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DETERMINATION OF HEAT TRANSFER COEFFICIENT IN A GUN BARREL FROM EXPERIMENTAL DATA

William F. Donovan

January 1985

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US ARMY BALLISTIC RESEARCH LABORATORY
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A review of the Schmidt technique for analyzing unsteady heat flow under the restrictions of idealized boundary conditions is presented. Algebraic formulations are used for estimating wall termperatures in a ballistics environment. The procedure is given in desk-top computer program for maximum utility.

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I. INTRODUCTION

The particular problem of estimating gun barrel temperature profiles in the vicinity of high energy gas flow has been examined experimentally by local investigators,[1] and the general problem of mathematically constrained, unsteady, high temperature heat flux is treated in the open literature.[2,3,4] Historically, the preliminary layout of liquid propellant rocket combustion chambers has often employed the Schmidt diagram[5,6] to estimate the critical metal temperatures in the structure. This is developed as a finite difference approximation of the second order, first degree partial differential equation of physics in heat flow context and is usually presented graphically. The graphical analysis is an alternative to the LaPlace Transform treatment[7] which is unwieldy in execution. It also competes with the differential analyzer technique. Rigorous solutions to the second order axisymmetric problem in the form of mapped finite element presentations of the detailed transient flow phenomena are promised[8] for the proximate future.

A specific and comprehensive solution to the barrel problem has recently been delivered in the form of mathematical presentation by Polk,[9] and this in the explicitly usable form of a moderate level computer program (Hewlett-Packard 9845) for direct application. The Polk treatise is not currently in BRL report format, however, and the program is still being refined. A

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1 *T. L. Brosseau, "An Experimental Method for Accurately Determining Temperature Distribution and the Heat Transferred in Gun Barrels," BRL-R-1740, September 1974. AD #B000171L.*

2 *Mark W. Zemansky, Heat and Thermodynamics, McGraw-Hill Book Company Inc., New York, 1957.*

3 *Max Jacob, Heat Transfer, Vol. 1, John Wiley and Sons, New York, 1949.*

4 *E. F. Quigley, "One Dimensional Transient Temperature and Stress Distribution Produced in 0.375- and 0.500- inch Thick 7075A1-T6 Flat Plates by Fourteen Nuclear Thermal Environment," BRL-MR-2173, April 1972. AD #901995.*

5 *Frank Kreith, Principles of Heat Transfer, International Text Book Company, Scranton, PA., 1963.*

6 *George P. Sutton, Rocket Propulsion Elements, John Wiley and Sons, New York, 1956.*

7 *Theodore V. Karman and Maurice A. Biot, Mathematical Methods in Engineering, McGraw-Hill Book Company Inc., New York 1940.*

8 *P. L. Versteegen and F. D. Varcolik, "Heat Transfer Studies in Gun Tubes," ARBRL-CR-00393 (Science Applications Inc., McLean, VA.) March 1979. AD #A069649*

9 *J. F. Polk, "An Algorithm for Heat Transfer in Gun Barrels," Transactions of the Twenty-Fifth Conference of Army Mathematicians, ARO Report 80-1, 1980.*

complete FORTRAN program, based on a Calspan reference,[10] is also available but has yet to be introduced into the local Control Data Corporation facility.

In direct graphical exposition, the Schmidt plot consists of a large scale drafting wherein the results are recovered as measured values rather than discrete numbers. This report offers the details of a desk top calculator (Hewlett-Packard 97) procedure algebraically transposed from the finite difference scheme to calculate heat transfer coefficients in a ballistics environment from an overdetermined set of equations. The data is from experiment, and an implicit assumption is that the driving gas temperature is constant over the selected time interval of the calculation. The appropriate driving gas temperature can then be determined by regressive substitution.

Thermodynamic units always deserve a special note. The examples are presented in the International System of Units, the British, and in normalized parameters. The British units are employed simply because the materials properties values are most widely available in this system, and it therefore offers direct and convenient application. The SI is proposed as a standard, and the normalized borrows from the Jacob-Hawkins[3] practice with an additional modification to eliminate geometric scale factors.

II. PROCEDURE

A. General Statement

The differential equation describing one dimensional, unsteady, conductive heat flow through a solid is

$$\frac{1}{a}\frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial x^2} \quad \text{(rectangular slab).} \qquad (1)$$

where a is thermal diffusivity,
T is temperature,
t is time,
x is distance.

Approximate solutions to the differential equations can be obtained by solving the finite difference analogue.

$$\frac{1}{a}\frac{\Delta_t T}{\Delta t} = \frac{\Delta_x^{\,2} T}{\Delta x^2} \qquad (2)$$

10 *"CYCOND" Program, and "HTC" Program, communication via Mr. Kovacs, DRDAR-SE, Picatinny Arsenal.*

with Δt equal to the finite difference in time,

Δx^2 equal to the finite difference in distance,

$\Delta_t T$ equal to the time variable effecting a change in T, and

$\Delta_x T$ equal to the distance variable effecting a change in T.

By then defining "t" as the number of time increments (Δt), and "n" as the distance increment, the direct application form of the equation becomes

$$\frac{1}{a}\ \frac{T_n(t+1) - T_n(t)}{\Delta t} = \frac{T_{(n+1)}(t) - 2\ T_n(t) + T_{(n-1)}(t)}{\Delta x^2} \tag{3}$$

Note: Throughout this report, the notation $T_n t$ and $T_n(t)$ will be used interchangeably. Parenthesis will be used only when added clarity is necessary.

Appendix A transcribes the finite differences to algebraic formulation and illustrates the Schmidt plot by numerical example.

In cylindrical coordinates the finite difference equation becomes

$$\frac{1}{a}\ \frac{T_n(t+1) - T_n(t)}{\Delta t} = \frac{1}{r^2}\ \left(\frac{T_{(n+1)}(t) + T_{(n-1)}(t) - 2\ T_n(t)}{\Delta j^2}\right) \tag{4}$$

where $r = e^j$,

and $\Delta j = \frac{\Delta r}{r}$,

for any radius "r" with "j" defined in terms of "r".

Appendix B presents the cylindrical transcription and a numerical example for illustration.

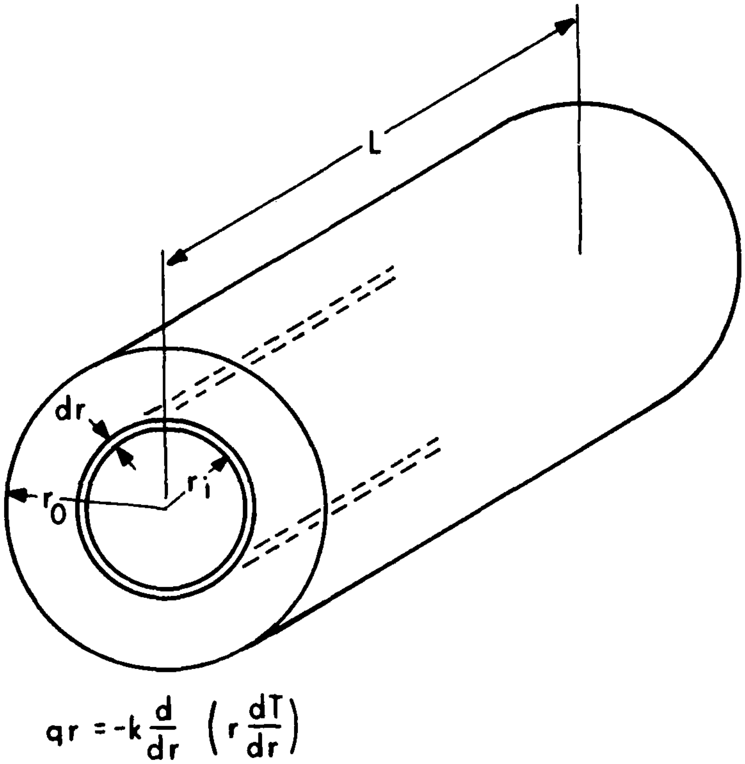
B. Application to a Gun Barrel

The gun barrel is represented as a long thick-walled annular cylinder, Figure 1, with a suddenly imposed high temperature gas flowing through the bore. To estimate the wall temperature gradient, three assumptions are required:

1. A gas temperature history.

2. Univalued properties of the medium.

3. Radiation and convection accountable by the modification of the value for unit surface conductance.



where

r_i is the inner cylinder radius

r_o is the outer cylinder radius

Figure 1. Schematic of Infinitely Long Cylinder

C. Wall Surface Temperature

The procedure used to develop the equations which specify the temperature of the face of a slab after exposure to a high temperature gas consists of:

1. Construction of the Schmidt plot, which is geometrically defined in terms of selected gas and metal properties; i.e., the heat transfer coefficient ($\bar{h}$), the reference temperature (T_o), the conductivities of the gas and the metal (k), and the diffusivity of the metal (a).

2. Transformation from the geometric to the equivalent algebraic expression.

In this context, as shown in Figure 2.

$$\Delta T_o = T_o o - T_n o.$$

$$m = \frac{\Delta x}{\Delta x + \Delta x_o}, \text{ or} \tag{5}$$

$$= \frac{1}{1 + (\Delta x_o / \Delta x)}; \tag{6}$$

where $\Delta x_o = \frac{k}{\bar{h}}$ by construction of the Schmidt plot and

$$\Delta x = (2a\Delta t)^{1/2}, \tag{7}$$

where $\Delta t = \frac{\text{duration of heat exposure}}{\text{number of time intervals}}$.

For the gun problem, the duration of the heat exposure may be determined from the in-bore projectile trajectory history and an adiabatic flame temperature established from a definition of the propellant composition. These are tenuous criteria, of course, but are within the existing "state of the art." From Figure 2 and Table 1, the temperatures at the wall (Plane 1) are algebraically fixed for the first ten time intervals in terms of equivalent Schmidt plot distances corresponding to the specific thermal resistances on each side of the wall plane. Table 1, obtained from Figures 3 through 7, presents the algebraic resolution of the temperature profile for the interstitial planes. Table 2 gives the corresponding temperatures for the second plane - based on the same reasoning. These particular formulae neglect the influence of curvature since the wall penetration (Δx) approaches zero.

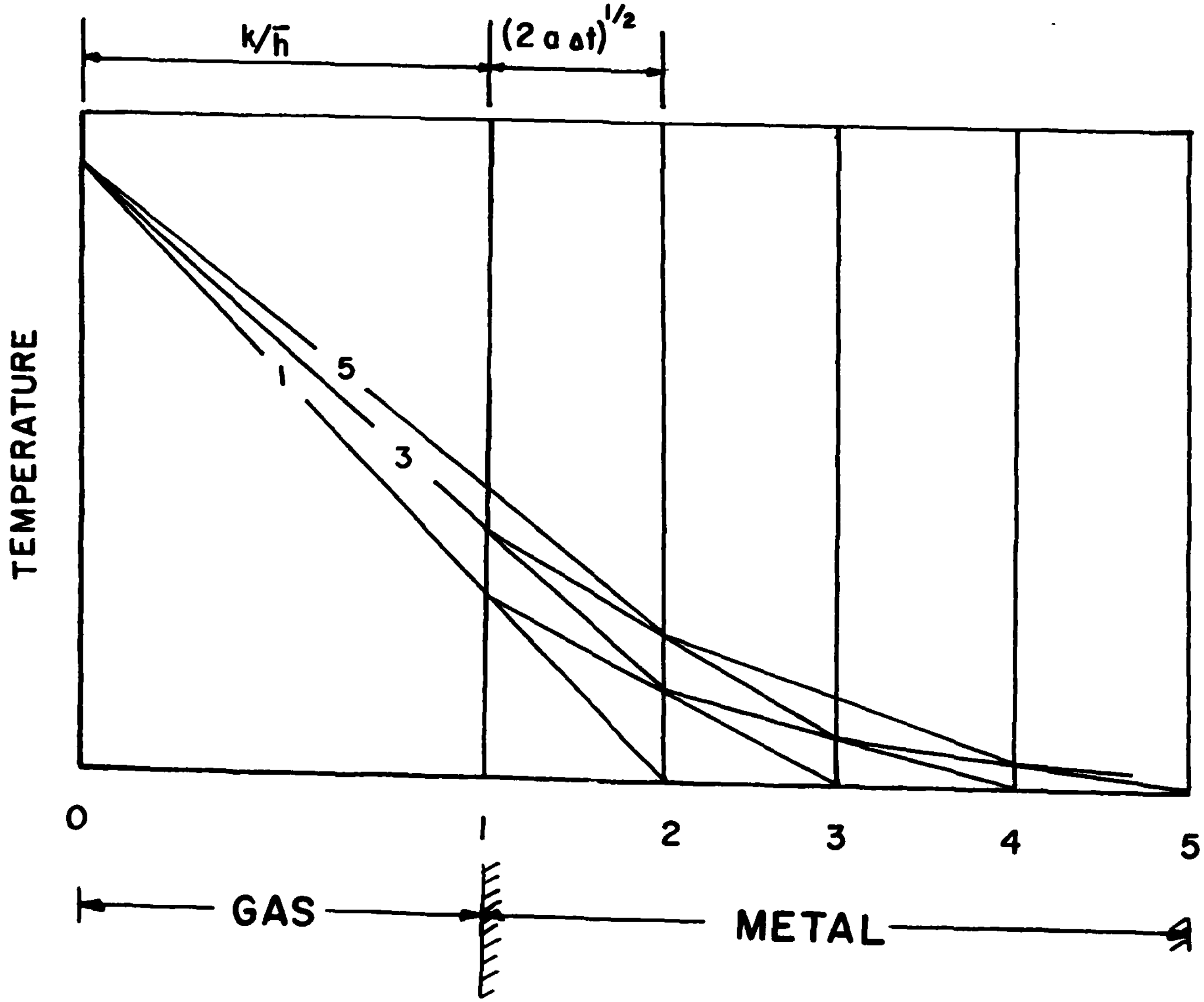


Figure 2. Wall Surface Temperature Gradient

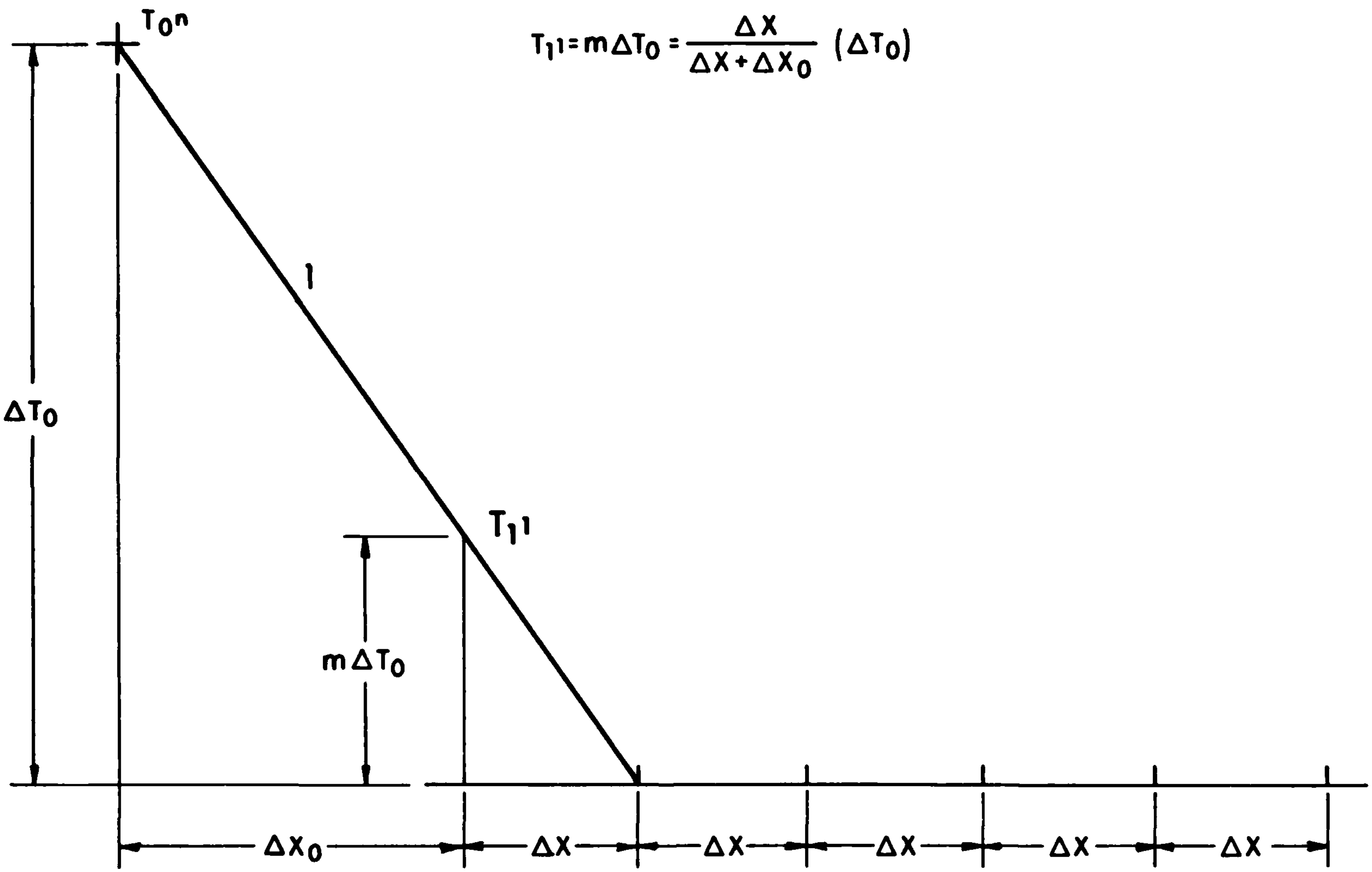


Figure 3. Graphical Determination of $T_1{}^1$

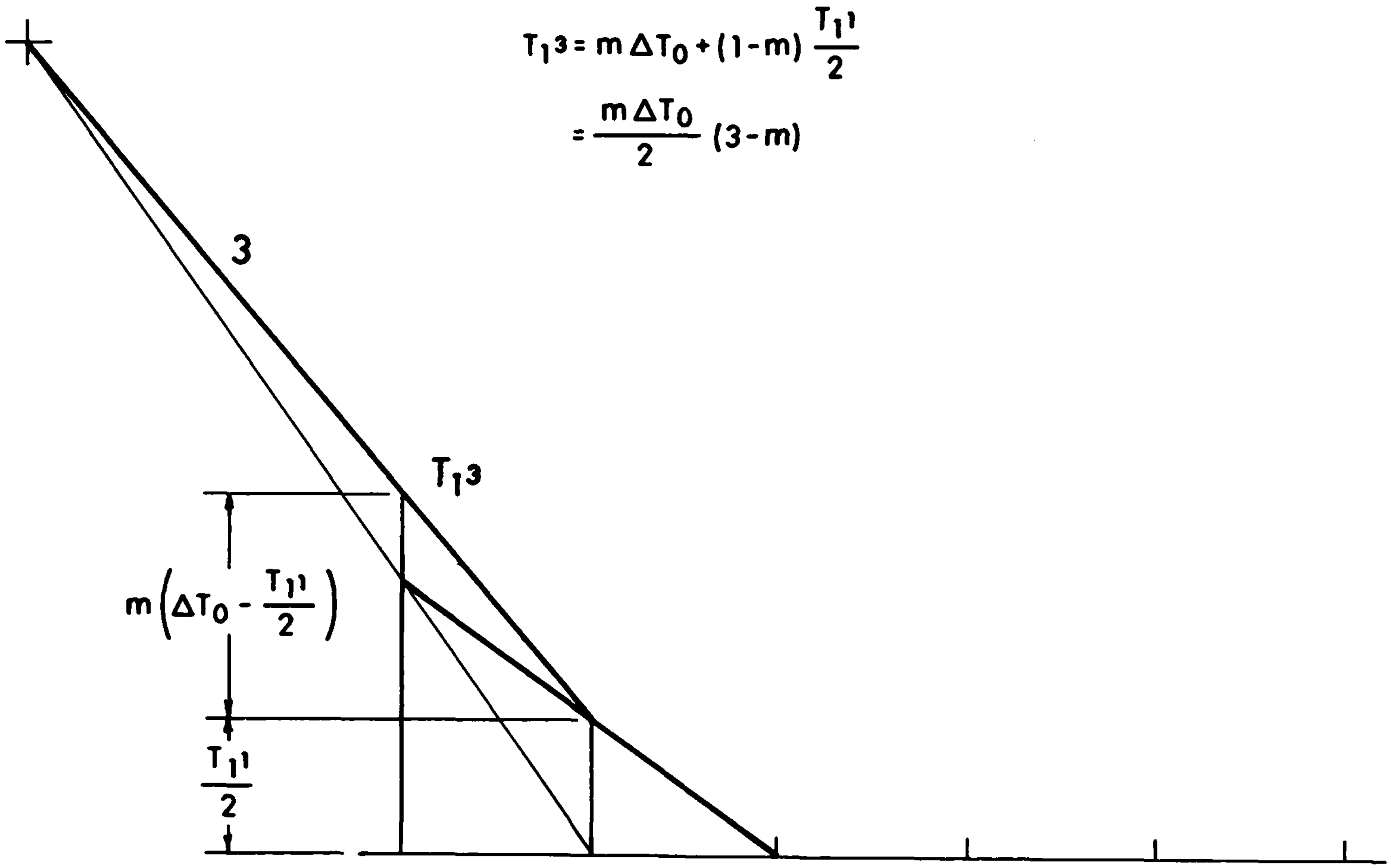


Figure 4. Graphical Determination of $T_1 3$

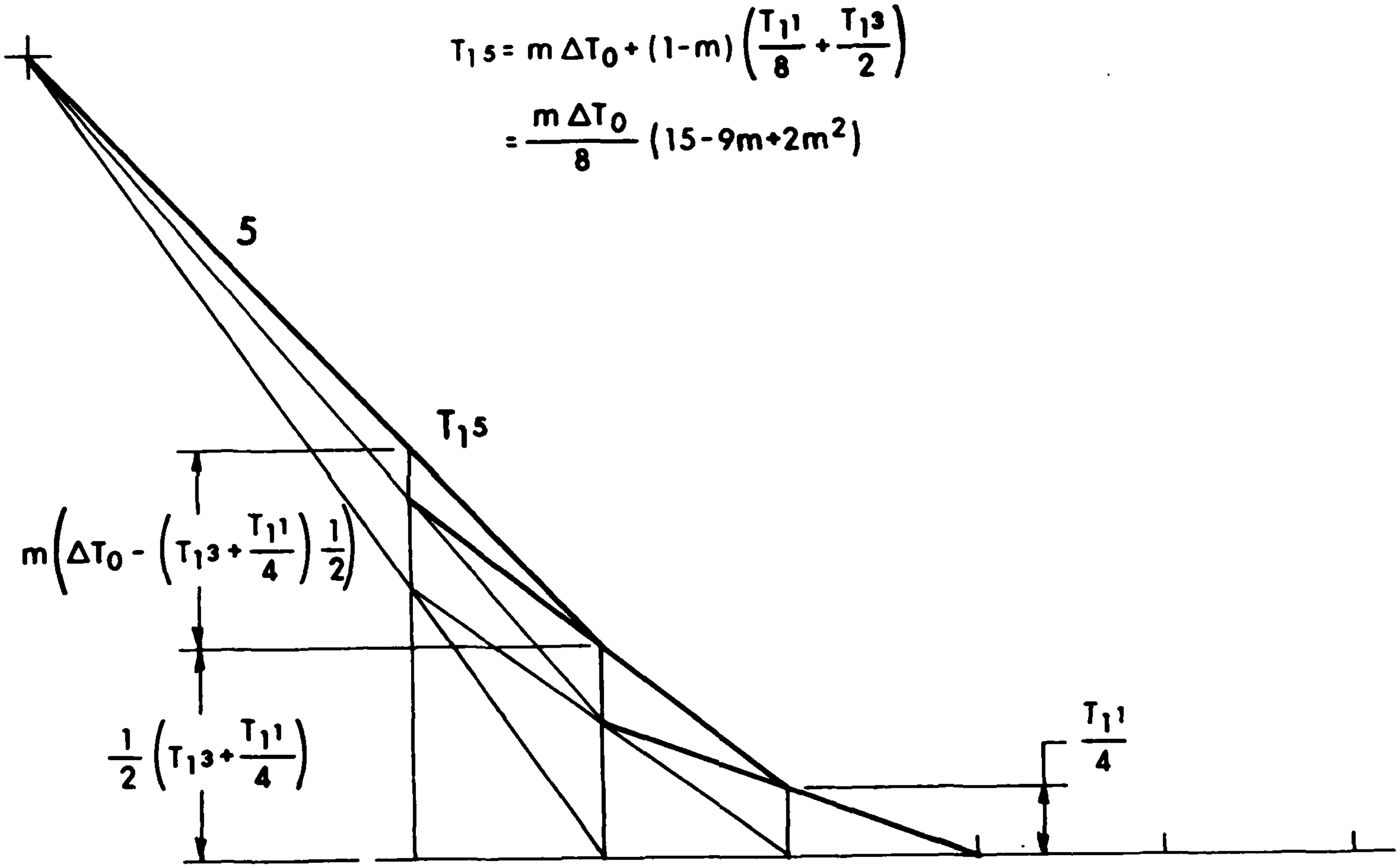


Figure 5. Graphical Determination of $T_1{}^5$

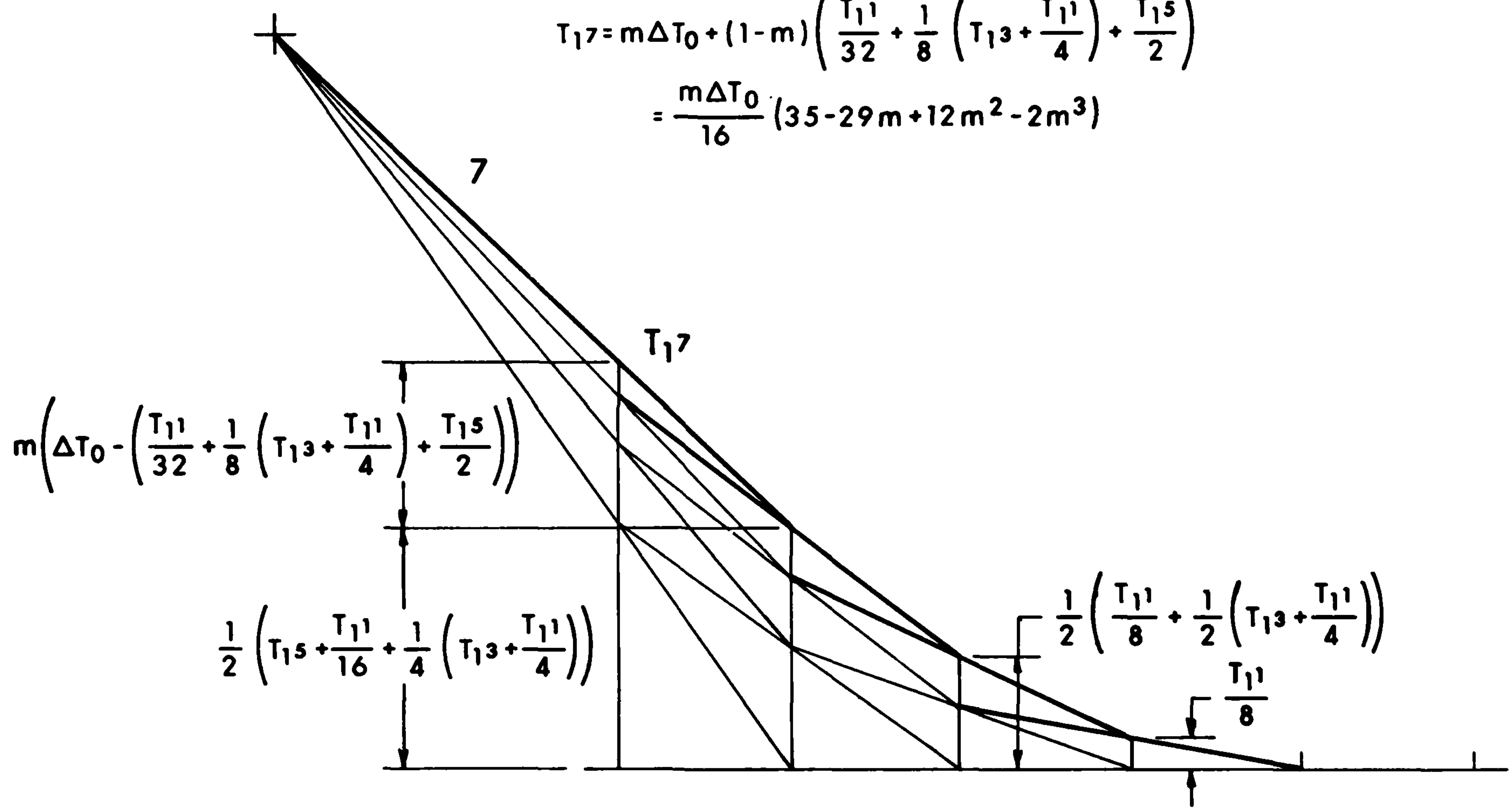


Figure 6. Graphical Determination of $T_1 7$

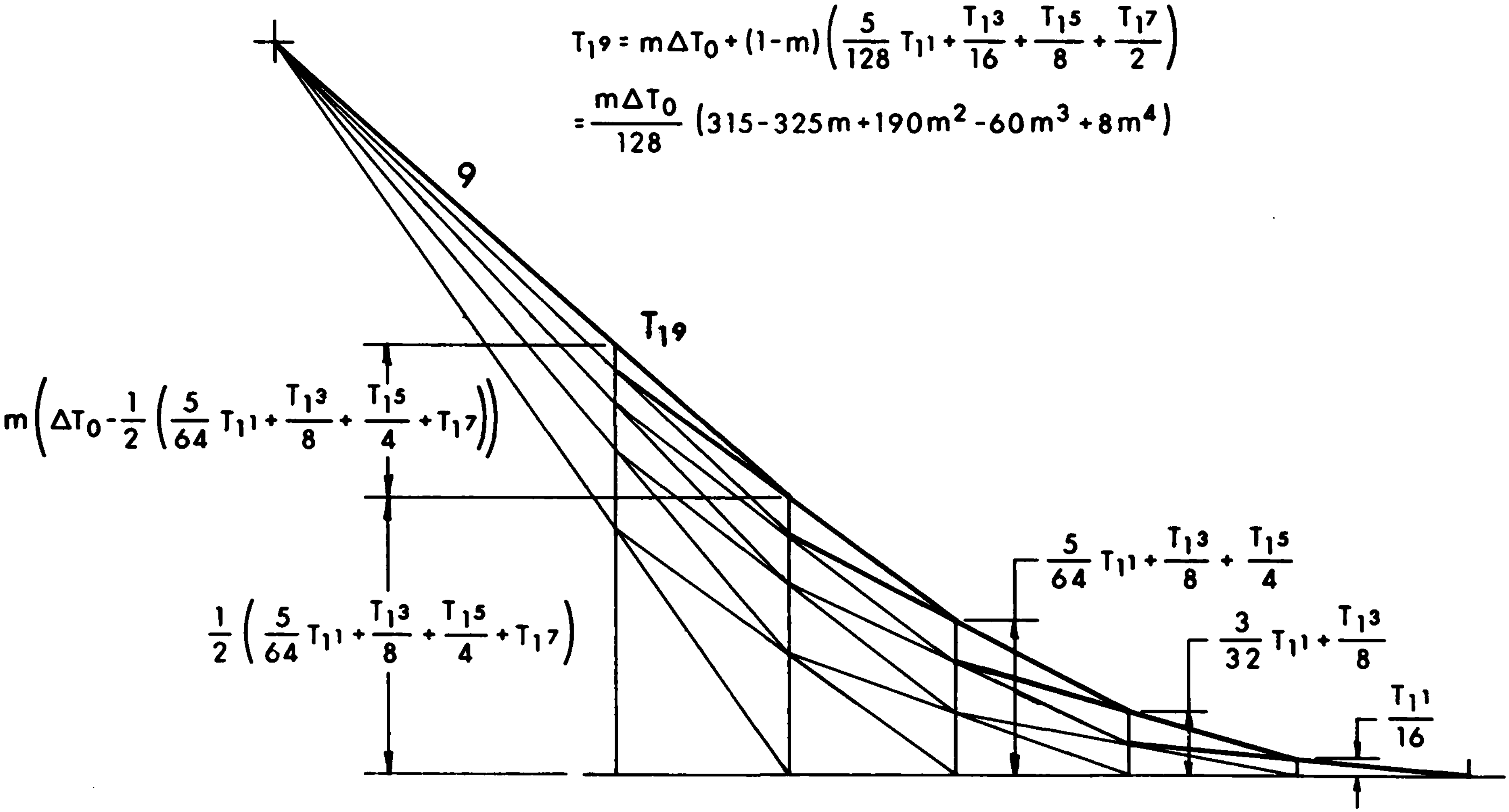


Figure 7. Graphical Determination of T_1^9

TABLE 1. ALGEBRAIC SPECIFICATION OF T_1n

$$T_1o = T_no$$

$$T_11 = T_12 = m\,\Delta\,T_o + T_1o \qquad = m\,\Delta T_o\phi_1 + T_1o$$

$$T_13 = T_14 = \tfrac{1}{2}\,m\,\Delta\,T_o(3-m) + T_1o \qquad = m\,\Delta T_o\phi_3 + T_1o$$

$$T_15 = T_16 = \tfrac{1}{4}\,m\,\Delta T_o(7.5-4.5m + m^2) + T_1o \qquad = m\,\Delta T_o\phi_5 + T_1o$$

$$T_17 = T_18 = \frac{1}{8}\,m\Delta T_o(17.5-14.5m + 6m^2 - m^3) + T_1o \qquad = m\,\Delta T_o\phi_7 + T_1o$$

$$T_19 - T_110 = \frac{1}{16}\,m\Delta T_o(39.375-40.625m + 23.75m^2-7.5m^3+m^4)+T_1o \qquad = m\,\Delta T_o\phi_9 + T_1o$$

where ϕ_n is as defined above.

TABLE 2. ALGEBRAIC SPECIFICATION OF T_2n

$$T_2o = T_21$$

$$T_22 = T_23 = \tfrac{1}{2}\,m\,\Delta T_o + T_2o \qquad = m\,\Delta T_o\theta_3 + T_2o$$

$$T_24 = T_25 = \tfrac{1}{4}\,m\,\Delta T_o(3.5-m) + T_2o \qquad = m\,\Delta T_o\theta_5 + T_2o$$

$$T_26 = T_27 = \frac{1}{8}\,m\Delta T_o(9.5-5m + m^2) + T_2o \qquad = m\,\Delta T_o\theta_7 + T_2o$$

$$T_28 = T_29 = \frac{1}{16}\,m\Delta T_o(24.25 - 19.75\,m + 6.5\,m^2-m^3) + T_2o \qquad = m\,\Delta T_o\theta_9 + T_2o$$

where θ_n is as defined above.

The calculating technique is demonstrated by example, using the HP-97, in Appendix C.

Evaluation of $\bar{h}$ from Experimental Data

If an experimental wall temperature distribution[1,4] can be established, it is possible to read the temperature rise at any two times and solve the algebraic set of equations of Table 1 to eliminate T_0. The resulting expressions are single valued in "m" and solvable. Using the most convenient divisor,

$$\frac{T_1 3 - T_1 o}{m \Delta T_o \phi_1} = \tfrac{1}{2}\ (3-m) = \phi_3(m) \quad , \tag{8}$$

$$\frac{T_1 5 - T_1 o}{m \Delta T_o \phi_1} = \tfrac{1}{2}\ (7.5-4.5m+m^2) = \phi_5(m) \quad , \tag{9}$$

$$\frac{T_1 7 - T_1 o}{m \Delta T_o \phi_1} = \frac{1}{8}\ (17.5-14.5m+6m^2-m^3) = \phi_7(m) \quad , \tag{10}$$

$$\frac{T_1 9 - T_1 o}{m \Delta T_o \phi_1} = \frac{1}{16}\ (39.375-40.625m+23.75m^2-7.5\ m^3 + m^4) = \phi_9(m) \ , \tag{11}$$

whereby
$$\frac{T_1 n - T_1 o}{\Delta T_o} = m\ \phi_n(m)\phi_1 . \tag{12}$$

The solution iterates "m" until the experimental and calculated $T_1 n$ differ by less than a pre-selected error bound. The heat transfer coefficient, $\bar{h}$, is then deduced from the definition of "m."

Appendix D contains a sample calculation and printout of the HP-97 program for illustration.

Determination of $T_0 n$ from Experimental Data

Plainly, the extension of the procedure is to examine the average value of the driving gas temperature over the subintervals, i.e., those within the elapsed time interval. Using values of $T_1 1$ and $T_1 3$, the intersection of the rays by the construction of Figure 4 locates the value of $T_0 n$ for the subinterval between time 1 and time 3. The remaining intersections are similarly determined except for the period between time 0 and time 1. $T_1 1$ is directly determined by the physical constraints of the finite difference equation. Figures 8 through 12 illustrate and Tables 3 and 4, based on the following identities, summarize the procedure. Table 3 considers m = constant while Table 4 allows a variable m.

$$\frac{T_1 1}{T_o n} = m, \quad 0<t<1 \quad , \tag{13}$$

$$\frac{T_1 3 - T_2 3}{T_o n - T_2 3} = m \quad , \quad 1<t<3 \quad , \tag{14}$$

$$\frac{T_1 5 - T_2 5}{T_o n - T_2 5} = m \quad , \quad 3<t<5 \quad , \tag{15}$$

$$\frac{T_1 7 - T_2 7}{T_o n - T_2 7} = m \quad , \quad 5<t<7 \quad , \tag{16}$$

$$\frac{T_1 9 - T_2 9}{T_o n - T_2 9} = m \quad , \quad 7<t<9 \quad . \tag{17}$$

$T_2 n$ is listed in Table 2 and $T_n 0$ commonly adds to zero.

For the subinterval temperature rise and a corresponding "m," the local heat transfer coefficient can be calculated by the methods demonstrated in Appendix D.

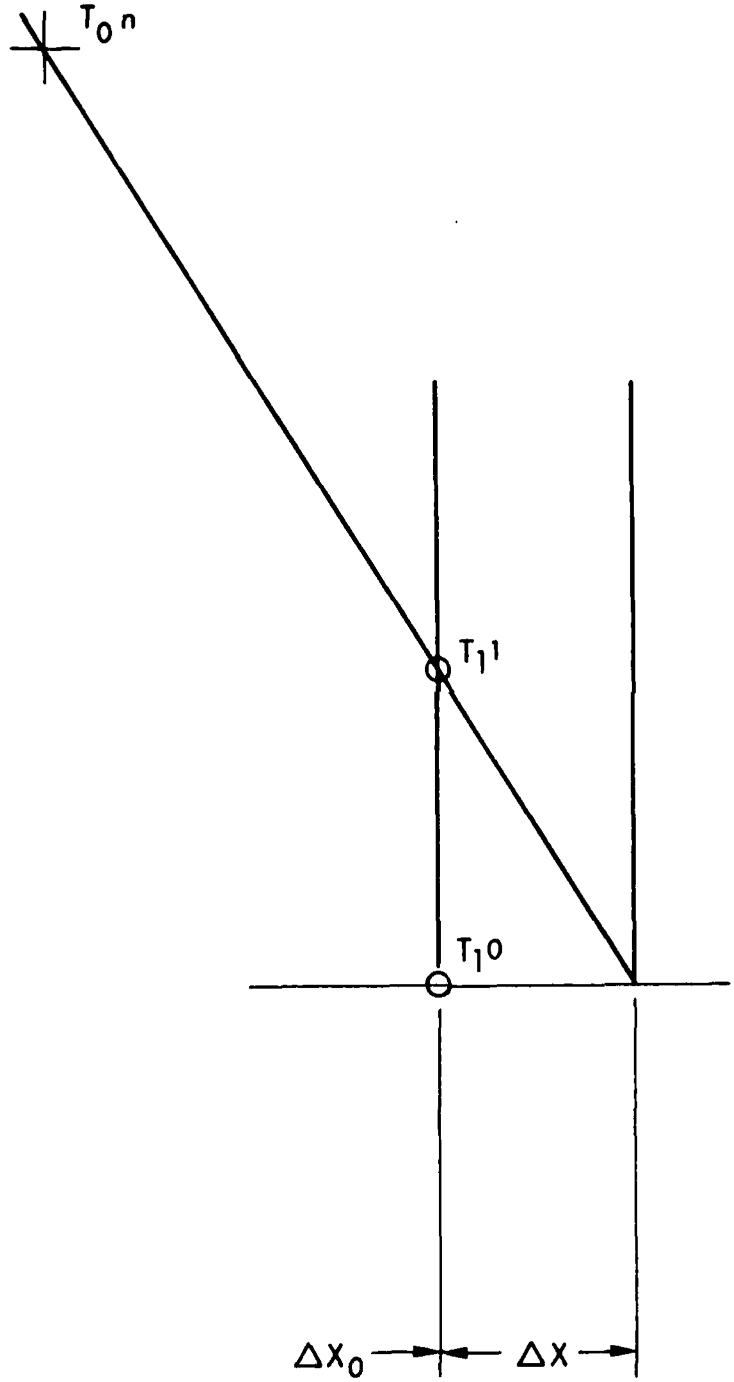


Figure 8. Schmidt Plot for Initial Subinterval from 0 to 1

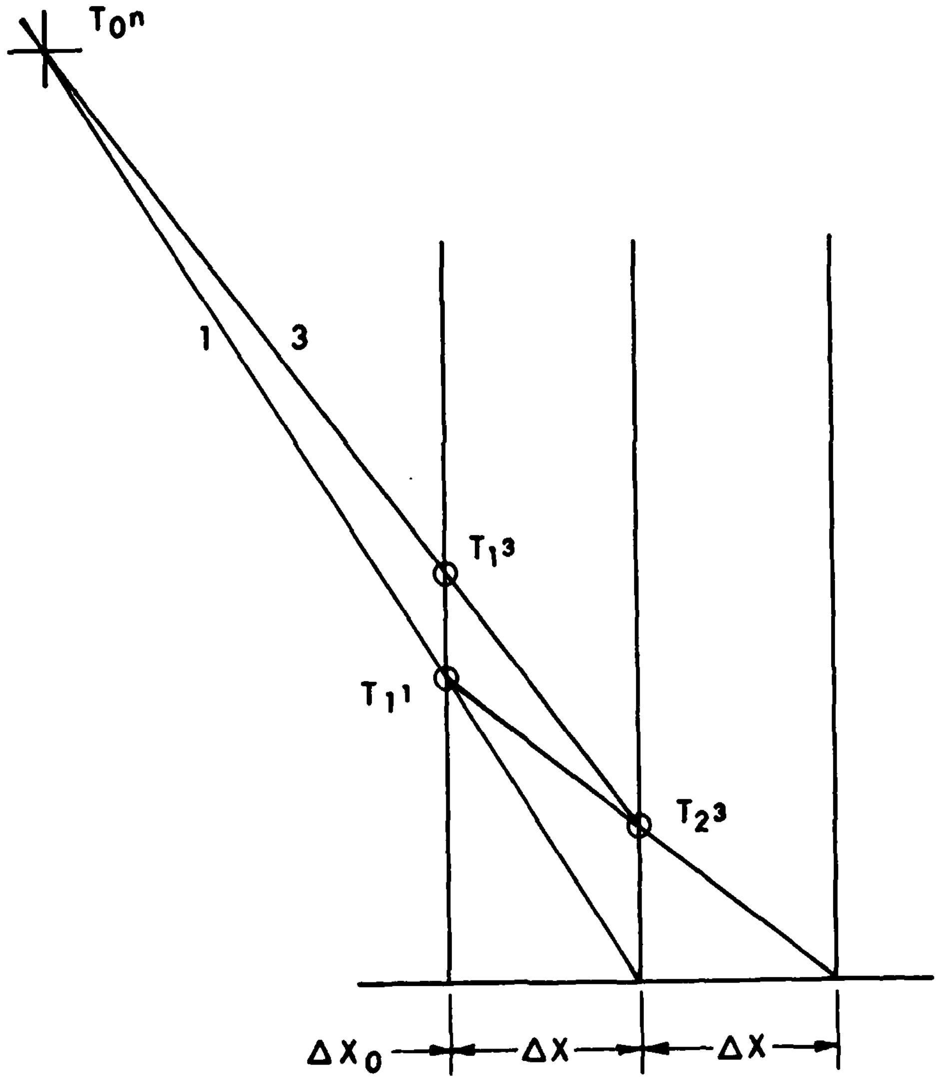


Figure 9. Schmidt Plot for Time Interval from 1 to 3

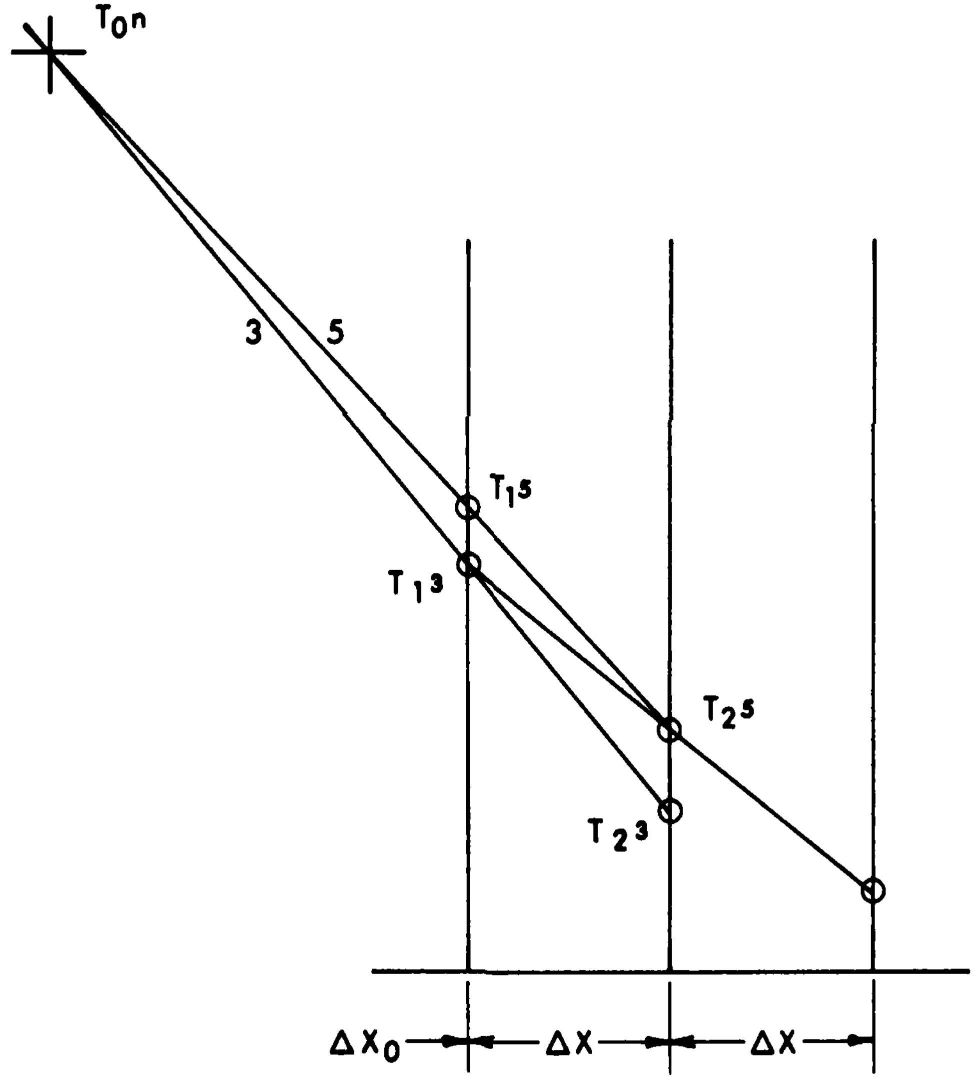


Figure 10. Schmidt Plot for Time Interval from 3 to 5

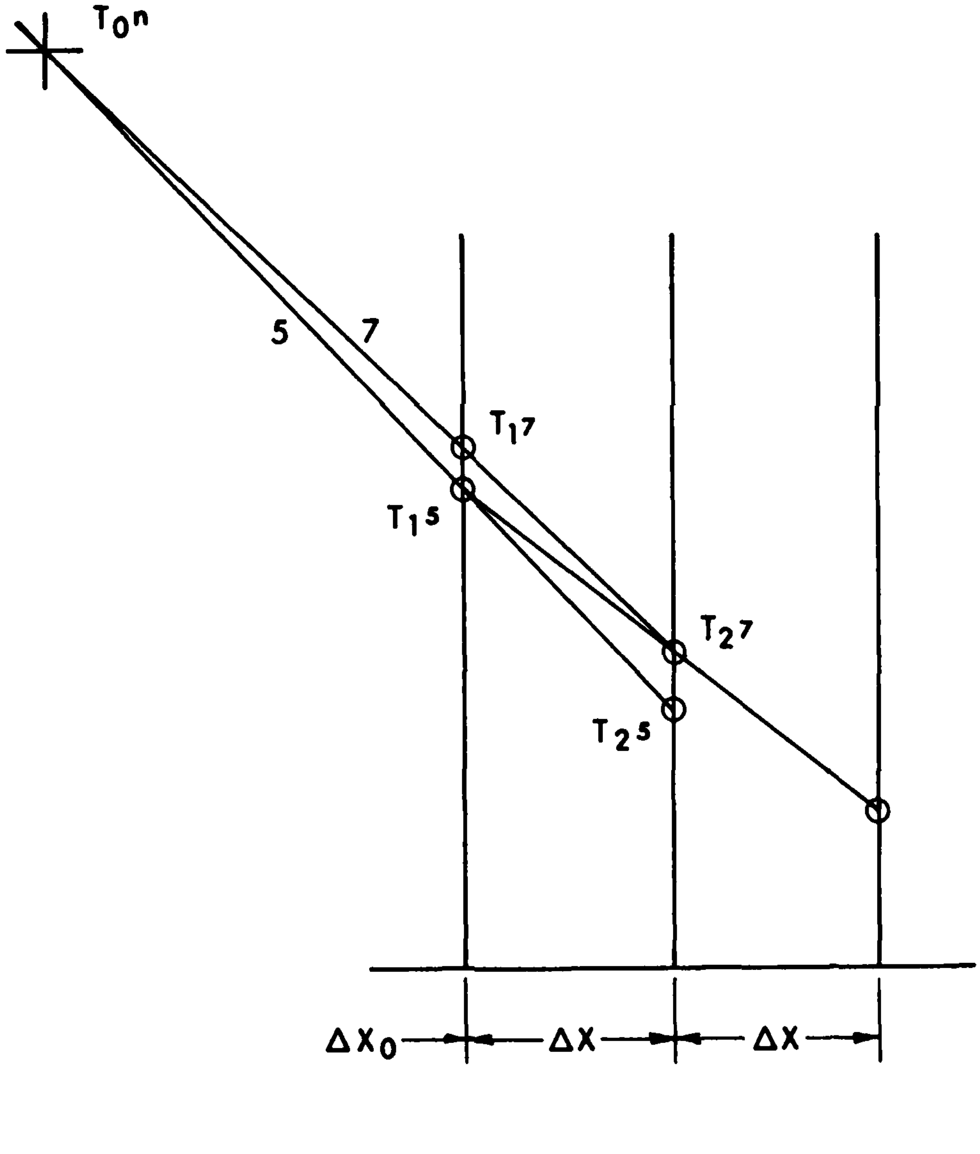


Figure 11. Schmidt Plot for Time Interval from 5 to 7

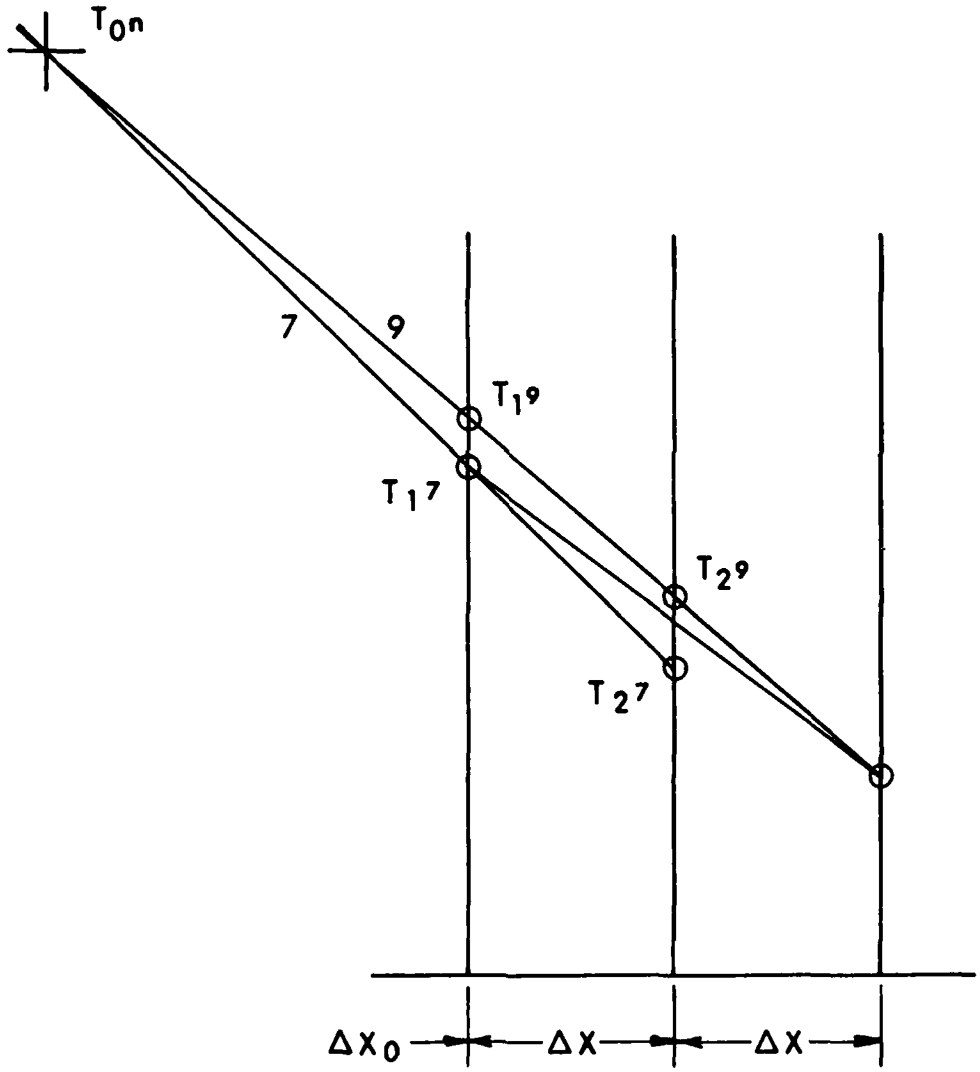


Figure 12. Schmidt Plot for Time Interval from 7 to 9

TABLE 3. ALGEBRAIC SPECIFICATION OF T_0n FOR m = CONSTANT

Time Interval		T_0n
From	To	For "m" = constant - $\dfrac{T_11}{T_0n}$
0	1	$2\ T_11$
1	3	$\dfrac{(T_11)^2}{3\ T_11 - 2\ T_13}$
3	5	$\dfrac{(T_11)^2 + 4T_11\ T_13}{9\ T_11 + 4\ T_13 - 8\ T_15}$
5	7	$\dfrac{(T_11)^2 + 2\ T_11\ T_13 + 8\ T_11\ T_15}{17\ T_11 + 2\ T_13 + 8\ T_15 - 16\ T_17}$
7	9	$\dfrac{5\ (T_11)^2 + 8\ T_11\ T_13 + 16\ T_11\ T_15 + 64\ T_11\ T_17}{133\ T_11 + 8\ T_13 + 16\ T_15 + 64\ T_17 - 128\ T_19}$

TABLE 4. ALGEBRAIC SPECIFICATION OF T_0n FOR m ≠ CONSTANT

Time Interval		T_0n
From	To	For "m" variable
0	1	$2\,T_1 1$
1	3	$\dfrac{(T_1 1)^2}{3\,T_1 1 - 2\,T_1 3}$
3	5	$\dfrac{4T_1 1 - 9\,T_1 1\,T_1 3 - 2(T_1 3)^2 + 8\,T_1 3\,T_1 5}{3\,T_1 1 - 10\,T_1 3 + 8\,T_1 5}$
5	7	$\dfrac{-\,T_1 1\,T_1 5 + 2\,T_1 1\,T_1 7 - 2\,T_1 3\,T_1 5 + 8\,T_1 3\,T_1 7 - 8\,(T_1 5)^2}{16\,T_1 7 - 16\,T_1 5 + T_1 1 + 6\,T_1 3 - 8\,T_1 5}$
7	9	$\dfrac{-5\,T_1 1\,T_1 7 - 8\,T_1 3\,T_1 7 - 16\,T_1 5\,T_1 7 - 64\,(T_1 7)^2 + 8\,T_1 1\,T_1 9 + 16\,T_1 3\,T_1 9 + 64\,T_1 5\,T_1 9}{128\,T_1 9 - 192\,T_1 7 + 3\,T_1 1 + 8\,T_1 3 + 48\,T_1 5}$

III. RESULTS

The data from References 11, 12 is used to examine some of the implications of the equations. Figure 13[12] provides the temperature history of the interior barrel surface of a bench mounted 20 mm weapon firing TB-1 propellant. The thermocouple was located immediately forward of the chamber. For the idealized conditions previously specified, i.e., constant T_o equal to the adiabatic flame temperature of the propellant and univalued $\bar{h}$ and with the properties given in Table D-1, the heat transfer coefficient corresponding to a maximum wall surface temperature rise of 842°F (450°C) is found to be 10,953 B/hr ft^2F (1656 cal/hr cm^2 C). This result is obtained by iterating "m" and $T_1 9 - T_1 o$ until the calculated agrees with the experimental temperatures. Figure 14 compares three different driving gas temperatures for the cases of an assumed constant heat transfer coefficient and adiabatic flame temperature source and the alternate cases where the heat transfer coefficient is allowed to vary within the subintervals--in one instance being an averaged constant over the full interval and in the other being coupled to the local effective gas temperature. Figure 15 shows the corresponding heat transfer coefficients for this example.

IV. CONCLUSIONS

With reliable digital temperature recording to one millisecond resolution and current desk top calculating facility, the Schmidt plot approach can be used to establish the effective heat transfer coefficient for experimental data where the transport properties of the system may be assumed constant.

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11 *Artillery Ammunition Master Calibration Chart, Material Testing Directorate Report 1375, 15th Revision, Aberdeen Proving Ground, MD 1973.*

12 *Unpublished Test Data, Interior Ballistics Division, Ballistic Research Laboratory APG, MD 1980.*

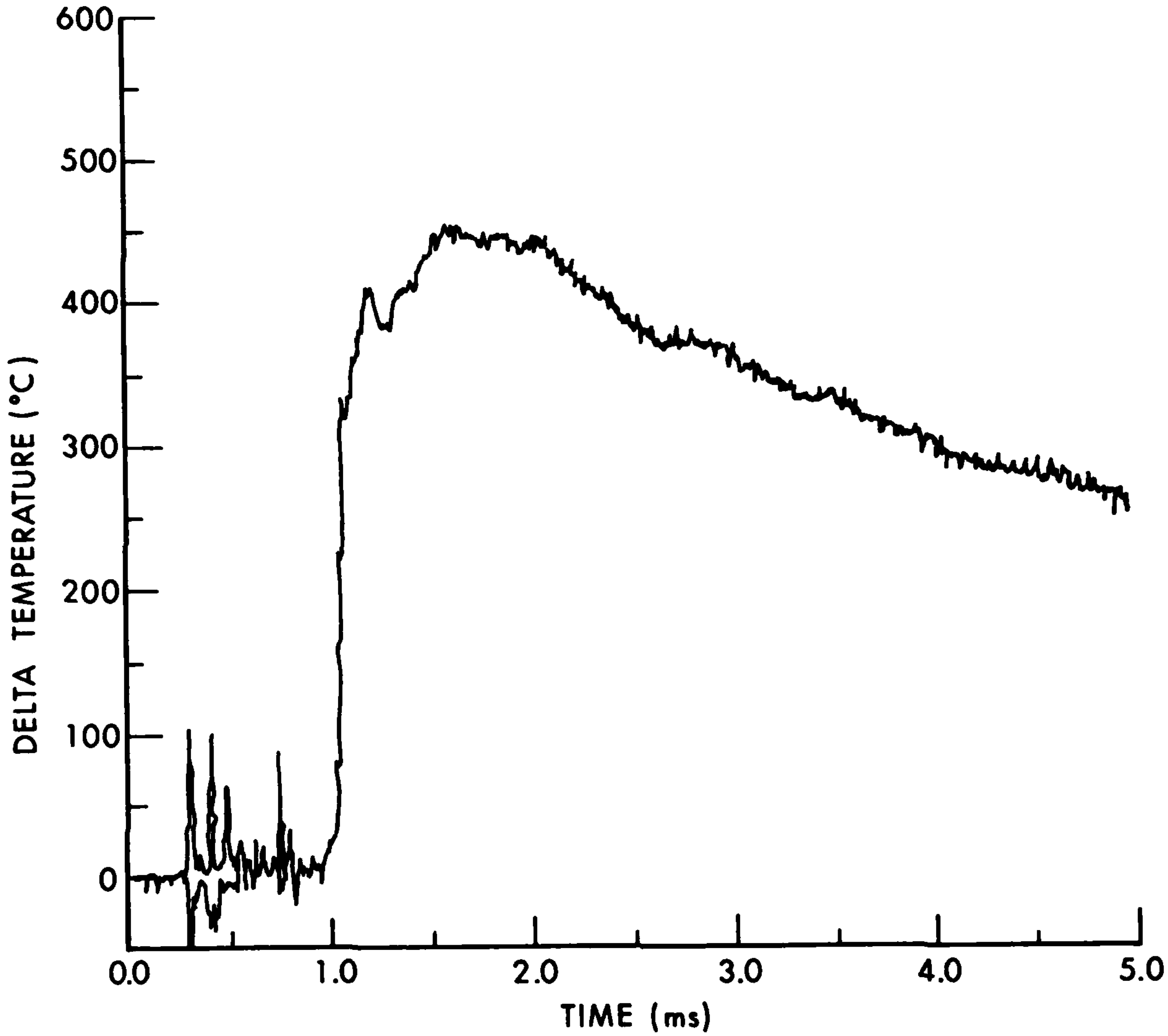

Figure 13. Experimental Wall Temperature History for Special Propellant Round

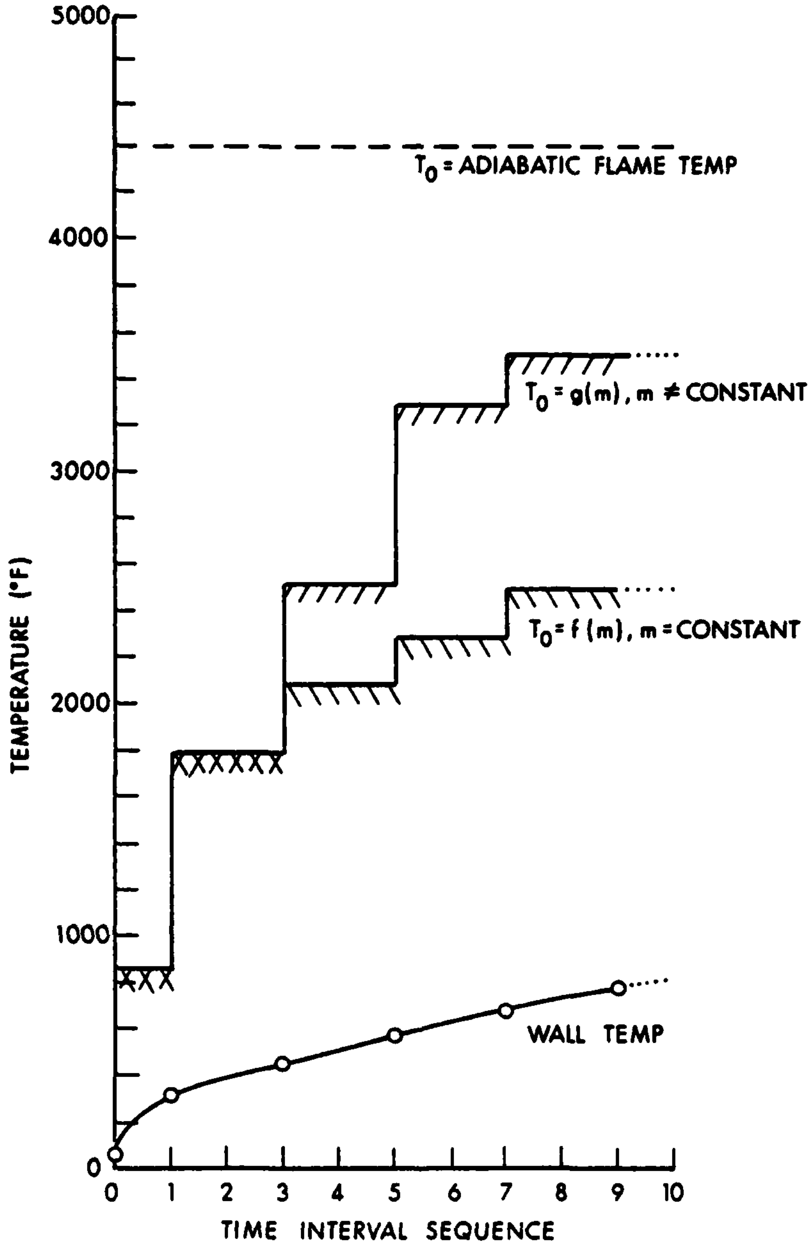

Figure 14. Driving Gas Temperature Profiles

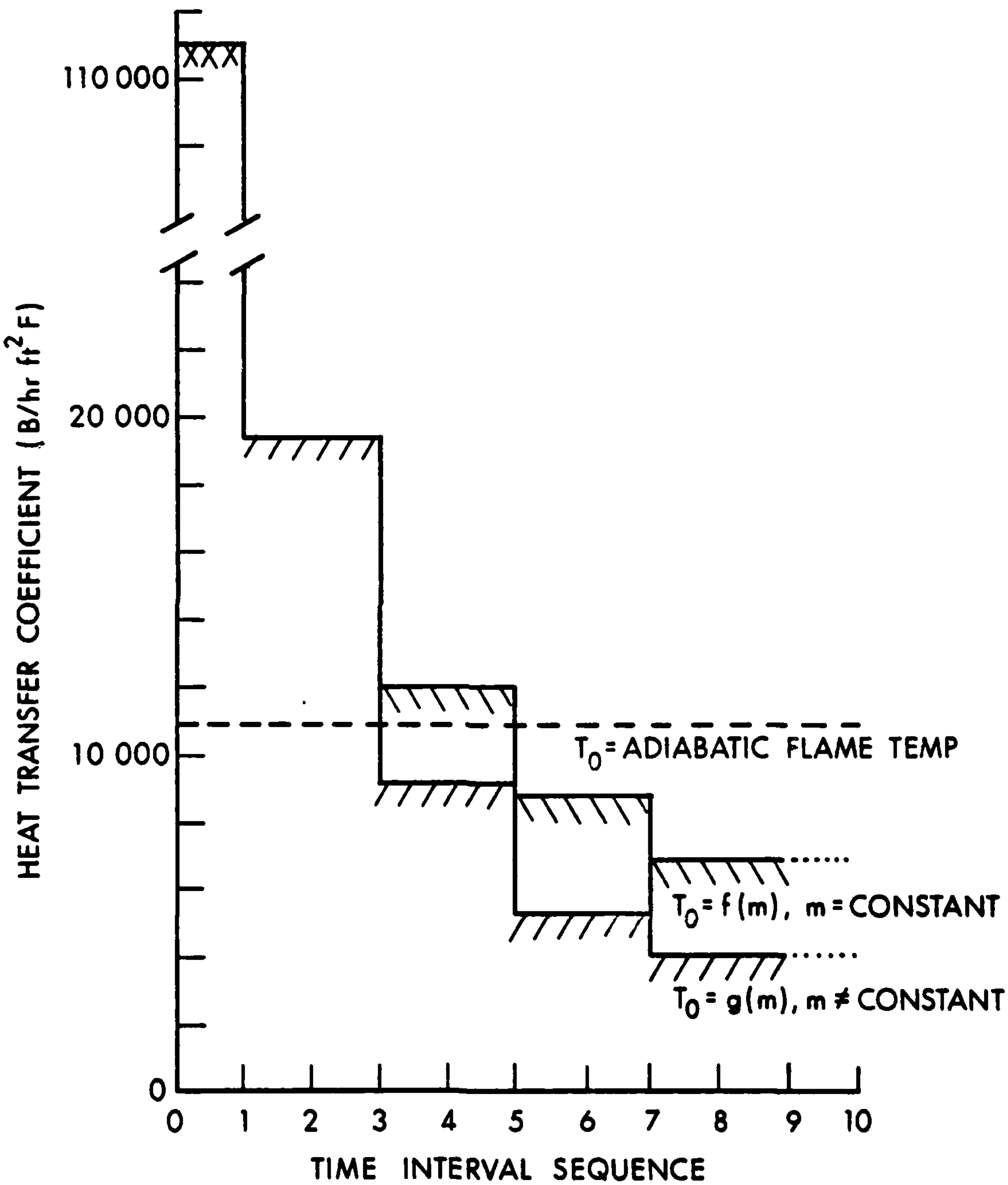

Figure 15. Heat Transfer Coefficient Profiles

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9. J. F. Polk, "An Algorithm for Heat Transfer in Gun Barrels," Transactions of the Twenty-Fifth Conference of Army Mathematicians, ARO Report 80-1, 1980.

10. "CYCOND" Program, and "HTC" Program, communication via Mr. Kovacs, DRDAR-SE, Picatinny Arsenal.

11. Artillery Ammunition Master Calibration Chart, Material Testing Directorate Report 1375, 15th Revision, Aberdeen Proving Ground, MD, 1973.

12. Unpublished Test Data, Interior Ballistics Division, Ballistic Research Laboratory, APG, MD, 1980.

F-1. W.F. Donovan, "One Factor Affecting the Dispersion of Long Rod Penetrators," ARBRL-MR-02846, June 1978. AD #A058596

APPENDIX A

TEMPERATURE GRADIENT IN A TWO DIMENSIONAL PLANE WALL

APPENDIX A

TEMPERATURE GRADIENT IN A TWO DIMENSIONAL PLANE WALL

For the simple geometry represented by a semi-infinite flat plate exposed to a suddenly imposed constant temperature gas on one surface, the Schmidt plot takes the form given on Figure A-1. The initial distance increment, Δx_0, is made proportional to the film coefficient conductance while the following nine, Δx, characterize the metal. Within the given calculation, the number of distance increments is determined by the storage capacity of the calculator. There is no limit to the number of time cycles.

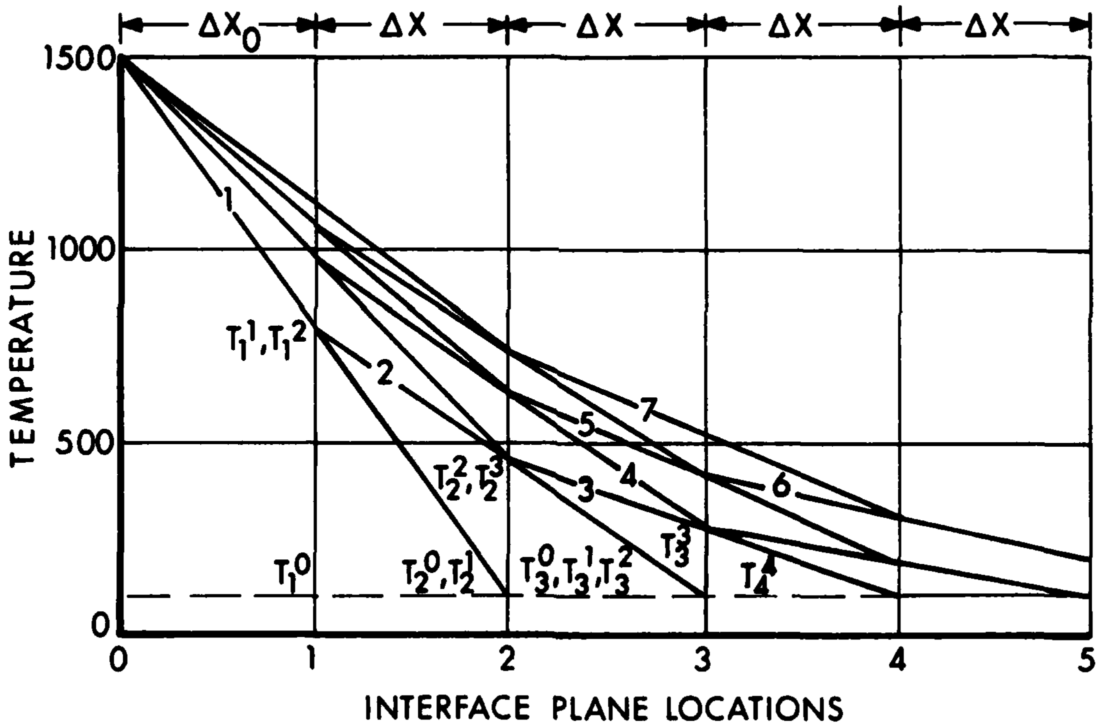

Figure A-1. Schmidt Plot for Schematic Slab Temperature Profile

The problem is to establish the interface time-temperatures in order to insure that the wall material strength criteria is not exceeded. Table A-1, following, illustrates the stepping progression in the finite difference parameters and Figure A-2 presents the results of a specific example.

TABLE A-1. STEPPING PROGRESSION IN TEMPERATURE TIME PARAMETERS FOR SLAB

		TIME PROGRESSION →										
DISTANCE		t^0	t^1	t^2	t^3	t^4	t^5	t^6	t^7	t^8	t^9	t^{10}
FILM	x_0	T_0	T_0^1	T_0^2	T_0^3	T_0^4	T_0^5	T_0^6	T_0^7	T_0^8	T_0^9	T_0^{10}
	x_1	T_1	$T_2^0+\Delta x\frac{T_0-T_2^0}{\Delta x_1+\Delta x}$	T_1^1	$T_2^2+\Delta x\frac{T_0-T_2^2}{\Delta x_1+\Delta x}$	T_1^3	$T_2^4+\Delta x\frac{T_0-T_2^4}{\Delta x_1+\Delta x}$	T_1^5	$T_2^6+\Delta x\frac{T_0-T_2^6}{\Delta x_1+\Delta x}$	T_1^7	$T_2^8+\Delta x\frac{T_0-T_2^8}{\Delta x_1+\Delta x}$	T_1^9
SLAB	x_2	T_2	T_2^0	$.5[T_1^1+T_3^1]$	T_2^2	$.5[T_1^3+T_3^3]$	T_2^4	$.5[T_1^5+T_3^5]$	T_2^6	$.5[T_1^7+T_3^7]$	T_2^8	$.5[T_1^9+T_3^9]$
	x_3	T_3	$.5[T_2^0+T_4^0]$	T_3^1	$.5[T_2^2+T_4^2]$	T_3^3	$.5[T_2^4+T_4^4]$	T_3^5	$.5[T_2^6+T_4^6]$	T_3^7	$.5[T_2^8+T_4^8]$	T_3^9
	x_4	T_4	T_4^0	$.5[T_3^1+T_5^1]$	T_4^2	$.5[T_3^3+T_5^3]$	T_4^4	$.5[T_3^5+T_5^5]$	T_4^6	$.5[T_3^7+T_5^7]$	T_4^8	$.5[T_3^9+T_5^9]$
	x_5	T_5	$.5[T_4^0+T_6^0]$	T_5^1	$.5[T_4^2+T_6^2]$	T_5^3	$.5[T_4^4+T_6^4]$	T_5^5	$.5[T_4^6+T_6^6]$	T_5^7	$.5[T_4^8+T_6^8]$	T_5^9
	x_6	T_6	T_6^0	$.5[T_5^1+T_7^1]$	T_6^2	$.5[T_5^3+T_7^3]$	T_6^4	$.5[T_5^5+T_7^5]$	T_6^6	$.5[T_5^7+T_7^7]$	T_6^8	$.5[T_5^9+T_7^9]$
	x_7	T_7	$.5[T_6^0+T_8^0]$	T_7^1	$.5[T_6^2+T_8^2]$	T_7^3	$.5[T_6^4+T_8^4]$	T_7^5	$.5[T_6^6+T_8^6]$	T_7^7	$.5[T_6^8+T_8^8]$	T_7^9
	x_8	T_8	T_8^0	$.5[T_7^1+T_9^1]$	T_8^2	$.5[T_7^3+T_9^3]$	T_8^4	$.5[T_7^5+T_9^5]$	T_8^6	$.5[T_7^7+T_9^7]$	T_8^8	$.5[T_7^9+T_9^9]$
	x_9	T_9	$.5[T_8^0+T_{10}^0]$	T_9^1	$.5[T_8^2+T_{10}^2]$	T_9^3	$.5[T_8^4+T_{10}^4]$	T_9^5	$.5[T_8^6+T_{10}^6]$	T_9^7	$.5[T_8^8+T_{10}^8]$	T_9^9
	x_{10}	T_{10}	T_9^0	T_9^1	T_9^2	T_9^3	T_9^4	T_9^5	T_9^6	T_9^7	T_9^8	T_9^9

There are two branches to the algorithm. One branch describes the conditions at odd-numbered time cycles such that

$$T_1t = T_2t\text{-}1 + \frac{\Delta x \left(T_o - T_2t\text{-}1\right)}{\Delta x + \Delta x_o} \qquad \text{(A-1)}$$

$$T_2t = T_2t\text{-}1 \qquad \text{(A-2)}$$

$$T_3t = \frac{T_2t\text{-}1 + T_4t\text{-}1}{2} \qquad \text{(A-3)}$$

$$T_4t = T_4t\text{-}1 \qquad \text{(A-4)}$$

$$T_5t = \frac{T_4t\text{-}1 + T_6t\text{-}1}{2} \qquad \text{(A-5)}$$

$$T_6t = T_6t\text{-}1 \qquad \text{(A-6)}$$

$$T_7t = \frac{T_6t\text{-}1 + T_8t\text{-}1}{2} \qquad \text{(A-7)}$$

$$T_8t = T_8t\text{-}1 \qquad \text{(A-8)}$$

$$T_9t = \frac{T_8t\text{-}1 + T_{10}t\text{-}1}{2} \qquad \text{(A-9)}$$

$$T_{10}t = T_9t\text{-}1 \qquad \text{(A-10)}$$

The second branch describes conditions at even-numbered time cycles such that

$$T_1t = T_1t\text{-}1 \quad \text{(A-11)}$$

$$T_2t = \frac{T_1t\text{-}1 + T_3t\text{-}1}{2} \quad \text{(A-12)}$$

$$T_3t = T_3t\text{-}1 \quad \text{(A-13)}$$

$$T_4t = \frac{T_3t\text{-}1 + T_5t\text{-}1}{2} \quad \text{(A-14)}$$

$$T_5t = T_5t\text{-}1 \quad \text{(A-15)}$$

$$T_6t = \frac{T_5t\text{-}1 + T_7t\text{-}1}{2} \quad \text{(A-16)}$$

$$T_7t = T_7t\text{-}1 \quad \text{(A-17)}$$

$$T_8t = \frac{T_7t\text{-}1 + T_9t\text{-}1}{2} \quad \text{(A-18)}$$

$$T_9t = T_9t\text{-}1 \quad \text{(A-19)}$$

$$T_{10}t = T_9t\text{-}1 \quad \text{(A-20)}$$

The magnitude of the time cycles is determined from the initial definitions such that

$$\frac{\Delta x^2}{2a\Delta t} = 1 \quad , \tag{A-21}$$

or

$$\Delta t = \frac{\Delta x^2}{2a} \quad ,$$

where

$$a = \frac{k}{\rho c} \quad , \tag{A-22}$$

and k is thermal conductivity,
c is specific heat, and
ρ is material density.

Whereby

$$\Delta t = \frac{c\Delta x^2}{2\rho k} \tag{A-23}$$

Example

The calculating technique is demonstrated by example. Assume a semi-infinite steel wall 12 inches (30.2 cm) thick suddenly exposed to a 5000°F (2760°C) gas flow over one face. After a measured interval of 26 sec, the gas temperature is reduced to ambient. Table A-2 defines the initial conditions and operating parameters for the program input. Figure A-2 presents a Schmidt plot of the results. The HP-97 listing follow Figure A-2.

TABLE A-2. DEFINITION OF BOUNDARY CONDITIONS FOR SAMPLE SLAB PROBLEM

	Heat Source	Film	Steel Slab	Reference
Initial Temperature (T_0)	5000 F 2760 C 2730 N	100 F 37.7 C 280 N	100 F 37.7 C 280 N	Assumed
Unit Film Conductance ($\bar{h}$)		25 B/hr ft^2 F 3.782 cal/hr cm^2 C 974.1 CAL^2/hr N		(5)
Thermal Conductivity (k)			10 B/hr ft F 46.11 cal/hr cm C 974.1 CAL^3/hr N	(5)
Diffusivity (a)			.48 ft^2/hr 446 cm^2/hr 3.0 CAL^2/hr	(5)
Equivalent Film Thickness ($k/\bar{h}$)		4.8 in 12.19 cm 1.0 CAL		Construction
Slab Thickness			12 in 30.19 cm 2.5 CAL	Assumed (n = 10)
Distance Increment (Δx_0) (Δx)		(Δx_0) 4.8 in 12.19 cm 1.0 CAL	(Δx) 1.0 in 2.54 cm .208 CAL	$\Delta x_0 = k/\bar{h}$
Time Increment e		26.[illegible] sec	26.04 sec	$\Delta t_c = n\Delta t$ $\Delta t = \Delta x^2/2a$

HP-97 INITIAL REGISTER CONTENTS

Registers R_0 through R_9 enter the initial temperatures at the interstitial planes.

R_A indicates the number of time cycles required to elapse 2.6 minutes (for this example).

R_B gives the distance increment for the steel.

R_C gives the sum of the steel and equivalent film increments.

R_D and R_E indicate the initial temperature at the terminal plane.

R_I enters the number of time increments completed (usually 0).

R_{S0} gives the effective ambient temperature of the gas during the cooling.

OUTPUT

The printed output presents the temperature in the enumerated sequence of the interstitial planes. The terminal plane temperature is given in the D register and the number of time cycles completed in the I register. The real time elapsed is this number times the time interval.

SAMPLE INPUT AND OUTPUT

The program entry employed with the HP-97 is given below. Any consistant system of units is suitable.

Primary Registers	Initial Values	Example
R_o	T_o	2730 N
R_1	T_1	280 N
R_2	T_2	280 N
R_3	T_3	280 N
R_4	T_4	280 N
R_5	T_5	280 N
R_6	T_6	280 N
R_7	T_7	280 N
R_8	T_8	280 N
R_9	T_9	280 N
R_A	Y	6
R_B	Δx	.208 CAL
R_C	$\Delta x_o + \Delta x$	1.208 CAL
R_D	T_{10}	208 N
R_E	T_{10}	280 N
R_I	0	0

Secondary Registers		
R_0	$T_{AMBIENT}$	280 N

Output after 15 time cycles

	T_o	280 N
	T_1	436 N
	T_2	469 N
	T_3	455 N
	T_4	440 N
	T_5	404 N
	T_6	367 N
	T_7	339 N

T_8	312 N
T_9	303 N
Intermediate Result	303 N
Intermediate Result	
Intermediate Result	
T_{10}	294 N
Intermediate Result	
Number of time cycles completed	15

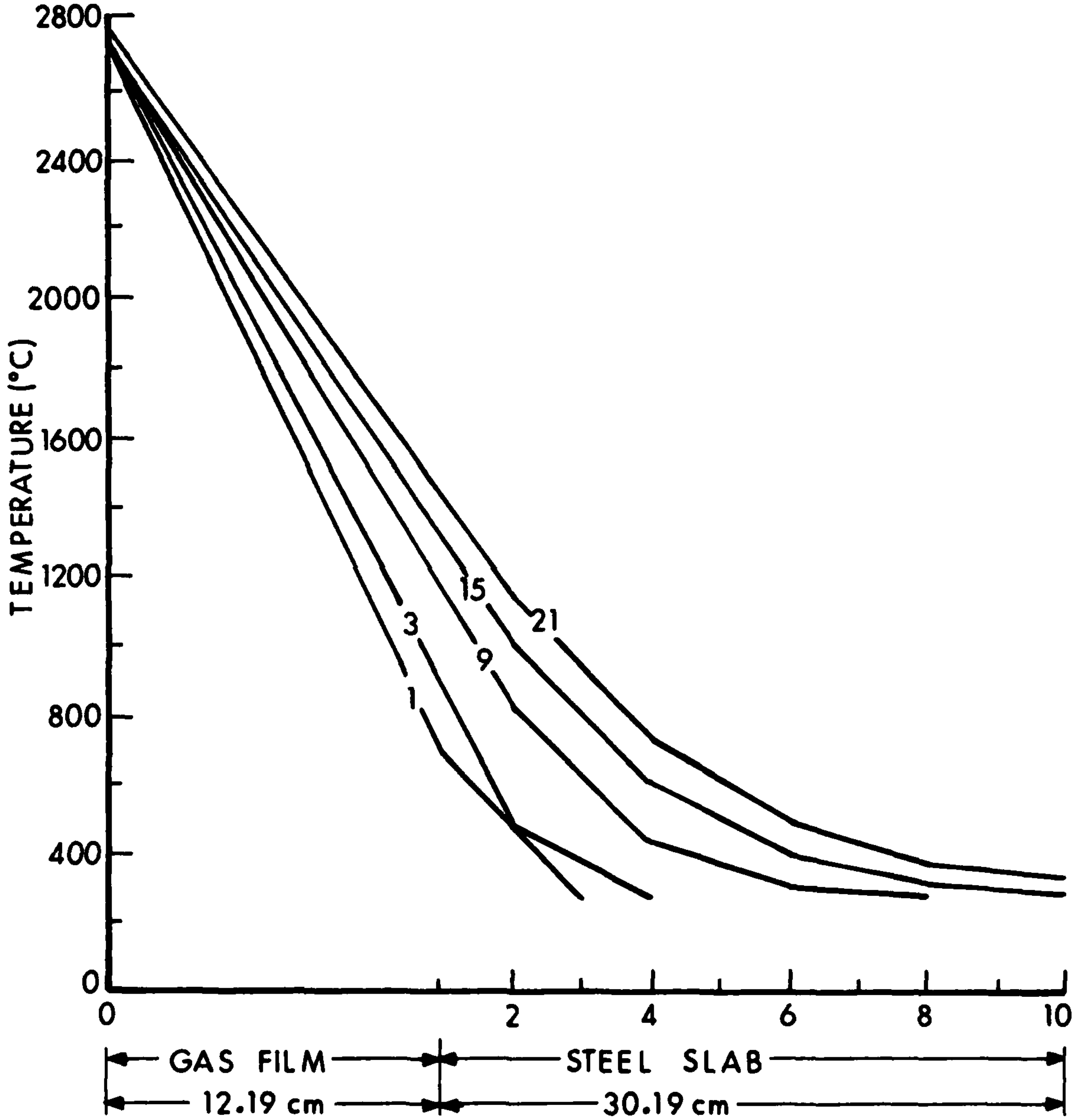

Figure A-2. Schmidt Plot for Sample Slab Problem

PROGRAM LISTING

```
*LBLB      21 12
PREG       16-13
*LBLA      21 11
RCL0       36 00
RCL2       36 02
-            -45
RCLC       36 13
÷            -24
RCLB       36 12
x            -35
RCL2       36 02
+            -55
ST01       35 01
CLX          -51
RCL2       36 02
RCL4       36 04
+            -55
2             02
÷            -24
ST03       35 03
CLX          -51
RCL4       36 04
RCL6       36 06
+            -55
2             02
÷            -24
ST05       35 05
CLX          -51
RCL6       36 06
RCL8       36 08
+            -55
2             02
÷            -24
ST07       35 07
CLX          -51
RCL8       36 08
RCLE       36 15
STOD       35 14
+            -55
2             02
÷            -24
ST09       35 09
STOE       35 15
CLX          -51
ISZI    16 26 46
SPC        16-11
PREG       16-13
*LBLC      21 13
RCL1       36 01
RCL3       36 03
+            -55
2             02
÷            -24
ST02       35 02
CLX          -51
RCL3       36 03
RCL5       36 05
+            -55
2             02
÷            -24
ST04       35 04
CLX          -51
RCL5       36 05
RCL7       36 07
+            -55
2             02
÷            -24
ST06       35 06
CLX          -51
RCL7       36 07
RCL9       36 09
+            -55
2             02
÷            -24
ST08       35 08
CLX          -51
ISZI    16 26 46
RCLI       36 46
RCLA       36 11
X≤Y?       16-35
GTOD       22 14
GTOA       22 11
*LBLD      21 14
P⇄S        16-51
RCL0       36 00
P⇄S        16-51
ST00       35 00
GTOA       22 11
R/S           51
```

APPENDIX B

TEMPERATURE GRADIENT IN A LONG THICK WALLED CYLINDER

APPENDIX B

TEMPERATURE GRADIENT IN A LONG THICK WALLED CYLINDER

In cylindrical coordinates the suddenly imposed temperature is shown in Schmidt plot presentation as Figure B-1, which is a transformation from the physical plane to that of the operating parameters. From the equations of II-A, the transformed equivalent film distance is equal to $\frac{k}{r\bar{h}}$ for equal radial increments transposed to logarithmically spaced increments in Δj.

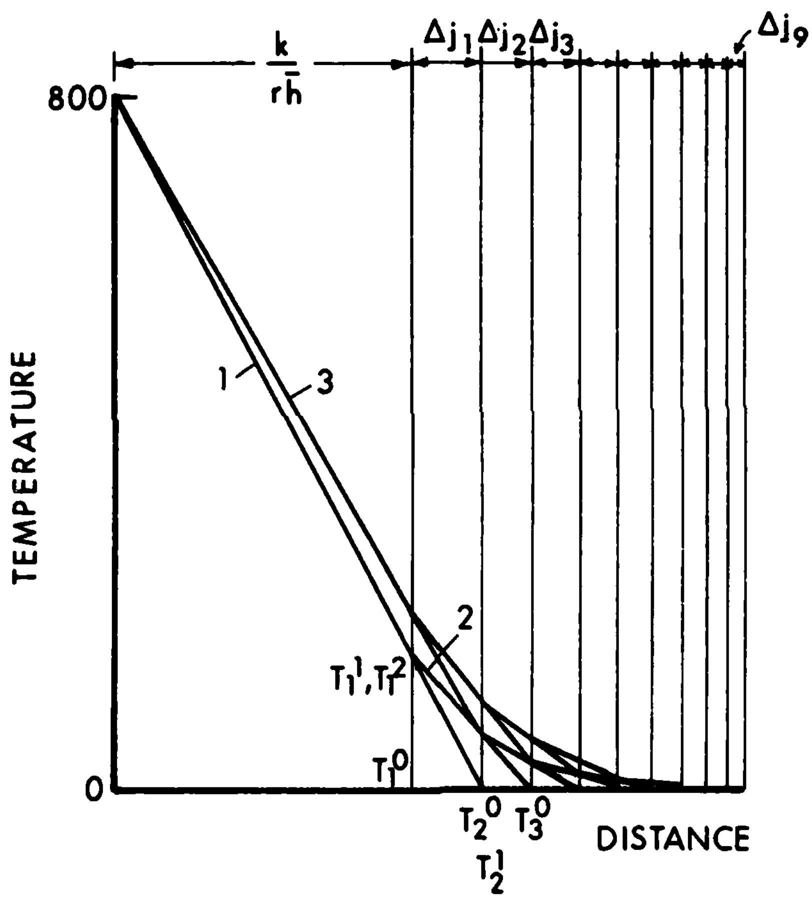

Figure B-1. Schmidt Plot Schematic for Cylindrical Wall Temperature Profile

The problem is that of the slab to establish the time-temperature distance gradient. Table B-1 illustrates the stepping procedure in finite difference parameters, and Figure B-1 plots the results of a specific example.

TABLE B-1. STEPPING PROGRESSION IN TEMPERATURE TIME PARAMETERS FOR CYLINDER

DISTANCE		TIME PROGRESSION → t_0	t_1	t_2	t_3	t_4	t_5	t_6	t_7	t_8	t_9	t_{10}
FILM	x_0	T_0	T_0^1	T_0^2	T_0^3	T_0^4	T_0^5	T_0^6	T_0^7	T_0^8	T_0^9	T_0^{10}
	x_1	T_1	$T_2^0+\frac{T_0^0-T_2^0}{2.445}$	T_1^1	$T_2^2+\frac{T_0^2-T_2^2}{2.445}$	T_1^3	$T_2^4+\frac{T_0^4-T_2^4}{2.445}$	T_1^5	$T_2^5+\frac{T_0^6-T_2^6}{2.445}$	T_1^7	$T_2^8+\frac{T_0^8-T_2^8}{2.445}$	T_1^9
BARREL	x_2	T_2	T_2^0	$T_3^1+\frac{T_1^1-T_3^1}{2.710}$	T_2^2	$T_3^3+\frac{T_1^3-T_3^3}{2.710}$	T_2^4	$T_3^5+\frac{T_1^5-T_3^5}{2.710}$	T_2^6	$T_3^7+\frac{T_1^7-T_3^7}{2.710}$	T_2^8	$T_3^9+\frac{T_1^9-T_3^9}{2.710}$
	x_3	T_3	$T_4^0+\frac{T_2^0-T_4^0}{2.410}$	T_3^1	$T_4^2+\frac{T_2^2-T_4^2}{2.410}$	T_3^3	$T_4^4+\frac{T_2^4-T_4^4}{2.410}$	T_3^5	$T_4^6+\frac{T_2^6-T_4^6}{2.410}$	T_3^7	$T_4^8+\frac{T_2^8-T_4^8}{2.410}$	T_3^9
	x_4	T_4	T_4^0	$T_5^1+\frac{T_3^1-T_5^1}{2.288}$	T_4^2	$T_5^3+\frac{T_3^3-T_5^3}{2.288}$	T_4^4	$T_5^5+\frac{T_3^5-T_5^5}{2.288}$	T_4^6	$T_5^7+\frac{T_3^7-T_5^7}{2.288}$	T_4^8	$T_5^9+\frac{T_3^9-T_5^9}{2.288}$
	x_5	T_5	$T_6^0+\frac{T_4^0-T_6^0}{2.222}$	T_5^1	$T_6^2+\frac{T_4^2-T_6^2}{2.222}$	T_5^3	$T_6^4+\frac{T_4^4-T_6^4}{2.222}$	T_5^5	$T_6^6+\frac{T_4^6-T_6^6}{2.222}$	T_5^7	$T_6^8+\frac{T_4^8-T_6^8}{2.222}$	T_5^9
	x_6	T_6	T_6^0	$T_7^1+\frac{T_5^1-T_7^1}{2.183}$	T_6^2	$T_7^3+\frac{T_5^3-T_7^3}{2.183}$	T_6^4	$T_7^5+\frac{T_5^5-T_7^5}{2.183}$	T_6^6	$T_7^7+\frac{T_5^7-T_7^7}{2.183}$	T_6^8	$T_7^9+\frac{T_5^9-T_7^9}{2.183}$
	x_7	T_7	$T_8^0+\frac{T_6^0-T_8^0}{2.155}$	T_7^1	$T_8^2+\frac{T_6^2-T_8^2}{2.155}$	T_7^3	$T_8^4+\frac{T_6^4-T_8^4}{2.155}$	T_7^5	$T_8^6+\frac{T_6^6-T_8^6}{2.155}$	T_7^7	$T_8^8+\frac{T_6^8-T_8^8}{2.155}$	T_7^9
	x_8	T_8	T_8^0	$T_9^1+\frac{T_7^1\ T_9^1}{2.132}$	T_8^2	$T_9^3+\frac{T_7^3\ T_9^3}{2.132}$	T_8^4	$T_9^5+\frac{T_7^5\ T_9^5}{2.132}$	T_8^6	$T_9^7+\frac{T_7^7\ T_9^7}{2.132}$	T_8^8	$T_9^9+\frac{T_7^9\ T_9^9}{2.132}$
	x_9	T_9	$T_{10}^0+\frac{T_8^0-T_{10}^0}{2.119}$	T_9^1	$T_{10}^2+\frac{T_8^2-T_{10}^2}{2.119}$	T_9^3	$T_{10}^4+\frac{T_8^4-T_{10}^4}{2.119}$	T_9^5	$T_{10}^6+\frac{T_8^6-T_{10}^6}{2.119}$	T_9^7	$T_{10}^8+\frac{T_8^8-T_{10}^8}{2.119}$	T_9^9
	x_{10}	T_{10}	T_9^0	T_9^1	T_9^2	T_9^3	T_9^4	T_9^5	T_9^6	T_9^7	T_9^8	T_9^9

The odd-numbered time cycles are described by

$$T_1t = T_2t\text{-}1 + \left[T_o - T_2t\text{-}1\right] \left[\frac{\Delta j_1}{1 + \Delta j_1}\right] \quad \text{(B-1)}$$

$$T_2t = T_2t\text{-}1 \quad \text{(B-2)}$$

$$T_3t = T_4t\text{-}1 + \left[T_2t\text{-}1 - T_4t\text{-}1\right] \left[\frac{\Delta j_3}{\Delta j_2 + \Delta j_3}\right] \quad \text{(B-3)}$$

$$T_4t = T_4t\text{-}1 \quad \text{(B-4)}$$

$$T_5t = T_6t\text{-}1 + \left[T_4t\text{-}1 - T_6t\text{-}1\right] \left[\frac{\Delta j_5}{\Delta j_4 + \Delta j_5}\right] \quad \text{(B-5)}$$

$$T_6t = T_6t\text{-}1 \quad \text{(B-6)}$$

$$T_7t = T_8t\text{-}1 + \left[T_6n\text{-}1 - T_8n\text{-}1\right] \left[\frac{\Delta j_7}{\Delta j_6 + \Delta j_7}\right] \quad \text{(B-7)}$$

$$T_8t = T_8t\text{-}1 \quad \text{(B-8)}$$

$$T_9t = T_{10}t\text{-}1 + \left[T_8n\text{-}1 - T_{10}n\text{-}1\right] \left[\frac{\Delta j_9}{\Delta j_8 + \Delta j_9}\right] \quad \text{(B-9)}$$

$$T_{10}t = T_9t\text{-}1 \quad \text{(B-10)}$$

The even-numbered time cycles are described by

$$T_1t = T_1t\text{-}1 \quad \text{(B-11)}$$

$$T_2t = T_3t\text{-}1 + \left[T_1t\text{-}1 - T_3t\text{-}1\right]\left[\frac{\Delta j_2}{\Delta j_1 + \Delta j_2}\right] \quad \text{(B-12)}$$

$$T_3t = T_3t\text{-}1 \quad \text{(B-13)}$$

$$T_4t = T_5t\text{-}1 + \left[T_3t\text{-}1 - T_5t\text{-}1\right]\left[\frac{\Delta j_4}{\Delta j_3 + \Delta j_4}\right] \quad \text{(B-14)}$$

$$T_5t = T_5t\text{-}1 \quad \text{(B-15)}$$

$$T_6t = T_7t\text{-}1 + \left[T_5t\text{-}1 - T_7t\text{-}1\right]\left[\frac{\Delta j_6}{\Delta j_5 + \Delta j_6}\right] \quad \text{(B-16)}$$

$$T_7t = T_7t\text{-}1 \quad \text{(B-17)}$$

$$T_8t = T_9t\text{-}1 + \left[T_7t\text{-}1 - T_9t\text{-}1\right]\left[\frac{\Delta j_8}{\Delta j_7 + \Delta j_8}\right] \quad \text{(B-17)}$$

$$T_9t = T_9t\text{-}1 \quad \text{(B-19)}$$

A fixed time increment is found such that

$$\Delta t = \frac{\Delta r^2}{2a}$$

The calculating technique is demonstrated by example. Assume an infinitely long stainless steel cylinder 1.0 inches (2.54 cm) inside diameter and 2.5 inches (6.35 cm) outside diameter suddenly exposed to a constant high temperature gas flow through the bore for 1.31 sec. The properties of the materials and the critical transport properties are given in Table B-2 with the HP-97 listing immediately following.

TABLE B-2. DEFINITION OF BOUNDARY CONDITIONS FOR SAMPLE CYLINDER PROBLEM

	Heat Source	Film	Steel Tube	Reference
Initial Temperature (T_0)	300 F 149 C 380 N	100 F 37.7 C 280 N	100 F 37.7 C 280 N	Assumed
Unit Film Conductance ($\overline{h}$)		250 B/hr ft^2 F 37.82 cal/hr cm C 8.84x10^5 CAL^2/hr N		(5)
Thermal Conductivity (k)			10.5 B/hr ft F 48.4 cal/hr cm C 8.84x10^5 CAL^{3}hr N	(5)
Diffusivity (a)			.197 ft^2/hr 183 cm^2/hr 4.69 CAL^2/hr	(5)
Equivalent Film Thickness ($k/r_i\overline{h}$)		1.008 ft 30.724 cm 1.0 CAL		Calculation
Maximum Wall Penetration			.038 ft 1.158 cm .038 CAL	$n\Delta r$ ($n = 10$)
Distance Increment (Δr_o) (Δr)		(Δr_o) 1.008 ft 30.724 cm 1.0 CAL	(Δr) .0038 ft .1154 cm .0038 CAL	$\Delta r_o = k/r_i\overline{h}$ $\Delta r = (2\, a\, \frac{\Delta t_e}{n})^{1/2}$
Time Increment $(\Delta t)_e$		1.31 sec	1.31 sec	Assumed

HP-97 REGISTER CONTENTS

The geometric radial boundaries are specified and equal radial increments are automatically converted to logarithmic parameters within the program. The corresponding initial temperatures must be entered as well as an elapsed time span.

Registers R_{S0} through R_{S9} enter the initial temperatures at the radial interstitial planes.

R_A indicates the number of time cycles required to elapse seconds (for this example).

R_B gives the inner radius.

R_C gives the outer radius.

R_D indicates the effective ambient temperature of the gas during the cooling phase.

R_E enters the initial temperature of the terminal plane.

R_I is given the number 1 for the initial calculation. The program automatically converts this figure to that of the number of time cycles completed at any stage in the calculation.

Note that the program starts with the secondary register filled and the primary register open.

OUTPUT

The printed output presents the temperature in the enumerated sequence of the radial interstitial planes. The terminal plane temperature is given in the D register and the number of time cycles completed in the I register. The real time elapsed is this number times the time interval.

The program entry employed with the HP-97 is given below. Any consistant system of units is suitable.

Primary Registers	Initial Values	Example
R_o	T_o	380 N
R_1	T_1	280 N
R_2	T_2	280 N
R_3	T_3	280 N
R_4	T_4	280 N
R_5	T_5	280 N
R_6	T_6	280 N
R_7	T_7	280 N
R_8	T_8	280 N
R_9	T_9	280 N
R_A	Y	3
R_B	r_{inner}	.5
R_C	r_{outer}	2.25
R_D	$T_{ambient}$	125 N
R_E	T_{10}	280 N
R_I	1	1

All primary registers are 0.

OUTPUT AFTER 6 TIME CYCLES

T_o	125 N
T_1	255 N
T_2	274 N
T_3	289 N
T_4	284 N
T_5	281 N
T_6	280 N
T_7	280 N
T_8	280 N
T_9	280 N
Intermediate Result	
Intermediate Result	
Intermediate Result	
T_{10}	280 N
Numer of time cycles completed	

PROGRAM LISTING

```
001  *LBLD      21 14
002   RCLB      36 12
003      9         09
004      x        -35
005   ST00      35 00
006   RCLC      36 13
007   RCLB      36 12
008      -        -45
009   RCL0      36 00
010      ÷        -24
011    1/X         52
012      .        -62
013      5         05
014      -        -45
015   ST09      35 09
016   RCLI      36 46
017      +        -55
018    1/X         52
019   ST01      35 01
020   ISZI   16 26 46
021   RCL9      36 09
022   RCLI      36 46
023      +        -55
024    1/X         52
025   ST02      35 02
026   ISZI   16 26 46
027   RCLI      36 46
028   RCL9      36 09
029      +        -55
030    1/X         52
031   ST03      35 03
032   ISZI   16 26 46
033   RCLI      36 46
034   RCL9      36 09
035      +        -55
036    1/X         52
037   ST04      35 04
038   ISZI   16 26 46
039   RCLI      36 46
040   RCL9      36 09
041      +        -55
042    1/X         52
043   ST05      35 05
044   ISZI   16 26 46
045   RCLI      36 46
046   RCL9      36 09
047      +        -55
048    1/X         52
049   ST06      35 06
050   ISZI   16 26 46
051   RCLI      36 46
052   RCL9      36 09
053      +        -55
054    1/X         52
055   ST07      35 07
056   ISZI   16 26 46
057   RCLI      36 46
058   RCL9      36 09
059      +        -55
060    1/X         52
061   ST08      35 08
062   ISZI   16 26 46
063   RCLI      36 46
064   RCL9      36 09
065      +        -55
066    1/X         52
067   ST09      35 09
068      0         00
069   STOI      35 46
070   RCLD      36 14
071   ST00      35 00
072   GTOB      22 12
073    P≠S      16-51
074  *LBLB      21 12
075    P≠S      16-51
076   RCL0      36 00
077   RCL2      36 02
078      -        -45
079    P≠S      16-51
080   RCL1      36 01
081    P≠S      16-51
082    1/X         52
083      1         01
084      +        -55
085      ÷        -24
```

```
130     +          -55
131   STO7       35 07
132   RCL8       36 08
133   RCLE       36 15
134     -          -45
135   P⇄S        16-51
136   RCL8       36 08
137   RCL9       36 09
138     ÷          -24
139     1           01
140     +          -55
141   P⇄S        16-51
142     ÷          -24
143   RCLE       36 15
144   STOD       35 14
145     +          -55
146   STO9       35 09
147   STOE       35 15
148   ISZI    16 26 46
149   PREG       16-13
150   RCL1       36 01
151   RCL3       36 03
152     -          -45
153   P⇄S        16-51
154   RCL1       36 01
155   RCL2       36 02
156     ÷          -24
157     1           01
158     +          -55
159   P⇄S        16-51
160     ÷          -24
161   RCL3       36 03
162     +          -55
163   STO2       35 02
164   RCL3       36 03
165   RCL5       36 05
166     -          -45
167   P⇄S        16-51
168   RCL3       36 03
169   RCL4       36 04
170     ÷          -24
171     1           01
172     +          -55
173   P⇄S        16-51

086   RCL2       36 02
087     +          -55
088   STO1       35 01
089   CLX          -51
090   RCL2       36 02
091   RCL4       36 04
092     -          -45
093   P⇄S        16-51
094   RCL2       36 02
095   RCL3       36 03
096     ÷          -24
097     1           01
098     +          -55
099   P⇄S        16-51
100     ÷          -24
101   RCL4       36 04
102     +          -55
103   STO3       35 03
104   RCL4       36 04
105   RCL6       36 06
106     -          -45
107   P⇄S        16-51
108   RCL4       36 04
109   RCL5       36 05
110     ÷          -24
111     1           01
112     +          -55
113   P⇄S        16-51
114     ÷          -24
115   RCL6       36 06
116     +          -55
117   STO5       35 05
118   RCL6       36 06
119   RCL8       36 08
120     -          -45
121   P⇄S        16-51
122   RCL6       36 06
123   RCL7       36 07
124     ÷          -24
125     1           01
126     +          -55
127   P⇄S        16-51
128     ÷          -24
129   RCL8       36 08

174     ÷          -24
175   RCL5       36 05
176     +          -55
177   STO4       35 04
178   RCL5       36 05
179   RCL7       36 07
180     -          -45
181   P⇄S        16-51
182   RCL5       36 05
183   RCL6       36 06
184     ÷          -24
185     1           01
186     +          -55
187   P⇄S        16-51
188     ÷          -24
189   RCL7       36 07
190     +          -55
191   STO6       35 06
192   RCL7       36 07
193   RCL9       36 09
194     -          -45
195   P⇄S        16-51
196   RCL7       36 07
197   RCL8       36 08
198     ÷          -24
199     1           01
200     +          -55
201   P⇄S        16-51
202     ÷          -24
203   RCL9       36 09
204     +          -55
205   STO8       35 08
206   ISZI    16 26 46
207   P⇄S        16-51
208   RCLI       36 46
209   RCLA       36 11
210   X>Y?       16-34
211   GTOB       22 12
212   RCL0       36 00
213   P⇄S        16-51
214   STO0       35 00
215   P⇄S        16-51
216   GTOB       22 12
```

APPENDIX C

PLANE WALL SURFACE TEMPERATURE

APPENDIX C

PLANE WALL SURFACE TEMPERATURE

Under the idealized conditions as previously specified, the wall temperature progression is established by algebraic determination as expressed in Table 1. For example, assuming an adiabatic flame temperature of M-2 propellant as the driving gas medium, and estimating the time required for the projectile to clear the barrel as .003 sec, and a heat transfer coefficient of 1845 B/hr ft^2 F, with the physicals as given in Table C-1, the wall temperature at the five time intervals can be read directly from the printout. The complete HP-97 listing follows on p.

INITIAL REGISTER CONTENTS

This program requires a constant driving gas temperature T_o, an initial wall temperature, the thermal conductance and diffusivity of the wall material, and a convective heat transfer coefficient corresponding to an assumed elapsed time increment. It presumes a slab wall and calculates the wall temperature growth from an initial slab depth of approximately .040 in (1 mm).

R_A indicates the driving gas temperature, T_on.

R_B gives the initial ambient temperature, T_no.

R_1 is the elapsed time in hours.

R_2 enters 10, the number of time increments for this example.

R_3 is the average convective heat transfer coefficient over this range.

R_4 gives the metal thermal conductivity.

R_5 is the metal thermal diffusivity.

OUTPUT

The printed output gives the wall temperature for ten sequential increments of time over the elapsed time interval.

TABLE C-1. DEFINITION OF BOUNDARY CONDITIONS FOR SAMPLE PLANE WALL PROBLEM

	Heat Source	Film	Steel Slab	Reference
Initial Temperature (T_0)	5596 F 3091 C 3028 N	70 F 21 C 265 N	70 F 21 C 265 N	(12)
Unit Film Conductance ($\bar{h}$)		1845 B/hr ft^2 F 279 cal/hr cm^2 C 1.53×10^8 CAL^2/hr N		(5)
Thermal Conductivity (k)			16 B/hr ft F 73.75 cal/hr cm C $1.53 \times 10^8 CAL^3$/hr N	(5)
Diffusivity (a)			.49 ft^2/hr 455 cm^2/hr 530 CAL^2/hr	(5)
Equivalent Film Thickness ($k/\bar{h}$)		8.67×10^{-3} ft .264 cm 1.0 CAL		Construction
Maximum Slab Penetration			2.87×10^{-3} ft 8.71×10^{-2} cm .33 CAL	$n\Delta x$ (n = 10)
Distance Increment (Δx_0) (Δx)		(Δx_0) 8.67×10^{-3} ft .264 cm 1.0 CAL	(Δx) 2.87×10^{-4} ft 8.71×10^{-3} cm .033 CAL	$\Delta x_0 = x/\bar{h}$ $\Delta x = (2\ a\ \Delta t_e)^{1/2}$
Time Increment $(\Delta t)_e$		.003 sec	.003 sec	(13)

The program entry employed with the HP-97 is given below. Any consistant system of units is suitable.

Primary Registers	Initial Values	Example
R_a	$T_o n$	5596 F
R_B	$T_n o$	70 F
R_1	t_e	$8.33x10^{-7}$ hr
R_2	10	10
R_3	$\overline{h}$	1845 B/hr ft^2 F
R_4	k	16 B/hr ft F
R_5	a	.49 ft^2/hr

Output after 3-millisecond time exposure

$T_1 1 = T_1 2$	246 F
$T_1 3 = T_1 4$	331 F
$T_1 5 = T_1 6$	394 F
$T_1 7 = T_1 8$	446 F
$T_1 9 = T_1 10$	490 F

PROGRAM LISTING

Step	Key	Code
001	*LBLA	21 11
002	PREG	16-13
003	RCL1	36 01
004	RCL2	36 02
005	÷	-24
006	STOC	35 13
007	RCL5	36 05
008	x	-35
009	2	02
010	x	-35
011	√X	54
012	STO1	35 01
013	RCL4	36 04
014	RCL3	36 03
015	÷	-24
016	STOD	35 14
017	RCL1	36 01
018	÷	-24
019	1	01
020	+	-55
021	1/X	52
022	STO2	35 02
023	RCLA	36 11
024	RCLB	36 12
025	-	-45
026	STO3	35 03
027	RCL2	36 02
028	x	-35
029	STO4	35 04
030	RCLB	36 12
031	+	-55
032	PRTX	-14
033	3	03
034	RCL2	36 02
035	-	-45
036	RCL4	36 04
037	x	-35
038	2	02
039	÷	-24
040	RCLB	36 12
041	+	-55
042	PRTX	-14
043	RCL2	36 02
044	X²	53
045	STO9	35 09
046	RCL2	36 02
047	4	04
048	.	-62
049	5	05
050	x	-35
051	CHS	-22
052	RCL9	36 09
053	+	-55
054	7	07
055	.	-62
056	5	05
057	+	-55
058	RCL4	36 04
059	x	-35
060	4	04
061	÷	-24
062	RCLB	36 12
063	+	-55
064	PRTX	-14
065	RCL9	36 09
066	RCL2	36 02
067	x	-35
068	STO8	35 08
069	CHS	-22
070	STO7	35 07
071	RCL9	36 09
072	6	06
073	x	-35
074	+	-55
075	STO7	35 07
076	RCL2	36 02
077	1	01
078	4	04
079	.	-62
080	5	05
081	x	-35
082	CHS	-22
083	RCL7	36 07
084	+	-55
085	1	01
086	7	07
087	.	-62
088	5	05
089	+	-55
090	RCL4	36 04
091	x	-35
092	8	08
093	÷	-24
094	RCLB	36 12
095	+	-55
096	PRTX	-14
097	RCL9	36 09
098	X²	53
099	8	08
100	x	-35
101	STO7	35 07
102	RCL9	36 09
103	CHS	-22
104	6	06
105	0	00
106	x	-35
107	RCL7	36 07
108	+	-55
109	STO7	35 07
110	RCL9	36 09
111	1	01
112	9	09
113	0	00
114	x	-35
115	RCL7	36 07
116	+	-55
117	STO7	35 07
118	RCL2	36 02
119	3	03
120	2	02
121	5	05
122	x	-35
123	CHS	-22
124	RCL7	36 07
125	+	-55
126	STO7	35 07
127	3	03
128	1	01
129	5	05
130	+	-55
131	RCL4	36 04
132	x	-35
133	1	01
134	2	02
135	8	08
136	÷	-24
137	RCLB	36 12
138	+	-55
139	PRTX	-14
140	RTN	24

APPENDIX D

HEAT TRANSFER COEFFICIENT FROM EXPERIMENTAL DATA

APPENDIX D

HEAT TRANSFER COEFFICIENT FROM EXPERIMENTAL DATA

Given the conditions recorded in Table D-1, the HP-97 input/output and the complete program listing follows. This program contains a recursive feature such that the initial estimate of $\bar{h}$ is used to calculate $T_1 9$, which is then compared with the experimental value, whereupon a recalculation using a modified value of $\bar{h}$ is used to reform $T_1 9$ until it is within the acceptable range selected. The cyclic converging values of $T_1 9$ are printed in order, and the result presents the final $\bar{h}$ with the corresponding temperature progression.

TABLE D-1. DEFINITION OF BOUNDARY CONDITIONS FOR EXPERIMENTAL PROPELLANT ROUND

	Heat Source	Film	Steel Slab	Reference
Initial Temperature (T_0)	4398 F 2425 C 2429 N	70 F 21 C 265 N	70 F 21 C 265 N	(12),(13)
Unit Film Conductance ($\bar{h}$)		12000 B/hr ft^2 F 1815 cal/hr cm^2 C $4.23x10^{10}$ CAL^2/hr N		Assumed
Thermal Conductivity (k)			16 B/hr ft F 73.75 cal/hr cm C $4.23x10^{10}$CAL/hr N	(5)
Diffusivity (a)			.49 ft^2/hr 455 cm^2/hr $2.77x10^5$ CAL^2/hr	(5)
Equivalent Film Thickness ($k/\bar{h}$)		$1.33x10^{-3}$ ft .0406 cm 1.0 CAL		Construction
Maximum Slab Penetration			$1.38x10^{-3}$ ft .042 cm 1.04 CAL	$n\Delta x$ (n = 10)
Distance Increment (Δx_0)		(Δx_0) $1.33x10^{-3}$ ft .0406 cm	(Δx) $1.38x10^{-4}$ ft .0042 cm	$\Delta x_0 = k/\bar{h}$ $\Delta x = (2\ a\ \frac{\Delta t_e}{n})^{1/2}$
Time Increment $(\Delta t)_e$		.0007 sec	.0007 sec	(13)

HP-97 INITIAL REGISTER CONTENTS

Register 0 enters the range of temperature convergence.

Register 1 is the elapsed time in hours.

Register 2 is 10

Register 3 allows the estimate of $\bar{h}$, the film coefficient.

Register 4 gives k, the conductivity of the metal wall.

Register 5 indicates a, the diffusivity of the metal wall.

Registers 6 - 9 are zero

Register A is the driving gas temperature $T_o n$, usually taken as the local adiabatic flame temperature.

Register B is the initial wall temperature $T_1 o$.

Registers 6, D and E are zero.

Register I gives the wall temperature rise in °C. If the calculation is not carried out in British units, steps 153 to 159 in the listing must be adjusted to delete the conversion of $T_{10} n$ from °F to °C.

OUTPUT

The intermediate printout is the temperature rise for each permutation of $\bar{h}$. For a slowly converging operation, the program can be stopped at any point and a revised $\bar{h}$ entered in Register 3.

The final printout gives the complete results of the terminal calculation.

Register 0 shows $\bar{h}$.

Register 1 gives $T_1 1$.

Register 2 gives $T_1 3$.

Register 3 gives $T_1 5$.

Register 4 gives $T_1 7$.

Register 5 gives $T_1 9$.

Register 6 repeats Register I.

Registers 7 - 9 are zero.

Register A repeats the driving gas temperature T_0n.

Register B repeats the initial wall temperature T_1o.

Registers C and D have no significance to the result.

Registers E and I present the final calculated and the reference temperature rise respectively.

The actual program entry is given below. With the exception noted for Register I, any consistant system of units is suitable.

Primary Registers	Initial Values	Example
R_o	T_z	2 C
R_1	t_e	1.94×10^{-7}hr
R_2	Constant	10
R_3	$\bar{h}$	12000 B/hr ft^{2}F
R_4	k	16 B/hr ft F
R_5	a	.49 ft^2/hr
R_6	0	0
R_7	0	0
R_8	0	0
R_9	0	0
R_A	$T_o n$	4398 F
R_B	$T_1 o$	70 F
R_C	0	0
R_D	0	0
R_E	0	0
R_I	$T_1 9 - T_1 0$	450 C

Intermediate values of

$T_1 9 - T_1 o$ (°C)

Output after iteration.

R_0	$\bar{h}$	10953.2 B/hr ft^{2}F
R_1	$T_1 1$	410 F
R_2	$T_1 3$	567 F
R_3	$T_1 5$	678 F
R_4	$T_1 7$	767 F
R_5	$T_1 9$	842 F
R_6	$T_1 9 - T_1 1$ (calc)	449.9 C
R_7		0

R_8	--	0
R_9	--	0
R_A	T_on	4398 F
R_B	T_10	70 F
$R_{\bar{C}}$	--	--
R_D	--	--
R_E	T_110 - T_10 (calc)	449.9 C
R_I	T_110 - T_10 (ref)	450 C

PROGRAM LISTING

001	*LBLA	21 11
002	PREG	16-13
003	*LBLC	21 13
004	RCL1	36 01
005	RCL2	36 02
006	÷	-24
007	STOC	35 13
008	RCL5	36 05
009	x	-35
010	2	02
011	x	-35
012	√X	54
013	STOC	35 13
014	RCL4	36 04
015	RCL3	36 03
016	P⇄S	16-51
017	STO0	35 00
018	P⇄S	16-51
019	÷	-24
020	STOD	35 14
021	RCLC	36 13
022	÷	-24
023	1	01
024	+	-55
025	1/X	52
026	STOD	35 14
027	RCLA	36 11
028	RCLB	36 12
029	-	-45
030	RCLD	36 14
031	x	-35
032	STOE	35 15
033	RCLB	36 12
034	+	-55
035	P⇄S	16-51
036	STO1	35 01
037	P⇄S	16-51
038	3	03
039	RCLD	36 14
040	-	-45
041	RCLE	36 15
042	x	-35
043	2	02
044	÷	-24
045	RCLB	36 12
046	+	-55
047	P⇄S	16-51
048	STO2	35 02
049	P⇄S	16-51
050	RCLD	36 14
051	X²	53
052	STO9	35 09
053	RCLD	36 14
054	4	04
055	.	-62
056	5	05
057	x	-35
058	CHS	-22
059	RCL9	36 09
060	+	-55
061	7	07
062	.	-62
063	5	05
064	+	-55
065	RCLE	36 15
066	x	-35
067	4	04
068	÷	-24
069	RCLB	36 12
070	+	-55
071	P⇄S	16-51
072	STO3	35 03
073	P⇄S	16-51
074	RCL9	36 09
075	RCLD	36 14
076	x	-35
077	STO6	35 06
078	CHS	-22
079	STO7	35 07
080	RCL9	36 09
081	6	06
082	x	-35
083	+	-55
084	STO7	35 07
085	RCLD	36 14
086	1	01
087	4	04
088	.	-62
089	5	05
090	x	-35
091	CHS	-22
092	RCL7	36 07
093	+	-55
094	1	01
095	7	07

```
096     .      -62
097     5       05
098     +      -55
099   RCLE   36 15
100     x      -35
101     8       08
102     ÷      -24
103   RCLB   36 12
104     +      -55
105   P⇄S    16-51
106   ST04   35 04
107   P⇄S    16-51
108   RCL9   36 09
109    X²       53
110     8       08
111     x      -35
112   ST07   35 07
113   RCL8   36 08
114    CHS     -22
115     6       06
116     0       00
117     x      -35
118   RCL7   36 07
119     +      -55
120   ST07   35 07
121   RCL9   36 09
122     1       01
123     9       09
124     0       00
125     x      -35
126   RCL7   36 07
127     +      -55
128   ST07   35 07
129   RCLD   36 14
130     3       03
131     2       02
132     5       05
133     x      -35
134    CHS     -22
135   RCL7   36 07
136     +      -55
137   ST07   35 07
138     3       03
139     1       01
140     5       05
141     +      -55
142   RCLE   36 15
143     x      -35
144     1       01
145     2       02
146     8       08
147     ÷      -24
148   RCLB   36 12
149     +      -55
150   P⇄S    16-51
151   ST05   35 05
152   P⇄S    16-51
153     3       03
154     2       02
155     -      -45
156     5       05
157     x      -35
158     9       09
159     ÷      -24
160   STOE   35 15
161   PRTX     -14
162   P⇄S    16-51
163   ST06   35 06
164   P⇄S    16-51
165   RCLI   36 46
166     -      -45
167   RCL0   36 00
168   X≤Y?   16-35
169   GTOB   22 12
170   GTOD   22 14
171  *LBLB   21 12
172   RCL3   36 03
173     .      -62
174     9       09
175     5       05
176     x      -35
177   ST03   35 03
178   GTOC   22 13
179  *LBLD   21 14
180   RCL3   36 03
181     .      -62
182     9       09
183     5       05
184     ÷      -24
185     1       01
186     0       00
187     0       00
188     -      -45
189   ST03   35 03
190   RCLE   36 15
191   RCLI   36 46
192     -      -45
193    X²       53
194    √X       54
195   RCL0   36 00
196   X≤Y?   16-35
197   GTOE   22 15
198   P⇄S    16-51
199   PREG   16-13
200   P⇄S    16-51
201    RTN      24
202  *LBLE   21 15
203   RCL3   36 03
204     7       07
205     0       00
206     -      -45
207   ST03   35 03
208   P⇄S    16-51
209   ST00   35 00
210   P⇄S    16-51
211   RCLE   36 15
212   RCLI   36 46
213     -      -45
214    X²       53
215    √X       54
216   RCL0   36 00
217   X≤Y?   16-35
218   GTOC   22 13
219   P⇄S    16-51
220   PREG   16-13
221    RTN      24
222    R/S      51
```

APPENDIX E

ALGEBRAIC SPECIFICATION OF T_0n

APPENDIX E

ALGEBRAIC SPECIFICATION OF T_0n

The program labeled E-3 refers to the determination of the driving gas temperature according to the requirements of Table 3. Program E-4 similarly refers to Table 4. The example and data of Appendix D are used for illustration.

INITIAL REGISTER CONTENTS

For both programs:

R_1 enters the wall temperature at time 1.

R_2 gives the wall temperature at time 3.

R_3 indicates the wall temperature at time 5.

R_4 is the wall temperature at time 7.

R_5 enters the wall temperature at time 9.

OUTPUT

The corresponding driving gas temperatures are given in stated sequence.

Primary Registers	Initial Values	Example
R_1	$T_1 1$	410 F
R_2	$T_1 3$	567 F
R_3	$T_1 5$	678 F
R_4	$T_1 7$	767 F
R_5	$T_1 9$	842 F

Output		
Program E-3	$T_0 1$	820 F
	$T_0 3$	1751 F
	$T_0 5$	2056 F
	$T_0 7$	2275 F
	$T_0 9$	2430 F
Program E-4	$T_0 1$	820 F
	$T_0 3$	1751 F
	$T_0 5$	2468 F
	$T_0 7$	3278 F
	$T_0 9$	3469 F

PROGRAM LISTING E - 3

```
001  *LBLE    21 15
002  RCL1     36 01
003     2        02
004     x       -35
005  PRTX       -14
006  RCL2     36 02
007     2        02
008     x       -35
009  STO7     35 07
010  RCL1     36 01
011     3        03
012     x       -35
013  RCL7     36 07
014     -       -45
015  STO0     35 00
016  RCL1     36 01
017    X²        53
018  RCL0     36 00
019     ÷       -24
020  PRTX       -14
021  RCL2     36 02
022     4        04
023     x       -35
024  RCL1     36 01
025     +       -55
026  RCL1     36 01
027     x       -35
028  STO0     35 00
029  RCL1     36 01
030     9        09
031     x       -35
032  STO7     35 07
033  RCL2     36 02
034     4        04
035     x       -35
036  RCL7     36 07
037     +       -55
038  STO7     35 07
039  RCL3     36 03
040     8        08
041     x       -35
042   CHS       -22
043  RCL7     36 07
044     +       -55
045  RCL0     36 00
046     ÷       -24
047   1/X        52
048  PRTX       -14
049  RCL2     36 02
050     2        02
051     x       -35
052  RCL1     36 01
053     +       -55
054  STO7     35 07
055  RCL3     36 03
056     8        08
057     x       -35
058  RCL7     36 07
059     +       -55
060  RCL1     36 01
061     x       -35
062  STO0     35 00
063  RCL1     36 01
064     1        01
065     7        07
066     x       -35
067  STO7     35 07
068  RCL2     36 02
069     2        02
070     x       -35
071  RCL7     36 07
072     +       -55
073  STO7     35 07
074  RCL3     36 03
075     8        08
076     x       -35
077  RCL7     36 07
078     +       -55
079  STO7     35 07
080  RCL4     36 04
081     1        01
082     6        06
083     x       -35
084   CHS       -22
085  RCL7     36 07
086     +       -55
087  RCL0     36 00
088     ÷       -24
089   1/X        52
090  PRTX       -14
091  RCL1     36 01
092     5        05
093     x       -35
094  STO7     35 07
095  RCL3     36 03
096     8        08
097     x       -35
098  RCL7     36 07
099     +       -55
100  STO7     35 07
101  RCL3     36 03
102     1        01
103     6        06
104     x       -35
105  RCL7     36 07
106     +       -55
107  STO7     35 07
108  RCL4     36 04
109     6        06
110     4        04
111     x       -35
112  RCL7     36 07
113     +       -55
114  RCL1     36 01
115     x       -35
116  STO0     35 00
117  RCL1     36 01
118     1        01
119     3        03
120     3        03
121     x       -35
122  STO7     35 07
123  RCL2     36 02
124     8        08
125     x       -35
126  RCL7     36 07
127     +       -55
128  STO7     35 07
129  RCL3     36 03
130     1        01
131     6        06
132     x       -35
133  RCL7     36 07
134     +       -55
135  STO7     35 07
136  RCL4     36 04
137     6        06
138     4        04
139     x       -35
140  RCL7     36 07
141     +       -55
142  STO7     35 07
143  RCL5     36 05
144     1        01
145     2        02
146     8        08
147     x       -35
148   CHS       -22
149  RCL7     36 07
150     +       -55
151  RCL0     36 00
152     ÷       -24
153   1/X        52
154  PRTX       -14
155   RTN        24
```

PROGRAM LISTING E-4

001	*LBLE	21 15
002	RCL1	36 01
003	2	02
004	x	-35
005	PRTX	-14
006	RCL1	36 01
007	3	03
008	x	-35
009	STO7	35 07
010	RCL2	36 02
011	2	02
012	x	-35
013	RCL7	36 07
014	-	-45
015	CHS	-22
016	STO0	35 00
017	RCL1	36 01
018	X²	53
019	RCL0	36 00
020	÷	-24
021	PRTX	-14
022	RCL1	36 01
023	4	04
024	x	-35
025	STO7	35 07
026	RCL1	36 01
027	RCL2	36 02
028	9	09
029	x	-35
030	CHS	-22
031	RCL7	36 07
032	+	-55
033	STO7	35 07
034	RCL2	36 02
035	X²	53
036	2	02
037	x	-35
038	CHS	-22
039	RCL7	36 07
040	+	-55
041	STO7	35 07
042	RCL2	36 02
043	RCL3	36 03
044	x	-35
045	8	08
046	x	-35
047	RCL7	36 07
048	+	-55
049	STO0	35 00
050	RCL1	36 01
051	3	03
052	x	-35
053	STO7	35 07
054	RCL2	36 02
055	1	01
056	0	00
057	x	-35
058	CHS	-22
059	RCL7	36 07
060	+	-55
061	STO7	35 07
062	RCL3	36 03
063	8	08
064	x	-35
065	RCL7	36 07
066	+	-55
067	RCL0	36 00
068	÷	-24
069	1/X	52
070	PRTX	-14
071	RCL1	36 01
072	RCL3	36 03
073	x	-35
074	CHS	-22
075	STO7	35 07
076	RCL1	36 01
077	RCL4	36 04
078	x	-35
079	2	02
080	x	-35
081	RCL7	36 07
082	+	-55
083	STO7	35 07
084	RCL2	36 02
085	RCL3	36 03
086	x	-35
087	2	02
088	x	-35
089	CHS	-22
090	RCL7	36 07
091	+	-55
092	STO7	35 07
093	RCL2	36 02
094	RCL4	36 04
095	x	-35
096	8	08
097	x	-35
098	RCL7	36 07
099	+	-55
100	STO7	35 07
101	RCL3	36 03
102	X²	53
103	8	08
104	x	-35
105	CHS	-22
106	RCL7	36 07
107	+	-55
108	STO6	35 06
109	RCL4	36 04
110	1	01
111	6	06
112	x	-35
113	STO7	35 07
114	RCL3	36 03
115	1	01
116	6	06
117	x	-35
118	CHS	-22
119	RCL7	36 07
120	+	-55
121	RCL1	36 01
122	+	-55
123	STO7	35 07
124	RCL2	36 02
125	6	06
126	x	-35
127	RCL7	36 07
128	+	-55
129	STO7	35 07
130	RCL3	36 03
131	6	06
132	x	-35
133	CHS	-22
134	RCL7	36 07
135	+	-55
136	RCL0	36 00
137	÷	-24

```
138   1/X     52
139   PRTX   -14
140   RCL1   36 01
141   5       05
142   x      -35
143   ST07   35 07
144   RCL2   36 02
145   8       08
146   x      -35
147   RCL7   36 07
148   +      -55
149   ST07   35 07
150   RCL3   36 03
151   1       01
152   6       06
153   x      -35
154   RCL7   36 07
155   +      -55
156   ST07   35 07
157   RCL4   36 04
158   6       06
159   4       04
160   x      -35
161   RCL7   36 07
162   +      -55
163   RCL4   36 04
164   x      -35
165   CHS    -22
166   ST07   35 07
167   RCL1   36 01
168   8       08
169   x      -35
170   ST08   35 08
171   RCL2   36 02
172   1       01
173   6       06
174   x      -35
175   RCL8   36 08
176   +      -55
177   ST08   35 08
178   RCL3   36 03
179   6       06
180   4       04
181   x      -35
182   RCL8   36 08
183   +      -55
184   RCL5   36 05
185   x      -35
186   RCL7   36 07
187   +      -55
188   ST00   35 00
189   RCL5   36 05
190   6       06
191   4       04
192   x      -35
193   ST07   35 07
194   RCL2   36 02
195   4       04
196   x      -35
197   RCL7   36 07
198   +      -55
199   ST07   35 07
200   RCL3   36 03
201   2       02
202   4       04
203   x      -35
204   RCL7   36 07
205   +      -55
206   ST07   35 07
207   RCL4   36 04
208   9       09
209   6       06
210   x      -35
211   CHS    -22
212   RCL7   36 07
213   +      -55
214   2       02
215   x      -35
216   RCL1   36 01
217   3       03
218   x      -35
219   +      -55
220   RCL0   36 00
221   ÷      -24
222   1/X     52
223   PRTX   -14
224   RTN     24
```

APPENDIX F

UNITS DESIGNATION

APPENDIX F

UNITS DESIGNATION

For the given examples, the properties are stated in the British, S.I. and normalized expression. The non-dimensional geometric "caliber" is most useful, with many applications in fluid dynamic analysis, and the extension to a normalized mass (weight) has been a convenience in rationalizing the performance of grossly different flight projectiles.[F-1] Such manipulative procedures have physical limitations which are defined in the framework of their employment.

The normalization parameters are given below. Dividing weight (force) quantities by the specific weight of water converts them to length quantities and the mechanical equivalent of heat transforms heat units into mechanical units. In addition, this report introduces a common temperature (°Normalized) based on the absolute scale.

Length	$\dfrac{\text{linear dimension}}{\text{reference dimension}}$	CAL
Force	$\dfrac{\text{force}}{\text{specific weight of water x ref dim}^3}$	CAL^3
Mass	$\dfrac{\text{mass}}{\text{specific mass of water x ref dim}^3}$	CAL^3
Heat	$\dfrac{\text{heat x mechanical equivalent}}{\text{specific weight of water x ref dim}^4}$	CAL^4
Temperature	°Normalized = .9 °Kelvin = .5°Rankine	

[F-1] W.F. Donovan, "One Factor Affecting the Dispersion of Long Rod Penetrators," ARBRL-MR-02846, June 1978. AD #A058596

LIST OF SYMBOLS

a	thermal diffusivity of metal wall
c	specific heat of metal wall
e	exponential constant
e	elapsed quantity (when appearing as a subscript)
$\bar{h}$	film heat transfer coefficient
j	natural logarithm of "r"
k	thermal conductivity of metal wall
m	$\frac{\Delta x}{\Delta x + \Delta x_o}$
n	sequential location of interstitial plane (appearing as subscript)
r	radius
t	time
x	distance
cal	calorie
cm	centimeter
ft	foot
hr	hour
sec	second
B	British Thermal Unit
C	Centigrade
F	Fahrenheit
L	Length
N	Normalized
T	Temperature
CAL	Caliber
Δ	Difference indicator
Δt	Finite difference in time
ΔT	Finite difference in temperature
Δx	Finite difference in distance
ϕ	Operator defined as used in text
θ	Operator defined as used in text

LIST OF SYMBOLS (continued)

ρ	Density of medium
$\dot{q}$	Heat flux
r_i	Inner radius
r_o	Outer radius
t_e	Elapsed time
t_n	Time at n^{th} interstitial plane
t_z	Range of convergence for temperature calculation
T_nt	Temperature at n^{th} interstitial plane at time "t"
Δx_o	Initial distance increment in Schmidt plot
ΔT_o	Initial driving temperature difference
HP-97	Hewlett Packard Model 97 Calculator
FORTRAN	Computer language acronym for formula translation
CDC	CONTROL DATA CORPORATION
CALSPAN	Formerly Cornell Aeronautical Laboratory
SI	STANDARD INTERNATIONAL
BRL	BALLISTIC RESEARCH LABORATORY

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