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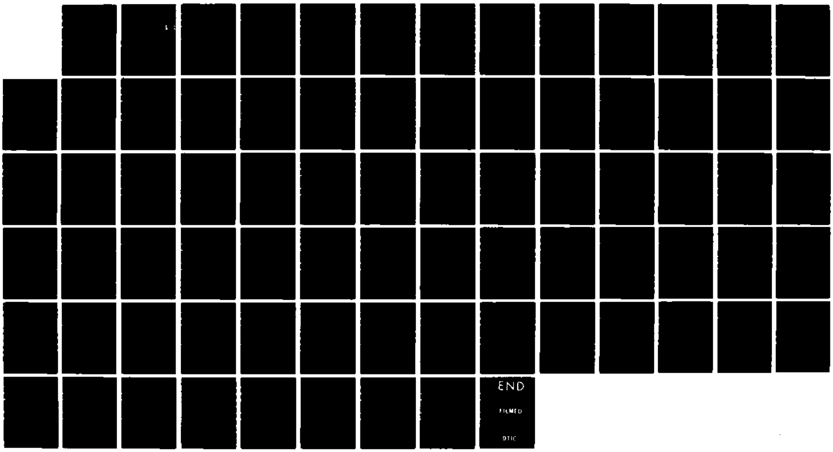

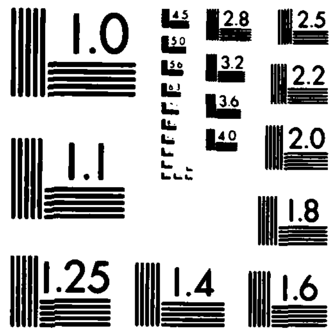


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In the introduction an overview is given of the status of 1/f noise at the end of the contract period. We indicate that the 1/f noise as observed in our laboratory, as well as in other places, quite often yields Hooge parameters of order $10^{-5}$ to $10^{-8}$, i.e., two to five orders less than a decade ago. Much of this noise can be seen as quantum 1/f noise which is the limiting noise that can be observed. Our measurements on submicron gallium arsenide devices, microwave narrow base transistors, and gold films at below the Debye temperature, and in radioactive decay and partition 1/f noise, all point in this direction. Over and above this limiting noise, additional 1/f-like noise is often observed. For this, the standard physical mechanisms, involving activation energy processes or tunneling processes, usually apply. More research on this larger 1/f noise is still needed.

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19. Abstract

In the section on experimental work, we discuss the status of 1/f noise in transistors and in gold films. Also, we discuss the high frequency intervalley scattering noise in gallium arsenide devices. Good agreement with Monte Carlo simulations is obtained. New results are also presented for noise in radioactive decay. Both 1/f noise and Lorentzian flicker noise are observed. The flicker floor is lower for lower α-particle energies, in agreement with the quantum theory of 1/f noise.

In the theory section the theory of quantum 1/f noise is applied to electron-phonon scattering. Explicit results for the resulting mobility-fluctuation noise and for the Hooge parameters involved are obtained. Numerical computations are in progress.

# Study of 1/f Noise in Solids

FINAL REPORT

to the

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## Abstract

In the introduction an overview is given of the status of 1/f noise at the end of this five-year contract period. We indicate that the 1/f noise as observed in our laboratory, as well as in other places, quite often yields Hooge parameters of order $10^{-5}$ to $10^{-8}$, i.e., two to five orders less than a decade ago. Much of this noise can be seen as quantum 1/f noise which is the limiting noise that can be observed. Our measurements on submicron gallium arsenide devices, microwave narrow base transistors, and gold films at below the Debye temperature, and in radioactive decay and partition 1/f noise, all point in this direction. Over and above this limiting noise, additional 1/f-like noise is often observed. For this, the standard physical mechanisms, involving activation energy processes or tunneling processes, usually apply. More research on this larger 1/f noise is still needed.

In the section on experimental work, we discuss the status of 1/f noise in transistors and in gold films. Also, we discuss the high frequency inter-valley scattering noise in gallium arsenide devices. Good agreement with Monte Carlo simulations is obtained. New results are also presented for noise in radioactive decay. Both 1/f noise and Lorentzian flicker noise are observed. The flicker floor is lower for lower α-particle energies, in agreement with the quantum theory of 1/f noise

In the theory section the theory of quantum 1/f noise is applied to electron-phonon scattering. Explicit results for the resulting mobility-fluctuation noise and for the Hooge parameters involved are obtained. Numerical computations are in progress.

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## A. Introduction and Summary of Work under Contract

A long introduction and summary of the status of 1/f noise in solids and solid state devices was given in the proposal for the first year of this grant, submitted in 1978. At that time we indicated that we were hopeful that a breakthrough in the 1/f noise enigma could be reached if a number of proper experiments were carried out, together with new theoretical developments. We believe that after five years of work on this project such a state has been reached. Noise measurements in transistors, in metal films, in gallium arsenide submicron devices, in radioactive decay, and of partition noise in pentodes have provided much insight into the problem.

The basic behavior of 1/f noise is contained in Hooge's law [1],

$$S_I(f)/I^2 = \alpha_H/fN \qquad (1)$$

where I is the average current, f the frequency, N the number of carriers involved, and $\alpha_H$ is Hooge's parameter. Originally, Hooge's parameter was thought to be a "universal" constant of magnitude $2 \times 10^{-3}$. Subsequent measurements (1970-1984) in our laboratory, as well as elsewhere [2], have indicated, however, that $\alpha_H$ is often very much smaller. Measurements by Kilmer on microwave transistors indicated values of order $10^{-7}$ [3]. In metal films, when the appropriate correction due to degeneracy is made, values of $10^{-6}$ to $10^{-7}$ were found at temperatures below the Debye temperature [4]. In GaAs submicron devices, values as low as $6 \times 10^{-8}$ were obtained [5]. All of this indicates that a considerable reduction in 1/f noise has been obtained in the last decade. Most significantly, this low 1/f noise now indicates that the fundamental limit, set by the quantum theory of 1/f noise proposed by Handel in 1975 [6,7], has now been reached. Thus, much of present-day 1/f noise work can be explained in terms of quantum 1/f noise. Explicit quantitative results of this theory for mobility fluctuations, caused by electron-phonon scattering, are, however, only recently forthcoming. Van der Ziel and Handel have recently proposed a simplified theory, involving both normal and Umklapp processes [8], which leads to quantitative results. A detailed theory, capable of explaining Hooge parameters of order $10^{-4}$ or less, has been started under this grant by Kousik and Van Vliet. Some preliminary results appear in this report (Section C.-I). These results will probably be completed early in 1985. We will then be able to make a quantitative comparison between experiments involving conductivity fluctuations and quantum 1/f noise.

Another avenue to test the validity of this theory was provided by measuring emission fluctuations of two kinds. First, as a nonelectric phenomenon, we proposed in 1980 to measure the 1/f fluctuations in alpha-particle radioactive decay. The results by Jeng Gong [9] indicated the occurrence of 1/f noise, as manifested by a flicker floor in the Allan variance of radioactive counting. The magnitude which was observed was in fair agreement with the results predicted by Handel's theory. This work has also aroused interest in the atomic energy community, such as in the Center of Chalk River [10], since it now is apparent that the predicted flicker flow sets a limit in various nuclear counting experiments.

Secondly, as another verification of quantum 1/f noise, van der Ziel suggested the measurement of partition 1/f noise in pentodes. These measurements were performed by C.J. Hsieh [11]. A very good agreement with theory was obtained. Altogether, the above three areas--mobility fluctuations (or diffusivity fluctuations) in films and devices, the flicker floor in $\alpha$-particle radioactive decay, and partition 1/f noise--indicate that quantum 1/f noise has been proven to exist, and most likely is the cause of limiting 1/f noise in various 1/f noise phenomena. We also mention that such noise can explain 1/f noise in phonon-assisted hopping processes in amorphous silicon, as shown in a very recent paper by d'Amico and Van Vliet [12].

The question now remains as to what causes the other observed 1/f noise with rather large Hooge parameter ($10^{-4}$ or higher) as observed in certain devices (long p-n diodes for example) and in metal or semiconductor films at room temperature or higher. First of all, we suggest that new experiments be performed to establish whether such noise is due to mobility or number fluctuations. Experiments on transistors, performed both in Florida and in Minnesota, have indicated that the current dependence for such noise is usually quadratic rather than linear, thus excluding mobility fluctuations. More experiments, possibly involving a repeat of Kleinpenning's experiments in the seventies of Hall effect and thermo power, should be performed. It is our surmise that this larger 1/f noise in all cases is due to number fluctuations. Mechanisms involving distributed tunnel ranges or distributed trapping energies, as suggested by various people in the fifties (see van der Ziel [13] and McWhorther [14]), are still the most likely candidates to explain such phenomena. The more precise nature of such physical mechanisms has in some cases been established (see, e.g., Dutta and Horn [15] for noise involving activated vacancy trapping in metal films at higher temperatures.

In conclusion, we believe that limiting 1/f noise in semiconductor devices is presently well understood. Over and above this noise, other 1/f-like phenomena can occur, the exact cause of which has been pinpointed in some occasions, while for others more study is definitely needed.

This report will focus on the various experiments, as well as on theoretical work, which were performed under the final year of this five-year grant period. We also emphasize that the study of submicron devices, as carried out under this grant by Schmidt et al. [5], has considerably aided in understanding the importance of mobility-fluctuation 1/f noise. Great thanks are due to the Cornell National Submicron Resource and Research Facilities (Drs. E. Wolf and L. Eastman) for providing the samples of this study.

## B. Experimental Work

### I. Study of 1/f Noise in Transistors (J. Kilmer)

The report of two years back (1981/1982) contained Kilmer's measurements on 1/f noise in transistors. It was shown that by proper choice of the source resistance, either the base current noise or the collector current noise could be isolated. The Hooge parameters for these noises were determined and, especially for the base current noise, it was found to be extremely low. It was not recognized at that time that a measurement of the current dependence of the noise could exclusively indicate whether the noise stems from mobility fluctuations (alias diffusivity fluctuations) or from other causes. In the former case, the dependence should be linear. Below we reproduce a recent paper by Kilmer, van der Ziel, and Bosman, reporting such measurements.

# Mobility Fluctuation 1/f Noise in Silicon $p^+$-n-p Transistors

by

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Summary

A study of the current dependence of base 1/f noise and of collector 1/f noise in $p^+$-n-p transistors shows that the former is most likely of the mobility fluctuation 1/f noise type and that the latter is most probably not of that type. The current dependence of 1/f noise in transistors is a powerful tool in the interpretation of the noise.

In recent papers[1,2] we showed that the low frequency 1/f noise in low-noise $p^{+}$-n-p transistors (GE 82-185) could be represented as mobility fluctuation 1/f noise. We found, respectively, for the Hooge parameter of the collector noise $\alpha_H = 1.92 \times 10^{-6}$ and for the Hooge parameter of the base noise $\alpha_H = 9.3 \times 10^{-8}$, and attributed these small values of $\alpha_H$ to the decrease in $\alpha_H$ at high doping

$$\alpha_H = \alpha_o \left(\frac{\mu}{\mu_{latt}}\right)^2 \qquad (1)$$

where $\mu_{latt}$ is the mobility due to phonon scattering.

But the theory of Kleinpenning[3] and van der Ziel[4] also predicts a specific <u>current</u> dependence. For the collector noise

$$S_{ifc}(f) = S_{I_{Ep}}(f) = 2qI_{Ep} \frac{\alpha_p}{4f\tau_{dp}} \ln \left(\frac{P(0)}{P(w_B)}\right) \qquad (2)$$

where $\alpha_p$ is the Hooge parameter for holes, $I_{Ep}$ the emitter current, $\tau_{dp} = w_B{}^2/2D_p$ is the diffusion time for holes through the base region, $w_B$ is the base width, $D_p$ the diffusion constant for holes and $P(0)$ and $P(w_B)$ are the hole concentrations for unit length at the emitter side and at the collector side of the base, respectively; here $I_{Ep} = I_c$ is the collector current. Moreover, for electron injection from base to emitter we have

$$S_{ifb}(f) = S_{I_{En}}(f) = 2qI_{En}\frac{\alpha_n}{4f\tau_{dn}}\ln\left(\frac{N(0)}{N(w_E)}\right) \qquad (3)$$

where $\alpha_n$ is the Hooge parameter for electrons, $\tau_{dn} = w_E^2/2D_n$, $w_E$ the width of the emitter region, $D_n$ the diffusion constant for electrons, whereas N(0) and $N(w_E)$ are the electron concentrations for unit length at the base side of the emitter and at the emitter contact, respectively; here $I_{En} = I_B$ is the base current. Neglecting the slow dependence of $P(0)/P(w_B)$ and of $N(0)/N(w_E)$ upon the current, $S_{I_{Ep}}(f)$ and $S_{I_{En}}(f)$ are proportional to $I_{Ep}$ and $I_{En}$ respectively.

Measurements by Zhu on NEC 57807 $n^+$-p-n microwave transistors[5] gave a different current dependence, indicating that the 1/f noise in these devices was probably not of the mobility fluctuation type. It therefore seemed worthwhile to investigate the current dependence of the 1/f noise in GE 82-185 transistors.

The results at 1 Hz are shown in Figs. 1 and 2 for $S_{ifb}(f)$ and $S_{ifc}(f)$, respectively. We see that $S_{ifb}(f)$ varies as $I_B$ to the power (1.14 ± 0.07), in agreement with (3), whereas $S_{ifc}$ varies as $I_c$ to the power (1.7 ± 0.3). We therefore conclude that $S_{ifb}(f)$ is most likely caused by mobility-fluctuation 1/f noise and that $S_{ifc}(f)$ is of a different origin. This seems to be corroborated by the fact that $S_{ifb}(f)$ differs little from transistor to transistor, as we observed by measuring the noise of three different devices. This is expected for fundamental 1/f noise mechanism described by (3). $S_{ifc}(f)$ differed strongly from unit to unit, as expected for a less fundamental noise mechanism than the one described by (2).

The authors are indebted to Mr. X.C. Zhu, University of Minnesota, for making his data available to us. The work was supported by the Air Force Office of Scientific Research under grant No. 82-0226.

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II. <u>1/f Noise in Gold Metal Films</u> (J. Kilmer)

The previous report (1982-1983) contained Kilmer's measurements on gold metal films presented at the Montpellier conference. The results have since been published in the open literature [4]. At that time, however, no correction was made for the temperature change due to heating by the d.c. current. The corrected results for a "quiet" 1 μm device are shown in Fig. 3. The top part of the figure indicates the relative noise, $S_I/I^2$. The bottom part gives the slope γ of the noise $(S_I/I^2 = \alpha_H/f^{\gamma}N_{actual})$. We note that due to heating it was not possible to measure below 50K. However, the trend of the temperature dependence of the noise is the same as previously reported. Below the Debye temperature (165K for gold) the noise changes in character and is most likely caused by quantum 1/f noise. The maximum occurs close to the Umklapp temperature (about 60K for gold). At low temperatures the noise decreases since Umklapp processes can no longer be excited. The deduced Hooge parameter is shown in Fig. 4. We note that the Hooge parameter is very low. It is in reasonable harmony with the value computed from the theory of quantum 1/f noise [4].

Other devices showed that the noise went up again at low temperatures, see Figs. 5 and 6. These devices were more noisy and have probably not reached the quantum 1/f noise limit.

A complete quantitative comparison with theory has to wait until G. Kousik's computations are completed, see Section C.-I.

III. <u>Intervalley Noise in GaAs Mesa Structures</u> (J. Andrian)

High frequency noise at 11 MHz and 17 MHz was measured in GaAs diodes of 1.1 μm contact spacing under pulsed conditions. In this frequency range the noise is flat, being due to velocity-fluctuation noise, or diffusion noise.

The latter is a function of the electric field E, and is caused by intervalley scattering. The noise source for such a process is given by, see Van Vliet [16],

$$S_H(x,x',j\omega) = 4q^2\langle n(x)\rangle D(E)\delta(x - x') \quad (2)$$

where D(E) is the diffusion coefficient defined by the Fourier-Laplace fluctuation correlation function

$$D(E) = \int_0^\infty dte^{-j\omega t}\langle\Delta v(t)\Delta v/0)\rangle \quad (3)$$

(we neglect the quantum correction factor, i.e., $h\omega \ll kT$, see [16]). Though at high fields far from thermal equilibrium no Einstein relation is valid (contrary to a claim by Nougier and Rolland [17], and earlier claims by Thornber [18]), we can <u>define</u> a noise temperature by

$$qD(E) = \mu^* kT_n \quad (4)$$

where $\mu^*$ is the differential mobility. From integration of the Langevin equation

$$\Delta J_n = qn\Delta[\mu_n(E)E] + q\Delta\left[D_n(E)\frac{dn}{dx}\right] + H(x,t) \quad (5)$$

it can now be shown [19] that the terminal noise becomes

$$S_{V_d} = \mathrm{Re}\int_0^L\int_0^L S_E(x,x',j\omega)dxdx' = 4kT_n r_d \Delta f \quad (6)$$

where $r_d$ is the differential resistance. Accordingly, from a measurement of $S_{V_d}$, $T_n$, and hence D(E), can be determined. The details are described in a paper by Andrian, Bosman, and Van Vliet, produced under the grant. This paper, which will shortly be submitted to Solid State Electronics in a slightly altered form (accounting for the inhomogeneity of E), follows below.

# Hot Electron Diffusion Noise Associated with Intervalley Scattering in Very Short GaAs Devices

by

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## Abstract

Noise measurements at 11 MHz and 17 MHz are reported for $N^+N^-N^+$ GaAs devices with an $N^-$ region thickness of 1.1 μm. The devices were fabricated by molecular beam epitaxy at Cornell University. The longitudinal diffusion coefficient calculated from these noise measurements is compared with that obtained by Monte Carlo simulation. A new method for high-frequency noise measurements of low impedance ($< 1\Omega$) devices is thoroughly discussed.

## I. Introduction

The progress in semiconductor device technology has enabled the industry to fabricate devices of ever-decreasing size. In small devices it is very important to understand how the transport coefficients, like mobility and diffusion coefficients, depend on the electric field.

Relatively low-bias voltages can easily produce high electric fields inside these small devices. Those coefficients are needed to evaluate the operating conditions of a device. In addition, the diffusion coefficient provides a measure of the velocity fluctuations and their contribution to the electrical noise in the device itself.

In the case of GaAs, many authors [1-3] have published theoretical results (analytical or computer simulations) calculated on the electric field dependence of the mobility and the diffusion coefficients, but except for the classic work of Ruch and Kino [4], little has been done on the experimental study of high field transport coefficients [5,6].

In this paper we present experimentl results for the longitudinal diffusion coefficient using noise measurements.

This method and the results will be described in Section II. In Section III results obtained using Monte Carlo simulations are given. The last section is devoted to discussions and comparisons between our experimental data and theoreticl computations.

## II. Device Description an Experimental Setup

The measurements have been performed on $N^{+}N^{-}N^{+}$ GaAs mesa structures. The active region ($N^{-}$ layer) has a thickness of 1.1 µm, a cross-sectional area of $7.8 \times 10^{-5} cm^{2}$ and a Si doping of $10^{15} cm^{-3}$. The $N^{+}$ layers are needed to provide good ohmic contacts at both ends of the device. The block diagram of the experimental setup is shown in Fig. 7. A pulsed bias method was used to avoid excessive Joule heating and possible destruction of the device at high electric fields. The pulse generator is synchronized with a switch at the input of the power detector so that one only measures the noise when the bias is applied.

Bareikis et al. [6] have used this method before; however, a detailed description of the step-up transformer at the input of the amplifier is given in this paper. This network is very important because the device has a very low impedance (< 1Ω) compared to the 50Ω input impedance of the amplifier. As a consequence, the device noise would drown in the amplifier noise without a step-up network.

## II.a. Step-up transformer

Since we have an amplifier with a noise equivalent resistance of 60Ω and a device with a resistance less than 1Ω, we need a tuned circuit to increase the resistance seen by the amplifier. The noise equivalent circuit is shown in Fig. 8, where r is the series resistance of the inductor, $r_d$ the device small signal resistance, and $\sqrt{4kTr\Delta f}$ and $\sqrt{4kTr_d\Delta f}$ (k is the Boltzmann constant) are the thermal noise sources associated with r and $r_d$ respectively.

The impedance at the terminals is given by

$$Z = \frac{j\omega L + r + r_d}{1 - \omega^2 CL + j\omega C(r+r_d)} \tag{1}$$

If we operate at the tuning frequency $\omega_0 = \frac{1}{\sqrt{LC}}$ and if we choose $\omega_0$ so that $r + r_d \ll |j\omega_0 L|$, then

$$Z \approx \frac{L}{C} \cdot \frac{1}{(r+r_d)} . \tag{2}$$

The spectral density S of the A.C. open-circuited voltage fluctuations at the output of the network is

$$S = 4k(T_0 r + T_d r_d) \left| \frac{\frac{1}{j\omega_0 C}}{\frac{1}{j\omega_0 C} + r + r_d + j\omega_0 L} \right|^2 \tag{3}$$

where $T_0$ is the inductor temperature and $T_d$ the device temperature. Carrying out the algebra and again using the fact that $\omega_0^2 CL = 1$, equation (3) becomes:

$$S = 4k(T_0 r + T_d r_d) \frac{L}{C} \frac{1}{(r+r_d)^2} . \tag{4}$$

The condition for "up" transformation of the noise equivalent resistance of the device is easily derived from equation (4) if we neglect r. Then S becomes

$$S = 4kT_d \frac{L}{Cr_d} . \tag{5}$$

"Up" transformation takes place if

$$\frac{L}{Cr_d} > r_d \quad \text{or} \quad \frac{L}{C} > r_d^2 . \tag{6}$$

For the series resonance step-up transformer, it holds that

$$\frac{L}{C} = r_d^2 Q^2 \tag{7}$$

where Q is the quality factor of the transformer. Hence, condition (6) is fulfilled as long as $Q > 1$. However, a high Q circuit has a narrow passband; therefore, we had to build transformers for each frequency of interest (11 and 17 MHz).

II.b. Determination of the series resistance of the inductor

If we replace the device by a short circuit, we obtain

$$S_1 = 4kT_0 \frac{L}{Cr} \tag{8}$$

Next, a known resistance ($r_0$) is used. Equation (4) gives

$$S_2 = 4kT_0 \frac{L}{C(r+r_0)}. \tag{9}$$

Equations (8) and (9) yield

$$\frac{S_1}{S_2} = 1 + \frac{r_0}{r} . \tag{10}$$

From equation (10) one obtains the value of the series resistance r of the inductor at the tuning frequency.

II.c. Determination of the device resistance with noise measurement

The actual device is then placed in series with the inductor. In thermal equilibrium with both the device and the inductor at room temperature $T_0$, we find

$$S_3 = 4kT_0 \frac{L}{C} \frac{1}{(r+r_d)} . \tag{11}$$

Equations (9) and (11) give:

$$\frac{S_2}{S_3} = \frac{r + r_d}{r + r_0} . \tag{12}$$

Every quantity in (12) is known except $r_d$. From the noise measurements we found $r_d = (.65 \pm .06)\Omega$ . A measurement using a DC ohm-meter gave $r_d = .58\Omega$ . The difference between these two results can be attributed to the skin effect in the thin copper wires of the inductor, which increses the resistance of the wires at higher frequencies.

II.d. Noise measurement with bias applied

All previous calculations are valid under thermal equilibrium conditions. However, we are interested in the high field behavior of the noise so we need to include a biasing circuit in the previous model. The circuit is shown in Fig. 9 and the noise equivalent circuit in Fig. 10. Let

$$v_s = \sqrt{4kT_0 R_s \Delta f} \ , \quad v_d = \sqrt{4kT_d r_d \Delta f} \ , \text{ and } \quad v_r = \sqrt{4kT_0 r \Delta f} \ .$$

We take into account possible excess noise of the device by using the noise temperature $T_d$ in the expression for $v_d$. A simple analysis of the circuit gives for the voltage fluctuations at the A.C. open-circuited doutput terminals at resonance frequency $\omega_0$

$$V = \frac{[r_d(v_s - v_r) + R_s(v_d - v_r)]}{j\omega_0 C(R_s r + R_s r_d + r r_d)} \ . \qquad (13)$$

Since the noise sources are not correlated, the mean cross-products $\overline{v_s v_d}$, $\overline{v_s v_r}$, and $\overline{v_d v_r}$ vanish. Hence we obtain

$$S(\omega_0) = \frac{L}{C} \frac{r_d^2 (S_{v_r} + S_{v_s})}{(R_s r + R_s r_d + r_d r)^2} + \frac{L}{C} \frac{R_s^2 (S_{v_d} + S_{v_r})}{(R_s r + R_s r_d + r_d r)^2}$$

$$+ \frac{L}{C} \frac{2 r_d R_s S_{v_r}}{(R_s r + R_s r_d + r_d r)^2} \ . \qquad (14)$$

Since $r_d \ll R_s$ and $r \ll R_s$ $(R_s \approx 50\Omega)$, we drop the terms with $r_d^2$ and $r_d R_s$ in the numerator. Equation (14) becomes,

$$S(\omega_0) = \frac{L}{C} \frac{1}{r_d^2} \frac{(S_{v_r} + S_{v_d})}{(1 + \frac{r}{r_d})^2} \qquad (15)$$

with $S_{v_r} = 4kT_0r$ and $S_{v_d} = 4kT_dr_d$. Equation (15) can be written as

$$S(\omega_0) = 4k\,\frac{L}{C}\,\frac{1}{r_d^2\left(1+\frac{r}{r_d}\right)^2}\,(T_0r+T_dr_d)\ . \qquad (16)$$

Equations (8) and (16) yield

$$\frac{S_1}{S} = \frac{r_d^2\left(1+\frac{r}{r_d}\right)^2T_0}{r(T_0r+T_dr_d)} \qquad (17)$$

or

$$T_dr_d + T_0r = \frac{S}{S_1}\,\frac{(r+r_d)^2T_0}{r}\ . \qquad (18)$$

We define the noise equivalent resistance of the device as

$$R_n = \frac{T_dr_d}{T_0}\ . \qquad (19)$$

From equation (18) we obtain

$$R_n = \frac{S}{S_1}\cdot\frac{(r+r_d)^2}{r} - r\ . \qquad (20)$$

In Figure 11 $R_n$ is plotted as function of the applied voltage for $\tau_0 = 300K$ and $f_0 = \frac{\omega_0}{2\pi} = 11.0$ MHz and $f_0 = 17.0$ MHz. We note that for bias voltages up to 0.2V the measured value of $R_n$ is equal to the small signal resistance of the device $r_d$, within the experimental error. For bias voltages above 0.2V, $R_n$ increases sharply with the voltage applied.

We define the noise temperature $T_d$ by [7] as $qD(E) = kT_d\mu_d$ where E is the mean d.c. field in the diode and $\mu_D$ is the differential mobility. It can then be shown that the terminal voltage noise is $4kT_n r_d \Delta f$; then $T_n \equiv T_d$. We now find for the diffusion coefficient

$$D(E) = \frac{kT_o}{q} \frac{R_n}{r_d} \mu_D \qquad (20a)$$

where we replaced $T_d$ by $\frac{R_n}{r_d} T_o$ (see equation (19). Measurements of the white noise and the current-voltage characteristic enable us to determine experimentally the diffusion coefficient as function of the applied electric field. In Figure 12 we plotted the diffusivity D(E) at 300K as following from Figure 11. In order to have some means of comparison with the theory, we performed computer simulations which we describe in the next section.

## III. Monte Carlo Simulation

To predict the high-field transport properties of a given semiconductor material, one has to solve the Boltzmann transport equation. This equation is very difficult to solve analytically unless drastic simplifications are introduced. Such simplifications, however, may alter very much the physical meaning of the solution.

To circumvent this difficulty, numerical techniques have been developed to obtain accurate results. One of these is the Monte Carlo simulation technique.

The principle of this method is to simulate on a computer the motion of one electron in the k-space through a large number of scattering processes. Excellent papers have been written on this subject by Fawcett et al. [1] and more recently by Boardman [2]. For more details we refer the reader to these articles.

Our simulation is based on the method developed by Boardman [4]. To explain the noise behavior of our device, we need to know the drift velocity and the diffusion coefficient of the electrons as a function of electric field strength. The drift velocity follows from the average displacement of an electron in a sufficient long-time interval. The diffusion coefficient is calculated as follows:

$$D = .5(\langle V^2\rangle - \langle V\rangle^2)T \tag{21}$$

where T is the sampling time, V is the average electron velocity for a sample over this time. The average is performed over an ensemble. In the computer simulation, however, we use time averaging. This is allowed because we deal with a stationary process, so the time average is equal to the ensemble average of a physical quantity. D(E) and V(E) are shown in Fig. 12 and Fig. 13 respectively.

## IV. Discussion

We obtained good agreements between the experimental and the computer simulation data. The intervalley deformation potential is the only parameter which we can adjust in our Monte Carlo simulation; all other parameters are generally well known. Pozela et al. [3] used a value of $10^8$eV/cm to match their simulation with the existing experimental data. In our case we use the value of $10^9$eV/cm. This value has also been used by Fawcett et al. [1]. We do not claim that this is an indirect determination of the intervalley deformation potential since more measurements in a wider range of temperature would be needed to obtain an accurate value for this parameter.

In Figure 14 we plotted the mean free path of an electron as function of the applied field, calculated using the Monte Carlo program for T = 300K. In the range of electric fields of our measurements, the mean free path is small compared to the length of the device. This assures that the measured data represent bulk behavior.

A difficulty one might encounter in the measurement of high field properties of GaAs is the possible occurrence of traveling dipole domains. These domains would make the electric field inside the device dependent on position and time. However, according to Kroemer [8], these dipole domains are formed only when the produce n (doping) by $\ell$ (length) exceeds $2 \times 10^{12} cm^{-2}$. This is not the case in our device which has a doping of $10^{15} cm^{-3}$ and a length of $1.1 \times 10^{-4} cm$.

Inside the device the electron experiences a higher effective electric field than that calculated from the applied voltage ($V/\ell$). This is due to the potential profile which exhibits a minimum whose position depends on the applied voltage. For high-bias voltages this potential minimum is located close to the cathode. The actual electric field is the gradientof this profile, and its value at the anode side of the potential minimum is probabl somewhat larger than $V/\ell$. As a consequence, the experimental curve has to be shifted slightly to higher electric field values.

We made sure that we measured velocity fluctuation noise at 11.0 and 17.0 MHz by measuring also the noise at lower frequencies. There we observed a small amount of 1/f noise, characterized by a Hooge parameter of $4 \times 10^{-7}$. No other excess noise sources were detected.

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## IV. Noise in Radioactive Decay (G. Kousik)

New measurements on counting statistics of α-particle radioactive decay in $_{95}Am$, $_{94}Pu$, $_{96}Cm$, and $_{64}Gd$ were made, in order to see whether deviations from Poisson statistics occurred. We measured up to intervals of 2000 minutes; however, since the counting equipment was only available to us for a limited period, not many long-time intervals could be measured. So the accuracy of the Allan variance for these intervals is rather poor. Nevertheless, it was clear that the noise is not purely Poissonian. Both a Lorentzian and a 1/f flicker noise component could be detected in these radioactive sources. The theory and experiment of these measurements are described below.

### Alpha Particle Counting Experiment

G. Kousik, W.H. Ellis, G. Bosman and C.M. Van Vliet

If we assume the noise spectrum of α-particle emission to consist of white noise, 1/f noise and a Lorentzian flicker noise, then

$$S_m(\omega) = 2m_0 + 2\pi C\ 1/|\omega| + 4B\,\frac{\alpha}{\alpha^2 + \omega^2}\,. \qquad (1)$$

Using the Allan variance theorem (1), we have for the relative Allan variance $[R(T) = \sigma^{A\,2}_{M_T}/\langle M_T\rangle^2]$ ,

$$R(T) = \frac{1}{m_0 T} + \frac{2C}{m_0^2} \log 2 + \frac{B}{m_0^2 T^2 \alpha^2} \left[4e^{-\alpha T} - e^{-2\alpha T} + 2\alpha T - 3\right] . \quad (2)$$

Here $m_0$ is the counting rate, $T$ is the time interval of counting, and $B$ and $\alpha$ are constants. For $T \ll \alpha^{-1}$,

$$4e^{-\alpha T} - e^{-2\alpha T} + 2\alpha T - 3 = 2/3\ \alpha^3 T^3 .$$

Therefore

$$R(T) = \frac{1}{m_0 T} + \frac{2C}{m_0^2} \log 2 + \frac{2B}{3m_0^2} \alpha T . \quad (3)$$

According to Handel's quantum 1/f noise theory, the flicker floor $2C \log 2/m_0^2$ is given by

$$\frac{2C \log 2}{m_0^2} = 33.28 \times 10^{-7} \log 2 \times \frac{E}{\kappa} . \quad (4)$$

Here E is the energy of the $\alpha$-particle in MeV. If the source has no Lorentzian flicker noise, then for large T the white noise term is much smaller than the flicker floor which is proportional to E.

$$R(T) \rightarrow 2C \log 2/m_0^2 \text{ (for large T)} .$$

## Experiment

Here two sources with four different energy $\alpha$-particles were used. The experimental setup is shown in Fig. 15. To count adjacent intervals a program was set up in the autosequence mode of the ND66, Multichannel Analyzer. This also contained a step for printing out the total number of counts in each peak after several interval counts.

| Source | Energy (MeV) | Half Life Time ($T_{1/2}$) Years |
|---|---|---|
| $_{94}Pu^{239}$ | 5.155 | $2.41 \times 10^4$ |
| $_{95}Am^{241}$ | 5.486 | 432 |
| $_{96}Cm^{244}$ | 5.805 | 18.1 |
| $_{64}Gd^{148}$ | 3.183 | 97.5 |

Since the average counting rate $m_0$ will be different for different time intervals T, all of the relative Allan variances were normalized as follows:

$$R_N(T) = R(T) - \frac{1}{m_0 T} + \frac{1}{m_0' T} \quad (5)$$

where $m_0'$ is chosen to the nearest 1000 or 500 counts. Here $m_0'$ are as follows:

| | | | |
|---|---|---|---|
| $_{94}Pu$ | 18500 | per | minute |
| $_{95}Am$ | 13000 | " | " |
| $_{96}Cm$ | 10000 | " | " |
| $_{64}Gd$ | 18000 | " | " |

Figs. 16 and 17 show $R_N(T)$ vs. 1/T for these sources.

The Pu and Cm sources show a trend towards a flick : floor with that of Cm being greater than that of Pu. This is in agreement with Handel's theory in that the flicker floor must be proportional to the energy of the α-particle. The Am source shows no flicker floor but a Lorentzian for the longer time intervals. The Gd peak was not sharp because this was a thick source and that led to an energy loss for the α-particles, broadening the peak on the lower energy side. This source also shows no distinct flicker floor, but a tendency towards a slow Lorentzian flicker noise.

C. Theoretical Work (G. Kousik)

Below we indicate computations of quantum 1/f noise for various scattering mechanisms. For that purpose we must first generalize Handel's expression for noise in a scattered beam, so as to include directional calculations.

Handel's original expression for the 1/f noise in a particle current K, which is scattered in a direction with velocity change $\Delta\bar{v}$, was

$$S_K/K^2 = 2\alpha A/f \tag{7}$$

where $\alpha$ is the fine structure constant $(137)^{-1}$,

$$A = \frac{8}{3\pi}\frac{|\Delta\bar{v}|^2}{c^2} = \frac{8\hbar^2}{3\pi}\frac{|\Delta\bar{k}|^2}{m^{*2}c^2} \tag{8}$$

and where m* is the effective mass and c the velocity of light. In phonon scattering we need the correlation between the particle current $w_{\underline{k}_1\underline{k}'}$, scattered from $\underline{k}_1$ to $\underline{k}'$, and the particle current $w_{\underline{k}_2\underline{k}''}$, scattered from $\underline{k}_2$ to $\underline{k}''$. A logical Ansatz for this correlation is

$$S_{w_{\underline{k}_1\underline{k}'},w_{\underline{k}_2,\underline{k}''}} = \frac{2\alpha A_{12}}{f} w_{\underline{k}_1\underline{k}'}\, w_{\underline{k}_2\underline{k}''}\, \delta_{\underline{k}_1,\underline{k}_2} \tag{9}$$

We took here the cross correlation to be bilinear in both fluxes, analogous to van der Ziel's expression for quantum partition 1/f noise, see [11]. The cross coupling constant should be given by

$$A_{12} = \frac{8\hbar^2}{3\pi}\frac{|\underline{k}_1 - \underline{k}'|\,|\underline{k}_2 - \underline{k}''|}{m^*c^2} = \frac{8\hbar^2}{3\pi}\frac{q'q''}{m^*c^2} \tag{10}$$

where

$$\underline{k}' = \underline{k}_1 \pm \underline{q}' \qquad \text{(+ sign phonon absorption)} \qquad (11a)$$

$$\underline{k}'' = \underline{k}_2 \pm \underline{q}' \qquad \text{(- sign phonon emission)} \qquad (11b)$$

with $\underline{k}$ representing the electron wave vector and $\underline{q}$ the phonon wave vector. The above holds for normal processes. For Umklapp processes,

$$\underline{k}' - \underline{k}_1 = \pm \underline{q}' + \underline{g}' \simeq \underline{g}' \qquad (12a)$$

$$\underline{k}'' - k_2 = \pm \underline{q}'' + \underline{g}'' \simeq \underline{g}'' \qquad (12b)$$

where $\underline{g}'$ and $\underline{g}''$ are reciprocal lattice vectors. Thus

$$A_{12,\text{Umklapp}} \simeq \frac{8\hbar^2}{3\pi} \frac{g'g''}{m^{*2}c^2} . \qquad (13)$$

For the smallest Umklapp process (adjacent Brillouin zones) $g' = g'' = 2\pi/a_{min} = 2\pi/a_0$, and we have

$$A_{12,Um} \simeq \frac{32\pi}{3} \frac{\hbar^2}{(m^* a_0)^2} . \qquad (14)$$

We now have complete expressions for the scattering correlation spectrum $S_{w_{k_1k'}, w_{k_2k''}}$ where the w's are the rates occurring in the matrix elements of the Boltzmann equation. Thus an exact theory for the fluctuations in mobility can be constructed. This has so far been carried out for acoustical and optical phonon scattering, involving only normal processes. The Umklapp processes will be considered in the near future.

Below we give a sketch of two detailed calculations, to be published at a later date.*

I. <u>Quantum 1/f Noise Due to Acoustical Phonon Scattering</u>
(G. Kousik, C.M. Van Vliet and G. Bosman)

We start with

$$\mu_n = \frac{e}{m^*} \frac{\langle v^2 \tau \rangle}{\langle v^2 \rangle} ,$$

where the averaging is over a Brillouin zone in k space. This yields for the mobility-fluctuation noise,

$$\frac{S_{\mu_n}(f)}{\bar{\mu}_n^2} = \{\sum_{\underline{k}_1} \sum_{\underline{k}_2} v_{\underline{k}_1}^2 \; v_{\underline{k}_2}^2 \, S_{\Delta\tau(\mathcal{E}_{\underline{k}_1})\Delta\tau(\mathcal{E}_{\underline{k}_2})} f(v_{\underline{k}_1}) f(v_{\underline{k}_2})\} / \{\sum_{k} v_{\underline{k}}^2 \tau(\mathcal{E}_k) f(v_k)\}^2 .$$

We have from the solution of the Boltzmann equation (see Figure 18),

$$\frac{1}{\tau(\mathcal{E}_{\underline{k}})} = \sum_{\underline{k}'} \{W_{\underline{kk}'}^{em} \left(1 - \frac{\cos\chi'}{\cos\chi}\right)_{em} + W_{\underline{kk}'}^{abs} \left(1 - \frac{\cos\chi'}{\cos\chi}\right)_{abs}\}$$

$$S_{\Delta\tau(\mathcal{E}_{\underline{k}_1})\Delta\tau(\mathcal{E}_{\underline{k}_2})} = \tau^2(\mathcal{E}_{\underline{k}_1})\tau^2(\mathcal{E}_{\underline{k}_2}) \sum_{\underline{k}'} \sum_{\underline{k}''} \{S_{W_{\underline{k}_1 k'}, W_{\underline{k}_2 k''}} \left(1 - \frac{\cos\chi'}{\cos\chi_1}\right)$$

$$\left(1 - \frac{\cos\chi'}{\cos\chi_2}\right) + S^{abs}_{W^{abs}_{\underline{k}_1 k''}} \left(1 - \frac{\cos\chi'}{\cos\chi_1}\right) \left(1 - \frac{\cos\chi''}{\cos\chi_2}\right) .$$

Here the noise spectrum $S_{\Delta\tau(\mathcal{E}_{\underline{k}_1})\Delta\tau(\mathcal{E}_{\underline{k}_2})}$ is written as a sum of contributions from absorption and emission. We write

*We note that in all cases Hooge's law $S_\mu/\mu^2 = \alpha/fH$ follows completely from the assumptions (9) and (10) of the theory. It is particularly gratifying that the factor N appears automatically in the denominator as a consequence of k-space integration. Explicit results for the Hooge parameter are deduced.

$$S_W^{em}(\underline{k}_1,\underline{k}_2,\underline{k}',\underline{k}'') = \frac{2\alpha A_{12}^{em}}{f} \bar{W}^{em}_{\underline{k}_1,\underline{k}'} \bar{W}^{em}_{\underline{k}_2,k''} \delta_{\underline{k}_1,\underline{k}_2}$$

$$S_W^{abs}(\underline{k}_1,\underline{k}_2,\underline{k}',\underline{k}'') = \frac{2\alpha A_{12}^{abs}}{f} \bar{W}^{abs}_{\underline{k}_1,k'} \bar{W}^{abs}_{\underline{k}_2,\underline{k}''} \delta_{\underline{k}_1,\underline{k}_2}$$

$$W^{em}_{\underline{k},\underline{k}'} = \sum_q \frac{2\pi}{\hbar} |\mathcal{F}(q)|^2 [F(q) + 1] \delta(\mathcal{E}_{\underline{k}'} - \mathcal{E}_{\underline{k}} + \hbar\omega_q) \; ; \; \underline{k}' = \underline{k} - \underline{q}$$

$$W^{abs}_{\underline{k},\underline{k}'} = \sum_q \frac{2\pi}{\hbar} |\mathcal{F}(q)|^2 F(q) \delta(\mathcal{E}_{\underline{k}'} - \mathcal{E}_{\underline{k}} - \hbar\omega_q) \; ; \; \underline{k}' = \underline{k} + q$$

$$F(q) = [\exp(\hbar\omega_q/k_B T) - 1]^{-1} \; ; \; |\mathcal{F}(q)|^2 = (\hbar c_1^2/2\rho V\omega_q)|q|^2$$

$$\left(1 - \frac{\cos\chi'}{\cos\chi}\right) = \pm \frac{q}{k}\cos\beta - \frac{q}{k}\tan\chi\sin\beta\cos\phi \begin{Bmatrix} + \text{ for emission} \\ - \text{ for absorption} \end{Bmatrix}$$

Let us assume that

$$\alpha A_{12} = \frac{8\alpha\hbar^2}{3\pi m^{*2} c^2} ,$$

see the introduction to this section. Now consider the emission term:

$$I_{em} = \sum_{\underline{k}'\underline{k}''} \sum_{\underline{q}',\underline{q}''} \{ (\tfrac{2\pi}{\hbar})^2 |\mathcal{F}(q')|^2 |\mathcal{F}(q'')|^2 [F(q') + 1] \delta(\mathcal{E}_{\underline{k}'} - \mathcal{E}_{\underline{k}_1} + \hbar\omega q')$$

$$\delta_{\underline{k}',\underline{k}_1 - q'} \delta(\mathcal{E}_{\underline{k}''} - \mathcal{E}_{\underline{k}_2} + \hbar\omega_{\underline{q}''}) \times \delta_{\underline{k}'',\underline{k}_2 - \underline{q}''} [\frac{q'}{k_1}(\cos\beta_1 - \tan\chi_1 \sin\beta_1 \cos\phi_1)]$$

$$[\frac{q''}{k_2}(\cos\beta_2 - \tan\chi_2 \sin\beta_2 \cos\phi_2)] \left(\frac{8\alpha\hbar^2}{3\pi m^{*2} c^2}\right) \frac{2}{f} \delta_{\underline{k}_1\underline{k}_2} q'q'' \} .$$

Summing over $\underline{k}'$ and $\underline{k}''$ we get,

$$I_{em} = (\tfrac{2\pi}{\hbar})^2 \left(\frac{16\alpha\hbar^2}{3\pi m^{*2} c^2}\right) \frac{\delta_{\underline{k}_1,\underline{k}_2}}{f k_1 k_2} \sum_{\underline{q}',\underline{q}''} \{ |\mathcal{F}(q')|^2 |\mathcal{F}(q'')|^2 [F(q') + 1][F(q'') + 1]$$

$$\delta(\mathcal{E}_{\underline{k}_1-q'} - \mathcal{E}_{\underline{k}_1} + \hbar\omega_{\underline{q}'})\delta(\mathcal{E}_{\underline{k}_2-q''} - \mathcal{E}_{\underline{k}_2} + \hbar\omega_{q''})$$

$$\times (\cos\beta_1 - \tan\chi_1 \sin\beta_1, \cos\phi_1)(\cos\beta_2. - \tan\chi_2 \sin\beta_2 \cos\phi_2)q'^2q''^2.$$

$$I_{em} = \left(\frac{2\pi}{\hbar}\right)^2\left(\frac{16\alpha\hbar^2}{3\pi m^{*2}c^2}\right)\frac{\delta_{\underline{k}_1,\underline{k}_2}}{fk_1k_2}\frac{V^2}{(8\pi^3)^2}\int\int d^3q'd^3q''\ \{q'^2q''^2|\mathcal{F}(q')|^2|\mathcal{F}(q'')|^2$$

$$[F(q')+1][F(q'')+1]\delta(\mathcal{E}_{\underline{k}_1-q'} - \mathcal{E}_{\underline{k}_1} + \hbar\omega_{\underline{q}'}) \times \delta(\mathcal{E}_{\underline{k}_2-\underline{q}''} - \mathcal{E}_{\underline{k}_2} + \hbar\omega_{\underline{q}})$$

$$(\cos\beta_1 \cos\beta_2 + \tan\chi_1 \tan\chi_2 \sin\beta_1 \sin\beta_2 \cos\phi_1 \cos\phi_2$$

$$- \cos\beta_1 \tan\chi_2 \sin\beta_2 \cos\phi_2 - \cos\beta_2 \tan\chi_1 \sin\beta_1 \cos\phi_1)\}\ .$$

We have

$$\iint d^3q'd^3q'' = \iint q'^2dq'q''^2dq'' \iint d\cos\beta_1\ d\cos\beta_2 \iint d\phi_1 d\phi_2\ .$$

Let us carry out the $\iint d\phi_1 d\phi_2$ . Writing only the last parenthesis we have

$$\int_0^{2\pi}\int_0^{2\pi} d\phi_1 d\phi_2[\cos\beta_1 \cos\beta_2 + \tan\chi_1 \tan\chi_2 \sin\beta_1 \sin\beta_2 \cos\phi_1 \cos\phi_2$$

$$- \cos\beta_2 \tan\chi_2 \sin\beta_2 \cos\phi_2 - \cos\beta_2 \tan\chi_1 \sin\beta_1 \cos\phi_1]\ .$$

Since

$$\int_0^{2\pi}\int_0^{2\pi} d\phi_1 d\phi_2(\cos\phi_1 \cos\phi_2 - \cos\phi_2 - \cos\phi_1) = 0\ ,$$

we are left with

$$\int_0^{2\pi}\int_0^{2\pi} d\phi_1 d\phi_2 = 4\pi^2 .$$

Therefore

$$I_{em} = \left(\frac{2\pi}{\hbar}\right)^2 \left(\frac{16\alpha\hbar^2}{3\pi m^{*2} c^2}\right) \frac{(4\pi^2)}{f} \frac{\delta_{\underline{k}_1,\underline{k}_2}}{k_1 k_2} \frac{V^2}{(8\pi^3)^2}$$

$$\int dq' d\cos\beta_1 \left\{q'^4 |\mathcal{F}(q')|^2 [F(q') + 1] \delta(\mathcal{E}_{\underline{k}_1 - q'} - \mathcal{E}_{\underline{k}_1} + \hbar\phi_{q'}) \cos\beta_1\right\}$$

$$\times \int dq'' d\cos\beta_2 \left\{q''^4 |\mathcal{F}(q'')|^2 [F(q'') + 1] \delta(\mathcal{E}_{\underline{k}_2 - q''} - \mathcal{E}_{\underline{k}_2} + \hbar\omega_{q''}) \cos\beta_2\right\}.$$

Let us write

$$\underline{k}_1 - \underline{q}' - \mathcal{E}_{\underline{k}_1} + \hbar\omega_{q'} = \frac{\hbar^2(\underline{k}_1 - q')^2}{2m^*} - \frac{\hbar^2 k_1^2}{2m^*} + \hbar c_\ell q'$$

$$= \frac{\hbar^2 q'^2}{2m^*} - \frac{\hbar^2 k_1 q'}{m^*} \cos\beta_1 + \hbar c_\ell q' ,$$

where we used $\omega_q{}' = c_\ell q'$ .

Define

$$y_1^e = \frac{\hbar q'}{2m^*} - \frac{\hbar k_1}{m^*} \cos\beta_1 + c_\ell$$

and

$$Q' = \hbar q''/2m^* .$$

With this change of variables,

$$dq'd\cos\beta_1 = \frac{dQ'dy_1^e}{\left|\frac{\partial(Q',y_1^e)}{\partial(q',\cos\beta_1)}\right|}$$

$$\frac{\partial(Q',y_1^e)}{\partial(q',\cos\beta_1)} = \begin{vmatrix} \frac{\hbar}{2m^*} & 0 \\ \frac{\hbar}{2m^*} & -\frac{\hbar k_1}{m^*} \end{vmatrix} = -\frac{\hbar^2 k_1}{2m^{*2}} \quad \text{and} \quad \frac{\partial(Q'',y_2^e)}{\partial(q'',\cos\beta_2)} = -\frac{\hbar^2 k_2}{2m^{*2}}.$$

In the integrals we get $\delta(\hbar q' y_1^e)$ and $e\delta(\hbar q'' y_2^e)$, which can be written as $\frac{1}{\hbar q'}\delta(y_1^e)$ and $\frac{1}{\hbar q''}\delta(y_2^e)$ respectively. Therefore

$$I_{em} = \left(\frac{2\pi}{\hbar}\right)^2\left(\frac{16\alpha\hbar^2}{3\pi m^{*2}c^2}\right)\frac{(4\pi^2)}{f}\frac{\delta_{\underline{k}_1,\underline{k}_2}}{k_1k_2}\left(\frac{V}{8\pi^3}\right)^2\int\frac{dQ'dy_1^e}{(\hbar^2k_1/2m^{*2})}$$

$$\{\left(\frac{2m^*}{\hbar}Q'\right)^4\left|\quad\left(\frac{2m^*Q'}{\hbar}\right)\right|^2\left[F\left(\frac{2m^*Q'}{\hbar}\right)+1\right]\frac{1}{2m^*Q'}\delta(y_1^e)\left[\frac{Q'+c_\ell-y_1^e}{(\hbar k_1/m^*)}\right]$$

$$\times\int\frac{dQ''dy_2^e}{(\hbar^2k_2/2m^{*2})}\{\left(\frac{2m^*Q''}{\hbar}\right)^4\left|\quad\left(\frac{2m^*Q''}{\hbar}\right)\right|^2\left[F\left(\frac{2m^*Q''}{\hbar}\right)+1\right]\frac{1}{2m^*Q''}\delta(y_2^e)\left[\frac{Q''+c_2-y_2^e}{(\hbar k_2/m^*)}\right]\}.$$

Since we have a $\delta(y_{1,2}^e)$, integration is confined to a strip at $y_{1,2}^e = 0$ along Q' and Q" (see Figure 19). Thus the limits are

$$0 < Q' < \frac{hk_1}{m^*} - c_\ell \quad\text{and}\quad 0 < Q'' < \frac{hk_2}{m^*} - c_\ell\ .$$

$$I_{em} = \left(\frac{2\pi}{h}\right)^2\left(\frac{16\alpha h^2}{3\pi m^{*2}c^2}\right)\frac{4\pi^2}{f}\frac{\delta_{\underline{k}_1,\underline{k}_2}}{k_1^3k_2^3}\left(\frac{\ }{8\pi^3}\right)^2\left(\frac{16m^{*6}}{\hbar^7}\right)^2$$

$$\{\int_0^{\left(\frac{\hbar k_1}{m^*}-c_\ell\right)} dQ'Q'^3\left|\quad\left(\frac{2m^*Q'}{\hbar}\right)\right|^2\left[F\left(\frac{2m^*Q'}{\hbar}\right)+1\right](Q'+c_\ell)\}$$

$$\times\{\int_0^{\left(\frac{\hbar k_2}{m^*}-c_\ell\right)} dQ''Q''^3\left|\quad\left(\frac{2m^*Q''}{\hbar}\right)+1\right](Q''+c_\ell)\}\ .$$

Therefore

$$I_{em} = (\frac{2\pi}{\hbar})^2 (\frac{16\alpha\hbar^2}{3\pi m^{*2} c^2}) \frac{4\pi^2}{f} \frac{\delta_{\underline{k}_1 \underline{k}_2}}{k_1^3 k_2^3} (\frac{V}{8\pi^3})^2 (\frac{16 m^{*6}}{\hbar^7})^2$$

$$\{ \int_0^{(\frac{\hbar k_1}{m^*} - c_\ell)} dQ' Q'^3 (\frac{m^* c_1{}^2}{\rho v c_\ell}) Q' (1 + \frac{k_B T}{2m^* c_\ell Q}) (Q' + c_\ell) \}$$

$$\times \{ \int_0^{(\frac{\hbar k_2}{m^*} - c_\ell)} dQ'' Q''^3 (\frac{m^* c_1^2}{\rho V c_\ell}) Q'' (1 + \frac{k_B T}{2m^* c_\ell Q''}) (Q'' + c_\ell) \} \ .$$

$$|\mathcal{F}(q)|^2 = (\frac{\hbar c_1^2}{2\rho V \omega_q}) q^2 = \frac{\hbar c_1^2}{2\rho V c_\ell q} q^2 = \frac{\hbar c_1^2}{2\rho V c_\ell} \frac{2m^* Q}{\hbar} = \frac{m^* c_1^2}{\rho V c_\ell} Q \ .$$

For $\hbar\omega_q / k_B T \ll 1$ we have

$$F(q) + 1 = (e^{\hbar c_\ell q / k_B T} - 1)^{-1} + 1 \simeq 1 + \frac{k_B T}{\hbar c_\ell q} = 1 + \frac{k_B T}{2m^* c_\ell Q}$$

$$I_{em} = (\frac{2\pi}{\hbar})^2 (\frac{16\alpha\hbar^2}{3\pi m^{*2} c^2}) \frac{4\pi^2}{f} \frac{\delta_{\underline{k}_1, \underline{k}_2}}{k_1^3 k_2^3} (\frac{V}{8\pi^3})^2 (\frac{16 m^{*6}}{\hbar^7}) (\frac{m^* c_1^2}{\rho V c_\ell})^2$$

$$\{ \int_0^{(\frac{\hbar k_1}{m^*} - c_\ell)} dQ' [Q'^5 + c_\ell Q'^4 + \frac{k_B T}{2m^* c_\ell} Q'^4 + \frac{k_B T \, Q'^3}{2m^*}]$$

$$\times \{ \int_0^{(\frac{\hbar k_2}{m^*} - c_\ell)} dQ'' [Q'^5 + c_\ell Q'^4 + \frac{k_B T}{2m^* c_\ell} Q'^4 + \frac{k_B T}{2m^*} Q'^3]\}$$

$$= (\frac{2\pi}{\hbar})^2 (\frac{16\alpha\hbar^2}{3\pi m^{*2} c^2}) \frac{4\pi^2}{f} \frac{\delta_{\underline{k}_1 \underline{k}_2}}{k_1^3 k_2^3} (\frac{V}{8\pi^3})^2 (\frac{16 m^{*6}}{\hbar^7})^2 (\frac{m^* c_1{}^2}{\rho V c_\ell})^2 [(\frac{\hbar k_1}{m^*} - c_\ell)^6 / 6$$

$$+ (c_\ell + \frac{k_B T}{2m^* c_\ell})(\frac{\hbar k_1}{m^*} - c_\ell)^5/5 + \frac{k_B T}{2m^*} \cdot (\frac{\hbar k_1}{m^*} - c_\ell)^4/4]$$

$$[(\frac{\hbar k_2}{m^*} - c_\ell)^6/6 + (c_\ell + \frac{k_B T}{2m^* c_\ell})(\frac{\hbar k_2}{m^*} - c_\ell)^5/5 + \frac{k_B T}{2m^*} (\frac{\hbar k_2}{m^*} - c_\ell)^4/4] \; .$$

The absorption term is

$$I_{abs} = \sum_{\underline{k}',\underline{k}''} \frac{S^{abs}}{W^{abs}_{\underline{k}_1 k'} W^{abs}_{\underline{k}_2 k''}} (1 - \frac{\cos\chi'}{\cos\chi_1})(1 - \frac{\cos\chi''}{\cos\chi_2}) = \sum_{\underline{k}',\underline{k}''} \sum_{\underline{q}',\underline{q}''}$$

$$\{(\frac{2\pi}{\hbar})^2 |\mathcal{F}(q')|^2 |\mathcal{F}(q'')|^2 F(q')F(q'')\delta(\mathcal{E}_{\underline{k}'} - \mathcal{E}_{\underline{k}_1} - \hbar\omega_{q'})\delta_{\underline{k}',\underline{k}_1+q'}$$

$$\delta(\mathcal{E}_{\underline{k}'} - \mathcal{E}_{\underline{k}_2} - \hbar\omega_{q''})\delta_{\underline{k}'',\underline{k}_2+q''} \times [\frac{q'}{k_1}(\cos\beta_1 + \tan\chi_1 \sin\beta_1 \cos\phi_1)]$$

$$[\frac{q''}{k_2}(\cos\beta_2 + \tan\chi_2 \sin\beta_2 \cos\phi_2)](\frac{8\alpha\hbar^2}{3\pi m^{*2} c^2}) \frac{2\delta_{\underline{k}_1\underline{k}_2}}{f} q'q'' \; .$$

Summing over $\underline{k}'$ and $\underline{k}''$ we get

$$I_{abs} = (\frac{2\pi}{\hbar})^2 (\frac{16\alpha\hbar^2}{3\pi m^{*2} c^2}) \frac{\delta_{\underline{k}_1,\underline{k}_2}}{f\underline{k}_1\underline{k}_2} \sum_{\underline{q}',\underline{q}''}$$

$$\{|\mathcal{F}(q')|^2 F(q')\delta(\mathcal{E}_{\underline{k}_1+q'} - \mathcal{E}_{\underline{k}_1} - \hbar\omega_{\underline{q}'})(\cos\beta_1 + \tan\chi_1 \sin\beta_1 \cos\phi_1\} q'^2$$

$$\times \{|\mathcal{F}(q'')|^2 F(q'')\delta(\mathcal{E}_{\underline{k}_2+q''} - \mathcal{E}_{\underline{k}_2} - \hbar\omega_{\underline{q}''})(\cos\beta_2 + \tan\chi_2 \sin\beta_2 \cos\phi_2\} q''^2 .$$

$$I_{abs} = (\frac{2\pi}{\hbar})^2 (\frac{16\alpha\hbar^2}{3\pi m^{*2} c^2}) \frac{\delta_{\underline{k}_1\underline{k}_2}}{f k_1 k_2} (\frac{V}{8\pi^3})^2 \iint d^3q' d^3q''$$

$$\{|\mathcal{F}(q')|^2 F(q')\delta(\mathcal{E}_{\underline{k}_1+\underline{q}'} - \mathcal{E}_{\underline{k}_1} - \hbar\omega_{\underline{q}'})(\cos\beta_1 + \tan\chi_1 \sin\beta_1 \cos\phi_1)\} q'$$

$$\times \{|\mathcal{H}(q'')|^2 F(q'')\delta(\mathcal{E}_{\underline{k}_2+q''} - \mathcal{E}_{\underline{k}_2} - \hbar\omega_{\underline{q}''})(\cos\beta_2 + \tan\chi_2 \sin\beta_2 \cos\phi_2)\}q''.$$

As in the emission case only the $\iint_0^{2\pi} \cos\beta_1 \cos\beta_2 d\phi_1 d\phi_2$ term survives. Therefore,

$$I_{abs} = \left(\frac{2\pi}{\hbar}\right)^2 \left(\frac{16\alpha\hbar^2}{3\pi m^{*2}c^2}\right) \frac{\delta_{\underline{k}_1,\underline{k}_2}}{fk_1k_2} \left(\frac{V}{8\pi^3}\right)^2 (4\pi^2)$$

$$\{\int dq' d\cos\beta_1 q'^4 |\mathcal{F}(q')|^2 F(q')\delta(\mathcal{E}_{\underline{k}_1+q'} - \mathcal{E}_{\underline{k}_1} - \hbar\omega_{\underline{q}'})\cos\beta_1\}$$

$$\times \{\int dq'' d\cos\beta_2 q''^4 |\mathcal{H}(q'')|^2 F(q'')\delta( {}_{\underline{k}_2+\underline{q}''} - \mathcal{E}_{\underline{k}_2} - \hbar\omega_{\underline{q}''})\cos\beta_2\}.$$

Again define

$$y_1^a = \frac{\hbar q'}{2m^*} + \frac{\hbar k_1}{m^*}\cos\beta_1 - c_\ell \text{ and } Q' = \hbar q'/2m^*; \left|\frac{\partial(Q', y_1^a)}{\partial(q', \cos\beta_1)}\right|.$$

Consider only the first integral:

$$\int dq' d\cos\beta_1 q'^4 |\mathcal{F}(q')|^2 F(q')\delta(hq' y_1^a)\cos\beta_1$$

$$= \int \frac{dQ' dy_1^a}{(\hbar^2 k_1/2m^{*2})} \left(\frac{2m^*}{\hbar} Q'\right)^4 \left|\mathcal{H}\left(\frac{2m^*Q'}{\hbar}\right)\right|^2 F\left(\frac{2m^*Q'}{\hbar}\right) \frac{1}{2m^*Q'} \delta(y_1^a)$$

$$\times (y_1^a + c_\ell - Q')/(\hbar k_1/m^*).$$

$$|\mathcal{H}(q')|^2 = \frac{m^* c_1^2}{\rho V c_\ell} Q' \; ; \; F(q') = \left[\exp\left(\frac{\hbar c_\ell q'}{k_B T}\right) - 1\right]^{-1} \simeq \frac{k_B T}{\hbar c_\ell q'} = \frac{k_B T}{2m^* c_\ell Q}.$$

Therefore

$$\int \frac{dQ'dy_1^a}{(\hbar^2 k_1/2m^{*2})} \left(\frac{2m^*}{\hbar}\right)^4 Q'^4\left(\frac{m^* c_1^2}{\rho V c_\ell}\right) Q'\left(\frac{k_B T}{2m^* c_\ell Q'}\right) \frac{1}{2m^* Q'} \delta(y_1^a)\left(\frac{y_1^a + c_\ell - Q'}{\hbar k_1/m^*}\right)$$

$$= \left(\frac{8m^{*6}}{\hbar^7}\right)\left(\frac{k_B T c_1^2}{k_1^2 \rho V c_\ell^2}\right) \int dQ'dy_1^a\, Q'^3(y_1^a + c_\ell - Q')\delta(y_1^a) \ .$$

From Figure 20 we have

$$\left(\frac{8m^{*6}}{\hbar^7}\right)\left(\frac{k_B T c_1^2}{k_1^2 \rho V c_\ell^2}\right) \int_0^{\left(\frac{\hbar k_1}{m^*} + c_\ell\right)} dQ'\ Q'^3(c_\ell - Q') = \left(\frac{8m^{*6}}{\hbar^7}\right)\left(\frac{k_B T c_1^2}{k_1^2 \rho V c_\ell^2}\right)$$

$$\left[c_\ell\left(\frac{\hbar k_1}{m^*} + c_\ell\right)^4/4 - \left(\frac{\hbar k_1}{m^*} + c_\ell\right)^5/5\right] \ .$$

$$I_{abs} = \left(\frac{2\pi}{\hbar}\right)^2\left(\frac{16\alpha\hbar^2}{3\pi m^{*2}c^2}\right) \frac{\delta_{\underline{k}_1,\underline{k}_2}}{f\, k_1^3 k_2^3} \left(\frac{V}{8\pi^3}\right)^2(4\pi^2)\left(\frac{8m^{*6}}{\hbar^7}\right)^2\left(\frac{k_B T c_1^2}{\rho V c_\ell^2}\right)^2$$

$$\left[c_\ell\left(\frac{\hbar k_1}{m^*} + c_\ell\right)^4/4 - \left(\frac{\hbar k_1}{m^*} + c_\ell\right)^5/5\right] \left[c_\ell\left(\frac{\hbar k_2}{m^*} + c_\ell\right)^4/4 - \left(\frac{\hbar k_2}{m^*} + c_\ell\right)^5/5\right] \ .$$

Therefore

$$S_{\Delta\tau(\mathcal{E}_{\underline{k}_1})\Delta\tau(\mathcal{E}_{\underline{k}_2})}\left(\frac{2\pi}{h}\right)^2\left(\frac{16\alpha h^2}{3\pi m^{*2}c^2}\right) \frac{4\pi^2}{f} \frac{\delta_{\underline{k}_1\underline{k}_2}}{k_1^3 k_2^3} \left(\frac{V}{8\pi^3}\right)^2\left(\frac{8m^{*6}}{h^7}\right)^2\left(\frac{c_1^2}{\rho V}\right)^2$$

$$\left\{\left(\frac{2m^*}{c_\ell}\right)^2\left[\left(\frac{\hbar k_1}{m^*} - c_\ell\right)^6/6 + \left(c_\ell + \frac{k_B T}{2m^* c_\ell}\right) \times \left(\frac{\hbar k_1}{m^*} - c_\ell\right)^5/5 + \frac{k_B T}{2m^*}\left(\frac{\hbar k_1}{m^*} - c_\ell\right)^4/4\right]\right.$$

$$\left[\left(\frac{\hbar k_2}{m^*} - c_\ell\right)^6/6 + \left(c_\ell + \frac{k_B T}{2m^* c_\ell}\right)\left(\frac{\hbar k_2}{m^*} - c_\ell\right)^5/5 + \frac{k_B T}{2m^*}\left(\frac{\hbar k_2}{m^*} - c_\ell\right)^4/4\right]$$

$$\left. + \left(\frac{k_B T}{c_\ell^2}\right)^2\left[c_\ell\left(\frac{\hbar k_1}{m^*} + c_\ell\right)^4/4 - \left(\frac{\hbar k_1}{m^*} + c_\ell\right)^5/5\right]\left[c_\ell\left(\frac{\hbar k_2}{m^*} + c_\ell\right)^4/4 - \left(\frac{\hbar k_2}{m^*} + c_\ell\right)^5/5\right]\right\}.$$

$$S_{\Delta\tau(\mathcal{E}_{\underline{k}_1})\Delta\tau(\mathcal{E}_{\underline{k}_2})} = \left(\frac{256\alpha m^{*6}\rho^2 c_\ell^8 \pi}{3c^2 c_1^4 k_B^4 T^4 h^2}\right) \frac{1}{f} \frac{\delta_{\underline{k}_1,\underline{k}_2}}{k_1^5 k_2^5} \left\{\left(\frac{2m^*}{c_\ell}\right)^2\left[\left(\frac{\hbar k_1}{m^*} - c_\ell\right)^6/6\right.\right.$$

$$+ \left(c_\ell + \frac{k_B T}{2m^* c_\ell}\right)\left(\frac{\hbar k_1}{m^*} - c_\ell\right)^5/5 + \frac{k_B T}{2m^*}\left(\frac{\hbar k_1}{m^*} - c_\ell\right)^4/4\right] \left[\left(\frac{\hbar k_2}{m^*} - c_\ell\right)^6/6$$

$$+ \left(c_\ell + \frac{k_B T}{2m^* c_\ell}\right)\left(\frac{\hbar k_2}{m^*} - c_\ell\right)^5/5 + \frac{k_B T}{2m^*}\left(\frac{\hbar k_2}{m^*} - c_\ell\right)^4/4\right] + \left(\frac{k_B T}{c_\ell^2}\right)^2 \left[c_\ell\left(\frac{\hbar k_1}{m^*} + c_\ell\right)^4/4$$

$$- \left(\frac{\hbar k_1}{m^*} + c_\ell\right)^5/5\right] \left[c_\ell\left(\frac{\hbar k_2}{m^*} + c_\ell\right)^4/4 - \left(\frac{\hbar k_2}{m^*} + c_\ell\right)^5/5\right]\Big\}.$$

$$\frac{S_\mu(f)}{\bar{\mu}^2} = \Big\{ \sum_{\underline{k}_1} \sum_{\underline{k}_2} v_{\underline{k}_1}^2 v_{\underline{k}_2}^2 S_{\Delta\tau(\quad \underline{k}_1)\Delta\tau(\quad \underline{k}_2)} f(v_{\underline{k}_1}) f(v_{\underline{k}_2}) \Big\} \Big/ \Big\{ \sum_{\underline{k}} v_{\underline{k}}^2 \tau(\mathcal{E}_{\underline{k}}) f(v_{\underline{k}}) \Big\}^2$$

The numerator has a form

$$\sum_{\underline{k}_1 \underline{k}_2} [R(k_1)R(k_2) + S(k_1)S(k_2)]\delta_{\underline{k}_1, \underline{k}_2}.$$

Going from

$$\sum_{\underline{k}_1 \underline{k}_2} \to \left(\frac{V}{8\pi^3}\right)^2 \iint d^3k_1 d^3k_2 \quad \text{and} \quad \delta_{\underline{k}_1 \underline{k}_2} \to \frac{\delta(\underline{k}_1 - \underline{k}_2)}{(V/8\pi^3) f(\mathcal{E})}$$

$$\to \left(\frac{V}{8\pi^3}\right) \iint [R(k_1)R(k_2) + S(k_1)S(k_2)] k_1^2 dk_1 d\cos\chi_1 \, d\cos\phi' \, k_2^2 dk_2 d\cos\chi_2 d\phi''$$

$$\frac{\delta(k_1 - k_2)}{k_2^2 f(V_{\underline{k}_2})} \delta(\cos\chi_1 - \cos\chi_2)\delta(\phi' - \phi'')$$

$$\to \left(\frac{V}{8\pi^3}\right) \iint [R(k_1)R(k_2) + S(k_1)S(k_2)] k_1^2 dk_1 dk_2 \delta \frac{(k_1 - k_2)4\pi}{f(v_{k_2})}.$$

Let

$$\left.\begin{aligned} x &= (k_1 - k_2) \\ k &= k_1 \end{aligned}\right\} \twoheadrightarrow dk_1 dk_2 \to \frac{dx dk}{1}$$

$$\rightarrow \left(\frac{V}{8\pi^3}\right)4\pi \iint [R(k)R(k-x) + S(k)S(k-x)]k^2 dk dx \delta(x)$$

$$= (4\pi)\left(\frac{V}{8\pi^3}\right)\int_0^{k_D} [R^2(k) + S^2(k)]k^2 dk \ .$$

$$\frac{S_\mu(f)}{\bar{\mu}^2} = \frac{\dfrac{256\alpha m^{*6}\rho^2 c_\ell^8 \pi}{3c^2 c_1^4 k_B^4 T^4 \hbar^2}}{\{\sum_{k} v_{\underline{k}}^2 \tau(\mathcal{E}_{\underline{k}}) f(v_{\underline{k}})\}^2} \ \frac{1}{f}\left(\frac{V}{2\pi^2}\right)\iint_0^{k_D} v_{\underline{k}}^4 \frac{1}{k^{10}} k^2 dk$$

$$\{(\frac{2m^*}{c_\ell})^2[(\frac{\hbar k}{m^*} - c_\ell)^6/6 + (c_\ell + \frac{k_B T}{2m^* c_\ell})(\frac{\hbar k}{m^*} - c_\ell)^5/5 + \frac{k_B T}{2m^*}(\frac{\hbar k}{m^*} - c_\ell)^4/4]^2$$

$$+ (\frac{k_B T}{c_\ell^2})^2 [c_\ell(\frac{\hbar k}{m^*} + c_\ell)^4/4 - (\frac{\hbar k}{m^*} + c_\ell)^5/5]^2\} f(v_{\underline{k}}) \ .$$

## II. Quantum 1/f Noise Due to Optical Phonon Scattering
(G. Kousik, C.M. Van Vliet and G. Bosman)

$$\mu = \frac{e}{m^*}\frac{\langle v^2\tau\rangle}{\langle v^2\rangle} \rightarrow \frac{S_\mu}{\mu^2} = \{\sum_{\underline{k}_1,\underline{k}_2} v_{\underline{k}_1}^2 v_{\underline{k}_2}^2 S_{\Delta\tau(\mathcal{E}_{\underline{k}_1})\Delta\tau(\mathcal{E}_{\underline{k}_2})} f(v_{\underline{k}_1}) f(v_{\underline{k}_2})\} /$$

$$/ \{\sum_{\underline{k}} v_{\underline{k}}^2 \tau(\mathcal{E}_{\underline{k}}) f(v_{\underline{k}})\}^2 \ .$$

The relaxation time is

$$\frac{1}{\tau(\mathcal{E}_{\underline{k}})} = \sum_{\underline{k}'}(W_{\underline{k},\underline{k}'}^{em} + W_{\underline{k},\underline{k}'}^{abs}) \rightarrow S_{\Delta\tau(\mathcal{E}_{\underline{k}_1})\Delta\tau(\mathcal{E}_{\underline{k}_2})} = \tau^2(\mathcal{E}_{\underline{k}_1})\tau^2(\mathcal{E}_{\underline{k}_2})$$

$$\sum_{\underline{k}',\underline{k}''} [S_{W_{\underline{k}_1\underline{k}'}^{em} W_{\underline{k}_2\underline{k}''}^{em}}^{em} + S_{W_{\underline{k}_1\underline{k}'}^{abs} W_{\underline{k}_2\underline{k}'}^{abs}}^{abs}] \ .$$

From Handel's theory

$$S^{em}_{W^{em}_{\underline{k}_1\underline{k}'} W_{\underline{k}_2 k''}} = \frac{2\alpha A_{12}}{f} \bar{W}^{em}_{\underline{k}_1\underline{k}'} \bar{W}^{em}_{\underline{k}_2,\underline{k}''} \delta_{\underline{k}_1,\underline{k}_2}$$

$$S^{abs}_{W^{abs}_{\underline{k}_1\underline{k}'} W^{abs}_{\underline{k}_2 k''}} = \frac{2\alpha A_{12}}{f} \bar{W}^{abs}_{\underline{k}_1\underline{k}'} \bar{W}^{abs}_{\underline{k}_2\underline{k}''} \delta_{\underline{k}_1,\underline{k}_2}$$

and

$$\alpha A_{12} = \frac{8\alpha\hbar^2}{3\pi m^{*2} c^2} q' q''$$

where

$$\underline{k}' = \underline{k}_1 \pm q' \quad \text{and} \quad \underline{k}'' = \underline{k}_2 \pm \underline{q}'' \quad \left\{ \begin{matrix} + \text{ absorption} \\ - \text{ emission} \end{matrix} \right\} .$$

$$W^{em}_{\underline{kk}'} = \left(\frac{2\pi}{\hbar}\right) \sum_q |\mathcal{F}(q)|^2 [F(q) + 1] \delta(\mathcal{E}_{\underline{k}'} - \mathcal{E}_{\underline{k}} + \hbar\omega_0)$$

$$W^{abs}_{\underline{kk}'} = \left(\frac{2\pi}{\hbar}\right) \sum_q |\mathcal{F}(q)|^2 F(q) \delta(\mathcal{E}_{\underline{k}'} - \mathcal{E}_{\underline{k}} - \hbar\omega_0) .$$

Here

$$|\mathcal{F}(q)|^2 = \left(\frac{\hbar \mathcal{D}_0^2}{2\rho V \omega_0}\right) \quad \text{and} \quad F(q) = (e^{\hbar\omega_0/kT} - 1)^{-1};$$

both independent of q.

Call $|\mathcal{F}(q)|^2 = C_0$ and $F(q) = F$. Consider the emission term in the sum:

$$I_{em} = \left(\frac{2\pi}{h}\right)^2 C_0^2 \sum_{\underline{k}',\underline{k}''} \sum_{\underline{q}',\underline{q}''} (F + 1)^2 \delta(\mathcal{E}_{\underline{k}'} - \mathcal{E}_{\underline{k}_1} + \hbar\omega_0)$$

$$\delta_{\underline{k}',\underline{k}_1-\underline{q}'} \delta(\mathcal{E}_{\underline{k}''} - \mathcal{E}_{\underline{K}_2} + h\omega_0) \delta_{\underline{k}'',\underline{k}_2-\underline{q}''} \left(\frac{8\alpha\hbar^2}{3\pi m^{*2} c^2}\right) \frac{\underline{q}'\underline{q}''}{f} \delta_{\underline{k}_1,\underline{k}_2} .$$

Sum over $\underline{k}',\underline{k}''$ to get,

$$I_{em} = (\frac{2\pi}{\hbar})^2 c_0^2(F + 1)^2(\frac{8\alpha\hbar^2}{3\pi^2 m^{*2} c^2}) \frac{\delta_{\underline{k}_1\underline{k}_2}}{f} \sum_{\underline{q}',\underline{q}''} \delta(\mathcal{E}_{\underline{k}_1-q'} - \mathcal{E}_{\underline{k}_1} + \hbar\omega_0)$$

$$\delta(\mathcal{E}_{\underline{k}_2-\underline{q}''} - \mathcal{E}_{\underline{k}_2} + \hbar\omega_0)q'q'' = (\frac{2\pi}{\hbar})^2 c_0^2(F + 1)^2(\frac{8\alpha\hbar^2}{3\pi^2 m^{*2} c^2}) \frac{\delta_{\underline{k}_1,\underline{k}_2}}{f} (\frac{V}{8\pi^3})^2$$

$$[\int q'^3 dq' d\cos\beta_1 d\phi_1 \delta(\mathcal{E}_{\underline{k}_1-\underline{q}'} - \mathcal{E}_{\underline{k}_1} + \hbar\omega_0)]$$

$$[\int q''^3 dq'' d\cos\beta_2 d\phi_2 \delta(\mathcal{E}_{\underline{k}_2-\underline{q}''} - \mathcal{E}_{\underline{k}_2} + \hbar\omega_0)] .$$

Here

$$\int_0^{2\pi} d\phi_1 d\phi_2 = 4\pi^2 .$$

The two integrals in $I_{em}$ look alike. Consider

$$\int q'3dq'd\cos\beta_1 \delta(\mathcal{E}_{\underline{k}_1-q'} - \mathcal{E}_{\underline{k}_1} + \hbar\omega_0) .$$

$$\mathcal{E}_{\underline{k}_1-\underline{q}'} - \mathcal{E}_{\underline{k}_1} + \hbar\omega_0 = \frac{\hbar^2(\underline{k}_1 - \underline{q}')^2}{2m^*} - \frac{\hbar^2 k_1^2}{2m^*} + \hbar\omega_0$$

$$= \frac{\hbar^2 q'^2}{2m^*} - \frac{\hbar^2 k_1 q'\cos\beta_1}{m^*} + \hbar\omega_0 .$$

Let

$$y_1^e = \frac{\hbar^2 q'^2}{2m^*} - \frac{\hbar^2 k_1 q'}{m^*}\cos\beta_1 + \hbar\omega_0 \quad \text{and} \quad Q' = \frac{\hbar q'}{\sqrt{2m^{*'}}} .$$

With this transformation, we get

$$\frac{\partial(Q',y_1^e)}{\partial(q',\cos\beta_1)} = \begin{vmatrix} \frac{\hbar}{\sqrt{2m^*}} & 0 \\ \frac{\hbar^2 q'}{m^*} - \frac{\hbar^2 k_1}{m^*}\cos\beta_1 & -\frac{\hbar^2 k_1 q'}{m^*} \end{vmatrix} = -\frac{\hbar^3 k_1 q'}{m^*\sqrt{2m^*}} = -\frac{\hbar^2 k_1 Q'}{m^*} .$$

Therefore

$$\int q'^3 dq' d\cos\beta_1 \delta(\mathcal{E}_{\underline{k}_1-\underline{q}'} - \mathcal{E}_{\underline{k}_1} + \hbar\omega_0) = \iint \left(\frac{\sqrt{2m^*}}{\hbar}\right)^3 Q'^3 \frac{dQ' dy_1^e}{(\hbar^2 k_1 Q'/m^*)} \delta(y_1^e)$$

$$= \frac{1}{k_1}\left(\frac{\sqrt{2m^*}}{\hbar}\right)^3 \frac{m^*}{\hbar^2} \iint dQ' Q'^2 dy_1^e \delta(y_1^e) .$$

Integrate along a strip (see Figure 22):

$$y_1^e = 0 \rightarrow Q'^2 - \frac{\sqrt{2}\hbar k_1 Q'}{\sqrt{m^*}} + \hbar\omega_0 = 0$$

$$\rightarrow Q' = \frac{\left[\sqrt{\frac{2}{m^*}}\hbar k_1 \pm \left(\frac{2\hbar^2 k_1^2}{m^*} - 4\hbar\omega_0\right)^{1/2}\right]}{2} .$$

Therefore we get

$$\frac{1}{k_1}\left(\frac{\sqrt{2m^*}}{\hbar}\right)^3 \frac{m^*}{\hbar^2} \int_{Q_1'}^{Q_2'} Q'^2 dQ' = \frac{1}{k_1}\left(\frac{\sqrt{2m^*}}{\hbar}\right)^3 \frac{m^*}{\hbar} \left[\frac{1}{3}\cdot\frac{1}{8}\left(\sqrt{\frac{2}{m^*}}\hbar k_1\right.\right.$$

$$\left.+\left(\frac{2\hbar^2 k_1^2}{m^*} - 4\hbar\omega_0\right)^{1/2}\right)^3 - \frac{1}{3}\cdot\frac{1}{8}\left(\sqrt{\frac{2}{m^*}}\hbar k_1 - \left(\frac{2\hbar^2 k_1^2}{m^*} - 4\hbar\omega_0\right)^{1/2}\right\}^3 \Bigg]$$

$$= \frac{1}{k_1}\left(\frac{\sqrt{2m^*}}{\hbar}\right)^3 \frac{m^*}{\hbar^2}\frac{1}{24}\left\{\left(\sqrt{\frac{2}{m^*}} hk_1\right)^3 + \left(\frac{2\hbar^2 k_1^2}{m^*} - 4\hbar\omega_0\right)^{3/2}\right.$$

$$+ 3\sqrt{\frac{2}{m^*}}\hbar k_1\left(\frac{2\hbar^2 k_1^2}{m^*} - 4\hbar\omega_0\right) + 3\cdot\frac{2\hbar^2 k_1^2}{m^*}\left(\frac{2\hbar^2 k_1^2}{m^*} - 4\hbar\omega_0\right)^{1/2}$$

$$- [(\sqrt{\frac{2}{m^*}}\hbar k_1)^3 - (\frac{2\hbar^2 k_1^2}{m^*} - 4\hbar\omega_0)^{3/2} + 3\sqrt{\frac{2}{m^*}}\hbar k_1(\frac{2\hbar^2 k_1^2}{m^*} - 4\hbar\omega_0)$$

$$- 3 \cdot \frac{2\hbar^2 k_1^2}{m^*}(\frac{2\hbar^2 k_1^2}{m^*} - 4\hbar\omega_0)^{1/2}]\} = \frac{1}{k_1}(\frac{\sqrt{2m^*}}{\hbar})^3 \frac{m^*}{24\hbar^2}\{2[(\frac{2\hbar^2 k_1^2}{m^*} - 4\hbar\omega_0)^{3/2}$$

$$+ 3 \cdot \frac{2\hbar^2 k_1^2}{m^*}(\frac{2\hbar^2 k_1^2}{m^*} - 4\hbar\omega_0)^{1/2}]\} = \frac{1}{k_1}(\frac{2\hbar^2 k_1^2}{\hbar} - 4\hbar\omega_0)^{1/2}[\frac{2\hbar^2 k_1^2}{m^*} - 4\hbar\omega_0 + \frac{6\hbar^2 k_1^2}{m^*}]$$

$$= \frac{1}{k_1}(\frac{\sqrt{2m^*}}{\hbar})^3 \frac{m^*}{3\hbar^2}(\frac{2\hbar^2 k_1^2}{m^*} - 4\hbar\omega_0)^{1/2}(\frac{2\hbar^2 k_1^2}{m^*} - \hbar\omega_0) = \frac{1}{k_1}(\frac{\sqrt{2m^*}}{\hbar})^3$$

$$\frac{2m^*}{3\hbar^2}(\frac{\hbar^2 k_1^2}{2m^*} - \hbar\omega_0)^{1/2}(4\frac{\hbar^2 k_1^2}{2m^*} - \hbar\omega_0) = \frac{4\sqrt{2}\, m^{*2}\sqrt{m^*}}{3\hbar^5 k_1}(\frac{\hbar^2 k_1^2}{2m^*} - \hbar\omega_0)^{1/2}$$

$$(4\frac{\hbar^2 k_1^2}{2m^*} - \hbar\omega_0) .$$

Therefore

$$I_{em} = (\frac{2\pi}{\hbar})^2 c_0^2 (F + 1)^2 (\frac{8\alpha\hbar^2}{3\pi m^{*2} c^2}) \frac{\delta_{\underline{k}_1 \underline{k}_2}}{f} (\frac{V}{8\pi^3})^2 4\pi^2 (\frac{4\sqrt{2}\, m^{*2}\, m^*}{3\hbar^5})^2$$

$$\frac{1}{k_1 k_2}(\frac{\hbar^2 k_1^2}{2m^*} - \hbar\omega_0)^{1/2}(4\frac{\hbar^2 k_1^2}{2m^*} - \hbar\omega_0)(\frac{\hbar^2 k_2^2}{2m^*} - \hbar\omega_0)^{1/2} \times (4\frac{\hbar^2 k_2^2}{2m^*} - \hbar\omega_0) .$$

Now consider the absorption term

$$I_{abs} = (\frac{2\pi}{\hbar})^2 c_0^2 F^2 \sum_{\underline{k}', \underline{k}''} \sum_{\underline{q}', q''} \delta(\mathcal{E}_{\underline{k}'} - \mathcal{E}_{\underline{k}_1} - \hbar\omega_0)\delta_{\underline{k}', \underline{k}_1 + \underline{q}'}$$

$$\delta(\mathcal{E}_{\underline{k}''} - \mathcal{E}_{\underline{k}_2} - \hbar\omega_0)\delta_{\underline{k}'', \underline{k}_2 + \underline{q}''}(\frac{8\alpha\hbar^2}{3\pi m^{*2} c}) q' q'' \frac{\delta_{\underline{k}_1, \underline{k}_2}}{f} .$$

Summing over $\underline{k}', \underline{k}'' \rightarrow$

$$I_{abs} = \left(\frac{2\pi}{\hbar}\right)^2 c_0^2 F^2 \left(\frac{8\alpha\hbar^2}{3\pi m^{*2} c^2}\right) \frac{\delta_{\underline{k}_1,\underline{k}_2}}{f} \sum_{\underline{q}',\underline{q}''} \delta(\mathcal{E}_{\underline{k}_1+\underline{q}'} - \mathcal{E}_{\underline{k}_1} - \hbar\omega_0)$$

$$\delta(\mathcal{E}_{\underline{k}_2+\underline{q}''} - \mathcal{E}_{\underline{k}_2} - \hbar\omega_0) q' q''$$

$$I_{abs} = \left(\frac{2\pi}{\hbar}\right)^2 c_0^2 F^2 \left(\frac{8\alpha\hbar^2}{3\pi m^{*2} c^2}\right) \frac{\delta_{\underline{k}_1,\underline{k}_2}}{f} \left(\frac{V}{8\pi^3}\right)^2 \iint q'^2 dq' d\cos\beta_1 d\phi_1 q''^2 dq'' d\cos\beta_2 d\phi_2$$

$$\delta(\mathcal{E}_{\underline{k}_1+\underline{q}'} - \mathcal{E}_{\underline{k}_1} - \hbar\omega_0)\delta(\mathcal{E}_{\underline{k}_2+\underline{q}'} - \mathcal{E}_{\underline{k}_2} - \hbar\omega_0) q' q'' .$$

Again

$$\int_0^{2\pi}\int_0^{2\pi} d\phi_1 d\phi_2 = 4\pi^2$$

and

$$I_{abs} - \left(\frac{2\pi}{\hbar}\right)^2 c_0^2 F^2 \left(\frac{8\alpha\hbar^2}{3\pi m^{*2} c^2}\right) \frac{\delta_{\underline{k}_1 \underline{k}_2}}{f} \left(\frac{V}{8\pi^3}\right)^2 4\pi^2$$

$$\{\int q'^3 dq' d\cos\beta_1 \delta(\mathcal{E}_{\underline{k}_1+\underline{q}'} - \mathcal{E}_{\underline{k}_1} - \hbar\omega_0)\}\{\int q''^3 dq'' d\cos\beta_2 \delta(\mathcal{E}_{\underline{k}_2+\underline{q}''} - \mathcal{E}_{\underline{k}_2} - \hbar\omega_0)\}$$

Let

$$y_1^a = \mathcal{E}_{\underline{k}_1+\underline{q}'} - \mathcal{E}_{\underline{k}_1} - \hbar\omega_0 = \frac{\hbar^2 q'^2}{2m^*} + \frac{\hbar^2 k_1 q'}{m^*} \cos\beta_1 - \hbar\omega_0$$

and

$$Q' = \frac{\hbar q'}{\sqrt{2m^*}} .$$

$$\frac{\partial(Q', y_1^a)}{\partial(q', \cos\beta_1)} = \begin{vmatrix} \frac{\hbar}{\sqrt{2m^*}} & 0 \\ \frac{\hbar^2 q'}{m^*} + \frac{\hbar^2 k_1 \cos\beta_1}{m^*} & \frac{\hbar^2 k_1 q'}{m^*} \end{vmatrix} = \frac{\hbar^3 k_1 q'}{m^* \ 2m} = \frac{\hbar^2 k_1 Q'}{m^*} .$$

Limits on Q':

$$\frac{1}{2}\left[-\sqrt{\frac{2}{m^*}}\,hk_1 + \left(\frac{2\hbar^2k_1^2}{m^*} + 4\hbar\omega_0\right)^{1/2}\right] < Q' < \frac{1}{2}\left[\sqrt{\frac{2}{m^*}}\,\hbar k_1 + \left(\frac{2\hbar^2k_1^2}{m^*} + 4\hbar\omega_0\right)^{1/2}\right].$$

as shown in Fig. 23. Therefore,

$$\int q'^3 dq' d\cos\beta_1 \delta(\mathcal{E}_{\underline{k}_1+\underline{q}'} - \mathcal{E}_{\underline{k}_1} - \hbar\omega_0) = \iint \left(\frac{\sqrt{2m^*}}{\hbar}\right)^3 Q'^3 \frac{dQ'dy_1^a}{(\hbar^2k_1Q'/m^*} \delta(y_1^a)$$

$$= \frac{1}{k_1}\left(\frac{\sqrt{2m^*}}{\hbar}\right)^3 \frac{m^*}{\hbar^2} \int_{Q_1'}^{Q_2'} Q'^2 dQ'$$

$$= \frac{1}{k_1}\left(\frac{\sqrt{2m^*}}{\hbar}\right)^3 \frac{m^*}{\hbar^2}\left[\frac{1}{3}\cdot\frac{1}{8}\left(\sqrt{\frac{2}{m^*}}\,\hbar k_1 + \left(\frac{2\hbar^2k_1^2}{m^*} + 4\hbar\omega_0\right)^{1/2}\right)^3\right.$$

$$\left. - \frac{1}{3}\cdot\frac{1}{8}\left(-\sqrt{\frac{2}{m^*}}\,\hbar k_1 + \left(\frac{2\hbar^2k_1^2}{m^*} + 4\hbar\omega_0\right)^{1/2}\right)^3\right] = \frac{1}{k_1}\left(\frac{\sqrt{2m^*}}{\hbar}\right)^3 \frac{m^*}{24\hbar^2}$$

$$\left\{\left(\sqrt{\frac{2}{m^*}}\,\hbar k_1\right)^3 + \left(\frac{2\hbar^2k_1^2}{m^*} + 4\hbar\omega_0\right)^{3/2} + 3\cdot\frac{2\hbar^2k_1^2}{m^*}\left(\frac{2\hbar^2k_1^2}{m^*} + 4\hbar\omega_0\right)^{1/2}\right.$$

$$+ 3\cdot\sqrt{\frac{2}{m^*}}\,\hbar k_1\left(\frac{2\hbar^2k_1^2}{m^*} + 4\hbar\omega_0\right) + \left(\sqrt{\frac{2}{m^*}}\,\hbar k_1\right)^3 - \left(\frac{2\hbar^2k_1^2}{m^*} + 4\hbar\omega_0\right)^{3/2}$$

$$\left. - 3\cdot\frac{2\hbar^2k_1^2}{m^*}\left(\frac{2\hbar^2k_1^2}{m^*} + 4\hbar\omega_0\right)^{1/2} + 3\cdot\sqrt{\frac{2}{m^*}}\,\hbar k_1\left(\frac{2\hbar^2k_1^2}{m^*} + 4\hbar\omega_0\right)\right\}$$

$$= \frac{1}{k_1}\left(\frac{\sqrt{2m^*}}{\hbar}\right)^3 \frac{m^*}{24\hbar^2}\left\{2\left(\sqrt{\frac{2}{m^*}}\,\hbar k_1\right)^3 + 6\sqrt{\frac{2}{m^*}}\,\hbar k_1\left(\frac{2\hbar^2k_1^2}{m^*} + 4\hbar\omega_0\right)\right\}$$

$$= \frac{1}{k_1}\left(\frac{\sqrt{2m^*}}{\hbar}\right)^3 \frac{m^*}{12\hbar^2}\left(\sqrt{\frac{2}{m^*}}\,\hbar k_1\right)\left\{4\,\frac{2\hbar^2k_1^2}{m^*} + 12\hbar\omega_0\right\}$$

$$= \left(\frac{\sqrt{2m^*}}{\hbar}\right)^3 \frac{m^*}{\hbar}\sqrt{\frac{2}{m^*}}\left\{\frac{4}{3}\left(\frac{\hbar^2k_1^2}{2m^*}\right) + \hbar\omega_0\right\} = \frac{4m^{*2}}{\hbar^4}\left\{\frac{4}{3}\left(\frac{\hbar^2k_1^2}{2m^*}\right) + \hbar\omega_0\right\}.$$

$$I_{abs} = \left(\frac{2\pi}{\hbar}\right)^2 c_0^2 F^2 \left(\frac{8\alpha\hbar^2}{3\pi m^{*2} c^2}\right) \frac{\delta_{k_1 k_2}}{f} \left(\frac{V}{8\pi^3}\right)^2 4\pi^2 \left(\frac{4m^{*2}}{\hbar^4}\right)^2$$

$$\left\{\frac{4}{3}\left(\frac{\hbar^2 k_1^2}{2m^*}\right) + \hbar\omega_0\right\} \left\{\frac{4}{3}\left(\frac{\hbar^2 k_2^2}{2m^*}\right) + \hbar\omega_0\right\} .$$

Therefore

$$S_{\Delta\tau(\mathcal{E}_{\underline{k}_1})\Delta\tau(\mathcal{E}_{\underline{k}_2})} = \tau^2(\mathcal{E}_{\underline{k}_1})\tau^2(\mathcal{E}_{\underline{k}_2}) \left(\frac{2\alpha c_0^2 V^2}{3\pi^3 m^{*2} c^2}\right) \frac{\delta_{\underline{k}_1,\underline{k}_2}}{f}$$

$$\left\{\left(\frac{4\sqrt{2}\ m^{*2}\ m^*}{3\hbar^5}\right)^2 \frac{(F+1)^2}{k_1 k_2} \left(\frac{\hbar^2 k_1^2}{2m^*} - \hbar\omega_0\right)^{1/2} \left(4\,\frac{\hbar^2 k_1^2}{2m^*} - \hbar\omega_0\right) \Theta\left(\frac{\hbar^2 k_1^2}{2m^*} - \hbar\omega_0\right)\right.$$

$$\times \left(\frac{\hbar^2 k_2^2}{2m^*} - \hbar\omega_0\right)^{1/2} \left(4\,\frac{\hbar^2 k_2^2}{2m^*} - \hbar\omega_0\right) \Theta\left(\frac{\hbar^2 k_2^2}{2m^*} - \hbar\omega_0\right) + \left(\frac{4m^{*2}}{\hbar^4}\right)^2 F^2$$

$$\left.\left(\frac{4}{3}\,\frac{\hbar^2 k_1^2}{2m^*} + \hbar\omega_0\right) \left(\frac{4}{3}\,\frac{\hbar^2 k_2^2}{2m^*} + \hbar\omega_0\right)\right\} .$$

$$S_{\Delta\tau(\mathcal{E}_{\underline{k}_1})\Delta\tau(\mathcal{E}_{\underline{k}_2})} = \tau^2(\mathcal{E}_{\underline{k}_1})\tau^2(\mathcal{E}_{\underline{k}_2}) \left(\frac{32\alpha c_0^2 V^2 m^{2*}}{3\pi^3 c^2 \hbar^8}\right) \frac{\delta_{\underline{k}_1\underline{k}_2}}{f} \left\{\left(\frac{2m^*}{9\hbar^2}\right) \frac{(F+1)^2}{k_1 k_2}\right.$$

$$\left(4\,\frac{\hbar^2 k_1^2}{2m^*} - \hbar\omega_0\right)^{1/2} \left(4\,\frac{\hbar^2 k_1^2}{2m^*} - \hbar\omega_0\right) \Theta\left(\frac{\hbar^2 k_1^2}{2m^*} - \hbar\omega_0\right)$$

$$\left.\times \left(\frac{\hbar^2 k_2^2}{2m^*} - \hbar\omega_0\right)^{1/2} \left(4\,\frac{\hbar^2 k_2^2}{2m^*}\right) \Theta\left(\frac{\hbar^2 k_2^2}{2m^*} - \hbar\omega_0\right) + F^2\left(\frac{4}{3}\,\frac{\hbar^2 k_1^2}{2m^*} + \hbar\omega_0\right)\left(\frac{4}{3}\,\frac{\hbar^2 k_2^2}{2m^*} + \hbar\omega_0\right)\right\} .$$

Using

$$\tau(\mathcal{E}_{\underline{k}}) = \left[\frac{\sqrt{2}\pi\hbar^3 \rho\omega_0}{(m^*)^{3/2} \mathcal{D}_0^2}\right] \left\{(F+1)\sqrt{\frac{\hbar^2 k^2}{2m^*} - \hbar\omega_0}\ \Theta\left(\frac{\hbar^2 k_1^2}{2m^*} - \hbar\omega_0\right)\right.$$

$$\left. + F\sqrt{\frac{\hbar^2 k^2}{2m^*} + \hbar\omega_0}\ \right\}^{-1} .$$

we find

$$\frac{S_\mu}{\bar{\mu}^2} = \frac{\left(\frac{32\alpha c_0^2 V^2 m^{*2}}{3\pi^3 c^2 \hbar^8}\right)\left(\frac{\sqrt{2}\pi\hbar^3\rho\omega_0}{m^{*3/2}\mathcal{D}_0^2}\right)^4 \frac{1}{f}}{\{\sum_k v_{\underline{k}}^2 \tau(\mathcal{E}_{\underline{k}}) f(v_{\underline{k}})\}^2} \times \sum_{\underline{k}_1 \underline{k}_2} \{\frac{2m^*}{9\hbar^2}(F+1)^2\left(\frac{\hbar^2 k_1^2}{2m^*} - \hbar\omega_0\right)^{1/2}$$

$$\left(4\frac{\hbar^2 k_1^2}{2m^*} - \hbar\omega_0\right)\Theta\left(\frac{\hbar^2 k_1^2}{2m^*} - \hbar\omega_0\right) \times \left(\frac{\hbar^2 k_2^2}{2m^*} - \hbar\omega_0\right)^{1/2} \times \left(4\frac{\hbar^2 k_2^2}{2m^*} - \hbar\omega_0\right)$$

$$\Theta\left(\frac{\hbar^2 k_2^2}{2m^*} - \hbar\omega_0\right)\frac{\delta_{\underline{k}_1,\underline{k}_2}}{k_1 k_2} + F^2\left(\frac{4}{3}\frac{\hbar^2 k_1^2}{2m^*} + \hbar\omega_0\right)\left(\frac{4}{3}\frac{\hbar^2 k_2^2}{2m^*} + \hbar\omega_0\right)\delta_{\underline{k}_1\underline{k}_2}\}$$

$$\{(F+1)\left(\frac{\hbar^2 k_1}{2m^*} - \hbar\omega_0\right)^{1/2}\Theta\left(\frac{\hbar^2 k_1^2}{2m^*} - \hbar\omega_0\right) + F\left(\frac{\hbar^2 k_1^2}{2m^*} + \hbar\omega_0\right)^{1/2}\}^{-2}$$

$$\{(F+1)\left(\frac{\hbar^2 k_2^2}{\quad} - \hbar\omega_0\right)^{1/2}\Theta\left(\frac{\hbar^2 k_2^2}{2m^*} - \hbar\omega_0\right) + F\left(\frac{\hbar^2 k_2^2}{2m^*} + \hbar\omega_0\right)^{1/2}\}^{-2} v_{\underline{k}_1}^2 v_{\underline{k}_2}^2$$

$$\times f(v_{\underline{k}_1}) f(v_{\underline{k}_2}) \ .$$

Here

$$\sum_{\underline{k}_1 \underline{k}_2} \to \frac{V^2}{(8\pi^3)^2} \iint d^3k_1 d^3k_2 \to \left(\frac{V}{8\pi^3}\right)^2 \iint k_1^2 dk_1 d\cos\chi_1 d\phi' k_2^2 dk_2 d\cos\chi_2 d\phi''$$

and

$$\delta_{\underline{k}_1\underline{k}_2} \to \frac{\delta(\underline{k}_1 - \underline{k}_2)}{(V/8\pi^3)} \to \frac{\delta(k_1 - k_2)}{(V/8\pi^3)k_2^2 f(v_{k_2})}\delta(\cos\chi_1 - \cos\chi_2)\delta(\phi' - \phi'') \ .$$

The integrals have the form

$$\int_0^{k_D}\int_0^{k_D} k_1^2 dk_1 k_2^2 dk_2 R(k_1)R(k_2)\frac{\delta(k_1 - k_2)}{k_2^2}\int_{-1}^{1}\int_{-1}^{1} d\cos\chi_1 d\cos\chi_2$$

$$\delta(\cos\chi_1 - \cos\chi_2) \int_0^{2\pi}\int_0^{2\pi} d\phi' d\phi'' \delta(\phi' - \phi'') .$$

Now let $x = k_1 - k_2$ and $z = k_1$.

$$dk_1 dk_2 = dxdz \Big/ \frac{\partial(x,z)}{\partial(k_1,k_2)} = dxdz .$$

Referring to Figure 24, we find

$$4\pi \int_0^{k_D} dz\ z^2 R(z) \int dx R(z - x)\delta(x) = 4\pi \int_0^{k_D} z^2 R^2(z) dz .$$

Therefore

$$\frac{S_\mu}{\bar{\mu}^2} = \frac{1}{f} \frac{\left(\frac{128\pi^2 \alpha g^6 \rho^2 v_0^2}{3c^2 m^{*4} \, {}_0^4}\right)\left(\frac{V}{8\pi^3}\right)}{\{\sum_{\underline{k}} v_{\underline{k}}^2 \tau(\mathcal{E}_{\underline{k}}) f(v_{\underline{k}})\}^2} \int_0^{k_D} dk \cdot k^2 \{ \frac{2m^*}{9h^2} \frac{(F + 1)^2}{k^2} \left(\frac{\hbar^2 k^2}{2m^*} - \hbar\omega_0\right)$$

$$\left(4 \frac{\hbar^2 k^2}{2m^*} - \hbar\omega_0\right)^2 \Theta\left(\frac{\hbar^2 j^2}{2m^*} - \hbar\omega_0\right) + F^2\left(\frac{4}{3}\frac{\hbar^2 k^3}{2m^*} + \hbar\omega_0\right)^2 \} \{ (F + 1)\left(\frac{\hbar^2 k^2}{2m^*} - \hbar\omega_0\right)^{1/2}$$

$$\Theta\left(\frac{\hbar^2 k^2}{2m^*} - \hbar\omega_0\right) + F\left(\frac{\hbar^2 k^2}{2m^*} + \hbar\omega_0\right)^{1/2} \}^{-4} v_{\underline{k}}^4 f(v_{\underline{k}}) .$$

D. Papers and Theses Published under the Contract

A. Papers

1. J. Kilmer, A. van der Ziel and G. Bosman, "Mobility Fluctuation 1/f Noise in p-n-p Transistors," Solid State Electr. (in press).

2. A. van der Ziel and P.H. Handel, "Quantum Partition 1/f Noise in Pentodes," Physica (in press).

3. J. Kilmer, C.M. Van Vliet, G. Bosman, A. van der Ziel and P.H. Handel, "Evidence of Electromagnetic Quantum 1/f Noise Found in Gold Thin Films," Phys. Stat. Solidi (b) 121, 429 (1984).

4. Jean Andrian, Gijs Bosman, A. van der Ziel and Carolyn Van Vliet, "Hot Electron Diffusion Noise Associated with Intervalley Scattering in Very Short GaAs Devices," Solid State Electr. (submitted).

Theses

1. Joyce P. Kilmer, "Identification of 1/f Noise Producing Mechanisms in Electronic Devices," U. of Florida, 1984.

2. Jean Andrian, "Noise Measurements and Bulk Properties of Gallium Arsenide" (in preparation--Spring, 1985).

3. Ganesh Kousik, "Mobility Fluctuations Associated with Various Scattering Processes Involving Quantum 1/f Noise," (in preparation--Spring, 1985).

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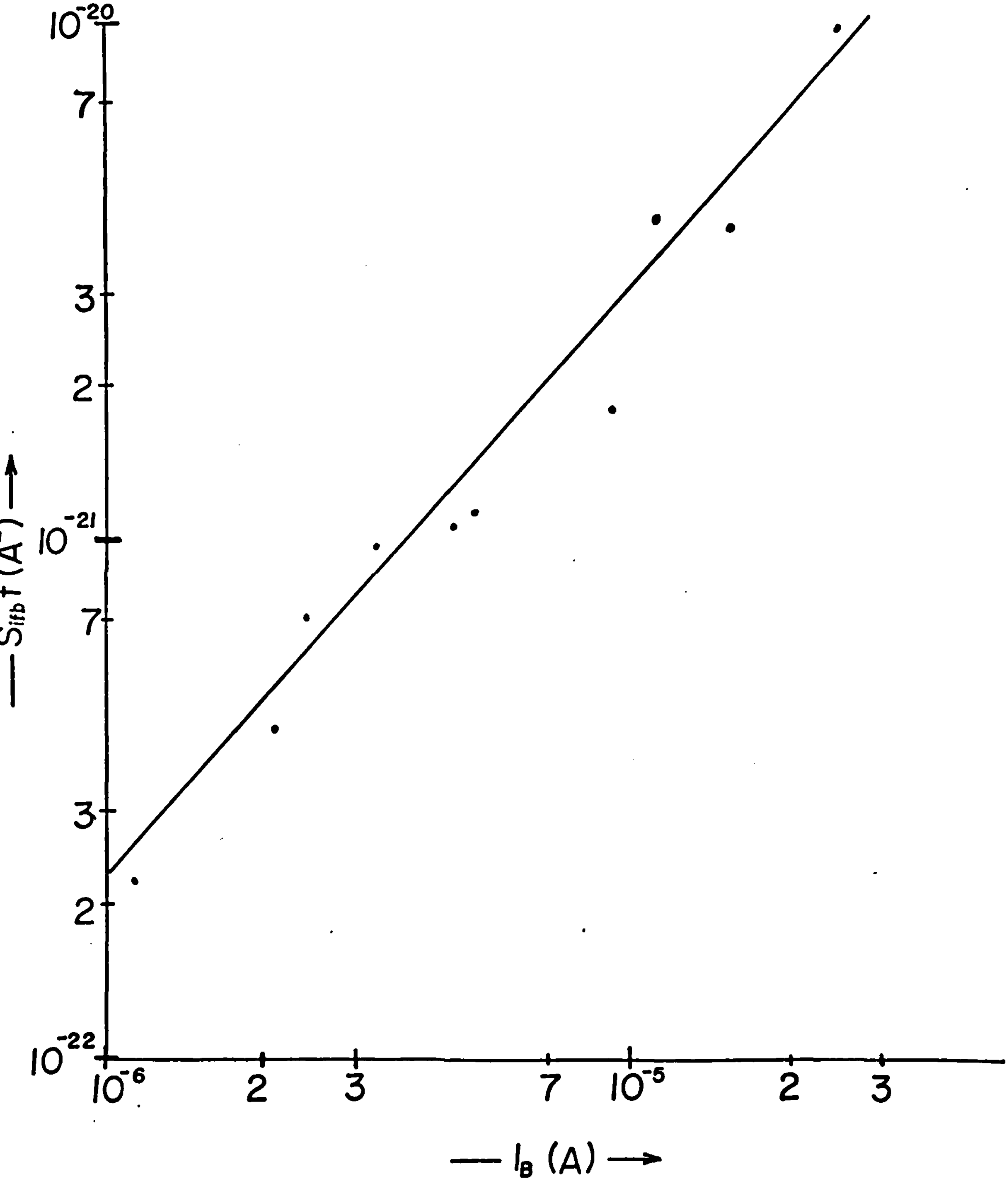



Figure 1

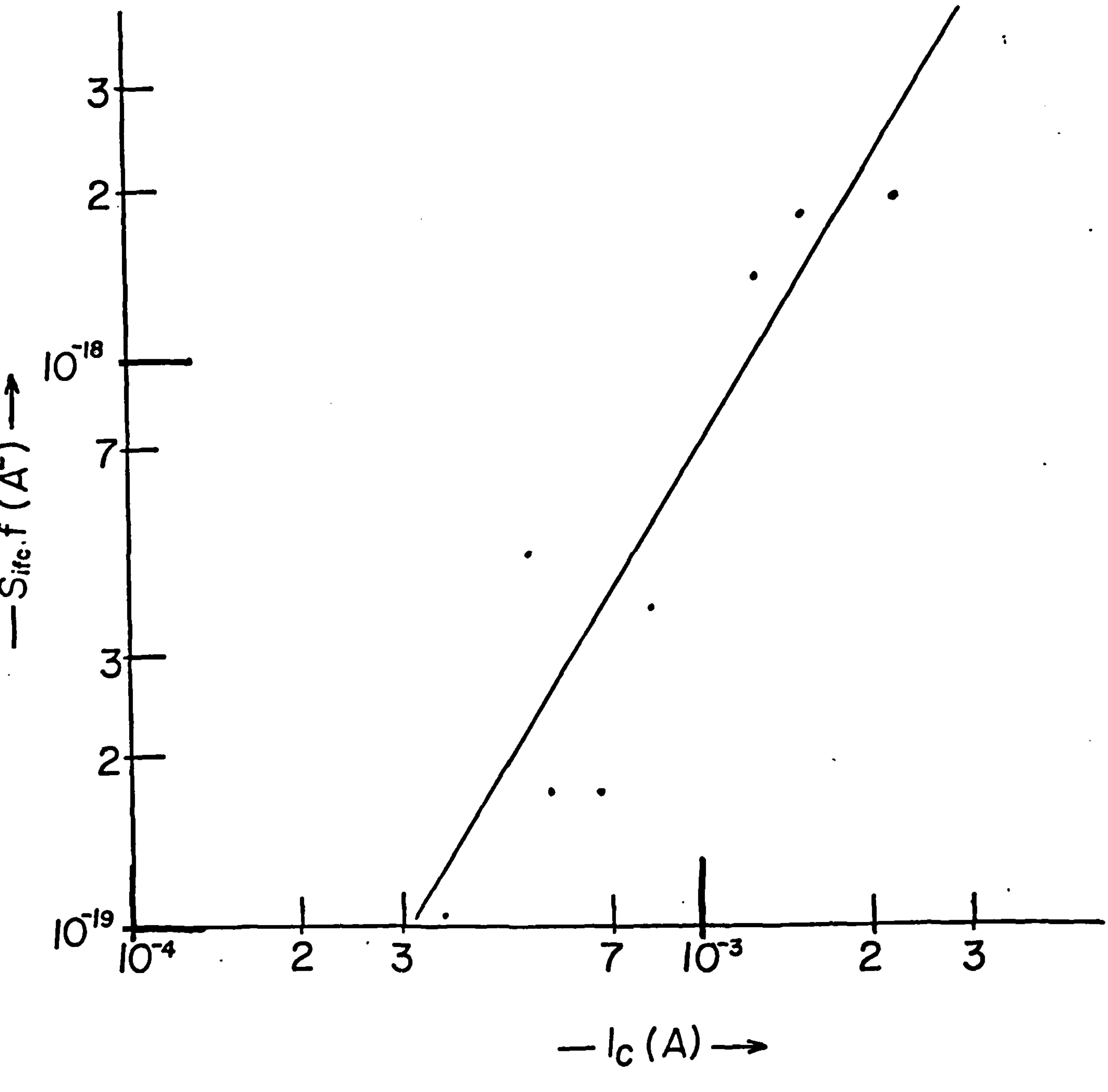



**Figure 2**

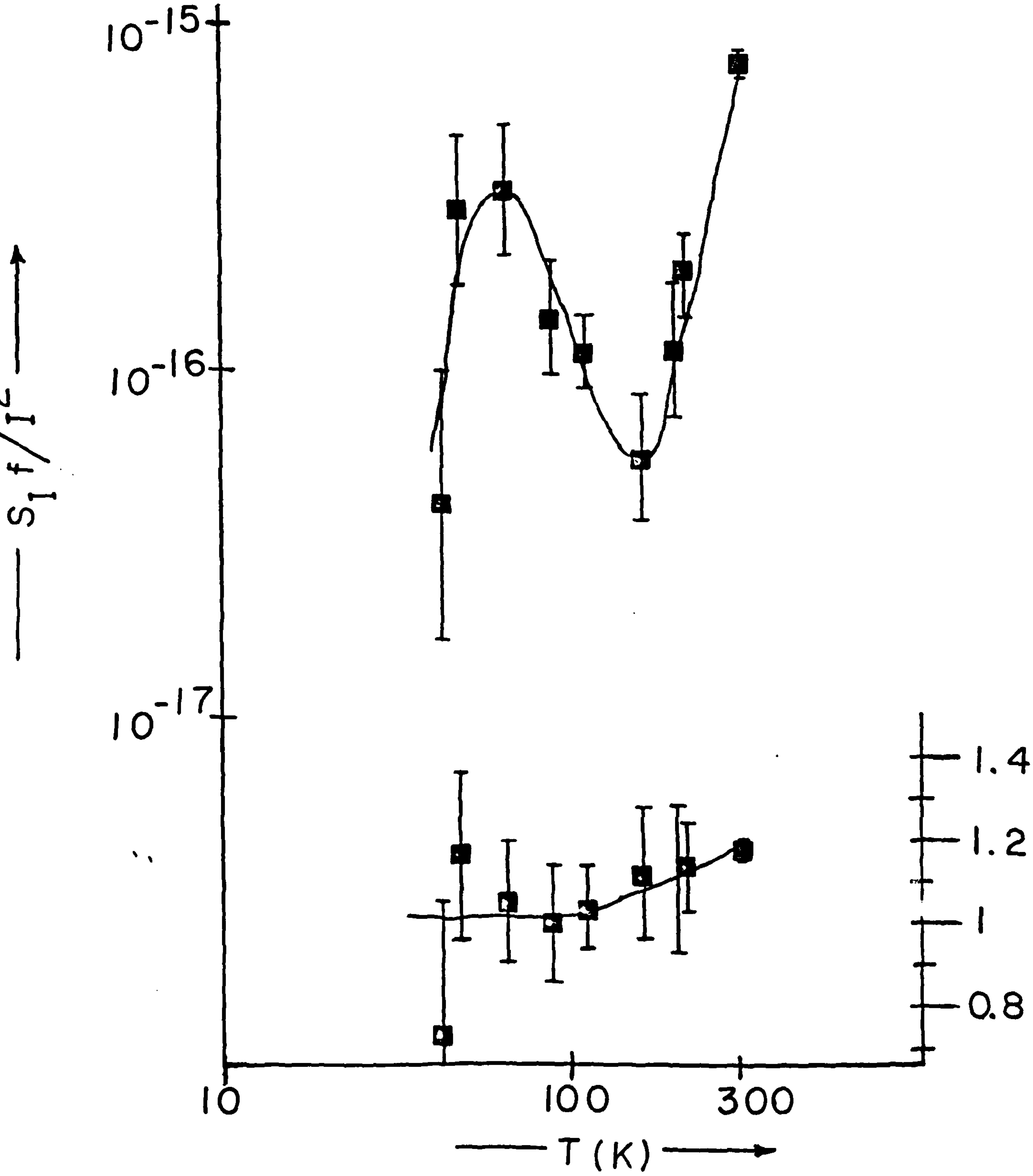



Figure 3

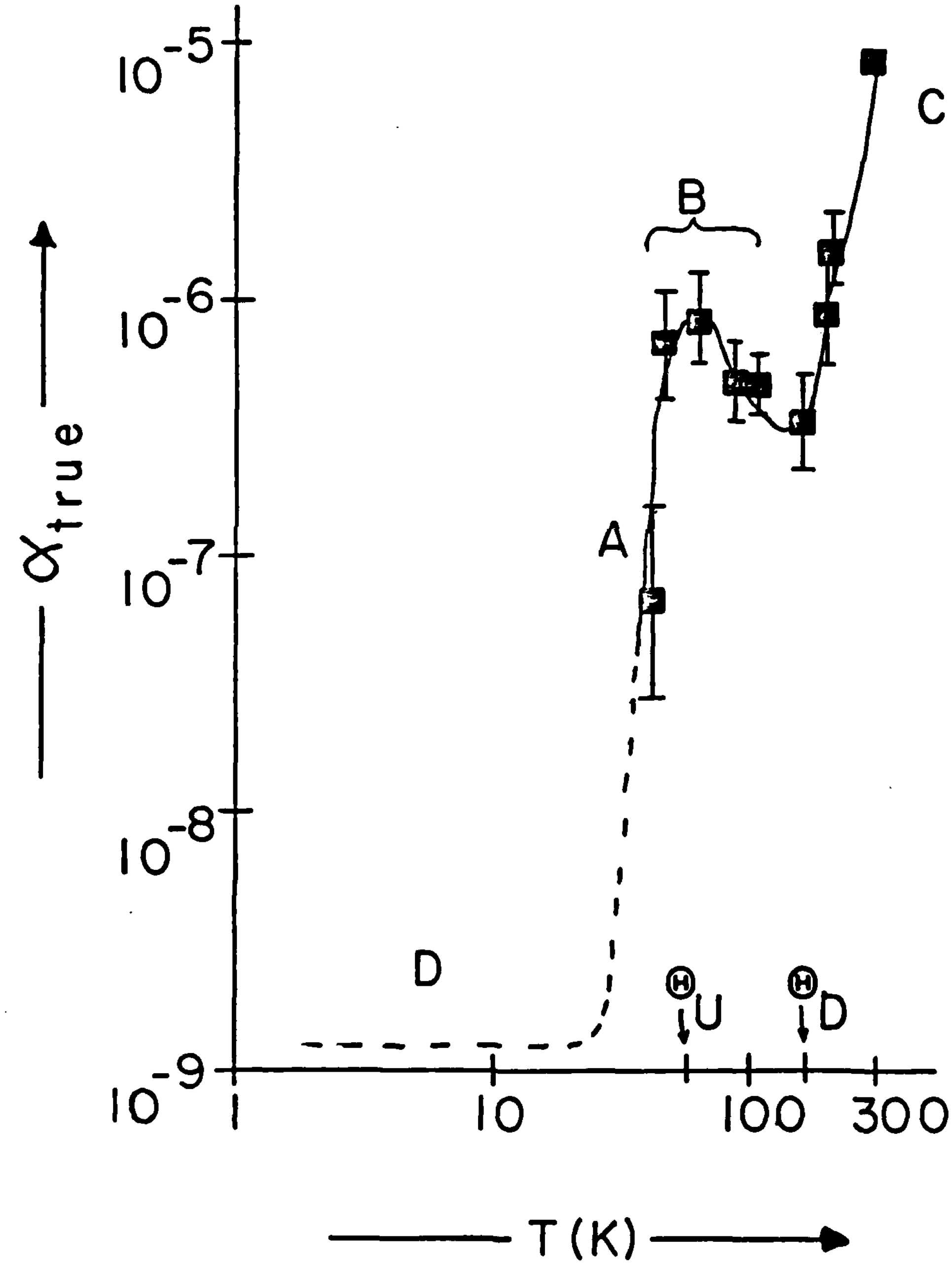



Figure 4

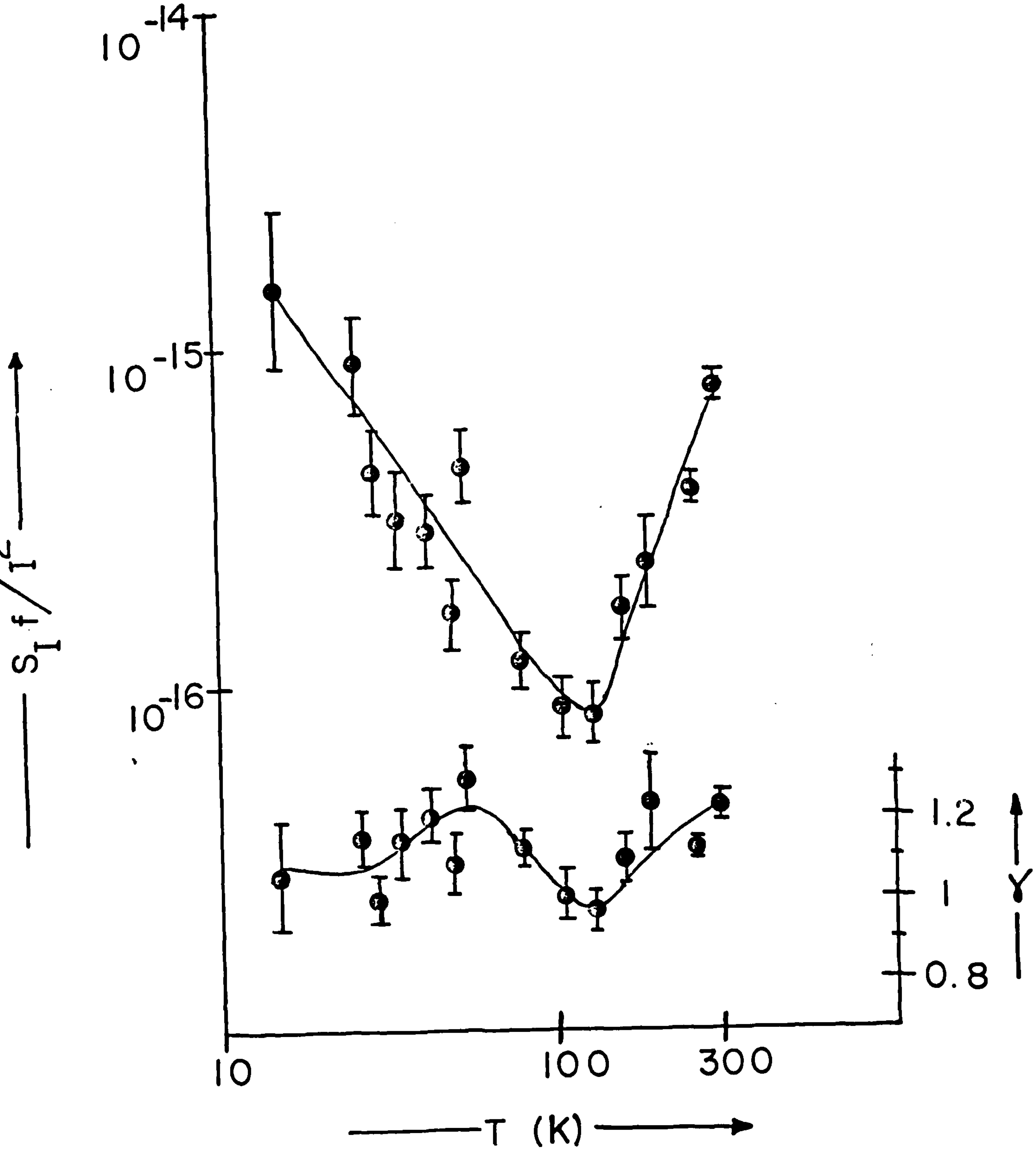



Figure 5

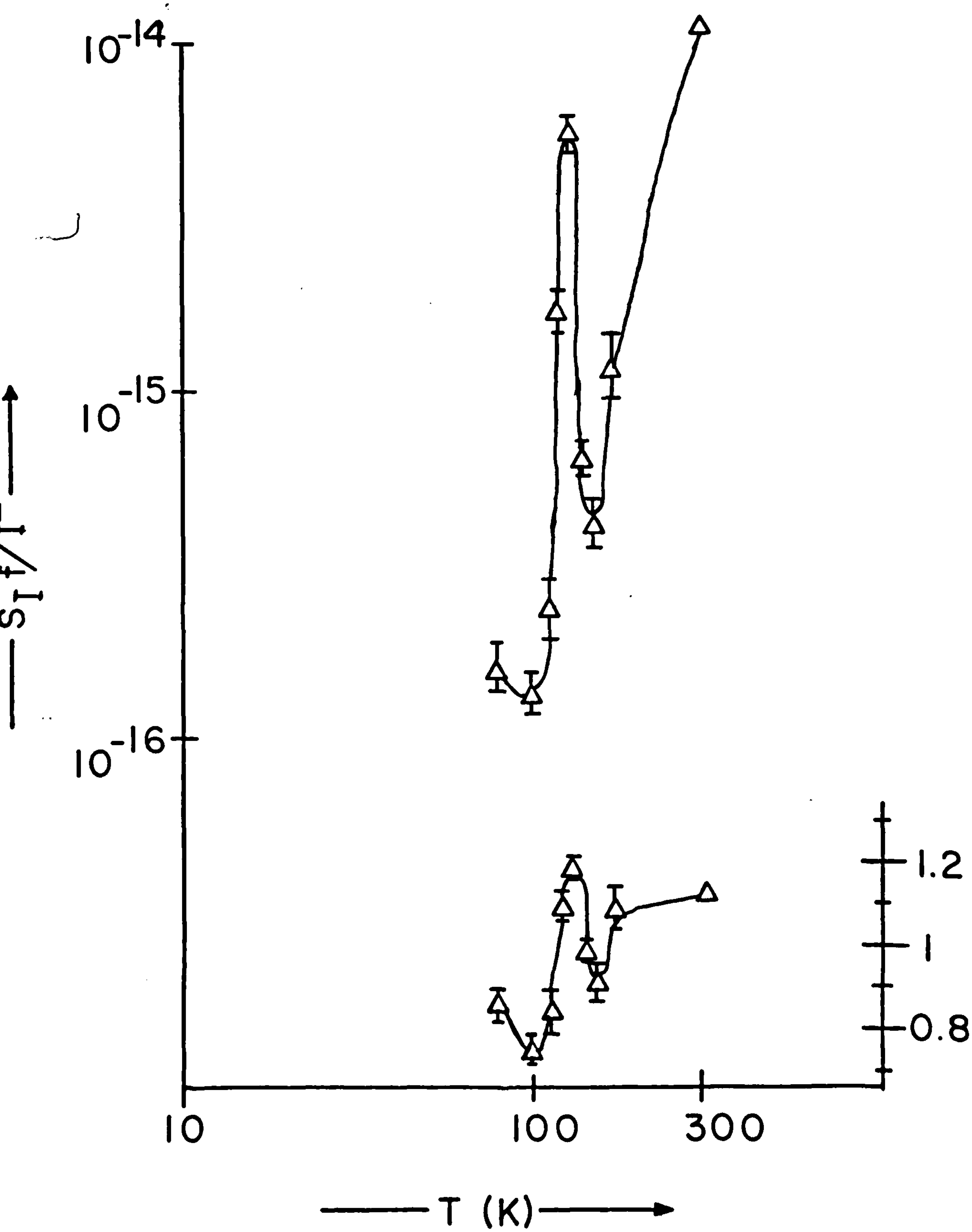



**Figure 6**

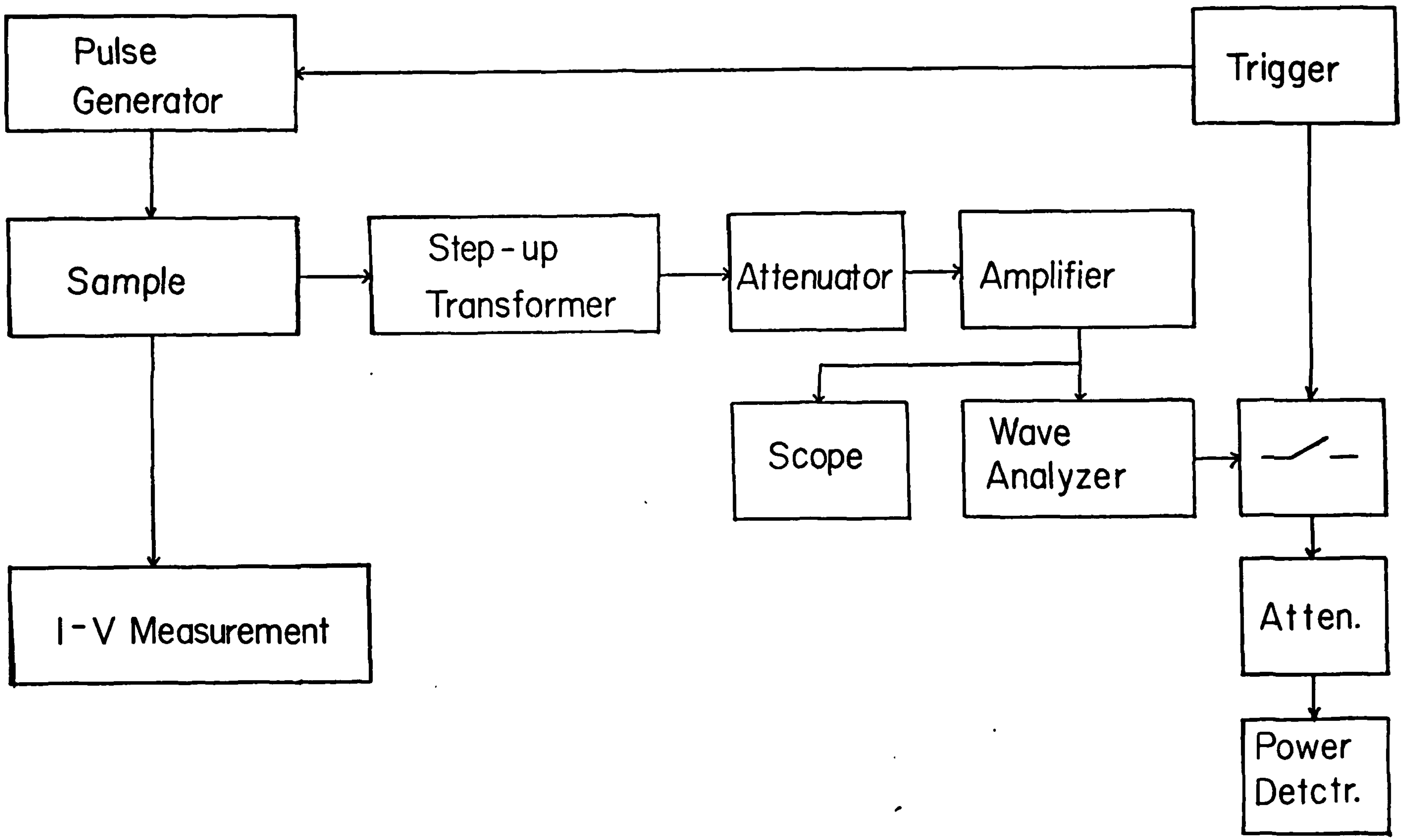

Pulse Generator
Trigger
Sample
Step-up Transformer
Attenuator
Amplifier
Scope
Wave Analyzer
Atten.
Power Detctr.
I-V Measurement

Figure 7

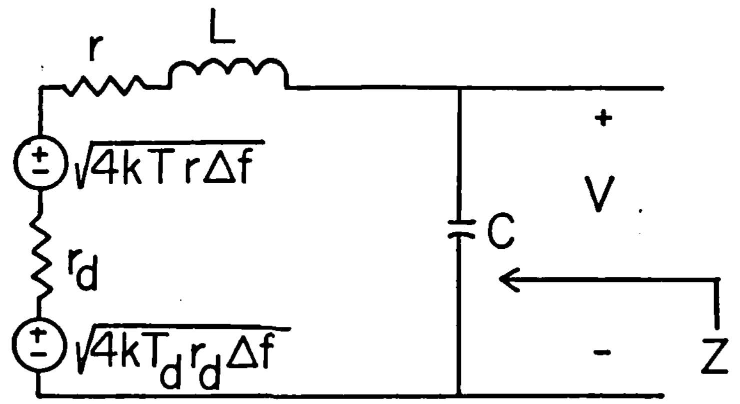



**Figure 8**

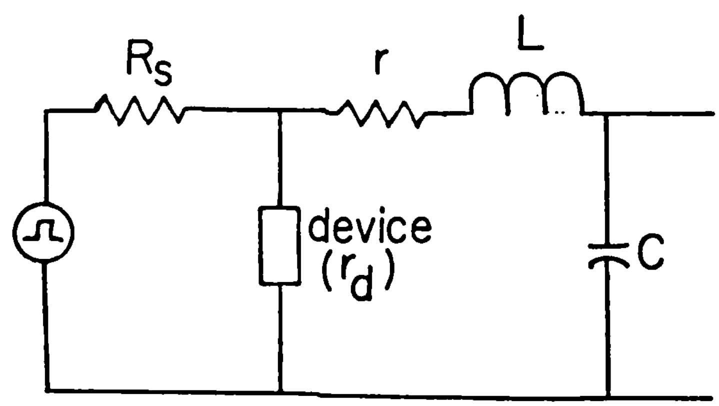



**Figure 9**

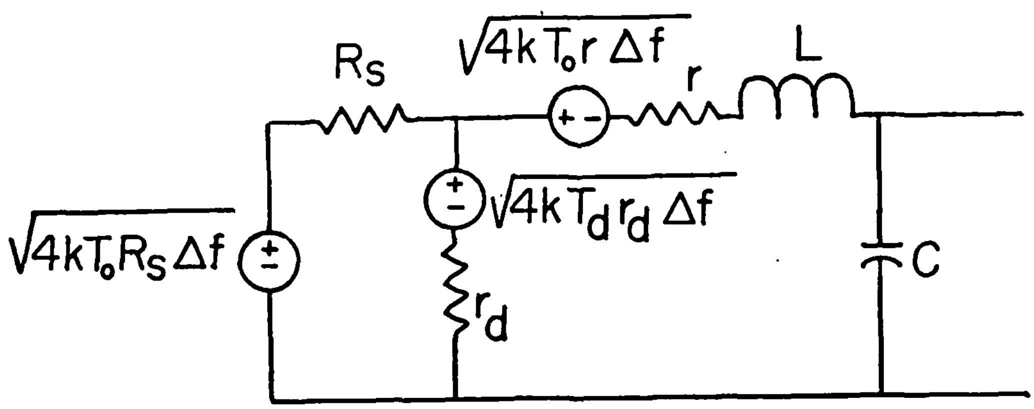



**Figure 10**

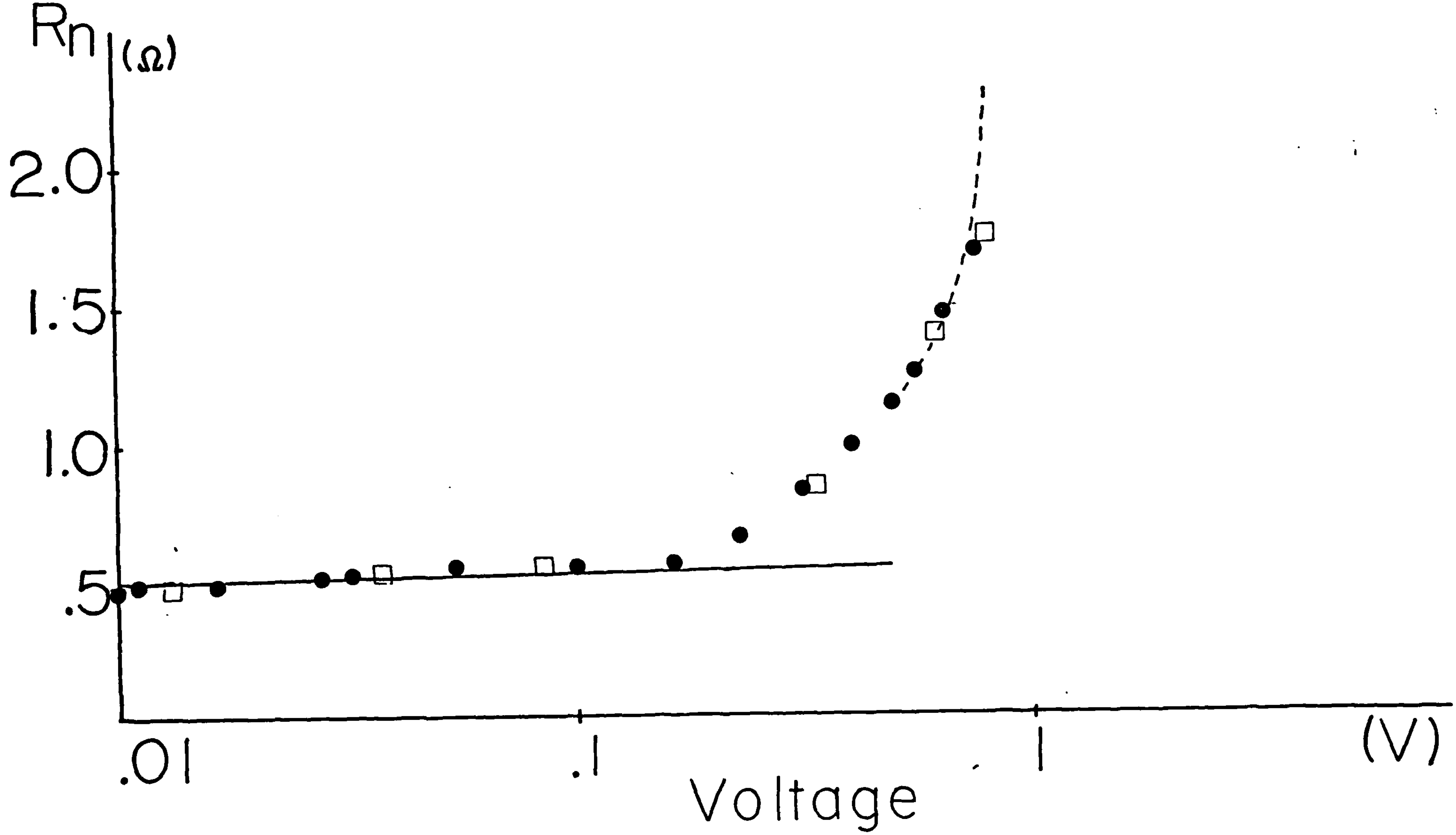



**Figure 11**

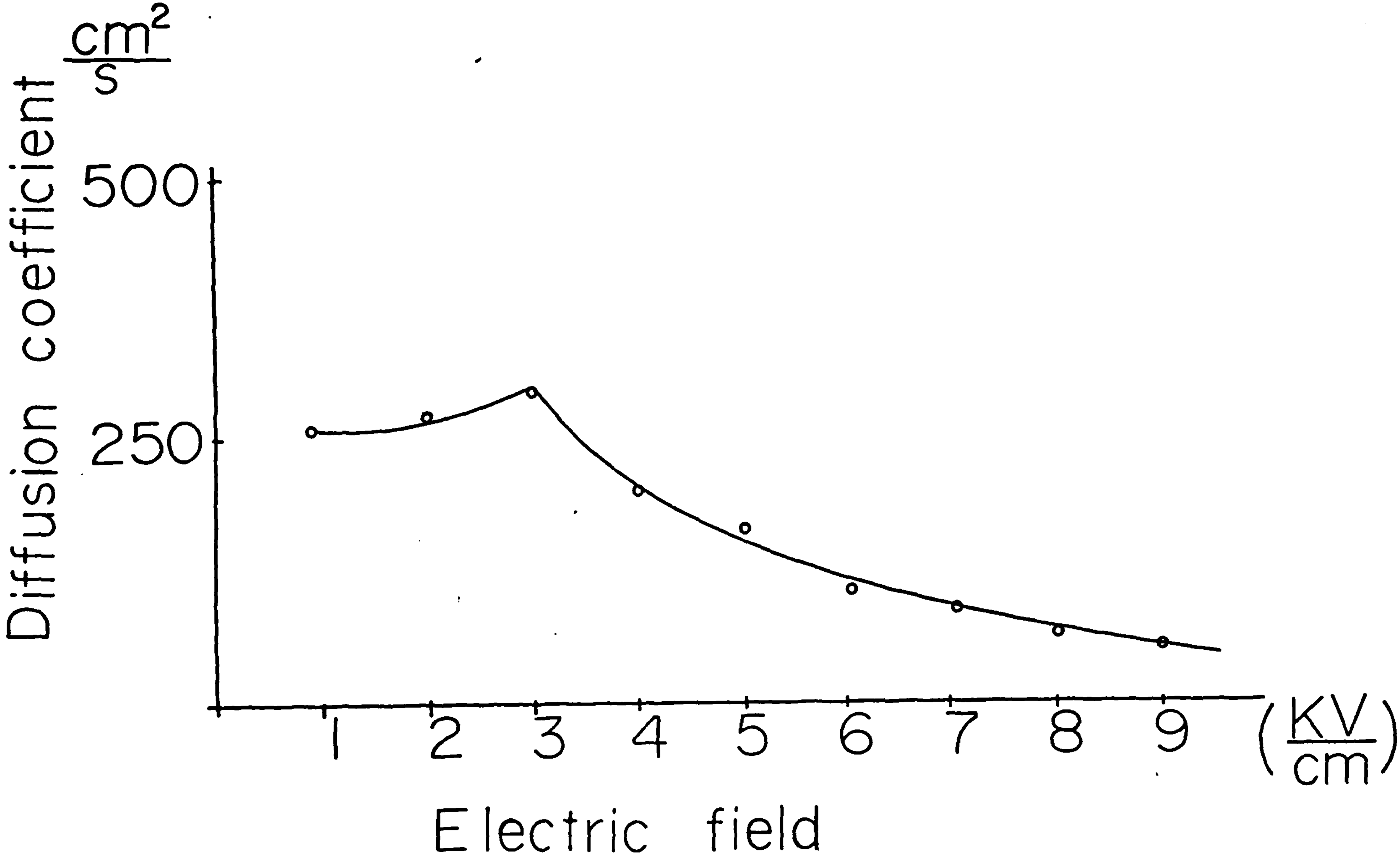

cm²/s
Diffusion coefficient
500
250
1
2
3
4
5
6
7
8
9
(KV/cm)
Electric field

**Figure 12**

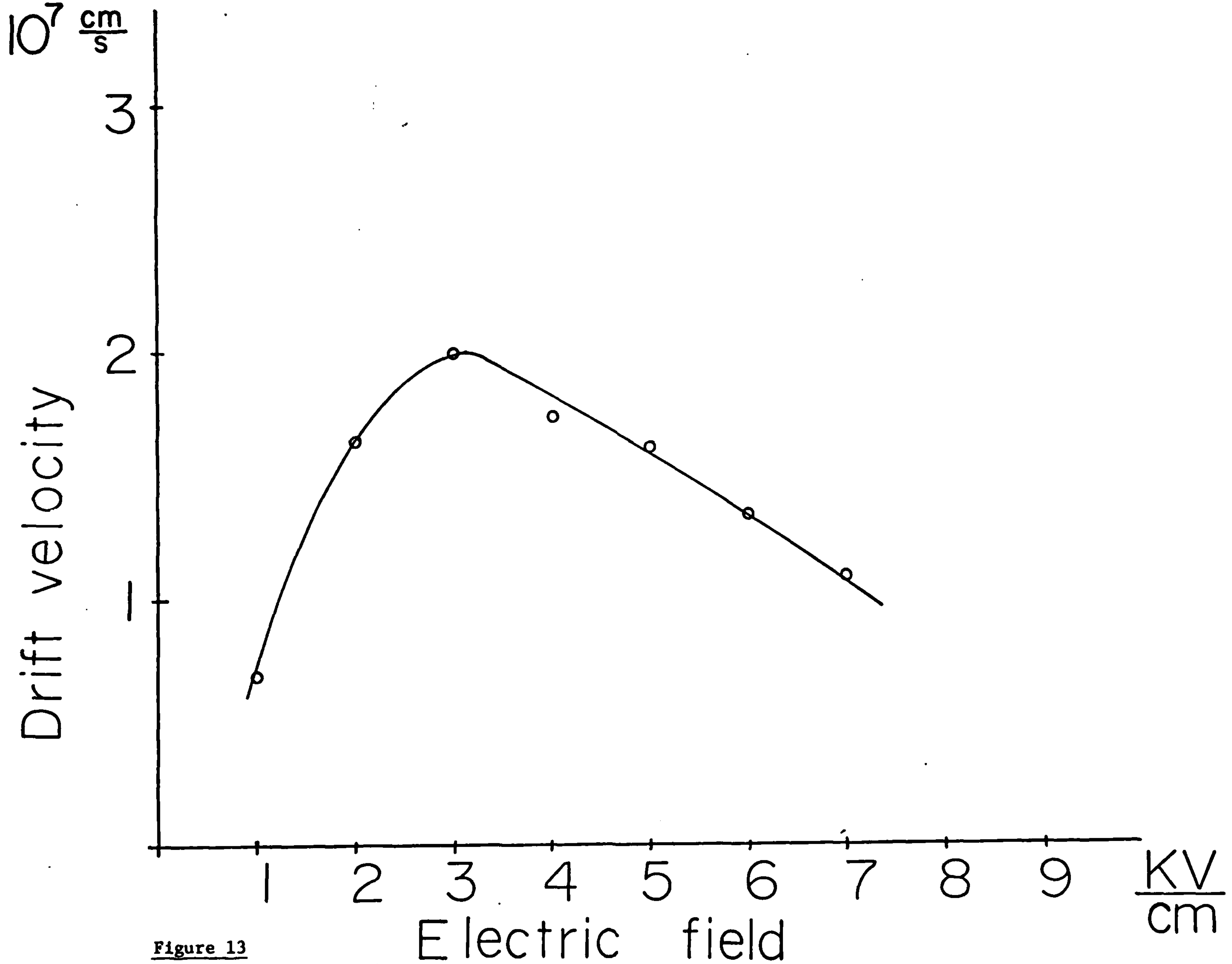



**Figure 13**

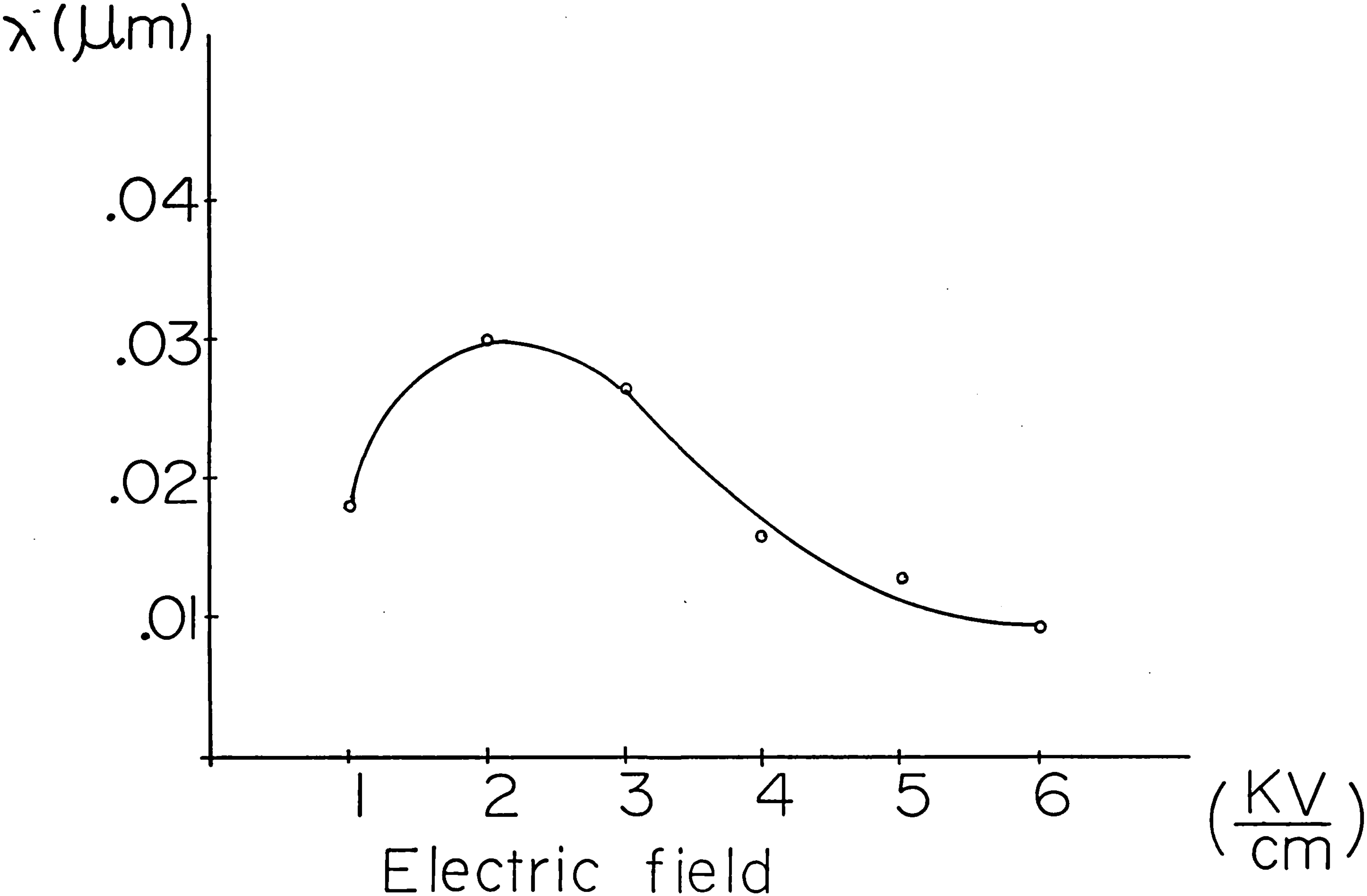



**Figure 14**

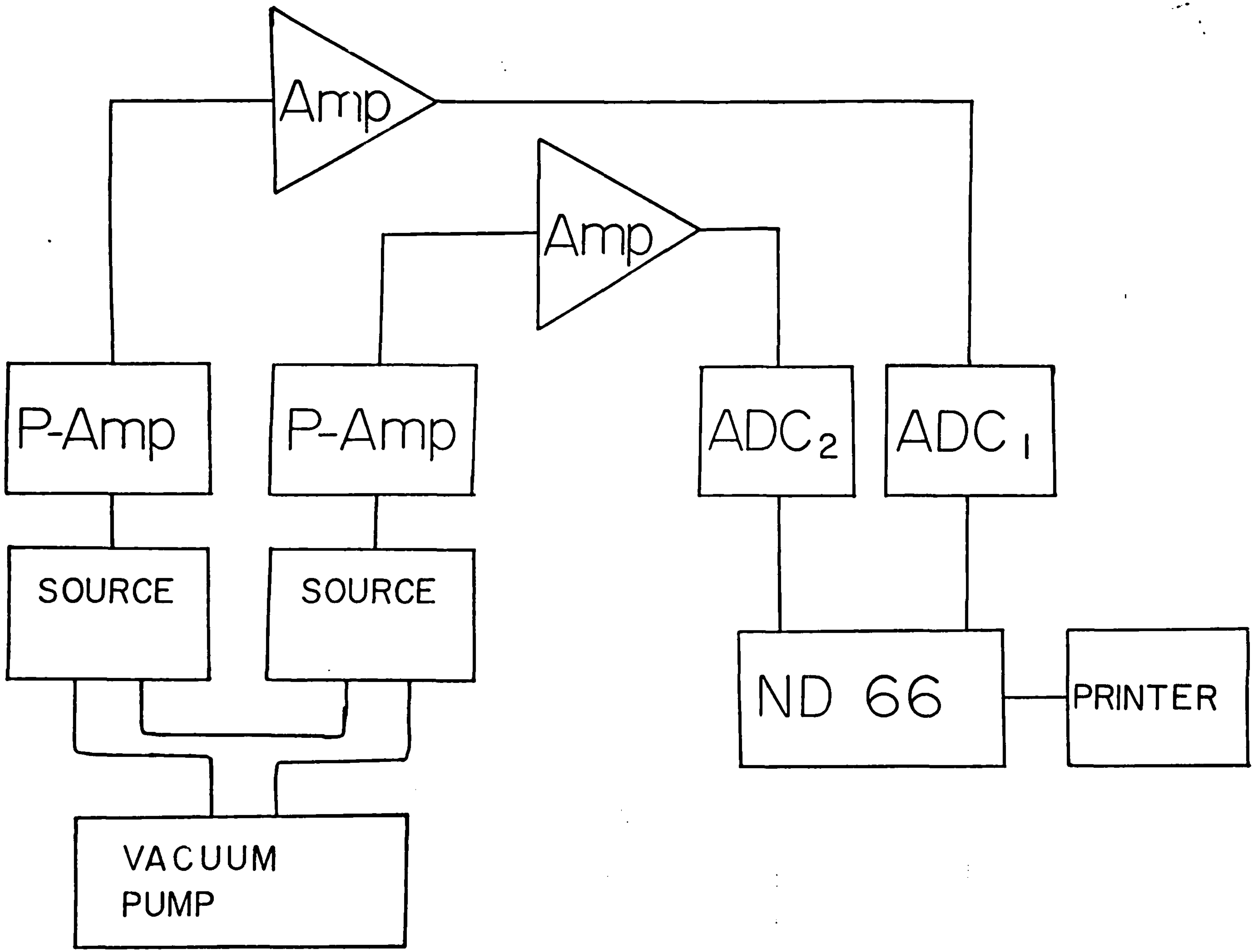
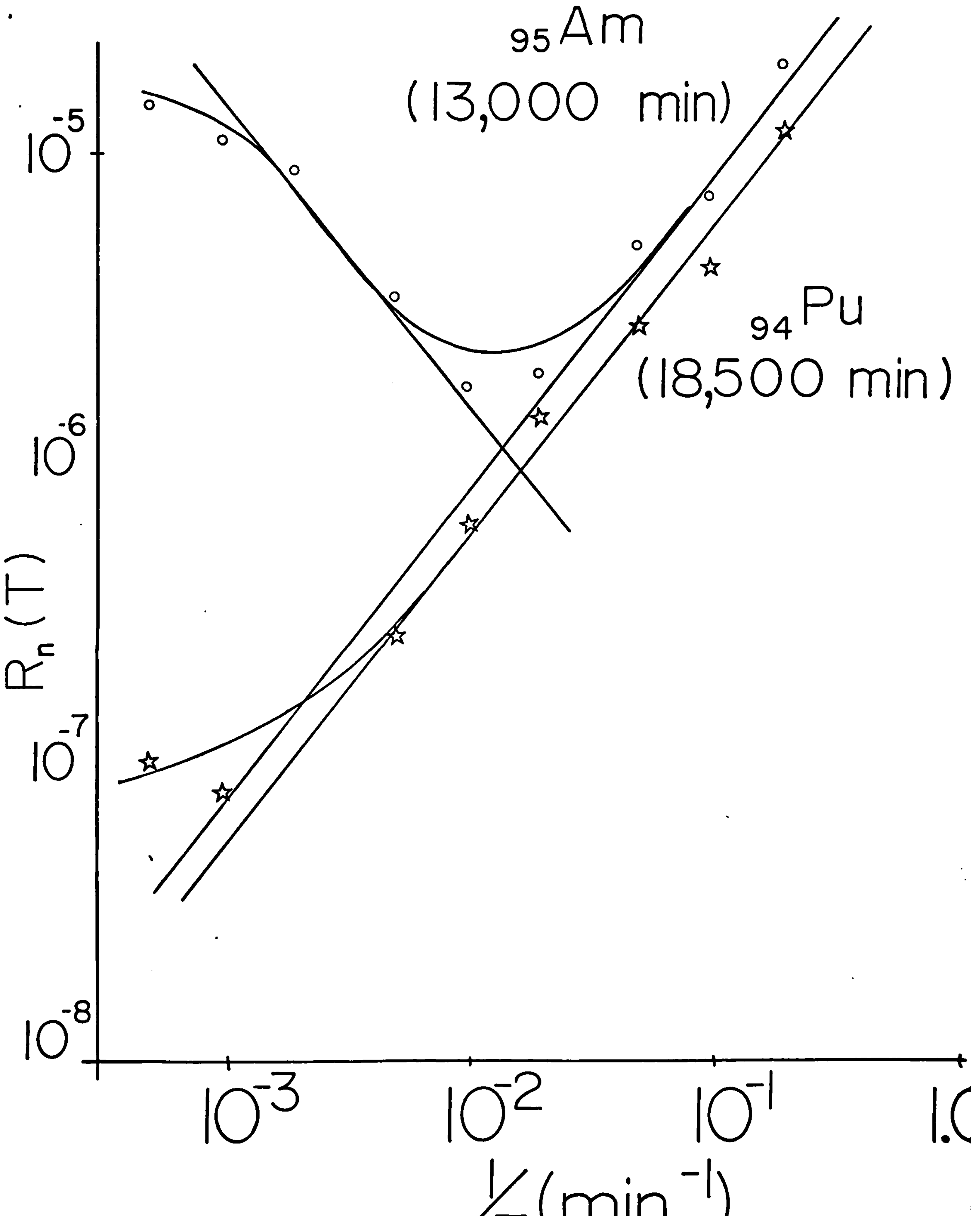
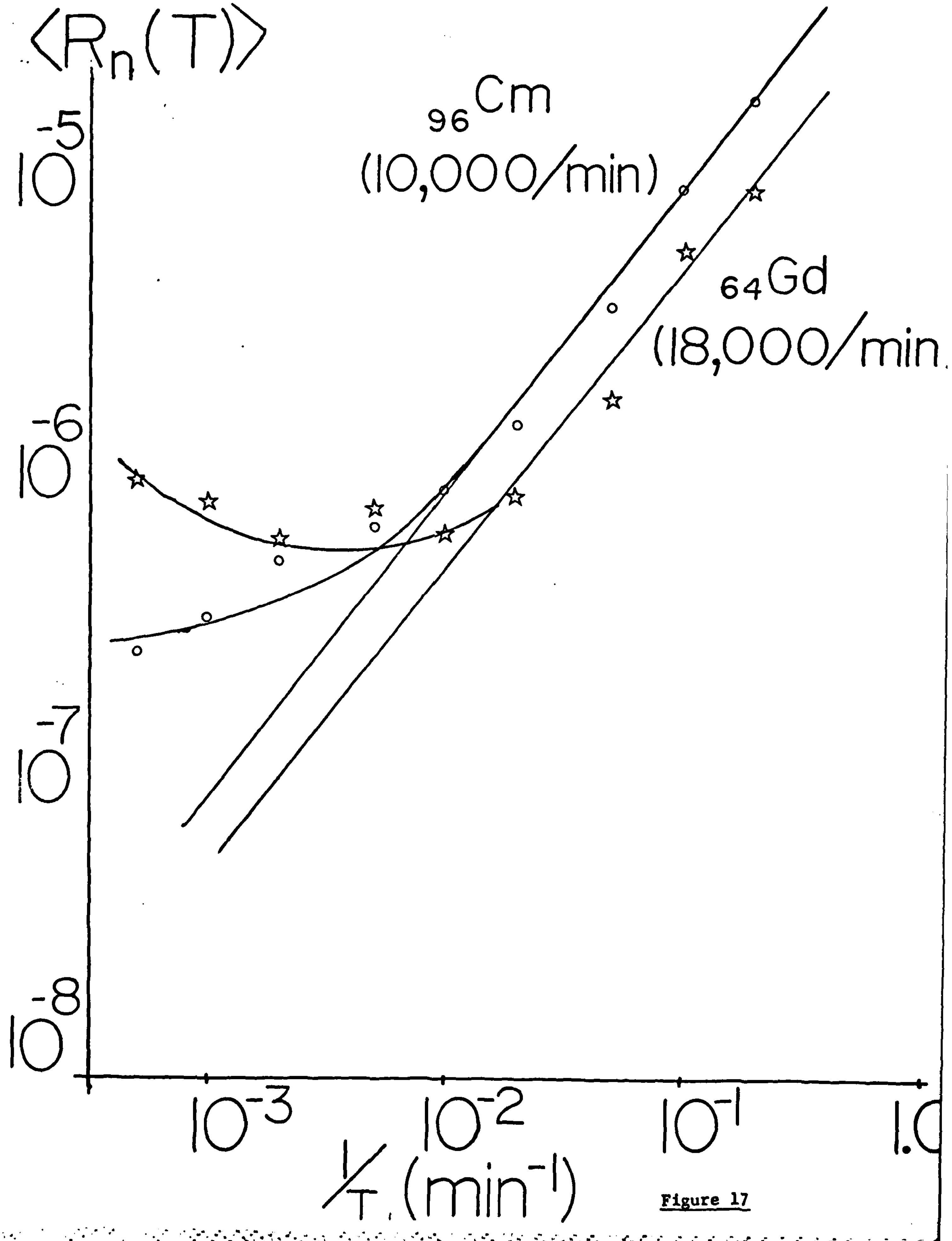

Amp
Amp
P-Amp
P-Amp
ADC2
ADC1
SOURCE
SOURCE
ND 66
PRINTER
VACUUM
PUMP

**Figure 15**



$_{95}$Am
(13,000 min)
$_{94}$Pu
(18,500 min)
$10^{-5}$
$10^{-6}$
$10^{-7}$
$10^{-8}$
$R_n$ (T)
$10^{-3}$
$10^{-2}$
$10^{-1}$
1.0
1/T (min$^{-1}$)


Figure 16



⟨Rn(T)⟩
96Cm
(10,000/min)
64Gd
(18,000/min
10-5
10-6
10-7
10-8
10-3
10-2
10-1
1.0
1/T (min-1)


**Figure 17**

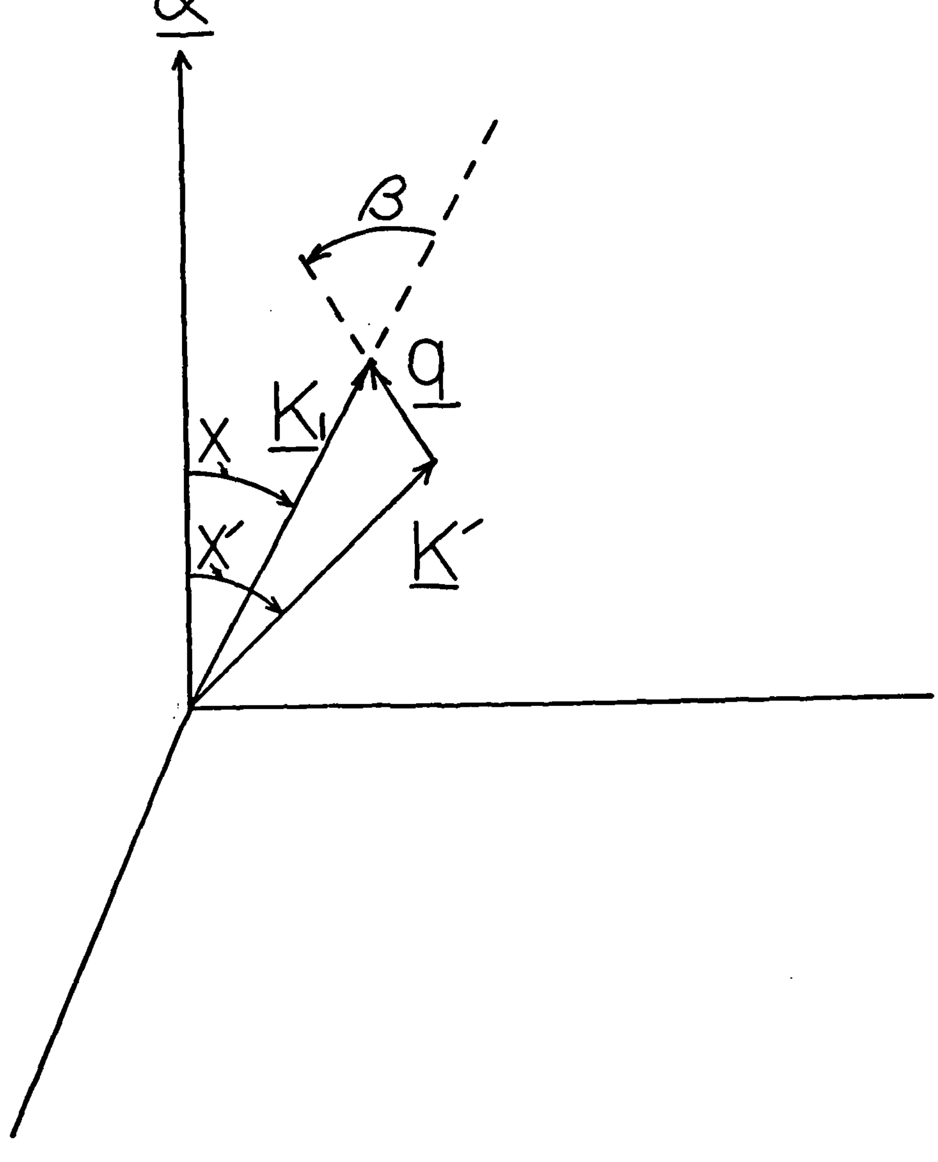



**Figure 18**

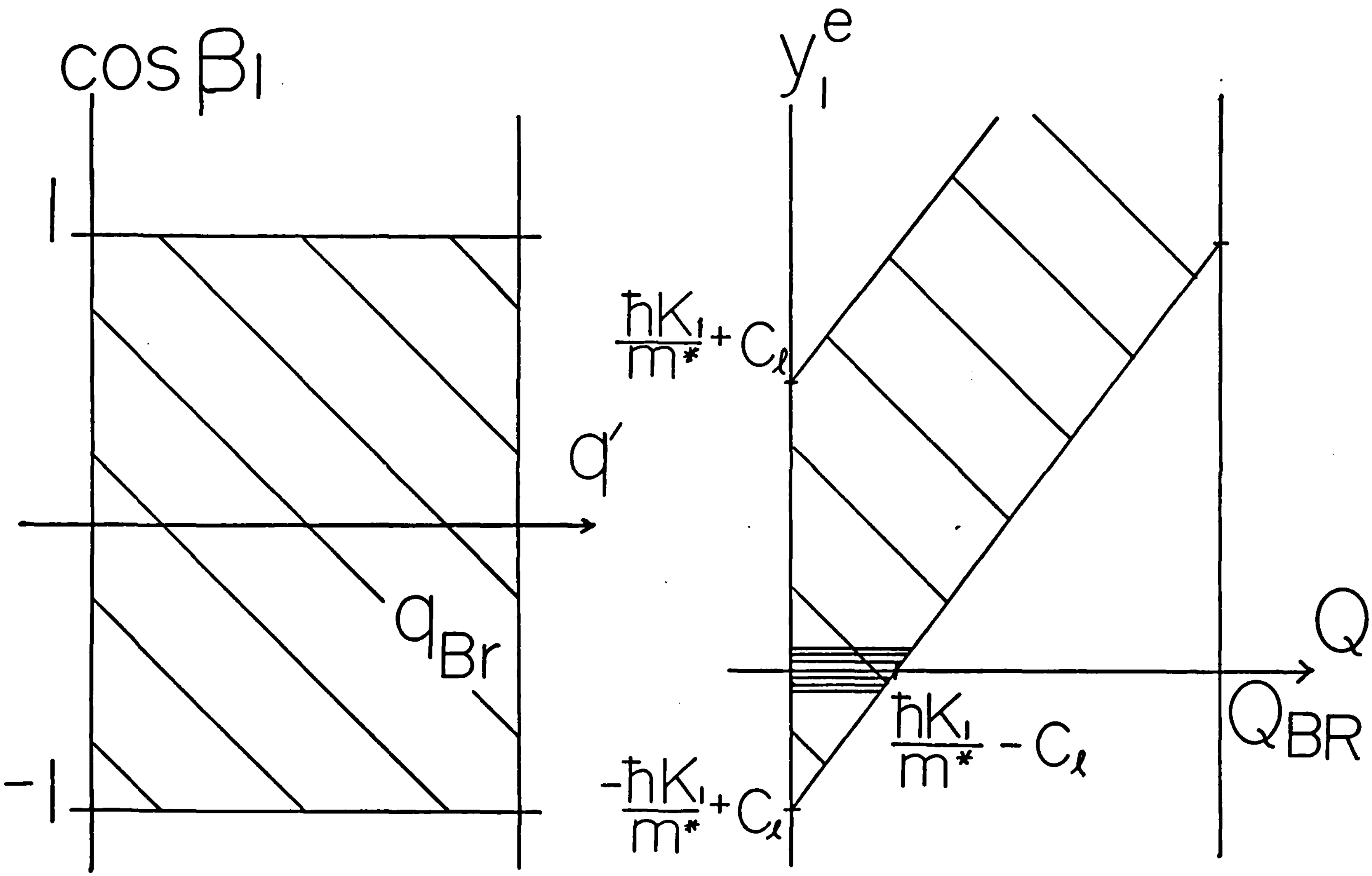



Figure 19

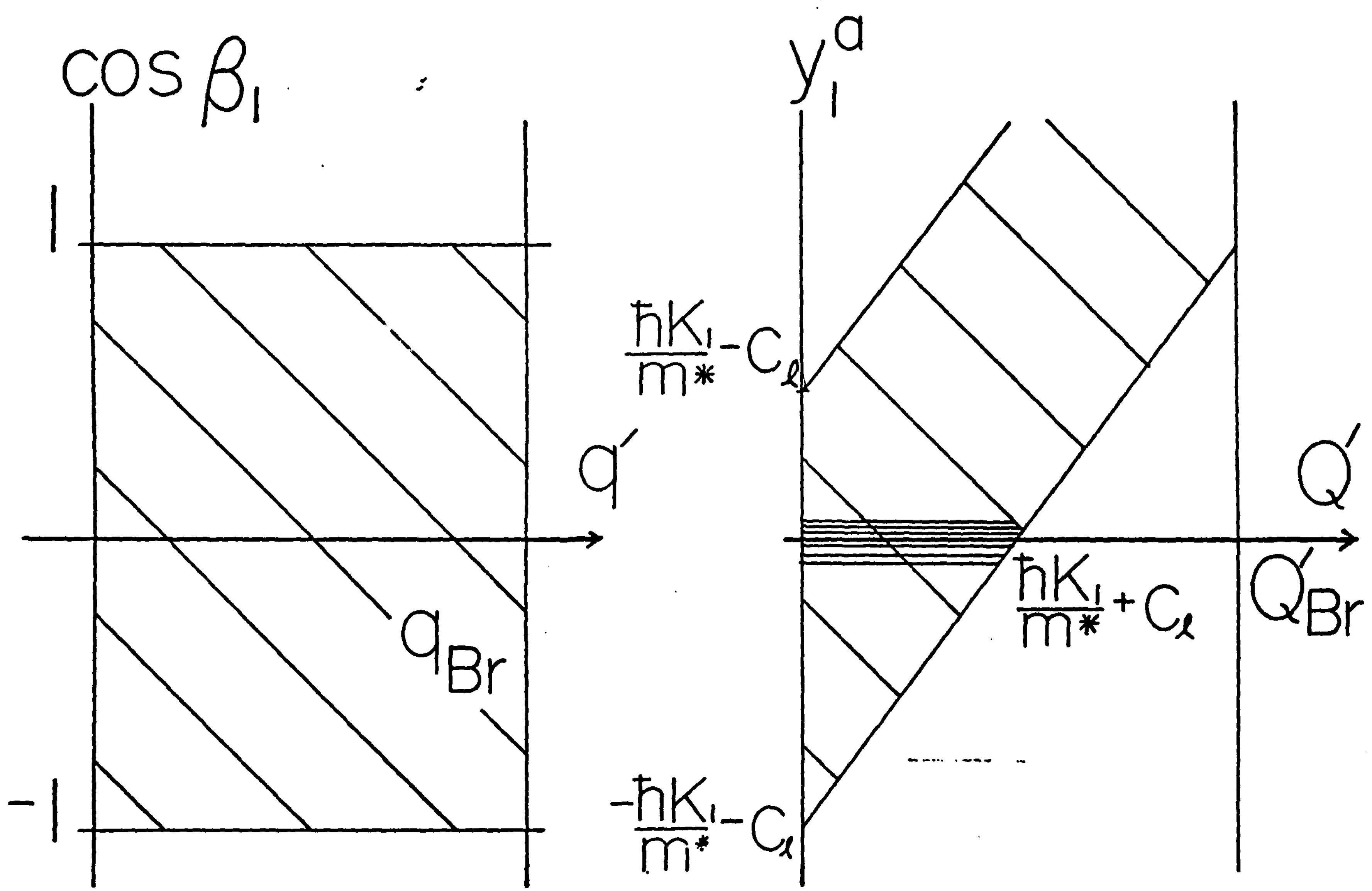



**Figure 20**

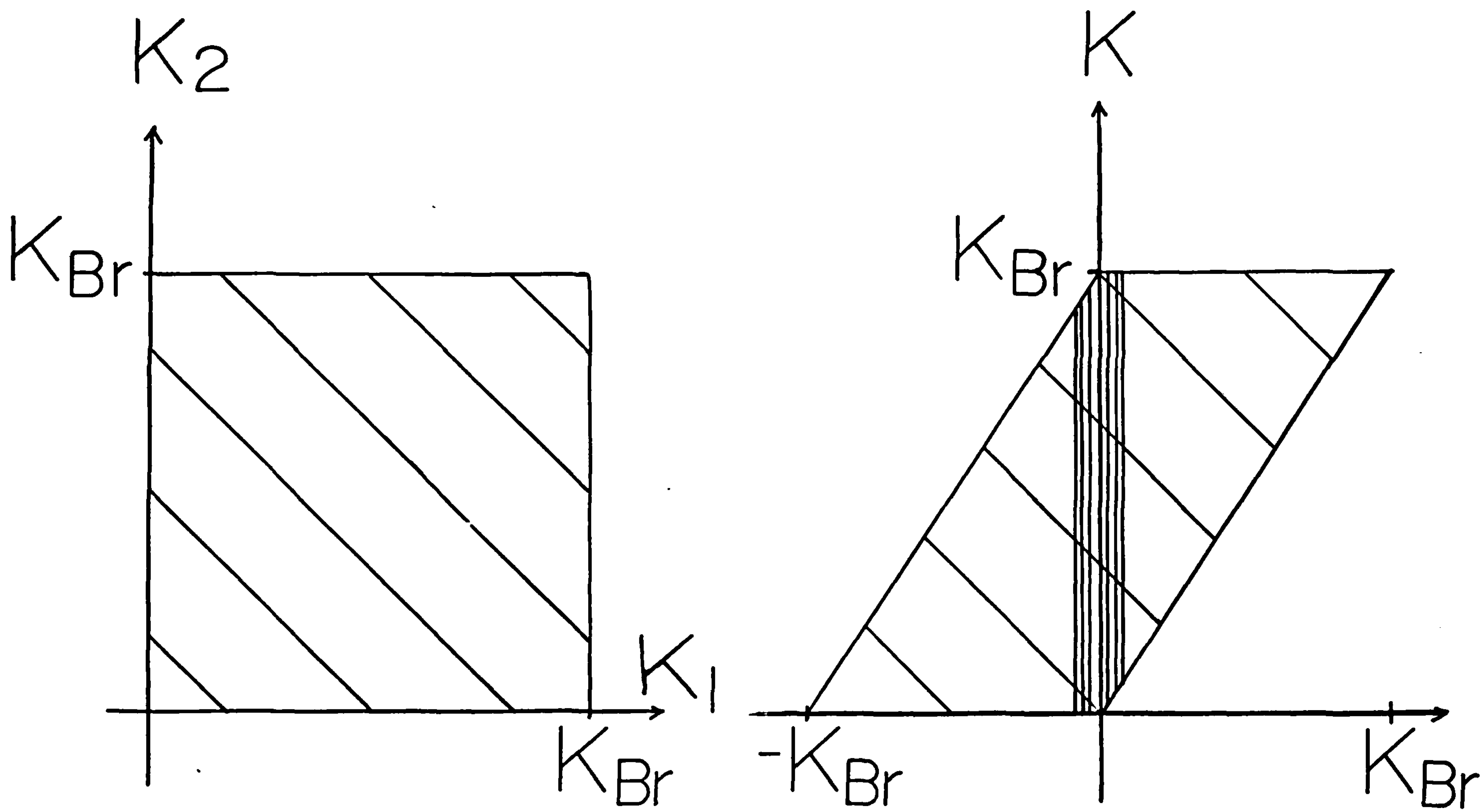



Figure 21

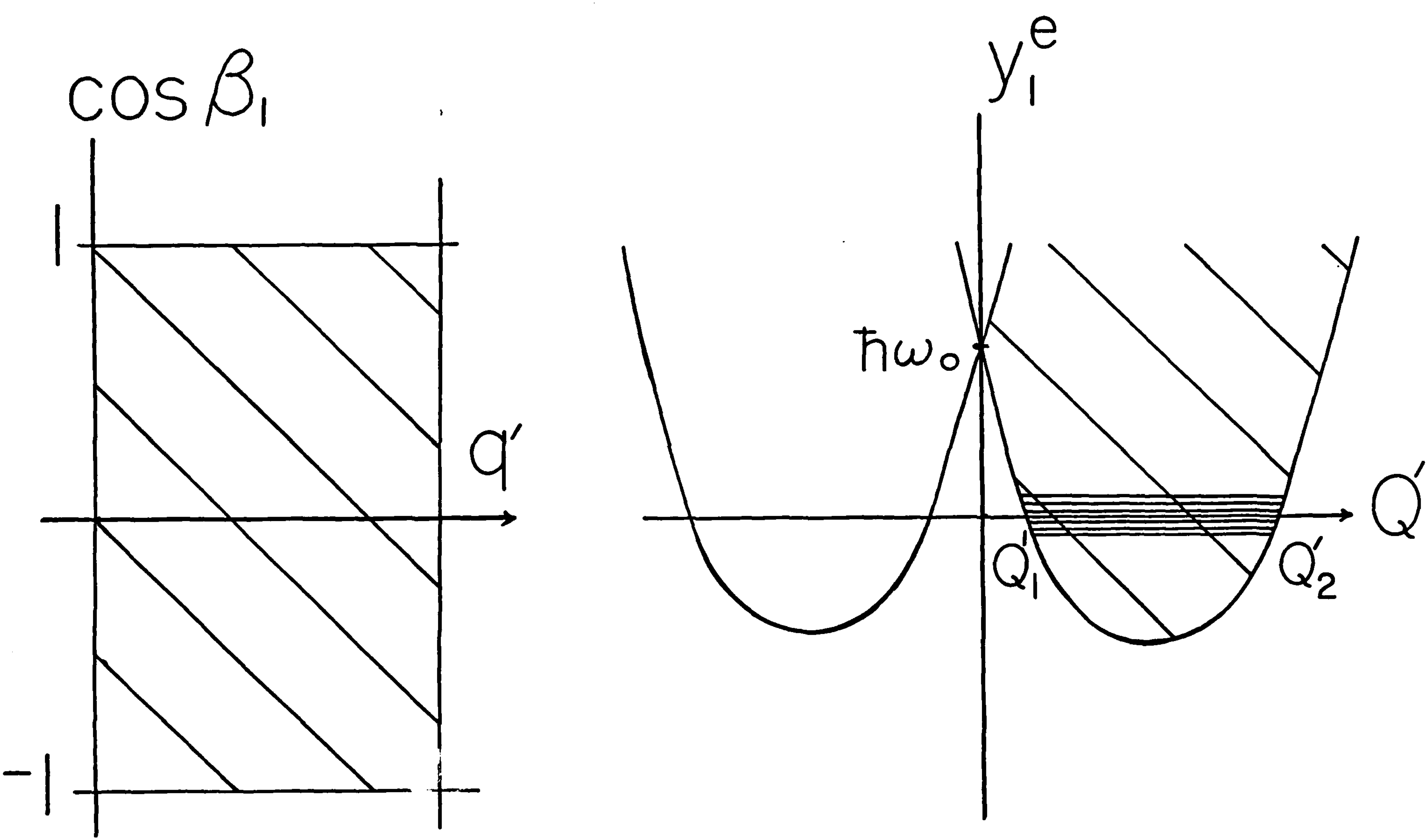



Figure 22

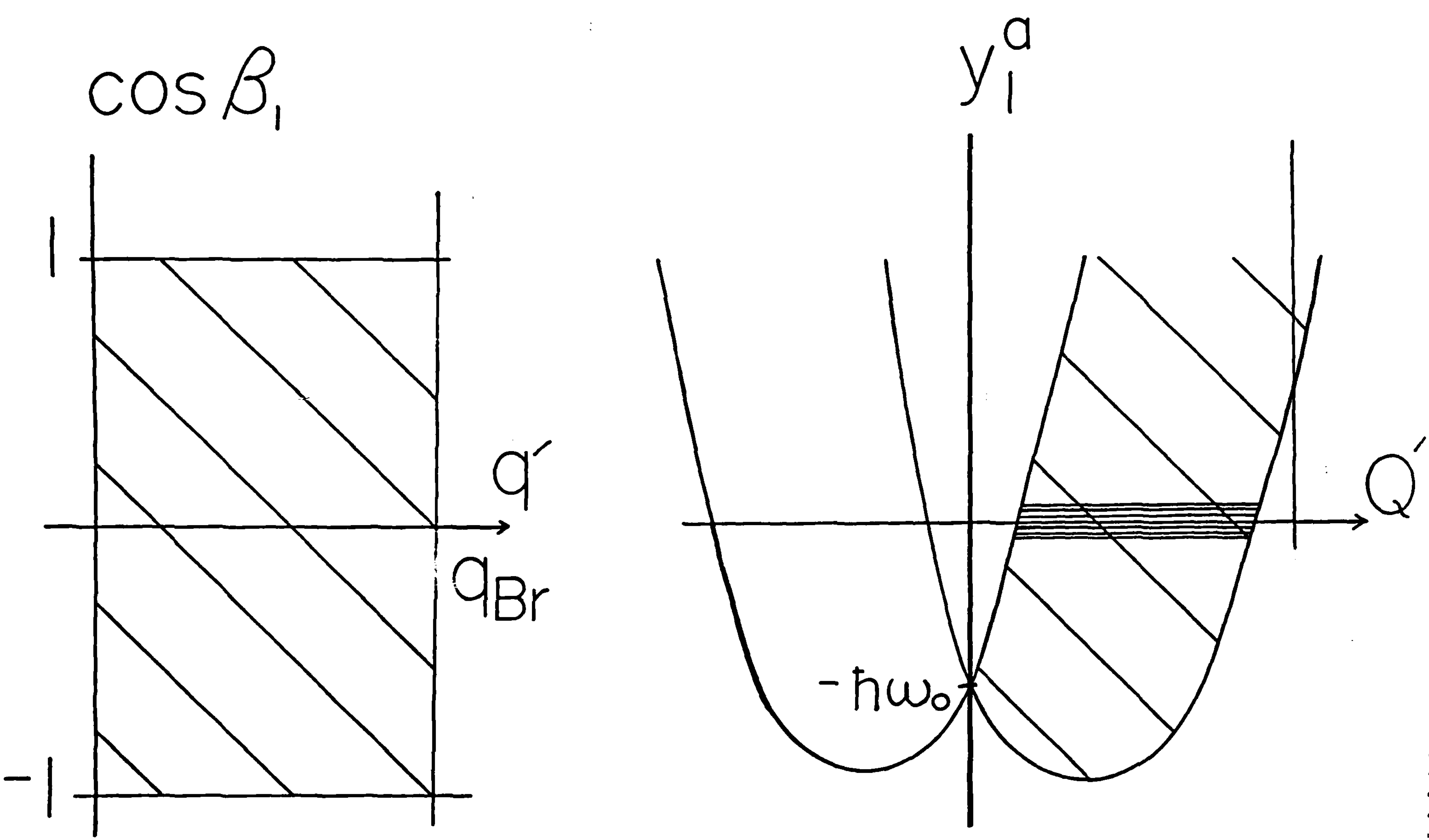



**Figure 23**

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