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these equations up to an overall scaling factor. Generalizations to the spatial dependent situation are discussed. We define a method for detecting material changes and illustrate a way of detecting the color of a material at its boundaries and propagating inwards.

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A Method for Computing Spectral Reflectance

A. Yuille

Abstract. Psychophysical experiments show that the perceived colour of an object is relatively independent of the spectrum of the incident illumination and depends only on the surface reflectance. We demonstrate a possible solution to this underdetermined problem by expanding the illumination and surface reflectance in terms of a finite number of basis functions. This yields a number of nonlinear equations for each colour patch. We show that given a sufficient number of surface patches with the same illumination it is possible to solve these equations up to an overall scaling factor. Generalizations to the spatial dependent situation are discussed. We define a method for detecting material changes and illustrate a way of detecting the colour of a material at its boundaries and propagating it inwards.

This report describes research done at the Artificial Intelligence Laboratory of the Massachusetts Institute of Technology. Support for the laboratory's artificial intelligence research is provided in part by the Advanced Research Projects Agency of the Department of Defense under Office of Naval Research contract N00014-80-C-0505.

A Method for Computing Spectral Reflectance

1. Introduction. The colour of an object depends on the spectral distributions of the incident light and the reflectance functions of the object. More precisely at each point x the receptors (for example, the rods and cone of the eye) measure a quantity l_μ given by

$$l_\mu(x) = \int a_\mu(\lambda) E(x,\lambda) S(x,\lambda) d\lambda \tag{1.1}$$

where $a_\mu(\lambda)$ is the absorption function of the receptors ($\mu = 1$ to 4 if we consider the rods and cones), $E(x,\lambda)$ is the incident illumination and $S(x,\lambda)$ the reflectance of the surface. In terms of the notation used by Hurlbert (1985) our function $S(x,\lambda)$ is the product of the albedo $\rho(x,\lambda)$ and the reflectance function $R(\underline{n},\underline{k},\underline{s})$.

These quantities $l_\mu(x)$ are then evaluated and combined to give the perception of colour. For the human eye it is assumed that the outputs of the three cones correspond to the perception of colour. The three cones corresponds to the three dimensions of the space of perceived colours (von Helmholtz 1866, or see Feynman 1965 for an interesting review).[1]

A number of experiments show that perceived colour of an object is relatively independent of the spectral distribution of the incident light . This is the colour constancy effect demonstrated most clearly by Land on Mondrians. Since the incident illumination and the surface reflectance are confounded in the input (1.1) some assumptions must be made to disentangle them and to obtain a colour which depends on the surface reflectance alone. This paper suggests a scheme for doing this. Alternative schemes are described in Rubin and Richards (1984), Hurlbert and Poggio (1985).

This paper assumes that both the colour, the incident illumination and the surface reflectance can be expressed as a finite sum of basis functions in spectral space.[2] It is then shown that given a sufficient number of patches of different surface reflectivity with roughly the same incident illumination it is possible to solve for the surface reflectance up to an overall scaling factor. In such a theory the colour of a patch would depend on the colour of the neighbouring patches, this is consistent with Land's experiments. Rubin and Richards (1984) have considered ways of distinguishing such "material" patches by considering the spectral properties of the observed intensities. They discuss "lawful processes" (such as

[1] The rods are not normally assumed to contribute to colour perception, however, we will argue later that although their inputs do not directly contribute to perceived colour they can still be used to "normalize" colour perception.

[2] This corresponds to the standard regularization approach (Poggio and Torre, 1984) of solving an underconstrained problem by restricting the space of solutions.

orientation and shadowing) which change the albedo while still corresponding to the same material and propose an "opposite sign slope condition" to detect such change.

The first part of this paper, although done independently, is similar to work of Wandell and Maloney (1984a, 1984b)[3]. In their work the illuminant and spectral surface reflectances are also expanded in terms of a finite number of basis functions in spectral space and neighbouring colour patches, with the same assumed illumination. Our formula for the number of patches required differs from theirs, however, and furthermore we demonstrate a simple way to solve the equations. Our scheme can also be extended using results from Hurlbert and Poggio(1985) to cases when the spectral components of the illuminant and the reflectance function vary with position. For this case we can find the colour of an object at its boundaries and then fill in the interior. This method will automatically find the material boundaries and thus will act as a material edge detector in the sense of Rubin and Richards (1984).

2. The Basis Functions. First we consider the case when there is no spatial dependence of the illuminant or the surface reflection. The inputs to the receptors are then

$$l_\mu = \int a_\mu(\lambda)E(\lambda)S(\lambda)d\lambda. \tag{2.1}$$

We now expand $E(\lambda)$ and $S(\lambda)$ in terms of basis functions $B_i(\lambda)$ and $C_j(\lambda)$

$$E(\lambda) = \sum_{i=1}^{n} \tau_i B_i(\lambda), S(\lambda) = \sum_{j=1}^{m} \gamma_j C_j(\lambda). \tag{2.2}$$

For the present we will set $n = m = 3$ and choose the same basis functions

$$E(\lambda) = \sum_{i=1}^{3} \tau_i B_i(\lambda), S(\lambda) = \sum_{j=1}^{3} \gamma_j B_j(\lambda). \tag{2.3}$$

This is merely for convenience and because of psychophysical evidence that perceived colours form a three dimensional space (Helmholtz, (1866), for a review see; Feynman 1965). It is straightforward to generalize the analysis to different values of n and m and for different numbers of rods and cones. Substituting (2.3) into (2.1) gives

$$l_\mu = \tau_i\gamma_j \int a_\mu(\lambda)B_i(\lambda)B_j(\lambda)d\lambda \tag{2.4}$$

where we use the summation convention over i and j.

We can write this as

[3] I am grateful to Ted Adelson for bringing their work to my attention.

$$l_\mu = T^\mu_{ij}\tau_i\gamma_j. \tag{2.5}$$

The (T^μ_{ij}) depend only on the absorption coefficients and the basis functions. They are therefore parameters of the system and hence are known (the system might be taught to learn the optimal (T^μ_{ij})). The (τ_i) and (γ_j) describe the illuminant and the surface reflection respectively. We therefore have four equations ($\mu = 1$ to 4) for six unknowns ($i = 1$ to 3 , $j = 1$ to 3). Note, however, that there are effectively only five unknowns because of the scaling ambiguity

$$\tau_i \mapsto p\tau_i, \gamma_j \mapsto \frac{\gamma_j}{p}. \tag{2.6}$$

This ambiguity[4] can be traced back to (2.1) where it corresponds to the scaling

$$E(\lambda) \mapsto pE(\lambda), S(\lambda) \mapsto \frac{S(\lambda)}{p}. \tag{2.7}$$

This scaling ambiguity cannot be dealt with by our theory and a further normalization is required. Note this is the only ambiguity in our theory and only one normalization is required. This is in contrast to the theories of Land(1983), Horn(1975) and Blake(1984) where colour is computed independently in seperate channels each of which has to be normalized seperately. Unlike these theories (for a review of these see Hurlbert 1985) our method allows interactions between channels and hence we only need one normalization.

To resolve this ambiguity we can normalize the light source by making (τ_i) a unit vector, or by setting $\tau_1 = 1$.

A single patch gives us four equations for five unknowns. Suppose we have a neighbouring patch with a different spectral surface reflectance. This second surface reflectance can be expressed in terms of basis functions by

$$S_2(\lambda) = \sum_{i=1}^{3} \varsigma_i B_i(\lambda). \tag{2.8}$$

Now if we make the reasonable assumption that the spectral distribution of the illuminant is unchanged on the second patch we measure four new quantities

$$k_\mu = \tau_i\varsigma_j T^\mu_{ij} \tag{2.9}$$

[4]whose nature was clarified during discussions with Hurlbert and Poggio

which involve only three new unknowns (ς_j). We combine this with (2.8) to get eight equations for eight unknowns. Number counting suggests this is enough to get a unique solution. The equations are non-linear however and, as we will show in the next section, in some cases there may be more than one solution.

For the human eye to use a colour scheme like this with four receptors, the rods are necessary as well as the cones. If we just use the cones and have three receptors it follows from (2.10) that we can only have two basis functions and hence only a two dimensional space of perceived colour. Observe that the output from the rods can be treated differently from the output from the cones. It is combined with them to solve (2.4) and (2.6) and determine the (ς_j) and the (γ_j) but it is only the inputs from the cones that gives the colours. The rods and the cones combine to determine the perception of colour from the spectral information at the cones.

We now consider the general case when there are p receptors, n basis functions for the illuminant and m for the reflectance function. Suppose we have q patches. Then the number of equations is pq. The number of variables is $n + mq - 1$ (the 1 comes from the scaling ambiguity). To obtain a unique solution[5] we need

$$pq \geq n + mq - 1. \tag{2.10}$$

This is a neccessary condition. To check it is sufficient we will show that the non-linear equations can be solved.

Note from (2.10) that we always need $p > m$, that is the number of different photoreceptors must be greater than the number of basis functions for the reflectance. Thus to obtain a three dimensional colour space with this scheme at least four photoreceptors are required.)

The number of patches required is

$$q \geq \frac{n-1}{p-m}. \tag{2.11}$$

So if there are two basis vectors for the illuminant and surface reflectances (n = m = 2) and three receptor fields (p = 3) we only need one patch.

3. Solving the Equations.

We now consider the equations in more detail. They are of the form

$$l_\mu = \tau_i \gamma_j T^\mu_{ij}, k_\mu = \tau_i \varsigma_j T^\mu_{ij} \tag{3.1}$$

[5] This equation has also been derived by J. Rubin (pers. comm.). It does not imply that $q \geq n$, a formula stated by Wandell and Maloney (1984a), and a counterexample is given in the next section.

The matrices T^{μ}_{ij} are parameters of the system given by

$$T^{\mu}_{ij} = \int a_{\mu}(\lambda) B_i(\lambda) B_j(\lambda) d\lambda. \tag{3.2}$$

If the basis functions $(B_i(\lambda))$ do not overlap then the off-diagonal terms of T^{μ}_{ij} are zero ($T^{\mu}_{ij} = 0, i \neq j$). This case is considered by Hurlbert and Poggio (1985). This simplifies the equations (3.1) but at the cost of preventing interactions between different colour channels (although interactions can be introduced later). These interactions are important for our purposes since they mean that only one colour normalization (the overall scaling given by (2.7)) is needed.

The basic strategy to solve the equations (3.1) is to take linear combinations of the matrices T^{μ}_{ij} to obtain simpler equations. We illustrate this by considering the case when there are three photoreceptors, and two basis functions ($n = m = 2, p = 3$). As shown in the previous section only one patch is needed to solve the equations. These are

$$l_{\mu} = \tau_i \gamma_j T^{\mu}_{ij}, i = 1, 2. j = 1, 2. \mu = 1, 2, 3. \tag{3.3}$$

The (T^{μ}_{ij}) are three two-by-two symmetric matrices. The space of such matrices is three dimensional and so the T^{μ}_{ij} form a basis for this space.[6] Hence we can find three vectors p^{a}_{μ} ($a = 1, 2, 3$) such that

$$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} = \sum p^1_{\mu} T^{\mu}_{ij}$$

$$\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} = \sum p^2_{\mu} T^{\mu}_{ij} \tag{3.4}$$

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \sum p^3_{\mu} T^{\mu}_{ij}.$$

Combining (3.4) with (3.3) defines three (known) numbers $\Lambda_1, \Lambda_2, \Lambda_3$ by

$$\Lambda_1 = \sum_{\mu=1}^{3} p^1_{\mu} l_{\mu} = \tau_1 \gamma_1, \Lambda_2 = \sum_{\mu=1}^{3} p^2_{\mu} l_{\mu} = \tau_2 \gamma_2, \Lambda_3 = \sum_{\mu=1}^{3} p^3_{\mu} l_{\mu} = \tau_1 \gamma_2 + \tau_2 \gamma_1. \tag{3.5}$$

We normalize the light source by setting $\tau_1 = 1$. This reduces (3.5) to

$$\gamma_1 = \Lambda_1, \gamma_2 \tau_2 = \Lambda_2, \Lambda_1 \tau_2 + \gamma_2 = \Lambda_3. \tag{3.6}$$

[6] We choose the (T^{μ}_{ij}) so that they are linearly independent. We are free to do this since they are the parameters of the system.

These can be solved to give

$$\tau_1 = 1, \tau_2 = \frac{A_3 \pm (A_3^2 - 4A_1A_2)^{1/2}}{2A_1}, \gamma_1 = A_1, \gamma_2 = \frac{A_3 \mp (A_3^2 - 4A_1A_2)^{1/2}}{2}. \tag{3.7}$$

This gives two possible solutions (the values of τ_2 and γ_2 are interdependent). The constraint that the illuminant and the reflectance function are both positive functions may not help to resolve this, the two roots of both τ_2 and γ_2 all have the same sign.

Note that only one patch is needed to solve for this case although there are two basis functions.

The General Case

The basic strategy to solve the equations consists of taking linear combinations of the (T^μ_{ij}) and reducing them to some algebraic equations having only a finite number of solutions.

For four photoreceptors and three basis functions we require two patches with the same illumination. We have two sets of equations, one from each patch,

$$l_\mu = \tau_i\gamma_j T^\mu_{ij}, k_\mu = \tau_i\zeta_j T^\mu_{ij}. \tag{3.8}$$

A simple way to solve these equations is by choosing the basis vectors $(B_i(\lambda))$ so that the (T^μ_{ij}) generate a space spanned by

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}. \tag{3.9}$$

The conditions on the basis vectors $(B_i(\lambda))$ which this requires are

$$\int a^\mu B_1(\lambda)B_3(\lambda)d\lambda = 0, \int a^\mu B_1(\lambda)B_2(\lambda)d\lambda = \int a^\mu B_2(\lambda)B_3(\lambda)d\lambda. \tag{3.10}$$

To see this observe that the space of symmetric three by three matrices is six dimensional. The matrices defined by (3.9) define a four dimensional subspace and to ensure that the (T^μ_{ij}) lie in this subspace we must impose the two conditions of (3.10). To check the exact form of (3.10) consider all possible matrices H_{ij} that can be generated by linear combinations of (3.9). First observe that it is impossible to get a matrix with $H_{13} \neq 0$, this gives us the first equation of (3.10). The second equation comes from noting that we must have $H_{12} = H_{32}$ for all H.

We now take linear combinations of (3.8) using four vectors g^a_μ $(a = 1, 2, 3, 4)$ as in (3.4) and (3.5) until we generate the basis matrices of (3.9). The equations (3.8) are now of form

$$\gamma_1 = A_1, \gamma_2 = \frac{A_2}{\tau_2}, \gamma_3 = \frac{A_3}{\tau_3}, \tag{3.11}$$

$$\varsigma_1 = B_1, \varsigma_2 = \frac{B_2}{\tau_2}, \varsigma_3 = \frac{B_3}{\tau_3}, \tag{3.12}$$

and

$$A_1(\tau_2^2\tau_3) + A_2(\tau_3 + \tau_3^2) + A_3\tau_2^2 = A_4\tau_2\tau_3, B_1(\tau_2^2\tau_3) + B_2(\tau_3 + \tau_3^2) + B_3\tau_2^2 = B_4\tau_2\tau_3, . \tag{3.13}$$

The (A_i) and (B_i) are linear combinations (using the g_μ^a)of the (l_μ) and the (m_μ). We have normalized by setting $\tau_1 = 1$ and divided some of the equations by τ_2 and τ_3.

This gives

$$\tau_2 = \frac{(A_4 - \delta B_4)\tau_3}{(A_1 - \delta B_1)\tau_3 + (A_3 - \delta B_3)}. \tag{3.14}$$

where $\delta = A_2/B_2$. We can substitute this into (3.13) to obtain a quartic equation for τ_3 which will have at most four solutions. Then we solve (3.14) for τ_2 and (3.11) and (3.12) for the (γ_i) and (ς_i).

The basis matrices defined by (3.9) were choosen arbitrarily. The same analysis could be applied for other basis matrices. For some choices of basis there is only one possible solution. An example is

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}. \tag{3.15}$$

This basis, however, requires that

$$\int a^\mu B_2^2(\lambda)d\lambda = 0, \int a^\mu B_3^2(\lambda)d\lambda = 0. \tag{3.16}$$

which seems an unnatural choice for the (B_i). This illustrates the important point that the choice of basis functions is crucial. By assuming that we can express our illumination and surface reflectance in terms of a finite set of basis functions we are performing an implicit type of regularization (Poggio and Torre, 1984) by restricting the space of possible solutions. Only if the basis functions are choosen so that linear combinations of them are able to represent most of the naturally occuring illuminants and surface reflectances will methods of this type give good solutions.

This strategy of taking linear combinations of the (T^{μ}_{ij}) will drastically simplify the equations. It is also possible to solve equations (3.8) directly. We write the equation as

$$l_{\mu} = \gamma_1(T^{\mu}_{11}\tau_1 + T^{\mu}_{12}\tau_2 + T^{\mu}_{13}\tau_3) + \gamma_2(T^{\mu}_{21}\tau_1 + T^{\mu}_{22}\tau_2 + T^{\mu}_{23}\tau_3) + \gamma_3(T^{\mu}_{31}\tau_1 + T^{\mu}_{32}\tau_2 + T^{\mu}_{33}\tau_3), \mu = 1, ..4 \tag{3.17}$$

We can use three of these four equations to solve for (γ_i) in terms of the (τ_i). This will leave us with two polynomial equations relating τ_2 and τ_3 (we have set $\tau_1 = 1$). Similarly we solve similar equations for (β_i) in terms of the (α_i) leaving another polynomial equation in τ_2 and τ_3. These two equations in τ_2 and τ_3 can be combined to give solutions for τ_2 and τ_3. Since these equations are polynomials there will usually be more than one solution, and in some cases an infinite number. The previous method involving basis matrices is a lot simpler. If it is impossible to choose a basis like (3.9) it will still be possible to simplify the equations by taking linear combinations of the (T^{μ}_{ij}) to reduce the matrices to simple forms.

Note that so far nothing has been said about the behaviour of the solution. It is even conceivable, though unlikely, that the solutions are sometimes not positive everywhere. They are guaranteed to be smooth provided the basis functions are. Instead of choosing three basis functions and solving for their coefficients it would be possible to have more basis functions and impose some a priori expectations on the solutions. Such an approach is discussed in Richards, Yuille and Poggio (1985).

The Spatial Extension

In this section we consider extensions to situations when the surface reflectance and the illumination are functions of space. Hurlbert and Poggio (1985) have shown that for many cases the spatial dependence of the illumination and surface reflectance can be factored out from the spectral dependence for colour patches. For these cases we can write

$$S(x, \lambda) = S(\lambda)f(x) \tag{4.1}$$

and

$$E(x, \lambda) = E(\lambda)g(x) \tag{4.2}$$

where $f(x)$ and $g(x)$ depend only on position.

With these assumptions the spatial dependence of the illuminant and the surface reflectance, when not due to material (colour) changes, is factored out and the analysis can proceed as in the previous section. There is an important difference, however, the scaling ambiguity

(2.7) will now have a spatial dependence. In other words we will not be able to distinguish the spatial dependences $f(x)$ and $g(x)$ of the surface reflectance and illumination. For many cases the illumination will have little spatial variation (for example, if the light sources are a long way away) and we can set $g(x) = 1$ and solve for $f(x)$. Otherwise we can consider the boundaries between two materials and assume that the spatial dependence of the illumination is negligible across the boundary. We can then solve for the colour on either side of the boundaries (and fill in if necessary). A third method is to note that if there is a single light source the spatial dependence $g(x)$ will have a definite form , typically it will fall off as $1/r^2$ (where r is the distance to the light source), and so $f(x)$ can be determined.

In the previous sections we were considering a Mondrian style world consisting of flat patches of uniform colour. Our method would sample two patches and obtain two equations (2.5) and (2.6). For the more general case we would sample neighbouring points. If the two points lie on the same material the l_μ will be proportional to the k_μ and the equations will be underdetermined. If the points occur at a material boundary, one lying in each material, then we have enough equations to solve for the colours. Since we are considering neighbouring points it is reasonable to assume that the illuminant is the same (including the scaling factor) for each point.

To summarize: we sample neighbouring points in the image and compare their values l_μ and k_μ. If these are proportional we conclude that the points have the same colour (though not necessarily the same brightness) and hence lie on the same material. If they are not then we are on a material boundary and can solve for the colours on either sides of the boundary. The colour cannot change inside the boundary, or else we would detect it, and so the colour on the boundaries determines the colour in the interior. However the brightness, or strength, of the colour is undetermined. If the illuminant is constant in space we can solve this by a single global normalization (such as setting $\tau_l = 1$, as in (3.5)). We suggest making this assumption even if the illuminant varies spatially, unless the variation is very large or there is strong evidence to the contrary from other souces. Thus spatial variation of the illuminant will be interpreted as due to changes in the reflectance function due to orientation and other changes.

Other ways of generalizing to allow for spatial dependence would include using the standard assumption that the spatial variation of the illuminant is smaller than the spatial variation of the surface reflectance (Land (1983), Horn (1974), Blake (1984)). New methods for doing this are discussed in Hurlbert and Poggio (1985) and extensions will be discussed in a forthcoming paper.

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