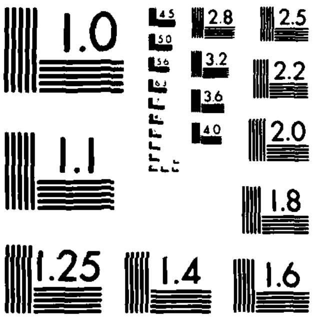
AD-A150 325 A NOTE ABOUT THE STRONG CONVERGENCE OF THE NONPARAMETRIC ESTIMATION OF A..(U) PITTSBURGH UNIV PA CENTER FOR MULTIVARIATE ANALYSIS Z FANG SEP 84 1/1

UNCLASSIFIED TR-84-45 AFOSR-TR-85-0005 F49620-82-K-0001 F/G 12/1 NL

END



MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS-1963-A

AD-A150 325

5

REPORT DOCUMENTATION PAGE

1a REPORT SECURITY CLASSIFICATION: UNCLASSIFIED

1b RESTRICTIVE MARKINGS:

2a SECURITY CLASSIFICATION AUTHORITY:

2b DECLASSIFICATION/DOWNGRADING SCHEDULE:

3 DISTRIBUTION/AVAILABILITY OF REPORT: Approved for public release; distribution unlimited.

4 PERFORMING ORGANIZATION REPORT NUMBER(S): TR-84-45

5 MONITORING ORGANIZATION REPORT NUMBER(S): AFOSR-TR- 85-0005

6a NAME OF PERFORMING ORGANIZATION: University of Pittsburgh

6b OFFICE SYMBOL (If applicable):

7a NAME OF MONITORING ORGANIZATION: Air Force Office of Scientific Research

6c ADDRESS (City, State and ZIP Code): Center for Multivariate Analysis, Pittsburgh PA 15260

7b ADDRESS (City, State and ZIP Code): Directorate of Mathematical & Information Sciences, Bolling AFB DC 20332-6448

8a NAME OF FUNDING/SPONSORING ORGANIZATION: AFOSR

8b OFFICE SYMBOL (If applicable): NM

9 PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER: F49620-82-K-0001

8c ADDRESS (City, State and ZIP Code): Bolling AFB DC 20332-6448

10 SOURCE OF FUNDING NOS.

PROGRAM ELEMENT NO.	PROJECT NO.	TASK NO.	WORK UNIT NO.
61102F	2304	A5	

11 TITLE (Include Security Classification): A NOTE ABOUT THE STRONG CONVERGENCE OF THE NONPARAMETRIC ESTIMATION OF A REGRESSION FUNCTION

12 PERSONAL AUTHOR(S): Zhaoben Fang*

13a TYPE OF REPORT: Technical

13b TIME COVERED: FROM ________ TO ________

14 DATE OF REPORT (Yr., Mo., Day): SEP 84

15 PAGE COUNT: 7

16 SUPPLEMENTARY NOTATION: *On leave of absence from China University of Science and Technology.

17 COSATI CODES

FIELD	GROUP	SUB. GR.

18 SUBJECT TERMS (Continue on reverse if necessary and identify by block number): Nonparametric regression; kernel estimation; large sample property.

19 ABSTRACT (Continue on reverse if necessary and identify by block number)

Consider the regression model ~~$Y_i^* = g(x_i^*) + e_i^*$ $i = 1, 2 \ldots n$.~~ that X_i^*'s are unordered design variables, g unknown function defined ~~on [0,1] $\{e_i^*\}$ i.i.d r.v.~~ with mean 0 and finite moment of order $p > 1$. The asymptotic behavior of estimator g_n are studied.

cont Keywords include

20 DISTRIBUTION/AVAILABILITY OF ABSTRACT: UNCLASSIFIED/UNLIMITED ☒ SAME AS RPT. ☐ DTIC USERS ☐

21 ABSTRACT SECURITY CLASSIFICATION: UNCLASSIFIED

22a NAME OF RESPONSIBLE INDIVIDUAL: MAJ Brian W. Woodruff

22b TELEPHONE NUMBER (Include Area Code): (202) 767- 5027

22c OFFICE SYMBOL: NM

DD FORM 1473, 83 APR — EDITION OF 1 JAN 73 IS OBSOLETE.

~~AFOSR-TR-85-0004~~

AFOSR-TR-

A NOTE ABOUT THE STRONG CONVERGENCE OF THE NONPARAMETRIC ESTIMATION OF A REGRESSION FUNCTION

Zhaoben Fang

Center for Multivariate Analysis

University of Pittsburgh

85 02 [illegible] 002

A NOTE ABOUT THE STRONG CONVERGENCE OF THE NONPARAMETRIC ESTIMATION OF A REGRESSION FUNCTION

Zhaoben Fang

China University of Science and Technology
and
Center for Multivariate Analysis
University of Pittsburgh

September 1984

Technical Report No. 84-45

Center for Multivariate Analysis
University of Pittsburgh
Room 515 Thackeray Hall
Pittsburgh, PA 15260

A NOTE ABOUT THE STRONG CONVERGENCE OF THE NONPARAMETRIC ESTIMATION OF A REGRESSION FUNCTION

by

Zhaoben Fang

China University of Science and Technology

ABSTRACT

Consider the regression model $Y_i^* = g(x_i^*) + e_i^*$ $i = 1, 2 \ldots n$. X_i^*'s are unordered design variables, g unknown function defined on $[0,1]$. $\{e_i^*\}$ i.i.d r.v with mean 0 and finite moment of order $p > 1$. The asymptotic behavior of estimator g_n are studied.

Key Words: Nonparametric regression, Kernel estimation, large sample property.

We consider the regression model

$$Y_i^* = g(x_i^*) + e_i^* \qquad i = 1, 2, \ldots, n \tag{1}$$

where $x_i^* \in [0,1]$ are unordered design variables. e_i^* are iid random variables with zero mean. $g(x)$ is an unknown function defined on the interval $[0,1]$. The problem is to estimate $g(x)$ through observable variables Y_i^*.

Denote ordered design variables by $x_i \le x_2 \le \ldots \le x_n$, and define $Y_i = Y_j^*$ and $e_i = e_j^*$ if $x_i = x_j^*$ and $\delta_n = \max_{1 \le i \le n+1} (x_i - x_{i-1})$ where $X_0 \equiv 0$, $X_{n+1} \equiv 1$. An estimator proposed by Lin and Cheng [1] is given by

$$g_n(x) = \sum_{i=1}^{n} Y_i \int_{x_{i-1}}^{x_i} a_n^{-1} k\left(\frac{x-z}{a_n}\right) dz \tag{2}$$

where $\{a_n\}$ is a sequence of positive constants converging to 0. as $n \to \infty$, $k(z)$ is a kernel density satisfying:

(i) $k(z) \ge 0$;

(ii) $k(z) = 0$ for $z \notin [-L, L]$ for some positive constant L;

(iii) $\int_{-L}^{L} k(z)dz = 1$.

In fact, it is obtained by the natural estimator

$$h(x) = \begin{cases} Y_1 & x \le x_1 \\ Y_{i+1} & x_i < x \le x_{i+1} \quad i = 1, \ldots, n-1 \\ Y_n & x > x_n \end{cases} \tag{3}$$

after smoothing with weight function $a_n^{-1}k[(x - z)/a_n]$. Lin and Cheng [1] discussed the strong convergence of g_n and the rates of uniform convergence under a condition

$E|e^*|^p < \infty$, $p \geq 2$ among other conditions. In this note we relax the restrictions on the moments of e_i^* and improve the results of [1] and [2]. The results are given in the following theorem:

<u>Theorem</u> Assume that k(z) is bounded, $E|e^*|^p < \infty$ for $p > 1$, $\delta_n = 0(n^{-1})$, $a_n = n^{-d}$, for $0 < d < 1 - \frac{1}{p}$. If g(x) is continuous in [0,1], then for all $x \in (0.1)$

$$g_n(x) \to g(x) \qquad \text{a.s.} \tag{4}$$

Proof: Denote $\tilde{e}_i = e_i I(|e_i| \leq i^{1/p})$.

$\tilde{g}_n = \sum_{i=1}^{n} \tilde{e}_i \int_{x_{i-1}}^{x_i} a_n^{-1} k\left(\frac{x-z}{a_n}\right) dz$, where $I(\cdot)$ is an indicator function. Then

$$\begin{aligned} & g_n(x) - g(x) \\ &= g_n(x) - \tilde{g}_n(x) + \tilde{g}_n(x) - E\tilde{g}_n(x) + E\tilde{g}_n(x) - Eg_n(x) + Eg_n(x) - g(x) \\ &\overset{\Delta}{=} I_1 + I_2 + I_3 + I_4. \end{aligned}$$

In the following we prove $I_i \xrightarrow{a.s.} 0$, $i = 1, 2$ and $I_j \to 0$, $j = 3, 4$ separately. First, when n is large enough

$$\begin{aligned} I_4 = Eg_n - g &= \sum_{i=1}^{n} g(x_i) \int_{x_{i-1}}^{x_i} a_n^{-1} k\left(\frac{x-z}{a_n}\right) dz - g(x) \\ &= \sum_{i=1}^{n} \int_{x_{i-1}}^{x_i} [g(x_i) - g(z)] a_n^{-1} \; k\left(\frac{x-z}{a_n}\right) dz \\ &\quad + \sum_{i=1}^{n} \int_{x_{i-1}}^{x_i} [g(z) - g(x)] a_n^{-1} k\left(\frac{x-z}{a_n}\right) dz \end{aligned} \tag{6}$$

By the uniform continuity of g(x) in [0.1] and the property of k(z), the first term above is less than $\varepsilon \int_0^1 \frac{1}{a_n} k\left(\frac{x-z}{a_n}\right) dz$, where ε is arbitrary positive constant. The second term in (6) is not greater than

$$\sup_{|z-x|<\delta} |g(z)-g(x)| + \frac{1}{\delta} \sup_{|t|>\frac{\delta}{a_n}} |tk(t)| \int_0^1 |g(z)|\,dz$$

$$+ \sup_x |g(x)| \int_{|u|>\frac{\delta}{a_n}} k(u)du.$$

So it is easy to see when $n \to \infty$,

$$I_4 \to 0 \tag{7}$$

by Holder and Chebyshev inequality.

$$|I_3| \leq \sum_{i=1}^{n} E\{|e_i| I(|e_i| > i^{1/p})\} \int_{x_{i-1}}^{x_i} a_n^{-1} k\left(\frac{x-z}{a_n}\right) dz$$

$$\leq a_n^{-1}\delta_n \|k\| \sum_{i=1}^{n} [P(|e_i| > i^{1/p})]^{1-\frac{1}{p}} (E|e_i|^p)^{\frac{1}{p}}$$

$$\leq a_n^{-1}\delta_n \|k\| \sum_{i=1}^{n} \frac{1}{i^{1-\frac{1}{p}}} (E|e_i|^p)^{1-\frac{1}{p}} (E|e_i|^p)^{\frac{1}{p}}$$

$$\leq c\, a_n^{-1}\delta_n \sum_{i=1}^{n} \frac{1}{i^{1-\frac{1}{p}}} \leq c\, a_n^{-1}\delta_n n^{\frac{1}{p}} \to 0, \qquad n \to \infty \tag{8}$$

where $\|k\| = \sup_x |k(x)|$, c a constant. Notice that if $1 < b \leq 2$, we have $e^z \leq 1 + z + |z|^b$ *when* $z \leq 1$, *so if r.v* $Z_i \leq 1$, $EZ_i = 0$ *then* $E(\exp\{Z_i\}) \leq \exp(E|Z_i|^b)$. Now, take $b < p$, $d_n = \ell_n n \ell_n \ell_n n$, $Z_i = d_n(\tilde{e}_i - E\tilde{e}_i) \int_{x_{i-1}}^{x_i} a_n^{-1} k\left(\frac{x-z}{a_n}\right) dz$. By the choice of d_n and the conditions on δ_n and a_n in the theorem we know that $Z_i \leq 1$ when n is large enough. Thus, we have

$$E\left(\prod_{i=1}^{n} \exp\{Z_i\}\right) = \prod_{i=1}^{n} E(\exp\{Z_i\}) \leq \prod_{i=1}^{n} \exp(E|Z_i|^b)$$

$$\leq \prod_{i=1}^{n} \exp\{d_n (d_n \|k\| \delta_n a_n^{-1})^{b-1} E|e_i|^b \int_{x_{i-1}}^{x_i} a_n^{-1} k\left(\frac{x-z}{a_n}\right) dz\}$$

$$\leq \exp\{c\, d_n (d_n \delta_n a_n^{-1})^{b-1}\}$$

For given $\varepsilon > 0$, when n is large enough,

$$P(I_2 > \varepsilon) \le e^{-d_n\varepsilon} E(\exp\{d_n I_2\})$$

$$= e^{-d_n\varepsilon} E\Big(\prod_{i=1}^{n} \exp\{Z_i\}\Big) < e^{-\frac{1}{2}d_n\varepsilon}.$$

So, we have

$$\sum_n P(I_2 > \varepsilon) < \sum_n e^{\frac{1}{2}d_n\varepsilon} < \infty$$

by Borel-Cantelli lemma

$$\limsup_{n\to\infty} I_2 \le 0 \qquad \text{a.s.}$$

Similarly, it is true that

$$\liminf_{n\to\infty} I_2 \ge 0 \qquad \text{a.s.}$$

Therefore

$$I_2 \to 0 \qquad \text{a.s.} \tag{9}$$

Since $E|e_i|^p < \infty$, we know that $\sum_n P(|e_i| > j^{\frac{1}{p}}) < \infty$. By Borel-Cantelli lemma we have $P(|e_i| > i^{\frac{1}{p}}, \text{ i.o}) = 0$. Hence

$$\sum_{i=1}^{\infty} e_i^2 I(|e_i| > i^{\frac{1}{p}}) < \infty \quad \text{a.s.} \tag{10}$$

Finally,

$$|I_1|^2 = |\sum_{i=1}^{n} (e_i - e_i) \int_{x_{i-1}}^{x_i} a_n^{-1} k\,(\frac{x-z}{a_n})\, dz|^2.$$

By Schwartz inequality,

$$|I_1|^2 \le \sum_{i=1}^{n} \Big(\int_{x_{i-1}}^{x_i} a_n^{-1}\Big)\, k\,(\frac{x-z}{a_n})\, dz\Big)^2 \sum_{i=1}^{n} e_i^2 I(|e_i| > i^{\frac{1}{p}})$$

$$\leq a_n^{-1}\delta_n \, \| k \| \int_0^1 a_n^{-1} k\left(\frac{x-z}{a_n}\right) dz \sum_{i=1}^{\infty} e_i^2 I(\| e_i \| > i^{\frac{1}{p}})$$

$$\leq c \; a_n^{-1}\delta_n \sum_i e_i^2 I(| e_i | > i^{\frac{1}{p}}).$$

By (10) we have

$$| I_1|^2 \to 0 \quad \text{a.s.} \tag{11}$$

From (5), (7), (8), (9), and (11), the theorem follows.

REFERENCES

[1] CHENG, Kuang-Fu and LIN, Pi-Erh, Z. Wahrscheinlichkeitstheorie venv. Gebiete, 57, 223-233 (1981).

[2] BENEDETTI, J. Roy. Statist. Soc. B39, 243-253, (1977).

END

FILMED

3-85

DTIC