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This report is a collection of published papers reporting on research supported by AFOSR. These papers deal primarily with theoretical aspects of adaptive control of systems which cannot be precisely modeled, e.g., unmodeled dynamics and disturbances. These latter characteristics are fundamental issues in adaptive (and nonadaptive) control design for large space structures (LSS). Some of the general topics covered include: LSS modeling and model error, decentralized control, robust adaptive control,

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SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered) global stability, local stability, and persistent excitation.

TABLE OF CONTENTS

PAGE

Introduction 1

ISSUES IN THE ADAPTIVE CONTROL OF LARGE SPACE STRUCTURES 3

1. Introduction 4

2. Background Discussion 5
   2.1 Adaptive Control 5
   2.2 LSS Modeling and Model Error 5
   2.3 Decentralized Control for LSS 7

3. Robust (Non-Adaptive) Control 12
   3.1 Application to LSS 12

4. Adaptive Control 16
   4.1 Robustness of Adaptive Control 20

5. Concluding Remarks 22

References 23

AN INPUT-OUTPUT VIEW OF ROBUSTNESS IN ADAPTIVE CONTROL 25

1. Introduction 25

2. Error System Formulation 26

3. Continuous-Time Adaptive Model-Following 29

4. Global Stability Conditions 32

5. Local Stability Conditions 34

6. Conclusion 36

References 36

A LOCAL STABILITY ANALYSIS FOR A CLASS OF ADAPTIVE SYSTEMS 38

References 49

ROBUST ADAPTIVE CONTROL: CONDITIONS FOR LOCAL STABILITY 50

1. Introduction 50

2. Notation 50

PAGE

3. Adaptive Error System. 50
 3.1 Parameter Variational Error System. 50
 3.2 Global Stability Conditions 51
 3.3 Full Variational Form 52

4. Conditions for Local Stability 52

5. Persistent Excitation 53

6. Concluding Remarks 54
 6.1 Test Procedures 54
 6.2 Other Mechanisms for Local Stability 54

Appendix A: Proof of Theorem 4.1. 54
Appendix B: Proof of Theorem 5.1. 55
Appendix C: Proof of Theorem 5.2. 55

References 56

CONDITIONS FOR LOCAL STABILITY AND ROBUSTNESS OF ADAPTIVE CONTROL SYSTEMS. 57

1. Continuous-Time Case 59

2. Discrete-Time Case 68

References 73

ROBUST ADAPTIVE CONTROL: CONDITIONS FOR GLOBAL STABILITY. 75

1. Introduction 76
 1.1 Background. 76

2. Some Preliminaries 80
 2.1 Notation 80
 2.2 Stability 81
 2.3 Passivity 81
 2.4 Model Error 83
 2.5 Persistent Excitation 84

3. Adaptive Error Model 86

4. Conditions for Global Stability. 94

5. Application to Model Reference Adaptive Control. 101

6. Limitations Imposed by the SPR Condition 108

7. SPR Compensation 112

8. Conclusions. 120

Appendix A: Proof of Theorems 1 and 2 123
Appendix B: Proof of Lemma 5.1. 133

References 136

INTRODUCTION

This report contains a collection of technical papers describing research in adaptive control supported by AFOSR contracts F49620-83-C-0107 and F49620-84-C-0054.

The basic objective of this research program is to establish the theoretical foundations and performance limitations for adaptive control applications to large space structures (LSS). An important element of the research is to examine implementation concepts which can lead to appropriate hardware development.

The program was originally formulated in late 1982 in response to the increasing concern that performance robustness of Air Force LSS type systems would be inadequate to meet mission objectives. In particular, uncertainties in both disturbance spectra and system dynamics characteristics (both time varying and stochastic uncertainty) usually significantly limit the performance obtainable with fixed gain, fixed architecture controls. The use of adaptive type controls, where disturbances and/or plant models are identified prior to or during control, gives systems designers more options for minimizing the risk in achieving performance benchmarks.

The research was originally directed toward real-time adaptation of the estimator form using variable order lattice filters to construct the desired compensation. Early in the research, however, lack of a well-developed robustness theory for adaptive mechanizations required a reexamination of the problem at a more fundamental level, i.e., development of model and disturbance uncertainty bounds for which adaptive algorithms would exhibit (stable) desired performance. Toward this end there have been two major accomplishments:

(1) Development of Theory: In examining the possible use of "rapid" adaptive control it was necessary to generate new theory of use on large space structures. This theory accounts for the effect of unmodeled dynamics with distributed parameter systems, such as flexible space structures, and extends current adaptive theory in several directions.

In the first place, current adaptive theory provides conditions for "global" stability, i.e., bounded-input, bounded-output stability with no limitation on the size (or spectrum) of the bounded-inputs (e.g., disturbances and references). Secondly, the theory is limited to finite-dimensional linear systems. This latter condition cannot be satisfied by a flexible space structure, which is a distributed parameter system. Also, the disturbance and reference inputs effecting the spacecraft have limited magnitudes and spectrums and these limits are known, although not precisely. The theory we have developed circumvents those difficulties by providing conditions for "local" stability, i.e., limitations in input size and spectrum are accounted. The theory also allows for a distributed system as well as providing quantifiable bounds on permissible model error. These results extend the state-of-the-art in adaptive theory beyond the current limits.

(2) Methodology Development: The use of "slow" adaptive control, which is more practical than rapid adaptive control in most space applications, necessitated a new methodology development merging key ideas in parameter estimation, system identification, and robust control design. By "slow" we mean that there is sufficient time to run batch identification before the control system is modified. The methodology we have developed resolves a long standing problem with adaptive systems of this type, namely, the means to provide a guaranteed level of performance given an "identified" model of the system together with the model error between the system and the identified model. In fact, our methodology generates performance vs. model error tables (to be stored in the computer) from which the control design is immediately obtained. Moreover, the order of the control design is determined strictly on the basis of model error and performance demand, rather than trial and error as has been suggested in the past.

Presented at 23rd IEEE Conf. on Decision and Control, Dec-1984, Las Vegas, NV - Submitted to AIAA Journal of Guidance and Control, Dec-1984.

ISSUES IN THE ADAPTIVE CONTROL OF LARGE SPACE STRUCTURES*

by

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Abstract

This paper examines some basic theoretical and practical issues in the adaptive control of large space structures (LSS). Particular attention is paid to the practical issues of model error, decentralization, and subsystem performance allocation. It is concluded that the currently available theory of adaptive control, which is based on global stability, centralized information, and perfect modeling, is not well suited for an LSS. A direction for future research is suggested which is based on a theory of local stability for the adaptive system.

*Research supported by AFOSR under contract F49620-84-C-0054.

1. INTRODUCTION

The high performance requirements of Large Space Structures (LSS) together with potentially large uncertainties in the system model, motivate the use of an adaptive control system. Although a great variety of adaptive control schemes exist for lumped parameter, small scale systems, e.g. [1], these methodologies cannot be directly applied to the LSS because of the following issues:

(1) Model error - The actual system is a distributed parameter system, theoretically of infinite dimension, whereas the adaptive scheme must be based on a reduced order model (ROM) of finite dimension. This discrepancy introduces one kind of model error, the effect of which is often referred to as 'spillover.' Another class of model errors are those attributable to uncertainty in parameters (e.g., mode shapes), neglected non-linearities, and other uncertain unmodeled phenomenon (e.g., residual modes included).

(2) Decentralized control - In some cases the physical size and complexity of the LSS makes it impractical to use a centralized control structure due to considerations of actuator/sensor costs, system reliability, and computational requirements, as well as the step-wise deployment (and removal) of sub-sections.

(3) Performance allocation - Since the performance requirements are stringent, it is necessary to allocate performance in an efficient manner so that sub-systems can help one another.

In this paper we explore the above issues from the point of view than an LSS can be represented as a large-scale interconnected system [2]. The interconnection model used is composed of a number of uncertain subsystems which are linked to other subsystems by an interconnection operator, which is also uncertain. Uncertainty in the subsystems and interconnections is expressed by using the notion of a conic model [3]-[4], i.e., representing a complicated uncertain dynamic system as belonging to a set of systems generated from simpler dynamic systems.

By using this representation the issues enumerated above can be brought within a single framework which facilitates the analysis and synthesis of adaptive controllers as discussed in [5]-[6].

2. BACKGROUND DISCUSSION

The development of a design methodology for adaptive control of LSS involves many different issues. A comprehensive discussion of the theoretical and practical problems involved in both LSS control and adaptive control is well beyond the scope of this paper. In this section we present a very selective discussion of the issues that seem most relevant.

2.1 Adaptive Control

Adaptive methods have achieved a great amount of success in producing stable, convergent adaptive controllers and adaptive observers for systems whose structure is known and whose parameters are constant but poorly known or slowly time-varying. Adaptive schemes may be direct, as shown in Fig. 1, i.e., the available control parameters are directly adjusted (adapted) to improve the overall system performance or indirect, as shown in Fig. 2, i.e., the system parameters are identified (based on the assumed system structure) and the control commands are generated from these parameter estimates as though they were the actual values.

The use of such methods on distributed parameter or large scale systems, like LSS, is greatly limited by the modeling problem—the adaptive scheme must be based on a ROM (Reduced Order Model) of the actual system and, hence, the order of the model is, and must remain, substantially lower than the controlled system [7]-[8].

The crux of the problem with adaptive control is to guarantee that the adaptive controller that is designed on the ROM will not, through a combination of spillover and model uncertainty, diverge and ultimately go unstable. This problem is extremely difficult and has only recently been addressed, even in the general context of adaptive system, e.g., [5],[6],[9].

2.2 LSS Modeling and Model Error

Traditionally, control design is based on models of the system which have been validated thoroughly by extensive testing. Since the structural integrity of LSS does not permit ground tests, the usual approach to modeling is not feasible for LSS. The primary difficulty is validating the model, i.e., determining a quantifiable measure of model uncertainty. In this

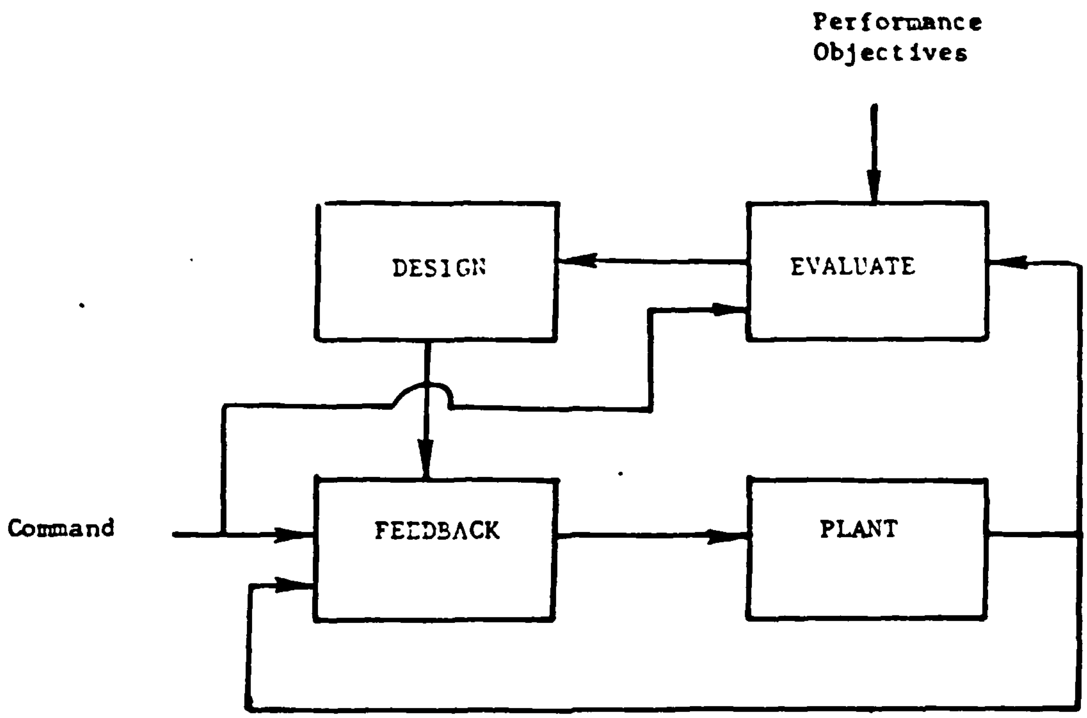


Figure 1. Direct Adaptive Control

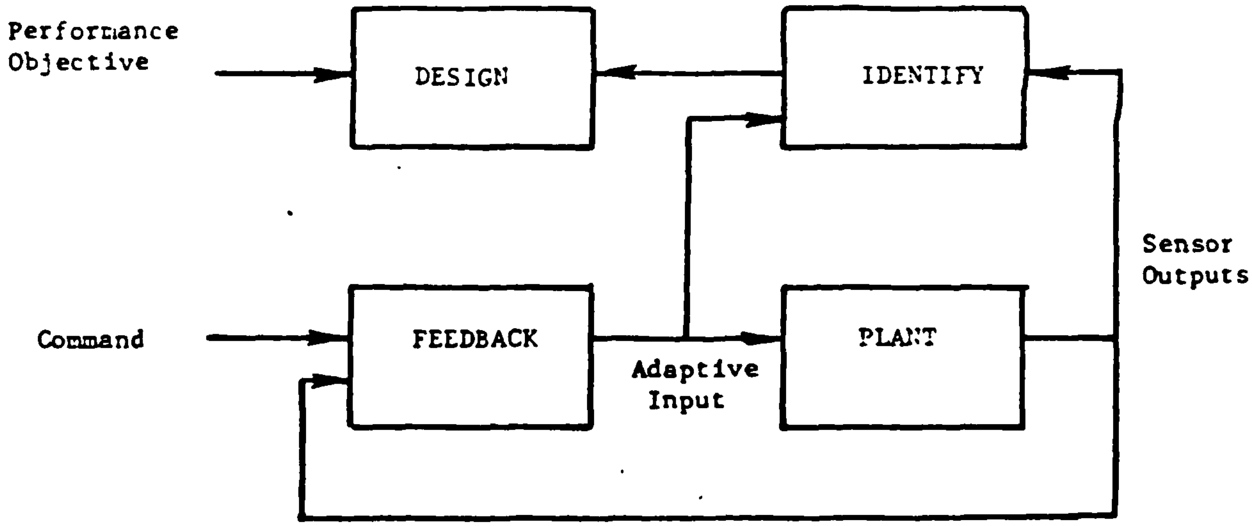


Figure 2. Indirect Adaptive Control

regard, the LSS has the interesting property of being an infinite dimensional system, at least theoretically so, but practically of very high order. Thus, the order of the design model and controller is not known a priori. In consequence, adaptive controllers for LSS should be not only parameter adaptive, but <u>order</u> adaptive as well. This leads naturally to the consideration of order-recursive lattice adaptive control [10].

A very natural means to determine model error (sometimes referred to as plant uncertainty) is to perform an experiment which compares the model with data from the actual system (or plant). If there is no error between the model and the plant, then we have perfect knowledge of the plant. Normally, the situation is the opposite--the error is non-zero and represents how close the model is to the plant. If we quantify this experiment, by defining a specific measure of the error size, then this gives a sensible statement as to model accuracy. For example, during experimental modeling using system identification methods the model uncertainty is measured as the difference between the measured output and the model output. Bounding this model error, for all possible input/output pairs, results in a set characterization of plant uncertainty. For example, a set description of an uncertain LTI plant is to define a ball in the frequency domain. The center of the ball is the nominal plant model, and the radius defines the model error. This set model description is one type of a more general set description, referred to as a <u>conic</u>-<u>sector</u> [4]. The uncertainty in the plant induces an uncertainty in the input/output map of the closed-loop system which can again be characterized by a conic sector. Performance requirements for the control system can be translated into statements on the conic sector which bounds the closed-loop systems, making it possible to check whether a given design meets specifications, and providing guidelines for robust controller design, e.g., [11].

2.3 <u>Decentralized Control for LSS</u>

In the context of LSS control design, what we mean by a decentralized control is the following: The control system is made up of a number of sub-controllers (local controllers) which have limited authority over the LSS and which use limited information about the LSS. The limitations on control authority and the information pattern are the main features of the decentralized control problem. The general structure of such a decentralized control system is illustrated in Figure 3. The dashed lines indicate a partial information exchange, e.g., the local controller receives reference commands (or discretes) from a higher level control (the coordinator) and/or information from other local controllers in the form of an 'aggregated' state.

In a decentralized control we also need to determine the effect of partial information on closed-loop performance. The two kinds of system ignorance, i.e., model uncertainty and partial information, can be viewed under one framework by considering the controlled LSS as an interconnected system, e.g., [2].

An interconnected system is a system which consists of several subsystems interacting through various interconnection operators. The key feature used here is that knowledge about the subsystems and interconnection operators is incomplete.

Techniques for decentralized synthesis more or less adhere to the following steps (see, e.g., [12]).

Step 1: (Decomposition) Identify the individual subsystems and the interconnection constraints between subsystems.

Step 2: (Local analysis) Design the local controller so that each individual subsystem satisfies specified local requirements.

Step 3: (Global analysis) Verify that the interconnection of the individual subsystems satisfies specified global requirements.

Step 4: (Robustness) Verify that the total system performance is robust with respect to failures, disconnections, parameter changes, etc.

In practice, these 'steps' overlap and iterations are required. However, the first step, decomposition, is necessary to begin the design process.

There are several methods available for decomposition. These may be broadly grouped into generic categories based on: time-scale separation, frequency separation, and performance properties (e.g., observability, controllability, quadratic cost, etc.). All of these can be viewed graphically as well as tabularly, and many physical systems, LSS included, possess all three types of decompositions (see, e.g., [13], Chapters I - III for complete exposition).

It is important to emphasize that many decomposition methods are purely mathematical and may decompose the system into simpler numerical problems convenient for parallel or distributed processing. In the LSS environment,

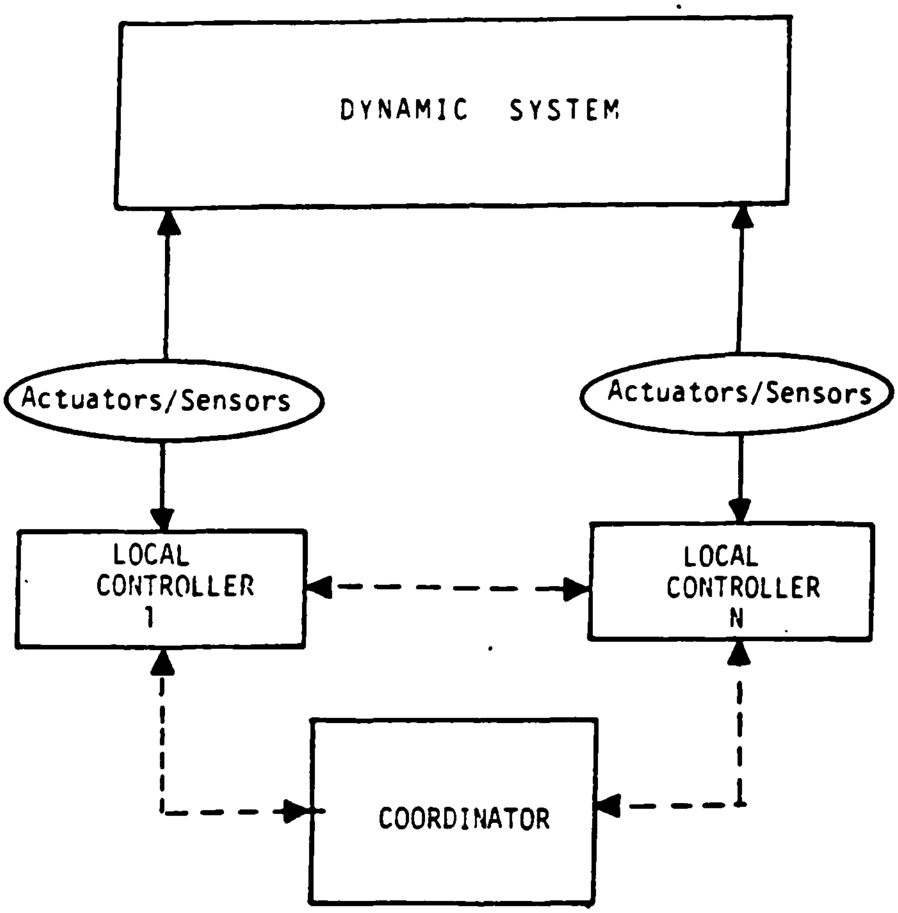


Figure 3. Decentralized LSS Control. The dashed lines indicate a partial exchange of information.

decompositions normally result because a natural separation is physically or geographically present between functional components of the system. For example, consider the following task oriented decomposition:

(1) High authority actuators and sensors with low bandwidth for rigid body control

(2) High authority actuators with bandwidths to 20 Hz for slewing, with possibly a series connected low authority actuator for final small motion slew correction

(3) Medium authority actuators and sensors for vibration isolation of disturbances

(4) Low authority actuators for isolation of critical structural subsystems (mirrors, focal plane, etc.)

(5) Very low authority actuators/sensors for active damping or resonant absorption.

Some decompositions result from spatial differences; weak dynamic interaction effects can be easily identified. A decomposition also occurs from temporal differences; phenomena occurring at different time-scales, e.g., a separation between fast and slow modes or between low frequency and high frequency effects. For example, groups of the modes can be separately controlled by separate controllers which do not destabilize each other. Specific combinations of weak dynamic coupling and separation of slow and fast modes can often be identified, e.g., Figure 4.

In many cases, delegation of control authority is 'politically' practical. It is unrealistic to assume that the manufacturer of one device will ever design another manufacturer's device, or even that both will delegate complete authority to a systems house. The only communication possible in this case is to assess each manufacturer with specifications so that the operating devices do not compete. For example, simultaneous on-orbit assembly of different parts of the LSS may be accomplished using temporary vibration control systems built by various manufacturers. This motivates a design specification which includes tolerances that allow for some variety, such that the overall differences do not impair on-going missions or constructions in other parts of the LSS.

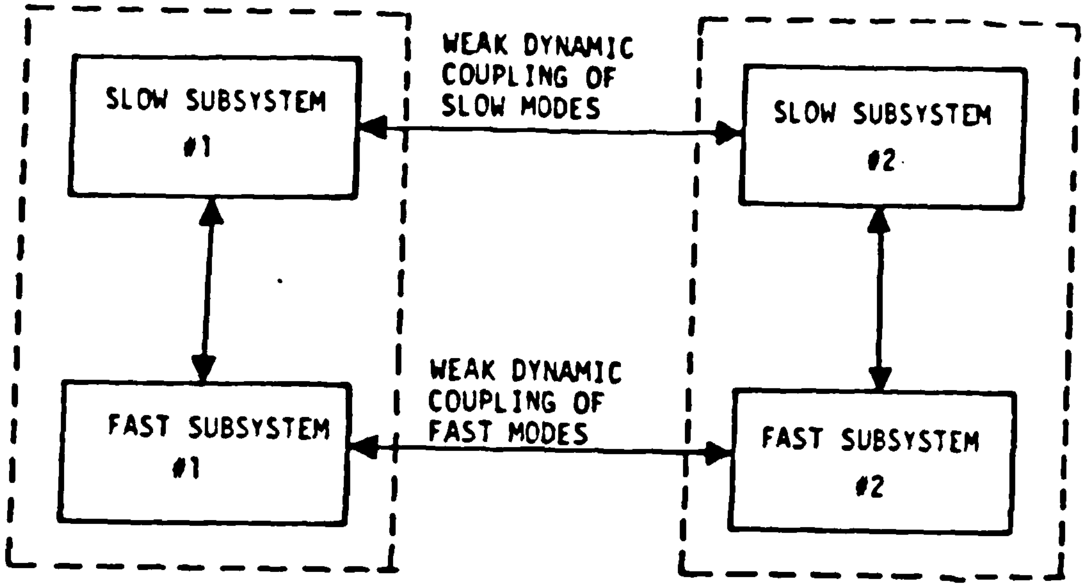


Figure 4. Weak Dynamic Coupling with Slow and Fast Modes

Robust Interconnected Systems Approach

The input/output view of interconnected systems is completely compatible with conic-sector model uncertainty descriptions. Representative theorems (see, e.g., [2]) refer to interconnected systems of the form

$$\left.\begin{aligned} e_i &= u_i - \sum_{j=1}^{m} H_{ij}\, y_j \\ y_i &= G_i e_i \end{aligned}\right\} \qquad i = 1, \ldots, m$$

where $G_i, \ldots, G_m$ are the subsystem operators, and $H_{11}, \ldots, H_{mm}$ are the interconnections. The key features of the theorems are:

(1) If the subsystems and interconnections have quantifiable input/output properties, e.g., conic-sector bounds and/or passivity, then the total system will exhibit performance properties directly attributable to the subsystems and interconnections, e.g., conic-sector bounds and/or passivity.

(2) The total system properties can be obtained by combinations of subsystem and interconnection properties.

This latter point is extremely important in the decentralized setting. This means that one subsystem can 'help' overcome the deficiencies of another system. Furthermore, this also gives a clue to the question of allocating subsystem performance in an efficient way so that a desired overall performance is achieved.

3. ROBUST (NON-ADAPTIVE) CONTROL

Before attempting to develop a methodology for adaptive control of uncertain (decentralized) systems, it is logical to consider the non-adaptive case first. In fact, the adaptive design procedure should build on the robust design procedure.

Historically, research in robust control theory has proceeded from an input/output view of systems, e.g., [3],[14]. The more recent of these results [4],[15] are variations on the Small Gain Theorem [3]. The theorem asserts that if the 'loop gain' of a feedback system is less than unity, then the closed-loop system is stable. However, to properly utilize the theorem it is necessary to isolate the source of the model error. This is accomplished by what is called a 'loop-transformation.'

Many variations on loop-transformations are available [11], [15], and small gain theory can be applied to analyze the robustness of criteria other than stability, e.g., tracking response transients. Moreover, the technique can also be used to assess the impact of various kinds of error sources. For example, typical sources of model error in spacecraft systems include:

(1) numerical errors due to approximate modeling techniques, e.g., high order NASTRAN models.

(2) actual parameter changes in the LSS ,e.g., thermal effects, gravity, spacecraft and antenna dimensions, mass distributions, etc.

(3) unmodeled dynamics, e.g., effect of reduced order modeling, neglected residual modes ('spillover'), neglected actuator/sensor dynamics, non-linearities, etc.

(4) incomplete data obtained from on-earth testing, e.g., partially assembled structures in simulated zero-g.

Therefore, uncertainty in the baseline model arises from both actual causes and intentional approximations of complicated phenomena. In many cases, these are indistinguishable.

Negative results from the robustness analysis may warrant redesign of the controller, or even upgrading the reduced order design model if adequate robustness cannot be achieved (e.g., [11]).

3.1 Application to LSS

Consider a controlled spacecraft, as depicted in Figure 5, with the following model:

Sensor Model

$$y_s = y + n \,, \qquad n = \text{sensor noise}$$

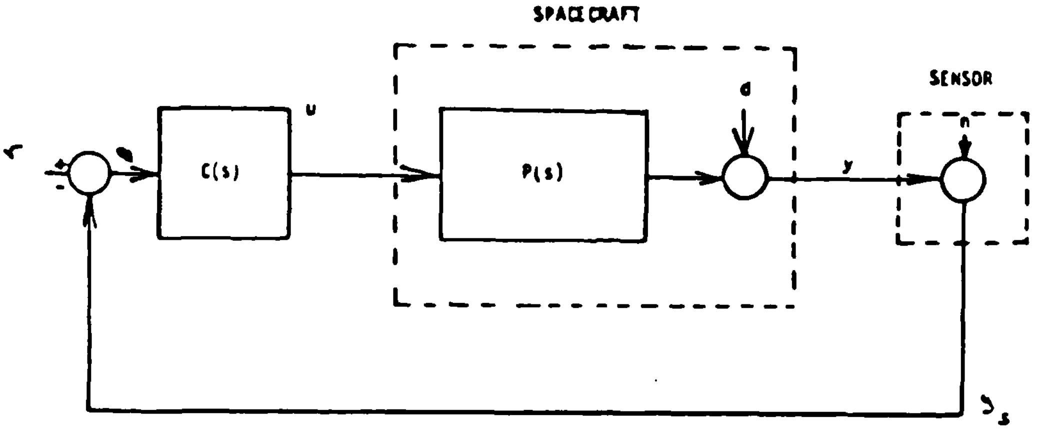


Figure 5. Block Diagram of Controlled Spacecraft

Dynamic Model

$$y = M(s)u + d \,, \qquad d = \text{disturbance}$$

Controller

$$u = C(s)(r-y_s) \,, \qquad r = \text{reference}$$

where $M(s)$ is a finite dimensional transfer function matrix model representing the dynamics of the actuators and spacecraft and $C(s)$ is the transfer function matrix of the controller.

Let the actual dynamics be represented by

$$y = P(s)u + d$$

where $P(s)$ is not necessarily finite dimensional. For example, $P(s)$ can be either a high order NASTRAN model or represent data from the actual system, whereas $M(s)$ is the reduced order control design model. Thus, the controlled output is:

$$y = \underbrace{(I + PC)^{-1}}_{H_{yd}} d + \underbrace{(I + PC)^{-1} PC}_{H_{yr} = I - H_{yd}} (r-n)$$

(The complex variable 's' has been suppressed for brevity of notation, unless needed for clarification.)

Suppose that M is a reduced order model of P. Let

$$P = M + \Delta_r$$

where Δ_r represents the effect of neglected residual modes. Thus, following [11], the closed-loop response is:

$$H_{yd} = \bar{H}_{yd} - (I + MC)^{-1}[I + \Delta_r C(I + MC)^{-1}]^{-1} \Delta_r C(I + MC)^{-1}$$
$$\bar{H}_{yd} = (I + MC)^{-1}$$

where $\bar{H}_{yd}$ is the nominal transfer function with no model error, i.e., $\Delta_r = 0$.

Similarly, we can examine the way in which other kinds of model error enter into the closed-loop dynamics. The spacecraft model, for example, may not include actuator dynamics. This omission can be represented by the model error form,

$$P = M(I + \Delta_a)$$

where Δ_a represents the deviation from the dynamics of an actuator with infinite bandwidth. In this case, the closed-loop response is:

$$H_{yd} = \bar{H}_{yd} - (I+MC)^{-1} M[I+\Delta_a(I+CM)^{-1}CM]^{-1}\Delta_a C(I+MC)^{-1}$$

Applying the Small Gain Theorem [11], [15], the <u>actual</u> spacecraft system is stable if:

(1) The spacecraft model M is stabilized by C, i.e., the transfer functions $(I + MC)^{-1}$, $C(I + MC)^{-1}$, $(I + MC)^{-1}M$, and $(I + CM)^{-1}CM$ are all stable.

and either:

(2) Reduced order model errors are bounded by

$$||\Delta_r(j\omega)|| < 1/ ||C(I+MC)^{-1}(j\omega)||, \quad \omega \geq 0$$

or

(3) Actuator model errors are bounded by

$$||\Delta_a(j\omega)|| < 1/||(I+CM)^{-1}CM(j\omega)||, \quad \omega \geq 0$$

where the norm $||.||$ can be <u>any</u> matrix norm. Typically, the maximum singular value $\bar{\sigma}(.)$ is used. However, this may be unnecessarily conservative; other measures are available, e.g., the Perron eigenvalue [4]. The selection of the appropriate matrix norm will be examined.

In [7], there are several examples of these robustness tests using the ACOSS model, CSDL-I. Table 1 summarizes these robustness tests for generic model errors bounded in singular value by

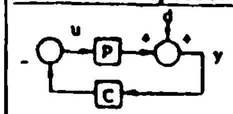
$$\bar{\sigma}[\Delta(j\omega)] \leq \delta(\omega), \quad \omega \geq 0$$

where $\delta(\omega)$ is determined from input/output tests, e.g., RMS tests. Table 1 shows the <u>stability margins</u>, denoted by δ_{sm}, defined as the maximum bound on model error, which (at the specified location, e.g., actuator, sensor, etc.) ensures stability. Thus,

$$\delta(\omega) < \delta_{sm}(\omega), \quad \omega \geq 0.$$

Note that the tests shown presume only <u>one</u> location for uncertainty. Bounds on simultaneous errors at different locations are easily obtained [4],

TABLE 1. STABILITY AND PERFORMANCE ROBUSTNESS MARGINS

SPECIFIC FORM OF MODEL ERROR	SOURCE OF MODEL ERROR IN SPACECRAFT SYSTEM	STABILITY MARGIN		PERFORMANCE MARGIN
P:= actual plant M:= model Δ:= model error		Guaranteed stability if $\bar{\sigma}(\Delta) < \delta_{SM}$	Controlled Spacecraft 	Nominal $(\Delta = 0)$: $\bar{H}_{vd} = (I + MC)^{-1}$ Guaranteed performance $\bar{\sigma}[H_{vd} - \bar{H}_{vd}] \le \rho\,\bar{\sigma}[\bar{H}_{vd}]$ if $\bar{\sigma}(\Delta) < \delta_{PM}$
Additive $P = M + \Delta$	• neglected residual modes, e.g. - spillover - reduced order modeling - uncertain interacting structural modes	$\delta_{SM} = 1/\bar{\sigma}[C(I + MC)^{-1}]$		$\delta_{PM} = \delta_{am}\,\rho(\rho + 1)^{-1}$
Output Multiplicative $P = (I + \Delta)M$	• sensor errors - misalignments - bandwidth - scale factors • neglected high frequency phenomena, e.g., - model approximations - friction - stiction	$\delta_{SM} = 1/\bar{\sigma}[(I + MC)^{-1}MC]$		$\delta_{PM} = \delta_{sm}\,\rho(\rho + 1)^{-1}$
Input Multiplicative $P = M(I + \Delta)$	• actuator errors - bandwidth - nonlinearities - parasitics - quantization (• neglected high frequency phenomena, as above)	$\delta_{SM} = 1/\bar{\sigma}[(I + CM)^{-1}CM]$		$\delta_{PM} = \delta_{sm}\,\rho\{\rho+\delta_{sm}\bar{\sigma}[(I+MC)^{-1}]$ $\cdot\bar{\sigma}[(I+MC)^{-1}M]/\bar{\sigma}[(I+MC)^{-1}]\}^{-1}$
Output Multiplicative $P = (I + \Delta)^{-1}M$	• low frequency parameter errors, e.g., - uncertain mode shapes - variations in mass distribution - thermal effects	$\delta_{SM} = 1/\bar{\sigma}[(I + MC)^{-1}]$		$\delta_{PM} = \delta_{sm}\,\rho(\rho + 1)^{-1}$
Input Multiplicative $P = M(I + \Delta)^{-1}$	• uncertain right half plane poles	$\delta_{SM} = 1/\bar{\sigma}[(I + CM)^{-1}]$		$\delta_{PM} = \delta_{sm}\,\rho\{\rho+\delta_{sm}\bar{\sigma}[(I+MC)^{-1}]$ $\cdot\bar{\sigma}[(I+MC)^{-1}M]/\bar{\sigma}[(I+MC)^{-1}]\}^{-1}$

[7]. Again, the upper singular value norm $\bar{\sigma}(.)$ can be replaced by any other matrix norm $||.||$.

Performance Robustness

A similar procedure will be used to determine an upper bound on model error to ensure a specified level of performance, i.e., performance robustness. Let desired performance be defined by

$$\bar{\sigma}[(H_{yd} - \bar{H}_{yd})(j\omega)] \leq \rho(\omega)\bar{\sigma}[\bar{H}_{yd}(j\omega)]$$

Thus, $\rho(\omega)$ specifies a bound on the relative deviation of H_{yd} about the nominal $\bar{H}_{yd}$. For example, if the effect of reduced order modeling is bounded by

$$\bar{\sigma}[\Delta_r(j\omega)] \leq \rho(\omega)[\rho(\omega)+1]^{-1}/\bar{\sigma}[C(I+MC)^{-1}(j\omega)], \quad \omega \geq 0$$

then the desired performance robustness is guaranteed.

Similar expressions can be obtained as a result of other sources of model error, e.g., sensor model error. Table 1 summarizes these performance robustness tests for generic model error. The table shows the performance margins, denoted by δ_{pm}, defined as the maximum bound on model error (at the specified location) which ensures the specified performance tolerance. Thus,

$$\delta(\omega) \leq \delta_{pm}(\omega) \quad , \quad \omega \geq 0 \ .$$

guarantees performance robustness. This also guarantees stability, since,

$$\delta_{pm}(\omega) < \delta_{sm}(\omega) \ , \quad \omega \geq 0.$$

4. ADAPTIVE CONTROL

It is compelling to pass directly from the preceding notions about robust control to the following indirect 'adaptive' control algorithm:

Identification

Step 1: Using input/output data estimates the free model parameters, thereby obtaining M.

Step 2: Using the same input/output data in Step 1 obtain the upper bound on the residual (unmodeled) dynamics.

Design

Step 3: Using the model M from Step 1 along with the model error bound in Step 2, determine a control compensator C such that performance is bounded above by a desired level. If no such control C can be found, return to Step 1 and upgrade the model fidelity.

Reconfiguration

Step 4: Reconfigure the existing control in accordance with Step 3.

Step 5: Return to Step 1 and repeat.

Although this process appears entirely reasonable, there are several open questions. In particular:

(1) What is the best identification procedure for Step 1? For example, what are the advantages of output error, ARMA models, lattice forms, etc?

(2) How is the control design in Step 3 actually implemented in Step 4? If, for the example, the new controller is put in place instantly, then there may very well be a transient introduced such that system performance, although stable, is unacceptable. If, on the other hand, the new controller is gradually phased in, e.g.,

$$u = (1-\alpha)u_{OLD} + \alpha\, u_{NEW}$$

where α varies slowly from '0' to '1', then the question is: how slowly?

(3) If in Step 3 no control is found to satisfy performance, then how is the model fidelity upgraded? Should we add modes to the model? Or perhaps the test for model error is too conservative. If so, then how can we build a hierarchy of model error tests?

(4) In Step 3, what is the design procedure for selecting the feedback given a nominal model M, a bound on model error, and a desired performance level?

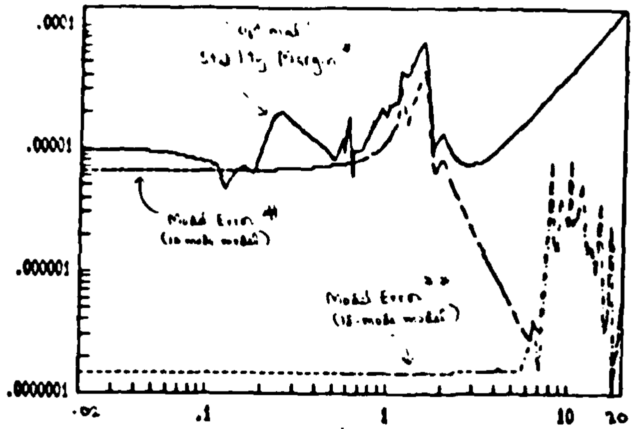
With the exception of (2), all the above issues pertain to robust (non-adaptive) design. This should not be surprising, since one must assume the existence of a tuned robust control, which could be attained by the adaptive system. Thus, in order to prove the existence of a tuned control, it must be possible to design one.

Application

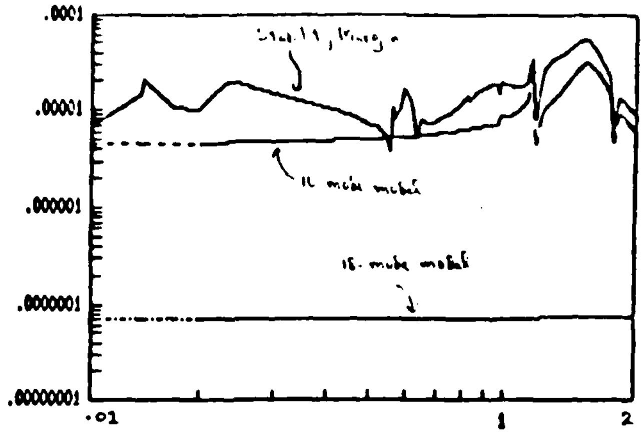
Graphs 1-5 show magnitude of model error vs. frequency us;ing various models compared to the input-output data for the CSDL #2 structure. In all the figures we have plotted the "optimal" stability margin determined on the basis of an accurate knowledge of the first 18 modes (up to 2 Hz). In order to achieve guaranteed performance levels near specification, it is necessary that model error be significantly smaller than the optimal margin in the 2 Hz range.

Graph 1 shows model errors for an accurate 10 mode model (0.64 Hz) and an accurate 18 mode model (1.77 Hz). To achieve performance it is necessary to identify modes 11 to modes 18 (0.81 to 1.77 Hz) in order to reduce the 10 mode model error. Graph 2 shows a blow-up of the 2 Hz region of interest.

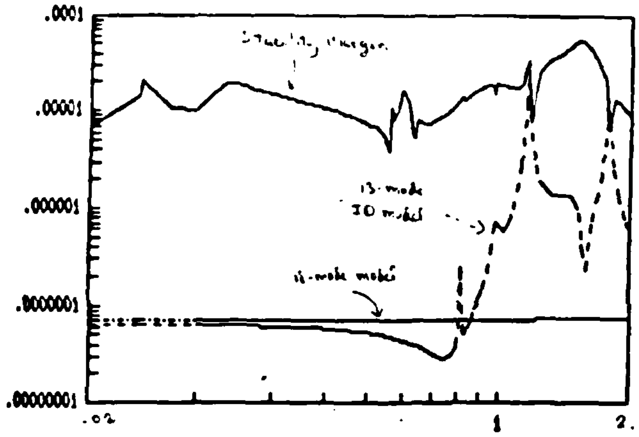
In Graphs 3 and 4, respectively, we show the result of two different 13 mode ID procedures. Both procedures use 2 mode models where the parameter estimates are obtained from data which is filtered over narrow frequency bands. The ID procedure in Graph 5 sweeps overlapping modal bands 11-14, 13-16, and 15-18. The models are then added together to form the 13-mode model. Although the model error is very close to ideal (18-mode model) below 1 Hz, there is considerable peaking near modes 17 and 18. The procedure used to obtain Graph 4, however, shows a more uniformly small error. In this case a 13-mode model is obtained by sweeping through narrow non-overlapping frequency bands, i.e., modal bands 11-14, 12-15, 13-16, and 14-18. Using this latter 13-mode model it is possible to obtain guaranteed performance very close to optimal even though modes 14, 15, and 17 are not completely known.



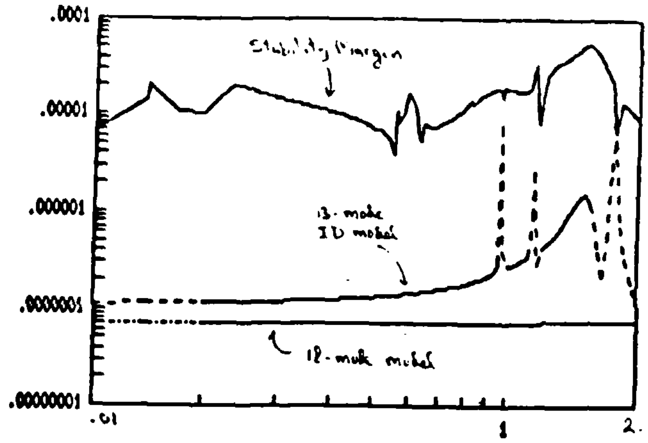
Graph 1



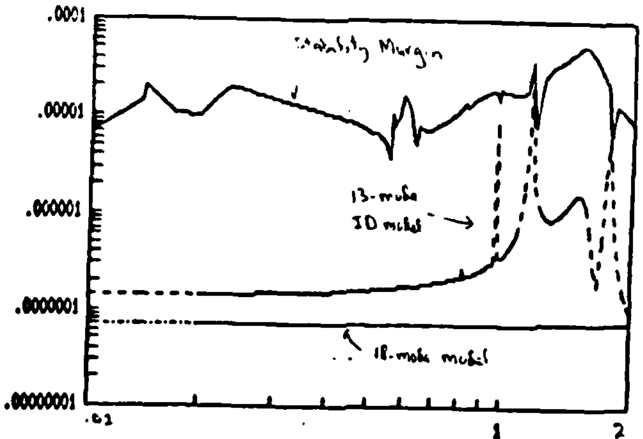
Graph 2



Graph 3



Graph 4



Graph 5

Graphs 1-5: Magnitude of Margins and Model Error vs. Frequency (Hz)

4.1 Robustness of Adaptive Control

In summary, a fundamental issue in the design of an adaptive controller for an LSS is robustness to reduced order modeling, parameter uncertainty, and unmodeled dynamics. Current theory, which provides conditions for stability (or convergence) of adaptive systems, is limited to global stability and relies on the passivity of a particular subsystem operator [1], [5]. In this regard, since the LSS has an inherent passivity with co-located actuators/sensors, it is natural to exploit this for adaptive control, e.g., [16], [17]. However, the presence of actuator dynamics easily violates the passivity requirement.

For example, consider actuator error

$$P = M(I + \Delta_a),$$

Δ_a unknown but stable where M, the nominal model, is in model form, i.e.,

$$M = sB'\Phi(s^2I + 2sZ\Omega + \Omega^2)^{-1}\Phi'B$$

with Ω and z diagonal matrices of modal frequencies and modal dampings, respectively, ϕ is an orthonormal matrix whose columns are the approximate mode shapes; and B is the actuator influence matrix. Thus, M is passive, in fact, M is positive real (PR), i.e., M is exponentially stable, and

$$\underline{\mu}[M(j\omega) := \frac{1}{2}\,\underline{\lambda}[M(j\omega) + M(-j\omega)'] \geq 0, \quad \omega \in R$$

where $\underline{\lambda}(.)$ is the smallest eigenvalue. For scalar systems, $\underline{\mu}[M(j\omega)] = \mathrm{Re}[M(j\omega)]$. In [5] it is shown that P remains passive if M is passive, Δ_a is stable, and Δ_a is bounded such that

$$\bar{\sigma}[\Delta_a(j\omega)] < \underline{\mu}[M(j\omega)]/\bar{\sigma}[M(j\omega)], \quad \omega \in R$$

As discussed in [5], this bound is very conservative and easily violated by even the most benign actuator dynamics, e.g., a second order actuator model. On the other hand, the SPR condition is a sufficient condition and not necessary for stability of the adaptive system. Further, practical evidence from actual applications supports the fact that SPR is not needed to provide high performance adaptive systems, e.g., [18].

The need for the SPR condition can be eliminated by considering <u>local</u> stability rather than global stability [6]. Local stability refers to stability where known restrictions exist for the system external inputs, uncertain parameters, and unmodeled dynamics. For example, persistent excitation induces exponential stability [18]. Since an exponentially stable system is inherently robust, it is logical to expect that unmodeled dynamics could be tolerated. In [6] several mechanisms—including persistent excitation— are examined which ensure stability of the adaptive system, without SPR, provided certain other restrictions are enforced, e.g., slowly varying signals, approximate SPR, and restricted signal magnitudes.

<u>Application to LSS</u>

Consider the LSS adaptive control system

$$y = d + Pu$$

where d is the disturbance; P is the $m\times m$ transfer function matrix across colocated actuators/sensors. Let each of the m control signals be given by

$$u_i = -\hat{\Theta}_i y_i \,, \qquad i = 1, \ldots, m$$

where $\hat{\Theta}_i(t)$ is the adaptive gain at each colocated station. The objective of the adaptive controller is to suppress the vibrations due to the disturbance, while achieving a specified damping in certain critical modes. Note that the adaptive control structure is, in effect, decentralized. A typical parameter update law [1] is,

$$\left.\begin{aligned} \dot{\hat{\Theta}}_i &= g_i e_i y_i , \; g_i > 0 \\ e_i &= W_i(s) y_i \end{aligned}\right\} \qquad i = 1, \ldots, m$$

Local stability conditions exist (see [6] for details) if the system $S: (u_1,u_2) \longmapsto (e_1,e_2,y_1,y_2)$, shown in Fig. 6, is stable where

$$L = \mathrm{diag}\left(\frac{g_1}{s}, \ldots, \frac{g_m}{s}\right)$$

$$M = y_* H_1 y_*' + e_* H_2 y_*'$$

$$H_1 = W(I + PC_*)^{-1} P$$

$$H_2 = (I + PC_*)^{-1}$$

$$W = \text{diag}\ (W_1, \ldots, W_m)$$

$$C_* = \text{diag}\ (\Theta_1^*, \ldots, \Theta_m^*)$$

$$y_* = H_2 d, \quad e_* = W y_*$$

The constants $\Theta_1^*, \ldots, \Theta_m^*$ are the tuned adaptive gains; y_* and e_* are the responses of the corresponding tuned system. Global stability is guaranteed if e_* and y_* approach zero asymptotically, H_1 and H_2 are exponentially stable, and H_1 is SPR. The SPR condition on H_1 is not needed for local stability. It turns out that S is exponentially stable if y_* is persistently exciting and e_* is sufficiently small [6].

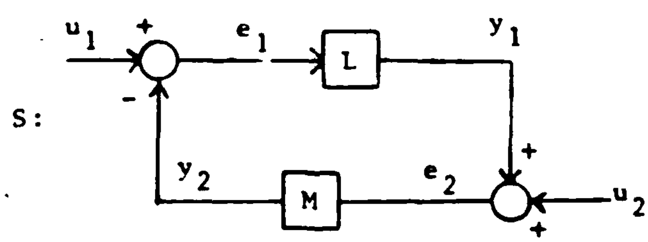


Figure 6. Feedback System S

5. CONCLUDING REMARKS

In this paper we have briefly discussed some of the practical issues involved in the adaptive control of an LSS, e.g., decentralization, model error, and performance allocation. The conclusion is that existing adaptive theory needs to be radically revised if useful engineering tools are to emerge. A particular direction for further research, as advocated here, is the further development of a theory of local stability for adaptive systems [6]. We have shown that such a theory is compatible with conic-model representations [4] and interconnected system theory [2], thus, providing the basis for resolving the issues enumerated above.

REFERENCES

[1] K.S. Narendra and L.S. Valavani, 'Stable Adaptive Controller Design - Direct Control,' IEEE Trans. Aut. Contr., Vol. AC-23, pp. 570-583, Aug. 1978.

[2] M. Vidyasagar, Input-Output Analysis of Large Scale Interconnected Systems, Springer-Verlag, New York, 1981.

[3] G. Zames, 'On the Input-Output Stability of Time-Varying Nonlinear Feedback Systems,' IEEE Trans. Aut. Contr., Part I: Vol. AC-11, No. 2, pp. 228-238, April 1966; Part II: Vol. AC-11, No. 3, pp. 465-476, July 1966.

[4a] M.G. Safonov, Stability and Robustness of Multivariable Systems, MIT Press, 1980.

[4b] M.G. Safonov, 'Stability Margins of Diagonally Perturbed Multivariable Feedback Systems,' Proc. 20th IEEE CDC, San Diego, CA, Dec. 1981, pp. 1472-1478.

[4c] M.G. Safonov, 'Tight Bounds on the Response of Multivariable Systems with Component Uncertainty,' Proc. 16th Allerton Conf., Oct. 1978.

[5a] R.L. Kosut and B. Friedlander, 'Performance Robustness Properties of Adaptive Control Systems,' Proc. IEEE Conf. on Dec. and Contr., Orlando, FL, Dec. 1982, pp. 18-23.

[5b] R.L. Kosut and B. Friedlander, 'Robust Adaptive Control: Conditions for Global Stability,' submitted to IEEE Trans. on Aut. Cont.

[6] R.L. Kosut, 'Robust Adaptive Control: Conditions for Local Stability,' Proc. 1982 ACC, San Francisco, CA, June 1982.

[7] R.L. Kosut, H. Salzwedel, and A. Emami-Naeini, 'Robust Control of Flexible Spacecraft,' AIAA Journal of Guid. and Control, Vol. 6, No. 2, pp. 104-111, March-April 1983.

[8] M. Balas, 'Some Trends in Large Space Structures Control Theory: Fondest Hopes; Wildest Dreams,' IEEE Trans. Aut. Contr., Vol. AC-27, No. 3, June 1982.

[9] C. Rohrs, L. Valavani, M. Athans and G. Stein, 'Analytical Verification of Undesirable Properties of Direct Model Reference Adaptive Control Algorithms,' Proc. 20th IEEE CDC, San Diego, CA, December 1981, pp. 1272-1284.

[10] N. Sandarajan and R.C. Montgomery, 'Adaptive Identification for the Dynamics of Large Space Structures,' Proc. AIAA Guid. and Contr. Conf., August 1982, pp. 379-387.

[11] Kosut, R.L., 'Analysis of Performance Robustness for Uncertain Multivariable Systems,' Proc. IEEE Conf. on Dec. and Contr., Orlando, Florida, December 1982, pp. 1289-1294.

[12] J. Willems, 'Stability of Large-Scale Interconnected Systems,' Directions in Large-Scale Systems, Plenum Press, NY, 1976.

[13] D.D. Siljak, Large-Scale Dynamic Systems: Stability and Structure, New York: North-Holland, 1978.

[14] C.A. Desoer and M. Vidysagar, Feedback Systems: Input-Output Properties, Academic, 1975.

[15] J.C. Doyle and G. Stein, 'Multivariable Feedback Design: Concepts for a Modern/Classical Synthesis,' IEEE Trans. Aut. Contr., AC-26, No. 1, pp. 4-17, February 1981.

[16] I. Bar Kana, H. Kaufman, and M. Balas, 'Model Reference Adaptive Control of Large Structural Systems,' AIAA J. of Guid. and Contr., Vol. 6, No. 2, Mar-Apr 1983, pp. 112-118.

[17] R.J. Benhabib, R.P. Iwens, and R.L. Jackson, 'Stability of Large Space Structure Control Systems Using Positivity Concepts,' AIAA Journal of Guid. and Control, Vol. 4, No. 5, September - October 1981, pp. 487-494.

[18] B.D.O. Anderson and C.R. Johnson, Jr., 'On Reduced-Order Adaptive Output Error Identification and Adaptive IIR Filtering,' IEEE Trans. Aut. Contr., Vol. AC-27, No. 4, Aug. 1981, pp. 927-933.

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An Input–Output View of Robustness in Adaptive Control*

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An input–output theory of adaptive control provides a means of determining the robustness properties of adaptive algorithms.

Key Words—Adaptive control; robustness; robust control; stability; model reference adaptive control.

Abstract—The stability and robustness properties of adaptive control systems are examined using input–output stability theory, i.e. passivity and small-gain theory. A generic adaptive error system is developed based on the concept of a tuned system—an ideal converged (nonadaptive) closed-loop system. Using this error system with passivity theory gives conditions for global stability where only boundedness (in norm) is required on the external inputs, e.g. disturbance, reference and initial conditions. Small gain theory is used to develop local stability results where the magnitudes of the external inputs are restricted. In the global case, a particular system operator (not the plant) is required to be strictly-passive, a condition which is unlikely to hold in actual use due to unmodeled dynamics. The local results, however, are not so restricted and allow for unmodeled dynamics. In this latter case an estimate of the stability margin is given under a persistent excitation condition.

1. INTRODUCTION

AT A VERY basic level, the issues involved in adaptive control design are no different from nonadaptive (robust) control design. In either case the goal is to maintain specified performance properties despite uncertainty about the dynamics of the plant to be controlled, as well as uncertainty about its environment. In the nonadaptive case the problem of robustness to unmodeled dynamics is well formulated (e.g. Doyle and Stein, 1981; Zames and Francis, 1983). However, research in adaptive control theory has focused almost exclusively on the case where the plant can be fully represented by some member of a family of linear finite-dimensional parametric models (e.g. Narendra, Lin and Valavani, 1980; Goodwin, Ramadge and Caines, 1980). Thus, the model error due to unmodeled dynamics is presumed to be zero. Unfortunately, unmodeled dynamics can cause adaptive controllers to exhibit significant performance degradation and instability, even with an initial controller parameterization that closely approximates the desired closed-loop response (Rohrs and co-workers, 1981, 1982; Ioannou and Kokotovic, 1983a,b). These simulated circumstances of undesirable behavior are in sharp contrast with successful applications of adaptive control where reduced-order modeling is unavoidable (e.g. Åström, 1983). This issue of model error, then, is of undeniable practical importance, because no actual plant is truly linear and finite-dimensional.

Perhaps the main reason for the lack of a robust/adaptive control theory is that the emphasis has been on *global* results. What we mean by 'global' is that the intent is to require as little *a priori* information about the plant parametrization and the external inputs as possible to prove stable behavior. Because of this, the resulting requirements (i.e. assumptions) are too strong, e.g. known plant order. Therefore, it is compelling to abandon the requirement of global stability—a requirement that may well be beyond the needs of most actual systems—and develop conditions for *local* stability. The term 'local' is used in the sense that the plant uncertainty and external inputs are limited in a defined way, e.g. by restricting the magnitude and spectrum of the reference commands and disturbances, as well as the initial adaptive parameter error.

In this paper we will present an input–output view§ of robustness in adaptive control. In particular, we shall draw attention to uncertain unmodeled plant dynamics—often referred to as model error—and to uncertain, but bounded, disturbances. Based on this view it may be possible to merge robust control theory with adaptive control theory.

The next section (Section 2) formalizes the conversion of a generic adaptive controller to an equivalent generic error system. The input output

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§ A good source book on this material is the text by Desoer and Vidyasagar (1975). The notation used there is also used throughout this paper.

properties of the error system relate the performance of the nominal control system to that of the adaptive control system. Section 3 applies this formulation for a specific continuous-time adaptive model-following algorithm. This permits the application in Section 4 of stability results for the continuous-time version of the generic error system. This section also includes a discussion of the strictly positive real (SPR) condition imposed on an operator within this error system. Finally, in Section 5, we will examine the issues involved in obtaining conditions for local stability and robustness. Though this paper concentrates on continuous-time systems (due to space limitations), this same input–output approach is applicable to robustness analysis of discrete-time adaptive control (e.g. Kosut, Johnson and Anderson, 1983; Ortega and Landau, 1983) as well as time-varying systems (Gomart and Caines, 1984).

2. ERROR SYSTEM FORMULATION

(A) Tuned control concept

Consider the nonadaptive control system of Fig. 1, described by

$$e = H_{ew}(\pi)\, w \tag{1}$$

where $e(t) \in R^{n_e}$ is the error output, $w(t) \in R^{n_w}$ is the external input, and $\pi \in R^{n_\pi}$ is a set of controller parameters to be selected. For our purposes, $H_{ew}(\cdot)$ represents a closed-loop parametric feedback system dependent on the adjustable parameters in π. The output e of $H_{ew}(\cdot)$ is the error the control system experiences in meeting its objective given the external input w. Portions of $H_{ew}(\cdot)$ are not entirely known, e.g. the open-loop plant imbedded in $H_{ew}(\cdot)$. The input $w(t)$ is also not entirely known but can be assumed to be in a subset W of bounded signals. For example, $w(t)$ can consist of a set of reference commands and bounded disturbances. If the imbedded controller were adaptive, it would adjust π continuously on-line so as to reduce the error; but for now assume that π is constant and will be selected off-line.

If the control designer had all the 'off-line' time in the world to 'fiddle' with the parameters π, then it is hoped that a satisfactory adjustment would be obtained. Many strategies can be envisioned for determining a satisfactory π. In fact, such a satisfactory parameterization may not be unique

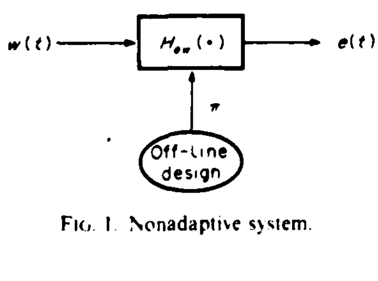

FIG. 1. Nonadaptive system.

but, rather, be any member of a set S, e.g.

$$S = \{\pi \in R^{n_\pi} | H_{ew}(\pi) \text{ has desired properties}\} \tag{2}$$

Certain parameter sets S correspond to well-defined design strategies. Specifically:

Matched. Let $\bar{S}$ denote the matched parameter set, i.e.

$$\bar{S} = \{\bar{\pi} \in R^{n_\pi} | H_{ew}(\bar{\pi}) = 0\}. \tag{3}$$

Robust. Let S^0 denote the robust parameter set, i.e.

$$S^0 = \{\pi^0 \in R^{n_\pi} | \, \|H_{ew}(\pi^0)\, w\|/\|w\| \le p^0, \ \forall w(\cdot) \in W\} \tag{4a}$$

where the norm $\|\cdot\|$ is defined on the underlying function space. The finite constant p^0 represents the robust performance specification. Note that S^0 includes the 'optimal' robust solution, i.e. those $\pi \in R^{n_\pi}$ that solve

$$\inf_{\pi} \sup_{w} (\|H_{ew}(\pi) w\|/\|w\|). \tag{4b}$$

Tuned. Let $S^*_{w(\cdot)}$ denote the tuned parameter set associated with a particular $w(\cdot) \in W$, i.e.

$$S^*_{w(\cdot)} = \{\pi^*_{w(\cdot)} \in R^{n_\pi} | \, \|H_{ew}(\pi^*_{w(\cdot)}) w\|/\| \le p^*\}. \tag{5a}$$

The finite constant p^* represents a tuned performance specification. In order for (4) and (5a) to be meaningful, it is necessary that

$$p^* < p^0 \tag{5b}$$

i.e. the desired tuned performance is better than the desired robust performance. Also, $S^*_{w(\cdot)}$ will include the 'optimal' tuned solution, i.e. for each $w \in W$, those $\pi \in R^{n_\pi}$ that solve

$$\inf_{\pi} (\|H_{ew}(\pi) w\|/\|w\|). \tag{5c}$$

Ideally, the adaptive control should converge to the optimal parametrization of (5c). Thus, the tuned parameter set, denoted by S^*, is given by

$$S^* = \bigcup_{w(\cdot) \in W} S^*_{w(\cdot)}. \tag{6}$$

Note that each element of S^* is satisfactory for a particular $w(\cdot) \in W$ and that no one element in the subset $S^*_{w(\cdot)} \subset S^*$ need provide satisfactory control for a different $w(\cdot)$. (Although $\pi^*_{w(\cdot)} \in S^*$ emphatically denotes the dependence of the tuned parameters on $w(\cdot)$, we will henceforth denote membership in S^* by the simpler notation $\pi^* \in S^*$, where the $w(\cdot)$ dependence is to be implied.)

The error signal corresponding to the matched case is identically zero. It is this particular case that has received practically all the attention in adaptive control research, and about which the strongest theoretical results are available. Unfortunately, in the first place, this case excludes unmeasurable bounded disturbances which are a virtual certainty in any actual system. By unmeasurable bounded disturbances we mean those disturbances which can not be totally rejected at the output of the plant. In the second place, there will always be unmodeled dynamics, i.e. there are never enough parameters in π to solve $H_{ew}(\pi) = 0$ in practice. These remarks apply equally in a stochastic environment. For example, whereas in the deterministic case $e = 0$, in the stochastic case $E\{e\} = 0$, with $E\{\ \}$ the expectation operator. Thus, the unmeasurable bounded disturbances alluded to above have their stochastic counterpart as processes which do *not* have zero mean, i.e. $E\{e\} \neq 0$ for any π.

The more appealing of the other two sets is the tuned set S^*, defined in (6). The associated error signal

$$e^* = H_{ew}(\pi^*)w \tag{7}$$

is referred to as the *tuned error* and $H(\pi^*)$ as the *tuned system*. Although $e^*(t) = 0$ is ruled out due to the impracticality of $\pi^* \in \bar{S}$, we do not preclude the case where $e^*(t) \to 0$. This latter case still presumes a degree of idealization. Consider the case where the external input w consists of a step reference command with no disturbance and e^* is the difference between the plant output and the reference command. Thus, $e^*(t) \to 0$ is the ideal output error for any stabilizing controller engendering unit d.c. gain. This class of tuned controllers can be quite large even if dim $(\pi^*) <$ dim $(\bar{\pi})$. Now consider the impact of a bounded disturbance, which is not necessarily of any particular functional form, such as a broadband bounded signal. Clearly, with such bounded disturbances present, $e^*(t) \nrightarrow 0$, and can only be assumed to be bounded.

An important comparison for the tuned set S^* is to the robust set S^0 (4). Let

$$e^0 = H_{ew}(\pi^0)w \tag{8}$$

denote the *robust error*. Recall from (5) and (6) that the tuned parameters π^* are dependent on a particular $w(\cdot) \in W$, whereas the robust parameters π^0 are not. Hence, the tuned error can never exceed the robust error, i.e. for a particular $w(\cdot) \in W$,

$$\|e^*\| = \|H_{ew}(\pi^*)w\| \leq \|e^0\| = \|H_{ew}(\pi^0)w\|. \tag{9}$$

Condition (9) also follows from the fact that $p^* < p^0$ (5b). Note that it is possible for the robust set S^0 to be empty even though the tuned set S^* is not. If S^0 is not empty, then consideration of an adaptive controller is justified if for some 'large' subset of $w(\cdot) \in W$, various tuned controllers exist such that *each* engenders

$$\|e^*\| < \|e^0\|. \tag{10}$$

If this were not the case, then a robust controller would suffice. This requirement (10) is weaker than the requirement $p^* < p^0$, which may not be attainable for all $w(\cdot) \in W$, since (10) is required only over a subset of W. However, even if (10) holds, adaptation may cause the error during adaptation to become either excessive or to otherwise exceed specifications.

The usefulness of defining the tuned parameter set will be borne out in the next subsection. The tuned set is used there to develop a generic adaptive error system. At this point, however, we remark that it is not necessary to solve the optimization problem defined implicitly in (5), rather we only need to know that a solution exists which is better than the robust solution (4).

(B) *Adaptive error system*

Now consider the adaptive version of (1), depicted in Fig. 2, and described by the input–output relations

$$\begin{bmatrix} e \\ \xi \end{bmatrix} = \begin{bmatrix} H_{ew}(\hat{\pi}) \\ H_{\xi w}(\hat{\pi}) \end{bmatrix} w = H(\hat{\pi})w \tag{11a}$$

$$\hat{\pi} = \Omega[\hat{\pi}(0), e, \xi] \tag{11b}$$

where $\hat{\pi}(t) \in R^{n_\pi}$ are the *adaptation parameters* which are generated from the parameter *adaptive algorithm* Ω, and $\hat{\pi}(0) \in R^{n_\pi}$ is the initial parameter estimate. The adaptive algorithm is driven by the output or *adaptation error* $e(t) \in R^{n_e}$ and the *regressor* $\xi(t) \in R^{n_\xi}$. The regressor is obtained from sensed signals within the feedback system.

We want to ultimately determine conditions under which the adaptive system (11) is stably attracted to the set of tuned systems (6). Recall that the tuned system set is likely to contain more than a single member, thus by stability we mean stability 'about' a (possibly disconnected) set rather than about a point.

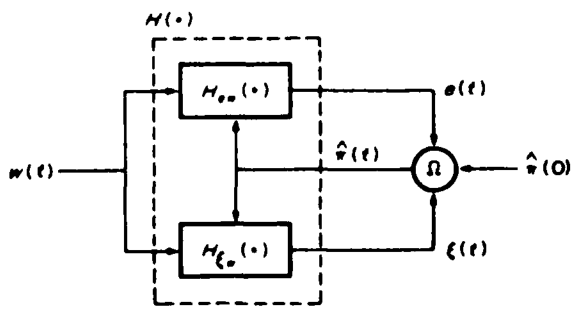

FIG. 2. Adaptive system

The analysis is facilitated by transforming the format of Fig. 2 into an *error system* format. To do this we must define the structure of the *adaptive control*. Consider a single-input single-output (SISO) plant embedded in $H(\cdot)$ whose input $u(t)$ is given by the *bilinear* expression

$$u(t) = -\xi(t)'\hat{\pi}(t). \tag{12}$$

Note that this development is not limited to SISO plants. The extension of (12) to the multivariable case involves a similar expression for each control channel, i.e.

$$u_i(t) = -\xi_i(t)'\hat{\pi}_i(t), \quad i = 1, \ldots, n_u \tag{13}$$

where $\xi_i(t)$ and $\hat{\pi}_i(t)$ are vectors consisting of elements from the regressor and parameter vectors, respectively. However, only the SISO case will be considered here to reduce the complexity of the development and allow sharper focus on the adaptive systems issues. Normally, $\xi(t)$ consists of the plant inputs and outputs, or filtered versions thereof. For example, in discrete-time systems $\xi(t)$ consists of a finite record of past plant inputs and outputs.

Although the bilinear structure in (12) and (13) remains the most widely used and studied format, nonetheless, other structures (as yet underdeveloped) may be more suitable to certain problems e.g. distributed and or nonlinear structures.

We will now make a strong assumption regarding the way in which $u(t)$ and $w(t)$ are transmitted through $H(.)$ into $e(t)$ and $\xi(t)$.

Assumption. The map $(w, u) \mapsto (e, \xi)$ is linear time-invariant (LTI), i.e.

$$\begin{bmatrix} e(t) \\ \xi(t) \end{bmatrix} = \begin{bmatrix} G_{ew}(s) & G_{eu}(s) \\ G_{\xi w}(s) & G_{\xi u}(s) \end{bmatrix} \begin{bmatrix} w(t) \\ u(t) \end{bmatrix} = G(s) \begin{bmatrix} w(t) \\ u(t) \end{bmatrix} \tag{14}$$

where $G(s)$ is the *open-loop interconnection* matrix whose elements are proper rational functions. (To simplify notation we will use s to denote either the Laplace transform variable or the differential operator, depending on the context.)

The adaptive system (11) with bilinear control (13) and LTI interconnections (14) is shown in Fig. 3. To transform this system to an error system, define the *parameter error*

$$\tilde{\pi}(t) = \hat{\pi}(t) - \pi^* \tag{15a}$$

with

$$\pi^* \in S^* \tag{15b}$$

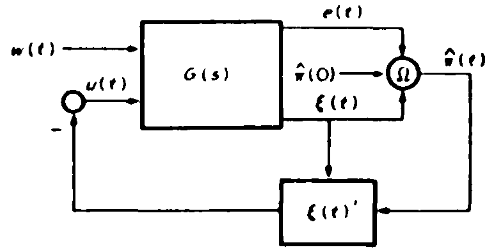

FIG. 3. Adaptive system with LTI interconnection.

and the *adaptive control error*

$$v(t) = \xi(t)'\tilde{\pi}(t). \tag{16}$$

Thus, Fig. 3 can be redrawn as shown in Fig. 4 and described by

$$\begin{bmatrix} e(t) \\ \xi(t) \end{bmatrix} = -G(s)\begin{bmatrix} 0 \\ \xi(t)'\pi^* \end{bmatrix} + G(s)\begin{bmatrix} w(t) \\ -v(t) \end{bmatrix}. \tag{17a}$$

Hence,

$$\begin{bmatrix} e(t) \\ \xi(t) \end{bmatrix} = \begin{bmatrix} H^*_{ew}(s) & H^*_{ev}(s) \\ H^*_{\xi w}(s) & H^*_{\xi v}(s) \end{bmatrix} \begin{bmatrix} w(t) \\ -v(t) \end{bmatrix}$$
$$= H^*(s) \begin{bmatrix} w(t) \\ -v(t) \end{bmatrix} \tag{18a}$$

where

$$H^*_{ew}(s) = G_{ew}(s) + G_{eu}(s)\pi^{*\prime}(I + G_{\xi u}(s)\pi^{*\prime})^{-1}G_{\xi w}(s) \tag{18b}$$

$$H^*_{ev}(s) = G_{eu}(s) + G_{eu}(s)\pi^{*\prime}(I + G_{\xi u}(s)\pi^{*\prime})^{-1}G_{\xi u}(s) \tag{18c}$$

$$H^*_{\xi w}(s) = (I + G_{\xi u}(s)\pi^{*\prime})^{-1}G_{\xi w}(s) \tag{18d}$$

$$H^*_{\xi v}(s) = (I + G_{\xi u}(s)\pi^{*\prime})^{-1}G_{\xi u}(s). \tag{18e}$$

The dashed box in Fig. 4 is $H^*(s)$. We will refer to $H(s)$ as the *tuned interconnections*. Note that the *tuned error* (7) is identical to

$$e^*(t) = H^*_{ew}(s)\, w(t). \tag{19}$$

We also make use of the *tuned regressor*, defined as

$$\xi^*(t) = H^*_{\xi w}(s)\, w(t). \tag{20}$$

Finally, the error system (Fig. 4) can be depicted as in Fig. 5, where

$$e(t) = e^*(t) - H^*_{ev}(s)\, v(t) \tag{21a}$$

$$\xi(t) = \xi^*(t) - H^*_{\xi v}(s)\, v(t) \tag{21b}$$

$$v(t) = \xi(t)'\tilde{\pi}(t) \tag{21c}$$

$$\hat{\pi}(t) = \Omega[\hat{\pi}(0), e(\cdot), \xi(\cdot)]. \tag{21d}$$

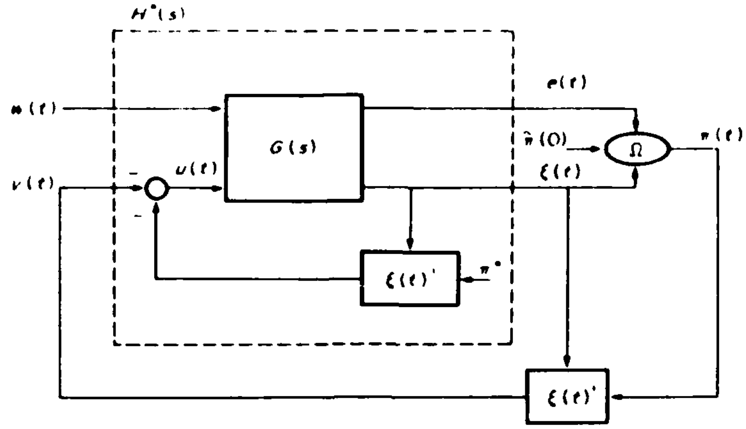

FIG. 4. Adaptive error system.

Figure 5 reveals that this error system is composed of a nonlinear system in the forward loop, denoted by N_{ve}, and the LTI system $H^*_{ev}(s)$ in the feedback path. Thus, the error system is driven externally by the tuned system outputs $e^*(t)$ and $\xi^*(t)$, and the initial parameter error $\tilde{\pi}(0) = \hat{\pi}(0) - \pi^*$.

(C) Existence of the tuned controller

The designation of the tuned controller is the concept most important to extracting a meaningful error system from the description of an adaptively controlled system. It might appear that the ability to specify this tuned controller presupposes our knowledge of an acceptable solution to the underlying adaptive control problem. This is not entirely the case. Given the parametric controller structure of $C(\cdot)$, we need only have an *approximate a priori* knowledge of the system behavior. Given a particular π^*, we will discover that the restrictions on H^*_{ev} and $H^*_{\xi v}$ can be assessed from knowledge of the tuned controller and bounds on the magnitude of the plant modeling error. Such information is a practical result of a thorough plant modeling study. Thus, the study of the stability of (21) will have practical meaningfulness. We will examine a particular continuous-time adaptive controller in the following section and derive the form of H^*_{ev} and $H^*_{\xi v}$.

3. CONTINUOUS-TIME ADAPTIVE MODEL-FOLLOWING

To characterize H^*_{ev} and $H^*_{\xi v}$ some designation of a tuned controller must be provided. We make the choice by assuming that no modeling error exists in the nominal plant parametric model. We close with consideration of the degree of plant mismodeling allowed such that this tuned controller is robust, i.e. maintains stable control of the actual system. Following this discussion, in Section 4 we consider the effect of the adaptive algorithm.

(A) Direct model reference adaptive control

Consider the model reference adaptive control (MRAC) system shown in Fig. 6, described by

$$y(t) = d(t) + P(s)u(t) \quad \text{(plant)} \tag{22a}$$

$$\bar{y}(t) = \bar{H}(s)r(t) \quad \text{(reference model)} \tag{22b}$$

$$e(t) = y(t) - \bar{y}(t) \quad \text{(tracking error)} \tag{22c}$$

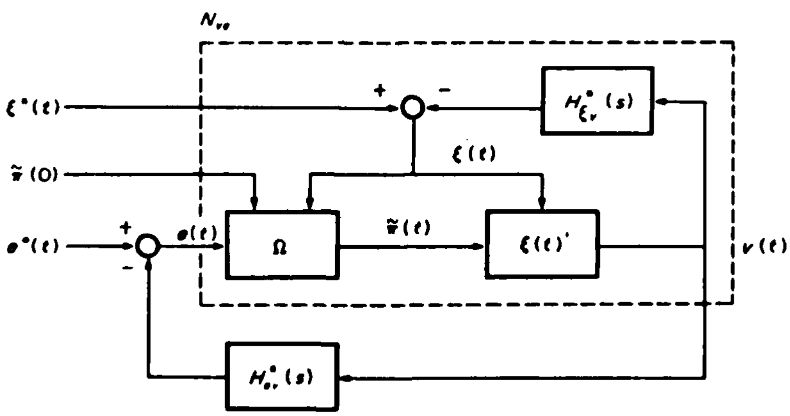

FIG. 5. Adaptive error system.

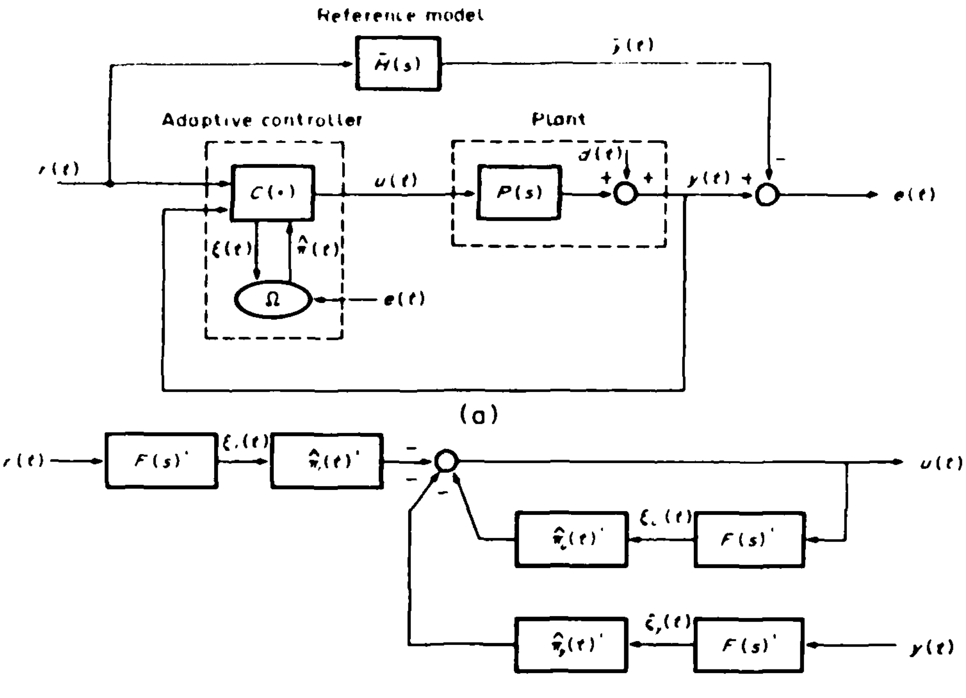

FIG. 6 Model reference adaptive system (a) block diagram, (b) controller detail.

where $d(t)$ consists of disturbances and plant initial conditions, and $r(t)$ is the reference command. Let $\{C(\cdot), \Omega\}$ denote the adaptive controller, where Ω is the parameter adaptive algorithm and $C(\cdot)$ is the parametric controller. Following Narendra, Lin, and Valavani (1980), let $C(\cdot)$ have the bilinear form

$$\begin{aligned} u(t) &= -\xi(t)'\hat{\pi}(t) \\ &= -\xi_u(t)'\hat{\pi}_u(t) - \xi_y(t)'\hat{\pi}_u(t) - \xi_r(t)'\hat{\pi}_r(t) \end{aligned} \quad (23a)$$

where the regressor is given by filtered versions of u, y and r

$$\begin{aligned} \xi(t)' &= [\xi_u(t)', \xi_y(t)', \xi_r(t)]' \\ &= [F(s)u(t), F(s)y(t), -F(s)r(t)] \end{aligned} \quad (23b)$$

with

$$F(s) = \left(\frac{1}{L(s)}, \ldots, \frac{s^{k-1}}{L(s)}\right) \quad (23c)$$

and

$$L(s) = s^k + \alpha_1 s^{k-1} + \ldots + \alpha_k. \quad (23d)$$

Thus, there are $3k$ adaptive parameters. Using the definition of adaptive control error in (21c), the MRAC control signal (22) can be expressed as

$$u(t) = -\frac{A_1^*(s)}{L(s)}u(t) - \frac{A_2^*(s)}{L(s)}y(t) + \frac{A_3^*(s)}{L(s)}r(t) - v(t) \quad (24)$$

where the tuned parametrization $\pi^*(=\hat{\pi}-\tilde{\pi})$ is distributed among the control elements as follows:

$$A_1^*(s) = \pi_1^* s^{k-1} + \ldots + \pi_k^* \quad (25a)$$

$$A_2^*(s) = \pi_{k+1}^* s^{k-1} + \ldots + \pi_{2k}^* \quad (25b)$$

$$A_3^*(s) = \pi_{2k+1}^* s^{k-1} + \ldots + \pi_{3k}^*. \quad (25c)$$

Thus, (24) becomes

$$u(t) = C_{ur}^*(s)\, r(t) - C_{uy}^*(s)\, y(t) - C_{uv}^*(s) v(t) \quad (26a)$$

where

$$C_{uy}^*(s) = \frac{A_2^*(s)}{L(s) + A_1^*(s)},$$

$$C_{ur}^*(s) = \frac{A_3^*(s)}{L(s) + A_1^*(s)},$$

$$C_{uv}^*(s) = \frac{L(s)}{L(s) + A_1^*(s)}. \quad (26b)$$

We will refer to $C^* = [C_{ur}^*, C_{uy}^*, C_{uv}^*]$ as the *tuned* controller. The adaptive error system (21) corresponding to the MRAC of Fig. 6 is shown in Fig. 5. The tuned signals (3.19) and (3.20) are

$$e^* = [(1 + PC_{uy}^*)^{-1}PC_{ur}^* - \bar{H}]\, r - (1 + PC_{uy}^*)^{-1}d \quad (27a)$$

$$\xi^* = \begin{bmatrix} (1 + PC_{uy}^*)^{-1}C_{ur}^*F'r - C_{uy}^*(1 + PC_{uy}^*)^{-1}F'd \\ (1 + PC_{uy}^*)^{-1}PC_{ur}^*F'r - (1 + PC_{uy}^*)^{-1}F'd \\ -F'r \end{bmatrix}. \quad (27b)$$

The tuned interconnections (18) are

$$H_{ry}^{*} = (I + PC_{uy}^{*})^{-1} PC_{ur}^{*} \tag{28a}$$

$$H_{w}^{*} = \begin{bmatrix} (I + PC_{uy}^{*})^{-1} C_{ur}^{*} I \\ (I + PC_{uy}^{*})^{-1} PC_{ur}^{*} I \\ 0 \end{bmatrix} \tag{28b}$$

(B) Tuned system control design

There are any number of ways to design the tuned controller C^*. The important point — no matter what design method is used — is that the tuned design must be robust, because the plant $P(s)$ in (27) and (28) is not entirely known. Recall from (5) that the tuned controller is dependent on the plant. For example, the $3k$ parameters in π^* cannot make $r \mapsto e$ in (27a) be identically zero. This can be viewed as a reduced order design problem or, as in the discussion that follows, a problem in robustness to unmodeled dynamics.

Suppose that the actual plant can be described by

$$P(s) = [1 + \Delta(s)]P^*(s) \tag{29a}$$

$$P^*(s) = \frac{b_0 B^*(s)}{A^*(s)}$$

$$= \frac{b_0(s^m + b_1 s^{m-1} + \ldots + b_m)}{s^n + a_1 s^{n-1} + \ldots + a_n}, \; m < n \tag{29b}$$

where $P^*(s)$ is a *tuned parametric model* of $P(s)$, i.e. the parameters $(b_0, \ldots, b_m, a_1, \ldots, a_n)$ provide a good fit, say at low frequencies. The transfer function $\Delta(s)$ represents unmodeled dynamics, i.e. those dynamics in $P(s)$ not accounted for by $P^*(s)$, e.g. high frequency efforts. Assume that $\Delta(s)$ is stable but is otherwise unknown except for a bound, i.e.

$$|\Delta(j\omega)| \le \delta(\omega), \; \forall \omega \in R. \tag{29c}$$

This type of modeling uncertainty is said to be *unstructured* (Doyle and Stein, 1981). In more general terms, (29) provides a *set description* of the plant rather than a single parametric model, such as P^* (Safonov, 1980).

We will now examine the impact of model error on a tuned control design based only on the parametric model. The model reference format suggests that we make e^* as small as possible. To eliminate the tracking error term in (29a) entirely, we will use the procedure described in Egardt (1979), which requires that the following information is known:

(1) $n > m$ ($P^*(s)$ is strictly proper)
(2) n and m are known
(3) $B^*(s)$ has all zeros strictly inside the left half plane.

Also, the reference model transfer function is assumed to be

$$\bar{H}(s) = \frac{\bar{B}(s)}{\bar{A}(s)} = \frac{\bar{b}_0 s^m + \bar{b}_1 s^{m-1} + \ldots \bar{b}_m}{s^n + \bar{a}_1 s^{n-1} + \ldots + \bar{a}_n} \tag{30}$$

where $\bar{b}_0, \ldots, \bar{b}_m, \bar{a}_1, \ldots, \bar{a}_n$ are preselected constants, and where $\bar{H}(s)$ is exponentially stable, i.e. all zeros of $\bar{A}(s)$ are strictly inside the left half plane. It is well to point out here that although assumption (2) above can be satisfied by the parametric model (29b), this is not the case for the actual plant (29a) due to the presence of the unstructured uncertainty (29c).

The tuned controller structure proposed in Egardt (1979) requires that

$$C_{uy}^{*} = -\frac{R^*}{b_0 B^* S^*} \tag{31a}$$

$$C_{ur}^{*} = \frac{\bar{B}T^*}{b_0 B^* S^*} \tag{31b}$$

where T^* is a stable monic polynomial of degree $n_T > n - m - 1$, and where the polynomials S^* and R^* uniquely solve the polynomial equation

$$T^*\bar{A} = R^* + AS^* \tag{31c}$$

with S^* monic of degree n_T, and R^* of degree $n - 1$. With no model error ($\Delta = 0$), this controller (31), in addition to stabilizing the tuned system, also makes the transfer function from r into y^* identical to the reference model $\bar{H}(s)$. Thus, the tuning of (31) is for the subset of W composed of bounded reference signals and zero disturbance. The effect of (31) on C_{ur}^{*} will shortly be made apparent. Comparing (33) with (26) motivates solving for π^* from

$$L + A_1^* = B^*S^*$$
$$A_2^* = \frac{1}{b_0} R^* \tag{32}$$
$$A_3^* = \frac{1}{b_0} \bar{B}T^*.$$

A solution for π^* exists provided that

$$k = n_T + m \ge n. \tag{33}$$

With this choice for (A_1^*, A_2^*, A_3^*), the tuned controller is given by (31) and by

$$C_{ur}^{*} = \frac{L}{BS^*}. \tag{34}$$

(C) The effect of model error on tuned system performance

It is convenient to define the transfer function

$$G^* = \frac{R^*}{T^*\bar{A}} \tag{35}$$

Using the tuned controller just described, the tuned signals (27) are then

$$e^* = (1 + \Delta G^*)^{-1}\left[\Delta(1 - G^*)Hr + \frac{A^*S^*}{AT^*}d\right] \quad (36)$$

$$\xi^* = \begin{bmatrix} (1 + \Delta G^*)^{-1}\left[\dfrac{A^*B}{b_0 B^* \bar{A}} F'r - \dfrac{A^*R^*}{b_0 B^* T^* \bar{A}} F'd\right] \\ (1 + \Delta G^*)^{-1}\left[(1 + \Delta)\dfrac{B}{A} F'r - \dfrac{S^*A^*}{T^*A} F'd\right] \\ -F'r \end{bmatrix} \quad (36b)$$

and the tuned interconnections (28) become

$$H_{e}^* = (1 + \Delta G^*)^{-1}(1 + \Delta)\frac{b_0 L}{T^*\bar{A}} \quad (37a)$$

$$H_{\xi}^* = \begin{bmatrix} (1 + \Delta G^*)^{-1}\dfrac{A^*L}{B^*T^*\bar{A}}F' \\ (1 + \Delta G^*)^{-1}(1 + \Delta)\dfrac{b_0 L}{T^*\bar{A}}F' \\ 0 \end{bmatrix}. \quad (37b)$$

The tuned system with no model error ($\Delta = 0$) is exponentially stable, since, by assumption, the poles of $(B^*)^{-1}$, $(\bar{A})^{-1}$, and $(T^*)^{-1}$ are in the open left half plane. Hence e^* and ξ^* are bounded if r and d are bounded. Thus, the stability of the *actual* tuned system is guaranteed if and only if

$$(1 + \Delta G^*)^{-1} \text{ and } (1 + \Delta G^*)^{-1}\Delta \text{ are exponentially stable.} \quad (38)$$

Note that under these conditions, the tuned interconnections in (37b) remain exponentially stable. However, it is not necessary (nor possible by assumption) to have a complete description of Δ in order to satisfy (38). For example, if Δ is known to be exponentially stable, then with G^* known to be exponentially stable, (38) holds if (e.g. Doyle and Stein, 1981)

$$|\Delta(j\omega)|\cdot|G^*(j\omega)| < 1, \ \forall \omega \in R. \quad (39)$$

Satisfaction of (39) requires that

$$|\Delta(j\omega)| < \delta(\omega) = 1/|G^*(j\omega)|, \ \forall \omega \in R. \quad (40)$$

We will show in Section 4 that $\delta(\omega) < 1$ is the limit imposed on $\delta(\omega)$ by the usual global stability results for continuous-time adaptive systems. Similar limits are also encountered with discrete-time adaptive systems.

4. GLOBAL STABILITY CONDITIONS

The purpose of this section is to introduce global stability conditions applicable to the generic error system of (21). In the preceding section, we specified an adaptive controller structure $C(.)$ from which we then developed the tuned system $(r, d) \rightarrow (e^*, \xi^*)$ and the interconnection operators H_e^* and H_ξ^*. We now need to characterize the adaptive law Ω in (21d). With this connection we will be able to interpret some conditions under which such a continuous-time adaptive controller possesses a (limited) degree of robustness. Our interpretive remarks will address the restrictiveness of the SPR condition on H_e^* that arises in practically all global stability theorems.

(A) *The adaptive algorithm*

We will begin by specifying the adaptive law(s) of interest. A large class of adaptive algorithms (21d) have the from

$$\dot{\hat{\pi}}(t) = A[\xi(\cdot), \omega(t)], \ \hat{\pi}(0) \in R^p \quad (41a)$$

$$\omega(t) = \xi(t)\, e(t). \quad (41b)$$

We will refer to $A(.,.)$ as the *adaptation gain*, which is a nonlinear operator. In general $A[\cdot,\cdot]$ can have memory, usually only in $\xi(t)$. The adaptive algorithm can also be expressed in terms of the parameter error $\tilde{\pi}(t)$ as

$$\dot{\tilde{\pi}}(t) = A[\xi(\cdot), \omega(t)], \ \tilde{\pi}(0) = \hat{\pi}(0) - \pi^*. \quad (42)$$

The complete adaptive error system (21), including the adaptive algorithm (42), is shown in Fig. 7.

The choice of algorithms, i.e. the variety of proposed adaptation gains, is virtually unlimited. The following two are our chosen representatives:

Constant gain (Narendra, Lin, and Valavani, 1980).

$$A[\xi(\cdot), \omega(t)] = A_0\omega(t) \quad \text{where } A_0 \in R^{p \times p}, A_0 = A_0' > 0. \quad (43)$$

Retarded gain (Kreisselmeier and Narendra, 1982).

$$A[\xi(\cdot), \omega(t)] = \begin{cases} A_0\omega(t), & |\tilde{\pi}(t)| < c \\ A_0[\omega(t) - (1 - |\tilde{\pi}(t)|/c)^2\tilde{\pi}(t)], & |\tilde{\pi}(t)| \geq c \end{cases} \quad (44)$$

where $A_0 \in R^{p \times p}$, $A_0 = A_0' > 0$, and $c \geq \max|\pi^*|$.

We will use the concept of persistent excitation that has proven important in adaptive control, as well as in adaptive system identification.

Definition (Anderson, 1977). A function $f(\cdot): R_+ \rightarrow R^n$ is *persistently exciting*, denoted by

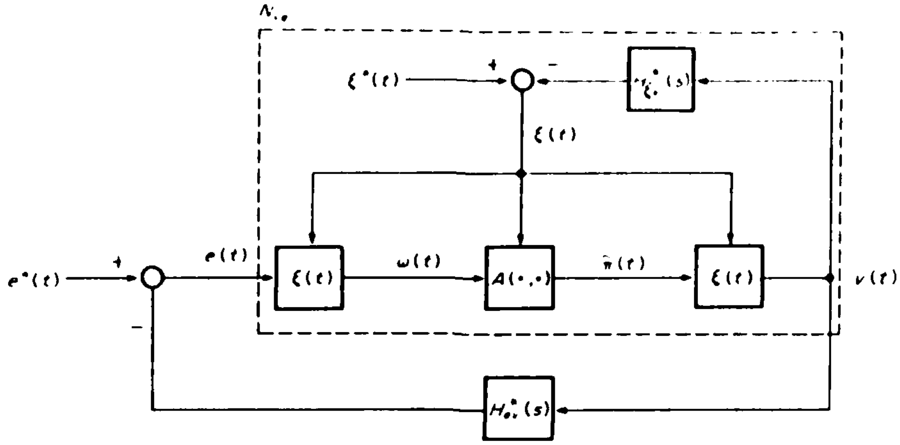

FIG. 7. MRAC error system.

$f \in$ PE, if there exists positive constants α_1, α_2 and α_3 such that

$$\alpha_2 I_n \geq \int_s^{s+\alpha_3} f(t) f(t)' dt \geq \alpha_1 I_n, \forall s \in R_+. \quad (45)$$

We will discuss the implications of persistent excitation on global stability below, as well as in Section 5, in regard to local stability.

(B) A global stability theorem

Theorem 1, which follows, gives conditions for global stability of the adaptive error system of Fig. 7. The term 'global' refers to the intention of seeking the minimal (reasonable) restrictions on the tuned signals $e^*(t)$ and $\xi^*(t)$, and the tuned interconnections $H_{ev}^*(s)$ and $H_{\xi v}^*(s)$ resulting in the proof that e and ξ remain bounded, i.e. (21) is stable, for any finite $\tilde{\pi}(0)$. (A detailed proof of Theorem 1 is given in Kosut and Friedlander (1983).) In particular, we will consider the following two tuned system signal sets as 'inputs' to the error system:

$$W_0^* = \{e^*, \xi^*, \tilde{\pi}(0) | e^*, \dot{e}^* \in L_2 \cap L_\infty, \xi^* \in L_\infty^p, \tilde{\pi}(0) \in R^p\} \quad (46)$$

$$W_B^* = \{e^*, \xi^*, \tilde{\pi}(0) | e^* \in L_\infty, \xi^* \in L_\infty^p, \tilde{\pi}(0) \in R^p\}. \quad (47)$$

Note that e^*, $\dot{e}^* \in L_2 \cap L_\infty$ essentially implies boundedness and ultimate decay to zero, whereas inclusion in L_∞ only implies boundedness.

Theorem 1. For the system of Fig. 7, assume that

(A1) $H_{\xi v}^*(s)$ is strictly proper and exponentially stable (48)

(A2) $H_{ev}^*(s)$ is strictly positive real (SPR), i.e. $H_{ev}^*(s)$ is strictly proper, exponentially stable, and there exists a finite constant $\rho > 0$ such that

$$\text{Re}\,[H_{ev}^*(j\omega)] \geq \rho |H_{ev}^*(j\omega)|^2, \forall \omega \in [0, \infty). \quad (49)$$

Under these conditions, algorithms (43) or (44) result in the following properties:

(i) If $(e^*, \xi^*, \tilde{\pi}(0)) \in W_0^*$ then, $\tilde{\pi}$, e, ξ, and v are bounded (in L_∞), $\dot{\tilde{\pi}}(t) \to 0$, and $e^*(t) - e(t) \to 0$. In addition, if $\xi^* \in$ PE, then $\tilde{\pi}(t) \to 0$ exponentially.

(ii) If $(e^*, \xi^*, \tilde{\pi}(0)) \in W_B^*$ and $\xi \in$ PE, then $\tilde{\pi}$, e, ξ, and v are bounded (in L_∞).

(iii) If $(e^*, \xi^*, \tilde{\pi}(0)) \in W_B^*$ and $\xi \notin$ PE, then the results of (ii) still follow by using the gain algorithm (44).

Comments on Theorem 1. Though theoretically significant, these results do not offer the engineering design guidelines we would like to obtain. The major reason is that $H_{ev}^*(s) \in$ SPR (condition (A_2)) is virtually impossible to achieve for any actual system. The primary culprit here is the effect of unmodeled dynamics. Details on this issue may be found in Rohrs and co-workers (1982). Further discussion will be provided in the following subsection.

Another technical hurdle is that the only realistic case, insofar as the tuned signals (e^*, ξ^*) are concerned, is when $(e^*, \xi^*) \in W_B^*$. This is the situation induced by continual bounded disturbances, such as would normally be encountered. But in this case the theory requires that either $\xi \in$ PE as in part (ii) or that the adaptation gain is retarded as in part (iii). With bounded disturbances present it is not known how to guarantee $\xi \in$ PE, since ξ is generated *inside* the adaptive loop. Note that part (i) only requires that the tuned regressor $\xi^* \in$ PE rather than the actual regressor $\xi \in$ PE as in part (ii). However, this requires $(e^*, \xi \in W_0^*$, which is only possible when the control structure provides asymptotic model following and disturbance rejection. This is the classic case studied in the literature. Obviously, unmodeled dynamics and bounded disturbances eliminate this ideal situation. A further difficulty regarding $\xi \in$ PE is that this occurs at the expense of any set-point regulation, which deteriorates in the presence of PE signals. Using gain retardation does not require persistent excitation, but does require some *a priori* information, i.e. as in (44), the foreknowledge of an upper bound on $|\pi^*|$, which is not too difficult to obtain. Although retardation does handle bounded disturbance, the SPR condition is still required.

(C) In pursuit of the SPR condition

> '*When a man points to the stars, only a fool looks at his finger.*'
>
> *Anonymous.*

The intent of this aphorism is to divert any lingering anxieties about the SPR condition. It—the SPR condition—simply will not do as a major building block in adaptive control theory. But that does not mean a total abandonment of our aim; it suggests, rather, a redirection. We should be establishing a different path to the 'stars.' For now, however, we will remain earthbound and address the restrictiveness of the SPR requirement.

A necessary condition for $H^*_{ev} \in$ SPR is that $H^*_{ev}(s)$ have a relative degree of one. As pointed out by Rohrs and co-workers (1982), this imposes the requirement that the relative degree of the plant is known, e.g. examine the effect on the plant P in (28a). This knowledge, however, is unavailable due to the presence of unmodeled dynamics, as assumed in (29).

The same type of restriction can also be seen as follows. From (37a)

$$H^*_{ev} = (1 + \Delta G^*)^{-1}(1 + \Delta)\bar{H}^*_{ev} \tag{50a}$$

$$\bar{H}^*_{ev} = \frac{b_0 L}{T^* \bar{A}}. \tag{50b}$$

If Δ is exponentially stable, but is otherwise unstructured, then conditions for $H^*_{ev} \in$ SPR include (Kosut and Friedlander, 1983)

$$(1)\ \bar{H}^*_{ev} \in \text{SPR} \tag{51a}$$

$$(2)\ |\Delta(j\omega)| < 1. \tag{51b}$$

Since $\bar{H}^*_{ev}$ is dependent only on the parametric model P^*, it is not difficult to find π^* such that $\bar{H}^*_{ev} \in$ SPR. Unfortunately, the drawback is that (51b) is a condition that is almost surely violated, due to typically unmodeled high frequency dynamics.

So, if $H^*_{ev} \in$ SPR can never hold, can we eliminate the SPR requirement or add some clever filtering to desirably alter H^*_{ev}? For the perfect modeling case ($\Delta = 0$) it is possible to obtain (Monopoli, 1974; Landau, 1978; Egardt, 1979)

$$H^*_{ev}|_{\Delta=0} = H^*_{ev} = \text{positive constant} \tag{52}$$

Although a positive constant is SPR, and hence, satisfies (51), condition (51b) is still required for $H^*_{ev}(\Delta \neq 0)$ to be SPR.

These disclaimers lead us away from the global approach typified by Theorem 1 to the establishment of local stability results which are robust to unmodeled dynamics and bounded disturbances.

5. LOCAL STABILITY CONDITIONS

In this section we indicate a means of obtaining local stability conditions. To clarify the distinction between local and global, consider, for example, result (ii) of Theorem 1. This result holds if $H^*_{ev} \in$ SPR, $H^*_{\xi v}$ exponentially stable, $(e^*, \xi^*) \in W^*_B$, $\xi \in$ PE, and $|\tilde{\pi}(0)| < \infty$. Aside from the difficulties in establishing SPR and PE, all the conditions are virtually free of any magnitude constraints, and hence, are 'global' conditions. In every practical case, it is more than likely that magnitude information is available, e.g. *a priori* bounds on $\|e^*\|$, $\|\xi^*\|$, and $|\tilde{\pi}(0)|$, as well as a bound on the gains of H^*_{ev} and $H^*_{\xi v}$. For example, Egardt (1979) shows robustness properties for minimum phase systems with bounded output disturbances. Dead-zone and projection mechanisms can handle small unmodeled dynamics as shown by Praly (1983) and Samson (1983). Ioannou and Kokotovic (1983a,b) are able to give an estimate of the region of attraction without SPR or PE in the case of high frequency parasitics. Persistent excitation, and the resulting exponential stability property (see equation (62) in this section) also leads to robustness (e.g. Anderson and Johnson, 1982a,b; Anderson and Johnstone, 1983; Kosut, 1983). Various other gain normalizations have also been suggested (e.g. Gawthrop and Lim, 1982; Ortega and Landau, 1983). These theoretical results remain incomplete, because they do not as yet provide a useful means of assessing the impact of unmodeled dynamics, e.g. a frequency domain bound on model error, dependent on the 'return-difference gain' (e.g. Doyle and Stein, 1981).

In this section we will show in Theorem 2, under mild magnitude bounds, that the adaptive system is (locally) L_∞-stable. This result is quite general because the conditions are independent of the nature of the adaptive algorithm, e.g. dead-zones, normalizations, or persistent excitation.

To facilitate the analysis we will only consider the continuous-time error system (21) with constant gain adaptation alorithm (43). It is convenient to transform (21) to the following variational form, which is more useful for local analysis:

$$\tilde{x} = \tilde{x}_L - \tilde{x}_{NL} \tag{53a}$$

$$\tilde{x}_{NL} = F f(\tilde{x}) \tag{53b}$$

where

$$\tilde{x} = (\tilde{\pi}, \tilde{e}, \tilde{\xi}) = (\pi - \pi^*, e - e^*, \xi - \xi^*) \tag{53c}$$

$$\tilde{x}_L = (\tilde{\pi}_L, \tilde{e}_L, \tilde{\xi}_L), f(\tilde{x}) = (\tilde{\xi}'\pi, \tilde{\xi}\tilde{e}). \tag{53d}$$

Details on transforming (21) to (53) are in Kosut (1983). This form of the adaptive error system is obtained by linearization of (21) about e^*, ξ^* and π^*. The linearized perturbation response is $\tilde{x}_L$, almost identical to the linearized system studied by Rohrs and co-workers (1981), which was arrived at by a 'final approach analysis.' The remaining nonlinear terms $\tilde{x}_{NL}$ are contained in $f(\tilde{x})$, a memoryless nonlinearity, and in F, a time-varying linear operator. The characteristics of F, as well as those of $\tilde{x}_L$, depend on the adaptation gain and the behavior of the tuned signals, e^* and ξ^*. For example, with the constant gain algorithm (43), the linearized perturbation response is

$$\tilde{\pi}_L = (I + LM)^{-1}\tilde{\pi}_0 + K\xi^* e^* \tag{54a}$$

$$\tilde{e}_L = -H^*_{e\iota}\xi^{*\prime}\tilde{\pi}_L \tag{54b}$$

$$\tilde{\xi}_L = -H^*_{\xi\iota}\xi^{*\prime}\tilde{\pi}_L \tag{54c}$$

with

$$F = \begin{bmatrix} KN & -K \\ H^*_{e\iota}(I - \xi^{*\prime}KN) & H^*_{e\iota}\xi^{*\prime}K \\ H^*_{\xi\iota}(I - \xi^{*\prime}KN) & H^*_{\xi\iota}\xi^{*\prime}K \end{bmatrix} \tag{55}$$

and where

$$L = \frac{1}{s}A_0 \tag{56a}$$

$$K = (I + LM)^{-1}L \tag{56}$$

$$M = \xi^* H^*_{e\iota}\xi^{*\prime} + e^* H^*_{\xi\iota}\xi^{*\prime} \tag{56c}$$

$$N = \xi^* H^*_{e\iota} + e^* H^*_{\xi\iota}. \tag{56d}$$

Since boundedness of (e^*, ξ^*) and stability of $(H^*_{e\iota}, H^*_{\xi\iota})$ are established by definition of the tuned system, it is not difficult to see that conditions for the stability of F and the boundedness of $\tilde{x}_L$ are identical. In fact, this follows if and only if the system $S:(x_0, W) \mapsto x$, described by

$$\dot{x} = A_0(w - Mx),\ x(0) = x_0 \in R^n \tag{57}$$

is stable (Kosut, 1983). Note that the system S is identical in form to the linearized parameter error system $(\tilde{\pi}_0, \xi^* e^*) \mapsto \tilde{\pi}_L$ in (54).

Assuming that $S \in L_\infty$-stable we obtain the following local stability.

Theorem 2

Suppose $F \in L_\infty$-stable and $\tilde{x}_L \in L_\infty$. Hence, there exists a constant c such that

$$\gamma_\infty(F) \le c < \infty. \tag{58}$$

Under these conditions, if, for some $\varepsilon < 2/c$,

$$\|\tilde{x}_L\|_\infty \le (1 - \varepsilon c/2)\varepsilon \tag{59}$$

then

$$\|\tilde{x}_L\|_\infty \le \varepsilon. \tag{60}$$

Proof. The proof is entirely analogous to the proof of the linearization theorem on p. 131 of Desoer and Vidyasagar (1975). Details for this case may be found in Kosut (1983).

Discussion

Theorem 2 asserts that the adaptive system is stable, i.e. bounded inside an ε-region, provided that $F \in L_\infty$-stable and the linearized response is bounded and sufficiently small, i.e. condition (59). *No claims are made about the mechanism that provides $F \in L_\infty$-stable and $\tilde{x}_L \in L_\infty$*. As mentioned earlier, these are insured if the map S defined in (57) is L_∞-stable. It is possible, of course, that $\tilde{x}_L \in L^n_\infty$ but $\|\tilde{x}_L\|_\infty$ exceeds the magnitude constraint of (59). Instability, however, does not follow because Theorem 2 only provides sufficient conditions.

In order for theorem 2 to be of practical use, it is necessary to provide stability of S without relying on passivity of $H^*_{e\iota}$. We will illustrate this by using persistent excitation. Consider the system

$$\dot{x} = -A f H f' x + u,\ x(0) \in R^n. \tag{61}$$

It is shown in Anderson (1977) that if $A \in R^{n \times n}$, $A = A' > 0$, $f \in$ PE and $H \in$ SPR then (61) is exponentially stable, i.e. there exists constants m, $\lambda > 0$ such that

$$|x(t)| \le m\, e^{-\lambda t}|x(0)| + \int_0^t m\, e^{-\lambda(t-\tau)}|u(\tau)|\,d\tau. \tag{62}$$

We will apply (62) to provide stability of S as follows. The system S can be written as

$$\dot{x} = -A_0\bar{\xi}\bar{H}_{e\iota}\bar{\xi}' x + A_0 w - Qx \tag{63a}$$

where ξ, H_{ei}, and Q are defined via

$$H^*_{ei} = H_{ei} + \tilde{H}_{ei} \quad (63b)$$

$$\xi^* = \xi + \bar{\xi} \quad (63c)$$

$$Q = A_0(M - \xi H_{ei} \xi) \quad (63d)$$

Comparing (63a) with (61), intuitively, if $\xi \in$ PE, $H_{ei} \in$ SPR, and Q sufficiently 'small' then the system (63) (equivalently the map S) remains exponentially stable. Thus, by Theorem 2, an ε-region of local stability exists. The precise conditions are stated as follows.

Corollary 2.1

Let $H_{ei} \in$ SPR and $\xi \in$ PE with corresponding positive constants λ and m as defined in (62). Then, $F \in L_\infty$-stable and $\tilde{x}_L \in L^n_\infty$ if

$$\lambda m > q = \|\xi\|_\infty \|\bar{\xi}\|_\infty (2 + \|\bar{\xi}\|_\infty)\gamma_\infty(H^*_{ei}) + \|e^*\|_\infty \|\xi^*\|_\infty \gamma_\infty(H^*_{\xi i}) \quad (64)$$

and

$$\gamma_\infty(\tilde{H}_{ei}\bar{\xi}) < (\lambda/m - q)\|\bar{\xi}\|^2_\infty. \quad (65)$$

Proof. Follows directly by application of Small Gain Theory (Zames, 1966) to (63). Details may be found in Kosut (193).

Discussion

Corollary 2.1 shows that persistent excitation is one mechanism which provides $S \in L_\infty$-stable, and hence, boundedness of $\tilde{x}_L$ and stability of F. Therefore, if in addition, $\tilde{x}_L$ is sufficiently small (59) then the adaptive system has a local stability.

Other mechanisms to provide stability of S include dead-zones, retardation functions, and signal normalizations. Their effect on S needs to be determined.

Corollary 2.1 also provides an upper bound on the effect of model error via (65). This is not yet in the frequency domain form we would like, but the bound can be quite large. Hence, H^*_{ei} *need not be* SPR, but only approximately so, e.g. $\bar{H}^*_{ei} \in$ SPR. Think of H^*_{ei} being SPR only at low frequencies. In the same way, the signal $\bar{\xi}$ can be viewed as the dominant part of ξ^* causing excitation in that part of the spectrum where the model error is small, e.g. also at low frequencies. Ioannou and Kokotovic (1983b) also discuss this type of frequency separation in the regressor in the presence of high-frequency parasitics.

These results still remain incomplete because we need to know the relationships between λ, m and the 'size' of $\bar{\xi}$, e.g. Theorem 2 requires a bound on $\|\tilde{\pi}_L\|_\infty$, which is a function of λ, m and consequently $\bar{\xi}$. Of further interest is the effect of dead-zones and signal normalizations on the variational form (53). Certainly the nature of the memoryless nonlinearity $f(x)$ changes, as well as the system S.

6. CONCLUSIONS

In this paper we have presented a framework for an input output theory of adaptive control. This viewpoint provides a means to realistically determine the robustness properties of adaptive algorithms. Moreover, input output concepts are closely related to measurement techniques, and hence, can lead to the determination of usable engineering techniques. In control design and analysis the most notable example is the use of Bode plots for scalar systems (Bode, 1945) and singular value plots for multivariable systems (Doyle and Stein, 1981). At the present time, no similar 'engineering theory' exists for adaptive control design. *En route* to establishing such a theory it will be necessary to resolve some of the open issues raised herein. The possible benefit to adaptive control engineering design is substantial.

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REFERENCES

Anderson, B. D. O. (1977). Exponential stability of linear equations arising in adaptive identification. *IEEE Trans. Aut. Control*, **AC-22**, 83–88.

Anderson, B. D. O. and C. R. Johnson, Jr (1982a). Exponential convergence of adaptive identification and control algorithms. *Automatica*, **18**, 1–13.

Anderson, B. D. O. and C. R. Johnson, Jr (1982b). On reduced order adaptive output error identification and adaptive IIR filtering. *IEEE Trans. Aut. Control*, **AC-27**, 927–933.

Anderson, B. D. O. and R. M. Johnstone (1983). Adaptive systems and time varying plants. *Int. J. Control*, **37**, 367–377.

Aström, K. (1983). Theory and applications of adaptive control—a survey. *Automatica*, **19**, 471–486.

Bode, H. W. (1945). *Network Analysis and Feedback Amplifier Design*. Van Nostrand, Princeton, NJ.

Desoer, C. A. and M. Vidyasagar (1975). *Feedback Systems: Input Output Properties*. Academic Press, New York.

Doyle, J. C. and G. Stein (1981). Multivariable feedback design: concepts for a modern classical synthesis. *IEEE Trans. Aut. Control*, **AC-26**, 4–17.

Egardt, B. (1979). *Stability of Adaptive Controllers*. Springer-Verlag, Berlin.

Gawthrop, P. J. and K. W. Lim (1982). Robustness of self-tuning controllers. *IEE Proc.*, **129**, 21–29.

Gomart, O. and P. Caines (1984). Robust adaptive control of time-varying systems. McGill University Research Report, Department of Electrical Engineering.

Goodwin, G. C., P. J. Ramadge and P. E. Caines (1980). Discrete time multivariable adaptive control. *IEEE Trans. Aut. Control*, **AC-25**, 449–456.

Ioannou, P. A. and P. V. Kokotovic (1983a). Improvement of robustness of adaptive schemes. *Proceedings of the 3rd Yale Workshop on Applications of Adaptive Systems Theory*, June 1983, pp. 148–153.

Ioannou, P. A. and P. V. Kokotovic (1983b). *Adaptive Systems with Reduced Models*. Springer-Verlag, Berlin.

Kosut, R. L. (1983). Robust adaptive control: conditions for local stability. *Proceedings of the 1983 Automatic Control Conference*, San Francisco, CA, June 1983.

Kosut, R. L. and B. Friedlander (1982). Performance robustness properties of adaptive control systems. *Proceedings of the 21st IEEE Conference on Decision and Control*, Orlando, FL, December 1982, pp. 18–23.

Kosut, R. L. and B. Friedlander (1983). Robust adaptive control: conditions for global stability. *IEEE Trans. Aut. Control*.

Kosut, R. L., C. R. Johnson, Jr and B. D. O. Anderson (1983). Conditions for local stability and robustness of adaptive control systems. *Proceedings of the 22nd IEEE Conference on Decision and Control*, San Antonio, TX, December 1983, pp. 972–976.

Kreisselmeier, G. and K. S. Narendra (1982). Stable model reference adaptive control in the presence of bounded disturbances. *IEEE Trans. Auto. Control*, **AC-27**, 1169–1175.

Landau, Y. D. (1978). Elimination of the real positivity condition in the design of parallel MRAS. *IEEE Trans. Aut. Control*, **AC-23**, 1015–1020.

Monopoli, R. V. (1974). Model reference adaptive control with an augmented error signal. *IEEE Trans. Aut. Control*, **AC-19**, 474–484.

Narendra, K. S., Y. H. Lin and L. Valavani (1980). Stable adaptive control design. *IEEE Trans. Aut. Control*, **AC-25**, 440–448.

Ortega, R. and I. D. Landau (1983). On the model mismatch tolerance of various parameter adaptation algorithms in direct control schemes: A sectoricity approach. *Proceedings of the IFAC Workshop on Adaptive Control*, San Francisco, CA, June 1983.

Praly, L. (1983). Robustness of indirect adaptive control based on pole placement design. *Proceedings of the IFAC Workshop on Adaptive Control*, San Francisco, CA, June 1983.

Rohrs, C. E., L. Valavani, M. Athans and G. Stein (1981). Analytical verification of undesirable properties of direct model reference adaptive control algorithms. *Proceedings of the 20th IEEE Conference on Decision and Control*, San Diego, CA, December 1981, pp. 1272–1284.

Rohrs, C. E., L. Valavani, M. Athans and G. Stein (1982). Robustness of adaptive control algorithms in the presence of unmodeled dynamics. *Proceedings of the 21st IEEE Conference on Decision and Control*, Orlando, FL, December 1982.

Safonov, M. G. (1980). *Stability and Robustness of Multivariable Systems*. MIT Press, Cambridge, Mass., 1980.

Samson, C. (1983). Stability analysis of adaptively controlled systems subject to bounded disturbances. *Automatica*, **19**, 81–86.

Zames, G. (1966). On the input–output stability of time-varying nonlinear feedback systems. *IEEE Trans. Aut. Control*, Part I: **AC-11**, 228–238; Part II: **AC-11**, 465–476.

Zames, G. and B. A. Francis (1983). Feedback, minimax sensitivity, and optimal robustness. *IEEE Trans. Aut. Control*, **AC-28**, 586–601.

A LOCAL STABILITY ANALYSIS FOR A CLASS OF ADAPTIVE SYSTEMS

by

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ABSTRACT

An analysis of adaptive systems is presented where a local L_∞-stability is insured under a persistent excitation condition.

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In this note we combine some earlier results [1] - [4] to provide a framework for stability analysis of adaptive systems. We consider here the continuous-time adaptive control of a scalar plant* with input u and output y, described by

$$\text{Plant:} \qquad y = d + Pu \tag{1a}$$

$$\text{Control:} \qquad u = -\hat{\theta}'z \tag{1b}$$

$$\text{Adaptation:} \qquad \dot{\hat{\theta}} = \gamma z e, \quad \hat{\theta}(0) \in R^p \tag{1c}$$

where P is linear with strictly proper transfer function $P(s)$, d is an external disturbance, $\hat{\theta}$ is the adjustable parameter vector, $\gamma>0$ is the constant adaptive gain, z is the regressor (information) vector consisting of filtered measurable signals, e.g., u, y, and references, and e is an error signal which drives the adaptation. System (1) can also be described in an error system form (e.g., [7], [8]) by proceeding as follows.

Define the parameter error by

$$\tilde{\theta} := \hat{\theta} - \theta_* \tag{2}$$

where $\theta_* \in R^p$ is a constant vector of tuned parameters i.e., the parameters that would be selected if the plant P were known. Using (2) we can rewrite (1b) as

$$\begin{aligned} u &= -\theta_*' z - v \\ v &:= \tilde{\theta}' z \end{aligned} \tag{3}$$

* Extension to MIMO plant is straightforward, e.g. [3].

where v is the *adaptive control error* signal. An equivalent representation of (1) is given by the adaptive error system depicted in Figure 1 and described by:

$$e = e_* - H_{ev} v \tag{4a}$$

$$z = z_* - H_{zv} v \tag{4b}$$

$$v = z'\tilde{\theta} \tag{4c}$$

$$\dot{\tilde{\theta}} = \gamma z e, \quad \tilde{\theta}(0) = \hat{\theta}(0) - \theta_* \tag{4d}$$

where (e_*, z_*) are the outputs of the tuned system which is defined as system (1) with control $u = -\theta_* z_*$. The operators H_{ev} and H_{zv} are linear with strictly proper transfer functions $H_{ev}(s)$ and $H_{zv}(s)$, respectively, which are dependent on the tuned parameter θ_*. From the definition of the tuned system [3], [4], it follows that $H_{ev}(s)$ and $H_{zv}(s)$ are exponentially stable. By the same reasoning the tuned signals $e_*(t)$ and $z_*(t)$ are bounded.

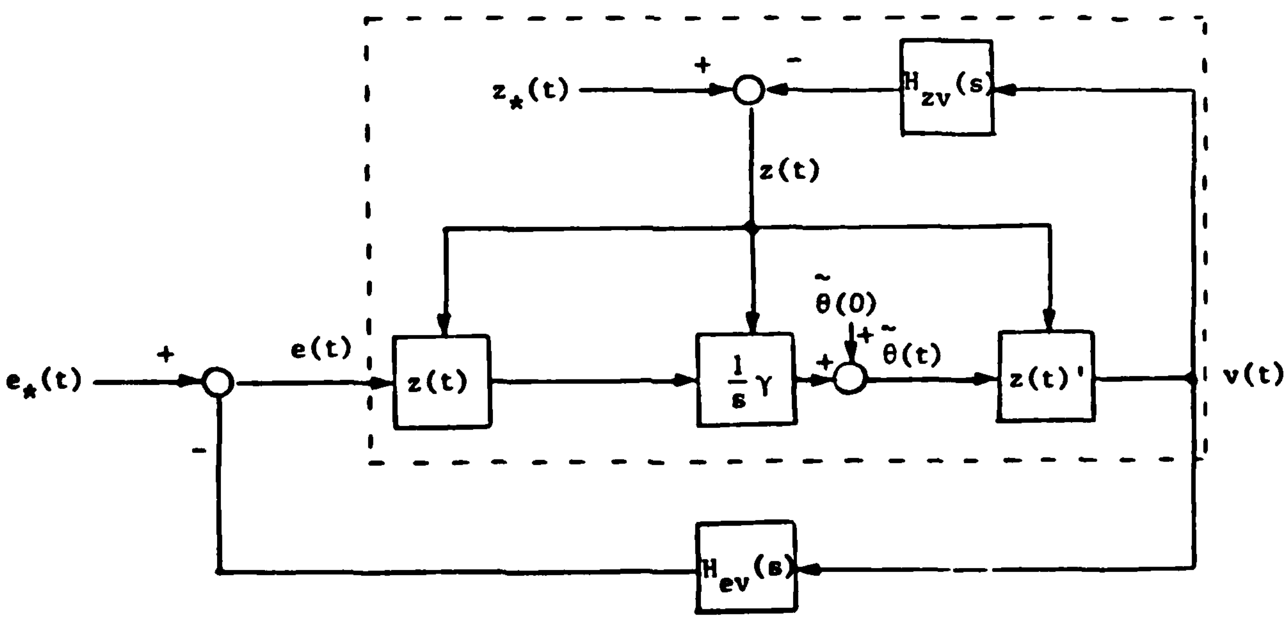

Figure 1. Adaptive Error System

One of the very useful features of this error system is that the nonlinear effect of the adaptive algorithm can be analyzed separately from the analysis of the tuned system. The tuned system represents an ideal which would be achieved with the given structure of the adaptive control. Hence, the algebraic design procedure is separated from the nonlinear stability analysis. It is convenient, therefore, to view e_*, z_*, and $\tilde{\theta}_0$ as 'inputs' to the error system. The assumption, naturally, is that e_* and z_* are well behaved with e_* small. Note that $\tilde{\theta}_0$ need not be small. In the ideal case, assuming perfect model following and no disturbances in the tuned system, $e_*(t) = 0$. If the disturbances are of a special kind then $e_*(t) \to 0$, i.e., the tuned system exhibits servo action. The more realistic case, however, is when $e_* \in L_\infty$ due to bounded disturbances which cannot be asymptotically rejected.

Global Stability Conditions

By global stability of (2) we mean that all bounded inputs e_*, z_*, and $\tilde{\theta}_0$ produce bounded outputs e, $\tilde{\theta}$, and $\tilde{z}$. In general, no restrictions are placed on the initial parameter error $\tilde{\theta}_0$ other than boundedness. Sufficient conditions for global stability can be obtained for (2) using passivity theory (e.g., [5], p. 182). A detailed analysis can be found in [3]-[4]. One of the conditions is that $H_{ev}(s)$ is strictly positive real (SPR), i.e., $H_{ev}(s)$ is strictly proper*, exponentially stable and there exist a positive constant ρ such that

$$\operatorname{Re} H_{ev}(j\omega) \geq \rho |H_{ev}(j\omega)|^2, \quad \forall\, \omega \in R \tag{5}$$

*When $H_{ev}(s)$ is proper but not strictly proper, then SPR is defined as $\operatorname{Re} H_{ev}(j\omega) \geq \varepsilon > 0$, $\forall\, \omega \in R$.

Unfortunately, $H_{ev}(s) \in SPR$ is not robust with respect to even mild modeling error, particularly high frequency unmodeled dynamics [6]. For example, $H_{ev}(s) \in SPR$ implies that the relative degree of $H_{ev}(s)$ cannot exceed one, from which it follows that applying this restriction to (1) imposes the same relative degree restriction on P(s) as well. This is unrealistic, even in this simple example.

Local Stability Conditions

Conditions for local stability require not only that the inputs e_*, z_*, and $\tilde{\theta}_0$ are bounded, but that these bounds are not arbitrary. The local analysis is facilitated by transforming the error system (4) to the variational form

$$x = x_L - Gf(x) \tag{6a}$$

where x, x_L, G, and $f(x)$ are defined by

$$x := \begin{pmatrix} \tilde{e} \\ \tilde{z} \\ \tilde{\theta} \end{pmatrix} := \begin{pmatrix} e - e_* \\ z - z_* \\ \hat{\theta} - \theta_* \end{pmatrix}, \qquad f(x) := \begin{pmatrix} \tilde{z}'\tilde{\theta} \\ \tilde{z}\tilde{e} \end{pmatrix} \tag{6b}$$

$$x_L := \begin{pmatrix} \tilde{e}_L \\ \tilde{z} \\ \tilde{\theta}_L \end{pmatrix} := \begin{pmatrix} -H_{ev} z_*' \tilde{\theta}_L \\ -H_{zv} z_*' \tilde{\theta}_L \\ (I + LM)^{-1}\tilde{\theta}_o + K\, z_* e_* \end{pmatrix} \tag{6c}$$

$$G := \begin{pmatrix} H_{ev}(1 - z_*' KN) & H_{ev} z_*' K \\ H_{zv}(1 - z_*' KN) & H_{zv} z_*' K \\ KN & -K \end{pmatrix} \tag{6d}$$

with

$$N := z_* H_{ev} + e_* H_{zv} \tag{6e}$$

$$M := N z_*' \tag{6f}$$

$$K := (I + LM)^{-1} L \tag{6g}$$

and where L has the transfer function,

$$L(s) = \frac{1}{s}\gamma \tag{6h}$$

with γ from the adaptive algorithm (1c). This error system (6) is arrived at by separating the nonlinear cross product terms in $f(x)$ from the linear terms in x_L. We shall refer to x_L as the response of the <u>linearized system</u>. This is almost identical to the linearized system studied by Rohrs, et al. [6a], which was arrived at by a 'final approach analysis.' Note that in this case the linearized system is the input to the nonlinear system. The operators K and G are linear and time-varying due to their dependence on the tuned signals. If the linearized response x_L in (6c) is small, and if the nonlinear term $f(x)$ is suitably restricted, then intuitively, x would be attracted to some neighborhood of x_L. The following theorem makes this notion precise. We use the notation $\gamma_\infty(.)$ and $\| \ \|_\infty$ to denote L_∞-gain and L_∞-norm, respectively.

<u>Theorem 1</u>: Suppose there exist finite positive constants g, ε, and $\delta(\varepsilon)$ such that

$$\gamma_\infty(G) \leq g < 1/\varepsilon \tag{7a}$$

$$|x| < \delta(\varepsilon) \Rightarrow |f(x)| < \varepsilon|x| \tag{7b}$$

Then

$$||x_L||_\infty \leq (1-g\ \varepsilon)\ \delta(\varepsilon) \tag{7c}$$

implies

$$||x||_\infty \leq \delta(\varepsilon). \tag{7d}$$

Theorem 1 follows directly from the linearization theorem of [5, p. 131]. Theorem 1 asserts that the error outputs x of the adaptive error system are L_∞-bounded in an ε-neighborhood of the linearized response, provided that the linearized response is small enough and that G ε L_∞-stable. Condition (7d) shows that the actual response can be arbitrarily close to the linearized response. Since Theorem 1 provides sufficient conditions, instability does <u>not</u> follow if $x_L \varepsilon L_\infty^n$ but exceeds the magnitude constraint of (7c).

The function $\delta(\varepsilon)$ in (7b) can be determined from the definition of $f(x)$ in (6b) and the norm selected. For example, if the norm on R^n is defined as $|x| = \underset{i}{\text{Max}}\ |x_i|$ and $||x||_\infty = \underset{t}{\text{sup}}|x(t)|$, then

$$\delta(\varepsilon) = \varepsilon \tag{8a}$$

and using the corresponding induced matrix norm, we obtain

$$\begin{aligned} g &= \max\ \{g_1,\ g_2\}, \\ g_1 &= g_0(1 + ||z_*||_\infty k(1 + n)) \\ g_2 &= k(1 + n) \end{aligned} \tag{8b}$$

where

$$\begin{aligned} g_0 &\geq \max\ \{\gamma_\infty(H_{ev}),\ \gamma_\infty(H_{zv})\} \\ n &\geq \gamma_\infty(N),\ k \geq \gamma_\infty(K) \end{aligned} \tag{8c}$$

Although Theorem 1 provides conditions for local L_∞-stability, these do not immediately provide a <u>region of attraction</u>, i.e., bounds on e_*, z_*, and $\tilde{\theta}_o$. These bounds in turn are determined from the set of allowable reference commands, plant initial conditions, and disturbances. Since e_* and z_* are bounded by predetermined performance goals of the tuned system, it follows that $\tilde{\theta}_o$ is the unknown driving factor governing the size of $||x_L||_\infty$. That the initial parameter error vector occupies this position of villainy should come as no surprise. One way to offset large initial parameter errors is to keep the adaptation gain γ small. This has the effect of reducing large system transients, however, this may be less than prudent if the system is initially unstable or lightly damped.

No claims are made in Theorem 1 about the mechanism that provides $x_L \varepsilon L_\infty^n$ and $G \varepsilon L_\infty$-stable. However, it follows from the definition of the tuned system that $e_* \varepsilon L_\infty$, $z_* \varepsilon L_\infty^n$ and H_{ev}, H_{zv} ε L_∞-stable, thus, M in (6f) is L_∞-stable. Hence, a term by term inspection of G (6d) and x_L (6c) reveals that $x_L \varepsilon L_\infty^n$ and $G \varepsilon L_\infty$- stable, if and only if $\tilde{\theta}_L \varepsilon L_\infty^p$. Looking at (6c) we can also describe $\tilde{\theta}_L(t)$ as the solution to the differential equation,

$$\xi(t) = -\gamma(M\xi)(t) + \gamma\, w(t) \tag{9}$$

with $w = z_* e_*$ and $\xi(0) = \tilde{\theta}_o$. Referring to (6) and (9), the operator K is equivalent to the mapping from w into ξ. Hence, the stability analysis of (9) is of fundamental importance.

<u>Persistent Excitation and Exponential Stability</u>

Equations similar to (9) have been studied by invoking a persistent excitation condition on $z_*(t)$. The following definition and lemma from [1] provides the basic result.

<u>Definition</u>: A regulated function $f(.):R_+ \rightarrow R^n$ is <u>persistently exciting</u>, denoted $f \varepsilon PE$, if there exist positive constants α_1, α_2, and α_3 such that

$$\alpha_1 I_n \leq \int_s^{s+\alpha_3} f(t)f(t)'dt \leq \alpha_2 I_n, \quad \forall s \in R_+ \tag{10}$$

<u>Lemma 1</u>: Consider the differential equation:

$$\dot{\xi}(t) = -\gamma f(t)\ (Hf'\xi)(t) + \gamma w(t), \quad t \geq 0 \tag{11a}$$

If $f \in PE$ and $H(s) \in SPR$ then the map $(\xi(0),w) \mapsto \xi$ is exponentially stable, i.e., there exist positive constants m and λ such that,

$$|\xi(t)| \leq me^{-\lambda t}|\xi(0)| + \int_0^t me^{-\lambda(t-\tau)}|w(\tau)|d\tau \tag{11b}$$

The usefulness of applying Lemma 1 to determine stability conditions *of (9) is made apparent by writing* H_{ev} as,

$$H_{ev} = \bar{H}_{ev} + \tilde{H}_{ev} \tag{12}$$

where $\bar{H}_{ev}$ is the nominal representation of H_{ev} and $\tilde{H}_{ev}$ is the deviation induced, for example, by modeling error. Combining (12) with (9), and using the definitions in (6) gives,

$$\xi = -\gamma z_e \bar{H}_{ev} z_e' \xi + \gamma Q\xi + \gamma w \tag{13a}$$

where

$$Q := M - z_e \bar{H}_{ev} z_e' = z_e \tilde{H}_{ev} z_e' + e_e H_{zv} z_e' \tag{13b}$$

If $\bar{H}_{ev}(s) \in SPR$ and $z_* \in PE$, then using Lemma 1 gives,

$$|\xi(t)| \leq me^{-\lambda t}|\xi(0)| + \int_0^t \gamma me^{-\lambda(t-\tau)}|(Q\xi)(\tau) + w(\tau)|d\tau \quad (14)$$

Hence, k from (8) is,

$$k = \frac{m}{\lambda} \sum \gamma_\infty(K) \quad (15a)$$

and from (14) with ξ replaced by $\tilde{\theta}_L$ we get,

$$||\tilde{\theta}_L||_\infty \leq (1 - \gamma mq)^{-1} [|\tilde{\theta}_0| + \gamma m||z_*e_*||_\infty/\lambda] \quad (15b)$$

provided $\gamma mq < 1$ where

$$q = ||z_*||_\infty^2 \gamma_\infty(\tilde{H}_{ev}) + g_0||z_*||_\infty||e_*||_\infty \sum \gamma_\infty(Q). \quad (15c)$$

Combining (8), (15), and Theorem 1 gives the following result.

Lemma 2: The adaptive system (1) or (2) is locally L_∞-stable if for some $s < 1/g$,

$$\sigma = 1 - \frac{|\tilde{\theta}_0| + \gamma m||z_*e_*||_\infty/\lambda}{(1 - g\ s)\ s} > 0 \quad (16a)$$

and

$$\gamma mq \leq \sigma \quad (16b)$$

Lemma 2 together with (15) and (8) provides an <u>explicit upper bound</u> on $\|\tilde{\theta}_L\|_\infty$, $|\theta_\theta|$, and the amount by which H_{ev} can deviate from a nominal $\bar{H}_{ev}$ which is SPR. If the bounds are satisfied then Theorem 1 asserts that the signals in the adaptive system (1) are all bounded.

Unlike the global stability case where the bound on the deviation $\tilde{H}_{ev}$ is severely restricted, the bound here can be large.

<u>Concluding Remarks</u>

The stability analysis provided here involves establishing the exponential stability of a differential equation (9) which arises in the study of most adaptive systems. Although the connection between exponential stability of (a) and persistent excitation is known [1], it is important here to obtain specific formulae for the rates and gains involved, e.g. (8), (15), (16). Other methods to obtain these values can be found in [9] and [10]. Note also that Theorem 1 only requires L_∞-stability which is certainly provided when (9) is exponentially stability. However, L_∞-stability can be obtained by using a nonlinear adaptation gain in (1c), i.e., $\dot{\hat{\theta}} = \gamma h(z,e)$. For example, h(z,e) can arise from using a dead-zone, leakage, or normalization [11]. Such schemes can be incorporated in the general framework presented here but require further analysis in order to obtain explicit signal bounds.

REFERENCES

[1] B.D.O. Anderson, "Exponential Stability of Linear Equations Arising in Adaptive Identification," IEEE Trans. Aut. Contr., Vol. AC-22, No. 1, pp. 83-88, February 1977.

[2] B.D.O. Anderson and C.R. Johnson, Jr., "Exponential Convergence of Adaptive Identification and Control Algorithms," Automatica, Vol. 18, No. 1, 1982.

[3] R.L. Kosut and B. Friedlander, "Robust Adaptive Control: Conditions for Global Stability," IEEE Trans. on Aut. Contr., to appear, July 1985.

[4] R.L. Kosut and C.R. Johnson, Jr., "An Input-Output View of Robustion in Adaptive Control", Automatica, Sept. 1984.

[5] C.A. Desoer and M. Vidyasagar, Feedback Systems: Input-Output Properties, Academic, 1975.

[6a] C. Rohrs, L. Valavani, M. Athans and G. Stein, "Analytical Verification of Undesirable Properties of Direct Model Reference Adaptive Control Algorithms" Proc. 20th IEEE CDC, San Diego, CA, December 1981.

[6b] C. Rohrs, L. Valavani, M. Athans and G. Stein, "Robustness of Adaptive Control Algorithm in the Presence of Unmodeled Dynamics," Proc. 21st IEEE CDC, Orlando, FL, December 1982.

[7] K.S. Narendra, Y. H. Lin and L.S. Valavani, "Stable Adaptive Controller Design, Part II: Proof of Stability," IEEE Trans. Aut. Contr., Vol. AC-25, pp. 440-448, June 1980.

[8] Y.D. Landau, Adaptive Control: The Model Reference Approach, Marcel Dekker, 1979.

[9] B.D.O. Anderson, R. Bitmead, C.R. Johnson, and R.L. Kosut, "Stability Theorems for the Relaxation of the SPR Condition in Hyperstable Adaptive Schemes", Proc. 23rd IEEE CDC, Las Vegas, NV, Dec. 1984.

[10] B. Riedle and P. Kokotovic, "A Stability-Instability Boundary for Disturbance-Free Slow Adaptation and Unmodeled Dynamics", Proc. 23rd IEEE CDC, Las Vegas, NV, Dec. 1984.

[11] G. Goodwin and K.S. Sin, Adaptive Filtering, Production, and Control, Prentice-Hall, 1984.

Proc. 23rd IEEE Conf. on Dec. & Control, Dec. 1984, Las Vegas, NV.

ROBUST ADAPTIVE CONTROL: CONDITIONS FOR LOCAL STABILITY

by

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ABSTRACT

The question is examined of when an adaptive control system is robust to unmodeled dynamics and unknown bounded disturbances. Conditions are presented that ensure the existence of such robustness properties, but only locally; i.e., restrictions are placed on the behavior of signals in the ideal, perfectly tuned adaptive system. Local L_∞-stability is investigated when certain tuned signals are assumed to be persistently exciting.

1. INTRODUCTION

Theoretical investigations on the stability of adaptive control systems have focused almost entirely on developing conditions that guarantee global stability, e.g., [1]-[3]. These results are global in the sense that initial conditions and external signal magnitudes need only be bounded. Specific bounds are not required. In addition, the results provide *sufficient conditions. One of* the conditions is that a particular subsystem operator be strictly passive with finite gain or, in the case of linear-time-invariant systems, the operator is strictly positive real (SPR). This condition results from application of the Passivity Theorem; specifically, the adaptive system can be reconfigured into two subsystems: a 'feedback' subsystem (the adaptation law) that is passive, and a 'feedforward' subsystem which is required to be SPR. This condition turns out to be quite restrictive. In the first place, the SPR condition necessitates that the system transfer function (in the scalar case) have a unitary relative degree. As pointed out by Rohrs, et al. [4], it is virtually impossible to guarantee unitary relative degree for an actual system. Secondly, the SPR condition has extremely limited robustness to unmodeled dynamics [5].

In this paper, conditions are developed that guarantee the existence of local stability and robustness properties of the adaptive system, i.e., conditions which take into acount the size of initial parameter error and external signal magnitudes. These conditions are imposed on certain subsystem operators, which have a time-varying dependence on signals that arise from an ideal fictitious system where the adaptive gains are perfectly tuned to the unknown t to be controlled. The mechanism for local sta ty which is examined here is that of persistent excitation [7], [8]. Under these conditions, we develop a specific bound on model error which ensures conditions for local stability.

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2. NOTATION

Let L_p^n denote the set of Lebesgue integrable functions $x(.): R_+ \to R^n$ with finite norm $\|x\|_p := (\int \|x(t)\|^p dt)^{1/p}$ for $p\ [1,\infty)$ and $\|x\|_\infty := \sup_{t\geq 0} |x(t)|$, where $|.|$ denotes a norm on R^n. Similarly, let L_{pe}^n denote the *extension* of L_p^n consisting of functions $x(.)$ such that $x_T \in L_p^n$, $\forall T \geq 0$, where $x_T(t)$ denotes the truncation of $x(t)$ at T, i.e., $x_T(t) = x(t)$ for $t \leq T$, and $x_T(t) = 0$ for $t > T$. The norm on L_{pe}^n is denoted by either $\|x_T\|_p$ or $\|x\|_{Tp}$.

3. ADAPTIVE ERROR SYSTEM

In this section we present an adaptive error system which is representative of a large class of adaptive control systems. The error system will be presented in two forms: a parameter variational form and a full variational form. The parameter variational form was developed in detail in [5b] and is used for global stability analysis. The full variational form, to be developed here, is used for local stability analysis.

3.1 Parameter Variational Error System

To facilitate the development of the error system, consider the simple model reference adaptive controller (MRAC) depicted in Fig. 3-1 with:

Uncertain Plant

$$y = d + Pu \tag{3.1a}$$

d := external disturbance + plant initial conditions

Reference Model

$$y_r = Hr \tag{3.1b}$$

r := reference command

Adaptive Control

$$u = -(\theta_1, \theta_2)\begin{pmatrix} y \\ -r \end{pmatrix} = -\theta' z \tag{3.1c}$$

θ := adaptive gains, z := regressor

Adaptation Law

$$\dot{\theta} = Bze, \ B = B' > 0 \tag{3.1d}$$

$e := y - y_r$

Define the *adaptive gain error* by

$$\tilde{\theta} := \theta - \theta_* \tag{3.2}$$

where $\theta_* = (\theta_{*1}, \theta_{*2})'$ is a constant vector of *tuned* gains; i.e., the values that would be selected if the plant P were known. Using (3.2) we can rewrite (3.1c) as

$$u = -\theta_*' z - v \tag{3.3}$$

$$v := \tilde{\theta}' z$$

where v is the *adaptive control error* signal. An equivalent representation of (3.1) is given by the adaptive error system depicted in Figure 3-2 and described by:

$$e = e_* - H_{ev} v \tag{3.4a}$$

$$z = z_* - H_{zv} v \tag{3.4b}$$

$$v = z'\tilde{\theta} \tag{3.4c}$$

$$\tilde{\theta} = \tilde{\theta}_0 + Lze \tag{3.4d}$$

where (e_*, z_*) are the outputs of the *tuned system*, as shown in Figure 3-3; $\tilde{\theta}_0$ is the initial value of the adaptive gain error, and H_{ev}, H_{zv}, and L are the *interconnection* operators. For the simple MRAC case considered here (Fig. 3-1), the tuned signals are:

$$e_* = (1 + P\theta_{*1})^{-1} d + [(1 + P\theta_{*1})^{-1} P\theta_{*2} - H_r] r \tag{3.5a}$$

$$z_* = (e_* + y_r, -r)' \tag{3.5b}$$

and the interconnections have transfer functions,

$$H_{ev}(s) = (1 + P(s)\theta_{*1})^{-1} P(s) \tag{3.6a}$$

$$H_{zv}(s) = [(1 + P(s)\theta_{*1})^{-1} P(s), 0]' \tag{3.6b}$$

$$L(s) = (1/s)B \tag{3.6c}$$

Although the error system (3.4) has been developed here for a very simple MRAC system, the form of (3.4) is generic and applies to practically all single-input-single-output adaptive controllers and filters [5]. Moreover, the extension of (3.4) to the multivariable case requires only that v and e are vectors and that H_{ev} and H_{zv} are multivariable of compatible dimensions. Specifically, (3.4c) and (3.4d) are replaced by

$$v = Z'\tilde{\theta} \tag{3.4c)'}$$

$$\tilde{\theta} = \tilde{\theta}_0 + LZe \tag{3.4d)'}$$

where Z is a block diagonal matrix of appropriate dimensions such that

$$Z = \mathrm{diag}\,(z_1, \ldots, z_m) \tag{3.4e)'}$$

where the regressor vector is

$$z' = (z_1', \ldots, z_m') \tag{3.4f)'}$$

Hence, each adaptive control channel is given by

$$u_i = -\theta_i' z_i, \quad i = 1, \ldots, m \tag{3.4g)'}$$

In this paper, the ensuing analysis will be illustrated by using the error system of (3.4). The extension to the multivariable case follows immediately.

One of the very useful features of this error system is that the nonlinear effect of the adaptive algorithm can be analyzed separately from the analysis of the tuned system. The tuned system represents an ideal which could be achieved with the given structure of the adaptive control. Hence, the algebraic design procedure is separated from the nonlinear stability analysis. It is convenient, therefore, to view e_*, z_*, and $\tilde{\theta}_0$ as 'inputs' to the error system. The assumption, naturally, is that e_* and z_* are well behaved with e_* small. Note that $\tilde{\theta}_0$ need not be small. In the classic case, assuming perfect model following and no disturbances in the tuned system, $e_*(t) = 0$. If the disturbances are of a special kind then $e_*(t) \to 0$, i.e., the tuned system exhibits servo action. The more realistic case, however, is when $e_* \in L_\infty$ due to bounded disturbances which cannot be asymptotically rejected.

3.2 Global Stability Conditions

Conditions for global stability require that $H_{ev}(s) \in SPR$. This arises because proofs of global stability utilize passivity theory (e.g., [9], p. 182). A detailed analysis can be found in [1]-[5]. Unfortunately, though of theoretical significance, these type of results do not offer any practical engineering guidelines. The major reason is that $H_{ev}(s) \in SPR$ is not robust with respect to even mild modeling error, particularly high frequency unmodeled dynamics [4]. Since $H_{ev}(s) \in SPR$ implies that the relative degree of $H_{ev}(s)$ cannot exceed one, it follows that applying this restriction to (3.6a) imposes a unitary relative degree restriction on $P(s)$ as well. This is unrealistic, even in this simple example.

Another view of the restrictiveness of $H_{ev} \in SPR$ is from robustness theory, e.g., [15]. Suppose that $P(s)$ in (3.1a) can be expressed as belonging to the set of transfer functions

$$P(s) = (1 + \Delta(s))\bar{P}(s) \tag{3.7a}$$

$$|\Delta(j\omega)| \le \delta(\omega), \quad \forall \omega \in R \tag{3.7b}$$

Hence, $\bar{P}(s)$ is a nominal model of $P(s)$ and $\Delta(s)$ represents modeling error, e.g., high frequency unmodeled dynamics. We can now write

$$H_{ev} = \bar{H}_{ev} + \tilde{H}_{ev} \tag{3.8a}$$

where the nominal is,

$$\bar{H}_{ev} = (1 + \bar{P}\theta_{*1})^{-1} \bar{P} \in SPR \tag{3.8b}$$

and the deviation induced by modeling error is,

$$\tilde{H}_{ev} = \bar{H}_{ev} [1 + (1 + \bar{P}\theta_{*1})^{-1} (1 + \bar{P}\theta_{*1})^{-1}\Delta \tag{3.8c}$$

It is shown in [5] that the largest tolerable $\delta(\omega)$ in (3.7b) to ensure $H_{ev} \in SPR$ is bounded by

$$\delta(\omega) < 1 . \tag{3.9}$$

Again, this is unrealistic and is violated even by the most mild form of unmodeled dynamics. Note that (3.9) and the unitary relative degree restriction both necessarily arise from the SPR condition.

3.3 Full Variational Form

The error system (3.4) can be transformed to the following variational form which is more useful for local stability analysis, i.e.,

$$x = x_L - Gf(x) \tag{3.10a}$$

where the quantities above are defined below by

$$x := \begin{pmatrix} \tilde{e} \\ \tilde{z} \\ \tilde{\theta} \end{pmatrix} := \begin{pmatrix} e - e_* \\ z - z_* \\ \theta - \theta_* \end{pmatrix}, \quad f(x) := \begin{pmatrix} \tilde{z}' & \tilde{\theta} \\ \tilde{z} & \tilde{e} \end{pmatrix} \tag{3.10b}$$

$$x_L := \begin{pmatrix} \tilde{e}_L \\ \tilde{z} \\ \tilde{\theta}_L \end{pmatrix} := \begin{pmatrix} -H_{ev} z_*' \tilde{\theta}_L \\ -H_{zv} z_*' \tilde{\theta}_L \\ (I + LM)^{-1} \tilde{\theta}_0 + K_* e_* \end{pmatrix} \tag{3.10c}$$

$$G := \begin{bmatrix} H_{ev}(1 - z_*' KN) & H_{ev} z_*' K \\ H_{zv}(1 - z_*' KN) & H_{zv} z_*' K \\ KN & -K \end{bmatrix} \tag{3.10d}$$

with

$$N := z_* H_{ev} + e_* H_{zv} \tag{3.10e}$$

$$M := N z_*' \tag{3.10f}$$

$$K := (I + LM)^{-1} L \tag{3.10g}$$

and where L has the transfer function,

$$L(s) = \frac{1}{s} B \tag{3.10h}$$

with B from the adaptive algorithm (3.1d).

The model (3.7) is arrived at by separating the nonlinear cross product terms in $f(x)$ from the linear terms in x_L. We shall refer to x_L as the response of the linearized system. This is almost identical to the linearized system studied by Rohrs, et al. [4a], which was arrived at by a 'final approach analysis.' Note that in this case the linearized system is the input to the nonlinear system (3.10a). The operators K and G are linear and time-varying due to their dependence on the tuned signals (e_*, z_*). This model (3.10) will now be utilized to develop local stability conditions.

4. CONDITIONS FOR LOCAL STABILITY

If the linearized response x_L in (3.10) is small, and if the nonlinear term $G\,f(x)$ is suitably restricted, then intuitively, x would be attracted to some neighborhood of x_L. The following theorem makes this notion precise.

Theorem 4.1

(i) If $\exists$ constant g_∞ such that,

$$\gamma_\infty(G) \le g_\infty < \infty \tag{4.1}$$

and if

$$\|x_L\|_\infty \le \epsilon_\infty (1 - g_\infty \epsilon_\infty/2), \quad \epsilon_\infty \in (0, 2/g_\infty) \tag{4.2}$$

then

$$\|x\|_\infty \le \epsilon_\infty \tag{4.3}$$

(ii) In addition, if, for some $p \in [1,\infty)$, $\exists$ constant g_p such that,

$$\gamma_p(G) \le g_p < 2/\epsilon_\infty \tag{4.4}$$

then

$$\|x\|_p \le (1 - g_p \epsilon_\infty/2)^{-1} \|x_L\|_p \tag{4.5}$$

Proof:

Theorem 4.1 is based on the linearization theorem of [9, p. 131]. The proof, as specialized here, is in Appendix A.

Remarks

(1) Theorem 4.1, part (i), asserts that the error outputs x of the adaptive error system are L_∞-bounded in an ϵ-neighborhood of the linearized response, provided that the linearized response is in L_∞ and is small enough (4.2), and that $G \in L_\infty$-stable (4.1). Condition (4.3) shows that the actual response can be arbitrarily close to the linearized response, if $\|x_L\|_\infty$ is small enough (4.2). Since Theorem 4.1, part (i), provides sufficient conditions, instability does not follow if $x_L \in L_\infty^n$ but exceeds the magnitude constraint (4.2).

(2) The results in part (ii) are stronger than in part (i) since they can only be applied when $x_L \in L_p^n$ for some $p \in [1,\infty)$. Looking at (3.10), this can only occur if $z_* e_* \in L_p^n$ which, in practical situations, almost never occurs due to the presence of disturbances in L_∞. Hence, part (ii) of Theorem 4.1 does not offer any practical advice and we will focus only on part (i).

(3) From (4.2), the largest upper bound on $\|x_L\|_\infty$ is $1/2g_\infty$ which occurs when $\epsilon_\infty = 1/g_\infty$.

(4) Although Theorem 4.1 part (i) provides conditions for local L_∞-stability, these do not immediately provide a region of attraction, i.e., bounds on e_*, z_*, and $\tilde{\theta}_0$. These bounds in turn are determined from the set of allowable reference commands, plant initial conditions, and disturbances. Since e_* and z_* are bounded by predetermined performance goals of the tuned system, it follows that $\tilde{\theta}_0$ is the unknown driving factor governing the size of $\|x_L\|_\infty$. That the initial parameter error vector occupies this position of villainy should come as no surprise. For example, if $\tilde{\theta}_0$ is small (order ϵ_∞) then the adaptive system stays near the tuned system for small (order ϵ_∞) inputs e_*.

(5) No claims are made in Theorem 4.1 part (i) about the mechanism that provides $x_L \in L_\infty^n$ and $G \in L_\infty$-stable. However, it follows from the definition of the tuned system (3.2) that $e_* \in L_\infty$, $z_* \in L_\infty^n$ and H_{ev}, $H_{zv} \in L_\infty$-stable, thus, M in (3.10f) is L_∞-stable. Hence, a term by term inspection of G (3.10d) and x_L (3.10c) reveals that $x_L \in L_\infty$ and $G \in L_\infty$-stable, if and only if:

$$(I + LM)^{-1} \tilde{\theta}_0 \in L_\infty^n, \quad \forall \tilde{\theta}_0 \in R^n \tag{4.6a}$$

and

$$K \in L_\infty\text{-stable} \tag{4.6b}$$

More precisely, we have the following result.

Lemma 4.1

Suppose that a tuned solution $\theta_* \in R^n$ exists. Let $\zeta(t)$ denote the solution at time t of the differential equation,

$$\dot{\zeta}(t) = -H(M\zeta)(t) + w(t) \qquad t \geq 0 \qquad (4.7)$$

Then, $z_1 \in L_\infty^n$ and $G \in L_\infty$-stable if and only if $\zeta \in L_\infty^n$ for all $\zeta(0) \in R^n$ and $w \in L_\infty^n$.

Proof:

Using the definitions of K and M in (3.10), if $\zeta(0) = 0$ then (4.7) represents $K : w \mapsto \zeta$. Thus, $K \in L_\infty$-stable if $w \mapsto \zeta \in L_\infty$-stable. Also, if $\zeta(0) = \tilde{\theta}_o$ and $w = z_* e_*$ then $\zeta = \tilde{\theta}_L$ from (3.10c). Hence, without any further restrictions on $\tilde{\theta}_o \in R^n$ or $z_* e_* \in L_\infty^n$, the result of Lemma 4.1 is established.

Remarks

Lemma 4.1 identifies the L_∞-stability of the system (4.7) as being crucial to obtaining local stability conditions from Theorem 4.1. This condition is not sufficient. Even if the conditions of Lemma 4.1 are satisfied the adaptive system is locally stable provided that $||z_L||_\infty$ is small enough, i.e., (4.2) must hold. Nonetheless, establishing (4.9) is a first step.

Recall that $||z_L||_\infty$ is small if $||\tilde{\theta}_L||_\infty$ is small, hence it is necessary to control the size of these signals. Comparing $\tilde{\theta}_L$ in (3.10c) to (4.7), some of these magnitude conditions can be secured by lowering the adaptation gain, i.e., the norm of the matrix B in (3.1d) or (3.10h).

5. PERSISTENT EXCITATION

In this section we examine persistent excitation as a mechanism to provide L_∞-stability of (4.7), and hence, local L_∞-stability of the adaptive system (3.10).

Definition [8]

A regulated function $f(.): R_+ \to R^n$ is persistently exciting, denoted $f \in PE$, if positive constants α_1, α_2, and α_3 such that

$$\alpha_1 I_n \leq \int_s^{s+\alpha_3} f(t)\, f(t)'dt \leq \alpha_2 I_n, \quad \forall s \in R_+ \qquad (5.1)$$

The relationship between persistent excitation and stability of (4.7) is given as follows.

Lemma 5.1 [8]

Consider the differential equation:

$$\dot{\zeta}(t) = -f(t)\,(Hf'\zeta)(t) + w(t), \quad t \geq 0 \qquad (5.2)$$

If $f \in PE$ and $H(s) \in SPR$ then the map $(\zeta(0), w) \mapsto \zeta$ is exponentially stable, i.e., there exist positive constants m and λ such that,

$$|\zeta(t)| \leq me^{-\lambda t}|\zeta(0)| + \int_0^t me^{-\lambda(t-\tau)}|w(\tau)|d\tau \qquad (5.3)$$

Conditions for Robustness

The usefulness of applying Lemma 5.1 to determine robust stability conditions of (4.7) is made apparent by proceeding as in Section 3, i.e.,

$$H_{ev} = \bar{H}_{ev} + \tilde{H}_{ev} \qquad (5.4)$$

where $\bar{H}_{ev}$ is the nominal representation of H_{ev} and $\tilde{H}_{ev}$ is the deviation induced by modeling error. Combining (5.4) with (4.7), and using the definitions in (3.10) gives,

$$\dot{\zeta} = -Bz_*\bar{H}_{ev}z_*'\zeta + Q\zeta + w \qquad (5.5)$$

where

$$Q := B(M - z_*\bar{H}_{ev}z_*') \qquad (5.6)$$

If $\bar{H}_{ev}(s) \in SPR$ and $z_* \in SPR$, then (5.3) of Lemma (5.1) applied to (5.5) gives,

$$|\zeta(t)| \leq me^{-\lambda t}|\zeta(0)| + \int_0^t me^{-\lambda(t-\tau)}|(Q\zeta)(\tau) + w(\tau)|d\tau \qquad (5.7)$$

Therefore, if Q has a sufficiently small gain then $(\zeta(0), w) \mapsto \zeta \in L_\infty$-stable, and hence, the adaptive system is locally L_∞-stable. Specific conditions are given as follows.

Theorem 5.1

Suppose $z_* \in PE$, $\bar{H}(s) \in SPR$, and hence, from Lemma 5.1 $\zeta(t)$ in (5.5) is bounded as shown in (5.7). Then, the adaptive system (3.10) is locally L_∞-stable if, for some $\alpha \in (0, \lambda)$,

$$\gamma_2(F) < 1 - q \qquad (5.8)$$

where

$$q = \frac{m}{\lambda - \alpha}\, ||e_*||_\infty ||Bz_*||_\infty \gamma_2(\tilde{H}_{zv}^\alpha) \qquad (5.9)$$

and the operator F has the integral form,

$$(Fu)(t) = \int_0^t me^{-(\lambda-\alpha)(t-\tau)}\, z_*(\tau)(\tilde{H}_{ev}^\alpha z_*'u)(\tau)d\tau \qquad (5.10)$$

Proof: See Appendix B.

Remarks

(1) The α-superscript notation H^α means that if H has transfer function $H(s)$, then H^α has transfer function $H(s-\alpha)$. Thus, $\alpha \in (0, \lambda)$ is further limited so that $\bar{H}_{ev}(s-\alpha)$ and $\tilde{H}_{ev}(s-\alpha)$ remain exponentially stable, otherwise the L_1-gains in (5.9), (5.10) are infinite.

(2) What Theorem 5.1 asserts is that if $z_* \in PE$, and if H_{ev} is close enough to being SPR, then under suitable small gain conditions (5.8), local stability can be guaranteed via Theorem 4.1. The crux of the matter is to establish that $\gamma_2(F)$ is sufficiently small despite a reasonably large $\tilde{H}_{ev}$.

Calculation of $\gamma_2(F)$

Intuitively, if the range of dominant frequencies of z_e is sufficiently separated from the frequencies where $\tilde{H}_{ev}(j\omega)$ is large, then $\gamma_2(F)$ would be small, e.g., z_e is persistently exciting at 'low' frequencies where H_{ev} is approximately SPR. We will formalize this notion in Theorem 5.2 below. First, however, we need the following results from [10] for determining a large class of persistently exciting signals.

Definition 5.1 [10]

A function $f(.): R_+ \rightarrow R^n$ has a *spectral line at frequency* ω *of amplitude* $a_f(\omega) \in C^n$ if

$$\lim_{\tau \to \infty} \frac{1}{\tau} \int_s^{s+\tau} f(t)e^{-j\omega t}\, dt = a_f(\omega), \quad \forall s \in R_+ \tag{5.11}$$

when $a_f(\omega) \neq 0$, f *has a spectral line at* ω.

Lemma 5.2 [10]

Suppose $f \in L_\infty^n$ has spectral lines at frequencies $\omega_1, \ldots, \omega_p$ of amplitudes $a_f(\omega_1), \ldots, a_f(\omega_p)$. Then $f \in PE$, if

$$\text{rank}\,[a_f(\omega_1), \ldots, a_f(\omega_p)] = n \tag{5.12}$$

Theorem 5.2

Suppose that:

(A1) $z_e \in PE \Leftarrow z_e$ has spectral lines at frequencies $\omega_1, \ldots, \omega_p$ with rank $[a_{z_e}(\omega_1), \ldots, a_{z_e}(\omega_p)] = n$.

(A2) $\bar{H}_{ev}(s) \in SPR$

Then, the adaptive system (3.10) is locally L_∞-stable *if, for some* $\alpha \in (0, \lambda)$, $\tilde{H}_{ev}(s-\alpha)$ is stable and bounded by,

$$|\tilde{H}_{ev}(j\omega-\alpha)| < \frac{1-q}{mp \sup_k |a_{z_e}(\omega_k)|} \inf_k \frac{[(\omega-\omega_k)^2 + (\lambda-\alpha)^2]^{1/2}}{|a_{z_e}(\omega_k)|}, \quad \forall \omega \in R_+ \tag{5.13}$$

with m, λ from (5.7) and q from (5.9).

Proof

See Appendix C.

Discussion

(1) Theorem 5.2 provides an explicit upper bound on the amount by which H_{ev} can deviate from a nominal $\bar{H}$ which is SPR. Thus, if (5.13) holds and if $\|\tilde{\xi}^{ev}\|_\infty$ is sufficiently small (4.2), then signals in the adaptive system are guaranteed to be bounded.

(2) Unlike the global stability case where the bound on the deviation $\tilde{H}_{ev}$ is severely restricted (3.9), the bound here can be quite large. Moreover, the bound can be determined from the spectral properties of z_e. Recall from Lemma 5.1 that m and λ in (5.13) are functions of the spectral properties of z_e.

(3) Since $\alpha \in (0, \lambda)$ in (5.13) can be arbitrarily small, and since $q \ll 1$ is likely due to $\|e_e\|_\infty$ being small, it follows that a reasonable approximation to the robustness test (5.13) is

$$|\tilde{H}_{ev}(j\omega)| < \frac{1}{mp \sup_k |a_{z_e}(\omega_k)|} \inf_k \frac{[(\omega-\omega_k)^2 + \lambda^2]^{1/2}}{|a_{z_e}(\omega_k)|}, \quad \forall \omega \in R_+ \tag{5.14}$$

6. CONCLUDING REMARKS

6.1 Test Procedures

The results of Theorem 5.2 can be of practical use since they provide the basis for developing robustness test procedures. The most obvious way to apply Theorem 5.2 is to determine the model error bound--the right hand side of (5.13)--by either analytical or empirical means. Once the bound is found, it remains to generate reasonable estimates of the model error and compare this to the bound.

An alternative procedure is to verify Lemma 4.1 by direct empirical means. In other words, we can utilize Theorem 5.2 to give qualitative guidelines on the required spectral characteristics of z_e and then simulate (4.7) for a variety of initial states $\xi(0)$ and inputs $w \in L_\infty^n$. This latter approach is not theoretically perfect, but is a practical means to gain understanding of the adaptive system behavior.

6.2 Other Mechanisms for Local Stability

Although we have focused on persistent excitation as a means to ensure local stability, this is by no means the only way. For example, if the adaptation algorithm (3.1d) is modified to include a retardation (see, e.g., [11], [12]) then $L(s)$ in (3.10b) will have the form [13]:

$$L(s) = \frac{a}{s+b} B, \quad B = B' > 0 \tag{6.1}$$

where (a,b) are positive constants. This means that if z_e is a constant vector, then the linearized system (3.10c) is L_∞-stable by passivity arguments [13]. (Note that it is not possible to prove (4.7) stable for z_e constant with $L(s) = (1/s)B$ as in (3.10h).) Hence, using (6.1) together with theorems on slowly varying systems (z_e stays near constant long enough), we can arrive at conditions for local L_∞-stability which are independent of persistent excitation (see [13] for preliminary results).

APPENDIX A
PROOF OF THEOREM 4.1

We first show that $f(z)$ in (3.10b) has the property that $\forall \varepsilon > 0$,

$$|z| < \varepsilon \Rightarrow |f(z)| < \frac{\varepsilon}{2} |z| \tag{A.1}$$

From (3.10b),

$$\begin{aligned} |f(z)| &= (|\tilde{z}'\bar{\theta}|^2 + |\tilde{z}\tilde{\theta}|^2)^{1/2} \\ &\leq |\tilde{z}|\, (|\bar{\theta}|^2 + |\tilde{\theta}|^2)^{1/2} \\ &= |\tilde{z}|\, (|z|^2 - |\tilde{z}|^2)^{1/2}, \quad \text{by (3.7b)} \end{aligned}$$

$$\le \frac{1}{2}\,|x|^2 . \quad \text{by holding } |x| \text{ fixed}$$

$$\le \frac{\varepsilon}{2}\,|x|$$

by $|x| \le \varepsilon$.

Now, assume *temporarily* that (A.1) holds for all $x \in R^n$, i.e.,

$$|f(x)| \le (\varepsilon/2)|x|, \quad \forall |x| \le \varepsilon \tag{A.2a}$$

$$|f(x)| \le (\varepsilon/2)\,|x|, \quad \forall |x| > \varepsilon \tag{A.2b}$$

From (3.10a)

$$\|x\|_{T\infty} \le \|x_L\|_{T\infty} + \|Gf(x)\|_{T\infty}$$

$$\le \|x_L\|_{T\infty} + g_\infty \|f(x)\|_{T\infty}, \quad \text{by (4.1)}$$

$$\le \|x_L\|_{T\infty} + (g_\infty\, \varepsilon/2)\,\|x\|_{T\infty} \tag{A.3}$$

using the temporary assumption (A.2). Since $x_L \in L_\infty$ (4.2) and $\|x_L\|_{T\infty} \le \|x_L\|_\infty$.

$$\|x\|_{T\infty} \le \|x_L\|_\infty + (g_\infty\, \varepsilon/2)\,\|x\|_{T\infty}$$

$$\le \varepsilon\,(1-g_\infty\, \varepsilon/2) + (g_\infty\, \varepsilon/2)\,\|x\|_{T\infty} \tag{A.4}$$

by (4.2). Since $g_\infty\, \varepsilon/2 < 1$ by assumption (4.2),

$$\|x\|_{T\infty} \le \varepsilon \tag{A.5}$$

and hence, $\|x\|_\infty \le \varepsilon$. Looking back over the proof we see that the temporary assumption (A.2) is never violated, i.e., the behavior of $f(x)$ for $|x| > \varepsilon$ (A.2b) is never needed under the assumptions of Theorem 4.1. This proves part (i).

Part (ii) proceeds analogously except now we use the L_p-norm. Note that part (ii) uses $\|x\|_\infty \le \varepsilon$ as an assumption.

APPENDIX B
PROOF OF THEOREM 5.1

We use the exponential weighting techniques from [9], [14]. Let y^α denote the exponential weighting operation,

$$(y^\alpha)(t) := y^\alpha(t) := e^{\alpha t}\, y(t) \tag{B.1}$$

If $y = Hu$ then let H^α denote the map $u^\alpha \mapsto y^\alpha$. For example, if H has transfer function $H(s)$, then H^α has transfer function $H(s-\alpha)$.

Definition B.1 [14]

An operator $H : L^m_{2e} \to L^n_{2e}$ has *decaying L_1-memory* if $\exists$ a nonnegative, nonincreasing function $\beta(.) \in L_1$ such that

$$|(Hu)(t)|^2 \le \int_0^t \beta(t-\tau)\,|u(\tau)|^2 d\tau, \quad \forall t \ge 0, \quad \forall u \in L^m_{2e}. \tag{B.2}$$

Lemma B.1 [14]

Suppose $H : L^m_{2e} \to L^n_{2e}$ has decaying L_1-memory. If, for some $\alpha > 0$, $H^\alpha \in L_2$-stable, then $H \in L_\infty$-stable.

Apply Lemma B.1 as follows: The exponentially weighted version of (5.7) is,

$$|\zeta^\alpha(t)| \le m e^{-(\lambda-\alpha)t}\zeta(0) + \int_0^t m e^{-(\lambda-\alpha)(t-\tau)}[w^\alpha(\tau) + (Q^\alpha\zeta^\alpha)(\tau)]\,d\tau \tag{B.3}$$

This, together with the definitions of F (5.10) and q (5.9) gives,

$$\|\zeta^\alpha\|_{T2} \le (m/\sqrt{2(\lambda-\alpha)})\,|\zeta(0)| + \frac{m}{\lambda-\alpha}\|w^\alpha\|_{T2} + (q+\gamma_2(F))\,\|\zeta^\alpha\|_{T2} \tag{B.4}$$

Using $q + \gamma_2(F) < 1$ from (5.8) gives,

$$\|\zeta^\alpha\|_{T2} \le [1-q-\gamma_2(F)]^{-1}\,[(m/\sqrt{2(\lambda-\alpha)})\,|\zeta(0)| + \frac{m}{\lambda-\alpha}\|w^\alpha\|_{T2}] \tag{B.5}$$

Hence, $w^\alpha \mapsto \zeta^\alpha \in L_2$-stable. Moreover, using (B.3) $w \mapsto \zeta$ is exponentially stable and hence, from definition B.1, $w \mapsto \zeta$ has decaying L_1-memory. Therefore, the conditions of Lemma B.1 are satisfied; consequently $w \mapsto \zeta \in L_\infty$-stable.

APPENDIX C
PROOF OF THEOREM 5.2

We need to calculate the L_2-gain of F (5.10) under the assumptions of the theorem. From (5.10), $F : x \mapsto y$ has the form,

$$y = Fx = Gz_e\, Hz_e' x \tag{C.1}$$

where G and H have transfer functions,

$$G(s) = \frac{m}{s+\lambda-\alpha} \tag{C.2}$$

$$H(s) = \tilde{H}_{ev}(s-\alpha) \tag{C.3}$$

Since F is causal, the L_2-gain of F is,

$$\gamma_2(F) = \sup_{x \in L^2_n/0} \frac{\|y\|_2}{\|x\|_2} \tag{C.4}$$

$$= \sup_{x \in L^n_2/0} \frac{\|y\|_2}{\|x\|_2} \quad \text{(by Parseval's Theorem)} \tag{C.5}$$

where $y(j\omega)$ and $x(j\omega)$ are the Fourier transforms of y and x, respectively. Using assumption (A1) of Theorem 5.2,

$$y(j\omega) = \sum_{k=1}^{p} \sum_{r=1}^{p} G(j\omega)H(j\omega - j\omega_k)a_{z}(\omega_k)\bar{a}_{z}(\omega_r)'x(j\omega - j\omega_k - j\omega_r) \tag{C.6}$$

Hence,

$$|y(j\omega)| \le c(\omega)\,[\sum_{k=1}^{p} \sum_{r=1}^{p} |x(j\omega - j\omega_k - j\omega_r|^2\,]^{1/2} \tag{C.7}$$

where

$$c(\omega) = (\sup_k |a_{z_e}(\omega_k)|)(\sup_k |G(j\omega)H(j\omega - j\omega_k)|.|a_{z_e}(\omega_k)|) \tag{C.8}$$

Thus,

$$\|y\|_2 = (\int_{-\infty}^{\infty} |y(j\omega)|^2 d\omega)^{1/2}$$

$$\leq [\sup_{\omega} c(\omega)] \left[\int_{-\infty}^{\infty} \sum_{k=1}^{p} \sum_{r=1}^{p} |s(j\omega, j\omega_k, j\omega_r)|^2 d\omega \right]^{1/2}$$

$$\leq p[\sup_{\omega} c(\omega)] \, ||s||_2 \qquad (C.9)$$

Using the definition of gain (C.5) together with (5.8) from Theorem 5.1 gives,

$$\gamma_2(F) \leq p \sup_{\omega} c(\omega) < 1 - q \qquad (C.10)$$

Condition (5.13) follows by substituting (C.2) and (C.3) into (C.10) and rearranging terms.

REFERENCES

[1] K.S. Narendra, Y.H. Lin and L.S. Valavani, 'Stable Adaptive Controller Design, Part II: Proof of Stability,' IEEE Trans. Aut. Contr., Vol. AC-25, pp. 440-448, June 1980.

[2] G.C. Goodwin, P.J. Ramadge and P.E. Caines (1980), 'Discrete Time Multivariable Adaptive Control,' IEEE Trans. Aut. Contr., Vol. AC-25, June 1980, pp. 449-456.

[3] Y.D. Landau, Adaptive Control: The Model Reference Approach, Marcel Dekker, 1979.

[4a] C. Rohrs, L. Valavani, M. Athans and G. Stein, 'Analytical Verification of Undesirable Properties of Direct Model Reference Adaptive Control Algorithms' Proc. 20th IEEE CDC, San Diego, CA, December 1981.

[4b] C. Rohrs, L. Valavani, M. Athans and G. Stein, 'Robustness of Adaptive Control Algorithm in the Presence of Unmodeled Dynamics,' Proc. 21st IEEE CDC, Orlando, FL, December 1982.

[5a] R.L. Kosut and B. Friedlander, 'Performance Robustness Properties of Adaptive Control Systems,' Proc. 21st IEEE CDC, Orlando, FL, December 1982.

[5b] R.L. Kosut and B. Friedlander, 'Robust Adaptive Control: Conditions for Global Stability,' IEEE Trans. on Aut. Contr., to appear.

[6] B. Wittenmark and K.J. Astrom, 'Implementation Aspects of Adaptive Controllers and Their Influence on Robustness,' Proc. 21st IEEE CDC, Orlando, FL, December 1982.

[7] B.D.O. Anderson and C.R. Johnson, Jr., 'Exponential Convergence of Adaptive Identification and Control Algorithms,' Automatica, Vol. 18, No. 1, 1982.

[8] B.D.O. Anderson, 'Exponential Stability of Linear Equations Arising in Adaptive Identification,' IEEE Trans. Aut. Contr., Vol. AC-22, No. 1, pp. 83-88, February 1977.

[9] C.A. Desoer and M. Vidyasagar, Feedback Systems: Input-Output Properties, Academic, 1975.

[10] S. Boyd and S. Sastry, 'On Parameter Convergence in Adaptive Control,' Memorandum No. UCB/ERL M83/83, Electronics Research Lab., University of California, Berkeley, June 1983.

[11] G. Kreisselmeier and K.S. Narendra (1982), 'Stable Model Reference Adaptive Control in the Presence of Bounded Disturbances,' IEEE Trans. Aut. Contr., Vol. AC-27, No. 6, December 1982, pp. 1169-1175.

[12] P.A. Ioannou and P.V. Kokotovic (1983b), Adaptive Systems with Reduced Models, Springer-Verlag, 1983.

[13] R.L. Kosut, 'Robust Adaptive Control: Conditions for Local Stability,' Proc. 1983 ACC, pp. 565-569, San Francisco, CA, June 1983.

[14] M. Vidyasagar, 'L_∞-Stability Criteria for Interconnected Systems Using Exponential Weighting,' IEEE Trans. Circuits Syst., Vol. CAS-25, No. 11, Nov. 1978.

[15] J.C. Doyle and G. Stein, 'Multivariable Feedback Design: Concepts for a Modern Classical Synthesis,' IEEE Trans. Aut. Contr., AC-26, No. 1, pp. 4-17, February 1981.

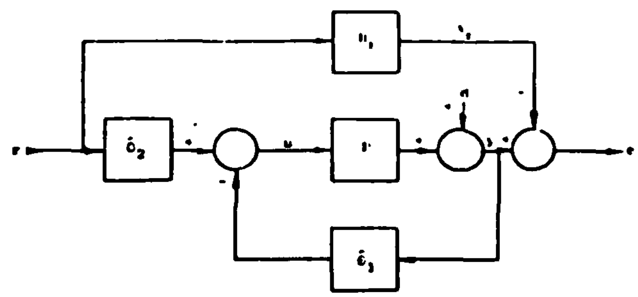

Figure 3-1. MRAC System

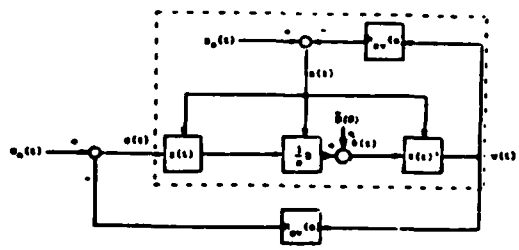

Figure 3-2. Adaptive Error System

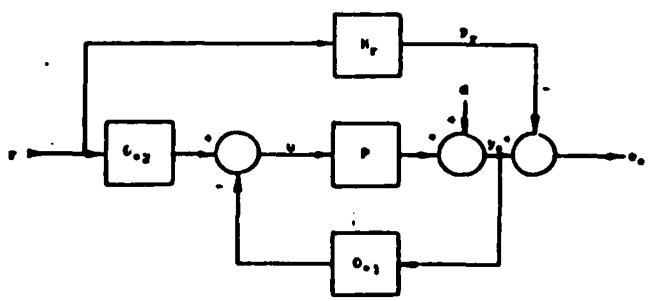

Figure 3-3. Tuned System

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CONDITIONS FOR LOCAL STABILITY AND ROBUSTNESS OF ADAPTIVE CONTROL SYSTEMS

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Abstract

This paper reports some preliminary results concerning robustness properties of adaptive control systems to unmodeled dynamics and bounded disturbances. The analysis is conducted from the viewpoint of input/output stability theory. Generic representations are proposed for both continuous-time and discrete-time adaptive systems and conditions for stability and robustness are developed for each case. These conditions require varying degrees of a priori knowledge about the plant, e.g., global conditions involving minimal knowledge and local conditions involving more restrictive assumptions.

1. Continuous-Time Case

A. Global Analysis

A large class of continuous-time adaptive systems can be represented by the nonlinear system (Fig. 1):

$$e_t = e_t^* - H(p)v_t, \quad v_t = \xi_t' \pi_t$$

$$\xi_t = \xi_t^* - G(p)v_t \qquad (S_c)$$

$$\dot{\pi}_t = A(\xi_t, \omega_t), \quad \omega_t = \xi_t e_t$$

where e_t, e_t^*, $v_t \in R$, and π_t, ξ_t, $\xi_t^* \in R^n$. The operators $H(p)$ and $G(p)$ are proper rational functions with real coefficients in the differential operator p, i.e., $(px)_t := \dot{x}_t$. We will refer to e_t as the <u>output error</u>, π_t as the <u>parameter error</u>, v_t as the <u>control error</u>, ξ_t as the <u>regressor</u> and $A(.,.)$ as the <u>adaptation gain</u>. In general, only e_t and ξ_t are available as measurements. The parameter error $\pi_t := \hat{\pi}_t - \pi^*$, where $\hat{\pi}_t$ is the adaptive estimate of the true, but unknown, parameter π^*. The signals e_t^* and ξ_t^* are referred to as the <u>tuned</u> output error and regressor, respectively, meaning that these signals are generated from an 'ideal' system with the desired parameters π^*, i.e., $\hat{\pi}_t = \pi^*$. Details on the relation between (S_c) and the actual system (unknown plant + adaptive controller) can be found elsewhere, e.g., [1]-[3]. In general, the unknown plant is imbedded in $G(p)$ and $H(p)$, which, incidentally, are also functions of the true parameter π^*.

Since π^* as well as the plant are unknown, it follows that $H(p)$ and $G(p)$ are unknown. However, in order to establish conditions for stability of (S_c), it is necessary to know something about $H(p)$ and $G(p)$. The same remark holds for knowledge about the tuned signals e_t^* and ξ_t^*. The following theorem gives conditions for <u>global</u> stability of (S_c). The term 'global' refers to the intention of requiring minimal, but reasonable, restrictions on $H(p)$, $G(p)$, e_t^* and ξ_t^*. Proof of Theorem 1 is given in [2].

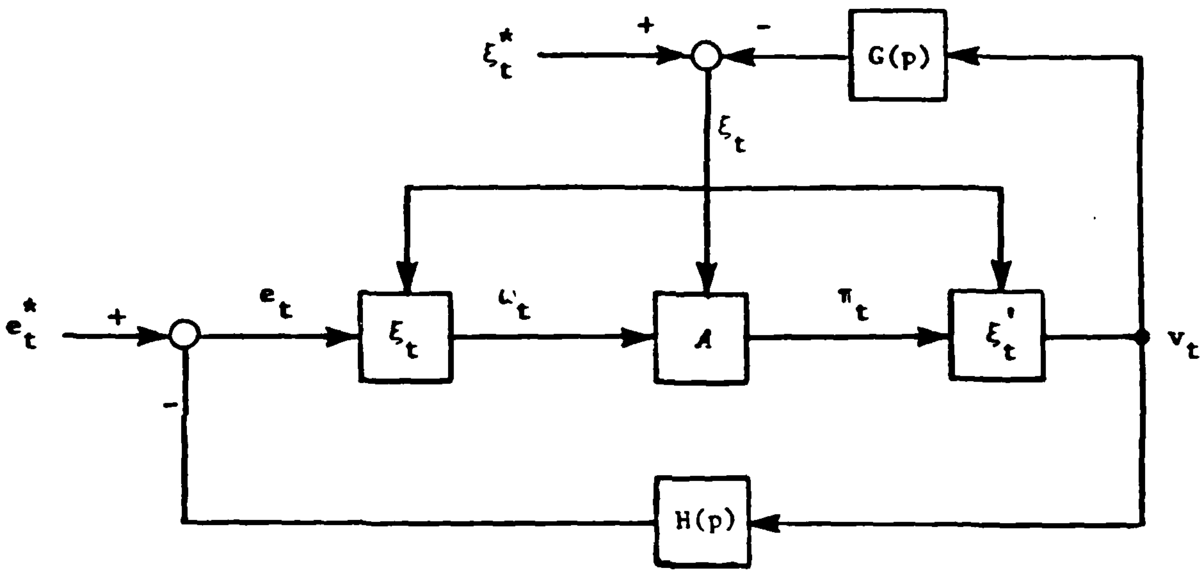

Fig. 1. Adaptive System.

Theorem 1: Global Stability

For the system (S_c) assume that:

(A1) The elements of $G(p)$ are strictly proper and exponentially stable (all poles strictly inside the left half plane)

(A2) $H(p)$ is strictly positive real (SPR), i.e., the elements of $H(p)$ are strictly proper, exponentially stable, and Re $[H(j\omega)]$ is positive for all $\omega \in [0,\infty)$.

(i) Suppose that the adaptation gain is constant, i.e.,

$$A(\xi_t, \omega_t) = A_o \omega_t, \; A_o = A_o' > 0 \tag{1}$$

Under these conditions, if $e^*, \dot{e}^* \in L_2 \cap L_\infty$ ($\Rightarrow e_t^* \to 0$), and $\xi^*, \dot{\xi}^* \in L_\infty^n$ then:

(i-a) $\pi \in L_\infty^n$, $\dot{\pi}$, $\pi \in L_2^n \cap L_\infty^n$, and $\dot{\pi}_t \to 0$ at the rate $e_t^* \to 0$.

(i-b) $e, \dot{e} \in L_2 \cap L_\infty$, and $e_t - e_t^* \to 0$ exp.

(i-c) $v \in L_2 \cap L_\infty$, $\dot{v} \in L_\infty$, and $v_t \to 0$.

(i-d) $\xi, \dot{\xi} \in L_\infty^n$, $\xi - \xi^*$, $\dot{\xi} - \dot{\xi}^* \in L_2^n \cap L_\infty^n$, and

$$\xi_t - \xi_t^* \to 0 \text{ exp.}$$

(ii) Suppose that ξ_t is persistently exciting [4], i.e., $\exists$ constants α_1, α_2, $\alpha_3 > 0$ such that

$$\alpha_1 I_n \le \int_s^{s+\alpha_3} \xi_t \xi_t' \, dt \le \alpha_2 I_n, \quad \forall s \in R^+ \tag{2}$$

(ii-a) Result (i) holds, and in addition, π_t, $v_t \to 0$ exp.

(ii-b) If the elements of e^*, $\dot{e}^*$, ζ^*, and $\dot{\zeta}^*$ are all in L_∞, then the elements of π, $\dot{\pi}$, $\ddot{\pi}$, e, $\dot{e}$, v, $\dot{v}$, ζ, and $\dot{\zeta}$ are all in L_∞.

(iii) Suppose that the adaptation gain is <u>retarded</u>, [5], i.e.,

$$A(\zeta_t, \omega_t) = \begin{cases} A_o \omega_t, & |\hat{\pi}_t| < c, \quad c \geq \max |\pi^*| \\ A_o(\omega_t - (1 - |\hat{\pi}_t|/c)^2 \hat{\pi}_t), & |\hat{\pi}_t| \geq c \end{cases} \tag{3}$$

(iii-a) Result (i) holds

(iii-b) Result (ii-b) holds.

<u>Remarks</u>

The major difficulties in applying Theorem 1 are that, in the first place, $H(p) \in SPR$ (condition (A2)) is an unlikely event in actual systems, due to the effect of unmodeled dynamics [6,7]. Secondly, the conditions on e^* as given in (i) are also unlikely, namely $e^*_t \to 0$. This condition rules out the presence of unmeasurable bounded disturbances. Thus, the conditions on e^*_t in (ii-b) remain the only realistic case insofar as the tuned signals are concerned. But, this raises another problem: ensuring that either ζ_t is persistently exciting (2) or that the adaptation gain is retarded (3). Notwithstanding the difficulty with the SPR condition on $H(p)$, there are specific problems related to (2) and (3). For example:

<u>Persistent Excitation (PE)</u>: With bounded disturbances (conditions (ii-b)) it is not known how to guarantee that $\zeta_t \in PE$. Recall that ζ_t is generated <u>inside</u> the adaptive loop, and thus, can only be controlled from the input, i.e., from either e^*_t or, more likely, from ζ^*_t. Since we do not know $G(p)$ and $H(p)$ it is not possible to conclude beforehand if $\zeta_t \in PE$ even if $\zeta^*_t \in PE$. In the special (unrealistic) case of no unmodeled dynamics and no bounded disturbances, $e^*_t = 0$, (ii-a) holds and $\zeta^*_t \in PE \Rightarrow \zeta_t \in PE$. Even though this latter situation is easily ruled out, it certainly makes sense that $\zeta^*_t \in PE$ implies a 'local' result. That is, with certain suitable restrictions on signal size and so forth, the system is

robust. These arguments were formalized to some extent in [3] and will be slightly extended here.

Retarded Update: In [5] it is suggested that the update algorithm be retarded as given by (3). Likewise, in [8], a slow or 'leaky' integrator is added. Although both these schemes (as well as similar ones) do give B.I.B.O. results, they both require additional information abut the plant, e.g., as in (3) an upper bound on $|\pi^*|$. These results can also be considered as 'local' results.

Slow Variations: Together with a retarded update, another mechanism for ensuring B.I.B.O. stability is to slow the variations in ξ_t (see [3]). The idea follows by examining the simple constant gain retarded algorithm,

$$\dot{\pi}_t = A_o \omega_t - \alpha \, \pi_t \qquad (\alpha > o)$$

$$= -A_o \xi_t \, H(p) \xi_t' \pi_t + A_o \xi_t e_t^* - \alpha \, \pi_t$$

If ξ_t is constant then exponential stability can be assured by direct LTI techniques. Thus, if ξ_t varies slowly enough with respect to the dynamics of $H(p)$ it is reasonable to expect a similar result. We will examine this more closely. However, control of ξ_t introduces the same difficulties as in requiring $\xi_t \in PE$, i.e., only 'local' results can be obtained.

B. Local Analysis

The system (S_c) can be transformed to a more useful form for local analysis:

$$\tilde{x} = \tilde{x}_L - \tilde{x}_{NL} \qquad (\tilde{S}_c)$$

$$\tilde{x}_{NL} = F \, f(\tilde{x})$$

where:

$$\tilde{x} := (\pi, \tilde{e}, \tilde{\xi}) := (\pi - \pi^*, e - e^*, \xi - \xi^*)$$

$$\tilde{x}_L := (\pi_L, \tilde{e}_L, \tilde{\xi}_L) \;, \quad f(\tilde{x}) := (\tilde{\xi}'\pi, \tilde{\xi}\tilde{e})$$

The system $(\tilde{S}_c)$ is obtained from (S_c) by linearization of (S_c) about e_t^*, ξ_t^*, and π^*, resulting in the linearized perturbation response $\tilde{x}_L$. The remaining nonlinear terms $\tilde{x}_{NL}$ are contained in $f(\tilde{x})$ where F is a time-varying linear operator. The characteristics of F, as well as those of $\tilde{x}_L$, depend on the adaptation gain and the behavior of the tuned signals, e_t^* and ξ_t^* (see [3]).

Consider the constant gain algorithm (1) with a retarded update. Fig. 2 depicts the resulting system $(\tilde{S}_c)$ where:

$$L := \frac{1}{p+\alpha} A_o, \; \alpha = \begin{cases} >0, \text{ with } (3) \\ 0, \text{ otherwise} \end{cases}$$

$$M := \xi^* H(p) \xi^{*\prime} + e^* G(p) \xi^{*\prime} \tag{4}$$

$$N := \xi^* H(p) + e^* G(p)$$

Thus, Fig. 2 reveals that $\tilde{x}_L$ is the response to $\mu_L := (\xi^* e^*, 0, 0)$ with $\pi_o \neq 0$, whereas $\tilde{x}_{NL}$ is the response to $\mu_{NL} := (0, \widetilde{\xi e}, \tilde{\xi}'\pi)$ with $\pi_o = 0$. Clearly, boundedness of the linearized response $\tilde{x}_L$ and stability of the operator F require stability of the map η, π_o into π, indicated in Fig. 2 by $K(\pi_o)$. It is shown in [3] that stability of $K(\pi_o): \eta \mapsto \pi$ ensures the existence of conditions for <u>local</u> stability of the adaptive system (S_c) or $(\tilde{S}_c)$.

Of particular interest is the degree to which it is possible to maintain stability despite arbitrary dynamics $H(p)$ and $G(p)$, i.e., robustness to model error. Primary consideration is given to unmodeled dynamics in $H(p)$. Let,

$$H(p) := \bar{H}(p) + \Delta_H(p) \tag{5}$$

where $\bar{H}$ denote the nominal dynamics obtained under ideal conditions, $\hat{\pi}_t = \pi^*$; consequently, we may consider $\bar{H}$ to be a fixed transfer function which is independent of π^*. All errors will be lumped into Δ_H. The

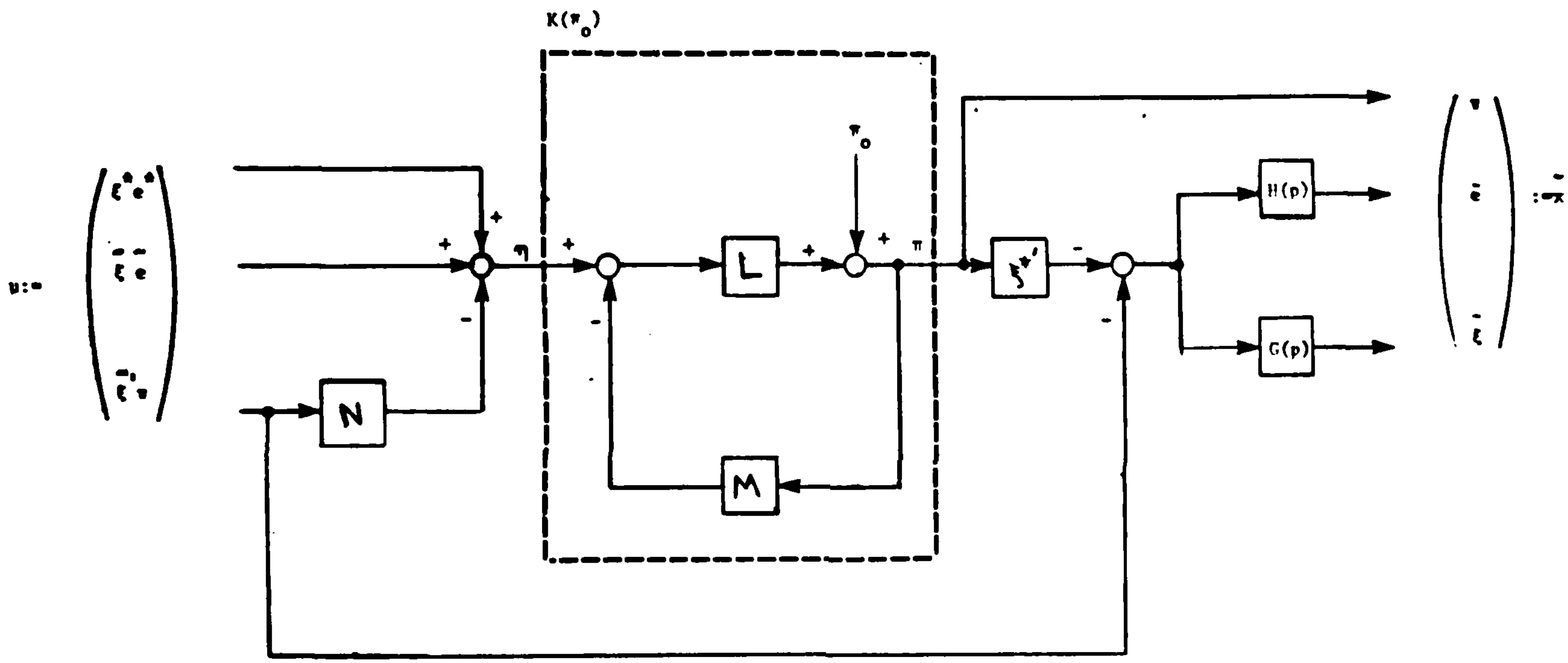

Fig. 2. Local System

desired result of the local analysis is to obtain a quantitative bound on the worst case model error for which stability of $(\tilde{S}_c)$ is guaranteed. We will do this by analyzing the stability robustness properties of the map $K(\pi_o)$, thus, the results obtained will <u>only</u> verify that local conditions exist.

B.1 <u>Local Stability by Persistent Excitation</u>

Assume that

$$\begin{aligned} &\xi^*_t \in PE \\ &\bar{H}(p) \in SPR \end{aligned} \tag{6}$$

Under these conditions, it follows from [4] that the system

$$\dot{x}_t = -\xi^*_t \bar{H}(p) \xi^{*\prime}_t x_t \tag{7}$$

is exponentially stable, i.e., there exists constants $m, \lambda > 0$ such that

$$|x_t| \leq m e^{-\lambda t} |x_o| \tag{8}$$

The following result gives a coarse bound on the model error Δ_H.

<u>Theorem 2</u>

The system $K(\pi_o)$ in Fig. 2 is L_∞-stable if:

$$\lambda/m > \sigma := ||e^*||_\infty ||\xi^*||_\infty \gamma_\infty(G) \tag{9}$$

and

$$\gamma_\infty(\Delta_H) < (\lambda/m - \sigma)/||\xi^*||^2_\infty \tag{10}$$

<u>Proof</u>: Follows directly from small gain theory (see e.g. [9]); details are in [3].

Remarks: Although sharper bounds can be obtained [10], the significance of Theorem 2 is that $H(p)$ need not be SPR if $\xi_t^* \varepsilon$ PE. The conditions of Theorem 2 can be determined experimentally by simulating $K(\pi_o)$ for a variety of $\pi_o \varepsilon R^n$, $\mu \varepsilon L_\infty^n$, and $\xi_t^* \varepsilon$ PE. This procedure can only yield an estimate.

B.2 Local Stability by Slow Variations with Retarded Update

In this case we will assume that ξ_t^* is not PE, but varies slowly, in a defined way, in relation to the known dynamics of $\bar{H}(p)$. Let ξ_τ^* denote ξ_t^* frozen at time $t = \tau$. Let $\bar{K}_\tau(p)$ denote the linear time-invariant operator given by,

$$\bar{K}_\tau(p) := [I_n + L(p)\bar{M}_\tau(p)]^{-1}L(p) \tag{11}$$

where

$$\bar{M}_\tau(p) := \xi_\tau^* \bar{H}(p) \xi_\tau^{*\prime} \tag{12}$$

Let R_τ denote the linear time-varying operator

$$R_\tau := \xi_t^* H(p)\xi_t^{*\prime} + e_t^* G(p)\xi_t^{*\prime} - \bar{M}_\tau(p) \tag{13}$$

The operator $\bar{K}_\tau(p)$ is simply $K(\pi_o)$ with $M = \bar{M}_\tau(p)$, i.e., ξ_t^* fixed at ξ_τ^*. Thus, R_τ represents the effect of how far ξ_t^* is from other values ξ_τ^* under the dynamics of $H(p)$ and $G(p)$.

Suppose that $\bar{K}_\tau(p)$ is exponentially stable, i.e., there exists constants m, $\lambda > 0$ such that,

$$|(\bar{K}_\tau(p)u)_t| \le m \int_0^t e^{-\lambda(t-s)} |u|_s ds, \quad u \varepsilon L_\infty[0,t], \ \forall \tau \varepsilon R^+ \tag{14}$$

The following result is analogous to Theorem 2.

Theorem 3:

The system $K(\pi_o)$ is L_∞-stable if:

$$\lambda/m > \sigma := ||e^*||_\infty ||\zeta^*||_\infty \gamma_\infty(G)$$

$$+ \sup_{\tau \geq 0} \gamma_\infty(\zeta^*\bar{H}\zeta^{*\prime} - \zeta_\tau^*\bar{H}\zeta_\tau^{*\prime}) \tag{15}$$

and

$$\gamma_\infty(\Delta_H) < (\lambda/m - \sigma)/||\zeta^*||_\infty^2 \tag{16}$$

Proof: Follows directly from small gain theory; details in [3],[10].

Remarks: As in Theorem 2, the conditions here for local stability do not depend on $H \in SPR$, and in this case do not depend on $\zeta_t^* \in PE$. Thus, Theorem 3 is weaker than Theorem 2. The key is to establish (14), i.e., exponential stability of $\bar{K}_\tau(p)$. Note that with $\bar{H} \in SPR$, and $L(p)$ given by (4) with $\alpha > 0$, (14) is established by passivity arguments. Tighter bounds on the norm operations can be obtained [10]. Also, the norms themselves can be estimated by simulating candidate actual systems (Fig. 2).

2. Discrete-Time Case

A. Global Analysis

The discrete-time version of S_c is somewhat different, due to the inherent system delay $k \geq 1$. The following discrete-time nonlinear system is representative of most discrete-time adaptive systems:

$$e_t = e_t^* - H_1(q^{-1})v_{1,t} - H_2(q^{-1})v_{2,t}$$

$$\xi_t = \xi_t^* - G_1(q^{-1})v_{1,t} - G_2(q^{-1})v_{2,t}$$

$$\pi_t = \pi_{t-1} + A(\xi_t, \omega_t), \quad \omega_t := \xi_t e_t \tag{$S_{d,k}$}$$

with

$$v_{1,t} := \xi_t'\pi_{t-1}, \quad v_{2,t} := \xi_t'\pi_{t-k}$$

The signal and operator dimensions are the same as those defined in (S_c), with q^{-1} the backward shift operator $(q^{-1}x)_t = x_{t-1}$. In general, with a unit delay $(k = 1)$, (S_d) collapses to the form of (S_c). Specifically, the unit delay adaptive system is:

$$e_t = e_t^* - H(q^{-1})v_t, \quad v_t := \xi_t'\pi_{t-1}$$

$$\xi_t = \xi_t^* - G(q^{-1})v_t \qquad (S_{d,1})$$

$$\pi_t = \pi_{t-1} + A(\xi_t,\omega_t), \quad \omega_t := \xi_t e_t$$

In this paper we will only examine $S_{d,1}$. Details on $S_{d,k}$ can be found in [10].

A.1. Adaptation Algorithms

It is an understatement to say that the choice of discrete-time algorithms is overwhelming. However, following [11],[12] they more or less belong to the following almost generic types:

Projection

$$A(\xi_t,\omega_t) = (1 + |\xi_t|^2)^{-1}\,\omega_t \qquad (P)$$

Recursive Least Squares

$$A(\xi_t,\omega_t) = S_t\omega_t$$

$$S_t^{-1} = S_{t-1}^{-1} + \xi_t\xi_t', \quad S_o = S_o' > 0 \qquad (RLS)$$

Stochastic Approximation

$$A(\xi_t,\omega_t) = s_t\omega_t$$

$$s_t^{-1} = s_{t-1}^{-1} + |\xi_t|^2, \quad s_o > 0 \qquad (SA)$$

Available stability results have dealt almost exclusively with $(S_{d,k})$ where, in the deterministic case, $e_t^* = 0$, with either $H_1(q^{-1}) = 1$ (or positive constant) and $H_2(q^{-1}) = 0$, or vice versa, e.g., [11]. (The stochastic version assumes e_t^* has zero mean with bounded variance, e.g., [12].) The following theorem extends the deterministic results to the case where $e^* \varepsilon \ell_2$. Thus, e_t^* approaches zero asymptotically, but is not identical to zero. Proof of theorem 2 is in [10].

Theorem 4: Global Stability

For the system $(S_{d,1})$ assume that:

(A1) The elements of $G(q^{-1})$ are proper and exponentially stable (all poles strictly inside the unit disc)

(A2) $H(q^{-1})$ is proper, exponentially stable, and for some constant $\delta > 0$,

$$| H(q^{-1}) - 1 | \leq \delta , \quad \forall |q| = 1 \tag{17}$$

Under these conditions, if $e^* \varepsilon \ell_2$ ($\Rightarrow e_t \to 0$) and $\xi^* \varepsilon \ell_\infty^n$ then using adaptation algorithm (P), (RLS), or (SA) results in $e, v \; \varepsilon \; \ell_2$ and $\xi, \pi \; \varepsilon \ell_\infty^n$ provided that

$$\delta < 1 \tag{18}$$

Remarks

(1) Theorem 4 offers no more than part (i) of Theorem 1 for continuous-time systems, in that it is not possible to insure an arbitrarily large model error. The bound (18) of $\delta < 1$ is as unrealistic as the requirement that $H(p) \; \varepsilon$ SPR in Theorem 1 (in fact, $H(p) \; \varepsilon$ SPR implies that $\delta < 1$; see [2]). Similar restrictive results for discrete-time adaptive systems have been reported in [13] and [14].

(2) It can be shown [10] that Theorem 2 is valid if, in (A2), H is either an LTI operator in the sector:

$$| H(q^{-1}) - \bar{H}(q^{-1}) | \leq \delta, \quad \forall |q| = 1$$

where (19)

$$\bar{H}(q^{-1}) \quad \varepsilon \text{ SPR}$$

or if $H - \bar{H}(q^{-1})$ is a slope-restricted memoryless nonlinearity, i.e.,

$$| (Hv)_t - \bar{H}(q^{-1})v_t | \leq \delta | v_t | \tag{20}$$

(3) With arbitrary sector conditions on H, Theorem 2 holds for $\delta < 1$ if the adaptation gain is modified, e.g.,

$$A(\xi_t, \omega_t) = m_t\omega_t, \quad m_t = (1 + || \xi ||^2_{t\infty})^{-1/2} \tag{21}$$

Other modifications like this can be constructed, provided m_t satisfies certain conditions, e.g., if m_t is a positive nonincreasing function, then sector properties on H apply to the operator $m_t H m_t^{-1}$. The required properties of m_t relate to the noncausal multiplier theory described in [9]. Picking the right multiplier - which is only needed in the proof of stability - is an artform akin to selecting a suitable Lyapunov function for a nonlinear system. The multiplier requirements do, however, motivate a myriad of modifications to adaptation gains (as proposed in (21)), for which multiplier selection is more easily facilitated, see e.g. [14]. It is unclear at this time whether these modifications can acheive practical sector conditions on H for global stability, i.e., where $\delta \gg 1$.

B. Local Analysis

Stability results dependent on persistent excitation or retarded update have a more 'local' character than their discontinuous-time counterparts, and thus, have been left out of the global analysis. As remarked before after Theorem 1, these are the known means to insure ℓ_∞-stability, which we have argued is the case most related to the actual system environment.

The local stability analysis for continuous-time systems can be developed analogously for the discrete-time case, with only minor modifications. Thus, Theorems 2-3 have their discrete-time counterparts. One major difference, however, is that the nonlinear term in $(\tilde{S}_c)$ is more complicated due to the complexity of the adaptation gain algorithms, e.g.,

(P) or (RLS). Other than that, similar reslults follow for the discrete-time case [10].

REFERENCES

[1] R.L. Kosut and B. Friedlander, 'Performance Robustness Properties of Adaptive Control Systems,' Proc. 21ar IEEE CDC, Orlando, FL, December 1982, pp. 18-23.

[2] R.L. Kosut and B. Friedlander, 'Robust Adaptive control: Conditions for Global Stability,' submitted to IEEE Trans. on Aut. Cont.

[3] R.L. Kosut, 'Robust Adaptive Control: Conditions for Local Stability,' Proc. 1982 ACC, San Francisco, CA, June 1982.

[4] B.D.O. Anderson, 'Exponential Stability of Linear Equations Arising in Adaptive Identification,' IEEE Trans. Aut. Contr., Vol. AC-22, February 1977, pp. 83-88.

[5] G. Kreisselmeier and K.S. Narendra, 'Stable Model Reference Adaptive Control in the Presence of Bounded Disturbances,' IEEE Trans. Auto. Contr., Vol. AC-27, No. 6, December 1982, pp. 1169-1175.

[6] C.E. Rohrs, L. Valavani, M. Athans, and G. Stein, 'Analytical Verification of Undesirable Properties of Direct Model Reference Adaptive Control Algorithms,' Proc. 20th IEEE CDC, San Diego, CA, December 1981, pp. 1272-1284.

[7] C.E. Rohrs, L. Valavani, M. Athans, and G. Stein, 'Robustness of Adaptive Control Algorithms in the Presence of Unmodeled Dynamics,' Proc. 21st IEEE Conference on Decision and Control, Orlando, FL, December 1982.

[8] P.A. Ioannou and P.V. Kokotovic, Adaptive Systems with Reduced Models, Springer-Verlag, 1983.

[9] C.A. Desoer, M. Vidyasager, Feedback Systems: Input-Output Properties, Academic Press, New York, 1975.

[10] R.L. Kosut and C.R. Johnson, Jr., 'An Input-Output View of Robustness in Adaptive Control,' in preparation.

[11] G.C. Goodwin, P.J. Ramadge, and P.E. Caines, 'Discrete Time Multivariable Adaptive Control,' IEEE Trans. Aut. Contr., Vol. AC-25, June 1980, pp. 449-456.

[12] G.C. Goodwin, P.J. Ramadge, and P.E. Caines, 'Discrete Time Stochastic Adaptive Control,' SIAM J. Contr., Vol. 19, No. 6, November 1981, pp. 829-853.

[13] P.J. Gawthrop and K.W. Lim, 'Robustness of Self-Tuning Controllers,' IEEE Proc., Vol. 129, Pt. D., No. 1, January 1982, pp. 21-29.

[14] R. Ortega and I.D. Landau, 'On the Model Mismatch Tolerance of Various Parameter Adaptation Algorithms in Direct Control Schemes: A Sectoricity Approach,' Proc. IFAC Workshop on Adaptive Control, San Francisco, CA, June 1983.

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ROBUST ADAPTIVE CONTROL: CONDITIONS FOR GLOBAL STABILITY

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ABSTRACT

An input-output approach is presented for analyzing the global stability and robustness properties of adaptive controllers to unmodeled dynamics. The concept of a tuned system is introduced, i.e., the control system that could be obtained if the plant were known. Comparing the adaptive system with the tuned system results in the development of a generic adaptive error system. Passivity theory is used to derive conditions which guarantee global stability of the error system associated with the adaptive controller, and ensure boundedness of the adaptive gains. Specific bounds are presented for certain significant signals in the control systems. Limitations of these global results are discussed, particularly the requirement that a certain operator be strictly positive real (SPR) -- a condition that is unlikely to hold due to unmodeled dynamics.

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1. INTRODUCTION

1.1 Background

The analysis and design of adaptive control systems has been the subject of extensive research in the past two decades [1]-[10]. Adaptive techniques provide a way of handling plant uncertainty by adjusting the controller parameters on-line to optimize system performance. An alternative method for handling uncertainty is to use a fixed structure controller designed to provide acceptable performance for a specified range of plant behavior. In principle, adaptive controllers can provide improved performance compared to fixed robust controllers, since they are tuned to the uncertain plant. However, adaptive controllers sometimes exhibit undesirable behavior during the tuning or adaptation process. For example, unmodeled dynamics can cause a rapid deterioration in performance and even instability [11],[12]. This problem is not resolved by increasing the order or complexity of the model. Since the model of any dynamic system, by definition, is <u>not</u> the actual system, it can therefore be argued that unmodeled dynamics are always present, <u>ad infinitum</u>.

The main reason for these difficulties with adaptive controllers seems to be that robustness to unmodeled dynamics was not considered as a design criterion in the development of the adaptive control algorithm. The design objective is global stability of the closed-loop system, e.g., [7], [9] and various assumptions on the structure of the plant are required to achieve that objective. In particular, it is necessary to assume that the plant is linear and time invariant (LTI), that the relative degree of the transfer function is known as well as the sign of the high frequency gain. Such requirements are not practical since real plants are often nonlinear and time-varying and can be accurately represented only by high order (sometimes infinite order [13]) complicated models.

The need for robustness to plant uncertainty is not unique to adaptive control. The problem of robustness is ubiquitous in control theory and has been studied in the context of fixed (nonadaptive) control [14]-[17]. These studies rely on the input/output properties of systems, e.g., [18],[19]. The predominant reason to examine robustness issues in this way is that the characteritics of unmodeled dynamics, such as uncertain model order, are easily represented. Lyapunov theory, on the other hand, is not well suited for this type of uncertainty. Typically, plant uncertainty is characterized by assuming that the plant belongs to a well defined set. For example, a set description of an uncertain LTI plant is to define a "ball" in the frequency domain. The center of the ball is the nominal plant model, and the radius defines the model error. This set model description is one type of a more general set description, referred to as a conic-sector [15]. The uncertainty in the plant induces an uncertainty in the input/output map of the closed-loop system which can, again be characterized by a conic sector. Performance requirements for the control system can be translated into statements on the conic sector which bounds the closed-loop systems, making it possible to check whether a given design meets specifications, and providing guidelines for robust controller design.

In this paper we use the input/output approach to analyze the global stability and robustness properties of continuous-time adaptive controllers with respect to unmodeled dynamics (although we consider only continuous-time algorithms, the input-output formalism can be readily extended to the discrete-time case). By global we mean that no specific magnitude constraint (other than boundedness) is placed on any of the external inputs or initial conditons. We develop an adaptive error system of a general form, by comparing the actual adaptive system with a tuned system, i.e., the control system that could be obtained if the plant were known. This error system is similar to the type used in [7],[8] where the tuned system error output is zero, due to the assumption of perfect modeling. By relaxing this assumption we show that the non-zero outputs of the error system are the inputs to a nonlinear feedback error system consisting of the adaptive algorithm and two feedback (interconnection) operators,denoted by H_{ev} and H_{zv} .

An important consequence of this structure is that the existence of solutions (e.g., tuned system performance) is separated from the stability analysis (e.g., stability of the nonlinear error system). In general, the adaptation law is passive; consequently, if H_{ev} is strictly positive real (SPR), then application of passivity theory [19]-[21], provides global

L_2-stability of the map from the tuned system output to the actual adaptive system output, even though the adaptive parameters may grow beyond all bounds. We provide other conditions (e.g., H_{zv} stable) to insure the L_∞ boundedness of the adaptive gains. Similar results are developed to insure L_∞-stability of the error system by using an exponentially weighted passivity theory [19]. These results are summarized in Theorems 1A and 1B.

As a by product of the input/output view we also obtain specific bounds on the L_2 and L_∞ norms of significant signals in the adaptive system. The results are summarized in Corollary 1.

The results in Theorem 1 and Corollary 1 are not essentially new (see e.g., [7],[8]), although they do provide some extentions to previous results. The main contribution, however, is the fact that all the results can be obtained from a generic error system and from the application of nonlinear stablity theorems based on input-output properties. As a consequence of this approach, it is to be expected that conditions for robustness will arise in a natural way. Such robustness results are obtained, but unfortunately,they have a limited practical use. The main limitation is that the global theory (Theorem 1) requires that $H_{ev} \varepsilon$ SPR , which in turn places an upper bound on the size of the unmodeled dynamics in the plant. The details are contained in Lemmas 4.1 and 5.2. This bound is quite restrictive and is easily violated by even the most benign model errors, thus, verifying the results obtained in [11], [12]. To over come this limitation, we construct an <u>SPR compensator</u>, based on the scheme proposed in [22] in the context of robust (non-adaptive) control. Although in the adaptive case the supporing arguments are heuristic, an example simulation shows a positive result.

The input/output analysis presented here provides a generic framework within which it is possible to analyze the robustness of adaptive robust controllers. We believe that this framework can be used to develop practical adaptive control algorithms that can be more readily applied to real systems, than the class of algorithms currently in use.

Since this paper merges ideas from several areas, it is necessary to introduce a number of definitions and concepts.

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2. SOME PRELIMINARIES

2.1 Notation

The input/output formulation of multivariable systems is the principal view taken throughout this paper and the notation and terminology used is standard (see e.g. [18],[19]). The input and output signals are assumed to be imbedded in either the normed function space

$$L_p^n = \{x : [0,\infty) \to R^n \mid \|x\|_p < \infty\} \tag{2.1a}$$

or its extention

$$L_{pe}^n = \{x : [0,T] \to R^n \mid \|x\|_{Tp} < \infty, \quad T < \infty\} \tag{2.1b}$$

The respective norms $\|\cdot\|_p$ and $\|\cdot\|_{Tp}$ are defined as follows:

$$\|x\|_p = \lim_{T\to\infty} \|x\|_{Tp} \tag{2.2a}$$

with

$$\|x\|_{Tp} = \begin{cases} \left(\int_0^T |x(t)|^p dt\right)^{1/p}, & p \in [1,\infty) \\ \sup_{t\in[0,T]} |x(t)|, & p = \infty \end{cases} \tag{2.2b}$$

where $|\cdot|$ is the Euclidean norm on R^n. Hence, L_{2e}^n is an inner product space, with inner product $\langle x,y\rangle_T$ of elements $x,\ y \in L_{2e}^n$ defined by

$$\langle x,y\rangle_T = \int_0^T x(t)'y(t)dt \tag{2.3}$$

and so $\|x\|_{T2} = (\langle x,x\rangle_T)^{1/2}$. If $T \to \infty$ then L_2^n is an inner-product space with inner product $\langle x,y\rangle = \lim_{T\to\infty}\langle x,y\rangle_T$.

2.2 Stability

Systems considered in this paper are described by input/output equations of the form $y = Gu$ where $G:L^m_{pe} \to L^n_{pe}$ is a causal map from u into y, also denoted $u \to y$. The system G is said to be L_p-stable (or simply stable) if G maps $u \in L^m_p$ into $y \in L^n_p$ and if there exists finite constants k and b such that $||Gu||_{Tp} \leq k\, ||u||_{Tp} + b$, for all $T > 0$ and all $u \in L^m_{pe}$. The smallest k that can be found is referred to as the L_p-gain (or simply gain) of G, denoted $\gamma_p(G)$.

Because we often encounter LTI systems it is convenient to introduce the following notation. Let $R(s)$ and $R_o(s)$ denote the proper and strictly proper rational functions, respectively. Let S and S_o denote functions in $R(s)$ and $R_o(s)$, respectively, whose poles all have negative real parts. Thus, S and S_o are the stable, lumped, LTI systems. Denote multivariable systems with transfer function matrices, by $R(s)^{n\times m}$, $S^{n\times m}$, etc. For example, $G \in S_o^{n\times m}$ means that <u>all</u> elements of G belong to S_o, and so on.

If $G \in S^{n\times m}$ then the following L_p-gains are obtained,

$$\gamma_1(G) \leq \gamma_\infty(G) = \int_0^\infty \bar{\sigma}[G(t)]dt \tag{2.4}$$

$$\gamma_2(G) = \sup_{\omega \in R} \bar{\sigma}[G(j\omega)] \tag{2.5}$$

where $\bar{\sigma}(A)$ denotes the maximum singular value of the matrix A, defined as the positive square root of the maximum eigenvalue of A*A, where * is the conjugate transpose of A. In (2.4), (2.5) G is the operator, $G(j\omega)$ the transfer function matrix, and $G(t)$ is the impulse response matrix.

2.3 Passivity

The following definitions follow those in [19],[21]. Let $G:L^m_{1e} \to L^m_{1e}$ and let μ, ρ be constants with $\mu > 0$. Then, $\forall\, u \in L^m_{2e}$:

G is passive if,

$$< u, G u>_T > \rho \tag{2.6}$$

G is input strictly passive if,

$$< u, Gu >_T > \rho + \mu \|u\|_{T2} \tag{2.7a}$$

G is output strictly passive if,

$$< u, Gu >_T > \rho + \mu \|Gu\|_{T2} \tag{2.7b}$$

(μ and ρ are not the same throughout). When $G \in S^{mxm}$ satisfies (2.7), G is said to be strictly positive real (SPR), denoted $G \in SPR^m$. Because SPR systems play a crucial role in the proof of stability of adaptive systems, we introduce the following subsets:

$$SPR_+^m = \{G \in S^{mxm} | \underline{\lambda}(\tfrac{1}{2} [G(j\omega) + G(-j\omega)'] - \mu I) > 0, \forall \omega \in R\} \tag{2.8a}$$

$$SPR_0^m = \{G \in S_0^{mxm} | \underline{\lambda}(\tfrac{1}{2} [G(j\omega) + G(-j\omega)'] - \mu\ G(-j\omega)'G(j\omega)) > 0, \forall \omega \in R\} \tag{2.8b}$$

where $\underline{\lambda}(A)$ denotes the smallest eigenvalue of A. Thus, whenever $G \in S^{mxm}$, conditions (2.7) can be tested in the frequency domain. Moreover, SPR_0^m and SPR_+^m, respectively, separate the strictly proper SPR functions from the proper, but not strictly proper, SPR functions. In the scalar case, the frequency domain conditions simplify because $\underline{\lambda}[G(j\omega) + G(-j\omega)'] = 2\ Re[G(j\omega)]$.

Certain unstable systems in $R(s)^{mxm}$ can be passive by virtue of (2.6). In particular, $G \in R(s)^{mxm}$ is passive if G(s) is positive real. The transfer function matrix G(s) is positive real if: (i) it has no poles in Re(s) > 0, (ii) poles on the $j\omega$ axis are simple with a non-negative residue, and (iii) for any $\omega \in R$ not a pole of $G(j\omega) + \underline{G}(-j\omega)' >$.

2.4 Model Error

The cornerstone of robust control design is a quantifiable bound on the error between the model used for control design and the actual plant to be controlled. In the adaptive control case considered here the model is a parametric model, where the parameters are not known exactly. The structure of the parametric model can be obtained analytically from physical laws, but this invariably results in a complicated model. Often a simple structure is selected because it is more convenient for analysis and synthesis.

Let P denote the plant to be controlled. In the broadest sense P is a relation in $L_{1e}^{m} \times L_{1e}^{n}$, i.e., the set of all possible ordered pairs $(u,y) \varepsilon L_{1e}^{m} \times L_{1e}^{n}$ of inputs $u \varepsilon L_{1e}^{m}$ and outputs $y \varepsilon L_{1e}^{n}$ that could be generated by the plant [18]. The uncertainty in the plant is denoted by $(u,y) \varepsilon P$.

Let $P_\alpha : L_{pe}^{m} \to L_{pe}^{n}$ denote a parametric model of the plant P with parameters $\alpha \varepsilon R^k$. The parameters can be selected so as to minimize any discrepancies between the model and the plant, i.e.,

$$\inf_{\alpha \varepsilon R^k} \| y - P_\alpha u \|_{Tp} = \| y - P_* u \|_{Tp} \tag{2.9}$$

We will refer to $\alpha_* \varepsilon R^k$ as the tuned model parameters and to $P_{\alpha_*} = P_*$ as the tuned parametric model of the plant. In general, P_* is dependent on the input/output sequence.

Most of the previous work on adaptive control deals with the case where for every $(u,y) \varepsilon P$ there exists a tuned parametric model P_*, such that $P_* = P$. In this paper we consider the presence of unmodeled dynamics, thus, the uncertain plant P cannot be perfectly modeled by any parametric model P_α . Since we will deal exclusively with LTI plants $P \varepsilon R(s)^{n \times m}$, it is convenient to describe this model error in the frequency-domain. Let $B_S(r)$ denote a "ball" in S of radius r, defined by

$$B_S(r) := \{ G \varepsilon S^{n \times m} | \, \bar{\sigma}[G(j\omega)] < r(\omega), \, \omega \varepsilon R \} \tag{2.10}$$

Let the plant to be controlled be described by

$$P = (I + \Delta)P_{\star} \tag{2.11a}$$

where $P \in R(s)^{nxm}$ is the plant, $P_{\star} \in R(s)^{nxm}$ is the tuned parametric model, and $\Delta \in S^{nxn}$ denotes the unmodeled dynamics. Further, the only knowledge available about Δ is that it is bounded such that

$$\Delta \in B_S(\delta) \tag{2.11b}$$

where $\delta(\omega)$ is known for all frequencies. In other words, while the operator Δ is not precisely known, we do know a bound on its effect. This model description (2.2) is used throughout the paper to precisely define the plant to be controlled in an adaptive system. Following Doyle and Stein [16] we will refer to (2.11b) as an unstructred uncertainty. Note that although Δ is stable, P and $P\star$ need not be stable. Hence, the parametric model is implicitly required to capture all unstable poles of the plant. Although this is not severly restrictive - at least on practical grounds - nonetheless, it can be eliminated by definng model error as (stable) deviations in (stable) coprime factors of the plant [23]. As the subsequent analysis is not substantially effected by this choice, we will remain with (2.11) for purposes of illustration.

2.5 Persistent Excitation

From [31], a regulated function $F(\cdot) = R_{+} \to R^{nxm}$ is persistently exciting, denoted $F \in PE$, if there exists finite positive constants α_1, α_2, and α_3 such that

$$\alpha_2 I_n > \int_s^{s+\alpha_3} F(t)F(t)'dt > \alpha_1 I_n , \quad \forall s \in R_{+} \tag{2.12}$$

The usefulness of a persistently exciting signal is in establishing the exponential stability of the following differential equation which arises in many adaptive and identification schemes, i.e.,

$$\dot{x} = -BFHF'x + w \,, \qquad x(0) \in R^n \tag{2.13}$$

It is shown in [31] that if $B \in R^{n \times m}$, $B = B' > 0$, $H \in SPR_0^m$ or SPR_+^m, and $F \in PE$, then $(w, x(0)) \mapsto x$ is exponentially stable, i.e., $\exists$ m, $\lambda > 0$ such that

$$|x(t)| < me^{-\lambda t} |x(0)| + \int_0^t me^{-\lambda(t-r)} |w(\tau)| d\tau \,. \tag{2.14}$$

We will utilize this latter result in section IV in our proof of stablity of the adaptive system.

3. ADAPTIVE ERROR MODEL

In this section we develop a generic adaptive error model which will be used in the subsequent analysis. This requires defining the notions of robust control and tuned control.

Robust and Tuned Control

Consider, for example, the model reference adaptive control (MRAC) depicted in Figure 3.1, consisting of the uncertain plant P, a reference model H_r, and an adaptive controller $C(\hat{\theta})$, where $\hat{\theta}$ is the adaptive gain vector, r is a reference input, d is a disturbance process, and n is sensor noise. Denote by $H(\hat{\theta})$ the closed-loop system relating the external inputs $w = (r', d', n')'$ to the output error e, as depicted in Figure 3.2.. Also, let $w \epsilon W$ denote the admissable class of input signals.

The objective of the adaptive controller is twofold: (1) adjust $\hat{\theta}$ to a constant $\theta_* \epsilon R^k$ such that $H(\theta_*)$ has desireable properties; and (2) during *adaptation*, as $\hat{\theta}$ *is adjusted*, the error is well behaved. In the usual formulations [7] only (1) is considered and further it is assumed that there exists a <u>matched gain</u>, denoted by $\bar{\theta} \epsilon R^k$, such that

$$H(\bar{\theta}) = 0 \qquad (3.1)$$

The presence of uncertain unmodeled dynamics in the plant eliminate the chance of satisfying the matching condition. Thus, it is more appropriate to define a <u>tuned gain</u>, denoted by $\theta_* \epsilon R^k$, corresponding to each $(u,y,w) \epsilon P \times W$, such that

$$H(\theta_*)w < H(\theta)w \quad , \quad \forall\, \theta \epsilon R^k \qquad (3.2)$$

The error signal $e_* := H(\theta_*)w$ is referred to as the <u>tuned error</u>. Note that each $(u,y,w) \epsilon P \times W$ engenders a possibly different θ_*. Also, it is important to distinguish the tuned gain θ_*, from the <u>robust gain</u> $\theta_o \epsilon R^k$, where

$$\sup_{P \times W} H(\theta_o)w < \sup_{P \times W} H(\theta)w \, , \quad \forall\, \theta \epsilon R^k \qquad (3.3)$$

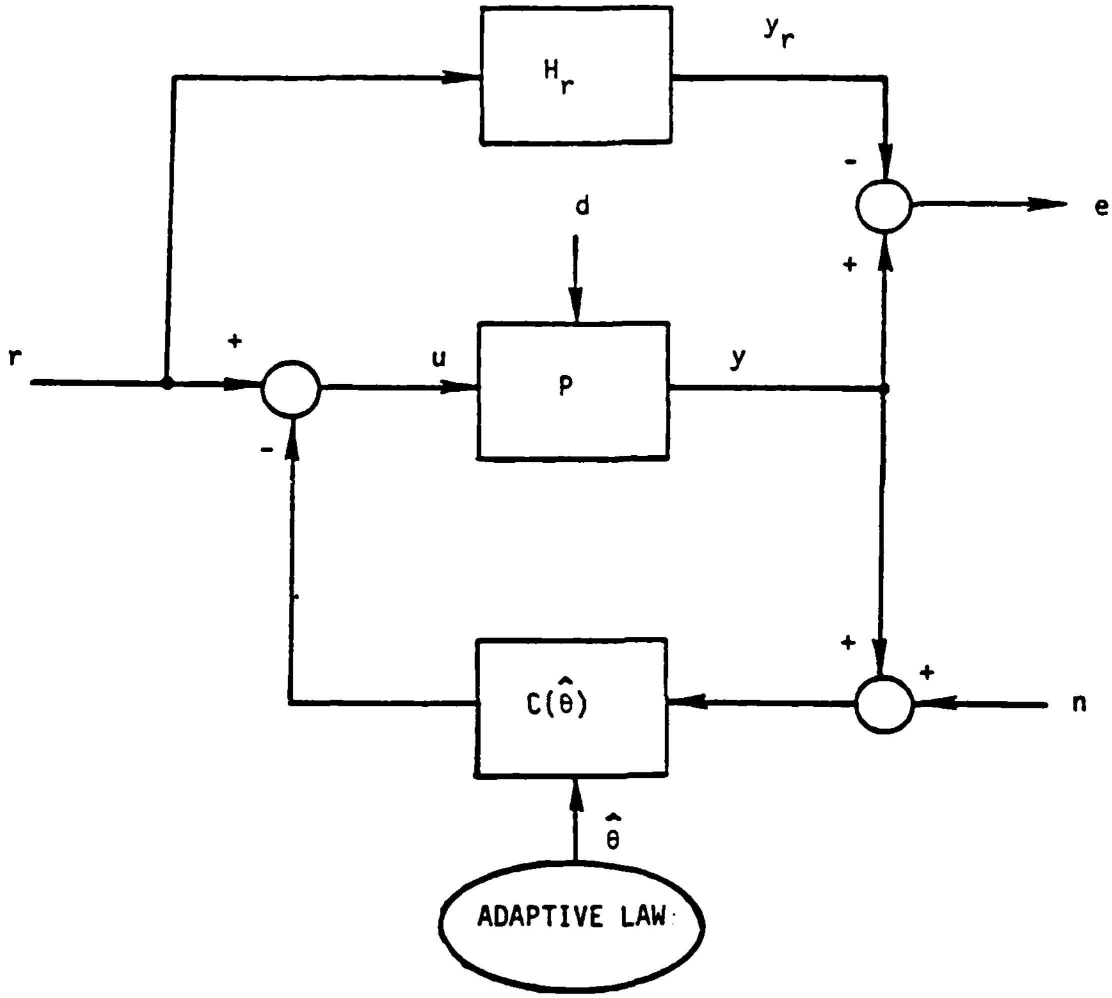

Figure 3.1 A Model Reference Adaptive Controller

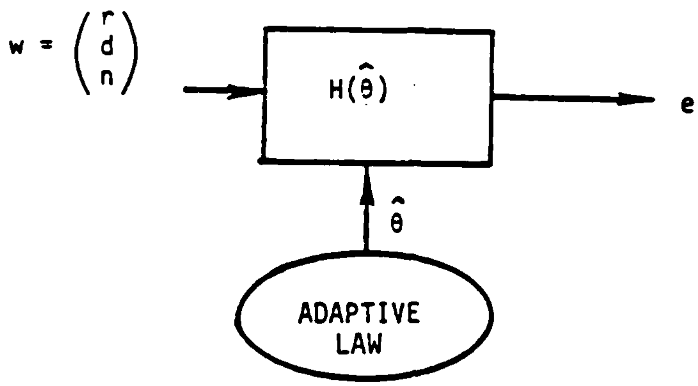

Figure 3.2 Closed-Loop System

The error signal $e_o := H(\theta_o)w$ is referred to as the robust error. It follows from these definitions that the tuned error is always smaller in norm than the robust error, thus $\forall\ w \in W$,

$$e_* = H(\theta_*)w < e_o = H(\theta_o)w , \tag{3.4}$$

The tuned controller is, unfortunately, unrealizable since it requires prior knowledge of the actual system $H(\theta)$ (or equivalently, the plant P) and the input w. A practical adaptive controller is likely to have a larger error norm.

Structure of the Adaptive Control

In summary, we consider the multivariable adaptive system, shown in Figure 3.2, and described by

$$e = H(\hat{\theta})w . \tag{3.5}$$

where $e(t) \in R^m$ is the error signal to be controlled, $w(t) \in R^q$ is the external input restricted to some set W, and $\hat{\theta}(t) \in R^k$ is the adaptive gain. The class of adaptive controllers considered here are such that the adaptive gains multiply elements of internal signals $z(t) \in R^k$, referred to as the regressor, to produce the adaptive control signals,

$$f_i = \hat{\theta}_i' z_i \quad , \quad i \in [1,m] \tag{3.6}$$

where $\hat{\theta}_i$ and z_i are k_i-dimensional subsets of the elements in $\hat{\theta}$ and z, respectively. Thus,

$$k = \sum_{i=1}^{m} k_i \tag{3.7}$$

Define the adaptive gain error,

$$\theta(t) := \hat{\theta}(t) - \theta_* \tag{3.8}$$

where $\theta_* \in R^k$ is the tuned gain (3.4). Also, define the adaptive control error signals,

$$v_i := \theta_i' z_i \quad , \quad i = 1, \ldots, m \tag{3.9}$$

An equivalent expression is,

$$v = Z'\theta \tag{3.10a}$$

where the time-varying matrix Z is defined by

$$Z = \text{block diag}(z_1, z_2, \ldots, z_m) \tag{3.10b}$$

To describe the relations among the signals e, z, v, and w we introduce the <u>interconnection system</u> $H_I : (w,v) \rightarrow (e,z)$, as shown in Figure 3.3. In particular, let $H_I \in R(s)^{(m+k)x(m+q)}$, and where H_I is defined by,

$$\begin{pmatrix} e \\ z \end{pmatrix} := H_I \begin{pmatrix} w \\ v \end{pmatrix} := \begin{pmatrix} H_{ew} \\ H_{zw} \end{pmatrix} \begin{pmatrix} -H_{ev} & w \\ -H_{zv} & v \end{pmatrix} \tag{3.11}$$

In effect, this structure serves to isolate the adaptive control error v, from the rest of the system. When the adpative control is tuned, $\theta = 0$ and $v = 0$; consequently, the tuned error signal (3.4) is,

$$e_* := H(\theta_*)w = H_{ew}w \tag{3.12}$$

We can also define a tuned regressor signal,

$$z_* := H_{zw}w \tag{3.13}$$

In general, all the subsystems in H_I are dependent on the tuned gains θ_* .

The interconnection system can also be written as,

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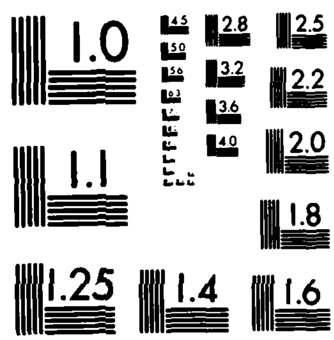

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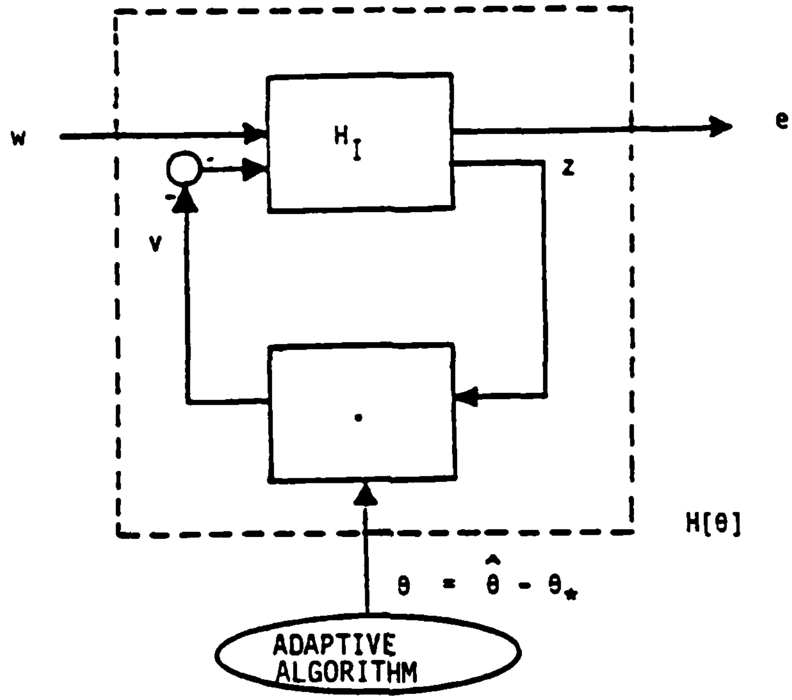

Figure 3.3 Interconnection Structure

$$e = e_* - H_{ev} v \tag{3.14a}$$

$$z = z_* - H_{zv}\, v \tag{3.14b}$$

with v given by (3.10). To complete the error model requires describing the adaptative algorithm, i.e., the means by which $\hat{\theta}(t)$ is generated. We will consider two typical algorithms. A constant gain (gradient) algorithm [7]:

$$\dot{\hat{\theta}} = \Gamma\, Z\, e \tag{3.15}$$

where $\Gamma \in R^{k\times k}$, $\Gamma = \Gamma' > 0$, and a similar but nonlinear gain algorithm:

$$\dot{\hat{\theta}} = \Gamma(Ze - \rho(\hat{\theta})\hat{\theta}) \tag{3.16a}$$

where $\rho : R^k \to R_+$ is a retardation function, whose purpose is to prevent $\hat{\theta}$ from growing too quickly in certain situations. Although many functions will suffice we will select the one proposed in [24], namely:

$$\rho(\hat{\theta}) := \begin{cases} (\|\hat{\theta}\|/c - 1)^2, & \|\hat{\theta}\| > c := \max\|\theta_*\| \\ 0 & \|\hat{\theta}\| \le c \end{cases} \tag{3.16b}$$

The complete adaptive error system, is shown in Figure 3.4. Note that the error system is composed of two subsystems: a linear subsystem Σ_L and a non-linear subsystem Σ_N .

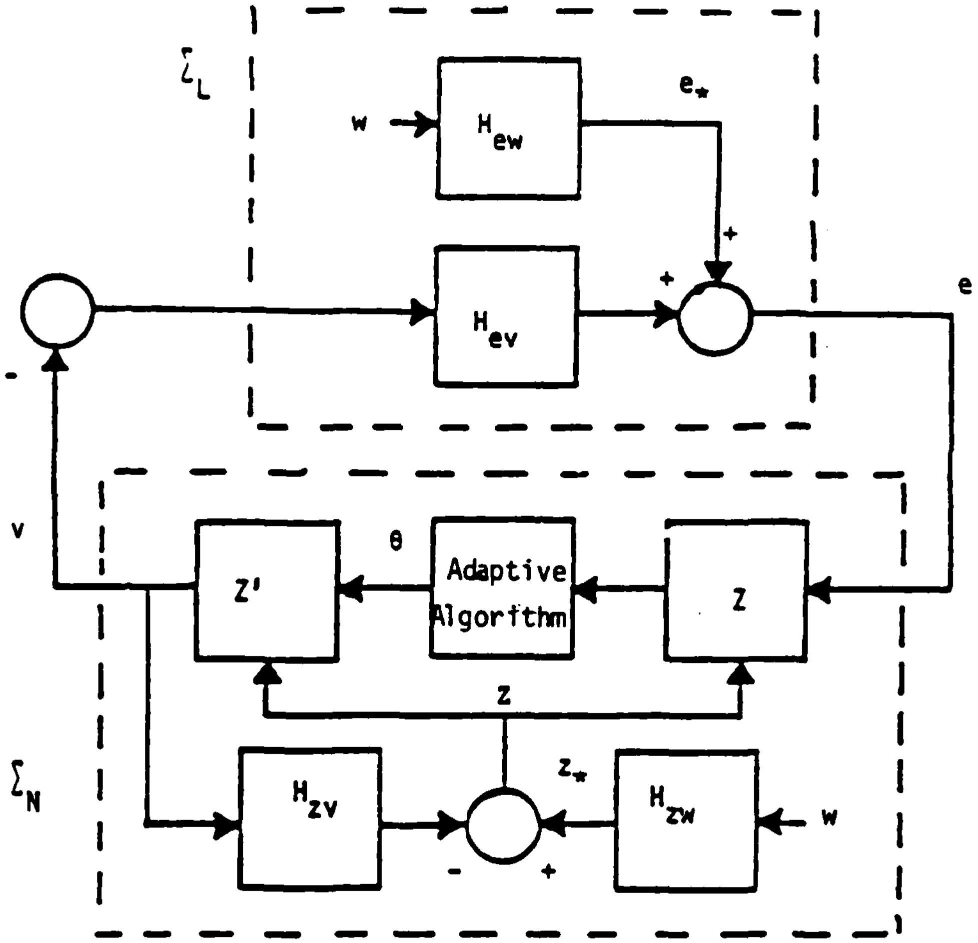

Figure 3.4 Adaptive Error System

4. CONDITIONS FOR GLOBAL STABILITY

The theorems stated below give conditions for which the adaptive error system (Fig. 3.4) is guaranteed to have certain stability and performance properties. Proofs are given in Appendix A. Heuristically, however, the basis for the proofs is application of the Passivity Theorem ([19], pg. 182). It turns out that the map $e \to v$ is passive. Thus, if H_{ev} is SPR^m, then the map $e_* \to (e,v)$ is L_2-stable even though z and/or θ can grow without bounds. Further restrictions, provided below, cause θ and z to be bounded. (We use the notation "$x \to 0$ (exp.)" to mean that $x(t) \to 0$ (exponentially) as $t \to \infty$.)

<u>Theorem A: Global Stability</u>

For the adaptive error system shown in Figure 3.4, assume that:

(A1) The system is well-posed in the sense that all inputs $w \in W$ produce signals e.v,z, θ, and $\dot{\theta}$ in $L_{\infty e}$. (4.1a)

(A2) $H_{zv} \in S_0^{k\times m}$ (4.1b)

(A3) $H_{ev} \in SPR_+^m$ (4.1c)

Under these conditions:

(i) If $(e_*, \dot{e}_*) \in L_2^m \cap L_\infty^m$ ($\Longrightarrow e_* \to 0$) and $(z_*, \dot{z}_*) \in L_\infty^k$ then with algorithm (3.15) or (3.16):

(i-a) $(\theta, \ddot{\theta}) \in L_\infty^k$, $\dot{\theta} \in L_2^k \cap L_\infty^k$, and $\dot{\theta} \to 0$. (4.a)

(i-b) $e \in L_2^m \cap L_\infty^m$, $\dot{e} \in L_\infty^m$, and $e - e_* \to 0$. (4.2b)

(i-c) $v \in L_2^m \cap L_\infty^m$, $\dot{v} \in L_\infty^m$, and $v \to 0$. (4.2c) (i-d) $(z,\dot{z}) \in L_\infty^k$, $(z-z_*, \dot{z}-\dot{z}_*) \in L_2^k \cap L_\infty^k$, and $z-z_* \to 0$ exp. (4.2d)

(i-e) If, in addition, $e_* = 0$ (matched) and $z_* \in PE$ then $(\theta, \dot{\theta}, e-e_*, v, z-z_*) \to 0$ exp. (4.2e)

(ii) If $(e_*, \dot{e}_*) \in L_\infty^m$ and $(z_*, \dot{z}_*) \in L_\infty^k$, then with algorithm (3.15):

(ii-a) $z \in L_\infty^k$ (4.3)

(ii-b) With the addition of either algorithm (3.16) or $z \in PE$ it follows that the elements of $\theta, \dot{\theta}, e, \dot{e}, v, \dot{v}$, and $\dot{z}$ are in L_∞ . (4.4)

<u>Theorem 1B: Global Stability</u>

Replace (A3) in Theorem 1 by

(A3)' $H_{ev} \in SPR_0^m$ (4.5)

(i) If $(e_*, \dot{e}_*) \in L_2^m \cap L_\infty^m$ ($\Rightarrow e_* \to 0$) , and $(z_*, \dot{z}_*) \in L_\infty^k$ then with algorithm (3.15) or (3.16)

(i-a) $(\theta, \ddot{\theta}) \in L_\infty^k$, $\dot{\theta} \in L_2^k \cap L_\infty^k$, $\dot{\theta} \to 0$ (4.6a)

(i-b) $e \in L_2^m \cap L_\infty^m$, $\dot{e} \in L_\infty^m$, $e - e_* \to 0$ (4.6b)

(i-c) $(v,\dot{v}) \in L_\infty^m$ (4.6c)

(i-d) $(z,\dot{z}) \in L_\infty^k$, $(z-z_*, \dot{z}-\dot{z}_*) \in L_2^k \cap L_\infty^k$, and $z-z_* \to 0$. (4.6d)

(i-e) If, in addition, $e_* = 0$ (matched) and $z_* \in PE$, then $(\theta, v) \to 0$ exp. (4.6e)

(ii) If $(e_*, \dot{e}_*) \varepsilon L_\infty^m$ and $(z_*, \dot{z}_*) \varepsilon L_\infty^k$, then with algorithm (3.15):

(ii-a) $z \varepsilon L_\infty^k$ (4.7d)

(ii-b) With the addition of either $z \varepsilon PE$ or algorithm (3.16), the elements of $\theta, \dot{\theta}, e, \dot{e}, v, \dot{v}$, and $\dot{z}$ are in L_∞. (4.7b)

Corollary 1: Performance Bounds

Suppose z_* and e_* satisfy the conditions in (i) of Theroems 1A or 1B.

(i) Let $H_{ev} \varepsilon SPR_+^m$, i.e., $\exists\, \mu, \gamma > 0$ such that $\forall \omega \varepsilon R$,

$$\bar{\sigma}[H_{ev}(j\omega)] < \gamma \text{ and } \frac{1}{2}[H_{ev}(j\omega) + H_{ev}(-j\omega)'] > \mu I_m \tag{4.8a}$$

Then, bounds on $\|e\|_2$ and $\|\theta\|_\infty$ can be obtained from:

$$\|e-e_*\|_2 < \frac{\gamma}{2\mu}\left[\|e_*\|_2 + \left(\|e_*\|_2^2 + 2\mu\ \theta(0)'\ \Gamma^{-1}\theta(0)\right)^{1/2}\right] \tag{4.8b}$$

$$\|\theta'\Gamma^{-1}\theta\|_\infty < \theta(0)'\ \Gamma^{-1}\theta(0) + 2\|e\|_2\|e-e_*\|_2/\gamma \tag{4.8c}$$

(ii) Let $H_{ev} \varepsilon SPR_0^m$, i.e., $\exists\, \mu, q, k > 0$ such that $\forall\, \omega \varepsilon R$,

$$\frac{1}{2}[H_{ev}(j\omega) + H_{ev}(-j\omega)'] > \mu\ H_{ev}(-j\omega)'\ H_{ev}(j\omega) \tag{4.9a}$$

$$\frac{1}{2}[G_{ev}(j\omega) + G_{ev}(-j\omega)'] > k\ I_m \tag{4.9b}$$

$$G_{ev}(s) := (1 + qs)\ H_{ev}(s) \tag{4.9c}$$

Then, bounds on $\|e_2\|$ and $\|\theta\|_\infty$ can be obtained from:

$$\|e\|_2 < \frac{1}{2\mu k}\left[\|e_*+q\dot{e}_*\| + \left(\|e_*+q\dot{e}_*\|_2^2 + 2k^2\mu\theta(0)'\Gamma^{-1}\theta(0)\right)^{1/2}\right] \tag{4.9d}$$

$$\|\theta'\Gamma^{-1}\theta\|_\infty < \theta(0)'\Gamma^{-1}\theta(0) + \frac{1}{k}\|e_*+q\dot{e}_*\|_2\|e\|_2 \tag{4.9c}$$

Discussion

(1) Theorems 1A and 1B give conditions under which the adaptive error system is globally stable. Essentially, conditions are imposed on the interconnection subsystems in H_I . In particular, $H_{ev} \varepsilon SPR^m$ and $H_{zv} \varepsilon S_0^{k \times m}$ are direct requirements, whereas the restrictions on the tuned signals e_* and z_* , indirectly impose requirements on H_{ew} and H_{zw} . These latter requirements are dependent on knowledge about $w \varepsilon W$. For example, if w is a constant, then the assumption that $e_* \to 0$ (Theorem 1A-i) requires that the tuned feedback system is a Type-I robust servomechanism, i.e., the transfer junction $H_{ew}(0) = 0$ for all $(u,y) \varepsilon P$.

(2) Corollary 1 gives explicit bounds on signals in the error system. These bounds can be used to evaluate the adaptive system design. Moreover, the bounds allow a coarse determination as to the efficacy of adaptive control vs. robust control. By comparing, for example, the adaptive error $\|e\|_2$ from (4.8) with the robust error $\|e_0\|_2$ from (1.5), it is possible to obtain a quantifiable measure of performance degradation during adaptation.

(3) Although Theorems 1A and 1B are essentially the same, there are slight difference worth noting. These differences arise because in 1A, $H_{ev} \varepsilon SPR_+^m \Rightarrow H_{ev}(s)$ is proper but not strictly proper, whereas in 1B, $H_{ev} \varepsilon SPR_0^m \Rightarrow H_{ev}(s)$ is strictly proper. Thus, comparing part (i) in 1A and 1B, we see that in 1B, $v, \dot{v} \varepsilon L_\infty^m$ whereas in 1A, v is additonally in L_2^m and $v \to 0$.

(4) The use of persistent excitation or gain retardation is seen in part (ii) of theorems 1A and 1B to provide the means to guaranty bounded signals. Other schemes based on signal normalizations or dead-zones can provide similar results, e.g. [32],[33]. The effect of these conditions is to provide an L_∞-stability which is not present otherwise. The persistent excitation condition actually supplies exponential stability, which is stronger than L_∞-stability, as provided, for example, by the gain retardation (see proof in Appendix A).

(5) The persistent excitation requirements in parts (i) and parts (ii)

are different. In parts (i), $z_* \varepsilon PE$, whereas in parts (ii), $z \varepsilon PE$. The different assumptions arise because in parts (i) we enforce the matched condition $e_*=0$. Hence, $z_* \varepsilon PE \Rightarrow z \varepsilon PE$. This follows from (i-d) where $z - z_* \to 0$ expoentially. Also, with $e_* = 0$, a bounded disturbance added to the reference can cause $z \varepsilon PE$ without forcing, $e_* \varepsilon L_\infty$. In parts (ii), which is more realistic, we disallow the matched condition, and hence, $e_* \varepsilon L_\infty$. Thus, $z \varepsilon PE$ is the weakest assumption to make. However, since z is inside the adaptive loop, it is very different to guarantee $z \varepsilon PE$ by injecting external signals. Note also (in both parts(ii)) that without retardation or PE it is possible for the regressor to remain bounded even though the adaptive parameters may grow unbounded. Similar results have been reported elsewhere, e.g. [24].

Robustness to Unmodeled Dynamics

Since the theorems impose requriements on the input/output properties of the interconnection system, it follows that the effect of model error on these properties determines the stability robustness of the adaptive system. For example, both theorems require that $H_{ev} \varepsilon SPR^m$. Suppose, however, that H_{ev} has the form,

$$H_{ev} = (I + \tilde{H}_{ev})\bar{H}_{ev} \tag{4.10}$$

where $\tilde{H}_{ev}$ is the projection onto H_{ev} of the plant uncertainy operator Δ ; and $\bar{H}_{ev}$ is the nominal transfer function when there is no uncertainty, i.e., when $\Delta = 0$. Thus, $\bar{H}_{ev}$ is a function of the tuned parametric model P_* and the tuned controller gains θ_* . (See Section V for more specific formulae, e.g. (5.5).)

Conditions to insure that $H_{ev} \varepsilon SPR_+^m$ despite uncertainty in H_{ev} is provided by the following:

Lemma 4.1: Let H_{ev} be given by (4.3). Then $H_{ev} \varepsilon SPR_+^m$ if the following conditions hold:

$$\text{(i)} \quad \bar{H}_{ev} \varepsilon SPR_+^m \tag{4.11a}$$

(ii) $\tilde{H}_{ev} \varepsilon\ B_S(k)$ where $\forall\ \omega\ \varepsilon\ R$, (4.11b)

$$k(\omega) < \frac{1}{2}\,\underline{\lambda}[\overline{H}_{ev}(j\omega) + \overline{H}_{ev}(-j\omega)']/\overline{\sigma}[\overline{H}_{ev}(j\omega)] \tag{4.11c}$$

Proof: Define $\underline{\mu}(\cdot)$: $C^{mxm} \to R$ by

$$\underline{\mu}\,(A) = \frac{1}{2}\,\underline{\lambda}(A+A^*)$$

where * denotes conjugate transpose. Then, using definition (2.8) with (4.10) - (4.11) we obtain

$$\underline{\mu}[H_{ev}(j\omega)] = \underline{\mu}[\overline{H}_{ev}(j\omega) + \tilde{H}_{ev}(j\omega)\overline{H}_{ev}(j\omega)]$$
$$> \underline{\mu}[\overline{H}_{ev}(j\omega)] - \overline{\sigma}(\tilde{H}_{ev}(j\omega)]\overline{\sigma}[\overline{H}_{ev}(j\omega)] > 0 \ .$$

Hnece, $H_{ev} \varepsilon\ SPR_+^m$.

Comments

(1) In order to apply Lemma 4.1 it is necessary to have a detailed description of how the plant uncertainty Δ propagates onto the interconnection uncertainty $\tilde{H}_{ev}$. This type of uncertainty propagation was explored in depth by Safonov [25] and more sophisticated expressions then (4.4b) are available to describe the uncertain operator $\tilde{H}_{ev}$. Section 5 contains more detail on this issue.

(2) In the scalar case (4.11c) becomes

$$\begin{aligned} k(\omega) &< \mathrm{Re}[\overline{H}_{ev}(j\omega)]/|\overline{H}_{ev}(j\omega)| \\ &= \cos \measuredangle\, [\overline{H}_{ev}(j\omega)] \end{aligned} \tag{4.12}$$

Since $\overline{H}_{ev}\ \varepsilon\ SPR^m$ by assumption, $k(\omega)$ is always positive for $\omega\ \varepsilon\ R$; but because of the cosine function, $k(\omega) < 1$. In Section 6 we show that this limitation on the effect of model error is easily violated by even the most benign type of unmodeled dynamics in the plant. Methods which overcome this limitation are discussed in Section 7. The requirement that $k(\omega) < 1$ also holds for any multivariable $\bar{H}_{ev} \varepsilon SPR^{m}$. To see this let $\bar{H}_{ev}$ have the polar decomposition,

$$\bar{H}_{ev} = G_{\ell} W_{ev} = W_{ev} G_{r} \tag{4.13}$$

where G_{ℓ} , G_r are Hermitian and W_{ev} is unitary. Since $\bar{\sigma}(\bar{H}_{ev}) = \bar{\sigma}(G_{\ell}) = \bar{\sigma}(G_{r})$, it follows that

$$k(\omega) < \bar{\sigma}[W_{ev}(j\omega)] < 1 \tag{4.14}$$

In the case of scalar systems, the condition $k(\omega) < 1$ can be interpreted in terms of a limitation on relative degree of $H_{ev}(s)$. A necessary condition for H_{ev} ε SPR is that the relative degree of $H_{ev}(s)$ does not exceed one i.e., phase limited to ±90°. Rohrs, et al. [12] show that this necessitates precise knowledge of plant order, and hence, is not a feasible requirement in the presence of an unstructured uncertainty (2.12), where the order is unknown. In the multivariable case it is awkward to talk about relative degree or phase, however, (4.14) expresses the same limitation.

(3) In several instances, e.g., [9],[26],[27], it has been reported that the SPR condition has been eliminated. In each case, however, it can be verified that the operator H_{ev} = positive constant , which is SPR. But, these studies do not account for unmodeled dynamics, thus, in the notation of (4.10), only $\bar{H}_{ev}$ = positive constant . Lemma 4.1 then provides the means to evaluate the effect of unmodeled dynamic.

5. APPLICATION TO MODEL REFERENCE ADAPTIVE CONTROL

Consider the model reference adaptive control (MRAC) system, shown in Figure 5.1, consisting of: an uncertain <u>scalar</u> plant $P \in R_0(s)$; a reference model $H_r \in S_0$; and filters with $F \in S_0^{\ell \times 1}$. The plant is affected by a disturbance d and a reference command r. The system equations are:

$$e = y - y_r \tag{5.1a}$$

$$y_r = H_r\, r \tag{5.1b}$$

$$y = d + Pu \tag{5.1c}$$

$$u = -\hat{\theta}'z = -(\hat{\theta}_1'z_1 + \hat{\theta}_2'z_2) \tag{5.1d}$$

$$z_1 = F\,u,\ z_2 = F(y-r) \tag{5.1e}$$

Assume that the adaptive law is given by (3.15), thus,

$$\dot{\hat{\theta}} = \Gamma\, z\, e \tag{5.1f}$$

Let the plant uncertainty be described by(2.12), i.e.,

$$\Delta := \frac{P-P_*}{P_*} \in B_S(\delta) \tag{5.1g}$$

where $P_* \in R_0(s)$ is a tuned parametric model for P. Let the filter dynamics be given by

$$F(s) = \left(\frac{1}{L(s)}, \frac{s}{L(s)}, \ldots, \frac{s^{\ell-1}}{L(s)}\right)' \tag{5.1h}$$

where L(s) is a stable monic polynomial of degree ℓ . Thus, $\hat{\theta}_1(t), \hat{\theta}_2(t) \in R^{\ell}$ and so $\hat{\theta}(t) \in R^{2\ell}$. Using the definition of tuned gain (3.2) we get,

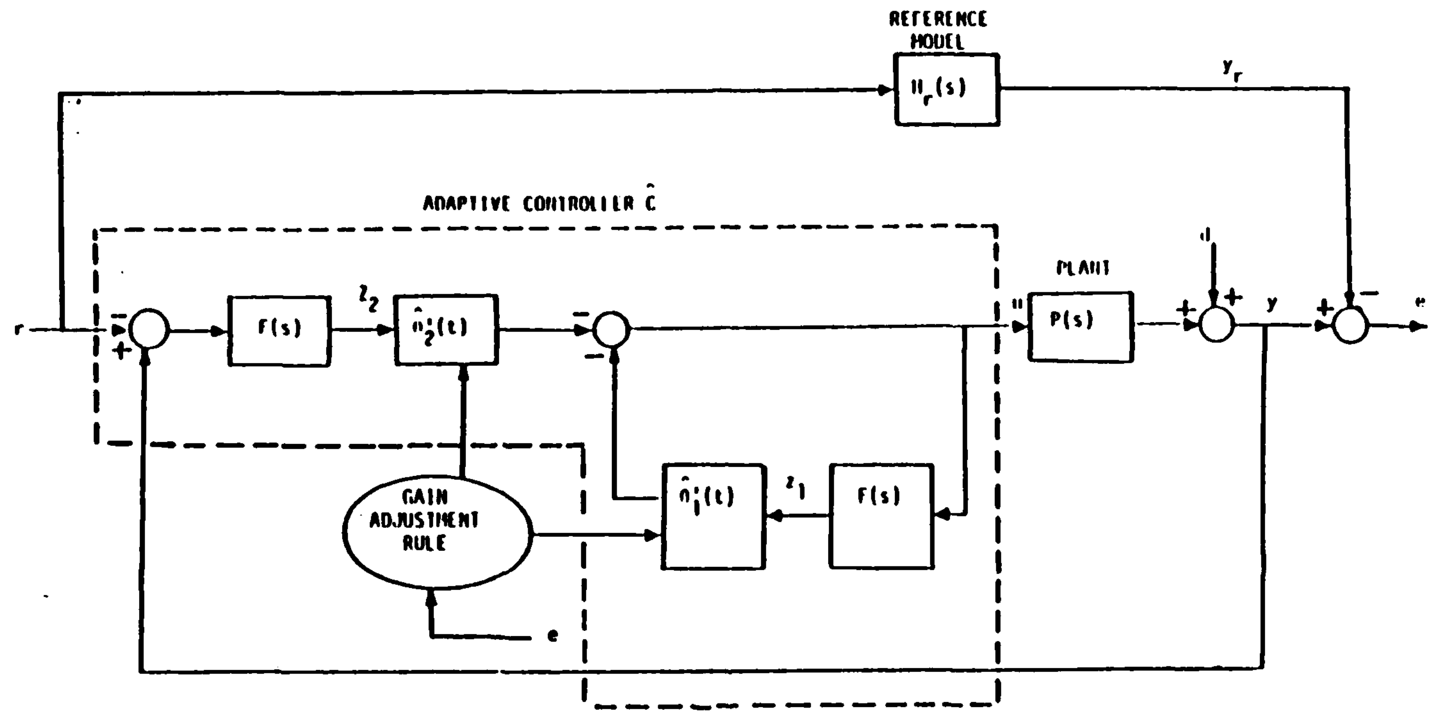

Figure 5.1 MRAC System With Scalar Plant

$$u = -\hat{\theta}'z = -(\theta_* + \theta)'z$$
$$= -(\theta_{*1}'z_1 + \theta_{*2}'z_2) - v\ , \quad v := \theta'z \quad \text{from (3.6)}$$
$$= -\frac{A_{*1}}{L}u + \frac{A_{*2}}{L}(r-y) - v$$

Finally,

$$u = \frac{A_{*2}/L}{1+A_{*1}/L}(r-y) - \frac{1}{1+A_{*1}/L}v := C_*(r-y) - \frac{1}{1+A_{*1}/L}v \tag{5.2}$$

where A_* and A_{*2} are polynomials, each of degree $\ell-1$, whose coefficients are the elements of the tuned gains θ_{*1} and θ_{*2} , respectively; and C_* denotes the tuned controller. The tuned system ($\theta=0$) is shown in Figure 5.2.

In terms of the uncertain plant P, the adaptive error system (Fig. 3.4) corresponding to this MRAC system, has tuned signals:

$$e_* = (1 + PC_*)^{-1}d + [(1+PC_*)^{-1}PC_* - H_r]r \tag{5.3a}$$

$$z_* = \begin{bmatrix} F(1+PC_*)^{-1}C_*(r-d) \\ \\ F(1+PC_*)^{-1}(d-r) \end{bmatrix} \tag{5.3b}$$

and interconnections:

$$H_{ev} = (1+PC_*)^{-1}P(1+A_{*1}/L)^{-1} \tag{5.3c}$$

$$H_{zv} = \begin{bmatrix} F(1+PC_*)^{-1}(1+A_{*1}/L)^{-1} \\ \\ F(1+PC_*)^{-1}P(1+A_{*1}/L)^{-1} \end{bmatrix} \tag{5.3d}$$

The error system can also be described so as to highlight the model error Δ . The following definitions are convenient:

$$T_* := (1+P_*C_*)^{-1}P_*C_* := 1 - S_* \tag{5.4a}$$

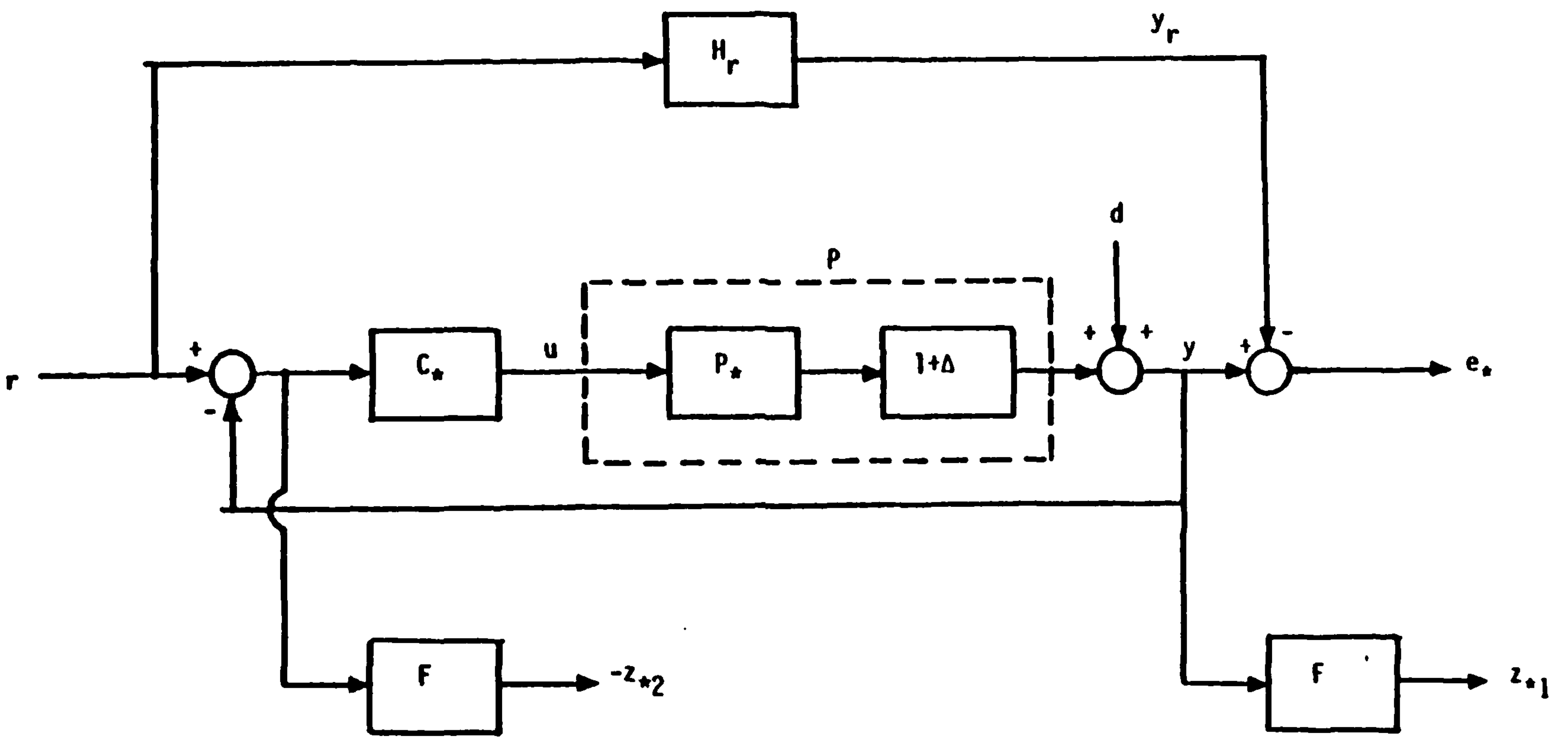

Figure 5.2 Tuned System

$$K_* := H_{ev}\big|_{\Delta=0} = (1+P_*C_*)^{-1}P_*(1+A_{*1}/L)^{-1} \tag{5.4b}$$

Thus, the error system (5.3) can be also be expressed as:

$$e_* = S_*(1+\Delta T_*)^{-1}d + (T_*(1+\Delta)(1+\Delta T_*)^{-1}-H_r)r \tag{5.5a}$$

$$z_* = \begin{bmatrix} F\ S_*C_*(1+\Delta T_*)^{-1}(r-d) \\ \\ F\ S_*(1+\Delta T_*)^{-1}(d-r) \end{bmatrix} \qquad)5.5b)$$

$$H_{ev} = K_*(1+\Delta)(1+\Delta T_*)^{-1} \tag{5.5c}$$

$$H_{zv} = \begin{bmatrix} F\ K_*P_*^{-1}(1+\Delta T_*)^{-1} \\ \\ F\ K_*(1+\Delta)(1+\Delta T_*)^{-1} \end{bmatrix} \tag{5.5d}$$

The result that follows in Lemma 5.1 gives conditions under which $H_{ev} \in SPR_0$ and $H_{zv} \in S_0^{2\ell x1}$, despite model error; thus conditions (A1)-(A3) of Theorems 1A and 2B are satisfied. Additional requirements are necessary to establish the class of tuned signals e* and z* as given by (5.5a) and (5.5b), respectively. These requirements are discussed following Lemma 5.1.

Lemma 5.1: For the adaptive system (5.3) or (5.5) $H_{ev} \in SPR_0$ and $H_{zv} \in S_0^{2\ell x1}$ if the following conditions are all satisfied:

(i) $$P_*(s) = \frac{g(s^{n-1}+\beta_1 s^{n-2}+\ldots+\beta_{n-1})}{s^n+\alpha_1 s^{n-1}+\ldots+\alpha_n} = \frac{gN_*(s)}{D_*(s)} \tag{5.6a}$$

(ii) $N_*(s)$ is a stable monic polynomial (5.6b)

(iii) $g > 0$ (5.6c)

(iv) $K_*(s) = \dfrac{g\,K_1(s)}{K_2(s)} \in SPR_0$ where $K_1(s)$ and $K_2(s)$ are monic stable polynomials.

(5.6d)

(v) $\quad \ell = \deg L(s) > n + \deg K_1(s) - 1$ (5.6e)

(vi) $\quad \Delta \in B_S(\delta)$ is such that

$$\left.\begin{array}{l}\delta(\omega) < \bar{\delta}(\omega) := \eta(\omega)[\eta(\omega)|T_*(j\omega)| + |S_*(j\omega)|]^{-1} \\ \quad \eta(\omega) := \cos \measuredangle [K_*(j\omega)]\end{array}\right\} \forall\, \omega \in R,$$

(5.6f)

Proof: See Appendix B.

Discussion

(1) Condition (i)-(v) of Lemma 5.1 are restatements of known results, but normally they apply to the actual plant P, e.g. [7]. In Lemma 5.1, however, these conditions apply to the parametric model P_* -- not to the actual plant. As such, they are easier to satisfy, since the parametric model is somewhat arbitrary. This flexibility is penalized by an increase in model error. For example, if the actual plant has a relative degree of 2, then choosing a parametric model of relative degree 1 -- as required by condition (i) -- incrases the high frequency model error.

(2) Condition (vi) imposes an upper bound $\bar{\delta}$ on the model error associated with the chosen parametric model. This condition simultaneously insures that $H_{ev} \in SPR_0$ despite model error, and that the tuned system is stable (see proof in Appendix B).

(3) It is easily verified that $\bar{\delta}(\omega) < 1$, as was discussed following Lemma 4.1. In fact, even the "optimally tight" bound (see [25] for details on this calculation) given by,

$$\bar{\delta} = \frac{1}{2|T|\eta}\left[-|1-T| + \left(|1+T|^2 + 4\eta\, \mathrm{Re}(KT/|K|)\right)^{1/2}\right] \tag{5.7}$$

is also restricted to be less than 1. This limitation severely restricts the type of admissable model error. This issue is pursued in Section 6.

(4) To guarantee global stability using the adaptive law (5.1f), property (i) of Theorem 1 requires that $e_* \to 0$ and $z_*, \dot{z}_* \in L_\infty^{2\ell}$ for all r and d. For example, let r and d be any bounded signals such that $r \to$ constant and $d \to$ constant as $t \to \infty$. Property (i) of Theorem 1 is satisfied if:

$$\delta(0) = 0 \tag{5.8a}$$

$$T_*(0) = H_r(0) = 1 \tag{5.8b}$$

Zero model error at DC (5.8a) is certainly to be expected from even the most crude tuned parametric model.

(5) Let r be bounded such that $r \to$ constant as $t \to \infty$, but let d be just bounded, i.e., $d \in L_\infty$. In this case it is not possible to guarantee $e_* \to 0$, but we can guarantee that $e_* \in L_\infty$. To obtain global stability in this case, requires the introduction of the retardation term (3.16) into the adaptive law (5.1f), see part (ii) of Theorems 1A or 1B.

(6) It is possible to obtain versions of Lemma 5.1 for adaptive systems of different forms, e.g., indirect adaptive [5]. Also, the use of "multipliers", e.g. [4], can be accounted for as well. The multiplier effectively makes use of the availability of $\hat{\theta}$ as a signal; and this allows rel deg $(P_*) = 2$ rather than 1 as required by condition (i) of Lemma 5.1.

6. LIMITATIONS IMPOSED BY THE SPR CONDITION

The fact that the model error bound given in condition (vi) of Lemma 5.1 can not exceed one has unfortunate consequences.

Example 1

Consider a plant with transfer function,

$$P(s) = P_*(s)\,\frac{ab}{(s+a)(s+b)} \tag{6.1}$$

where P_* is the parametirc model, with two unmodeled stable poles at -a and -b. Suppose,also, that b is much greater than a, and that a is much greater than the bandwidth of $P_*(s)$. This situation seems benign -- and most likely a certainty. Comparing (6.1) with (5.1g) gives,

$$\delta(\omega) = \omega\left[\frac{\omega^2 + (a+b)^2}{(\omega^2+a^2)(\omega^2+b^2)}\right]^{1/2} > 1$$

for all frequencies $\omega > (ab/2)^{1/2}$, thus, condition (vi) of Lemma 5.1 is violated, and global stability cannot be guaranteed. The following example illustrates this point.

Example 2

Consider the example MRAC system (Fig. 5.1) studied by Rohrs et al. [12], where:

$$P(s) = \frac{2}{s+1}\,\frac{229}{(s+15)^2 + 4}$$

$$H_R(s) = \frac{3}{s+3}$$

$$u = -\hat{\theta}_1 y + \hat{\theta}_2 r$$

$$\dot{\hat{\theta}}_1 = ye, \ \hat{\theta}_1(0) = .65$$

$$\dot{\hat{\theta}}_2 = -r\,e, \ \hat{\theta}_2(0) = 1.14$$

Let r = constant and d = 0. Thus, $e_* \to 0$ exponentially when the tuned gains are such that (5.8) is satisfied, i.e.,

$$T_*(0) = \frac{2\theta_{*2}}{1+2\theta_{*1}} = H_r(0) = 1$$

Even though $(\theta_{*1}, \theta_{*2})$ exist to satisfy this, $H_{ev}(s)$ is not SPR, and so global stability is not guaranteed. Simulation runs with r = .4 and r = 4.0 are shown in Figures 6.1 and 6.2, respectively. With the small input (Fig. 6.1) we see a stable response which tracks the reference very well. With the large input (Fig. 6.2) the response is still stable, but large oscillations are taking place. Larger inputs will eventually drive the system unstable, e.g. [12].

In this example, if the tuned model is taken to be $P_*(s) = 1/(s+1)$ then it is easily verified that model error $\delta(\omega)$ is greater than one at some frequency.

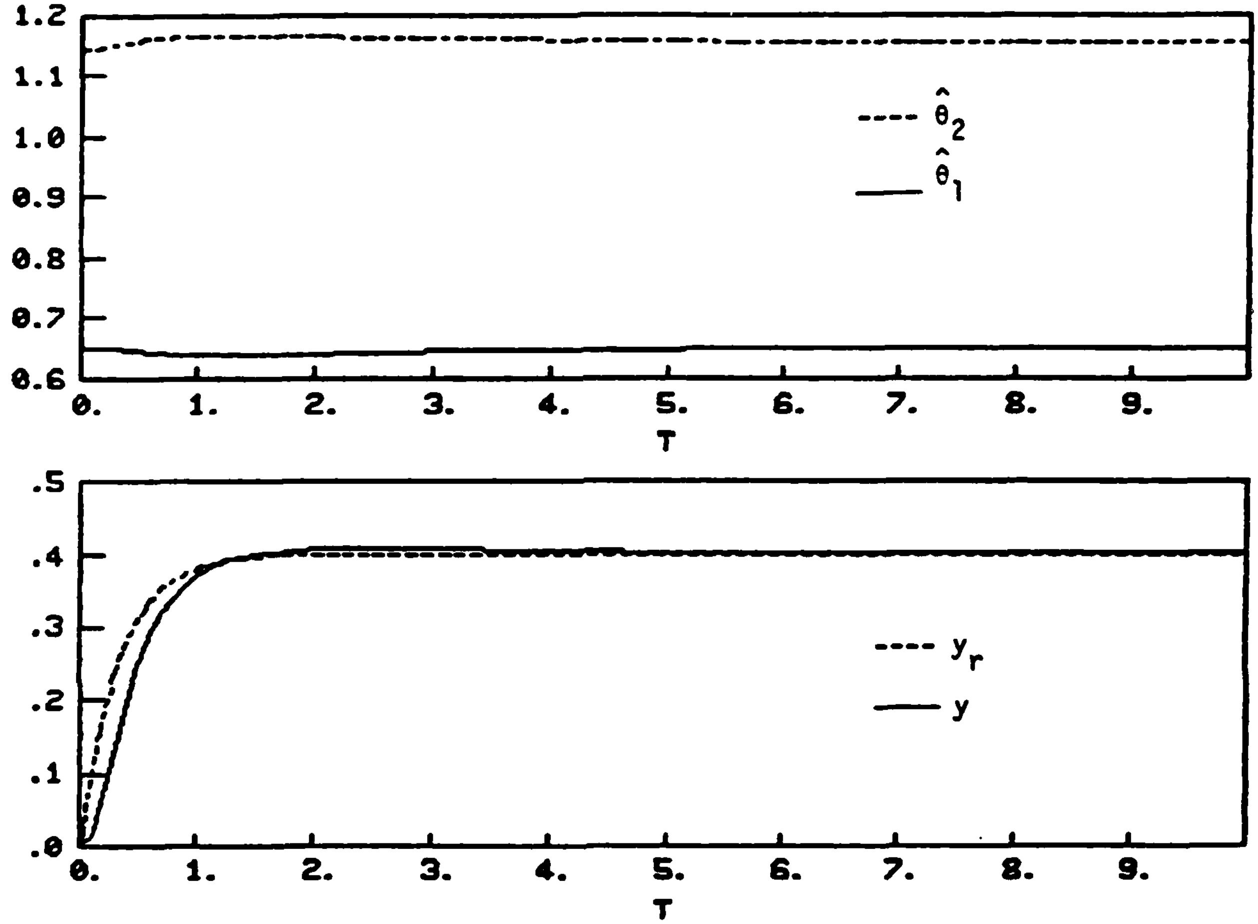

Figure 6.1 Response to r = .4

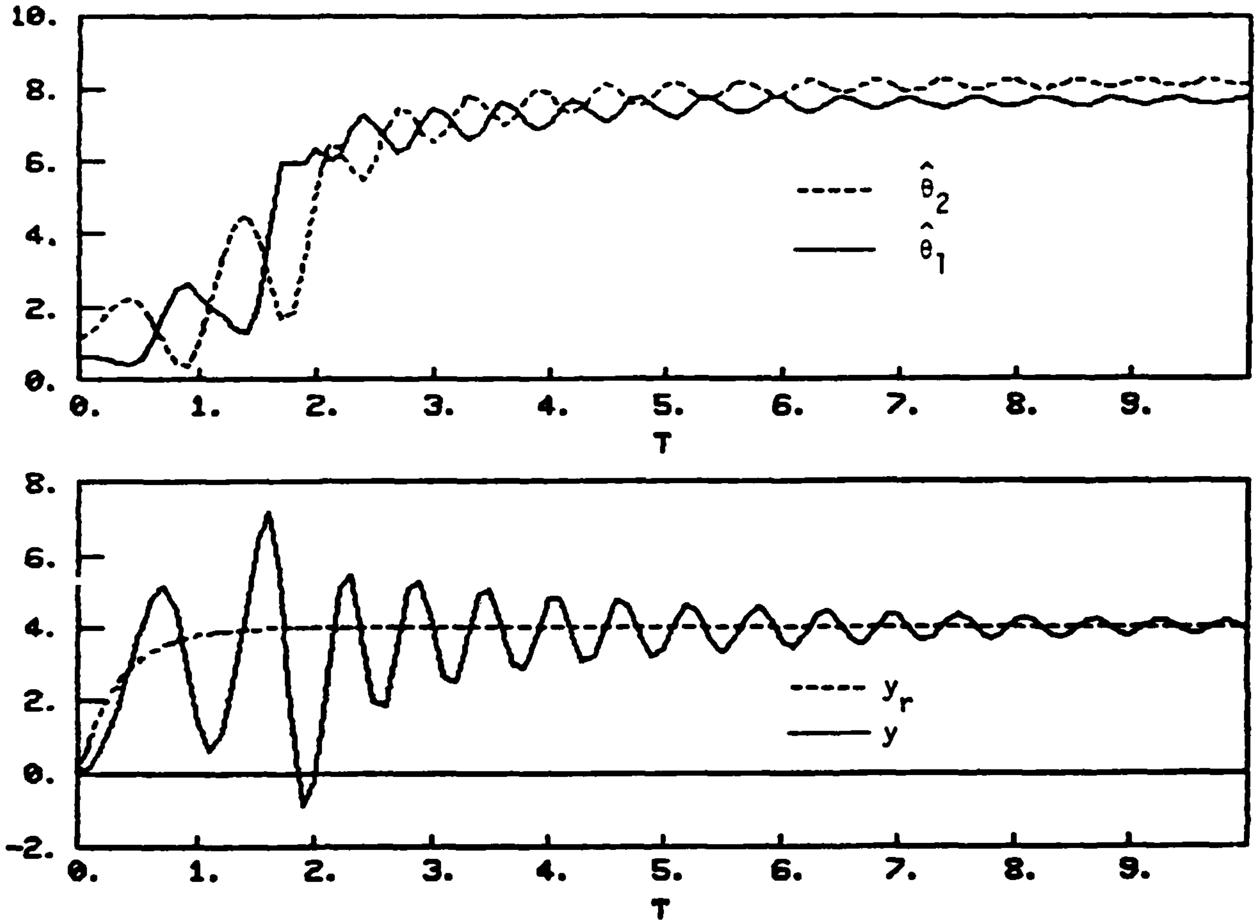

Figure 6.2 Response to r = 4.0

7. SPR COMPENSATION

In this section we heuristically develop a means to obtain global robust adaptive control. Since the SPR condition is violated whenever model error exceeds one, a natural scheme is to construct an <u>SPR compensator</u> which alleivates the problems by "filtering" the plant output; thus, avoiding the trouble. However, direct filtering does not change the size of model error. For example, with the plant $P = (1+\Delta)P_*$, let y_W denote the output of the filtered plant, where

$$y_w := Wy = Wd + (1+\Delta)WP_*u \tag{7.1}$$

Thus, model error is uneffected. Even filtering H_{ev} directly by W offers no help, since the bound (4.4c) is still less than one, i.e.,

$$|\tilde{H}_{ev}| < \mathrm{Re}(W\,\bar{H}_{ev})/|W\,\bar{H}_{ev}| < 1 \tag{7.2}$$

for any stable W. What we seek is an SPR compensator which only effects the unmodeled dynamics, but leaves the paramtric model intact.

A compensation scheme, which offers some promise as an SPR compensator, is that proposed in [22], as shown in Figure 7.1. To see the desired result suppose that $P = (1+\Delta)P_m$ with $\Delta \in B_S(\delta)$. Then, the compensator is equivalent to a plant which maps (u,d) into y_c where

$$y_c = Wd + P_c u \tag{7.2a}$$

$$\Delta_c := \frac{P_c - P_m}{P_m} \in B_S(W\delta) \tag{7.2b}$$

Thus, whenver $\delta(\omega) > 1$, select W(s) such that $|W(j\omega)|\delta(\omega) < 1$. The filter W acts like a "frequency switch" whose function is to insure condition (vi) of Lemma 5.1.

There are two ways to implement this compensator in an adaptive system. The first way is to use a fixed model of the plant for P_m, i.e., $P_m = \bar{P}$. The second way is to replace P_m with an adaptive observer, i.e., $P_m = \hat{P}$.

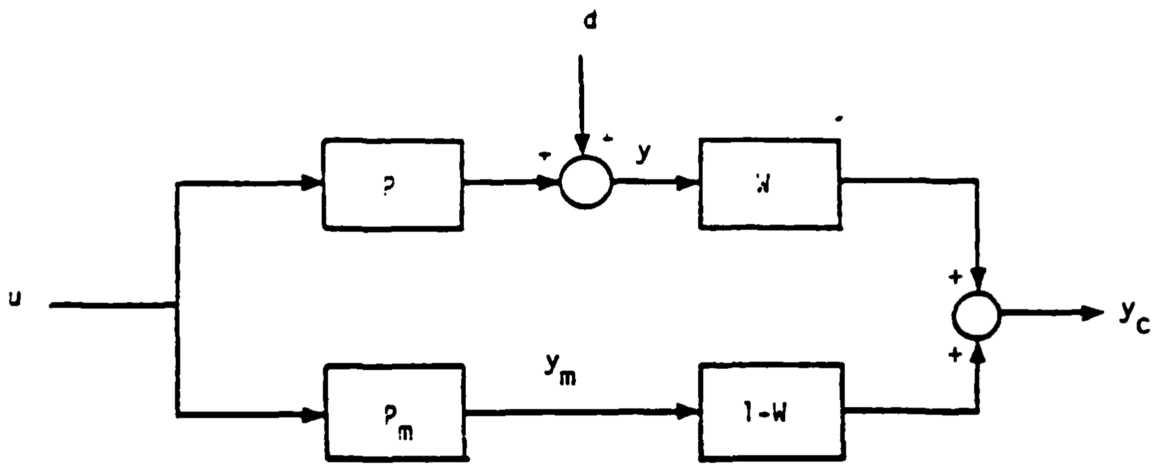

Figure 7.1 SPR Compensation

In either case, to obtain the benefit of the SPR compensator, the signal to be controlled is the compensator output y_c, not the plant output y. Both of these compensators will now be examined.

Fixed SPR Compensator

Let $P_m = \bar{P}$, a fixed model, and let the actual plant be given by (2.17), $P = (1+\Delta)P_*$ with $\Delta \in B_S(\delta)$. Then the fixed compensator plant equivalent model error (7.2b) is:

$$\Delta_c := \frac{P_c - P_*}{P_*} \in B_S(\delta_1) \tag{7.3a}$$

where

$$\delta_1(\omega) := |W(j\omega)|\delta(\omega) + |1 - W(j\omega)| \cdot \left|\frac{\bar{P}(j\omega) - P_*(j\omega)}{P_*(j\omega)}\right| \tag{7.3b}$$

This scheme is motivated by the fact that at low frequencies the tuned parametric model P_* is close to P; thus δ is small and $W \approx 1$. At high frequencies δ is large but $(\bar{P} - P_*)/P_*$ is small, $W \approx 0$ and so δ_1 is small. Of course the compensator is limited if there is large model error at intermediate frequencies.

Example 2

Example 1 is modified to include a fixed SPR compsnator with $W(s) = 1/(s+1)$ and $\bar{P}(s) = 2/(s+1)$. Simulation results with the large step command (r=4) are shown in Figure 7.2. Comparing these to Figure 6.2, without compensation, it is readily verified that the instability tendencies are eliminated. Also, direct calculations reveal that $H_{ev} \in SPR_o$, thus global stability is insured.

Adaptive SPR Compensation

An adaptive SPR compensator, together with the adaptive controller, is shown in Figure 7.3. The adaptive controller is described by,

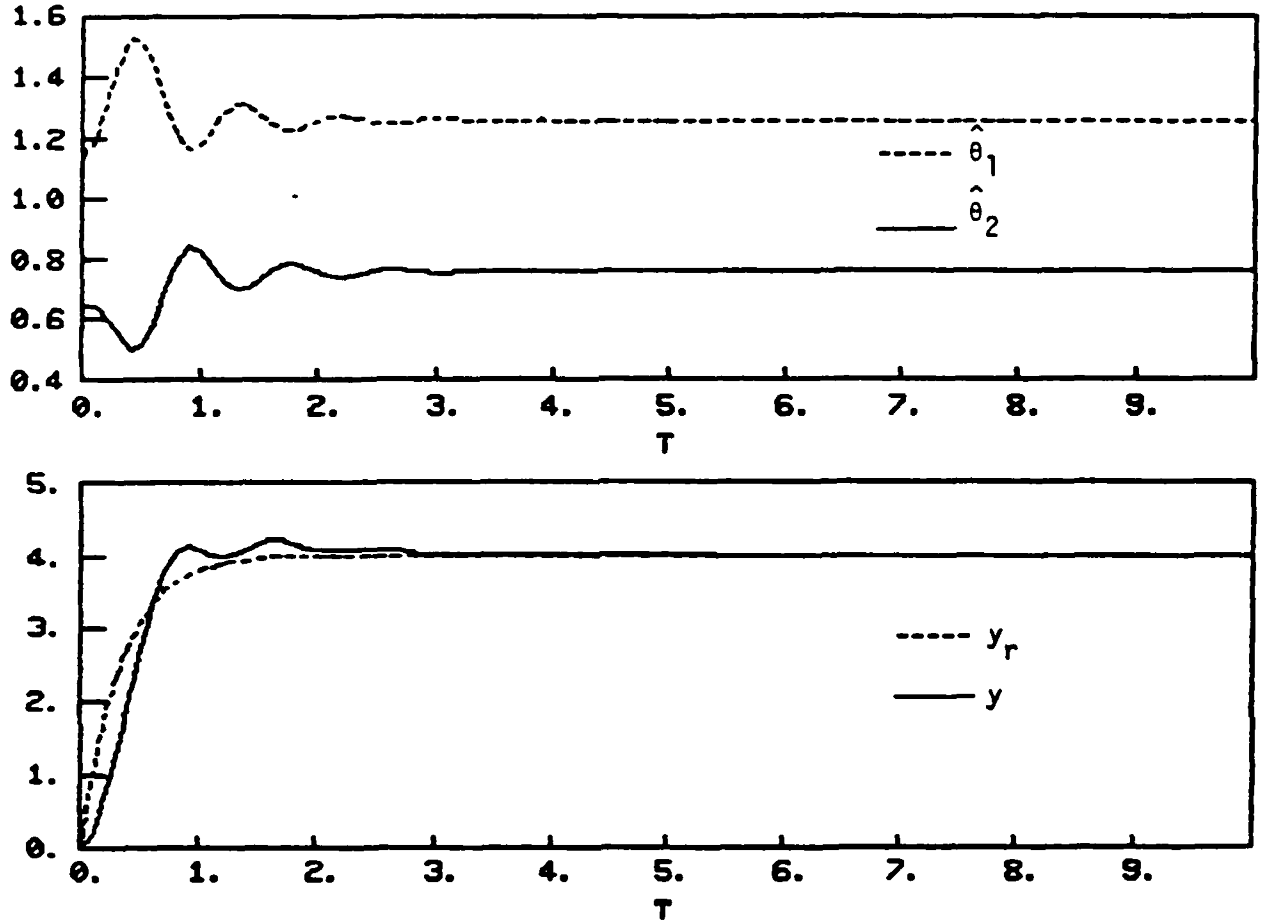

Figure 7.2 MRAC with SPR Compensator, r = 4.0

$$u = -\hat{\theta}_c' z_c \quad , \quad z_c' = \left(F_c' u \ , \ F_c'(y_c - r)\right) \tag{7.4a}$$

$$\dot{\hat{\theta}}_c = \Gamma_c \, z_c \, e_c \ , \quad e_c = y_c - y_r \tag{7.4b}$$

$$F_c'(s) = \left(1/L_c(s), \ldots, s^{n_c - 1}/L_c(s)\right) \ , \ n_c = \deg L_c(s) \tag{7.4c}$$

and the adaptive observer is described by,

$$\hat{y} = \hat{\theta}_o' \, z_o \quad , \quad z_o' = (F_o' \, u. \ -F_o' \, y) \tag{7.4d}$$

$$\dot{\hat{\theta}}_o = \Gamma_o z_o e_o \quad , \quad e_o = y - \hat{y} \tag{7.4d}$$

$$F_o'(s) = \left(1/L_o(s), \ \ldots, \ s^{n_o - 1}/L_o(s)\right) \ , \ n_o = \deg L_o(s) \tag{7.4f}$$

where $L_o(s)$ and $L_c(s)$ are both monic and stable. To generate the error system interconnection operators associated with this system, let θ_{*c} and θ_{*o} denote the tuned parameters with respective gain errors, θ_c and θ_o ; and let $v_c := \theta_c' z_c$ and $v_o := \theta_o' z_o$ be the corresponding adaptive control errors (3.6). By analogy with the procedure used in Section 5 we get,

$$u = C_*(r - y_c) - \frac{1}{I + A_{*1}/L_c} v_c \tag{7.5}$$

$$\hat{y} = -\frac{B_{*1}}{L_o} d + \left(1 - \frac{B_{*1}}{L_o} \Delta\right) P_* u + v_o \tag{7.6}$$

where

$$C_* = \frac{A_{*2}/L_c}{I + A_{*1}/L_c} \tag{7.7}$$

$$P_* = \frac{B_{*2}/L_o}{I + B_{*1}/L_o} = \frac{gN_*}{D_*} \tag{7.8}$$

and where (A_{*1}, A_{*2}) are polynomials whose coefficients are the parameters in θ_{*c} ; (B_{*1}, B_{*2}) are polynomials whose coefficients are the parameters in θ_{*o} ; and N_*, P_* and g are as defined by (5.6a). The adaptive error model is given below in terms of T_*, S_*, and K_* as defined in (5.4). In additon, define:

$$R := 1 + (W-1)\frac{D_*}{L_o} \tag{7.9}$$

The tuned signals are:

$$e_{*c} = S_*(1+\Delta RT_*)^{-1}R\ d + \left(T_*(1+\Delta R)(1+\Delta RT_*)^{-1}-H_r\right)r \tag{7.10a}$$

$$e_{*o} = D_*L_o^{-1}(1+\Delta RT_*)^{-1}d + D_*L_o^{-1}T_*\Delta(1+\Delta RT_*)^{-1}r \tag{7.10b}$$

$$z_{*c} = \begin{bmatrix} F_cA_{*2}L_c^{-1}P_*^{-1}K_*(1+\Delta RT_*)^{-1}(r-Rd) \\ \\ F_cS_*(1+\Delta RT_*)^{-1}(Rd-r) \end{bmatrix} \tag{7.10c}$$

$$z_{*o} = \begin{bmatrix} F_oA_{*2}L_c^{-1}P_*^{-1}K_*(1+\Delta RT_*)^{-1}(r-Rd) \\ \\ F_oT_*(1+\Delta RT_*)^{-1}(d - (1+\Delta)r) \end{bmatrix} \tag{7.10d}$$

The interconnections are:

$$H_{ev} = \begin{bmatrix} K_*(1+\Delta R)(1+\Delta RT_*)^{-1} & -(1-W)S_*(1+\Delta RT_*)^{-1} \\ \\ K_*D_*L_o^{-1}\Delta(1+\Delta RT_*)^{-1} & 1+(1-W)T_*D_*L_o^{-1}(1+\Delta RT_*)^{-1} \end{bmatrix} \tag{7.11a}$$

$$H_{z_cv} = \begin{bmatrix} F_cP_*^{-1}K_*(1+\Delta RT_*)^{-1} & F_cA_{*2}L_c^{-1}P_*^{-1}K_*(1-W)(1+\Delta RT_*)^{-1} \\ \\ F_cK_*(1+\Delta R)(1+\Delta RT_*)^{-1} & -F_cS_*(1-W)(1+\Delta RT_*)^{-1} \end{bmatrix} \tag{7.11b}$$

$$H_{z_o v} = \begin{bmatrix} F_o P_*^{-1} K_* (1+\Delta R T_*)^{-1} & F_o A_{*2} L_c^{-1} P_*^{-1} K_* (1-W)(1+\Delta R T_*)^{-1} \\ -F_o K_* (1+\Delta)(1+\Delta R T_*)^{-1} & -F_o T_* (1-W)(1+\Delta)(1+\Delta R T_*)^{-1} \end{bmatrix} \tag{7.11c}$$

The factor $(1+\Delta RT_*)^{-1}$ appears in all the terms above. The transfer function R (7.9) reduces the effect of unmodeled dynamics; however not exactly by the amount anticipated, vis a vis (7.2). This is due to additonal model error introduced by the adaptive observer. Nonetheless, the model error attenuation is greater than with the fixed SPR compensator. In particular, at low frequencies $\Delta \approx 0$ and at high frequencies $R \approx 0$, since $W \approx 0$ and $D_* L_o^{-1} \approx 1$. Without further testing of H_{ev} (7.11a) it is not possible to state that $H_{ev} \in SPR_o$ at intermediate frequencies. Note, however, that the nominal value of H_{ev} is:

$$\bar{H}_{ev} = \begin{bmatrix} K_* & -(1-W)S_* \\ 0 & 1 \end{bmatrix} . \tag{7.12}$$

which is SPR_o provided that $K_* \in SPR$ and

$$\mathrm{Re}\, K_*(j\omega) > \frac{1}{4} |(1-W(j\omega))S_*(j\omega)|^2 \;,\; \omega \in R \tag{7.13}$$

Applying (4.11) to (7.11a), a tedious procedure, would give an upper bound on model error to insure $H_{ev} \in SPR_o$.

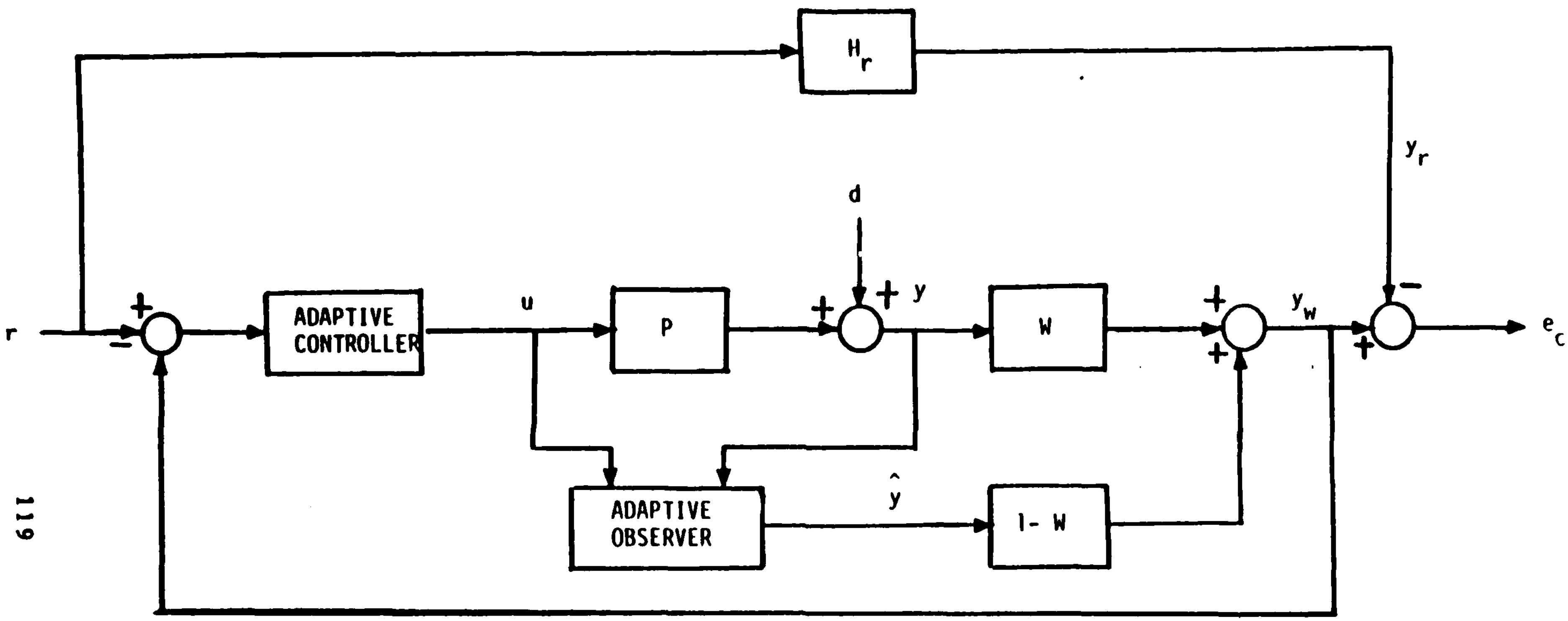

Figure 7.3 Adaptive SPR Compensator

8. CONCLUSIONS

This paper has presented an input/output view of multivariable adaptive control for uncertain linear time invariant plants. The essence of the results are captured in Theorems 1A and 2B which provide conditions that guarantee global stability. Corollary 1 also give specific L_2 and L_∞ bounds on significant signals in the adaptive control system. These bounds, for example, can be used to guarantee that the adaptive system performs as well as a robust (non-adaptive) system using the same structure, but with fixed gains. By distinguishing between a tuned system and a robust system, we establish formulae which can be used to restrict the minimum performance improvement possible with the same control structure.

Although the stability results (Theorem 1A, 1B) are not entirely new (see e.g., [7],[8]), the input/output setting provides the means to directly determine the system robustness properties with respect to model error. The type of model error examined can arise from a variety of causes, such as unmodeled dynamics and reduced order modeling. It is very difficult to treat this type of "unstructured" dynamic model error by using Lyapunov theory, since the system order may not be known -- in fact, it may be infinite. Although infinite dimensional (distributed) systems were not considered here, Theorem 1 can be modified to include them, e.g., [26].

The structure of Theorems 1A and 1B require that a particular subsystem operator, denoted H_{ev} , is strictly positive real (SPR). This requirement is not unique to this presentation - passivity requirements, in one form or another, dominate proofs of global stability for practically all adaptive control systems, including recursive identification algorithms. Unfortunately, although $H_{ev} \in$ SPR is robust to model error (Lemma 4.1), the bound on the model error is too small to be of practical use. Even the most benign neglected dynamics violate the bound.

Although this paper is concerned with continuous-time systems, the theorems carry over virtually intact to discrete-time systems. This is a direct consequence of the portable nature of the input/output view. However, there is an important issue unique to discrete-time systems: plant uncertainty is critical to where performance is actually measured, which is in continuous-time, not at the sampled-data points. As a consequence, it may be necessary to map the discrete portions of the adaptive system (most likely the controller) into continuous-time, i.e., the L_2-gains of the discrete-time operators in the interconnection map, which are associated with the adaptive discrete-time controller, would be needed rather the discrete-time ℓ_2-gains .

Another area worth pursuing is the adaptive control of non-linear plants. The plant uncertainty description (2.11) does not exclude non-linear plants. Note that slowly drifting parameters in an otherwise perfectly known LTI plant could yield the same uncertainty description as a non-linear plant approximated by a parametric LTI model. All that is required is that there exists a (possibly) infinite dimensional LTI system which matches the input/output behavior of the plant for each possible input/output pair. Of course, if the plant is truly non-linear, then the tuned control is likely to be non-linear, which raises some very interesting issues for further research.

One final remark: the stability results presented here, as well as other known results, provide global stability. This is achieved by requiring $H_{ev} \in SPR$, a condition which is difficult to maintain in normal circumstances. On the other hand, this is a sufficient conditon; violation of which does not necessarily lead to instability. The simple example presented here in Figure 6.1-6.2, illustrates the point. Other examples of this phenomena abound, e.g., [12]. It would appear then, that a more valid approach to providing a system-theoretic setting for adaptive control is to develop local stability conditions, which, hopefully, do not require that $H_{ev} \in SPR$. Preliminary results on local stability support this hope, e.g., [33], [34].

ACKNOWLEDGEMENT

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APPENDIX A

PROOF OF THEOREMS 1 AND 2

Preliminaries

The main ingredient in the proof is to show stability by means of passivity. Although there are many variations on this theme, a general result is given by the following.

Theorem A.1 ([21], [35]

Consider the feedback system of Figure A.1 below with causal operators G_1 and G_2 .

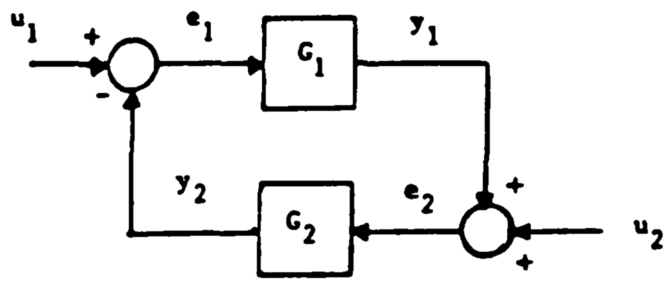

Figure A.1 Feedback System

Suppose there exists real constants ε_i, δ_i, α_i, $i=1,2$, such that

$$\langle x,G_i x\rangle_t > \varepsilon_i \|x\|_{t2}^2 + \delta_i \|G_i x\|_{t2}^2 + \alpha_i, \ \forall t > 0, \ \forall\, x \in L_2[0,t] \tag{A.1}$$

for $i=1,2$. Then the following holds $\forall\, t > 0$,

$$(\varepsilon_2+\delta_1)\|y_1\|_{t2}^2 + (\varepsilon_1+\delta_2)\|y_2\|_{t2}^2 < \|y_1\|_{t2}(\|u_1\|_{t2} + 2|\varepsilon_2|\cdot\|u_2\|_{t2})$$

$$+ \|y_2\|_{t2}(\|u_2\|_{t2} + 2|\varepsilon_1|\cdot\|u_1\|_{t2}) + |\varepsilon_1|\cdot\|u_1\|_{t2}^2 + |\varepsilon_2| \cdot \|u_2\|_{t2}^2$$

$$+ |\alpha_1| + |\alpha_2| \tag{A.2}$$

Proofs of both theorems also rely on well known results for systems $H \in S_o^{n \times m}$. The results required here are summarized in the following.

<u>Theorem A-2</u> [see [19], Thm. 9, pg. 59]

Let $H \in S_o^{n \times m}$; then:

(i) If $u \in L_2^m$, then $y = Hu \in L_2^n \;\; L_\infty^n$, $\dot{y} \in L_2^n$, y is continuous, and $y(t) \to 0$ as $t \to \infty$.

(ii) If $u \in L_\infty^m$, then $y = Hu \in L_\infty^n$, $\dot{y} \in L_\infty^n$, and y is uniformly continuous.

(iii) If $u \in L_\infty^m$ and $u(t) \to$ constant $c \in R^m$ as $t \to \infty$, then $y(t) \to H(0)c$ exponentially as $t \to \infty$.

In order to simplify notation we drop the superstrict on L_p^n which *indicates vector size.*

We will establish Theorem 1A first. Some of the steps will be repeated for 1B. Also, without loss of generality, the matrix Γ in the adaptation law (3.15),(3.16) is set to identity. Corollary 1 is established as a by-product.

<u>Proof of Theorem 1A</u>

<u>Part (i)</u>

Identify G_1, G_2 in Figure A.1 with $e \to v$ and H_{ev} respectively. Also, let $u_1 = e_*$, $u_2 = 0$, $e_1 = e$, $y_1 = e_2 = v$, and $y_2 = H_{ev}v$. . Using adaptive law (3.15) we obtain,

$$\langle e,v\rangle_T = \langle e,Z'\theta\rangle_T = \langle Ze,\theta\rangle_T = \langle\dot{\theta}, \theta\rangle_T \tag{A.4}$$

$$= \frac{1}{2}\|\theta(T)\|^2 - \frac{1}{2}\|\theta(0)\|^2 \tag{A.5}$$

$$> -\frac{1}{2}\|\theta(0)\|^2 \tag{A.6}$$

Thus, using (A.1) gives,

$$\varepsilon_1 = \delta_1 = 0, \; \alpha_1 = -\frac{1}{2}\|\theta(0)\|^2 \tag{A.7}$$

Since $G_2 = H_{ev} \varepsilon_2 SPR_+$ by assumption, $\exists \mu, \gamma > 0$ such that $\forall x \varepsilon L_{2e}$, $\langle x, H_{ev}x\rangle_T > \mu\|x\|_{T2}^2$, $\|H_{ev}x\|_{T2} < \gamma\|x\|_{T2}$. Hence, from (A.1),

$$\varepsilon_2 = \mu, \; \delta_2 = \alpha_2 = 0 \tag{A.8}$$

Using Lemma A.1, together with (A.4)-(A.8) gives,

$$\|v\|_{T2} < \frac{1}{2\mu}[\|e_*\|_{T2} + (\|e_*\|_{T2}^2 + 2\mu|\theta(0)|^2)^{1/2}] \tag{A.9}$$

$$\|e-e_*\|_{T2} < \gamma\|v\|_{T2} \tag{A.10}$$

$$|\theta(T)|^2 < |\theta(0)|^2 + 2\|e\|_{T2}\|v\|_{T2} \tag{A.11}$$

The bounds shown in (4.8) follow using the assumption $e_* \varepsilon L_2$. Hence, $e, v \varepsilon L_2$ and $\theta \varepsilon L_\infty$.

Having established that $v \varepsilon L_2$, Theorem A-2 $\Longrightarrow \tilde{z} := z - z_* \varepsilon L_2 \cap L_\infty$, $\dot{\tilde{z}} \varepsilon L_2$, $\tilde{z} \to 0$, and $\tilde{z}$ is continuous. Since $z_*, \dot{z}_* \varepsilon L_\infty$ by assumption, it follows that $z \varepsilon L_\infty$ and $\dot{z} \varepsilon L_\infty (\Longrightarrow z$ is uniformly continuous). Using $v = Z'\theta$ with $z, \theta' \varepsilon L_\infty \Longrightarrow v \varepsilon L_\infty$. Using $e = e_* - H_{ev}v$ with $e_* \varepsilon L_\infty$ and $H_{ev} \varepsilon S$ (by assumption), and $v \varepsilon L_\infty \Longrightarrow e \varepsilon L_\infty$. Hence, $\dot{\theta} = Ze \varepsilon L_\infty \Longrightarrow \theta$ is uniformly continuous $\Longrightarrow v = Z'\theta$ is uniformly continuous (since z is) $\Longrightarrow v \to 0$ since $v \varepsilon L_2$ is established. Using $v \to 0 \Longrightarrow e - e_* \to 0$, and since $e_* \to 0$ by assumption, $e \to 0$. Furthermore, $v \to 0 \Longrightarrow \tilde{z} \to 0$ exp. and $\dot{\theta} = Ze = \tilde{Z}e + Z_*e \to 0$, because $\tilde{z}$ and $e \to 0$. Using $\dot{v} = \dot{Z}'\theta + Z'\dot{\theta}$ with $\dot{z}, \theta, \dot{\theta} \varepsilon L_\infty \Longrightarrow \dot{v} \varepsilon L_\infty$. Hence, $e^\bullet = \dot{e}_* - H_{ev}\dot{v} \varepsilon L_\infty$, because $e_* \varepsilon L_\infty$ by assumption. Thus, $\ddot{\theta} = \dot{Z}e + Z\dot{e} \varepsilon L_\infty$. This establishes properties (i-a)-(i-d).

To show (i-e) consider (3.15) written as:

$$\dot{\Theta} = -Z_* H_{ev} Z_*' \theta + w$$

$$w := -(Z_* H_{ev} \tilde{Z}' + \tilde{Z} H_{ev} Z_*' + \tilde{Z} H_{ev} \tilde{Z}')\theta \tag{A.12}$$

Since we have already established that $\tilde{z} \to 0$ exp. and $\theta \in L_\infty$, it follows that $w \to 0$ exp. Since $z_* \in PE$ by assumption (provided $e_* = 0$), $w \mapsto \theta$ is exp. stable by (2.15). Hence, $\theta \to 0$ exp. $\Rightarrow \dot{\theta}$, $v \to 0$ exp. $\Rightarrow e - e_* \to 0$ exp. This completes the proof of part (i) with adaptive law (3.15).

To show that (i-a)-(i-a) hold with adaptive law (3.16) requires showing that $G_1 : e \to v$ is passive. Consider the typical time interval,

$$I = \begin{cases} I_1 = \{t \in [t_0, t_1) \mid \|\hat{\theta}(t)\| < c\} \\ I_2 = \{t \in [t_1, t_2) \mid \|\hat{\theta}(t)\| > c > \max\|\theta_*\|\} \end{cases} \tag{A.13}$$

Hence,

$$\langle e, v\rangle_I = \langle e, v\rangle_{I_1} + \langle e, v\rangle_{I_2} \tag{A.14}$$

Thus,

$$\langle e, v\rangle_{I_1} = \langle \dot{\theta}, \theta\rangle_{I_1} = \frac{1}{2}\|\theta(t_1)\|^2 - \frac{1}{2}\|\theta(t_0)\|^2 \tag{A.15}$$

$$\langle e, v\rangle_{I_2} = \langle \dot{\theta} + (1 - \|\hat{\theta}\|/c)^2 \hat{\theta}, \theta\rangle_{I_2} \tag{A.16}$$

$$= \frac{1}{2}\|\theta(t_2)\|^2 - \frac{1}{2}\|\theta(t_1)\|^2 + (1 - \|\hat{\theta}\|/c)^2 \langle \hat{\theta}, \theta\rangle_{I_2} \tag{A.17}$$

$$> \frac{1}{2}\|\theta(t_2)\|^2 - \frac{1}{2}\|\theta(t_1)\|^2 \tag{A.18}$$

because $\langle \hat{\theta}, \theta\rangle_{I_2} > 0$ from,

$$\begin{aligned}\hat{\theta}(t)'\,\theta(t) &= \hat{\theta}(t)'[\hat{\theta}(t)-\theta_*] \\ &= \|\hat{\theta}(t)\|^2 - \hat{\theta}(t)'\theta_* \\ &> \|\hat{\theta}(t)\|^2 - \|\hat{\theta}(t)\|c \\ &= \|\hat{\theta}(t)\|(\|\hat{\theta}(t)\| - c) > 0, \qquad \forall t \in I_2 .\end{aligned} \tag{A.19}$$

Thus,

$$\langle e,v\rangle_I > \frac{1}{2}\|\theta(t_2)\|^2 - \frac{1}{2}\|\theta(t_0)\|^2 \tag{A.20}$$

Repeating the above procedure recursively,we eventually conclude that $\langle e,v\rangle_T > -\frac{1}{2}\|\theta(0)\|^2$ as before (A.6), and hence, $G_1 e \mapsto v$ is passive. The results in (i) now repeat for adaptive law (3.16). This completes the proof of part(i).

Proof of Theorem 1A,Part (ii)

Theorem 1A, Part (ii) is essentially an L_∞-stability result. The method of proof requires the notion of "exponential weighting" which is a means to obtain L_∞-stability of a system from the L_2-stability of an exponentially weighted version of the system (see e.g., [19], Chapter 9). We require the following:

Definition: Given a real number α define the exponential weighting operator by

$$x^\alpha(t) := \varepsilon^{\alpha t}x(t) \tag{A.21}$$

Consider the system $y = Gu$. An exp. weighted version of this system is denoted by $y^\alpha := G^\alpha u^\alpha$. Note that if G is a convolution operator with transfer function G(s) then G^α is also a convolution operator with transfer function $G(s-\alpha)$. Thus, the corresponding exponentialy weighted error system corresponding is described by

$$
\begin{aligned}
e^{\alpha} &= e_{*}^{\alpha} - H_{ev}^{\alpha}\, v^{\alpha} \\
z^{\alpha} &= z_{*}^{\alpha} - H_{zv}^{\alpha}\, v^{\alpha} \\
v^{\alpha} &= Z'\theta^{\alpha} \\
\dot{\theta}^{\alpha} &= \alpha\theta^{\alpha} + Ze^{\alpha} - \rho(\hat{\theta})\hat{\theta}^{\alpha}
\end{aligned}
\tag{A.22}
$$

e

where $\alpha > 0$ such that

$$
H_{ev}^{\alpha} \in SPR_{+}^{m} \text{ and } H_{zv}^{\alpha} \in S_{o}^{k\times m} \tag{A.23}
$$

Using Theorem A-1, identify G_1 with $e^{\alpha} \to v^{\alpha}$ and G_2 with H_{ev}^{α}. Note that it is always possible to find some $\alpha > 0$ such that (A.23) holds. We now examine the passivity of G_1: $e^{\alpha} \to v^{\alpha}$. Thus,

$$
\begin{aligned}
\langle e^{\alpha}, v^{\alpha}\rangle_T &= \langle e^{\alpha}, Z'\theta^{\alpha}\rangle_T = \langle Ze^{\alpha}, \theta^{\alpha}\rangle_T \\
&= \langle \theta^{\alpha}, \dot{\theta}^{\alpha} - \alpha\theta^{\alpha} + \rho(\hat{\theta})\hat{\theta}^{\alpha}\rangle_T \\
&= \frac{1}{2}\varepsilon^{2\alpha T}\|\theta(T)\|^2 - \frac{1}{2}\|\theta(0)\|^2 + \langle\rho(\hat{\theta})\hat{\theta}^{\alpha}, \theta^{\alpha}\rangle_T - \alpha\|\theta^{\alpha}\|_{T2}^2 \\
&> \frac{1}{2}\varepsilon^{2\alpha T}\|\theta(T)\|^2 - \frac{1}{2}\|\theta(0)\|^2 - \alpha\|\theta^{\alpha}\|_{T2}^2
\end{aligned}
\tag{A.24}
$$

The last line follows from (A.19), hence, (A.24) holds with or without the retardation term in the adaptive law. At this point there are two possibilities: either $\theta \in L_{\infty}$ or $|\theta(t)| \to \infty$ as $t \to \infty$. If $\theta \in L_{\infty}$ then $\exists$ constant $c_o < \infty$ such that $\|\theta\|_{\infty} < c_o$. Then,

$$
\begin{aligned}
\langle e^{\alpha}, v^{\alpha}\rangle_T &> \frac{1}{2}\varepsilon^{2\alpha T}(\|\theta(T)\|^2\ c_o^2) - \frac{1}{2}\|\theta(o)\|^2 \\
&> -\frac{1}{2}\varepsilon^{2\alpha T}c_o^2 - \frac{1}{2}\|\theta(0)\|^2
\end{aligned}
\tag{A.25}
$$

If $|\theta(t)| \to \infty$ as $t \to \infty$ then it is always possible to select an arbitrarily large T such that $\|\theta(T)\| = \|\theta\|_{T\infty}$. Hence, for this T, (A.24) becomes,

$$
\begin{aligned}
\langle e^{\alpha}, v^{\alpha}\rangle_T &> \frac{1}{2}\varepsilon^{2\alpha T}(\|\theta(T)\|^2 - \|\theta\|_{T\infty}^2) - \frac{1}{2}\|\theta(0)\|^2 \\
&= -\frac{1}{2}\|\theta(0)\|^2
\end{aligned}
\tag{A.26}
$$

Thus, for some arbitrarily large T, (A.25) and (A.26) have the general form, i.e.,

$$\langle e^{\alpha}, v^{\alpha}\rangle_T \geq -c_1 \varepsilon^{2\alpha T} - c_2 := -c(\alpha T) \tag{A.27}$$

where c_1, c_2 are non-negative constants. Hence,

$$\varepsilon_1 = \delta_1, = 0, \ \alpha_1 = -c(\alpha T) \tag{A.28}$$

Since $G_2 = H^{\alpha}_{ev} \in SPR_+$, $\exists$ constants μ, $\gamma > 0$ such that

$$\begin{aligned} &\langle x, H^{\alpha}_{ev}\, x\rangle_T \geq \mu \| x\|^2_{T2} \\ &\| H^{\alpha}_{ev}\, x\|_{T2} \leq \gamma \, \| x\|_{T2} \end{aligned} \tag{A.29}$$

Then,

$$\varepsilon_2 = \mu, \ \delta_2 = \alpha_2 = 0 \tag{A.30}$$

Using (A.2), we get

$$\| v^{\alpha}\|_{T2} \leq \frac{1}{2\mu}[\| e^{\alpha}_{*}\|_{T2} + (\| e^{\alpha}_{*}\|^2_{T2} + 2\mu \ c(\alpha T))^{1/2}] \tag{A.31}$$

Since $e_* \in L_\infty$ by assumption,

$$\| e^{\alpha}_{*}\|_{T2} \leq \varepsilon^{\alpha T}(2\alpha)^{-1/2}\| e_{*}\|_{\infty} \tag{A.32}$$

Thus,

$$\| v^{\alpha}\|_{T2} \leq \frac{\varepsilon^{\alpha T}(2\alpha)^{-1/2}}{2\mu}[\| e_{*}\|_{\infty} + (\| e_{*}\|^2_{\infty} + 4\alpha \ \varepsilon^{-2\alpha T}\mu c(\alpha T))^{1/2}] \tag{A.33}$$

Since $H^{\alpha}_{zv} \in S_0^{kxm}$, we obtain

$$|\tilde{z}(T)| = | \int_0^T H_{zv}(T-\tau)v(\tau)d\tau| \tag{A.34}$$

$$= |e^{-\alpha T} \int_0^T H^{\alpha}_{zv}(T-\tau)v^{\alpha}(\tau)d\tau| \tag{A.35}$$

$$< e^{-\alpha T} \| H^{\alpha}_{zv}(\cdot) \|_1 \cdot \| v^{\alpha} \|_{T2} \tag{A.36}$$

where $H^{\alpha}_{zv}(t)$ is the impulse response matrix associated with H^{α}_{zv}. Substituting (A.33) and (A.27) into (A.36) and noting that $e^{-2\alpha T} c(\alpha T) < c_1 + c_2$, we obtain,

$$|\tilde{z}(T)| < \frac{(2\alpha)^{-1/2}}{\mu} \| H^{\alpha}_{zv}(\cdot) \|_1 \cdot \left[\| e_* \|_{\infty} + \left(\| e_* \|_{\infty}^2 + 4\alpha\mu(c_1+c_2) \right)^{1/2} \right] \tag{A.37}$$

Since the right hand side is independent of T, and since T can be selected to be arbitrarily large, it follows that $z \in L_{\infty}$. Assuming there is no retardation or persistent excitation, this completes the proof of (ii-a) to (ii-d).

Assume now that $z \in PE$, which is a noncontradictory assumption since we have already shown that $z \in L_{\infty}$. Hence,

$$\dot{\theta} = -Z\, H_{ev}\, Z'\, \theta + Ze_* \tag{A.38}$$

Since $z \in PE$, $H_{ev} \in SPR_+$ and $z, \epsilon_* \in L_{\infty}$, it follows from (2.15) that $(Ze_*, \theta(0)) \mapsto \theta$ is exp. stable, thus, $\theta, \dot{\theta} \in L_{\infty}$. The remaining results in (ii-e) follow immediately.

Suppose now that the adaptive law is given by (3.16). Then, we can write,

$$\begin{aligned}\dot{\hat{\theta}} &= Z\, e - \rho(\hat{\theta})\hat{\theta} = Z[e_* - H_{ev}Z'(\hat{\theta}-\theta_*)] - \rho(\hat{\theta})\hat{\theta} \\ &= w - Z\, H_{ev}Z'\, \hat{\theta} - \rho(\hat{\theta})\hat{\theta}\end{aligned} \tag{A.39}$$

where $w := Z\, e_* + Z\, H_{ev}Z'\theta_* \in L_{\infty}$, because $z, e_* \in L_{\infty}$. Consider the candidate Lyapunov function $V: t \mapsto \|\hat{\theta}(t)\|^2$. Hence,

$$\dot{V} = 2\, w'\hat{\theta} - \hat{\theta}'\, Z\, H_{ev}Z'\hat{\theta} - \rho(\hat{\theta})V \tag{A.40}$$

Suppose $\|\hat{\theta}(t)\| \to \infty$ as $t \to \infty$. Then there exists a time $T > 0$ such that

$\|\hat{\theta}(T)\| = \|\hat{\theta}\|_{T\infty} = V_T^{1/2} > c$. Hence,

$$\dot{V}_T < 2\|w\|_\infty V_T^{1/2} + \|z\|_\infty^2 \gamma_\infty(H_{ev})V_T - (1 - V_T^{1/2}/c)^2 V_T \tag{A.41}$$

Clearly, there exists a finite constant c_1 such that when $V_T > c_1$, $\dot{V}_T < 0$. Therefore, $\hat{\theta}$ can not grow beyond all bounds, and hence, $\hat{\theta} \in L_\infty$. So then is θ and $\dot{\theta}$, and again the result of (ii-e) follow. This completes the proof of Theorem 1A. Note that in this case we do not obtain specific bounds on e, because the proof proceeds by contradition.

Proof of Theorem 1B

Part (i)

Since $H_{ev} \in SPR_0$, there exists $q > 0$ such that $G_{ev} := (1 + qs)H_{ev} \in SPR_+$, and furthermore, $G_{ev}^{-1} \in S$. As a result we can write (3.14a) as,

$$e = -H_{ev}\, y,\; y = v - G_{ev}^{-1}(e_* + q\, \dot{e}_*) \tag{A.42}$$

Referring to Lemma A-1, let $G_1 : v \mapsto e$, $G_2 = H_{ev}$, $u_1 = 0$, and $u_2 = -G_{ev}^{-1}(e_* + q\dot{e}_*)$. Using (A.2) together with (A.42) and the passivity properties of H_{ev} gives,

$$\|e\|_{T2} < \frac{1}{2\mu}[\|u_2\|_{T2} + (\|u_2\|_{T2}^2 + 2\mu|\theta(0)|^2)^{1/2}] \tag{A.43}$$

$$|\theta(T)| < |\theta(0)| + 2\|e\|_{T2} \cdot \|u_2\|_{T2} \tag{A.44}$$

where μ is defined in (4.9a). Using (4.9b) gives, $\|u_2\|_{T2} < (1/k)\|e_* + q\dot{e}_*\|_{T2}$. This together with (A.43), (A.44) and the assumption e_*, $\dot{e}_* \in L_2$ gives the bounds shown in (4.9). Hence, $e \in L_2$, $\theta \in L_\infty$. However, we can not conclude that $v \in L_2$ as in Theorem 1A, part (i). From (A.42), we can conclude that $(1 + qs)^{-1}\, v \in L_2$. Since $\underline{G}_{zv} := (1 + qs)H_{zv} \in S_{0\ell}$, it follows from Lemma A-2 that $z := z - z_* \in L_2 \cap L_\infty$, $\dot{z} \in L_2$ and $\tilde{z} \to 0$. Repeated use of Lemma A-2 and the error equations (3.14) gives the results (i-a) - (i-d). (i-e) follows from the arguments in the proof of Theorem 1A, part (i).

Part (ii)

The proof is entirely analgous to that of Theorem 1A, part (ii), where again we use exponential weighting.

APPENDIX B

PROOF OF LEMMA 5.1

The proof utilizes the following known results:

Definition: Let J denote a subset of S, consisting of functions in S whose inverse is also in S.

Fact [29]: If G is any scalar transfer function in R(s), then G has a coprime factorization in S, i.e., there exists N, D, A, and B in S such that $G = N/D$ and $AN + BD = 1$.

Lemma B-1: Consider the tuned adaptive system of Figure 5.2. Let $P_* \in R_0(s)$ and $C_* \in R_0(s)$ have coprime factorizations in S given by $P_* = N_p/D_p$ and $C_* = N_c/D_c$, respectively. Then, the elements of the transfer matrix from (r,d) into (e_*, z_*, y, u) all belong to S, if:

$$\text{(i)} \quad Q := D_pD_c + N_pN_c \in J , \qquad \text{(from [29])} \tag{B.1}$$

and

$$\text{(ii)} \quad \delta(\omega)|T_*(j\omega)| < 1, \ \forall\ \omega \in R , \qquad \text{(from [16])}$$

where

$$T_* := N_pN_c/Q := P_*C_*(1+P_*C_*)^{-1} \tag{B.2}$$

Using the definition of Q we can write H_{ev} and H_{zv} from (5.5) as,

$$H_{ev} = N_pQ^{-1}(1+\Delta)(1+\Delta T_*)^{-1} \tag{B.3}$$

$$H_{zv} = \begin{bmatrix} FD_pQ^{-1}(1+\Delta T_*)^{-1} \\ FN_pQ^{-1}(1+\Delta)(1+\Delta T_*)^{-1} \end{bmatrix} \tag{B.4}$$

From the definition of K_* (5.4b), we also obtain

$$Q = N_p K_*^{-1} \tag{B.3}$$

Proof of Lemma 5.1

We first show that (i), (ii), and (iv) => $Q \in J$. Let $P_* = N_p/D_p$ be a coprime factorization of P* such that rel deg $D_p(s) = 0$. Since (i) => rel deg $P_*(s) = 1$, it follows that rel deg $N_p(s) = 1$. Moreover, (iv) => rel deg $K_*(s) = 1$, and that $K_1(s)$ and $K_2(s)$ are stable. This, together with (ii) and (B.3) establishes that $Q \in J$.

$H_{zv} \in S_o$ follows immediately by inspection of (B.2), since: $F \in S_o$ by assumption; $D_p, N_p \in S$; $Q \in J$; $\Delta \in S$ by assumption (vi); and finally (vi) => (ii) of Lemma B-1 => $(1+\Delta T_*)^{-1} \in S$.

Conditions (iv) and (vi) => $H_{ev} \in SPR_o$. This follows from Lemma 4.1 by letting $\bar{H}_{ev} = K_*$ and letting $1 + \bar{H}_{ev} = (1+\Delta)(1+\Delta T_*)^{-1}$. Thus, (4.4a) is satisfied since $K_* \in SPR_o$ from (iv). Also, from (4.4b),

$$k(\omega) = |\bar{H}_{ev}(j\omega)| = |\Delta(j\omega)S_*(j\omega)[1-\Delta(j\omega)T_*(j\omega)]^{-1}| \tag{B.4}$$

$$< \frac{\delta(\omega)|S_*(j\omega)|}{1-\delta(\omega)|T_*(j\omega)|} < \bar{k}(\omega) = \eta(\omega) \tag{B.5}$$

The last inequality comes from conditions (vi) and the definition of $\bar{k}(\omega)$ from (4.4b).

The final step in the proof of Lemma 5.1 is to show that there are a sufficient number of parameters in θ_* to insure a solution exists. This is guaranteed by satisfaction of condition (v). To see this combine (B.3) with the definition of Q from (B.1) to get

$$Q := N_c N_p + D_p D_c = N_p K_*^{-1} \tag{B.6}$$

From (5.2), let $N_c = A_{*2}/L$ and $D_c = 1 + A_{*1/L}$ be a coprime factorization of $C*$, and let $N_p = g\, N*/L$ and $D_p = 1 + D_*/L$ be a coprime factorization of P_*, where P_* is as defined in (i). With $K*$ given by (iv), (B.6) becomes the polynomial equation,

$$A_{*1}\, K_1 D_* + A_{*2} K_1 N_* = L(K_2 N_* - K_1 D_*) \tag{B.7}$$

Since $\deg(K_2 N_*) = \deg(K_1 D_*)$ and K_1, K_2, N_*, and D_* are all monic, it follows that $\deg[L(K_2 N_* - K_1 D_*)] = \deg(L) + \deg(K_1) + \deg(D_*) - 1$. Then, using known results on polynomial equations, e.g. [30], it can be shown that (v) implies that (B.7) has a solution (A_{*1}, A_{*2}) .

REFERENCES

[1] R. Bellman, Adaptive Control Processes: A Guided Tour, Princeton, 1961.

[2] P. C. Parks, "Liapunov Redesign of Model Reference Adaptive Control Systems," IEEE Trans. Aut. Contr., vol. AC-11, July 1966, pp. 362-367.

[3] I. D. Landau, "A Hyperstability Criterion for Model Reference Adaptive Control Systems," IEEE Trans. Aut. Contr., Vol. AC-14, No. 5, Oct. 1969, pp. 552-555.

[4] R. V. Monopoli, "Model Reference Adaptive Control with an Augumented Error Signal," IEEE Trans. Aut. Contr., Vol. Ac-19, Oct. 1974, pp. 474-484.

[5] K. J. Astrom and B. Wittenmark, "On Self Tuning Regulators," Automatica, Vol. 9, pp. 185-199, 1973.

[6] L. Ljung, "On Positive Real Transfer Functions and the Convergence of Some Recursive Schemes," IEEE Trans. Aut. Contr., Vol. AC-22, No. 4, Aug. 1977, pp. 539-551.

[7] K.S. Narendra, L.S. Valavani, "Direct and Indirect Model Reference Adaptive Control," Automatica, Vol. 15, pp. 653-664, 1979.

[8] Y.D. Landau, Adaptive Control: The Model Reference Approach, Dekker, NY, 1979.

[9] G.C. Goodwin, P.J. Ramadge, and P.E. Caines, "Discrete Time Mutivariable Adaptive Control," IEEE Trans. Aut. Contr. , Vol. AC-25, June 1980, pp. 449-456.

[10] G. Kreisselmeier, "On Adaptive State Regulation," Vol. AC-27, No. 1, Feb. 1982, pp. 3-17.

[11] C.E. Rohrs, L. Valavani, M.Athans, and G. Stein, "Analytical Verification of Undesirble Properties of Direct Model Reference Adaptive Contrl Algorithms," Proc. 20th IEEE CDC, pp. 1272-1284, Dec. 1981,San Diego, CA.

[12] __________, "Robustness of Adaptive Control Algorithms in the Presence of Unmodeled Dynamics," Proc. 21st IEEE Conference on Decision and Control, Orlando, Florida, Dec. 1982, pp.

[13] B.D.O. Anderson and C.R. Johnson, Jr., "On Reduced Order Adaptive Output Error Identification and Adaptive IIR Filtering," IEEE Trans. Aut. Contr., Vol. AC-27, No.4, pp. 927-932, Aug. 1982.

[14] I. Horowitz, Synthesis of Feedback Systems, NY, Academic, 1963.

[15] M.G. Safonov, Stability Robustness of Multivariable Feedback Systems, MIT Press, 1980.

[16] J.C. Doyle and G. Stein, "Multivariable Feedback Design: Concepts for a

Modern/Classical Synthesis," IEEE Trans. Aut. Contr., Vol. AC-26, No. 1, pp. 4-17, Feb. 1981.

[17] G. Zames and B. A. Francis, "A New Approach to Classical Frequency Methods: Feedback and Minimax Sensitivity," Proc. 20th CDC, pp. 867-874, Dec. 1981, San Diego, CA.

[18] G. Zames, "On the Input-Output Stability of Time-Varying Nonlinear Feedback Systems," IEEE Trans. on Aut. Contr., Part I: Vol. AC-11, No. 2, pp. 228-238, April 1966; Part II: Vol. AC-11, No. 3, pp. 465-476, July 1966.

[19] C.A. Desoer, M. Vidyasager, Feedback Systems: Input-Output Properties, Academic Press, New York, 1975.

[20] B.D.O. Anderson, "A Simplified Viewpoint of Hyperstability," IEEE Trans. Aut. Contr., Vol. AC-13, pp. 292-294, June, 1968.

[21] D.J. Hill and P.J. Moylan, "Dissipative Dynamical Systems: Basic Input-Output and State Properties," J. Franklin Inst., Vol. 309, No. 5, 1980.

[22] F. Donati and M. Vallauri, "Guaranteed Control of Almost Linear Plants," IEEE Trans. Aut. Contr, Vol. AC-29, No. 1, January 1984.

[23] M. Vidyasager, H. Schneider, and B.A. Francis, "Algebraic and Topological Aspects of Feedback Stabilization," IEEE Trans. Aut. Contr., Vol. AC-27, No.4, pp. 880-894, August 1982.

[24] G. Kreisselmeier and K.S. Narendra, "Stable Model Reference Adaptive Control in the Presence of Bounded Disturbances," IEEE Trans. Aut. Contr, Vol. AC-27, No. 6, pp 1169-1175, Dec 1982.

[25] M.G. Safonov, "Tight Bounds on the Response of Multivariable Systems with Component Uncertainty," Proc. Allerton Conf. on Comm., Contr., and Comput., Monticello, Ill, October, 1978.

[26] I.P. Landau, "Elimination of the Real Positivity Condition in the Design of Parallel MRAS," IEEE Trans. Aut. Contr., Vol. AC-23, No. 6, pp. 1015-1020, Dec. 1978.

[27] B. Egardt, Stability of Adaptive Controllers, Springer-Verlag, 1979.

[28] R.L. Kosut and B. Friedlander, "Performance Robustness Properties of Adaptive Control Systems," 21st IEEE Conference on Dec. and Control, pp 18-23, Orlando, Florida, Dec. 1982.

[29] C.A. Desoer, R.W. Liu, J. Murray, and R. Saeks, "Feedback System Design: The Fractional Representation Approach to Analysis and Synthesis", IEEE Trans. Aut. Contr., Vol. AC-25, No. 3, pp 399-412, June 1980.

[30] V. Kucera, Discrete Linear Control: The Polynomial Equation Approach, Wiley-Interscience, NY.

[31] B.D.O. Anderson, "Exponential Stability of Linear Equations Arising in

[31] B.D.O. Anderson, "Exponential Stability of Linear Equations Arising in Adaptive Identification," IEEE Trans. Aut. Contr., Vol. AC-22, pp. 83-88, February 1977.

[32] B. Peterson and K.S. Narendra, "Bounded Error Adaptive Control," IEEE Trans. Aut. Contr., Vol. AC-27, December 1982.

[33] P. Ioannou and P. Kokotovic, Adaptive Systems with Reduced Models, Springer-Verlag, 1983.

[34] R.L. Kosut and B.D.O. Anderson, "Robust Adaptive Control: Conditions for Local Stability," IEEE Trans. Aut. Contr., submitted, and Proc. 23rd CDC, Las Vegas, Nevada, December 1984.

[35] M. Vidyasager, Input-Output Analysis of Large-Scale Interconnected Systems, Springer-Verlag, 1981.

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