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MANY-VERSUS-MANY STOCHASTIC DUELS

FINAL REPORT

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19. KEY WORDS (Continue on reverse side if necessary and identify by block number) → Stochastic, Duels, Many-Versus-Many, Bibliography, ←		
20. ABSTRACT (Continue on reverse side if necessary and identify by block number) → The free-world literature on Many-Versus-Many Stochastic models of combat fire-fights (excluding pre-fire-fight optimization models) has been exhaustively searched, collected and reviewed. A bibliography has been prepared. → cont. <i>References include:</i>		

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MANY VERSUS MANY STOCHASTIC DUELS

The existing free-world literature on many-versus-many stochastic combat models has been studied. The area of concern was limited to the following:

- a. Analytical models (not simulations except where simulation results shed light on theoretical matters).
- b. Stochastic models.
- c. There are at least two contestants engaged on at least one side.
- d. Fire-fight models only; this precludes any optimization models designed to aid in pre-fire-fight decisions or in determining termination rules. Optimization models usually employ Differential Game Theory, Dynamic Programming, various mathematical programming techniques, Control Theory, etc.
- e. Must contribute to theory and not purely special applications of known results.

This literature has never been collected and thoroughly evaluated in a systematic manner.

The principal work accomplished has been the following:

- a. About 150 to 200 reports, papers and other documents were obtained by an exhaustive search of the free world literature followed by an effort to obtain copies of the relevant documents. All but a very small number were acquired and efforts to obtain the missing documents continue.
- b. The pool of acquired documents were carefully evaluated and reduced to 110 which met the criteria listed above.
- c. the relevant documents have been partially annotated but the annotations are not included in this report as we do not consider them to be fully useful until complete.

The full annotation and publication of the bibliography in the Defense Technical Information Center is expected to be accomplished in the future under the auspices of U.S. Army TRASANA White Sands Missile Range, NM. This will make the results of the study available in useful form to the defense research and user communities.

A subset of these documents is already being used in an extensive examination of the validity of current practices in utilizing Lanchester attrition coefficients in a great many Army (and other DOD agencies) simulations and analytical studies. This work will be published shortly in a readily available TRASANA document.



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ABBREVIATIONS;

CNA	Center for Naval Analyses, Arlington, VA.
DOAE	Defence Operational Analysis Establishment, West Byfleet UK.
DTIC	Defense Technical Information Center, Alexandria, VA, 22314.
IDA	Institute for Defense Analyses, Arlington, Va. 22202.
MOD	Ministry of Defence, West Byfleet, UK.
NPGS	Naval Postgraduate School, Monterey, CA.
NRLQ	Naval Research Logistics Quarterly,
OEG	Operations Evaluation Group, CNA
OR	Operations Research, Journal of the Operations Research Society of America.
RARDE	Royal Armament Research and Development Establishment, Fort Halstead, Sevenoaks, Kent, UK.
RMCS	Royal Military College of Science, Shrivenham, UK.
UMic	University Microfilms, Inc.,
UK	United Kingdom (England).

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