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Adaptive Filtering of Signals from Noise

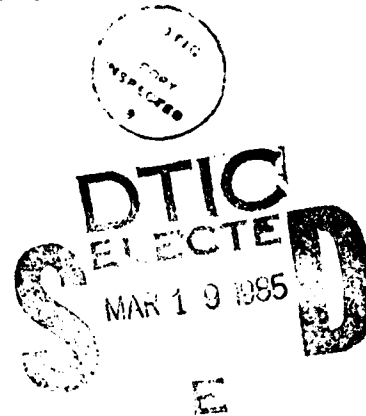
Henry D. I. Abarbanel

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number) This is a primer on extracting signals from noise. Two situations are considered: (1) a reference signal is coming directly from the noise source itself (e.g., "own ship" noise generated by the machiner on a ship towing a sonar array); (2) a signal which has a correlation function in time, which falls off slowly in time compared to the correlation function of the noise. A new idea in adaptive correlation, called instantaneous correlation, is introduced.			

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19. KEY WORDS (Continued)

20 ABSTRACT (Continued)

This is a primer on extracting signals from noise which has one potentially new idea and otherwise has as its goal a review - very brief review - of a subject with hundreds of papers to introduce it, explore it, develop it, and exploit it. This little primer is not a substitute for them. (Some are listed at the end of this note, i.e., the ones we found useful.)

The general problem is that a signal detected at times $k\Delta t$, call it $D(k)$, is composed of the signal $s(k)$ one wishes to observe and unwanted additive noise, $n(k)$, one would like to remove. To accomplish this we want to feed into $D(k)$ a signal $y(k)$ with characteristics which isolate $s(k)$ from $n(k)$ according to some performance criterion. A typical performance criterion is that

$$P = \sum_{k=1}^N (D(k) - y(k))^2 \quad (1)$$

be a minimum. N is the total number of measurements of D .

Two cases arise naturally, one in which $y(k)$ is driven toward $n(k)$ as P is minimized (this is called case R) and a second in which $y(k)$ is driven to $s(k)$ (case B):

(1) Case R. In case R we have a reference signal coming directly from the noise source itself. An important example of this

would be "own ship" noise generated by machinery on a surface ship or submarine which propagated from the turbines or whatever through elastic or waterborne paths to contaminate a signal, $s(k)$, which an onboard or towed sonar array is trying to hear. In case R we choose for $y(k)$ a linear combination of our knowledge of the reference noise $r(k-1)$, $r(k-2)$, . . . $r(k-L)$

$$y(k) = \sum_{a=1}^L W(a) r(k-a) , \quad (2)$$

where $r(k)$ is some linear combination of the noise samples themselves. In case R we want to choose the weights $W(a)$ so P is minimum, i.e.

$$\frac{\partial P}{\partial W(a)} = -2 \sum_{k=1}^N (D(k) - y(k)) r(k-a) = 0; a=1, \dots, L \quad (3)$$

If we succeed in choosing the W 's to minimize P in case R, then y has become our noise estimator chosen to minimize the correlation between D and y . The part of D correlated with y is the noise (since we may assume the signal $s(k)$ and the noise $n(k)$ are uncorrelated), so $D(k) - y(k)$ is our estimate of the signal alone.

(2) Case B. In this case we have a signal which has a correlation function in time

$$C_s(\ell) = \frac{1}{N} \sum_{k=1}^N s(k + \ell) s(k) \quad (4)$$

which falls off very slowly in time compared to the correlation function of the noise

$$C_n(\ell) = \frac{1}{N} \sum_{k=1}^N n(k + \ell) n(k) . \quad (5)$$

If the envelope of C_s or C_n behave as $e^{-\ell/\Delta_s}$ or $e^{-\ell/\Delta_n}$ respectively, case B is $\Delta_s \gg \Delta_n$. Stated in other words, we have broadband (in frequency) noise.

In case B we want to choose as our reference signal, r , the detected signal lagged by a time Δ which satisfies

$\Delta_n \ll \Delta \ll \Delta_s$. Then $r(k - \Delta) = s(k - \Delta) + n(k - \Delta)$ is correlated with s but uncorrelated with n . If we choose the weights $W(a)$, $a=1, \dots, M$ so

$$y(k) = \sum_{a=1}^M W(a) r(k - a - \Delta) \quad (6)$$

and minimize P , we are in effect minimizing any correlation between D and y . Since the part of D still correlated, after lag Δ , with y is the signal s , it follows that y itself is our estimate of the signal.

Case R and B are indicated in Figure A.

The solution to the optimum filtering problem is that set of $W_o(a)$, $a = 1, \dots, M$ satisfying (3):

$$C_{Dr}(a) = \sum_{b=1}^M W_o(b) C_D(b,a) , \quad (7)$$

where

$$C_{Dr}(a) = \frac{1}{N} \sum_{k=1}^N D(k) r(k-a) , \quad (8)$$

and

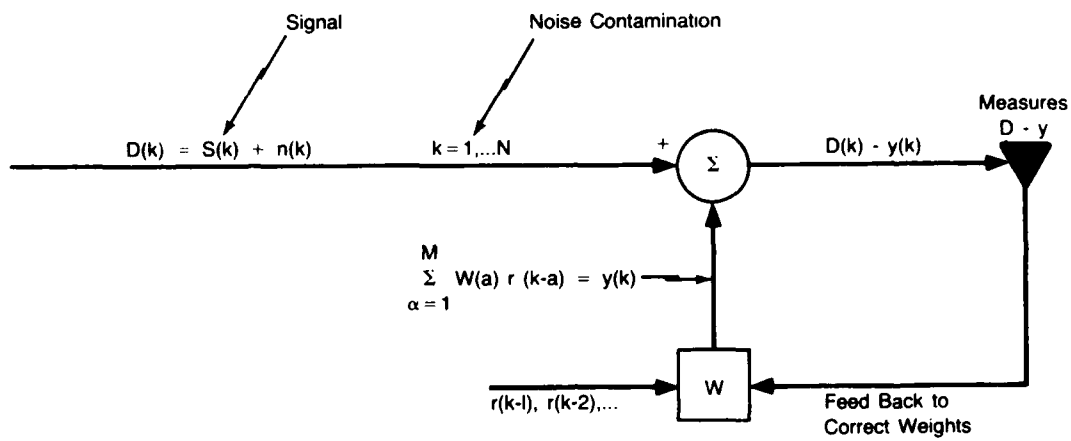
$$C_r(b,a) = \frac{1}{N} \sum_{k=1}^N r(k-a) r(k-b) \quad (9)$$

are the obvious correlation functions. If the processes involved are stationary in time, then C_D is a function only of $|a-b|$.

Clearly at the optimum

$$W_o(b) = \sum_{a=1}^M C_r^{-1}(b,a) C_{Dr}(a) , \quad (10)$$

which is the classical Wiener filtering solution. To use this solution one needs to accurately compute the correlation functions



- a) Reference Signal from Probe on Noise Source
- b) Detected Signal $D(k)$ Delayed by Δ
- R) Case R – Reference Signal is Filtered Noise.
- B) Case B – Reference Signal is Time Delay of Detected Signals.

Figure A.

C_{Dr} and C_r for as many lags as filter weights and then to do a $M \times M$ matrix inversion followed by a multiplication. All this is costly in computing time. Furthermore, if the nature of the signal or the noise changes during the observation time, the correlation functions remember too much about the past signal - so (10) is not likely to be very useful for time varying situations; it doesn't learn enough.

The, by now almost traditional, approach is to correct the weights at each time step by adding an amount proportional to the instantaneous slope $\frac{\partial P}{\partial W(a)}$ in weight space. If we give a time label to the weights so $W_k(a)$ is the a^{th} weight at time step k , then the so-called LMS adaptive algorithm chooses $W_{k+1}(a)$ as

$$W_{k+1}(a) = W_k(a) + \mu(D(k) - y(k)) r(k-a) . \quad (11)$$

The parameter μ is up to the user. When the sequence of

$W_k(a)$ converges, one can expect it to find $W_0(a)$ given by (10).

Much less computation is involved.

We'll give an example of the use of (11) in a few paragraphs. Let us now comment that the use of the LMS prescription is numerically tricky. The time (in number of "cycles" of the basic signal) to learn a new signal and choose weights accordingly is proportional to μ^{-1} ; this encourages the user to choose μ

large. The convergence properties of the sequence, however, rapidly go bad as μ becomes too large. Unfortunately, this may happen at very small μ . Although some of the references give some bounds on μ for convergence, in practice it is somewhat cut and try and depends in a sensitive way on the scale of the noise. If the noise-- or $r(k)$ --in (11) is large, μ must be correspondingly decreased.

My colleague A. Peterson in JASON indicated to me that the hardware versions of the LMS algorithm set μ quite large to reduce the learning time and then monitor the size of the $W(a)$, $a = 1$ to check for instabilities. If the $W(a)$ get outside an acceptable bound, they are reinitialized.

The second method of adaptive cancellation we suggest is the one original piece of this note. The information in the correlation functions C_r and C_{Dr} in (8) and (9) is contained in their correlation lengths. This is the number of time steps in which $C_{Dr}(\ell)$, for example, falls by $1/e$ of its value at zero lag. So

$$C_{Dr}(\ell) \sim e^{-\ell/L} \quad (12)$$

and L is the correlation length.

Our Instantaneous Correlation (IC) algorithm does the following: At time step k compute

$$C_{Dr}^{(k)}(a) \equiv \frac{1}{2L_1 + 1} \sum_{j = k - 2L_1}^k D(j) r(j - a) , \quad (13)$$

and

$$C_r^{(k)}(b,a) = \frac{1}{2L_2 + 1} \sum_{j = k - 2L_2}^k r(j - a) r(j - b) , \quad (14)$$

where L_1 and L_2 are correlation lengths to be adjusted.

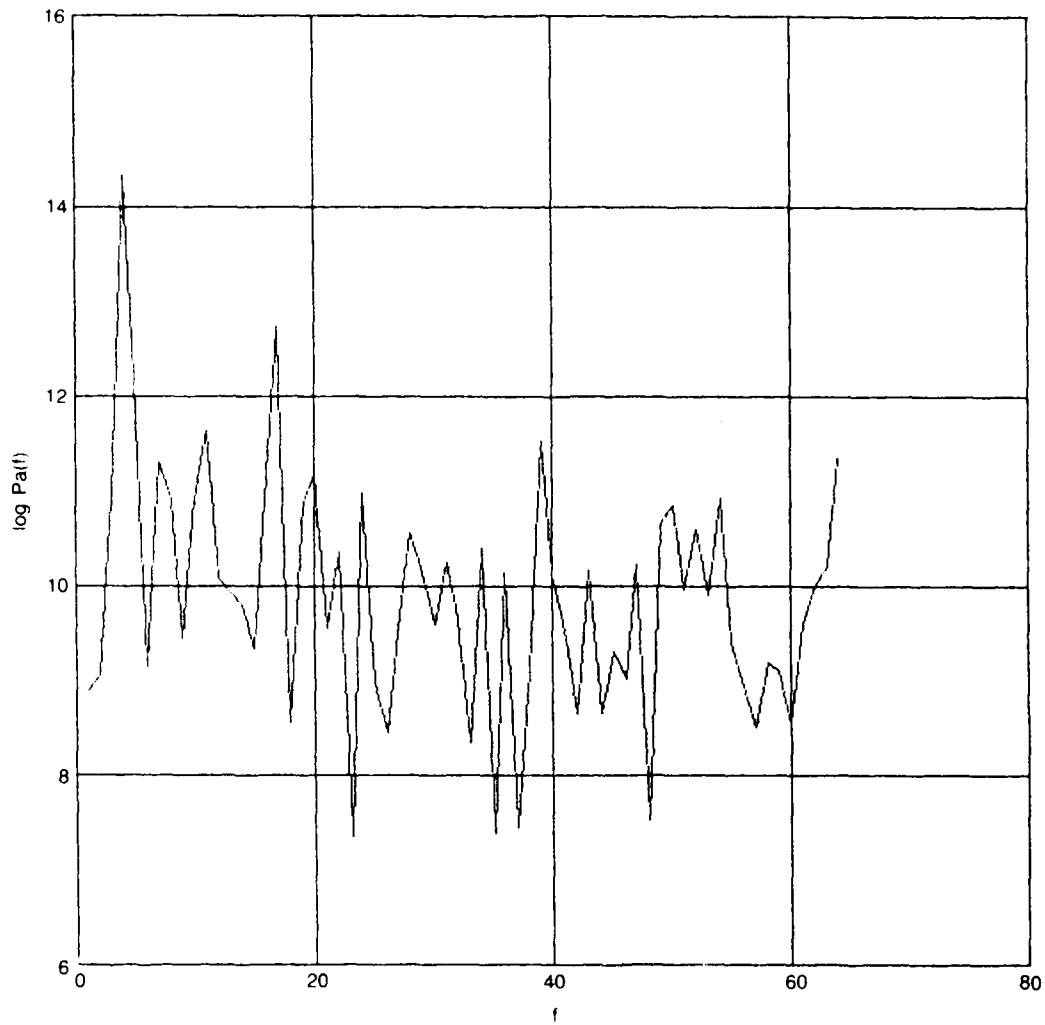
Next form

$$W^{(k)}(a) = \sum_{b = 1}^M C_r^{(k)-1}(a,b) C_{Dr}^{(k)}(b) , \quad (15)$$

which are the filter weights entering our determination of the signal.

One must learn by experimentation which L_1 and L_2 are appropriate. Since the correlation length of the noise is much less than that of the signal, we can expect $L_1 \approx L_2$.

The IC method may require a lot of computing, but it clearly learns very well. As the signal or the noise changes, so will the



Power Spectrum of
Noisy Signal

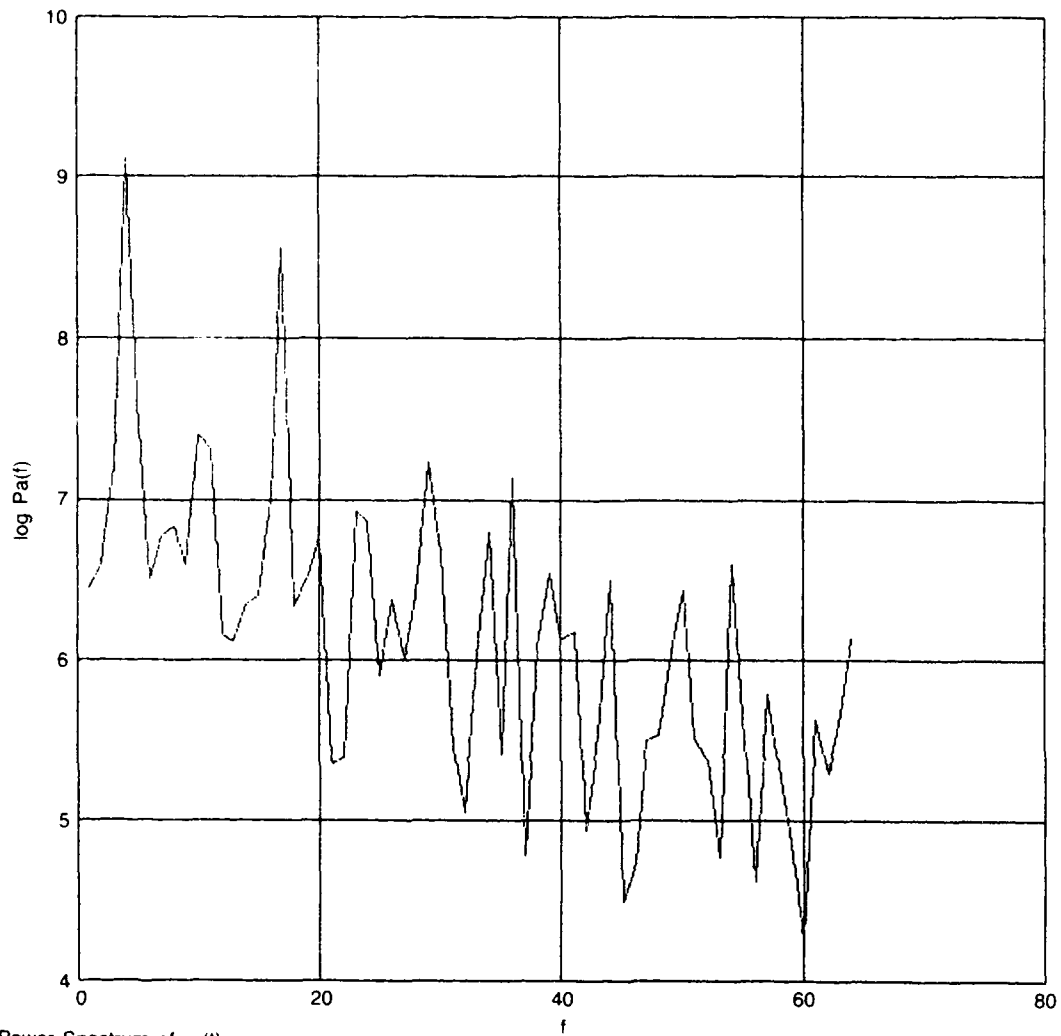
g 3 041

Gain 30

2ⁿ Points FT

2⁶ Points Shown

Figure 10



Power Spectrum of $a_c(t)$

$g = 3.041$

2ⁿ Points FT

2⁶ Shown

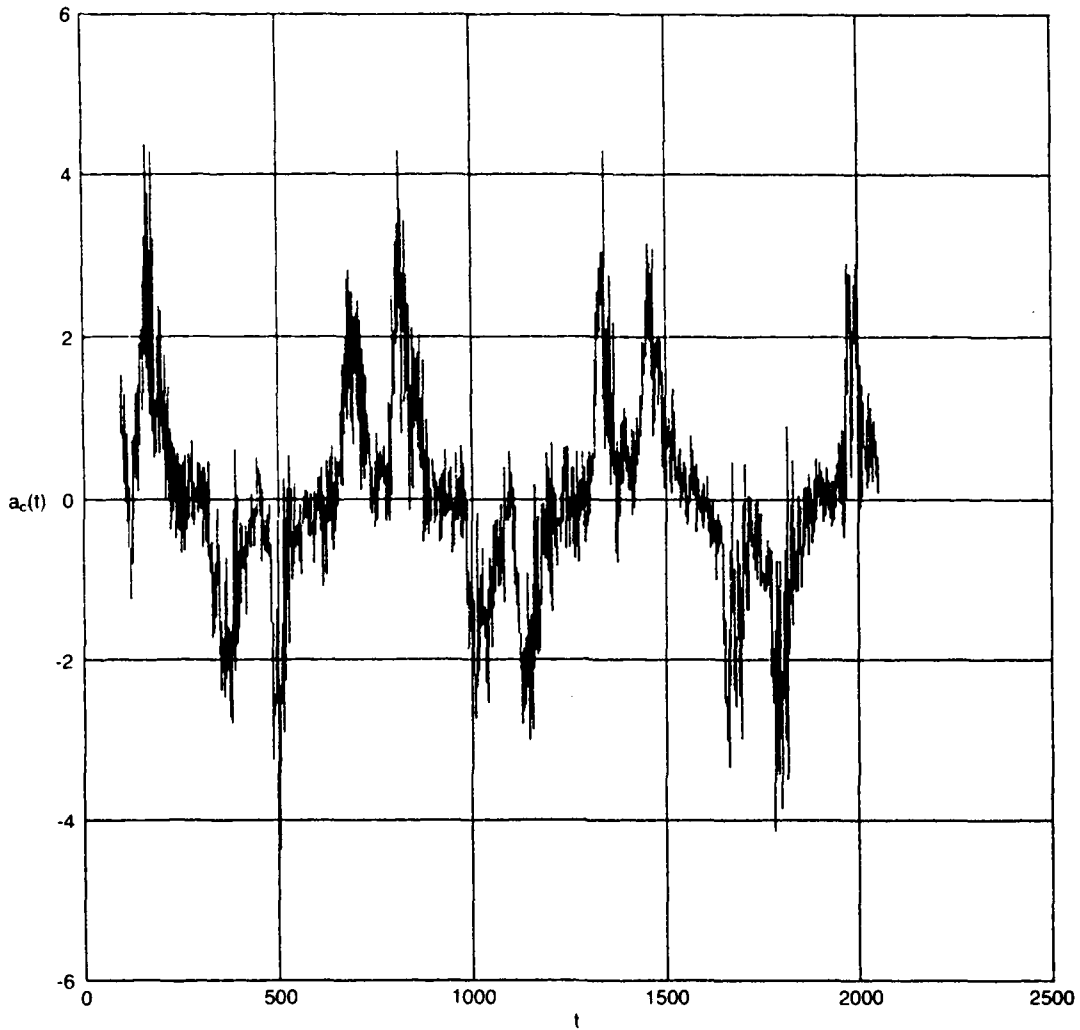
Gain 1.0

$\mu = 0.01$

Lag 3

16 Weights

Figure 9



Corrected Time Series

2' Points

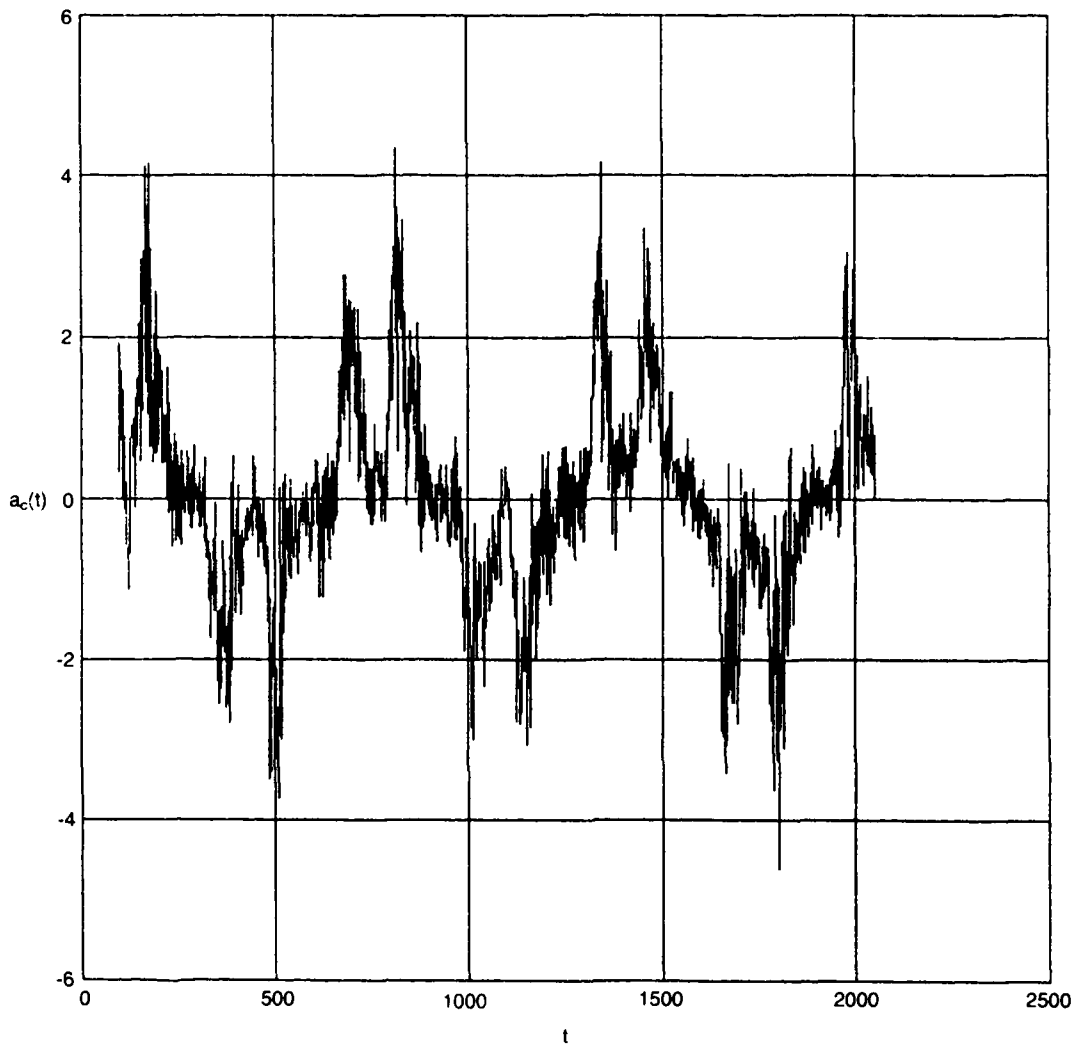
16 Weights

μ 0.01

Lag 5

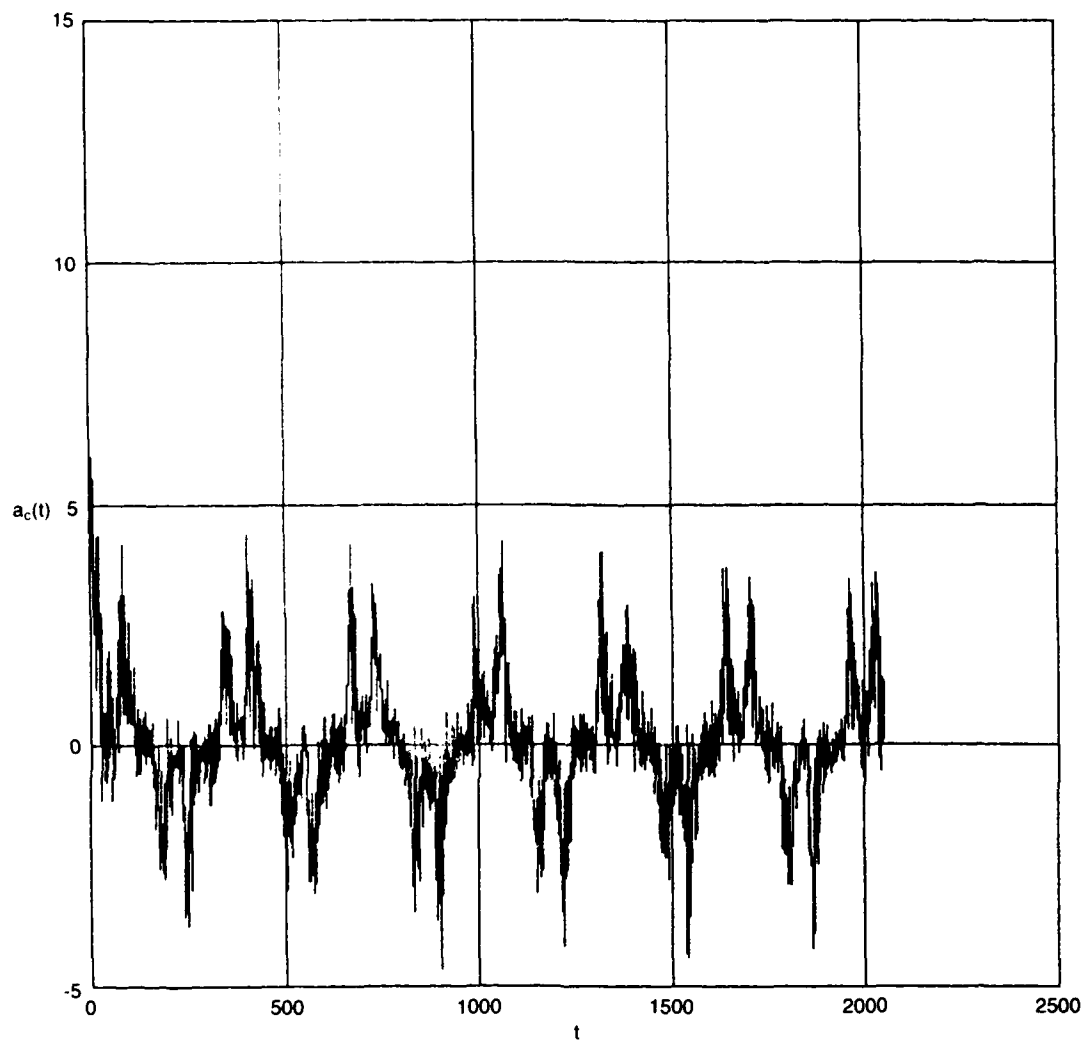
Gain 1.0

Figure 8



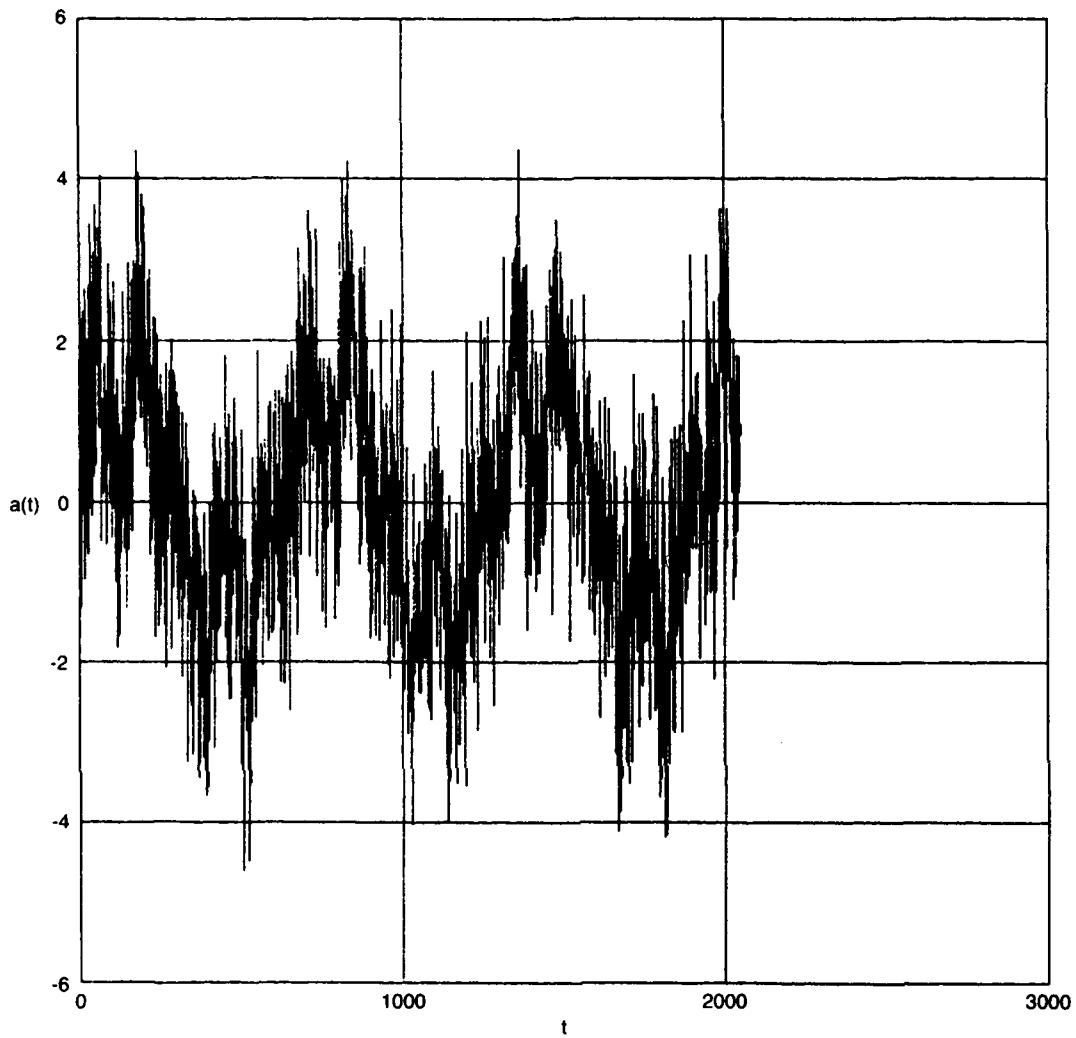
Corrected Time Series
2" Corrected Points
16 Weights
 $\mu = 0.01$
Lag = 3
Noise Gain = 1.0

Figure 7



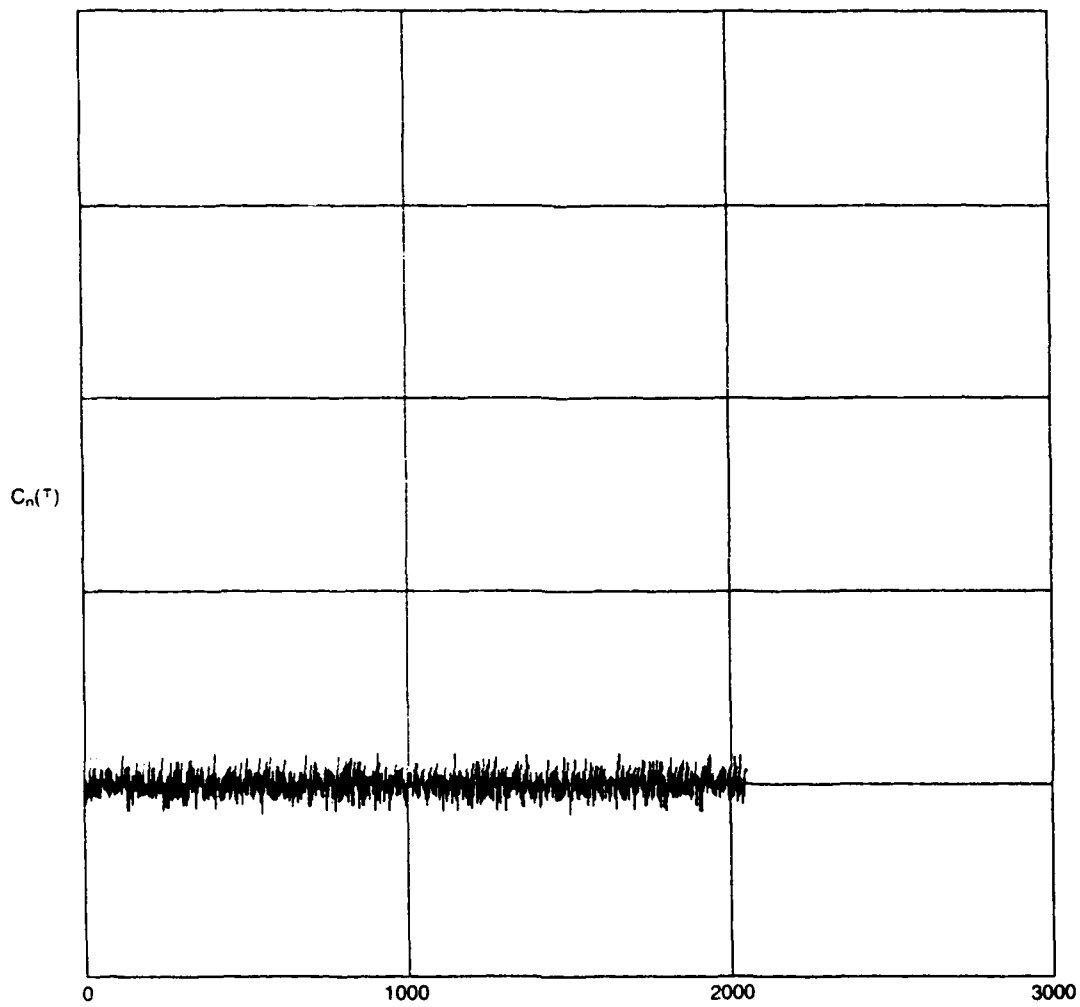
Corrected Time Series
2' + 2500 + 4nw Points on Orbit
2' Corrected Points
16 Weights
 μ 0.01
lag 3
Noise Gain 1.0

Figure 6



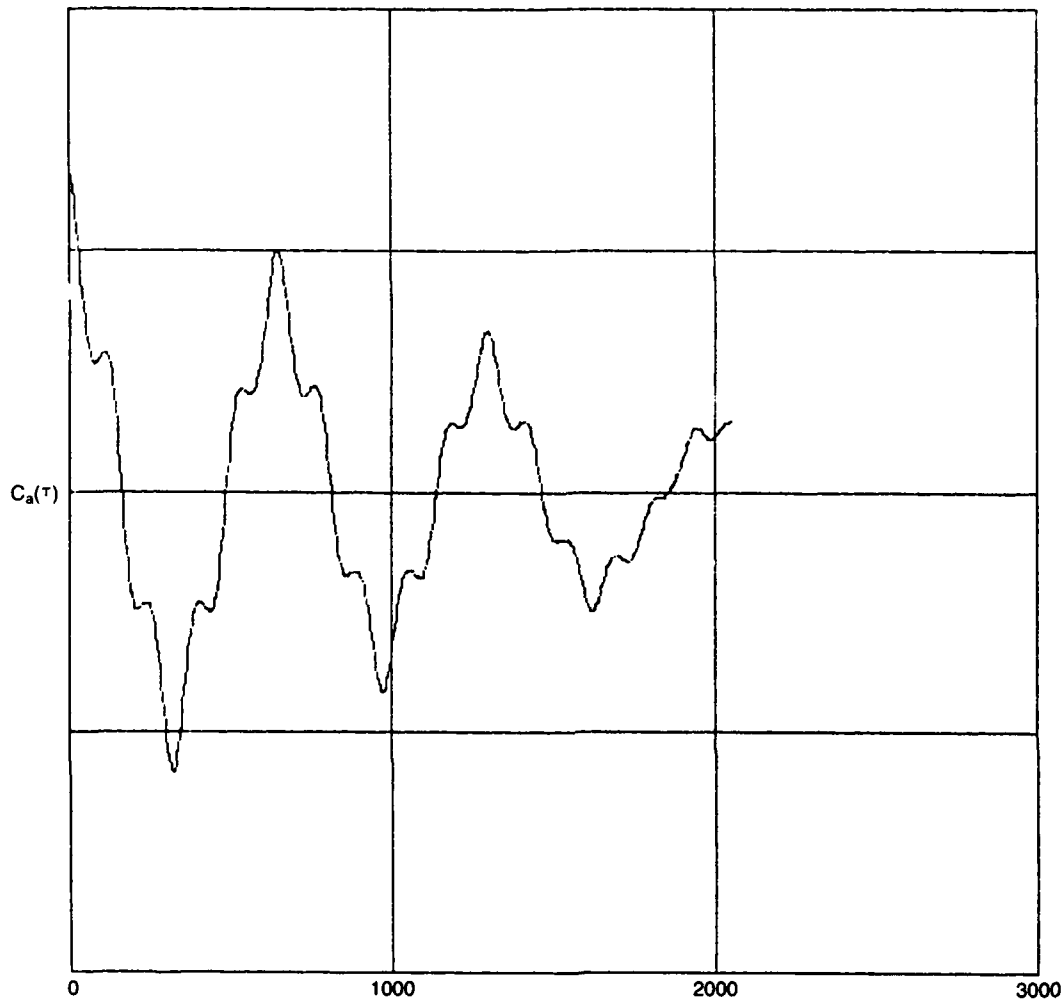
$g = 3.041$
2' Points
Gain = 1.0

Figure 5



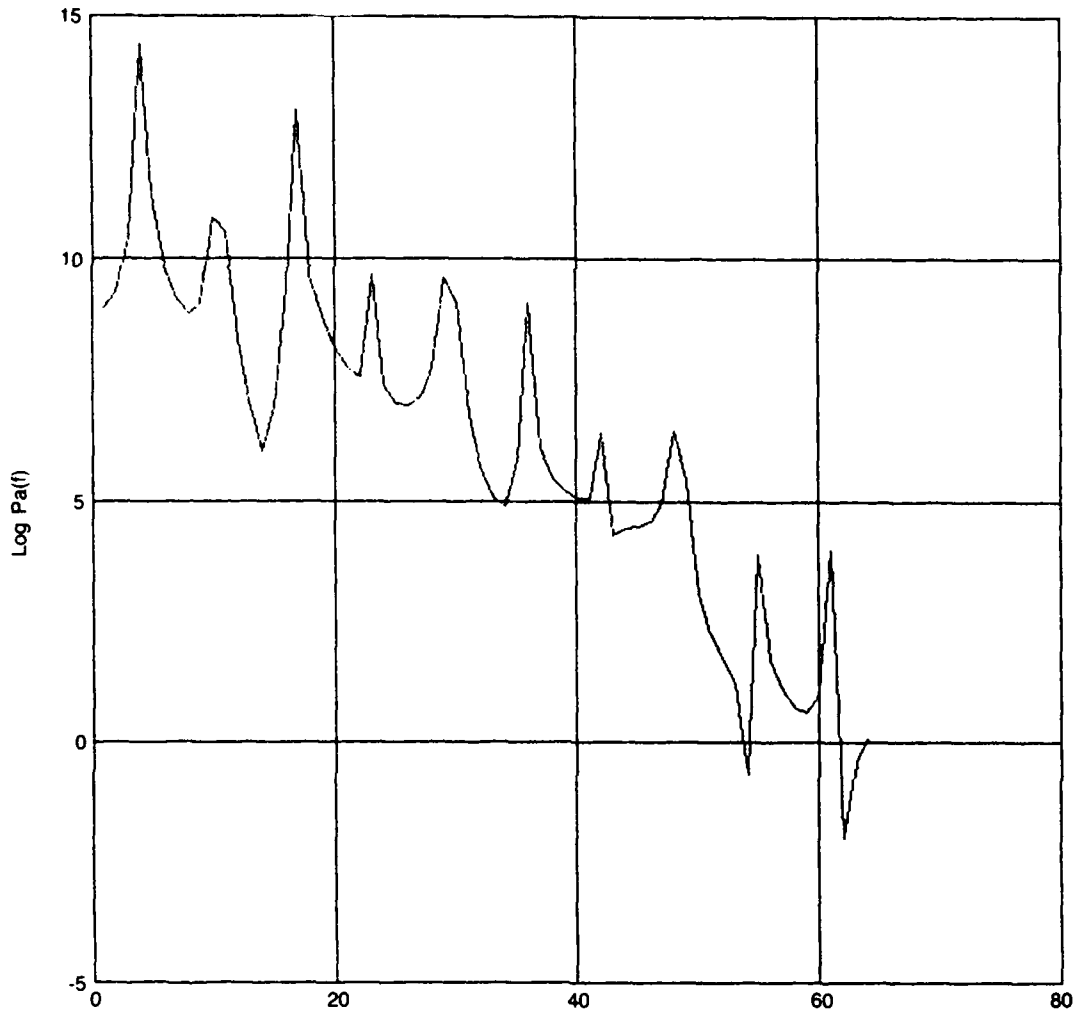
Noise Correlation
Function
 2^{13} Points on Orbit
 2^7 Points of $C_n(\tau)$

Figure 4



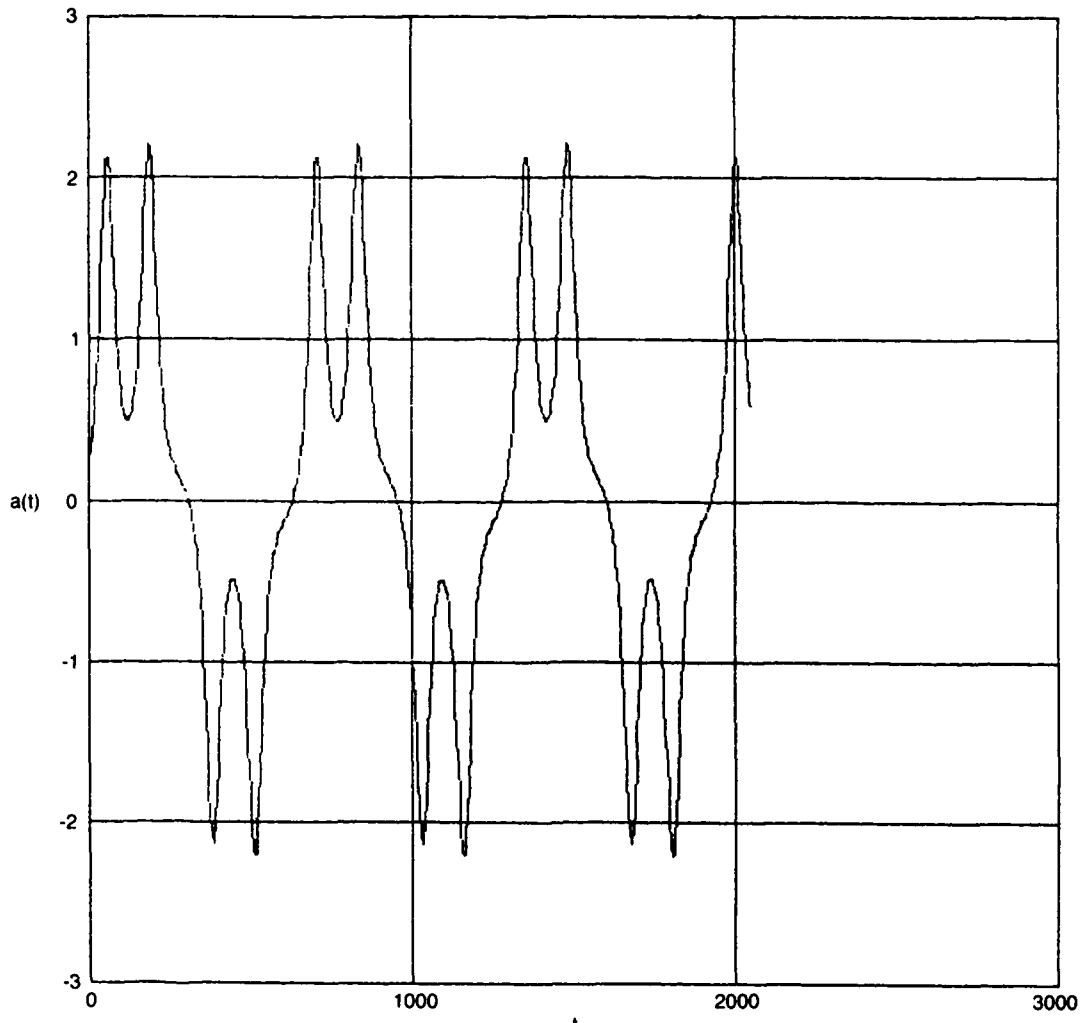
$g = 3.041$
 2^{12} Points on Orbit
 2^7 Points of $C_a(\tau)$ Shown
 Correlation Function

Figure 3



$g = 3.041$
Power Spectrum
Gain = 0.0
2⁷ Points FT
2⁸ Points Displayed

Figure 2



g = 3.041
2' Points
Gain = 0.0

Figure 1

the adaptive filter through more cycles of the signal to let the filter learn better. We had to choose μ small and therefore adaptation times long to insure stability of the algorithm. This is the fundamental snag of the LMS technique. One should be aware of it, not deterred by it.

(2) We could have taken a number of samples of noisy signals, filtered them, and then averaged the power spectra. This will quite likely improve matters, and is very straightforward we simply didn't have time.

Furthermore and finally, time prevented us from exploring the IC algorithm in any detail. It should be easy to do - we did it running in a few hours, but hadn't enough experience by this writing to "learn" from it.

Next we corrupted the signal with gaussian random noise with zero mean and variance unity. The noisy signal is shown in Figure 5. Using the LMS algorithm we then corrected the noisy signal using 16 weights, an LMS parameter $\mu = 0.01$ (chosen by trial and error to be small enough to be stable), and a lag $= \Delta = 3$. The result of the calculation shown in Figure 6 shows the initial learning and then the corrected signal. In Figure 7 we suppressed the learning period and exhibit only the corrected signal. This is to be directly compared to the noisy signal in Figure 5. Our final timeseries display, Figure 8, is identical to Figure 7 but with the lag $= \Delta = 5$ which does not produce a visible improvement.

Next we show three power spectra. Figure 9 is the power spectrum of Figure 7. It should be compared with the spectrum in Figure 2 of the clean signal. A few lines are still visible. In the last two figures (10 and 11) we show the power spectra of the even more noisy signal (signal variance = 3 times larger) first without correction, then LMS corrected with $\mu = 0.008$, 16 weights, and a lag of 25.

What conclusions shall we draw from this. Clearly, to the eye, the LMS algorithm has cleaned up the time series by reducing the variance of the noise. Nevertheless, the visible improvement in the power spectra is not much. Several things should be tried: (1) run

might not. Using one of the filtering algorithms (LMS or IC or whatever) guarantees that the appropriate correlation is driven as small as possible, thus providing one with an objective goal which will serve even when box averaging fails. Perhaps the way to look at the weighted average is that it knows a little bit more about the noise than its average. It also knows that it must kill its correlations with the signal and that it must reduce the variance due to the noise which creeps, unwanted, into the estimated signal.

A LITTLE EXPERIMENT

To examine the ease with which one can use these ideas we created a signal from the solution of a set of nonlinear differential equations and then, by hand, added gaussian random noise to it. Using the LMS algorithm we then tried to extract the signal back out of noise.

The signal is shown in Figure 1. It is periodic but not sinusoidal. Its power spectrum is given in Figure 2. Clearly it has many harmonics. To see that we can apply Case B to this signal we evaluated its autocorrelation function (unnormalized) and show it in Figure 3. On the same horizontal scale we show our noise autocorrelation function in Figure 4. An expanded view of Figure 4 indicates that $\Delta_n \approx 2$ while from Figure 3 we see $\Delta_s \approx 10^3$. Clearly Case B applies.

$c^{(k)}$ and the $w^{(k)}$, and within a correlation time of the changed signal. Further the method is intermediate between the LMS algorithm and the full Wiener W_0 in amount of computation required. It is stable, while the LMS approach may not be.

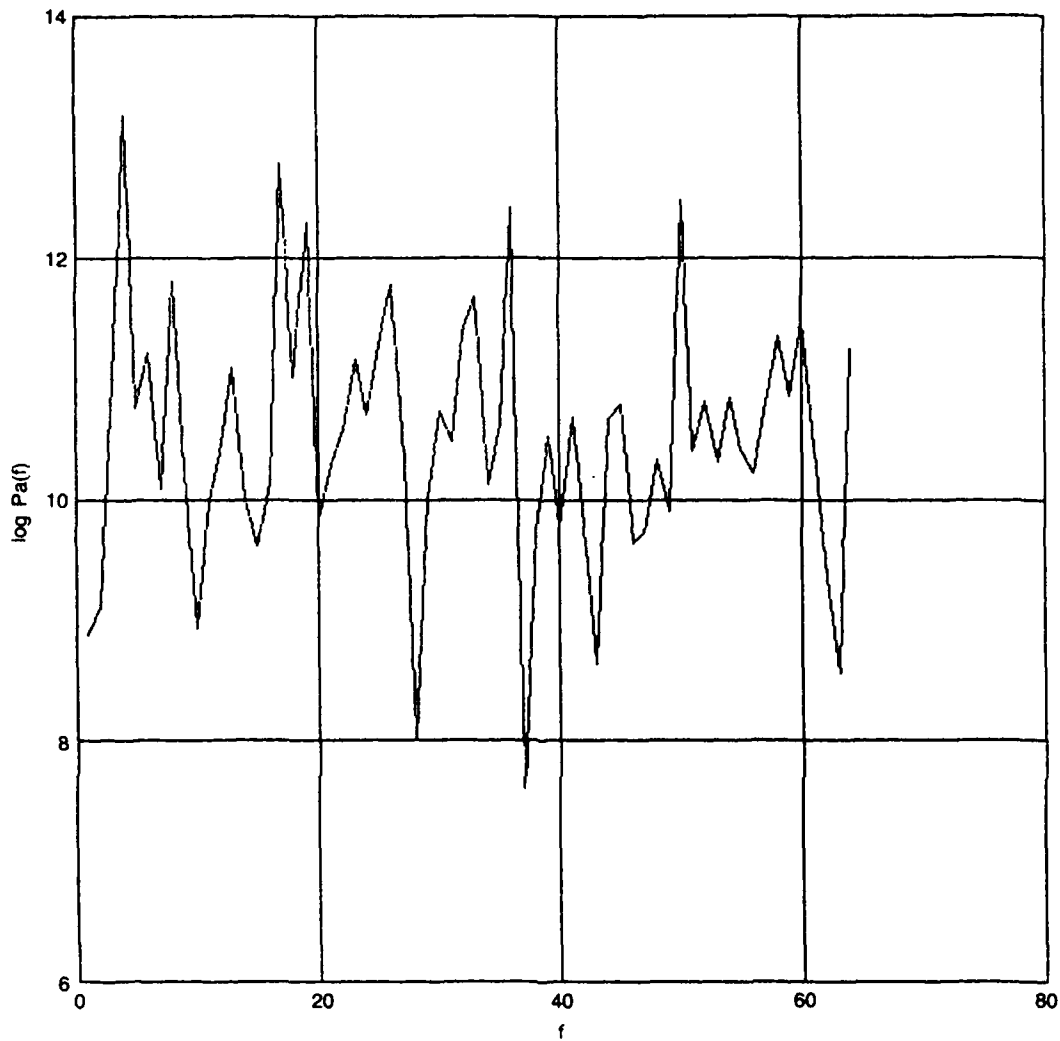
A final remark: in case B the signal estimate is

$$y(k) = \sum_{a=1}^M W(a) [s(k - \Delta - a) + n(k - \Delta - a)]. \quad (16)$$

One might have thought to "smooth" the detected signal by extracting the broadband noise through an averaging procedure over the "recent" past. Namely, one might take as the the estimate for the signal

$$s_e(k) = \frac{1}{K} \sum_{j=1}^K [s(k - j) + n(k - j)]. \quad (17)$$

This kind of "box" averaging will eliminate high frequency noise and is really a choice of uniform weights $W(a) = 1/K$ in our optimization scheme. The procedure outlined chooses $W(a)$ according to a prescription which filters out noise at different rates at different frequencies to smooth the detected signal in an optimum way over the whole spectrum without a priori bias as to the relative magnitude of the $W(a)$. As given, the prescription (17) might well work - and it



Power Spectrum of
 Corrected Signal
 $g = 3.041$
 2^7 Points FT
 2^6 Points Shown
 $\mu = 0.008$
 16 Weights
 Lag = 25 Gain = 3.0

Figure 11

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