191E-01	0.40102E+01	0.13484E+04	0.13850E+04	0.14308E+04
0.15698E-01	0.42395E+01	0.13683E+04	0.14089E+04	0.14579E+04
0.16204E-01	0.44720E+01	0.13869E+04	0.14317E+04	0.14839E+04
0.16711E-01	0.47076E+01	0.14042E+04	0.14534E+04	0.15089E+04
0.17217E-01	0.49460E+01	0.14203E+04	0.14740E+04	0.15329E+04

TABLE 1

EMACK shot number 2

$L0 = 0.500E-05H$, $L1 = 0.585E-06H$, $g = 8.55m$
 $R0+Rp = 0.271E-04ohm$, $R1 = 0.450E-04ohm$, $t1 = 0.185E+00s$
 $mass = 0.225E+00Kg$, barrel length = 5.00m
 $I0 = 0.800E+06A$, $v0 = 0.000E+00m/s$, $vf = 3771.24m/s$
 $epsilon = .0245674$

time	position	spd R-I	spd second	spd first
0.00000E+00	0.00000E+00	0.00000E+00	0.00000E+00	0.00000E+00
0.33995E-03	0.53386E-02	0.94184E+02	0.94183E+02	0.94224E+02
0.67991E-03	0.21332E-01	0.18802E+03	0.18802E+03	0.18817E+03
0.10199E-02	0.47928E-01	0.28126E+03	0.28127E+03	0.28162E+03
0.13598E-02	0.85046E-01	0.37370E+03	0.37370E+03	0.37433E+03
0.16998E-02	0.13258E+00	0.46509E+03	0.46511E+03	0.46610E+03
0.20398E-02	0.19040E+00	0.55523E+03	0.55528E+03	0.55671E+03
0.23797E-02	0.25836E+00	0.64390E+03	0.64401E+03	0.64599E+03
0.27197E-02	0.33627E+00	0.73093E+03	0.73113E+03	0.73374E+03
0.30596E-02	0.42395E+00	0.81612E+03	0.81647E+03	0.81982E+03
0.33995E-02	0.52116E+00	0.89931E+03	0.89989E+03	0.90409E+03
0.37394E-02	0.62768E+00	0.98036E+03	0.98125E+03	0.98641E+03
0.40794E-02	0.74326E+00	0.10591E+04	0.10605E+04	0.10667E+04
0.44193E-02	0.86763E+00	0.11355E+04	0.11374E+04	0.11449E+04
0.47592E-02	0.10005E+01	0.12094E+04	0.12121E+04	0.12208E+04
0.50992E-02	0.11416E+01	0.12807E+04	0.12844E+04	0.12945E+04
0.54391E-02	0.12907E+01	0.13494E+04	0.13543E+04	0.13660E+04
0.57790E-02	0.14474E+01	0.14155E+04	0.14217E+04	0.14351E+04
0.61190E-02	0.16114E+01	0.14788E+04	0.14868E+04	0.15020E+04
0.64589E-02	0.17824E+01	0.15394E+04	0.15494E+04	0.15666E+04
0.67988E-02	0.19602E+01	0.15973E+04	0.16096E+04	0.16289E+04
0.71387E-02	0.21443E+01	0.16526E+04	0.16675E+04	0.16889E+04
0.74787E-02	0.23346E+01	0.17051E+04	0.17230E+04	0.17468E+04
0.78186E-02	0.25307E+01	0.17551E+04	0.17763E+04	0.18025E+04
0.81585E-02	0.27323E+01	0.18025E+04	0.18274E+04	0.18561E+04
0.84985E-02	0.29391E+01	0.18473E+04	0.18763E+04	0.19077E+04
0.88384E-02	0.31508E+01	0.18897E+04	0.19231E+04	0.19572E+04
0.91784E-02	0.33673E+01	0.19297E+04	0.19678E+04	0.20049E+04
0.95185E-02	0.35881E+01	0.19675E+04	0.20107E+04	0.20507E+04
0.98586E-02	0.38131E+01	0.20030E+04	0.20516E+04	0.20946E+04
0.10199E-01	0.40420E+01	0.20363E+04	0.20907E+04	0.21369E+04
0.10539E-01	0.42745E+01	0.20676E+04	0.21281E+04	0.21775E+04
0.10879E-01	0.45105E+01	0.20970E+04	0.21638E+04	0.22165E+04
0.11219E-01	0.47497E+01	0.21244E+04	0.21979E+04	0.22540E+04
0.11559E-01	0.49919E+01	0.21501E+04	0.22304E+04	0.22899E+04

TABLE 2

EMACK shot number 3

$L0 = 0.300E-05H$, $L1 = 0.585E-06H$, $g = 8.55m$
 $R0+Rp = 0.271E-04ohm$, $R1 = 0.450E-04ohm$, $t1 = 0.185E+00s$
 $mass = 0.275E+00Kg$, barrel length = 5.00m
 $i0 = 0.120E+07A$, $v0 = 0.000E+00m/s$, $vf = 5116.82m/s$
 $epsilon = .0181069$

time	position	spd R-H	spd second	spd first
0.00000E+00	0.00000E+00	0.00000E+00	0.00000E+00	0.00000E+00
0.25056E-03	0.53392E-02	0.12781E+03	0.12780E+03	0.12784E+03
0.50111E-03	0.21337E-01	0.25518E+03	0.25518E+03	0.25534E+03
0.75166E-03	0.47943E-01	0.38181E+03	0.38181E+03	0.38216E+03
0.10022E-02	0.95087E-01	0.50736E+03	0.50737E+03	0.50800E+03
0.12528E-02	0.12265E+00	0.63155E+03	0.63158E+03	0.63257E+03
0.15034E-02	0.19053E+00	0.75409E+03	0.75415E+03	0.75559E+03
0.17539E-02	0.25856E+00	0.87468E+03	0.87482E+03	0.87680E+03
0.20045E-02	0.32657E+00	0.99308E+03	0.99335E+03	0.99597E+03
0.22550E-02	0.42438E+00	0.11090E+04	0.11095E+04	0.11129E+04
0.25055E-02	0.52175E+00	0.12223E+04	0.12231E+04	0.12273E+04
0.27561E-02	0.62847E+00	0.13328E+04	0.13340E+04	0.13392E+04
0.30066E-02	0.74429E+00	0.14402E+04	0.14420E+04	0.14482E+04
0.32571E-02	0.86895E+00	0.15444E+04	0.15470E+04	0.15544E+04
0.35077E-02	0.10022E+01	0.16453E+04	0.16489E+04	0.16576E+04
0.37582E-02	0.11437E+01	0.17428E+04	0.17476E+04	0.17578E+04
0.40088E-02	0.12932E+01	0.18367E+04	0.18432E+04	0.18549E+04
0.42593E-02	0.14504E+01	0.19271E+04	0.19355E+04	0.19489E+04
0.45098E-02	0.16150E+01	0.20139E+04	0.20246E+04	0.20398E+04
0.47604E-02	0.17867E+01	0.20970E+04	0.21104E+04	0.21276E+04
0.50109E-02	0.19652E+01	0.21766E+04	0.21930E+04	0.22123E+04
0.52615E-02	0.21501E+01	0.22525E+04	0.22725E+04	0.22940E+04
0.55120E-02	0.23413E+01	0.23248E+04	0.23489E+04	0.23727E+04
0.57625E-02	0.25384E+01	0.23937E+04	0.24222E+04	0.24485E+04
0.60131E-02	0.27410E+01	0.24591E+04	0.24926E+04	0.25214E+04
0.62636E-02	0.29480E+01	0.25211E+04	0.25601E+04	0.25916E+04
0.65141E-02	0.31621E+01	0.25798E+04	0.26247E+04	0.26590E+04
0.67647E-02	0.33799E+01	0.26353E+04	0.26867E+04	0.27239E+04
0.70154E-02	0.36022E+01	0.26877E+04	0.27460E+04	0.27862E+04
0.72660E-02	0.38287E+01	0.27371E+04	0.28028E+04	0.28460E+04
0.75167E-02	0.40593E+01	0.27836E+04	0.28571E+04	0.29036E+04
0.77674E-02	0.42936E+01	0.28274E+04	0.29091E+04	0.29588E+04
0.80180E-02	0.45315E+01	0.28695E+04	0.29589E+04	0.30119E+04
0.82687E-02	0.47727E+01	0.29070E+04	0.30065E+04	0.30629E+04

TABLE 3

EMACK shot number 4

L0 = 0.500E-05H,

R0+Rp = 0.271E-04ohm,

mass = 0.317E+00Kg,

i0 = 0.210E+07A,

epsilon = .0111088

L1 = 0.585E-06H,

R1 = 0.450E-04ohm,

barrel length = 5.00m

v0 = 0.000E+00m/s,

g = 8.55m

t1 = 0.1E5E+00s

vf = 8340.17m/s

time	position	spd R-V	spd second	spd first
0.00000E+00	0.00000E+00	0.00000E+00	0.00000E+00	0.00000E+00
0.15372E-03	0.53398E-02	0.20836E+03	0.20836E+03	0.20840E+03
0.30744E-03	0.21342E-01	0.41608E+03	0.41608E+03	0.41624E+03
0.46116E-03	0.47960E-01	0.62266E+03	0.62266E+03	0.62301E+03
0.61488E-03	0.85123E-01	0.82757E+03	0.82758E+03	0.82821E+03
0.76861E-03	0.13273E+00	0.10303E+04	0.10304E+04	0.10314E+04
0.92234E-03	0.19067E+00	0.12305E+04	0.12306E+04	0.12320E+04
0.10761E-02	0.25878E+00	0.14275E+04	0.14277E+04	0.14297E+04
0.12298E-02	0.33690E+00	0.16211E+04	0.16215E+04	0.16241E+04
0.13835E-02	0.42484E+00	0.18108E+04	0.18115E+04	0.18149E+04
0.15372E-02	0.52239E+00	0.19962E+04	0.19974E+04	0.20016E+04
0.16909E-02	0.62933E+00	0.21771E+04	0.21790E+04	0.21841E+04
0.18446E-02	0.74542E+00	0.23531E+04	0.23559E+04	0.23622E+04
0.19983E-02	0.87039E+00	0.25239E+04	0.25281E+04	0.25355E+04
0.21520E-02	0.10040E+01	0.26895E+04	0.26953E+04	0.27040E+04
0.23057E-02	0.11459E+01	0.28496E+04	0.28574E+04	0.28676E+04
0.24594E-02	0.12959E+01	0.30041E+04	0.30144E+04	0.30262E+04
0.26131E-02	0.14537E+01	0.31528E+04	0.31663E+04	0.31798E+04
0.27669E-02	0.16187E+01	0.32957E+04	0.33129E+04	0.33282E+04
0.29206E-02	0.17913E+01	0.34329E+04	0.34544E+04	0.34717E+04
0.30743E-02	0.19706E+01	0.35642E+04	0.35907E+04	0.36101E+04
0.32280E-02	0.21565E+01	0.36897E+04	0.37220E+04	0.37436E+04
0.33817E-02	0.23486E+01	0.38095E+04	0.38483E+04	0.38722E+04
0.35354E-02	0.25468E+01	0.39236E+04	0.39697E+04	0.39960E+04
0.36891E-02	0.27506E+01	0.40322E+04	0.40863E+04	0.41153E+04
0.38428E-02	0.29599E+01	0.41353E+04	0.41903E+04	0.42299E+04
0.39965E-02	0.31743E+01	0.42331E+04	0.43057E+04	0.43402E+04
0.41503E-02	0.33936E+01	0.43258E+04	0.44009E+04	0.44462E+04
0.43040E-02	0.36176E+01	0.44134E+04	0.45077E+04	0.45481E+04
0.44578E-02	0.38458E+01	0.44953E+04	0.46026E+04	0.46461E+04
0.46116E-02	0.40783E+01	0.45744E+04	0.46934E+04	0.47401E+04
0.47654E-02	0.43146E+01	0.46480E+04	0.47806E+04	0.48305E+04
0.49192E-02	0.45545E+01	0.47173E+04	0.48640E+04	0.49174E+04
0.50730E-02	0.47979E+01	0.47826E+04	0.49441E+04	0.50008E+04

TABLE 4

EMAC! shot number 5

L0 = 0.500E-05H,

R0+Rp = 0.271E-04ohm,

mass = 0.318E+00kg,

i0 = 0.160E+07A,

epsilon = .0146033

L1 = 0.585E-06H,

R1 = 0.450E-04ohm,

barrel length = 5.00m

v0 = 0.000E+00m/s,

vf = 6344.41m/s

g = 8.55m

t1 = 0.185E+00s

time	position	spd R-F	spd second	spd first
0.00000E+00	0.00000E+00	0.00000E+00	0.00000E+00	0.00000E+00
0.20208E-03	0.53395E-02	0.15849E+03	0.15849E+03	0.15853E+03
0.40415E-03	0.21379E-01	0.31646E+03	0.31646E+03	0.31662E+03
0.60622E-03	0.47952E-01	0.47353E+03	0.47353E+03	0.47388E+03
0.80830E-03	0.85103E-01	0.62931E+03	0.62932E+03	0.62975E+03
0.10104E-02	0.13269E+00	0.78342E+03	0.78346E+03	0.78445E+03
0.12125E-02	0.17060E+00	0.93551E+03	0.93559E+03	0.93703E+03
0.14145E-02	0.25967E+00	0.10852E+04	0.10854E+04	0.10874E+04
0.16166E-02	0.33674E+00	0.12323E+04	0.12326E+04	0.12352E+04
0.18187E-02	0.42461E+00	0.13763E+04	0.13769E+04	0.13802E+04
0.20207E-02	0.52207E+00	0.15171E+04	0.15180E+04	0.15222E+04
0.22228E-02	0.62890E+00	0.16543E+04	0.16558E+04	0.16610E+04
0.24249E-02	0.74486E+00	0.17878E+04	0.17900E+04	0.17963E+04
0.26269E-02	0.86967E+00	0.19175E+04	0.19206E+04	0.19281E+04
0.28290E-02	0.10031E+01	0.20430E+04	0.20474E+04	0.20562E+04
0.30310E-02	0.11448E+01	0.21643E+04	0.21703E+04	0.21805E+04
0.32331E-02	0.12945E+01	0.22813E+04	0.22892E+04	0.23010E+04
0.34352E-02	0.14520E+01	0.23939E+04	0.24042E+04	0.24177E+04
0.36372E-02	0.16167E+01	0.25021E+04	0.25152E+04	0.25305E+04
0.38393E-02	0.17870E+01	0.26058E+04	0.26222E+04	0.26395E+04
0.40413E-02	0.19679E+01	0.27050E+04	0.27253E+04	0.27447E+04
0.42434E-02	0.21533E+01	0.27990E+04	0.28245E+04	0.28461E+04
0.44455E-02	0.23450E+01	0.28902E+04	0.29199E+04	0.29438E+04
0.46475E-02	0.25426E+01	0.29763E+04	0.30115E+04	0.30379E+04
0.48496E-02	0.27459E+01	0.30581E+04	0.30995E+04	0.31284E+04
0.50516E-02	0.29545E+01	0.31350E+04	0.31839E+04	0.32155E+04
0.52537E-02	0.31682E+01	0.32094E+04	0.32649E+04	0.32993E+04
0.54558E-02	0.33867E+01	0.32791E+04	0.33425E+04	0.33798E+04
0.56580E-02	0.36099E+01	0.33449E+04	0.34169E+04	0.34572E+04
0.58601E-02	0.38373E+01	0.34070E+04	0.34882E+04	0.35316E+04
0.60623E-02	0.40689E+01	0.34656E+04	0.35564E+04	0.36030E+04
0.62644E-02	0.43041E+01	0.35207E+04	0.36218E+04	0.36716E+04
0.64666E-02	0.45430E+01	0.35725E+04	0.36844E+04	0.37376E+04
0.66687E-02	0.47853E+01	0.36213E+04	0.37444E+04	0.38010E+04

TABLE 5

43-AP-012

Minigrant Proposal to Air Force Office of
Scientific Research

Attention: Richard W. Kopka, Major, USAF
Department of the Air Force
Air Force Office of Scientific Research (AFSC)
Attention: XOT
Bolling AFB, D.C. 20332

Proposing Institution: University of Miami
Department of Physics
Coral Gables, FL 33124

Title: The Electromagnetic Rail Launcher as a Circuit Element

Budget: \$11,998

Proposal Duration: January 1, 1983 to December 31, 1983

Principal Investigator: Dr. Manuel A. Huerta
Associate Professor of Physics
[REDACTED]
Telephone No. (305)284-2323

Manuel A. Huerta
Manuel A. Huerta
Principal Investigator
Chairman, Department of Physics
Date 10/6/82

Ronald E. Siegwald
Ronald E. Siegwald
Authorized Official
Director, Sponsored Programs/
Fiscal Management
Date 10/1/82

Introduction

This proposal is a follow-up to the research the P.I. performed at the AFATL/DL DL at Eglin AFB under the 1982 USAF-SCREE Summer Faculty Research Program. That research identified the basic research issues involved in improving the models for the arc plasma produced in electromagnetic rail launchers with plasma driven projectiles. The results of that research are summarized in the final report entitled Basic Research Issues in Electromagnetic Rail Launchers with Plasma Driven Projectiles. A copy of that report is attached as the Appendix to this proposal.

The small scale of a minigrant dictates that a problem of the appropriate scope be considered. I propose to study the behavior of rail launchers as circuit elements. The circuit properties most directly describe how the stored electrical energy is converted into kinetic energy of the projectile and determine the overall efficiency.

Previous Work by Other Authors

The current variations in rail launchers are predominantly of low frequencies. This enables us to limit our treatment to the case where the length of the rails is much shorter than the wavelength of the electromagnetic fields present. Under these conditions the current is uniform as a function of position along the rails and the distributed capacitance is negligible. We are then able to speak of the relationship between the voltage at the breech and the current through the rails and plasma.

A further simplification is possible regarding the skin depth. It is shown in the Appendix that provided that the rail thickness is much smaller than the rail separation one may take the rail resistance per unit length R' and the inductance per unit length L' to be constants independent of the frequency. From Eq. (56) of the Appendix we get that the breech voltage $V_b(t)$ is related to the current $i(t)$ by

$$V_b(t) = i(t) R_b + d/dt(L_b i) \quad (1)$$

$$\text{where } R_b = R_p + R'x \text{ and } L_b = L'x, \quad (2)$$

with R_p equal to the plasma resistance. Here x is the distance from the breech to the plasma as shown in Fig. 1 of the Appendix. The i - V relation involves the variable x . The equation for it is

$$m d^2x/dt^2 = 1/2 L' i^2 - f. \quad (3)$$

This equation is the statement that the magnetic force $\frac{1}{2} L' i^2$ accelerates the mass m against a retarding friction f .

The form of Eq. (1) can be understood simply. The voltage drop at the breech is made up of the resistive drop plus the rate of change of the magnetic flux $\Phi = L_b i = L'xi$. The fact that the force on the armature is $1/2 L' i^2$ can be understood by considering the energy into the breech. The

power into the breech is given by iV_b . From Eq. (1) one readily gets that

$$iV_b = i^2 R_b + d/dt (1/2 L' x i^2) + 1/2 L' i^2 dx/dt. \quad (4)$$

We see that the power into the breech goes into a resistive heat loss, an increase of the magnetic energy $(1/2 L_b i^2)$ stored in the rails, and a third term that gives the rate of doing work on the armature. From this term we deduce that the magnetic force on the armature is given by $1/2 L' i^2$.

The circuit behavior of the rail launcher is described by Eqs. (1)-(3). Of course the rails will be part of a complete circuit. The simplest launcher arrangement consists of a storage inductor L_0 , with its resistance R_0 , connected in series with the breech as shown in Fig. 1. Previous steps have energized the inductor L_0 so that at $t = 0$ the current has the value i_0 .

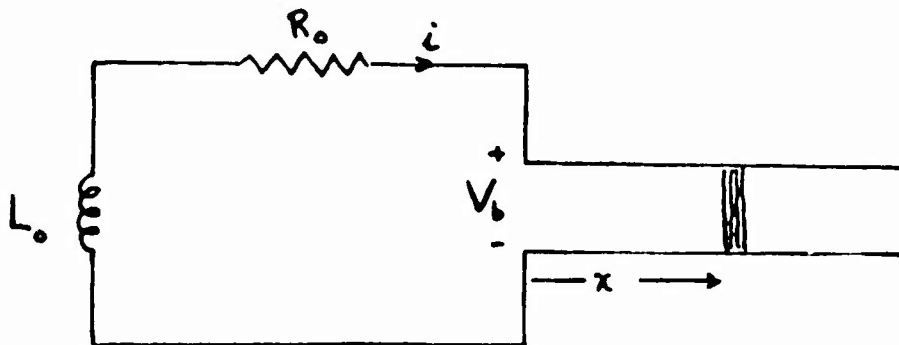


Fig. 1 Schematic of rail gun. V_b is the breech voltage. The arc has moved to a distance x from the breech. L_0 is a storage inductor with internal resistance R_0 .

The complete circuit equations are then

$$d/dt[(L_0 + L'x)i] + (R_0 + R_p + R'x)i = 0, \quad (5)$$

$$\text{and } d^2x/dt^2 = L'i^2/2m, \quad (6)$$

where the force of friction has been neglected in eq. (6). The initial conditions are

$$i(t = 0) = i_0, \quad x(t = 0) = 0,$$

$$\text{and } dx/dt(t = 0) = v_0. \quad (7)$$

The initial velocity allows for a preaccelerated projectile that is injected into the launcher with a speed v_0 . The system of Eqs. (5) and (6) is nonlinear and has not been solved analytically in nontrivial cases. Numerical results are easy to obtain. In fact, Mr. Kenneth Cobb, of AFATL/DL DL, already has a program to integrate them.

Previous work by the P.I.

Some of the work under this proposal would go into developing further the numerical results. The main effort, however, will go into developing analytical results. Analytical results are essential to understand how the solution scales as the various parameters are varied. The P.I. has already made sufficient progress on this problem that valuable results are almost assured.

We first note that multiplying Eq. (5) by i and using Eq. (6) yields a single third order equation for $x(t)$,

$$(L_0 + L'x) \frac{d^3x}{dt^3} + 2L' \frac{dx}{dt} \frac{d^2x}{dt^2} + 2(R_0 + R_T + R'x) \frac{d^2x}{dt^2} = 0. \quad (8)$$

This equation is to be solved with the boundary conditions of Eq. (7) together with

$$\frac{d^2x}{dt^2}(t=0) = \frac{L' i_0^2}{2m}. \quad (9)$$

We note that Eq. (8) can be rewritten as

$$\frac{d}{dt} \left[(L_0 + L'x) \frac{d^2x}{dt^2} \right] + \frac{L'}{2} \frac{d}{dt} \left[\left(\frac{dx}{dt} \right)^2 \right] + (R_0 + R_T) \frac{d}{dt} \left[\frac{dx}{dt} \right] + 2R' \frac{d}{dt} \int x \frac{d^2x}{dt^2} dt = 0. \quad (10)$$

Obviously Eq. (10) can be integrated once if the R' term is negligible. This term would be zero in an ideal rail launcher with supercooled rails. We will carry along the R' integral term and treat it as a perturbation. Integrating Eq. (10) yields

$$(L_0 + L'x) \frac{d^2x}{dt^2} + \frac{L'}{2} \left[\frac{dx}{dt} \right]^2 + 2(R_0 + R_p) \frac{dx}{dt} + 2R' \int_0^t x \frac{d^2x}{dt^2} dt =$$

$$\frac{L'}{m} \left[\frac{L_0 i_0^2}{2} + \frac{mv_0^2}{2} \right] + 2(R_0 + R_p)v_0 = b' \quad (11)$$

Eq. (11) follows immediately from conservation of energy, that is

$$\frac{1}{2} L_0 i_0^2 + \frac{1}{2} mv_0^2 = \frac{1}{2} (L_0 + L'x) i^2 + \frac{1}{2} mv^2 + \int_0^t i^2 (R_0 + R_p + R_1 x) dt. \quad (12)$$

Eqs. (11) and (12) can easily be shown to be equivalent by making use of Eq. (6). Eq. (11) cannot be integrated further for $x(t)$ even if one neglects the R' term. Progress is possible, however, if we go to the energy form. That is, we introduce the velocity v as a dependent variable,

$$v = \frac{dx}{dt}, \quad \frac{dv}{dt} = \frac{d^2x}{dt^2}, \quad (13)$$

and furthermore use that

$$\frac{dv}{dt} = \frac{dv}{dx} \frac{dx}{dt} = v \frac{dv}{dx},$$

to get

$$(L_0 + L'x)v \, dv/dx + L'/2 \, v^2 + 2(R_0 + R_p)v + 2R' \int_{v_0}^v x \, dv = b'. \quad (15)$$

Eq. (15) is in a form suitable for a nonsingular perturbation expansion of the solution where the lowest order can be found exactly. For this we introduce dimensionless variables using the principal parameters of the problem as scales. The length of the rails, l , plays an important role in limiting the

final projectile velocity. We introduce a dimensionless length X by

$$x = \ell X. \quad (16)$$

We introduce a velocity scale v_f obtained by letting the initial acceleration

$$a = \frac{L' i_0^2}{2m} \quad (17)$$

apply for the distance ℓ to give

$$v_f = \sqrt{2a\ell} = \sqrt{L'\ell i_0^2/m}. \quad (18)$$

We then introduce a dimensionless velocity V by

$$v = v_f V, \quad (19)$$

and a time scale t_f by

$$t_f = \frac{\ell}{v_f} \sqrt{\frac{\ell}{2a}} = \sqrt{\frac{m}{2L' i_0^2}}. \quad (20)$$

A dimensionless parameter h that gives the ratio of rail inductance to external inductance is

$$h = L'\ell/L_0. \quad (21)$$

Another dimensionless parameter r is defined by

$$r = 2 t_f/t_c, \quad (22)$$

where t_c is the time scale of the external R-L circuit,

$$t_c = \frac{L_0}{R_0 + R_p}. \quad (23)$$

Finally the expansion parameter is given by the ratio of rail resistance to external and plasma resistance

$$\varepsilon = \frac{R' l}{R_0 + R_p} \quad (24)$$

In terms of the dimensionless variables X and V and the dimensionless parameters defined above, Eq. (15) is rewritten as

$$(1 + hX) V dV/dX + h/2 V^2 + rV + \varepsilon r \int_{v_0/v_f}^V X dV = b, \quad (25)$$

$$\text{where } b = \frac{l b'}{L_0 v_f^2} = \frac{1}{2} + \frac{h}{2} \left(\frac{v_0 t_f}{l} \right)^2 + r \frac{v_0 t_f}{l}.$$

A nonsingular perturbation expansion is set up by substituting the series

$$V(X) = V_0(X) + \varepsilon V_1(X) + \varepsilon^2 V_2(X) + \dots \quad (26)$$

into Eq. (25) with the initial conditions

$$V_0(0) = v_0/v_f, V_i(0) = 0, i = 1, 2, \dots \quad (27)$$

The results are

$$(1 + hX) V_0 dV_0/dX + h/2 V_0^2 + rV_0 = b, \quad (28)$$

$$(1 + hX)(V_1 dV_0/dX + V_0 dV_1/dX) + hV_0 V_1 + rV_1 + r \int_{v_0/v_f}^{V_0} X dV_0 = 0, \quad (29)$$

$$(1 + hX)(V_0 dV_2/dX + V_1 dV_1/dX + V_2 dV_1/dX) + h/2 (V_1^2 + 2V_0 V_2) + rV_2 + r \int_0^{V_1} X dV_1 = 0,$$

for the terms of order ε^0 , ε^1 , and ε^2 respectively.

Proposed Work

The sequence of Eqs. (28) - (30) is to be solved analytically as far as practical. The analytical solution is to be compared with a numerical solution of Eq. (25).

The analytical solution is off to a solid start because the P.I. has obtained a closed form solution for Eq (28) in the form $X = X(V_0)$. This is easily obtained by rewriting Eq. (28) as

$$\int_{v_0/v_f}^{V_0} \frac{v_0 dv_0}{b - (h/2) v_0^2 - r v_0} = \int_0^X \frac{dX}{1 + hX} \quad (31)$$

A simple integration gives

$$1 + hX = \frac{a^2 - 1}{a^2 - \beta^2 (1 + hV_0/r)^2} \left(\frac{a + 1}{a - 1} \cdot \frac{a - \beta(1 + hV_0/r)}{a + \beta(1 + hV_0/r)} \right)^{\beta/a} \quad (32)$$

$$\text{where } \beta = (1 + hv_0/rv_f)^{-1} \text{ and } a = (1 + 4h\beta^2/r^2)^{1/2} \quad (33)$$

Eq. (32) gives us $X(V_0)$ for an ideal launcher. Since one cannot invert for $V_0(x)$, we simply modify Eq. (29) to continue with the asymptotic expansion. All that is needed is to consider V_1 , V_2 , etc as functions of V_0 .

Budget for Minigrant Proposal

Duration: January 1, 1983 to December 31, 1983

Salaries, Wages and Fringe Benefits

Mannuel A. Huerta, Principal Investigator	
1983 - 1984 estimated salary \$35049/9 months	
100% time during 1 summer month	3894
Graduate Student Research Assistant	
100% time during 3 summer months	2000
Fringe Benefits: 17.33% of faculty salary	675

<u>Total of Salaries, Wages and Benefits</u>	<u>6569</u>
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Other Direct Costs

Travel costs	
to attend meeting of subject related to this proposal	250
Publication Costs	200
Disc Drive (permanent equipment)	550
Software, and/or other equipment to	
upgrade microcomputer	136

<u>Total Other Direct Costs</u>	<u>1136</u>
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<u>Total Direct Costs</u>	<u>7705</u>
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<u>Indirect Costs: 60% of MTDC(7155)</u>	<u>4293</u>
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<u>Total Budget</u>	<u>\$11,998</u>
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*The Coral Gables campus negotiated indirect cost rate for the University of Miami for fiscal year 6/1/82 to 5/31/83 is 131.0% of Modified Total Direct Costs (MTDC). The University is willing to accept an indirect cost rate that is lower than the negotiated rate. However, if the proposal is awarded, the University retains the right to exercise any cost clause or cost regulation made a part of the signed agreement.