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THESIS

TURBULENCE EFFECTS ON A
GLOW DISCHARGE AS DERIVED FROM
CONTINUITY AND ENERGY CONSIDERATIONS

by

William R. Oker

September 1984

Thesis Advisor: Oscar Biblarz

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#20 - ABSTRACT - (CONTINUED)

A mathematical model is developed which is solved numerically for the ambipolar diffusion and an overall energy equation in an unsteady, cylindrical coordinate system. Strong perturbations of the electric field are introduced which disturb an otherwise stable configuration and the effects of turbulence on the time evolution of the perturbation are observed.

It is shown that the modification of the ambipolar diffusion coefficient and the thermal conductivity is a reasonably sufficient model to introduce the turbulence effects. It is found that the charged particle density is effectively the sole source of heating in the gaseous environment of the discharge. It is then shown that turbulence acts to supress the temperature instability introduced by the discharge streamer.

Turbulence Effects on a
Glow Discharge as Derived from
Continuity and Energy Considerations

by

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ABSTRACT

Effects of a turbulent gas flow on the stability of a glow discharge are investigated. While it is known that turbulence affects the stability of glow discharges, the mechanism is not clear. Primarily, the problem lies in the very long characteristic times of turbulence phenomena when compared to the glow discharge instability times.

A mathematical model is developed which is solved numerically for the ambipolar diffusion and an overall energy equation in an unsteady, cylindrical coordinate system. Strong perturbations of the electric field are introduced which disturb an otherwise stable configuration and the effects of turbulence on the time evolution of the perturbation are observed.

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LIST OF SYMBOLS

A = area

α = $\alpha_3 n_o / \alpha_2$

α_2 = two-body recombination coefficient

α_3 = three-body recombination coefficient

β = inverse turbulence parameter due to ambipolar diffusion

β_A = inverse turbulence parameter due to thermal conductivity

D_a = ambipolar diffusion coefficient

e = internal energy, or elementary electron charge

E, E_o = electric field strength, initial value

G = conductance

I = current

j = current density

K = thermal conductivity

k = Boltzman's constant

ℓ = length of streamer (or distance between electrodes)

m_e = electron mass

μ_e = electron mobility

n, n_o = charged particle density, initial value

$n_{\mathcal{C}}$ = n value on streamer centerline

$\hat{n}$ = non-dimensionalized n

n_e = number of electrons

N = total number of particles

ν_i = ionization coefficient

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ν_e = electron collision frequency

Ω = resistance

P = pressure

q = heat flux

r, r_o = streamer channel radius, initial value

$\hat{r}$ = non-dimensionalized radius

ρ = density

σ, σ_o = conductivity, initial value

T, T_o = temperature, initial value

$\hat{T}$ = non-dimensionalized temperature

τ_R = characteristic recombination time

TR = same as τ_R

τ_A = characteristic time for energy equation

t, t_o = time, initial value

$\hat{t}$ = non-dimensionalized time

t_A = t associated with τ_A

U = gas flow velocity

v_d = electron drift velocity

V = voltage

Z = electric field perturbation parameter for continuity equation

Z_A = "Z" for energy equation

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Without her constant love, support, and understanding, this
work would not have been possible.

I. INTRODUCTION

If a gas such as nitrogen or argon, or a mixture of gases such as air, is subjected to an electrical potential, a sequence of phenomena take place as a result of a continuously increasing applied voltage and/or current. Initially, for a small voltage, a very weak current on the order of 0.1 picoamperes develops. This current will increase slightly with increasing voltage until all the naturally occurring charge carriers are being utilized. A current saturation condition now prevails until the applied voltage reaches about 6 kV in air at which time a further voltage rise rapidly increases the current flow to about one picoampere. This is referred to as the Townsend Discharge Region and it is due to new charge carriers being created by collisions between the highly energetic electrons (fast moving) and the neutral (slow moving) atoms. This process can have several effects on the individual gas molecules. Essentially, it can excite them, dissociate them, and/or ionize them. This ionization effect is higher for lower gas pressures since the farther the electron travels between collisions the higher will be its kinetic energy which is imparted by the acceleration produced by the applied electric field. When the electron's energy is greater than that required for ionization of the neutral gas particles the collisions will

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produce additional electrons which in turn gain energy,
produce collisions, and therefore, increase the total number
of electrons present. This process is called an electron
avalanche.

From this point, slight increases in voltage produce
current increases of several orders of magnitude at which
time the current becomes self sustaining. Reducing the
external circuit resistance now increases the current flow
and a steady glow is produced in the gas. This glow-
discharge is the familiar phenomenon which occurs in neon
lights and has other practical applications such as a
pumping scheme for gas lasers.

Finally, when the current is increased still further, a
highly ionized channel is formed which collapses the glow.
If there is sufficient current carrying capacity in the
external circuit a steady current is established which
represents a plasma state called an arc.

A full understanding of the arcing mechanism is diffi-
cult due to the extremely short formation times involved.
However, since delaying the point of transition to arcing is
a highly practical problem, a method for modeling it is
needed. A blend of theory and experiment is usually re-
quired. Such a model could then be used to study the
effects of the arc on and by its environment.

It has been observed that this glow-to-arc transition
can be delayed by utilizing gas dynamic stabilization

techniques in flowing gas systems [Ref. 1]. The purpose of this work is to establish the extent of fluid dynamic stabilization that results from the introduction of varying levels of fluid dynamic turbulence into the flow. It will be shown that this method has beneficial effects on the stability of the glow discharge process and on the temperature distribution in the gas produced by the presence of discharge streamers. For further discussion and background information relating to ionization and the breakdown processes refer to any standard reference in the field or to Wallace, R.J. [Ref. 2].

An introductory skill level in atomic physics plus fundamental proficiencies in mechanics, electrical principles and numerical analysis should provide the reader with adequate preparation for understanding this material.

The principles discussed here have many practical engineering applications. For maximum power output of an electrically pumped gas laser, such as a carbon dioxide laser, the glow discharge must be held essentially at the limit of arcing. Since turbulent flows tend to delay this glow-to-arc transition, the laser can be allowed to operate at either the highest possible power level or with a greater margin of safety at lower power levels. Another application where a stabilized high frequency electrical spark discharge could be used as a means for injecting heat into a propulsive duct (like a pulsejet) has been suggested by C.E. Tharratt [Ref. 3].

This type of a device is theoretically capable of unlimited thrust. It would also have a low specific fuel consumption and could operate outside the audible range. The possibility of nearly silent operation would be highly advantageous as an aircraft propulsion device for the future.

Two major cases will be discussed. The first case concerns the behavior of a highly transient discharge in a non-ionized gaseous environment such as the lightning discharge. This first case concerns a pre-glow discharge type phenomenon since the streamer propagates in a non-ionized medium. However, it serves to establish the effects of turbulence in a simpler situation in order to form a starting place for this research. The second case will look at discharge streamers in a partially ionized gas which might be used for the pumping mechanism of a gas laser. The method of approach to these problems will be developed in the next chapter where a mathematical model will be presented. The governing partial differential equations for the model presented will be solved by numerical integration techniques utilizing a computer. A working Fortran program is presented which can be used to investigate a wide variety of the aspects of this problem. Significant results are then presented in graphical format to demonstrate the stabilizing features of the turbulent flow.

II. ANALYSIS

A. THE CONTINUITY EQUATION

Highly ionized channels which develop at the final stages of a glow discharge just prior to arc breakdown are called streamers. The transition to arcing is then often due to streamer-breakdown. It should be noted here that the type of streamers to be analyzed are the non-self-sustaining discharge streamers that precede the arc, rather than the electron avalanche (onset streamers) type that occur earlier in the glow discharge phenomena.

Streamers are seen to issue from either electrode and appear always to precede the breakdown. These streamers propagate virtually instantaneously for our purposes as an ultrafast ionization wave. They consist of a partially ionized plasma column for which the conductivity is dependent directly on the charged particle density within the streamer [Ref. 4]. It will be shown that the collisions made by the charged particles within these streamers are the main source of heating of the surrounding gas. The value of this charged particle density may be determined by the application of the following continuity equation to the breakdown process.

$$\frac{\partial n}{\partial t} = \frac{D_a}{r} \frac{\partial}{\partial r}(r \frac{\partial n}{\partial r}) - U \frac{\partial n}{\partial r} - \alpha_2 n^2 - \alpha_3 n^3 \qquad (2.1)$$

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Equation (2.1) gives the time rate of change of the charged

particle density, n, as a function of radial position, r,

and time, t, for the decaying streamer. It is written in

cylindrical coordinates due to the natural form of the

streamer. A schematic drawing of the streamer geometry is

depicted in Figure 2.1. The right hand side (RHS) of

Equation (2.1) consists of three classes of terms. The first

term represents the effect of diffusion due to the high

density gradient within the plasma channel. D_a is the ambi-

polar diffusion coefficient. This ambipolar diffusion results

in the electrons and ions diffusing out of the original

channel at essentially the same rate. Briefly, the lighter

electrons having higher thermal velocities leave the plasma

ahead of the slower, heavier ions. This produces a local

positive space charge which retards the loss of electrons and

accelerates the loss of the ions. The net effect is that the

ambipolar diffusion coefficient is approximately twice the

diffusion coefficient of the ions alone [Ref. 4].

Next consider terms 3 and 4 of the RHS of Equation (2.1)

which represent two and three-body recombination respectively.

Recombination is a de-ionization process which is then

responsible here for the reduction in conductivity of the

partially ionized gas. These processes occur when an electron

and an ion collide at relatively low velocity and recombine

to form a neutral atom. In order to conserve linear momen-

tum a third body must be present. In two-body recombination

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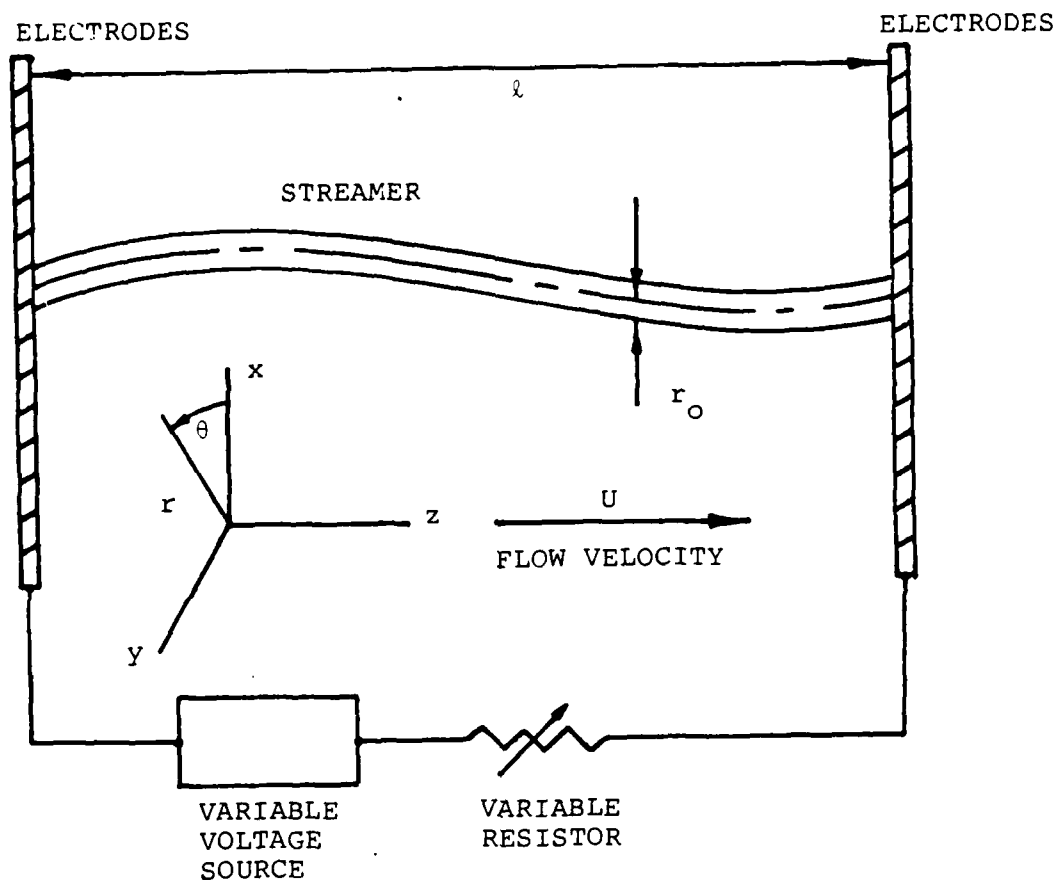
Figure 2.1. Streamer Geometry

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this third body is a photon and, therefore, this process is sometimes called radiative recombination. For three-body recombination the third body is usually an ion or neutral atom [Ref. 4]. Due to the large complexity of electron loss mechanisms accurate values for the recombination coefficients are difficult to obtain. The values used in this thesis will be discussed in the next subsection.

The second term on the RHS is the convection term. This term is neglected since the characteristic time of convection (length/flow velocity) is much less than the characteristic recombination times associated with terms 3 and 4 [Ref. 2].

While the convection term in Equation (2.1) can be neglected, the diffusion term cannot since the diffusion coefficient D_a can increase by about three orders of magnitude between laminar and turbulent flow [Ref. 2]. This will cause the characteristic diffusion time to be on the same order of magnitude as the recombination time and some relationship will exist between diffusion and recombination which will affect the charged particle density distribution, n, in space and time.

Once Equation (2.1) is solved for $n(r,t)$ the resulting distribution profile can be integrated to determine a conductance of the decaying streamer as a function of the streamer radius and time. This can be accomplished under varying turbulence level conditions by changing the effective value of the diffusion coefficient.

Equation (2.1) describes the lightning-discharge type phenomenon in a non-ionized medium. A more interesting situation is the case of the streamer as it appears in an ionized-sustainer discharge for the pumping of a gas laser [Ref. 5]. For this case, Equation (2.1) may be written as,

$$\frac{\partial n}{\partial t} - \frac{D_a}{r} \frac{\partial}{\partial r} \left(r \frac{\partial n}{\partial r}\right) = \nu_i n - \alpha_2 n^2 - \alpha_3 n^3 \tag{2.2}$$

where now a new first term is added to the RHS which represents a production of electrons and ions. The ionization coefficient, ν_i, is a function of the sustainer E/N and can be determined from Figure 2.2. The electric field referred to here is an existing axial electric field, that is, the "pumping field" that maintains the glow. Equations (2.1) and (2.2) are non-linear partial differential equations which will be solved numerically by the use of a finite difference approximation scheme.

B. THE ENERGY EQUATION

In addition to the continuity equation, a suitable energy equation must be considered in order to describe the temperature distribution introduced into the gas by the glow discharge streamer. For thermal stability to exist, a balance must be created such that the time rate of change of the internal energy is zero because the loss of heat through thermal conductivity and the addition of energy from Ohmic

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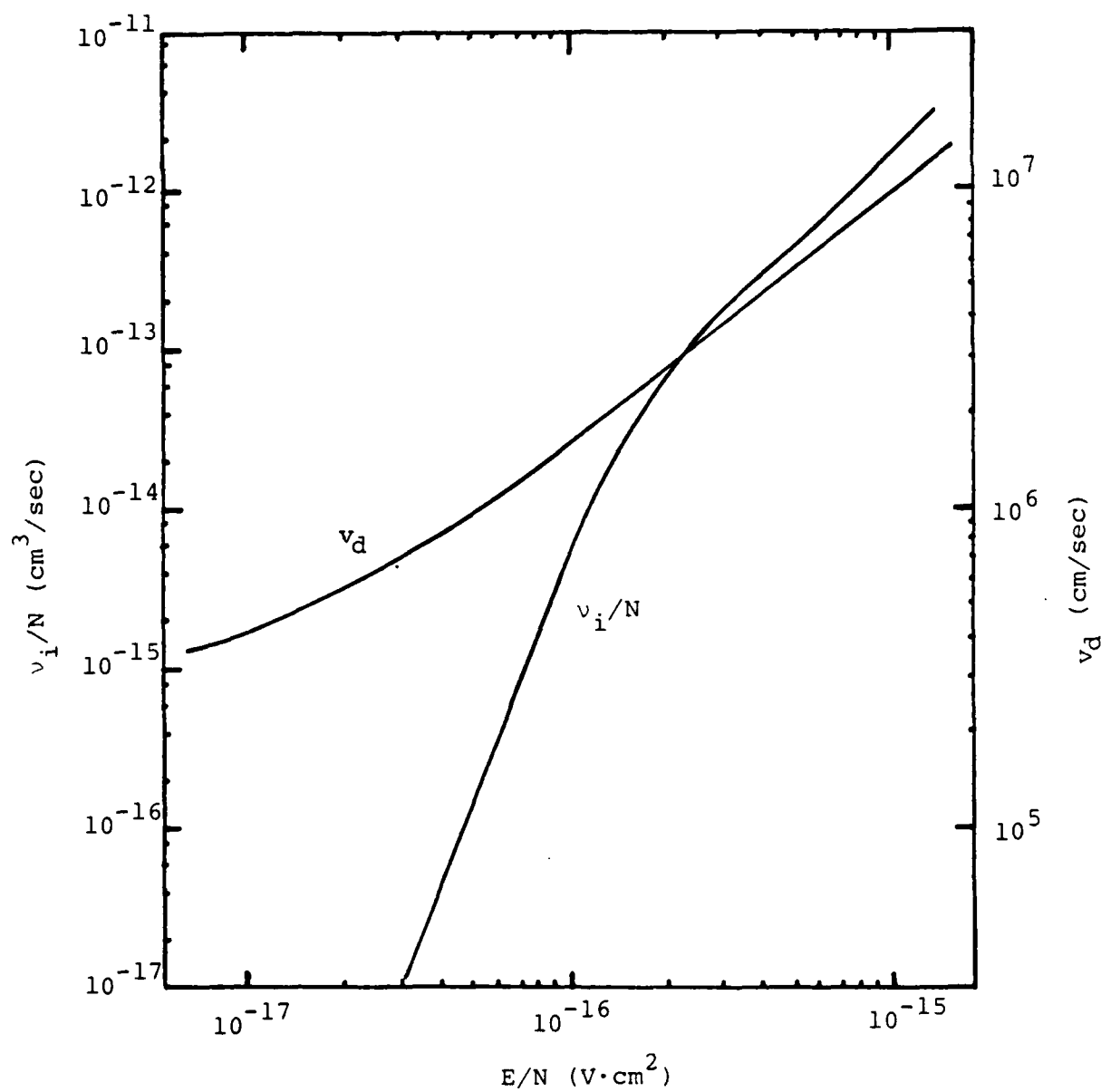
Figure 2.2.   Relevant Properties of Nitrogen

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heating must balance each other. The following equation
represents this relationship.

$$\rho \, \frac{De}{Dt} \;=\; \vec{E} \cdot \vec{j} - \nabla \cdot \vec{q} \qquad\qquad (2.3)$$

The first term on the RHS of Equation (2.3) represents
Joule or Ohmic heating where, $\vec{E}$, is the electric field
strength and, $\vec{j}$, is the current density. The second term
is the heat flux loss due to thermal conductivity and
radiation. The total derivative,

$$\frac{De}{Dt} \;=\; \frac{\partial e}{\partial t} + v_r \, \frac{\partial e}{\partial r} + v_\theta \, \frac{\partial e}{\partial \theta} + v_z \, \frac{\partial e}{\partial z} \qquad\qquad (2.4)$$

reduces to, $\frac{\partial e}{\partial t}$, since the model under consideration here
allows for conduction *only in the "z" direction*. It is
further assumed that the change of the internal energy along
the length of the streamer is small, the radiation emitted
primarily perpendicular to the streamer centerline has a
negligible effect and, due to the short characteristic times
involved, the heating occurs at an essentially constant
volume. With these assumptions in mind, Equation (2.3)
becomes,

$$\rho C_v \, \frac{\partial T}{\partial t} - K \nabla^2 T \;=\; \sigma E_z^2 \qquad\qquad (2.5)$$

where, ρ, is the gas density, σ, is the electrical conduc-
tivity of the plasma (recalling that $j = \sigma E_z$) and K is the

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thermal conductivity. Equation (2.5) gives the temperature, T, as a function of the radius and time.

The conductivity, σ, is given by,

$$\sigma = e\, n_e\, \mu_e \qquad (2.6)$$

where, e, is the elementary electronic charge in coulombs, n_e, is the number of electrons and, μ_e, is the electron mobility. The mobility is also a function of E/N and is defined as the ratio of the electron drift velocity, v_d, to the applied external electric field. The value of v_d is given in Figure 2.2 as a function of E/N.

It will be shown later that for the σE_z^2 term from Equation (2.5), E_z is essentially constant and since σ is directly proportional to n the solution of Equation (2.2) represents the single source of heating that drives the temperature distribution obtained from Equation (2.5). The consequence of this is that a mathematical model describing the temporal and spatial characteristics of the streamer can now be developed. Furthermore, the effects of turbulence as a stabilizing medium can then be studied since the thermal conductivity, K, and the ambipolar diffusion coefficient, D_a, are the only parameters affected by turbulence in Equations (2.1), (2.2) and (2.5).

C. SOLVING THE EQUATIONS

An effective means of generalizing a problem such as this is to non-dimensionalize the equations. To begin then,

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after some algebraic manipulation, Equation (2.1) can be written in the form,

$$\frac{1}{\alpha_2 n_o} \frac{\partial \hat{n}}{\partial t} - [\frac{D_a}{n_o \alpha_2 r_o^2}] \frac{\partial}{\partial \hat{r}} (\hat{r} \frac{\partial \hat{n}}{\partial \hat{r}}) = -\hat{n}^2 - \frac{\alpha_3 n_o}{\alpha_2} \hat{n}^3 \qquad (2.7)$$

where $\hat{n} = n/n_o$ and $\hat{r} = r/r_o$. "n_o" and "r_o" are the initial values for the charged particle density and the streamer radius. The following additional definitions will prove useful in this model,

$$\beta \equiv \frac{\alpha_2 n_o r_o^2}{D_a} \qquad (2.8)$$

$$\alpha \equiv \frac{\alpha_3 n_o}{\alpha_2} \qquad (2.9)$$

$$\hat{t} \equiv \frac{t}{\tau_R} \qquad (2.10)$$

The parameter, β, is an inverse measure of the turbulence level since the ambipolar diffusion coefficient is smaller in laminar flow (large β) and larger in a turbulent flow (small β). If one neglects the diffusion term in Equation (2.1), then for two-body recombination, direct integration yields,

$$n = \frac{n_o}{1 + \alpha_2 n_o t} \qquad (2.11)$$

This ion loss formula [Ref. 2] then shows that the characteristic recombination time, τ_R, is simply $1/\alpha_2 n_o$.

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Incorporating the above definitions allows Equation (2.7)
to be written in the non-dimensionalized form,

$$\frac{\partial \hat{n}}{\partial \hat{t}} - \frac{1}{\beta} \frac{\partial}{\partial \hat{r}}(\hat{r} \frac{\partial \hat{n}}{\partial \hat{r}}) = -\hat{n}^2 - \alpha\hat{n}^3 \qquad (2.12)$$

which lends itself to a numerical solution by substituting
the appropriate finite difference approximation formula
[Ref. 6] for the differentials. After some manipulation,
Equation (2.12) can be approximated by,

$$\hat{n}_{i,j+1} = \{\Delta\hat{t}[\frac{1}{\beta}(A + B) - C]\} + \hat{n}_{i,j} \qquad (2.13)$$

where,

$$A = \frac{1}{\Delta\hat{r}^2}(\hat{n}_{i+1,j} - 2\hat{n}_{i,j} + \hat{n}_{i-1,j}) \qquad (2.14)$$

$$B = \frac{1}{2\hat{r}\Delta\hat{r}}(\hat{n}_{i+1,j} - \hat{n}_{i-1,j}) \qquad (2.15)$$

$$C = (\hat{n}_{i,j}^2 + \alpha\hat{n}_{i,j}^3) \qquad (2.16)$$

In this scheme, 'i' represents position perpendicular to
the streamer centerline and 'j' represents relative position
in time after some initial condition. Equations (2.13)
through (2.16) form the basic algorithm used by the Fortran
program discussed in Appendix A.

The accuracy of the charged particle density obtained by this type of differencing scheme depends on the step sizes $\Delta\hat{t}$ and $\Delta\hat{r}$. The steps must be smaller than the smallest characteristic time of the phenomena described. Therefore, $\Delta\hat{t}$ is taken as 0.1 or in real time $\tau_R/10$ [Ref. 6]. Primarily for programming convenience, $\Delta\hat{r}$ is also taken as 0.1. Additionally, the value of β is limited by the following stability criterion [Ref. 2].

$$\beta \;>\; \frac{2\Delta\hat{t}}{\Delta\hat{r}^2} \tag{2.17}$$

Returning to Equation (2.2), it is seen that the addition of the ionization term to the RHS requires a modification to Equation (2.16) which now becomes,

$$C_1 \;=\; [\hat{n}_{i,j}^2 + \alpha\hat{n}_{i,j}^3 - \hat{n}_{i,j}] \tag{2.18}$$

Equation (2.5) can be non-dimensionalized in a similar manner by writing it in the form,

$$\frac{T_o}{\tau_A}\frac{\partial\hat{T}}{\partial\hat{t}_A} - [\frac{KT_o}{\rho C_v r_o^2}]\frac{1}{\hat{r}}\frac{\partial}{\partial\hat{r}}[\hat{r}\,\frac{\partial\hat{T}}{\partial\hat{r}}] \;=\; \frac{\sigma E_z^2}{\rho C_v} \tag{2.19}$$

where τ_A is defined as,

$$\tau_A \;\equiv\; \frac{T_o \rho C_v}{\sigma_o E_{z_o}^2} \tag{2.20}$$

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A check of Equation (2.20) reveals that τ_A has the units of seconds and, therefore, is a characteristic time for the energy equation. If one now defines the following additional relations,

$$\hat{t}_A \equiv \frac{t}{\tau_A} \qquad (2.21)$$

$$\hat{T} \equiv \frac{T}{T_o} \qquad (2.22)$$

$$\beta_A \equiv \frac{\sigma E_{z_o}^2 r_o^2}{K T_o} \qquad (2.23)$$

then, it can be shown that, Equation (2.19) becomes the following non-dimensional relation,

$$\frac{\partial \hat{T}}{\partial \hat{t}_A} - \frac{1}{\beta_A \hat{r}} \frac{\partial}{\partial \hat{r}} (\hat{r} \frac{\partial \hat{T}}{\partial \hat{r}}) = \frac{\sigma}{\sigma_o} (\frac{E_z^2}{E_{z_o}^2}) \qquad (2.24)$$

where, T_o, is the initial temperature, which for this work, is taken as 300° Kelvin. It should be clear here that, β_A, is now a non-dimensional parameter whose value depends only on the thermal conductivity, K. This parameter then serves the same function for the energy equation as does β for the continuity equation, that is, it allows the influence of turbulence to be introduced into the model. The value of the electric field strength, E_z, from the RHS of Equation (2.24) can be estimated by assuming that the initial value of the externally applied field, E_o, is given by [Ref. 8],

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$$E_O = E + \frac{I\Omega}{\ell} \qquad (2.25)$$

A reasonable value for Ω/ℓ might be on the order of 10,000 Ω/m, for which Equation (2.25) becomes,

$$E_O = E[1 + \frac{I}{E}(10,000)] \qquad (2.26)$$

where I/E for the streamer can be shown to be,

$$\frac{I}{E} = \frac{e(\mu_e N)}{N} \int_0^r 2\pi r n_e \, dr \qquad (2.27)$$

and,

$$N = \frac{P}{kT} \qquad (2.28)$$

"N" is the total number of particles present in the gas at a pressure, P, and a temperature, T. The Boltzman constant is $k = 1.38E-23$ J/°K. The integral in Equation (2.27) is a measure of the conductance of the streamer and is evaluated in Chapter III by integrating the charged particle density distribution obtained from Equation (2.1). The initial value of this integral is simply $\pi n_o r_o^2$. Using estimates of n_o and r_o to be $3.125E18$ m^{-3} and $4.0E-4$ m respectively [Ref. 7], Equation (2.26) can be solved for E_z becoming,

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$$E_{z_O} = \frac{E_O}{1 + 0.0006} \approx E_O \qquad (2.29)$$

where the value of $\mu_e N$ in Equation (2.27) can be obtained from reference tables to be 5.257E24 $(v-m-s)^{-1}$ for nitrogen. Results to be discussed in Chapter III show that the value of the integral drops rapidly to a small and nearly steady value (on the order of 0.01) such that the denominator of Equation (2.29) approaches unity in very short times.

The results of the previous paragraph then show that the RHS of Equation (2.24) is simply $\hat{n}$ since σ/σ_O is $(en_e\mu_e/en_O\mu_e)$ and $E_z^2/E_{z_O}^2$ is approximately one. So now finally, Equation (2.24) can be written as,

$$\frac{\partial \hat{T}}{\partial \hat{t}_A} - \frac{1}{\beta_A \hat{r}} \frac{\partial}{\partial \hat{r}} [\hat{r} \frac{\partial \hat{T}}{\partial \hat{r}}] = \cdot \hat{n} \qquad (2.30)$$

Equation (2.30) then shows that the temperature distribution due to the presence of the streamer is affected only by the charged particle density.

As for the continuity equations, Equation (2.30) can be solved by introducing the appropriate finite difference formula as follows (after some rearranging of terms),

$$\hat{T}_{i,j+1} = \Delta\hat{T}[\frac{1}{\beta_A}(A+B) + n_{i,j}] + \hat{T}_{i,j} \qquad (2.31)$$

where,

$$A = \frac{1}{\Delta\hat{r}^2}[\hat{T}_{i+1,j} - 2\hat{T}_{i,j} + \hat{T}_{i-1,j}] \qquad (2.32)$$

and,

$$B = \frac{1}{2\hat{r}\Delta\hat{r}}[\hat{T}_{i+1,j} - \hat{T}_{i-1,j}] \qquad (2.33)$$

The Fortran program discussed in Appendix A solves
Equation (2.31) simultaneously with Equation (2.1) or (2.2)
depending on the case being considered by using the new
value of $\hat{n}$ calculated from each step as an input to the
calculation of the new temperature value. The time step for
the temperature calculation is adjusted such that the real
time step for the two equations is equal.

D. MODELING THE STREAMER

The results of Chapter III show that appropriate selec-
tion of the controlling parameters for Equations (2.2) and
(2.30) will produce a stable system in space and time. Then
the question is: what effect does the momentary appearance
of a streamer have on this stable situation? If a local
perturbation of the electric field occurs, then the ioniza-
tion coefficient will momentarily be raised which will in
turn raise the charged particle density in the region of the
perturbation. Figure 2.2 provides the data for estimating
the magnitude of this perturbation by noting the relative
increase in E/N for a corresponding increase in ν_i/N. The

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ratio of these values can then be used to determine a simple multiplier (labelled Z and Z_A) for the $\hat{n}$ terms appearing on the RHS of Equations (2.2) and (2.30) to simulate the effect of the streamer. Therefore, one can write,

$$\frac{\partial \hat{n}}{\partial \hat{t}} - \frac{1}{\beta_A \hat{r}} \frac{\partial}{\partial \hat{r}} (\hat{r} \frac{\partial \hat{n}}{\partial \hat{r}}) = Z\hat{n} - \hat{n}^2 - \alpha \hat{n}^3 \qquad (2.34)$$

and

$$\frac{\partial \hat{T}}{\partial \hat{t}_A} - \frac{1}{\beta_A \hat{r}} (\hat{r} \frac{\partial \hat{T}}{\partial \hat{r}}) = Z_A \hat{n} \qquad (2.35)$$

where Z and Z_A are the multipliers discussed. For the finite difference approximations, these two additions are easily handled for Equations (2.2) and (2.30) respectively, as follows.

$$C_2 = [\hat{n}^2_{i,j} - \alpha \hat{n}^3_{i,j} - Z_i \hat{n}_{i,j}] \qquad (2.36)$$

$$\hat{T}_{i,j+1} = \Delta \hat{t} [\frac{1}{\beta_A}(A+B) + Z_A \hat{n}_{i,j}] + \hat{T}_{i,j} \qquad (2.37)$$

The Fortran program incorporates this concept by utilizing a separate subroutine called "STREMR" which is identical to the subroutine "DENSTY" except for the addition of Equations (2.36) and (2.37). In this scheme then, the streamer is modeled by calling up this subroutine when desired and then observing the reaction of the system over time.

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III. RESULTS AND DISCUSSION

A. NON-IONIZED ENVIRONMENT CASE

In this section the effects of turbulence on the
streamer in a non-ionized environment are considered.
Equation (2.12) applies to this situation for which Figure
3.1 shows a typical plot of the charged particle density
versus radial position perpendicular to the centerline in a
fully laminar flow. Note that the abscissa and the ordinate
are expressed in the non-dimensional coordinates of $\hat{r} = r/r_o$
and $\hat{n} = n/n_o$, respectively. There are five curves plotted
which show the charged particle density distribution profile
from the initial conditions at time zero in steps as specified
in the legend to the profile at 3000 recombination times (TR).
"TR" is the computer's version of "τ_R" and will be used here
in the text. Now, for this plot, and except for Figures 3.11
and 3.12, for all the figures in this section, n_o = 3.125E18
m^{-3}. and α_2 = 2.0E-12 m^3/sec [Ref. 7]. These values then
equate to a recombination time of 0.16 microsecond, and so,
1000TR is equivalent to 0.16 millisecond in real time.

In the laminar case depicted in Figure 3.1 it is clearly
seen that the centerline charged particle density drops off
in time in accordance with Equation (2.11) which in non-
dimensional form would be,

$$\hat{n}_{\mathfrak{C}} = \frac{1}{1 + \hat{t}} \tag{3.1}$$

31

CHARGED PARTICLE DENSITY PROFILE

IN LAMINAR FLOW

LEGEND
□ = TIME ZERO
○ = 100 TR
△ = 1000 TR
+ = 2000 TR
× = 3000 TR

FIGURE 3.1

32
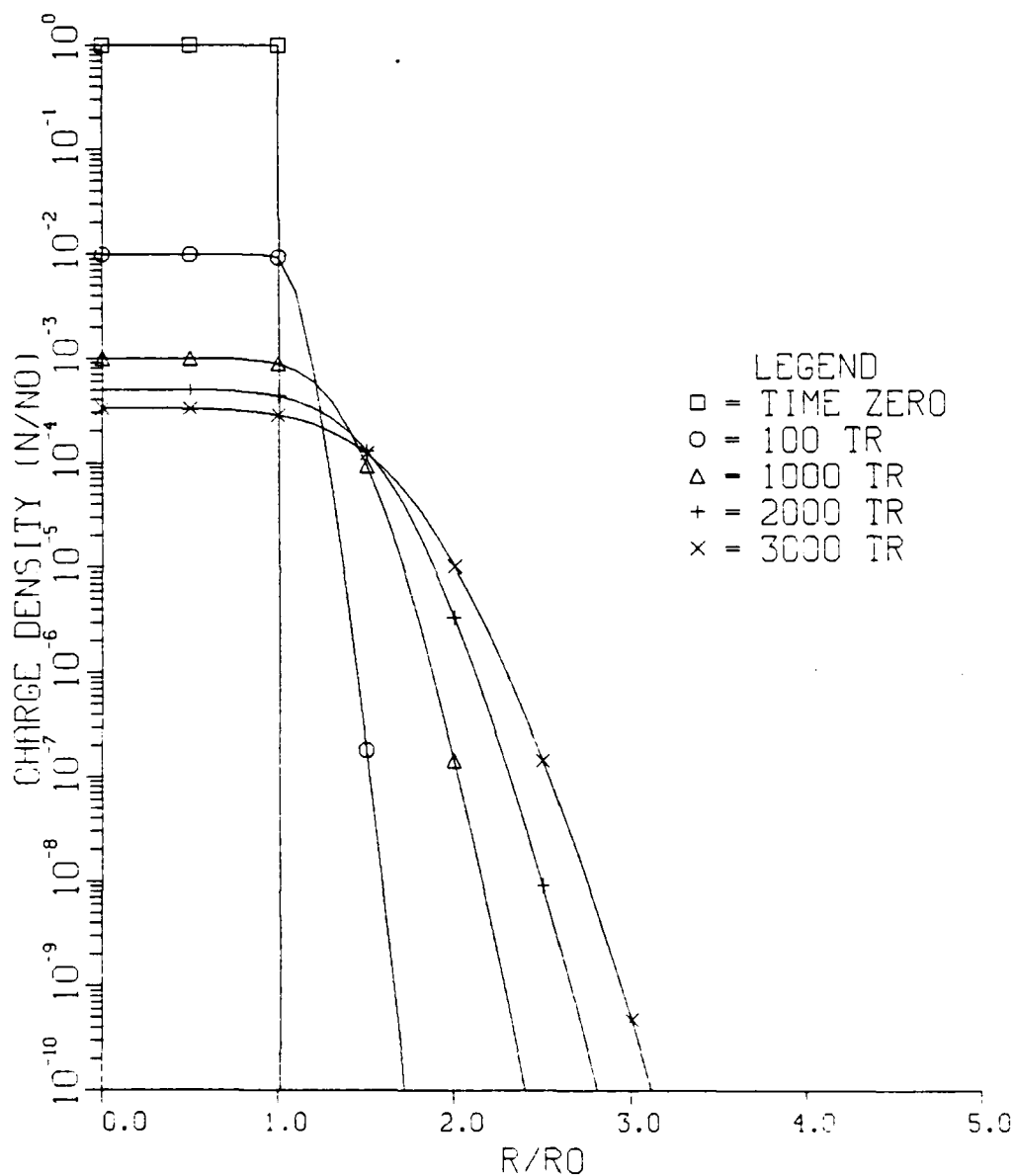
For example, at 1000TR, $\hat{n}_{\mathfrak{c}}$ = 1.0E-3. This decay is due primarily to recombination only since, in this case, beta = 50,000, and therefore, the diffusion term becomes very small. For longer times, diffusion comes into play and some spreading of the charge profile is seen.

Figure 3.2 shows the effect of introducing increasing amounts of turbulence into the flow. This plot compares the different charged particle profiles as they would appear at 0.32 msec for beta values of 50 (fully turbulent), 500, 5000 and 50,000 (fully laminar). Note that the 2000TR curve from Figure 3.1 is identical to the beta = 50,000 curve in Figure 3.2. The effect of the turbulence is clearly seen to spread out the profile and to lower $\hat{n}$ on the centerline as well [Ref. 2].

As was mentioned previously, the conductance of the decaying streamer can be determined by integrating the charged particle density over the radius. This can be derived by "inverting" Ohm's law and introducing the conductivity expression for a partially ionized plasma for which the current can now be expressed as [Ref. 4],

$$ I = GV = (\frac{\sigma A}{\ell})V = (\frac{e^2 n_e A}{m_e \nu_e \ell})V \qquad (3.2) $$

where G is the conductance, or the reciprocal of the resistance, in the familiar relation, V = IR. 'σ' is the conductivity, 'A' the cross-sectional area and 'ℓ' is the

33

CHARGED PARTICLE DENSITY PROFILE

IN TURBULENT FLOW

T/TO = 2000TR

LEGEND
□ = BETA = 50000
○ = BETA = 5000
△ = BETA = 500
+ = BETA = 50

FIGURE 3.2

34
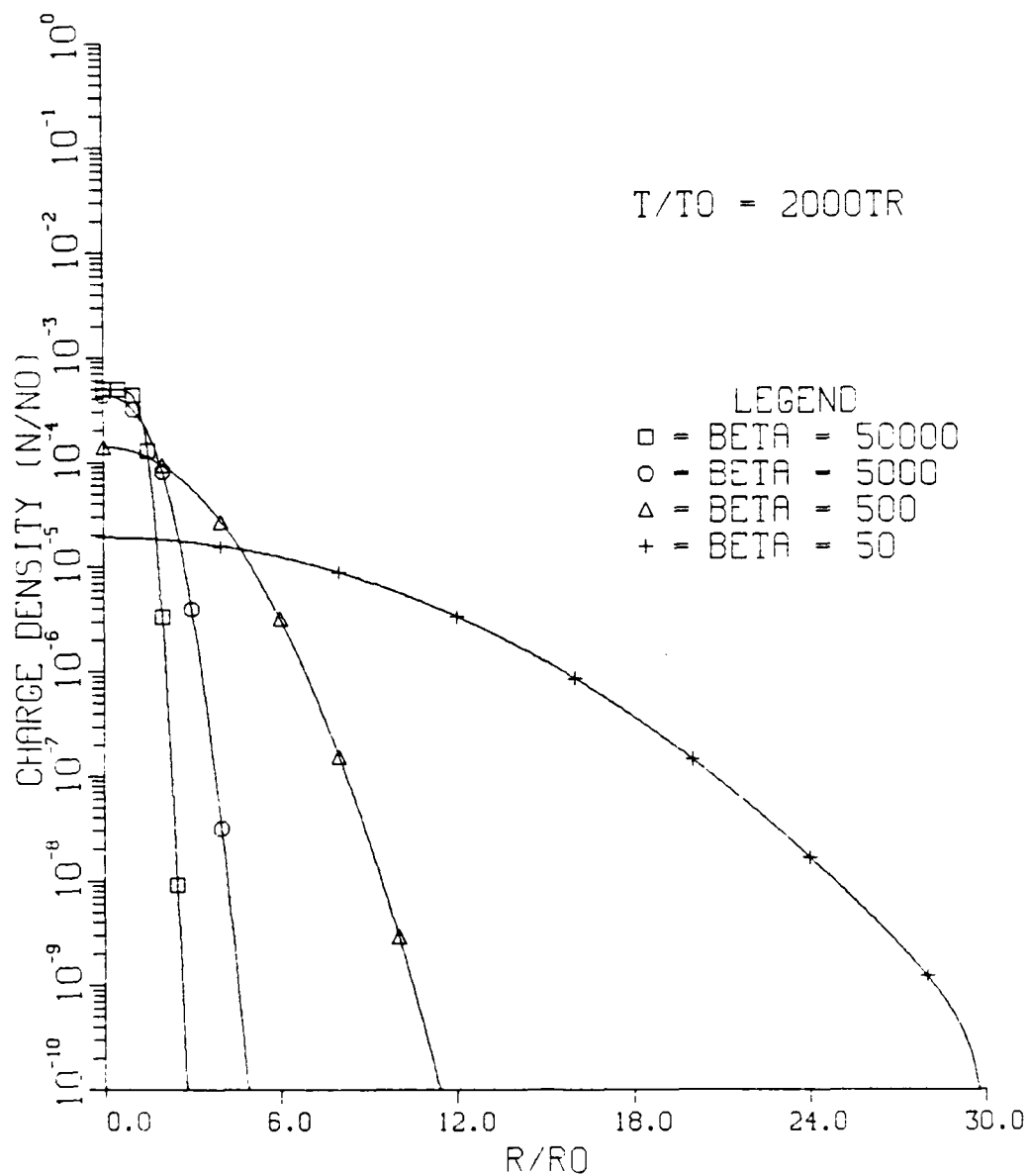
length of the streamer (or the distance between the elec-
trodes in Figure 2.1 for this case). Additionally, m_e and
ν_e are the electron mass and the electron collision frequency,
respectively.

For the decaying streamer, the radius, and therefore the
area, is changing but the length is constant. Since all the
other factors within the brackets of Equation (3.2) are also
constant, 'G' can then be expressed as,

$$G \; = \; \frac{e^2}{\ell m_e \nu_e} \int_0^{\hat{r}} \hat{n} \; 2 \pi \; \hat{r} \; d\hat{r} \tag{3.3}$$

where gamma is now defined as the value of the integral in
the above equation. Gamma has the units of reciprocal
meters, and is determined in the program, "CHARGE" (Appendix
A), by the subroutine, "CNDUCT." Gamma is then directly
proportional to the conductance.

Figure 3.3 shows gamma plotted over time out to 100TR.
Note that gamma drops in value by 90% in only about 10TR
(1.6 microsec). After this time the value levels off to
about 0.04 at 100TR and then beyond the range of Figure 3.3
the value drops very slowly to about 0.001 at 10,000TR (1.6
msec).

Figure 3.3 is for fully laminar flow. Introducing
turbulence into the flow causes a very slight increase in
gamma over all time for the complete streamer. Figure 3.4

35

GAMMA VS TIME

IN LAMINAR FLOW
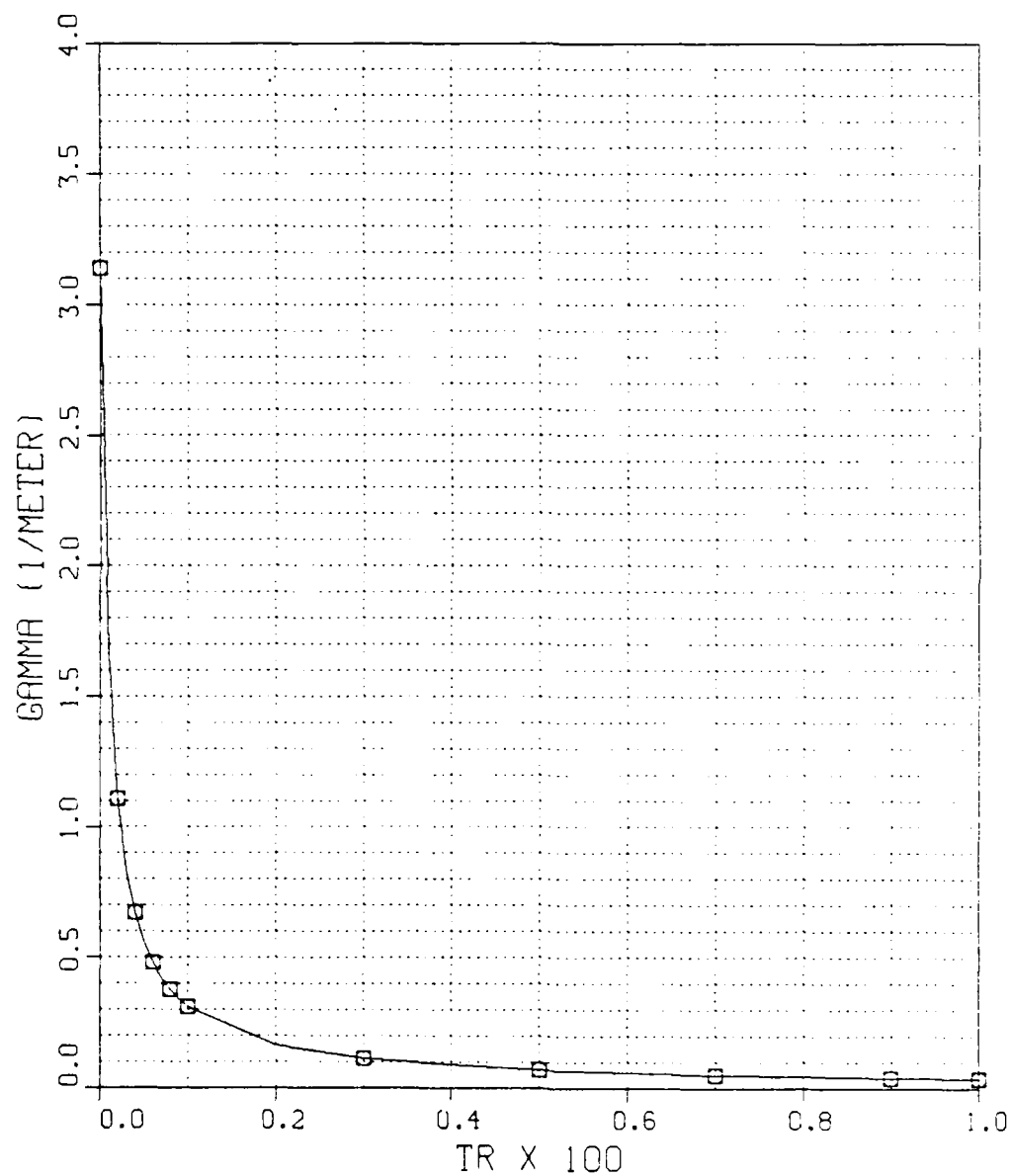
FIGURE 3.3

GAMMA VS BETA

FIGURE 3.4

37
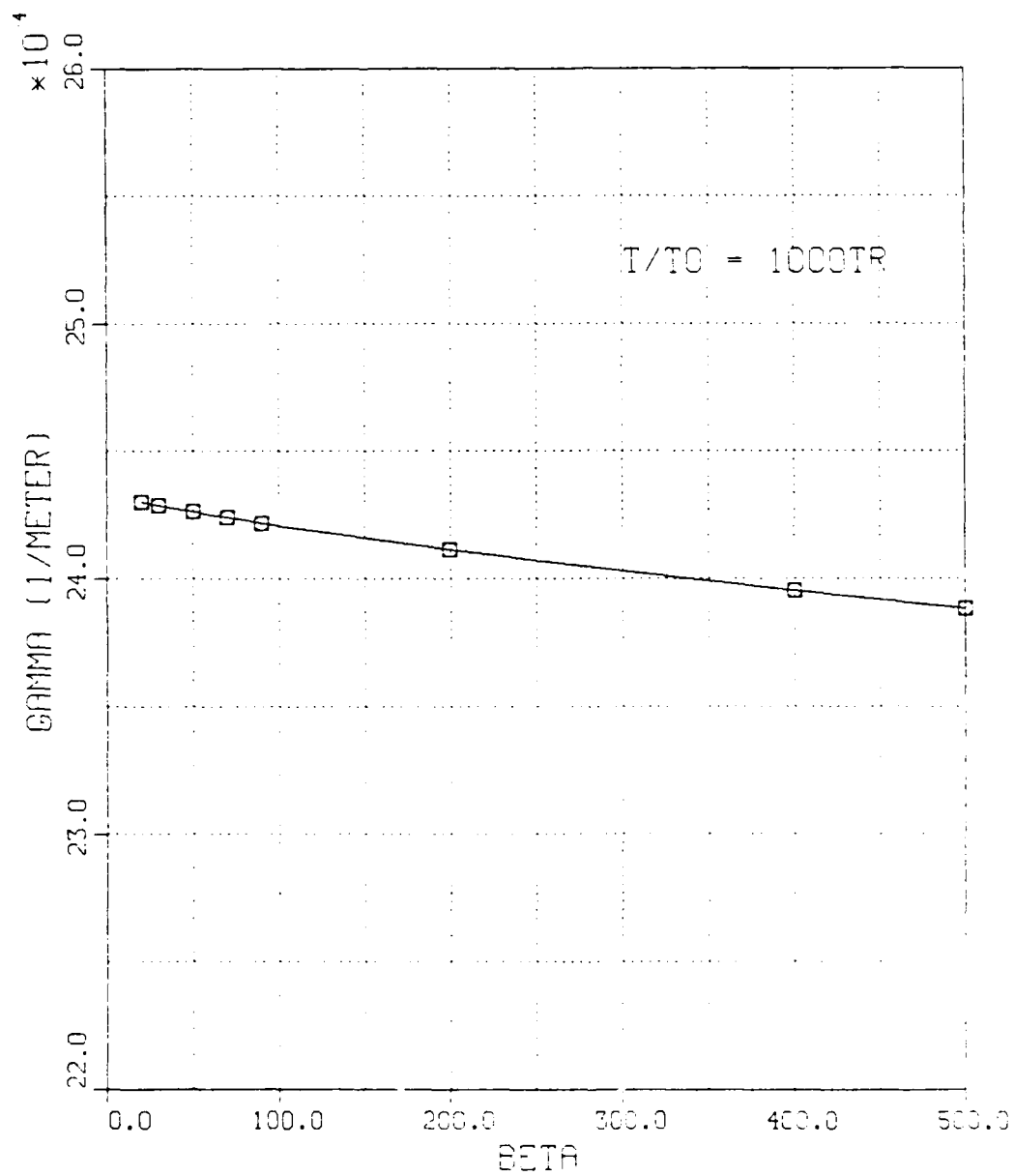
shows this effect at 1000TR. Figure 3.5 shows the value of gamma within the original streamer tube channel only. Here it is seen that for small beta (high turbulence) the conductance is lower than in the laminar flow condition.

Referring back to Figure 3.2, and with these conductance characteristics in mind, it seems rather apparent that one effect of the turbulence is to delay the possibility of a transition to arcing since the conductance is lower along the original channel. Another consideration is what happens to a follow-on streamer appearing after the charged particle density profile of the first streamer has died away for say, 2000TR as in Figure 3.2. In a laminar flow there would be a distinct path of least resistance along the original channel. However, in the turbulent flow, the profile is spread out over a much larger radius and then, this "follow-on streamer" essentially has no favored "route" for arcing.

Equation (2.30) showed that the charged particle density was the sole source of heating in this model. Figure 3.6 shows the result of solving Equation (2.30) in a fully laminar flow for an E/N of $1.635E-17$ $V-cm^2$, at one atmosphere, and 300°K for an electric field strength of $4.0E4$ V/m. Note that the temperature rise amounts to less than about 0.05% in 1000TR and then rises very slightly over time eventually peaking out at about 4000TR and then falling back slowly. This would seem to be a logical behavior for this case since the charged particle density that is heating the gas is also

38

GAMMA VS BETA

FOR STREAMER TUBE
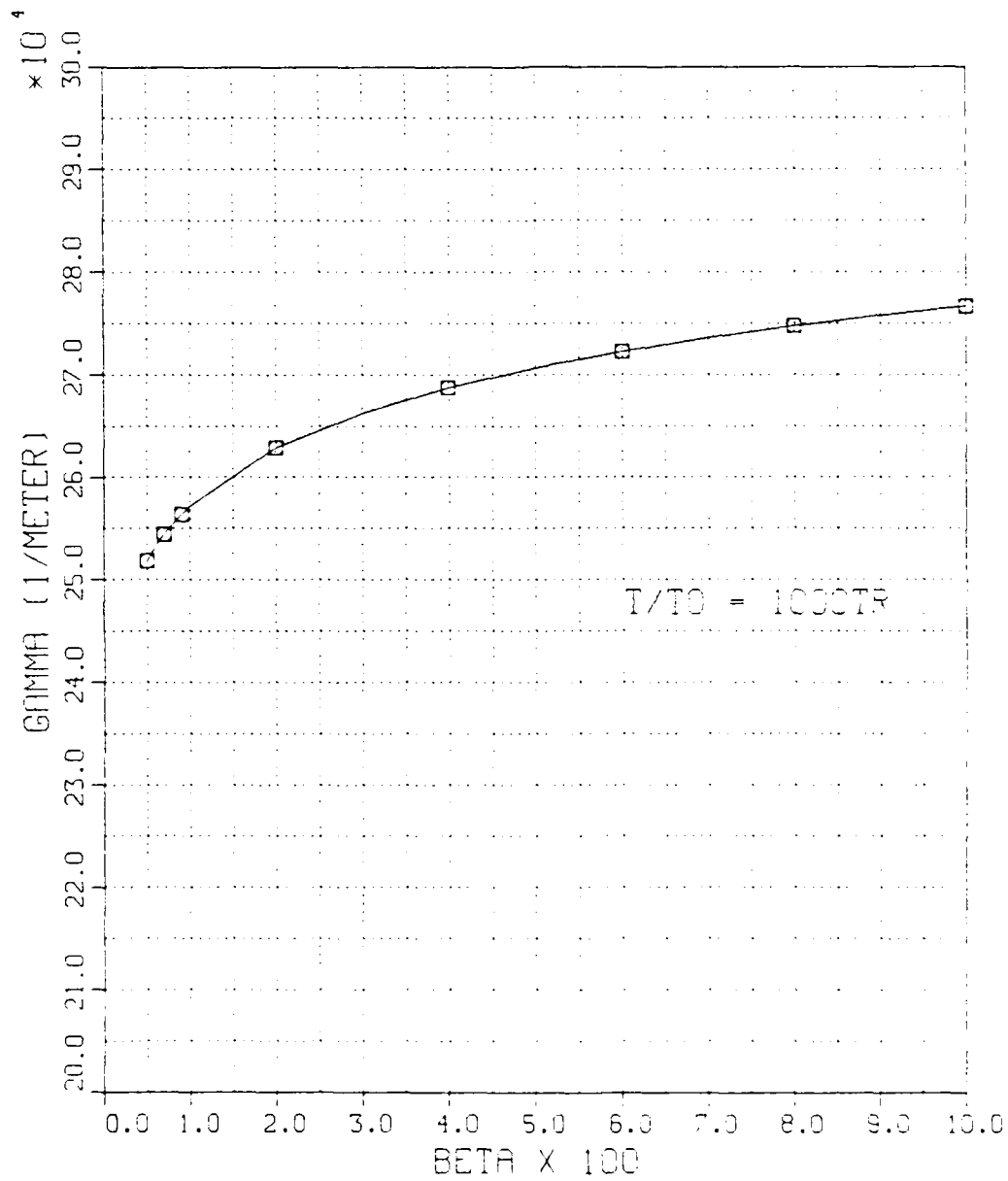
FIGURE 3.5

TEMPERATURE PROFILE

FOR E = 4.0E4 V/M

FIGURE 3.6

40
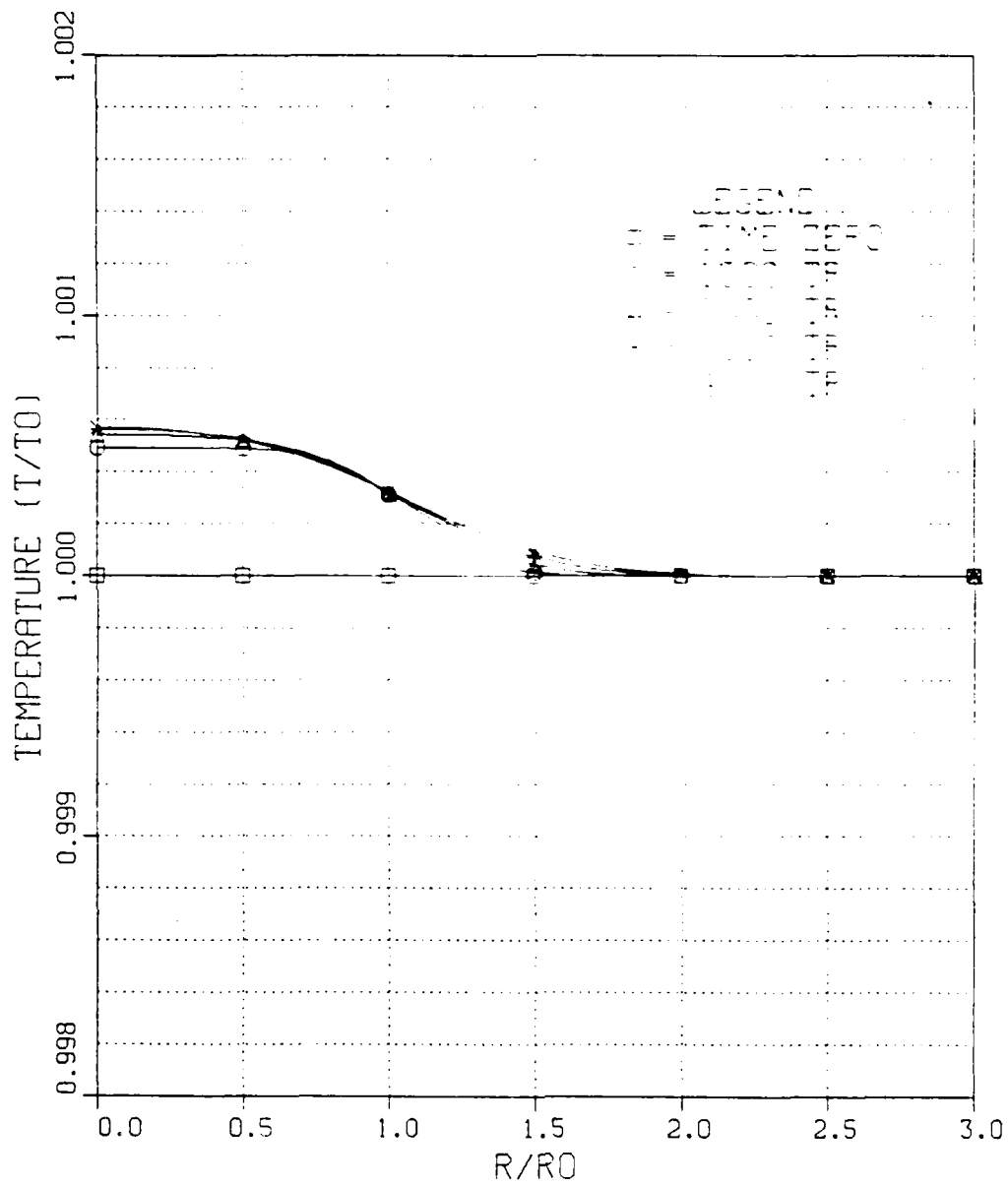
dying away. The question is then; what happens if one increases the electric field?

Figures 3.7 through 3.10 show the effects of increasing E_O from 1.0E6 V/m (a typical value of interest for lasers) to 8.0E6 V/m. Note that β_A changes for each of these cases. This is due to the fact that the electron drift velocity is a function of E/N, which is plotted in Figure 2.2. Since the drift velocity changes, the electron mobility changes which, in turn, produces a new conductivity value for each different electric field strength case. Recalling Equations (2.20) and (2.23), one can see that each case has a different characteristic time and a different β_A. For increasing E_O, τ_A gets smaller while β_A gets larger.

The main difficulty with this is that the time step for the energy equation solution must be adjusted to ensure that the continuity equation is "feeding" the charged particle density values into the energy equation at the correct rate. For the continuity equation, the non-dimensionalized time step is 0.1 ($\tau_R/10$ in real time). Therefore, the true time for one time step (one "DT" in the program) is simply $DT \times \tau_R$. Similarly, the true time for the energy equation is $DTA \times \tau_A$. It follows then that the value needed for DTA is simply $(DT \times \tau_R)/\tau_A$.

Returning to Figures 3.7 through 3.10, it can be clearly seen that increasing E_O increases the temperature rise, as would be expected. The effects of longer times is the same as the case for Figure 3.6 but requires much longer computing

41

TEMPERATURE PROFILE

FOR E = 1.0E6 V/M

FIGURE 3.7

42
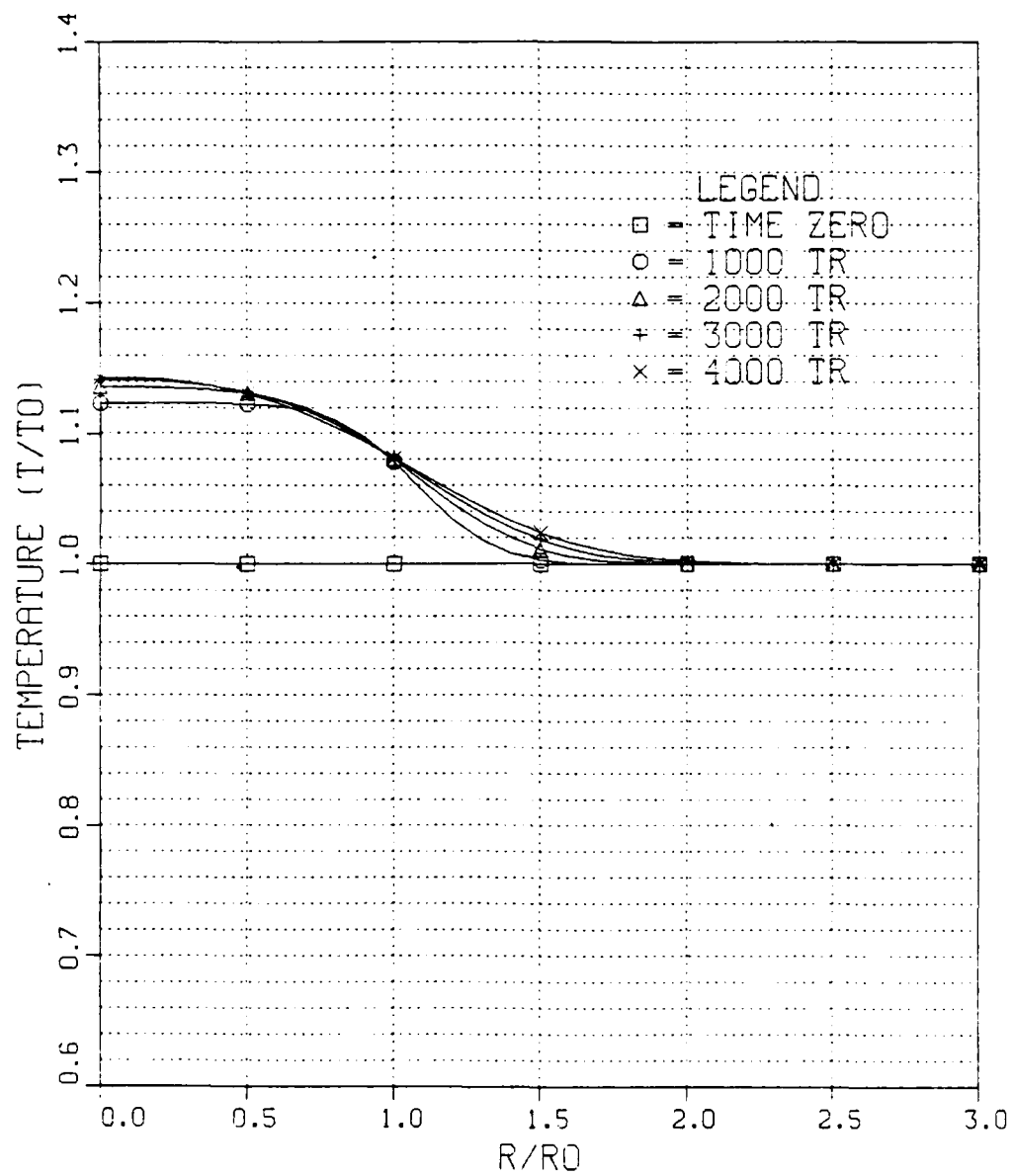
TEMPERATURE PROFILE

FOR E = 2.0E6 V/M
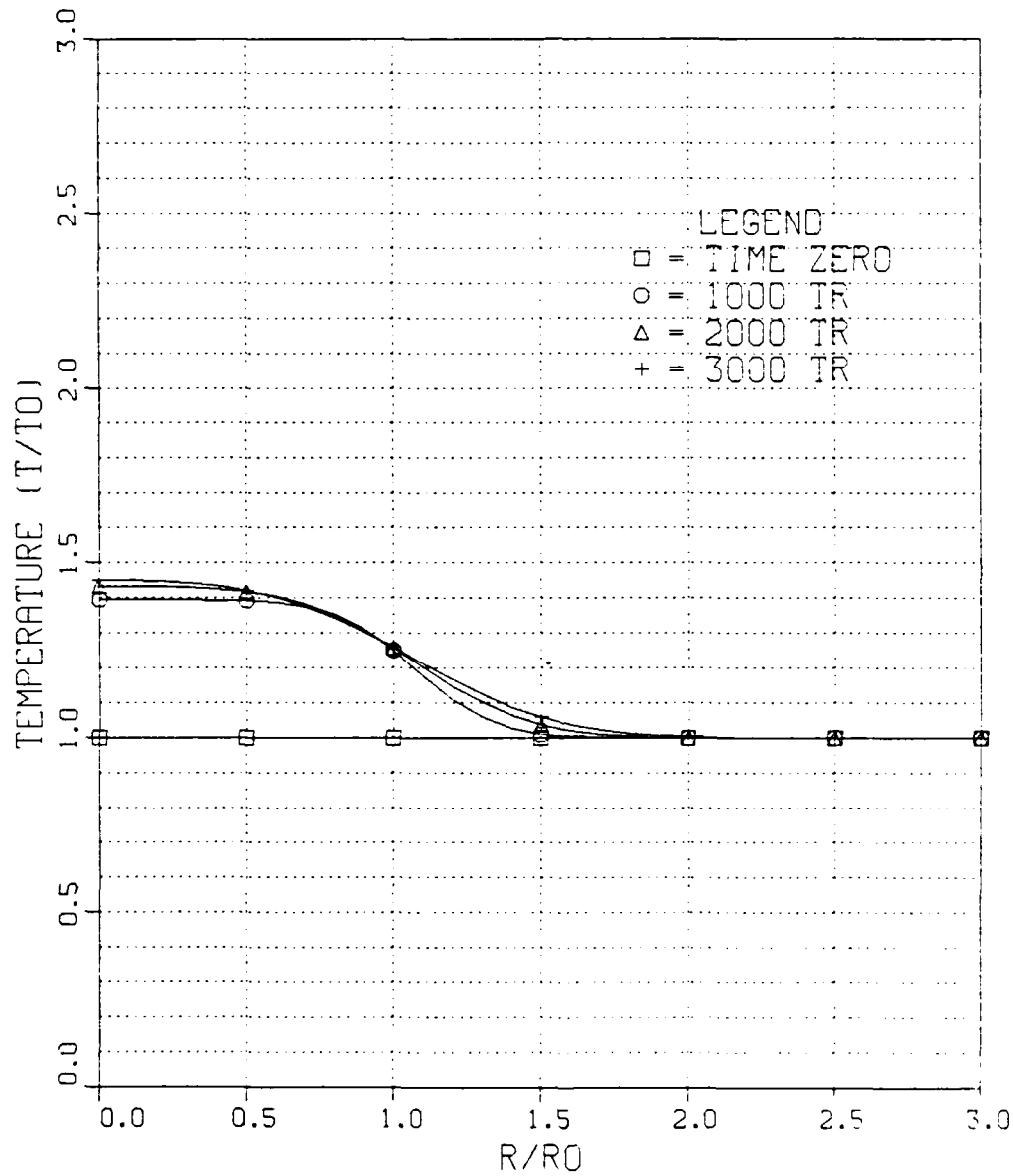
FIGURE 3.8

TEMPERATURE PROFILE

FOR E = 4.0E6 V/M

FIGURE 3.9

44
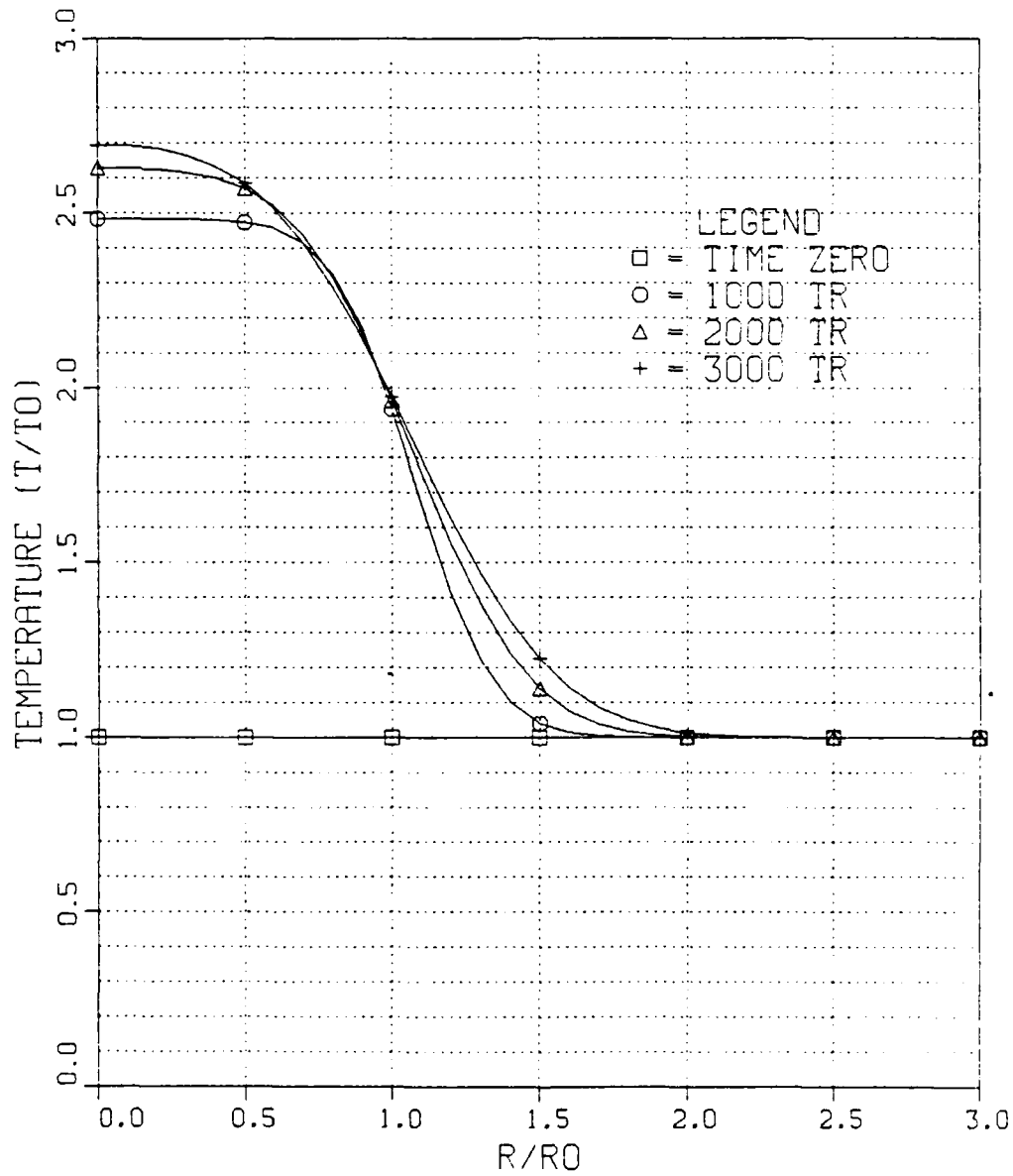
TEMPERATURE PROFILE

FOR E = 8.0E6 V/M

FIGURE 3.10

45
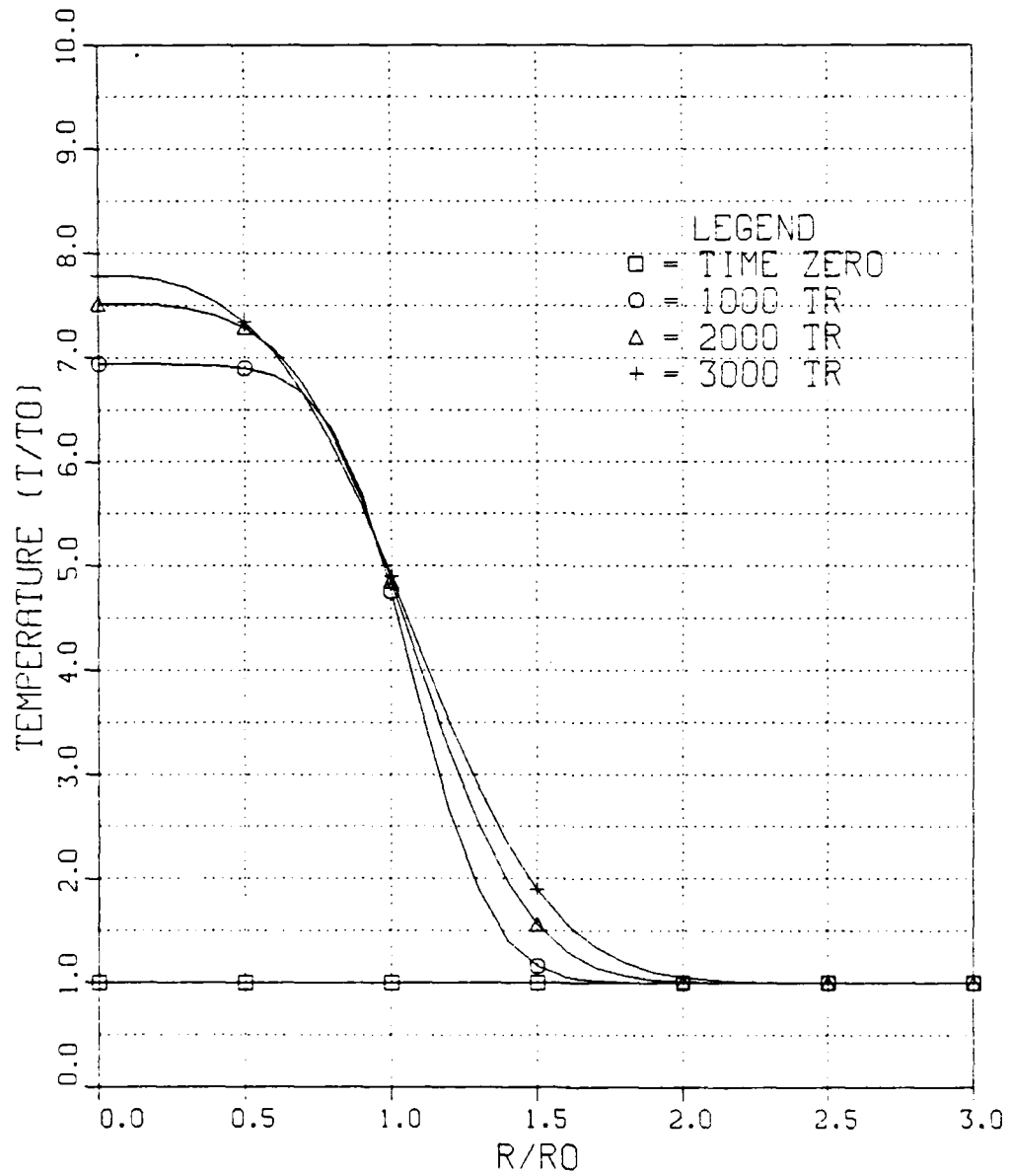
times to reach the peak and to begin falling back. The
effect of turbulence on the cases of interest, say 4.0E4
V/m to 1.0E6 V/m, is to essentially eliminate the temperature
rise.

One remaining question for this non-ionized case is what
effect would increasing n_o have? Figure 3.11 shows that at
E_o = 1.0E6 V/m and n_o increased by a factor of 10 to $3.125E19/m^3$
produces a more squared off profile than that depicted in
Figure 3.7 suggesting that the temperature dies rapidly
outside of the immediate influence of the streamer. Figure
3.12 shows the effect of another 10-fold increase in N.
Here the resulting temperature rise drops off at an even
steeper rate at 2000TR than for Figures 3.7 and 3.11.
Recalling that $\tau_R = 1/\alpha_2 n_o$, it is clear that increasing n_o
decreases the characteristic recombination time significantly
which, in turn, lowers $\hat{n}$ such that there is a noticeable
reduction in the temperature profile outside of this immedi-
ate streamer influence.

B. IONIZED ENVIRONMENT CASE

In this section the effects of a glow discharge in an
environment where a steady glow exists is considered. For
this situation, Equation (3.4) is the applicable non-dimensional
form of the governing continuity equation.

$$\frac{\partial \hat{n}}{\partial \hat{t}} - \frac{1}{\beta \hat{r}} \frac{\partial}{\partial \hat{r}}(\hat{r} \frac{\partial \hat{n}}{\partial \hat{r}}) = \hat{n} - \hat{n}^2 - \alpha \hat{n}^3 \qquad (3.4)$$

46

TEMPERATURE PROFILE

FOR NO = 3.125E19 M

FIGURE 3.11

47

TEMPERATURE PROFILE

FOR NO = 3.12E20 M

FIGURE 3.12

48
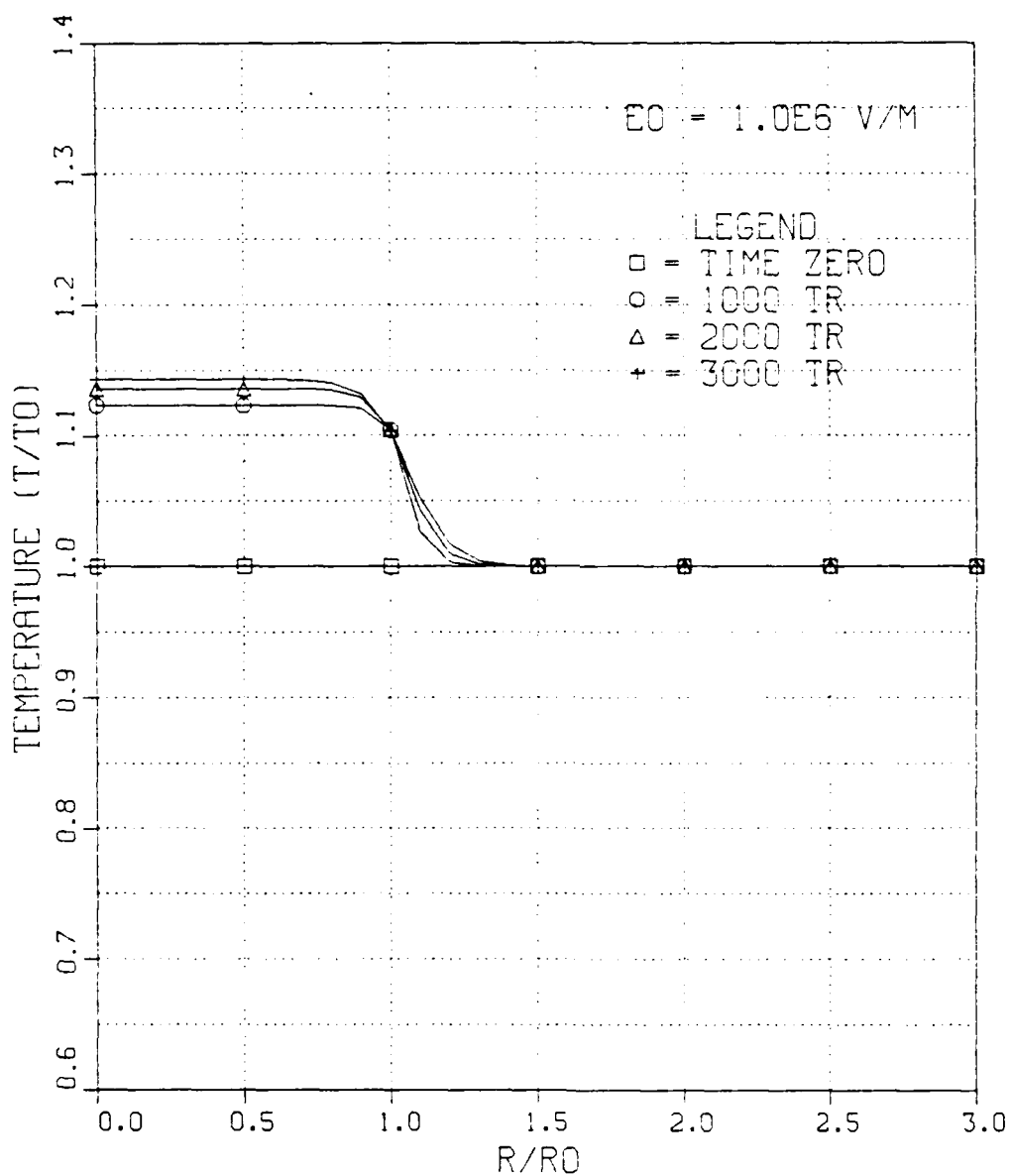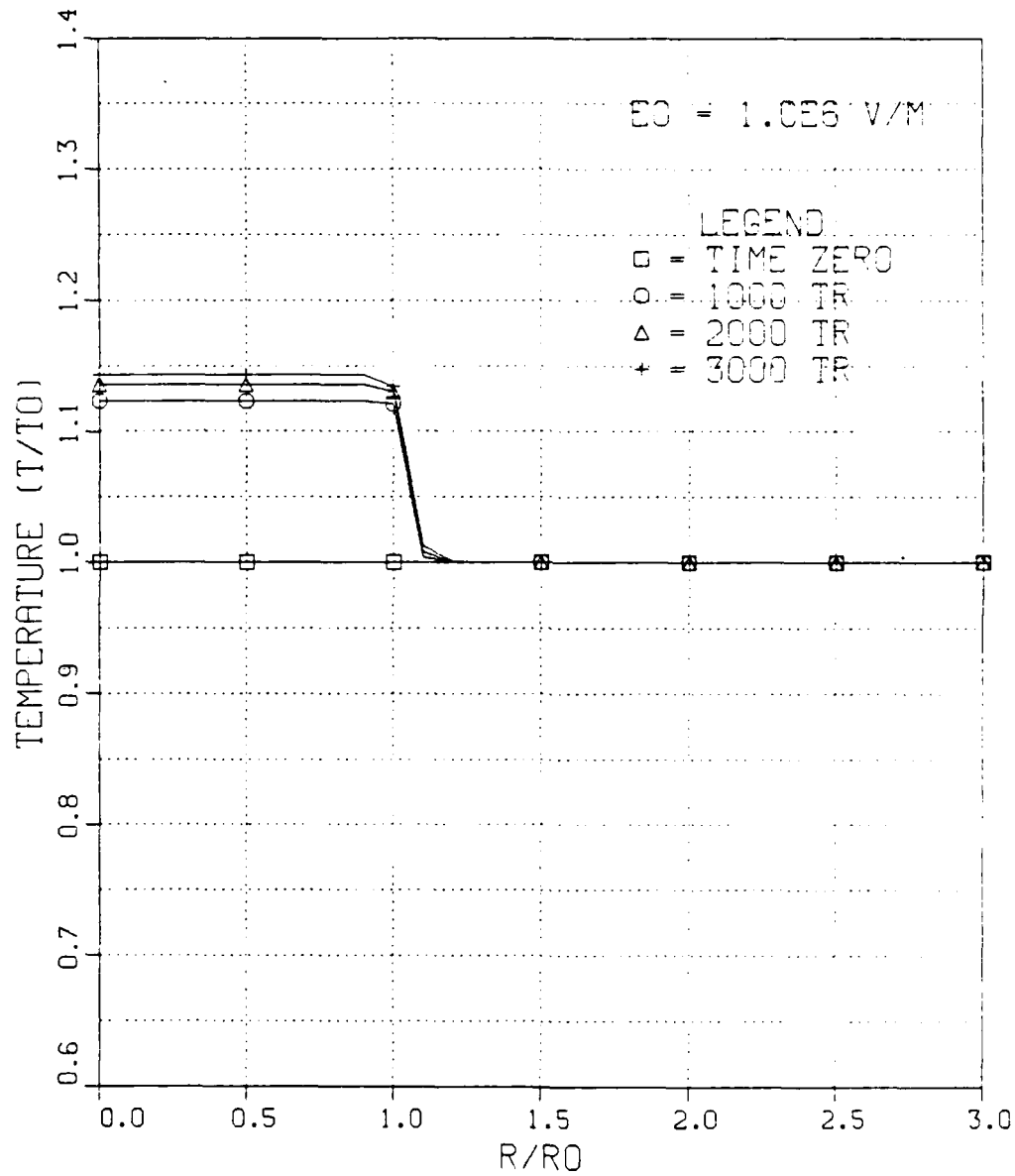
The temperature effects are then determined by Equation (2.30) where $\hat{n}$ is obtained by solving Equation (3.4). The first question in this case might be; what effect does turbulence have on the temperature profile due to this steady glow discharge? At sub-atmospheric pressure, it is known that the charged particle density profile takes on a Bessel function distribution [Ref. 9]. Therefore, to investigate this situation, Equation (2.30) can be solved independently from Equation (3.4) by calling up the subroutine "DENSTY" in the program and sending it a constant, for all time, value for $\hat{n}$. The Bessel function serves as the initial condition profile for the value of $\hat{n}$. This distribution, labeled "Nl" in the program, allows $\hat{n}$ to go to zero at $r = 4.8$. Figure 3.13 shows the results of such a case for strictly laminar flow. The values of the controlling parameters for Figures 3.13 through 3.16 are: $n_o = 3.125E18 \text{ m}^{-3}$, $\alpha_2 = 2.0E-12 \text{ m}^3/\text{sec}$ and $E_o = 1.0E6$ V/m. It is clearly evident that the temperature for such a laminar case would rise to approximately 5400°K (recall that $T/T_o = 1$ is equivalent to 300°K) in only about 1000TR. Since molecular nitrogen (N_2) begins to dissociate at about 4000°K, the situation depicted in Figure 3.13 is undesirable. However, introducing turbulence into the flow tends to significantly reduce this temperature distribution. Figure 3.14 shows that a moderate degree of turbulence can be selected which, essentially, holds the temperature profile near the laminar profile without
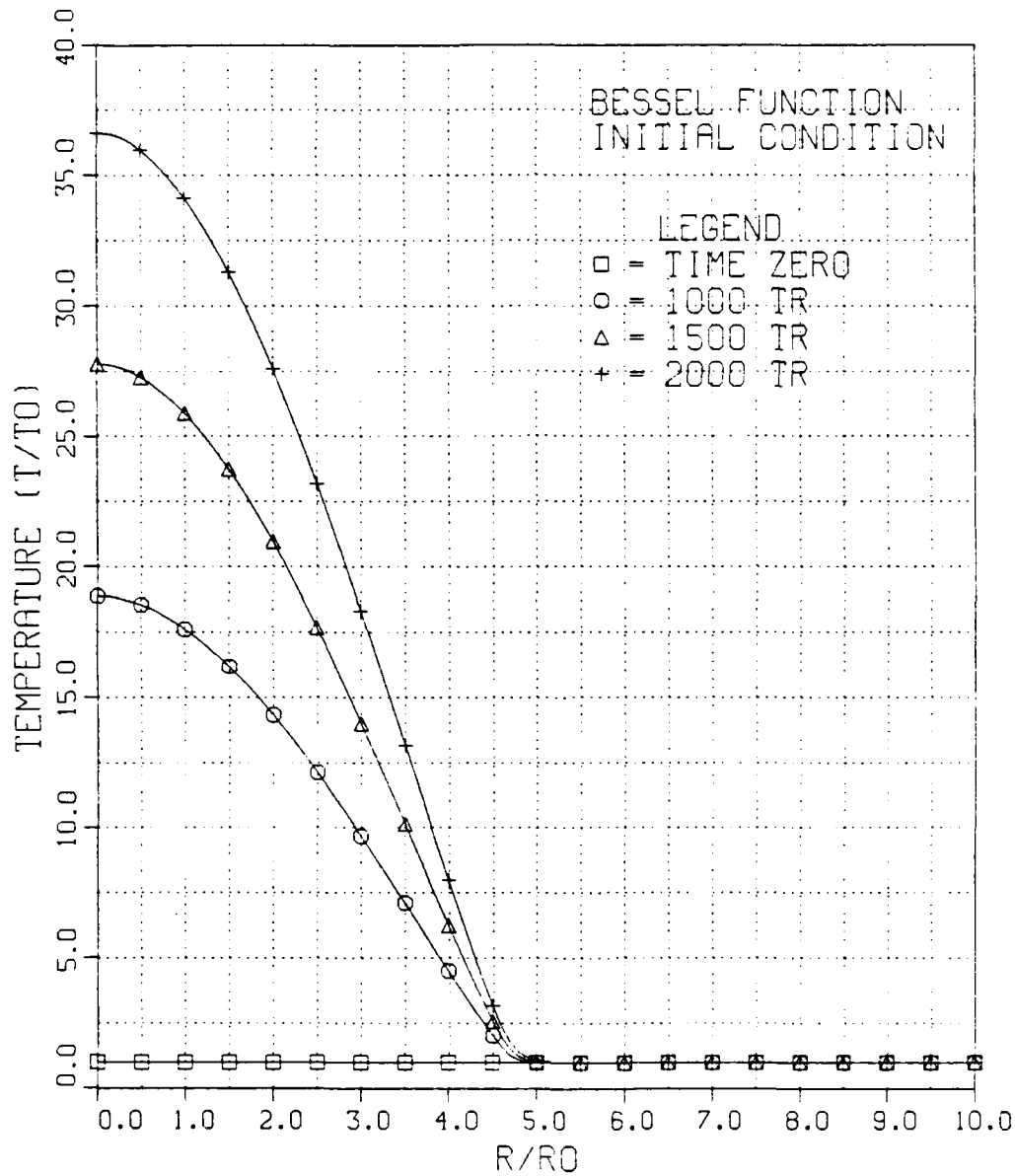
49

TEMPERATURE PROFILE

FOR BAMIN = 800

FIGURE 3.13

50

TEMPERATURE PROFILE

FOR BPMIN = 3

FIGURE 3.14

51

TEMPERATURE PROFILE

FOR BAMIN = 1

FIGURE 3.15

52

TEMPERATURE PROFILE

FOR BAMIN = 0.4

FIGURE 3.16

53
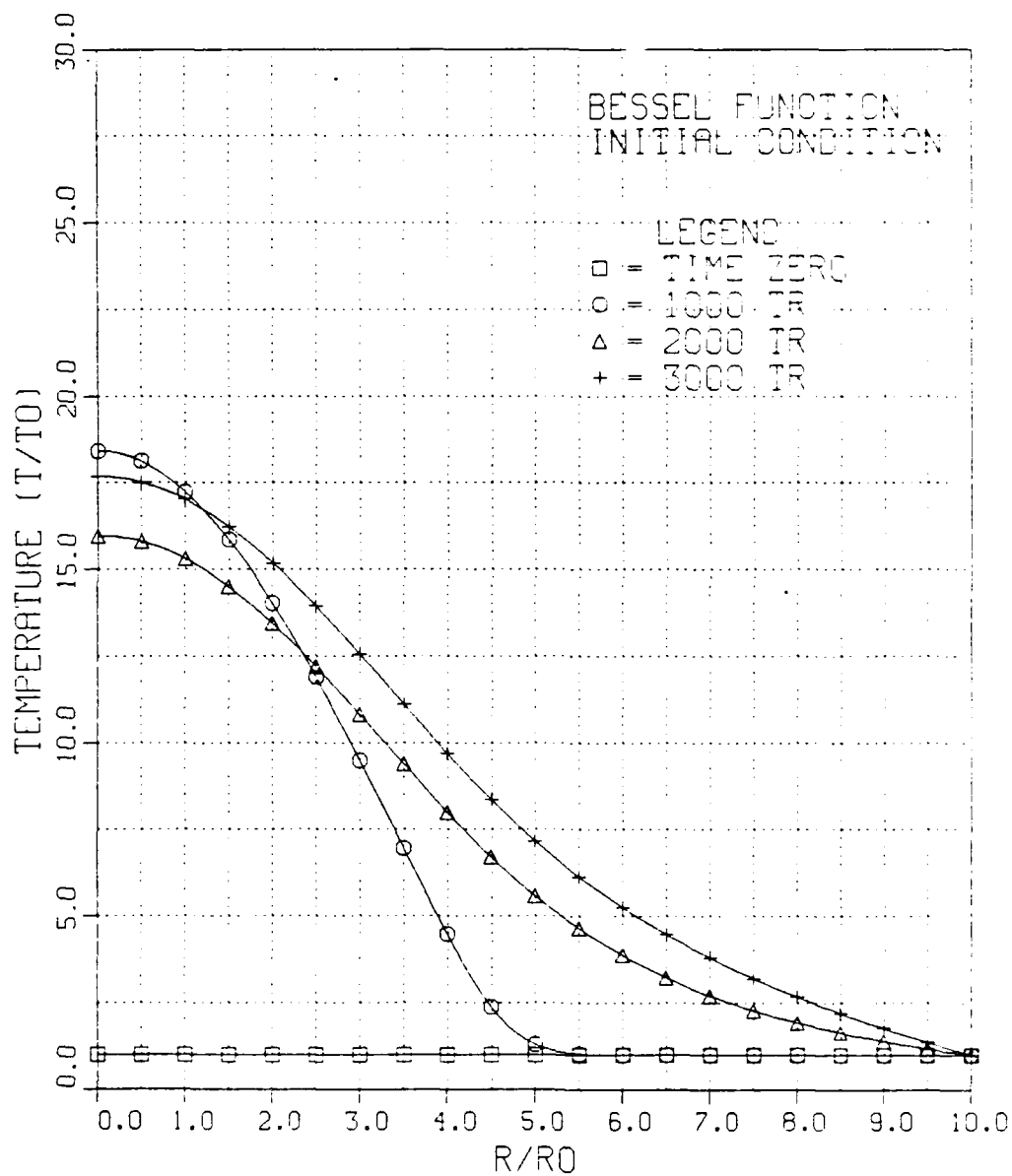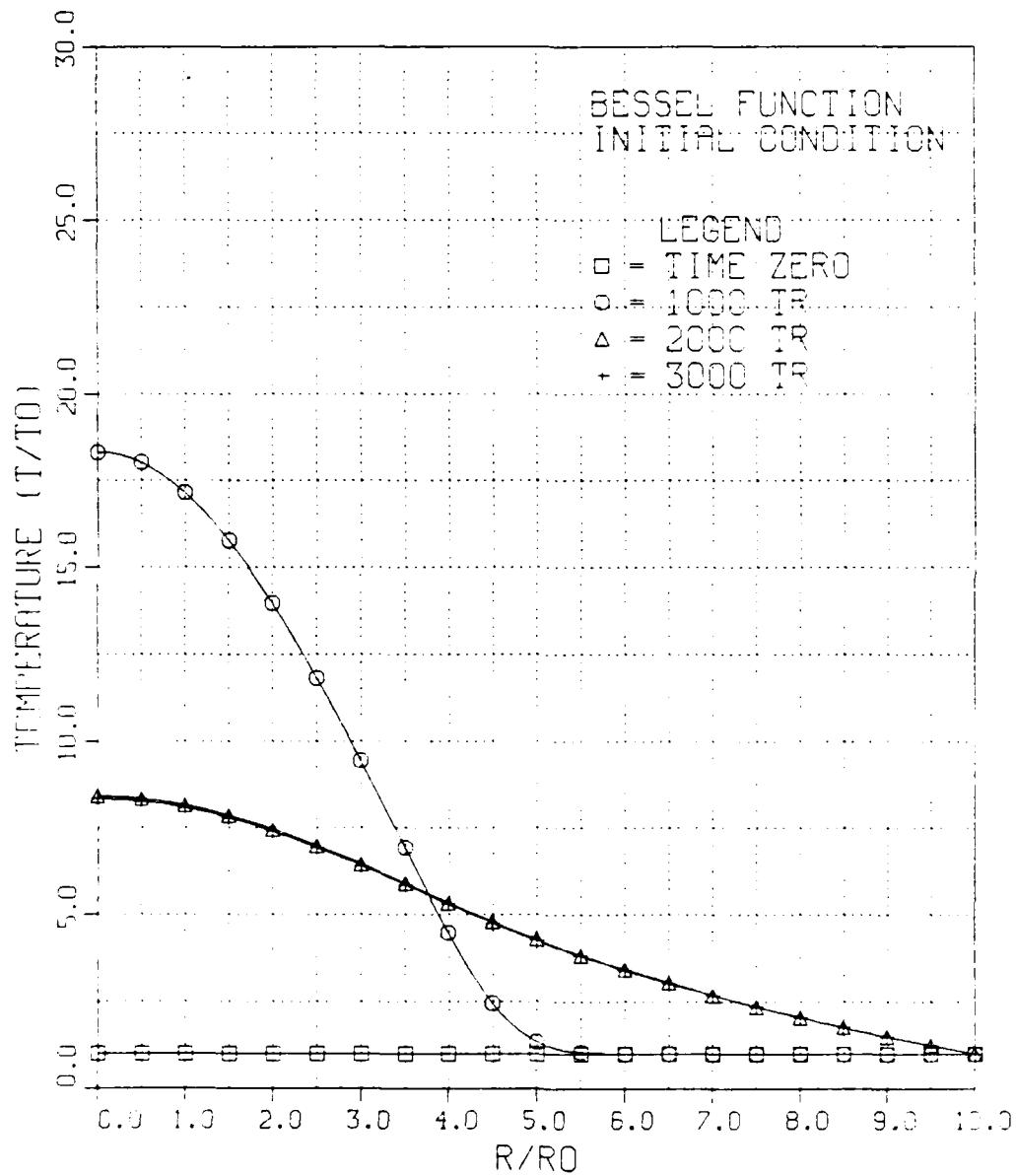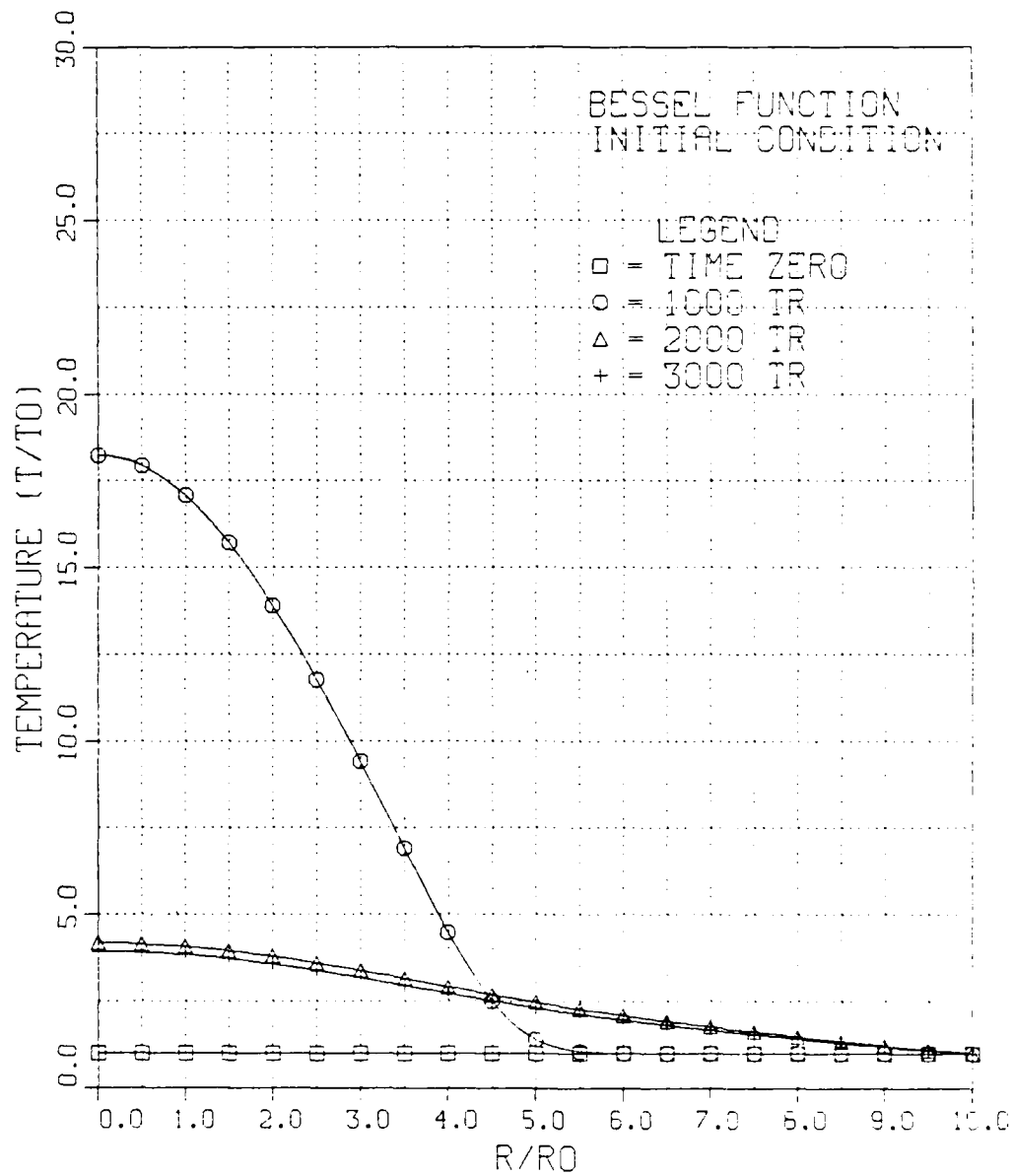
going unstable as in Figure 3.13. Figures 3.15 and 3.16 show the effects of increasing the turbulence level still further. Note that in Figure 3.15, the temperature profile is nearly "frozen out" at about 2400°K, while Figure 3.16 is holding steady at around 1200°K.

This asymptotic behavior of the temperature rise in reaching this stable profile appears to be related to the characteristic time associated with Equation (2.30). Observe that for all three figures (3.14, 3.15 and 3.16) the temperature rise stabilizes between about 2000TR and 3000TR. This effect will continue to be evident in the remaining cases to be discussed. The parameters for the case presently being considered are such that the true time of 2500TR is 0.4 msec. This is approximately 4.5 times the characteristic time step in seconds for the energy equation, which is consistent with the concept of a rise time in an asymptotic system.

The final case to be discussed here is perhaps the most interesting. This will be the case where there is a steady level of ionization in the gas and a perturbation of the electric field is introduced to model the effects of the streamer. For the steady state situation at the centerline, $\frac{\partial n}{\partial t} = 0$, so one can write Equation (3.5).

$$- \frac{D_a}{r} \frac{\partial}{\partial r}(r \frac{\partial n}{\partial r})_{\mathcal{C}} = 0 = \nu_i n_{\mathcal{C}} - \alpha_2 n_{\mathcal{C}}^2 - \alpha_3 n_{\mathcal{C}}^3 \qquad (3.5)$$

Neglecting the three-body recombination term (α_3 is on the order of 1.0E-34 m^6/sec), and noting that $\frac{\partial n}{\partial r}$ on the centerline

54

is also zero, the ionization term is then equal to $\alpha_2 n_{\text{C}}$. Since Figure 2.2 gives ν_i/N, the charged particle density for the steady state condition can be obtained by,

$$n_{\text{C}} \approx \frac{\nu_i}{\alpha_2} = \left(\frac{\nu_i}{N}\right)\left(\frac{N}{\alpha_2}\right) \qquad (3.6)$$

where N is given by Equation (2.28). Figure 3.17 shows this steady state charged particle distribution where, for this model, the glow discharge is contained in a region of a radius equal to 10 r_o. At r_o, the boundary condition is set such that $\hat{n}$ is zero at the "wall." The left hand boundary condition assumes that the slope of the charged particle density is zero. See Appendix A for details concerning how the program meets these boundary conditions. Note that in Figure 3.17, the steady state condition is reached in less than 100TR, and is constant at the longer times. The first vertical line, at $r/r_o = 1.0$, is the initial condition distribution which assumes $\hat{n} = 1.0$ out to $\hat{r} = 1.0$ and zero beyond this point.

The steady state solution for Equation (2.30) is given in Equation (3.7).

$$\hat{T} = -(\beta_A \hat{n})\,\frac{\hat{r}^2}{4} + \beta_A\,\frac{\hat{nb}^2}{4} + \hat{T}_o \qquad (3.7)$$

The boundary conditions are $\hat{T} = T_o$ at $\hat{r} = b$ and $\dfrac{\partial \hat{T}}{\partial \hat{r}} = 0$ at $\hat{r} = 0$. Figure 3.18 shows this result for $E_o = 1.47E5$ V/m. For this case $\beta_A = 0.2219$, and so at the centerline $\hat{r} = 0$

55

CHARGED PARTICLE DENSITY PROFILE

WITH IONIZATION TERM

EO = 1.7E5 V/M

LEGEND
□ = TIME ZERO
○ = 100 TR
△ = 200 TR
+ = 300 TR

FIGURE 3.17

56
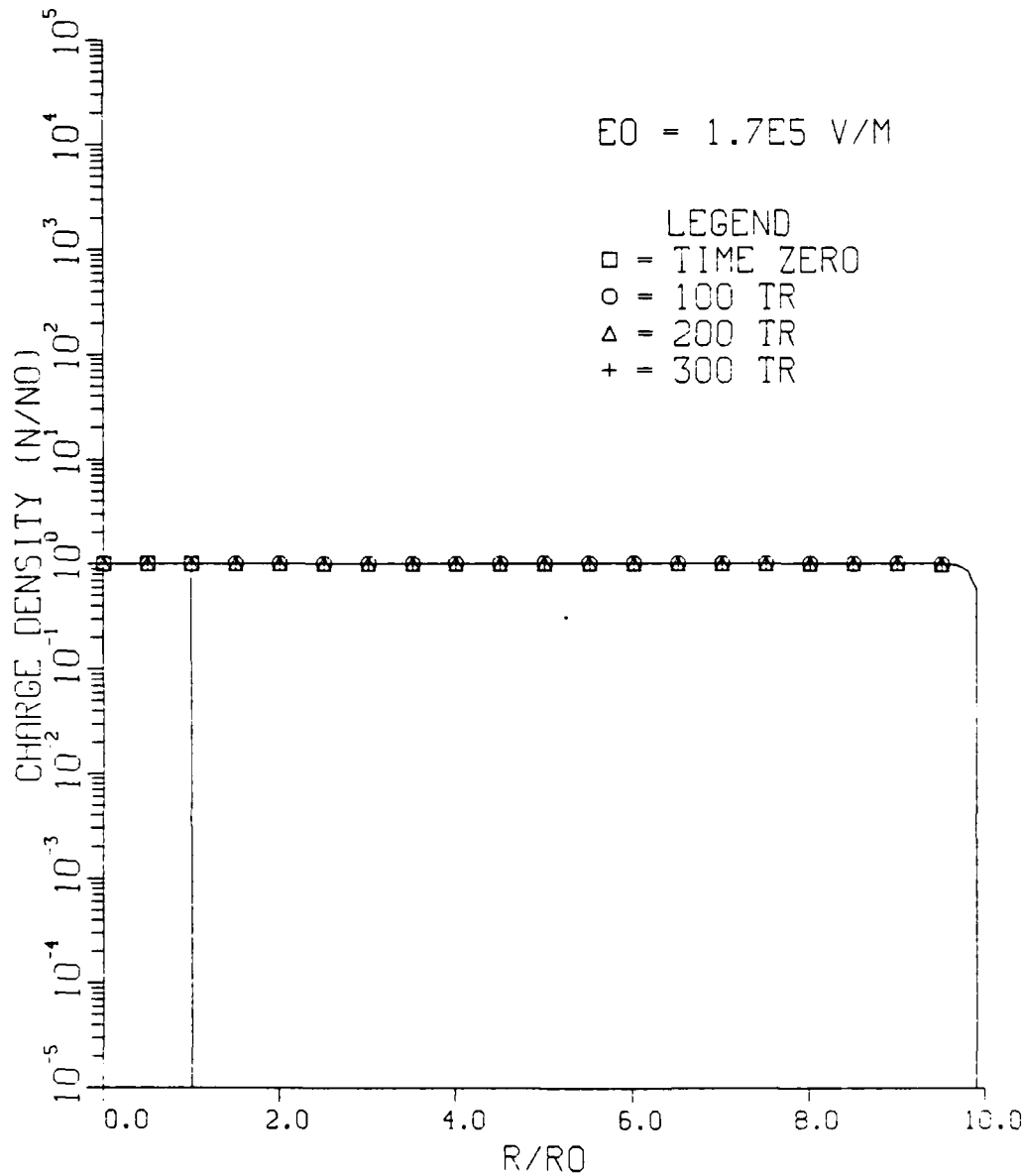
TEMPERATURE PROFILE

IN TURBULENT FLOW

FIGURE 3.18

57
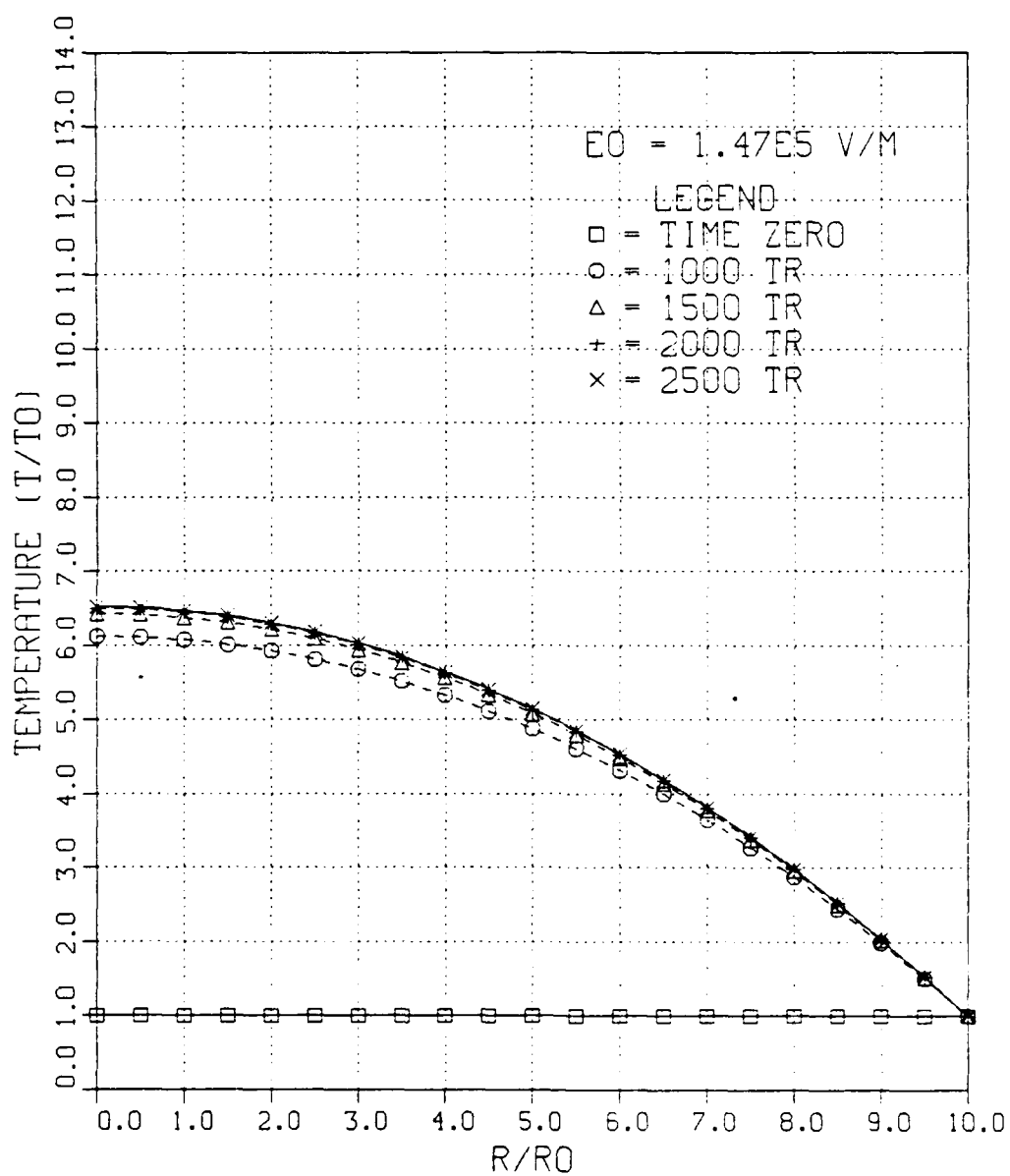
and substituting in the appropriate values into Equation

(3.7) one obtains $\hat{T}$ = 6.548 which is confirmed at $\hat{r}$ = 0 in

Figure 3.18. Note that this steady state condition is

reached at approximately 2500TR which is about the same time

period required for stabilization from the previous case

for the Bessel function initial condition profile (Figures

3.14, 3.15 and 3.16) and for the case showing the temperature

profile of a lone streamer in a non-ionized environment

(Figures 3.6 through 3.12).

Figures 3.19 through 3.28 show the results of introducing

the electric field perturbation into the model. Each plot

shows the situation of the streamer appearing in a steady

state turbulent environment and lasting for increasing

periods of time beginning at 5 recombination times and being

incremented by 5TR up to a final value of 50TR. In real time,

this would be equivalent to 0.6 msec to 6.0 msec. For these

plots, E_o = 1.47E5 V/m which produces an E/N of 6.0E-17

V-cm^2. This case was chosen for example purposes since the

steady state temperature profile is convenient at a centerline

value of T/T_o = 6.5 and the rise due to the perturbation

reaches a maximum of about 13.75 (4125°K) at 50TR. Each figure

shows five curves, 3 are solid and 2 are dashed. The solid

lines represent the initial temperature distribution ($\hat{T}$ = 1.0),

the steady state profile, and then, the profile of the streamer

at the end of its stay time. The dashed lines are the decay-

ing, or transient, profiles produced after the appearance of

TEMPERATURE PROFILE
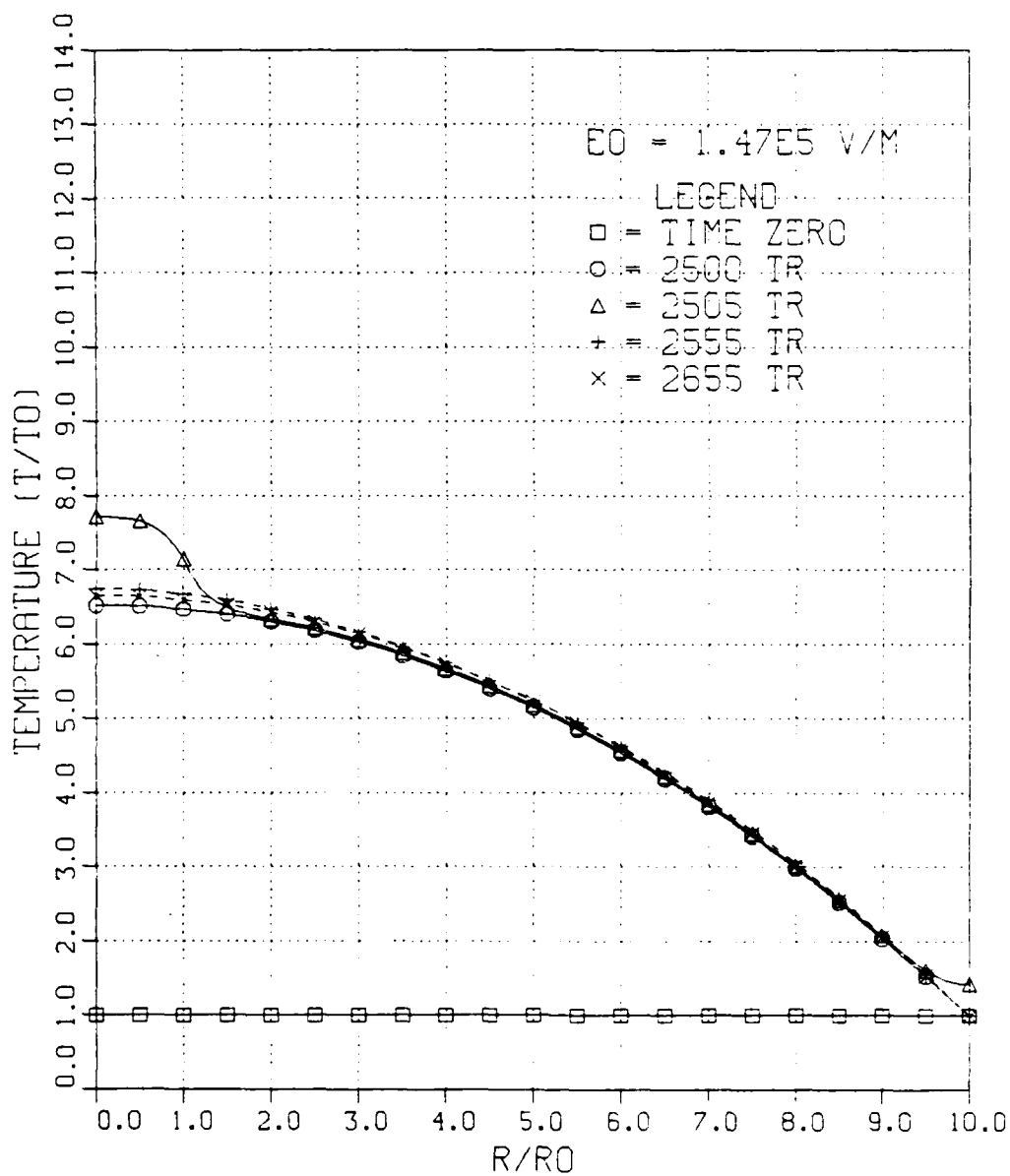
WITH STREAMER ON 5 TR

FIGURE 3.19

59

TEMPERATURE PROFILE

WITH STREAMER ON 10 TR
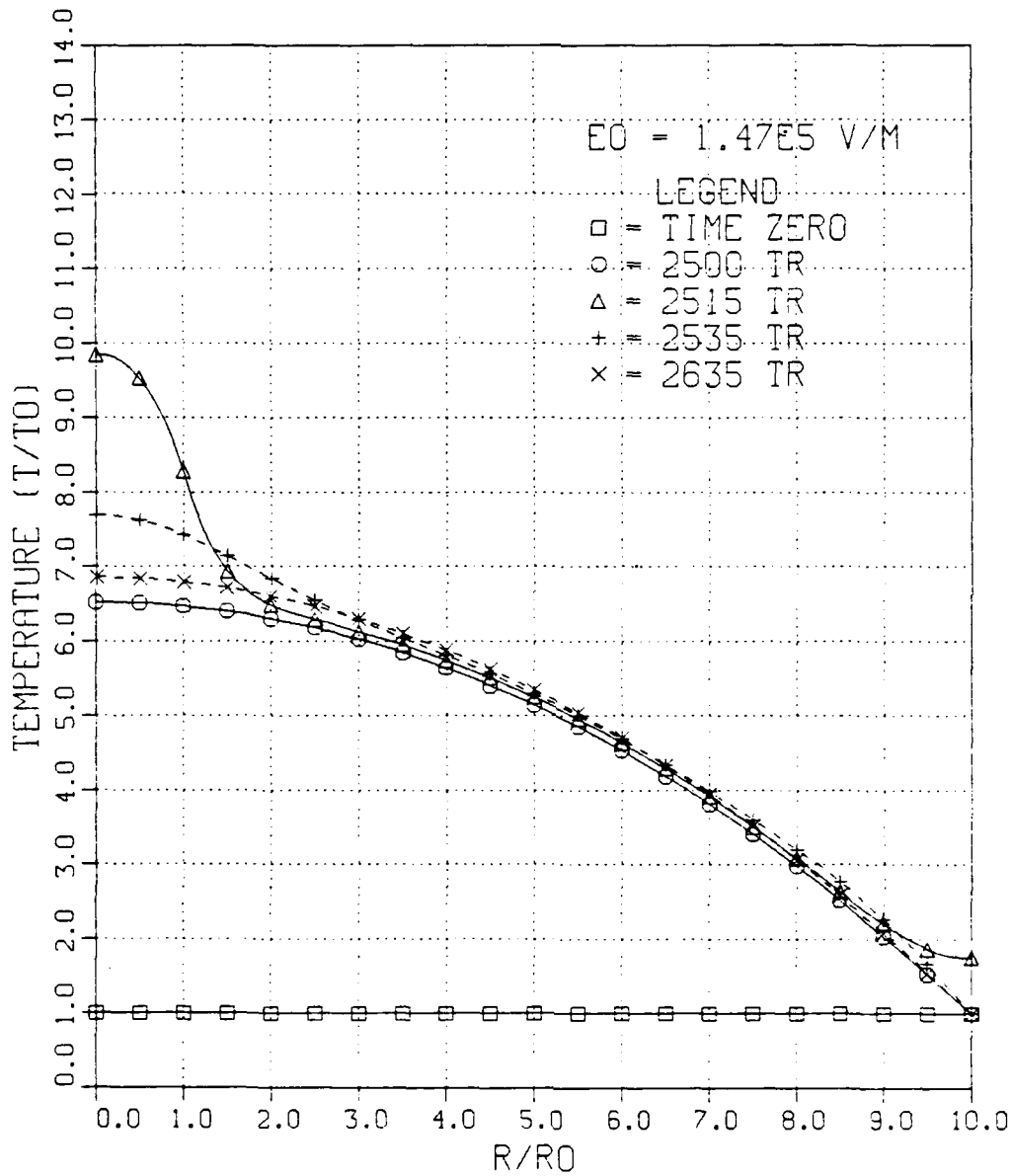
$EO = 1.47E5$ V/M

LEGEND
□ = TIME ZERO
○ = 2500 TR
△ = 2510 TR
+ = 2530 TR
× = 2630 TR

FIGURE 3.20

60

TEMPERATURE PROFILE

WITH STREAMER ON 15 TR

FIGURE 3.21

61

TEMPERATURE PROFILE

WITH STREAMER ON 20 TR

EO = 1.47E5 V/M

LEGEND
□ = TIME ZERO
○ = 2500 TR
△ = 2520 TR
+ = 2540 TR
× = 2640 TR

FIGURE 3.22

62
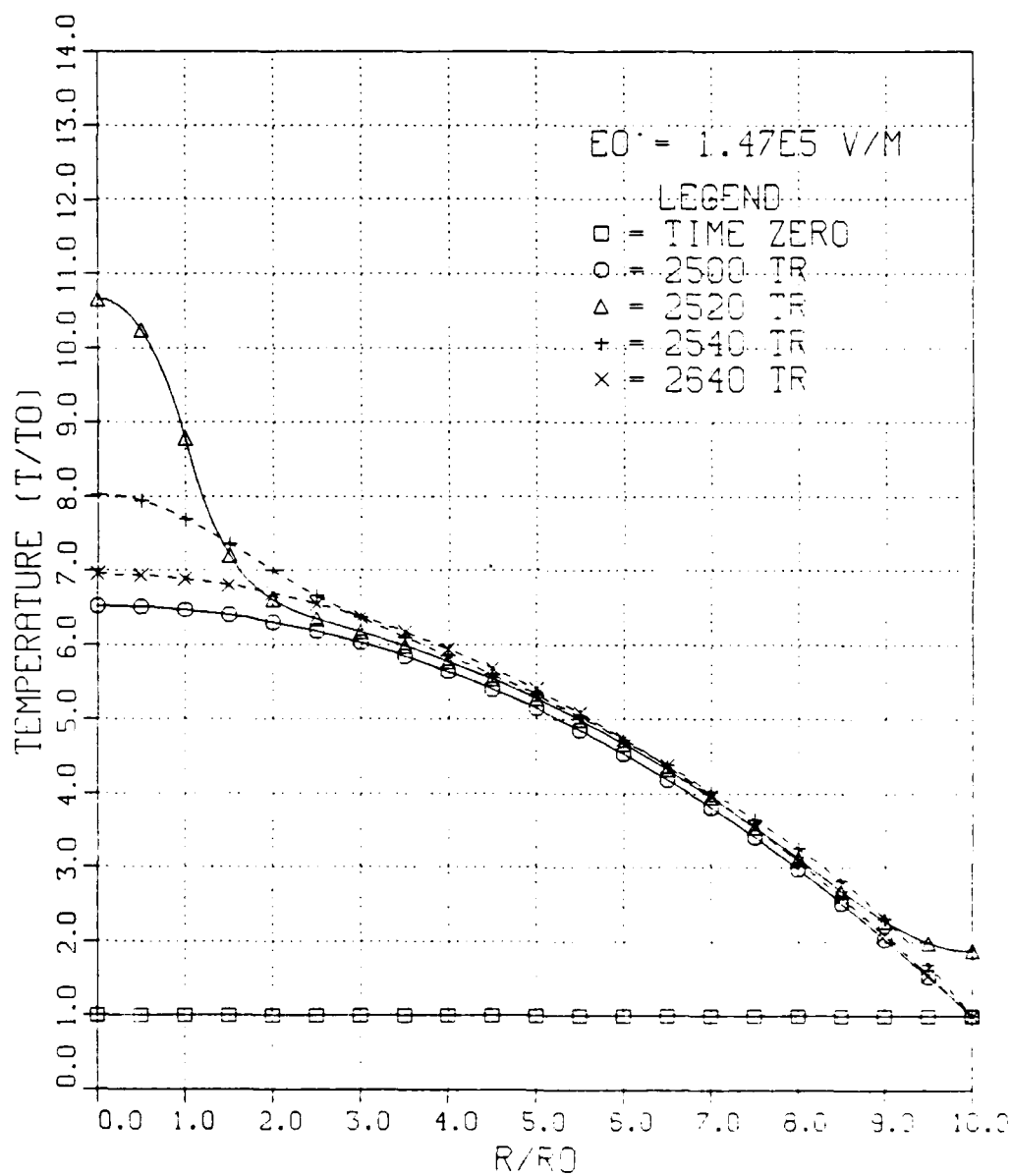
TEMPERATURE PROFILE

WITH STREAMER ON 25 TR
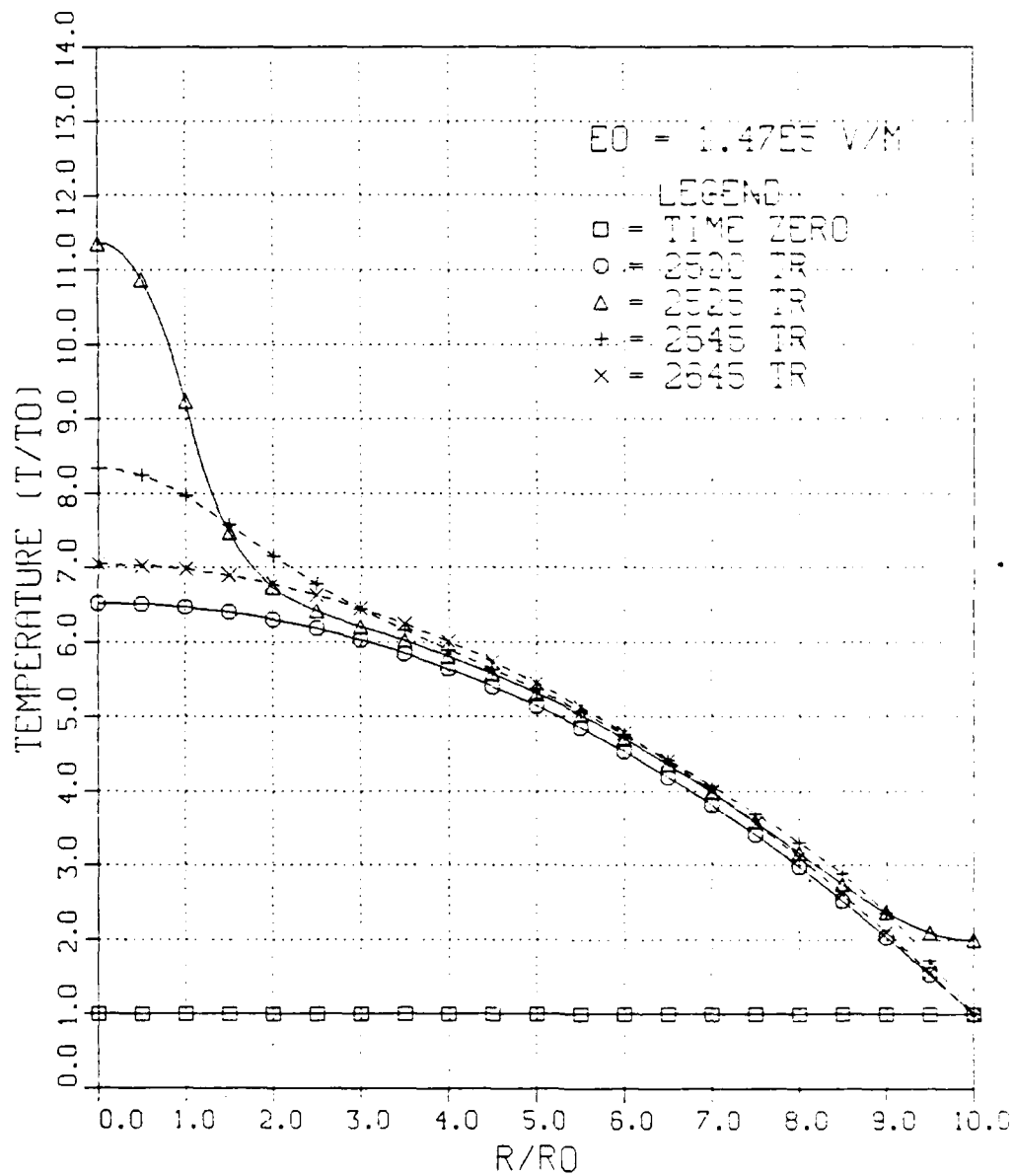
FIGURE 3.23

TEMPERATURE PROFILE

WITH STREAMER ON 30 TR

FIGURE 3.24

64
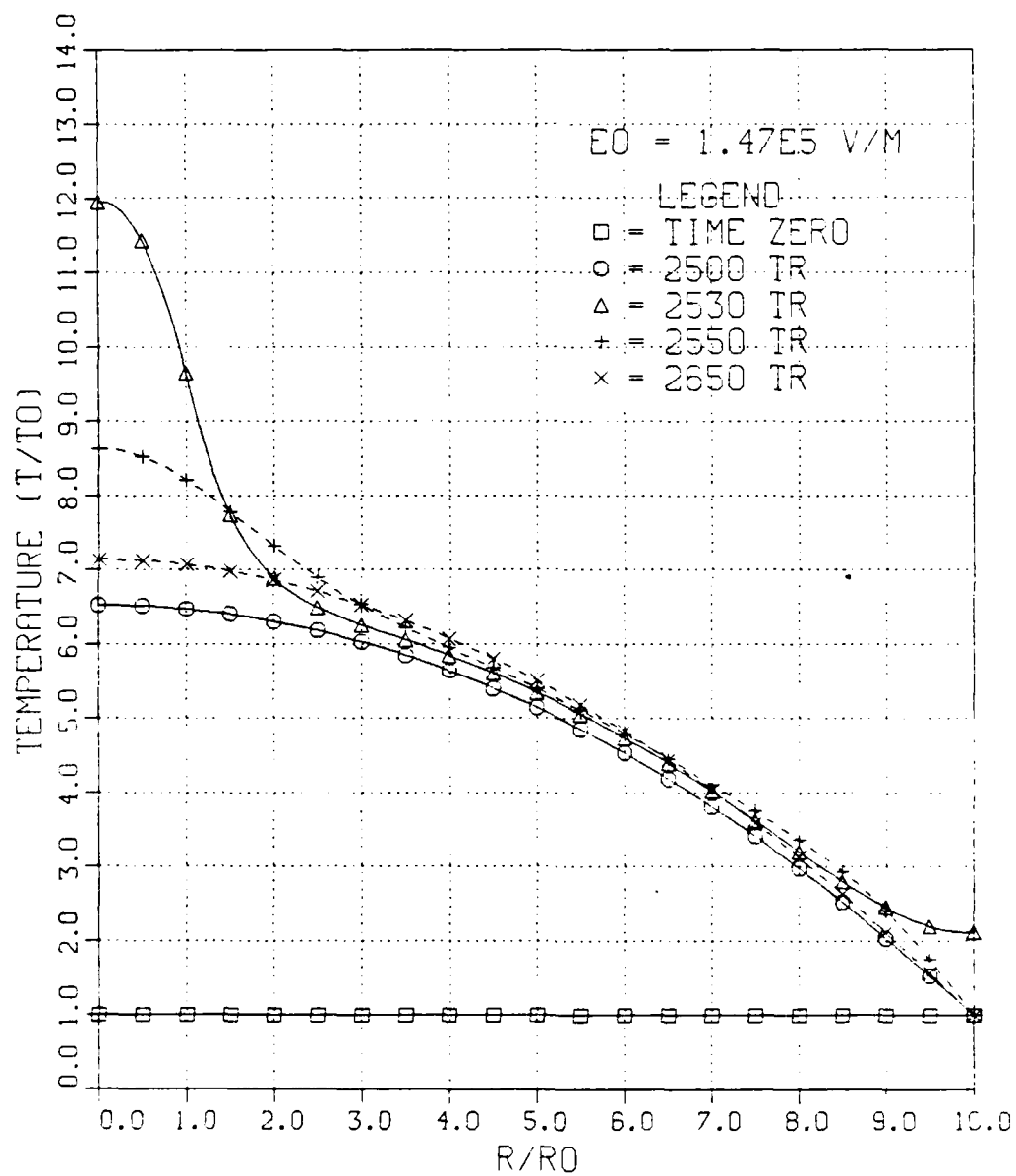
TEMPERATURE PROFILE

WITH STREAMER ON 35 TR

FIGURE 3.25

65
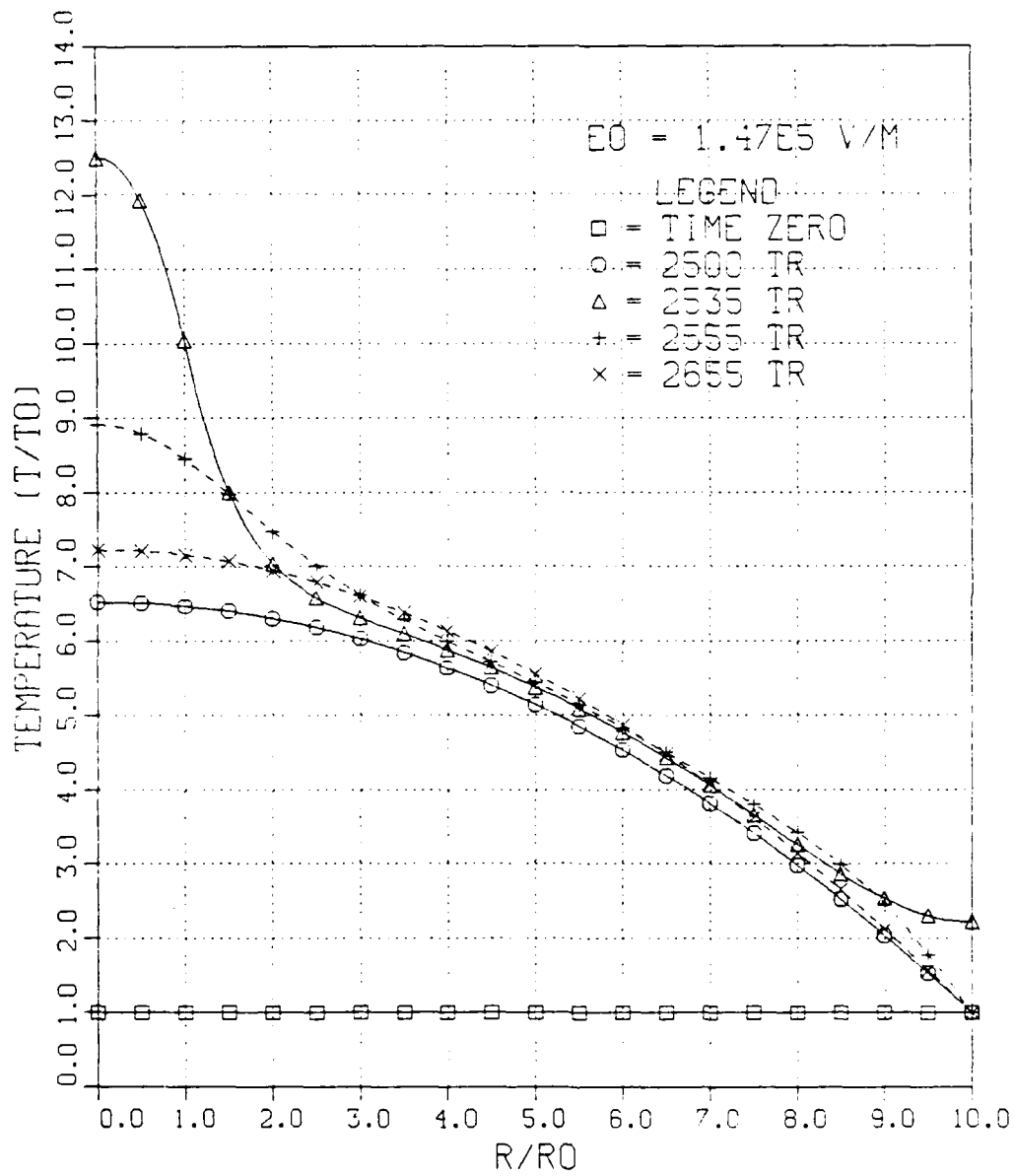
TEMPERATURE PROFILE

WITH STREAMER ON 40 TR

FIGURE 3.26

66
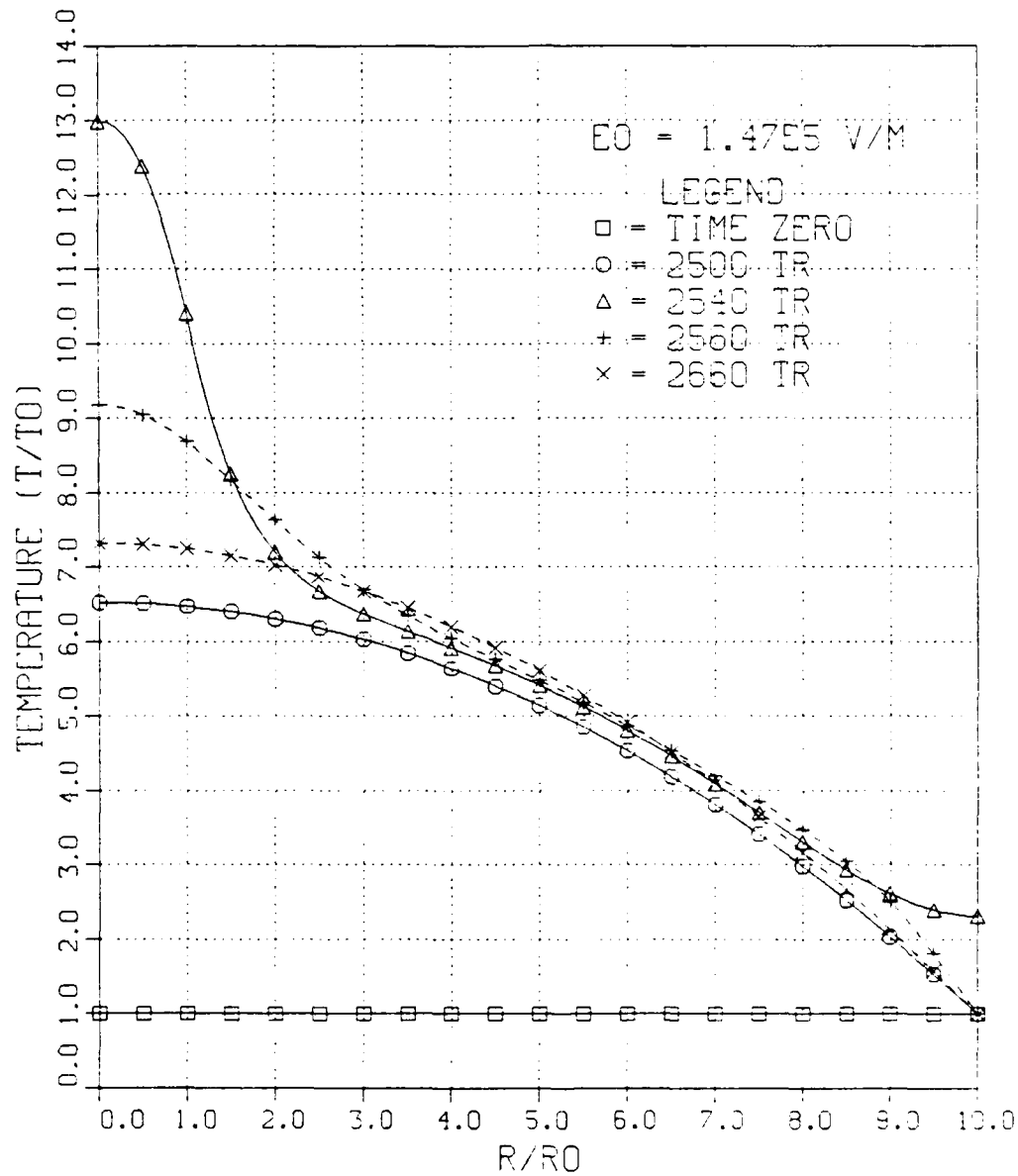
TEMPERATURE PROFILE

WITH STREAMER ON 45 TR

FIGURE 3.27

67

TEMPERATURE PROFILE

WITH STREAMER ON 50 TR

EO = 1.47E5 V/M

LEGEND
□ = TIME ZERO
○ = 2500 TR
△ = 2550 TR
+ = 2570 TR
× = 2670 TR

TEMPERATURE (T/TO)

R/RO
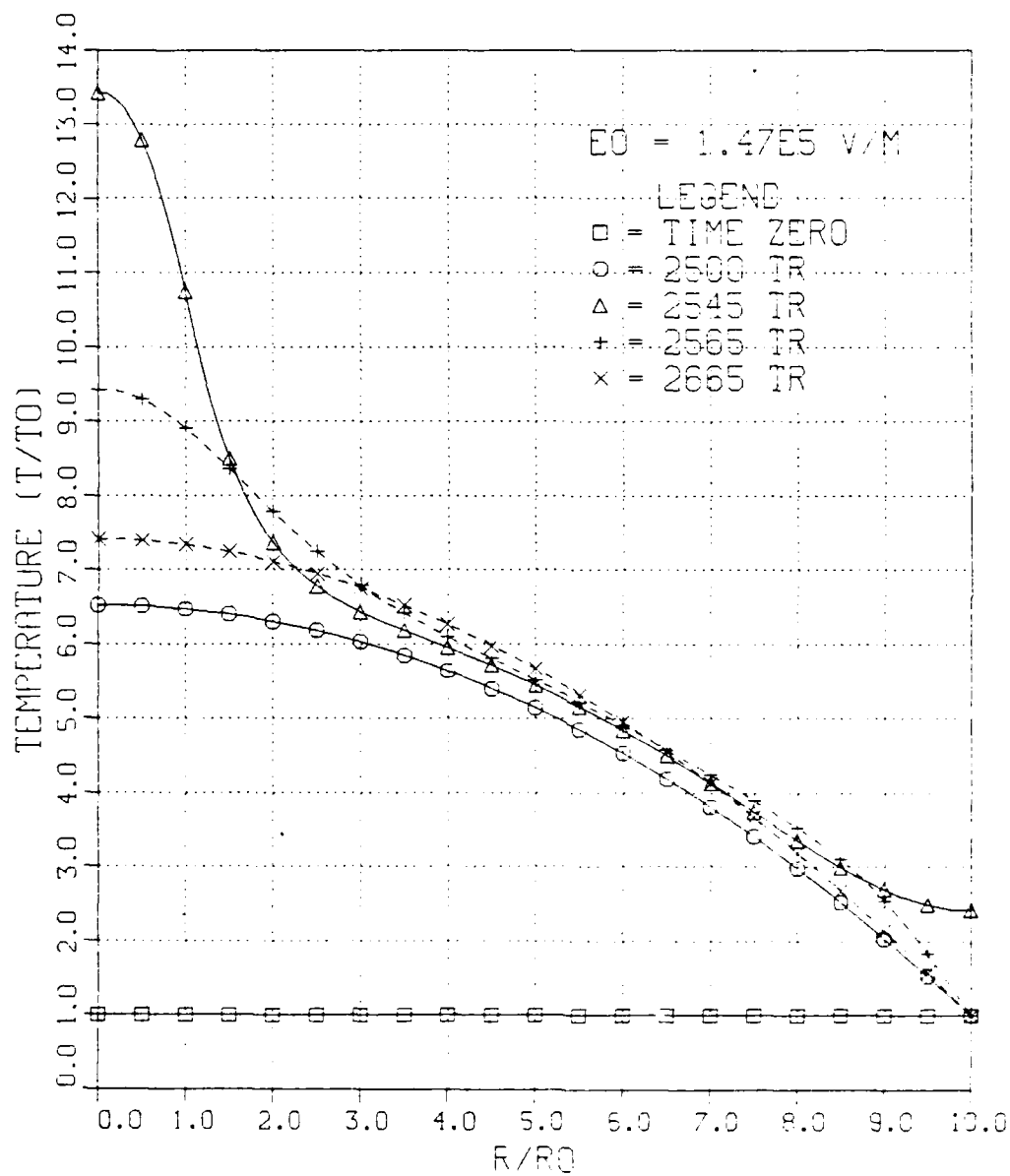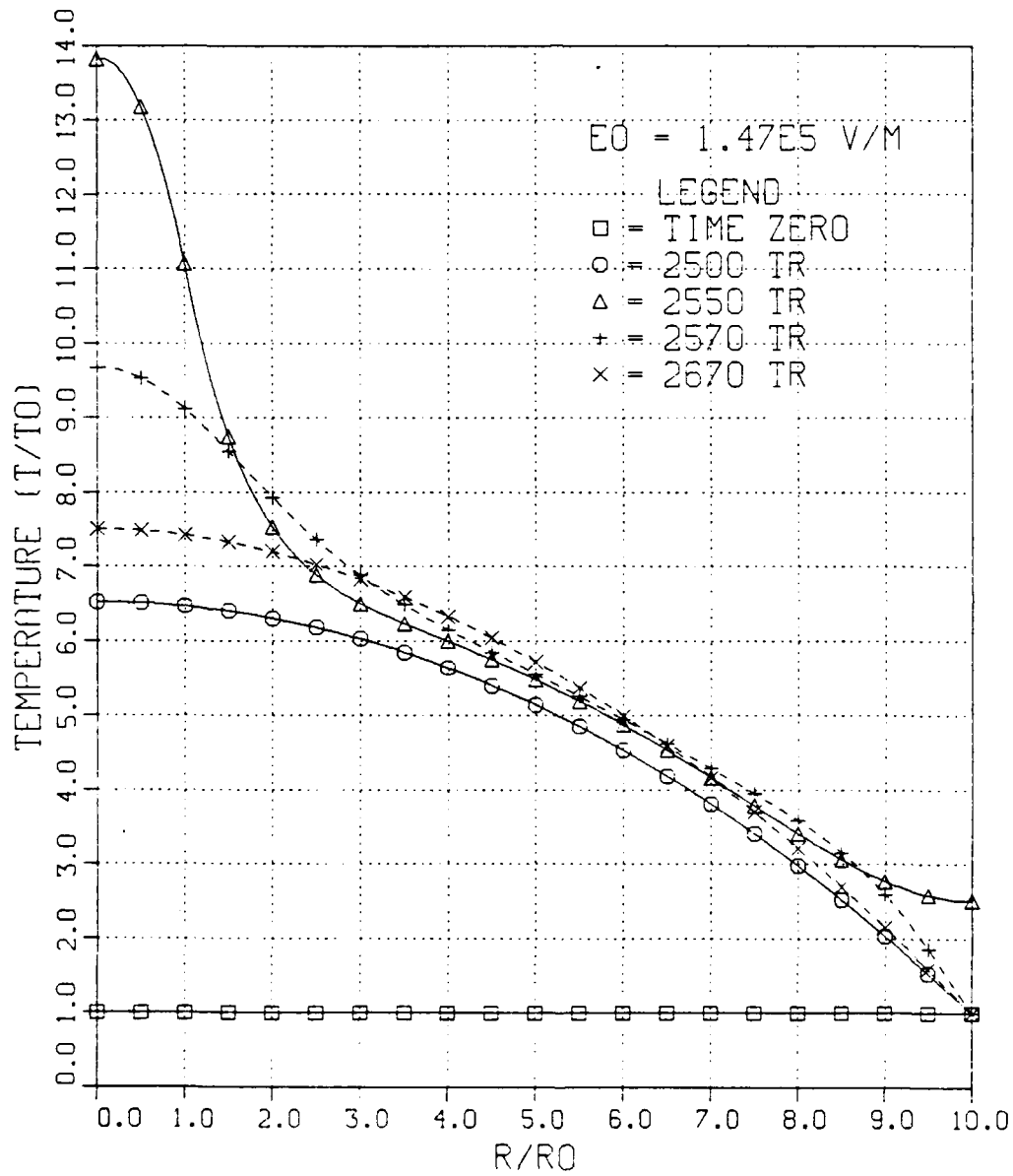
FIGURE 3.28

68

the streamer. Note that the centerline temperature drops to about half of the rise value in only 20TR and has nearly reached steady state in another 100TR.

Figures 3.29 and 3.30 show a case for a higher E/N value of 7.0E-17 V-cm^2. Here the steady state temperature profile has a centerline value of about 10.0 and then a streamer "on time" of only 8TR sends the profile above 4000°K. Cases for smaller E/N values produce lower steady state temperature profiles. The streamer effects are similar except that much longer "on times" are needed to produce much of a temperature rise. This appears consistent in the model since at the lower temperature it would take a much larger perturbation to upset the steady state condition, while at the higher energy levels a comparatively smaller perturbation is all that is required.

TEMPERATURE PROFILE

WITH STREAMER ON 5 TR

EO = 1.7E5 V/M

LEGEND
□ = TIME ZERO
○ = 2500 TR
△ = 2505 TR
+ = 2525 TR
× = 2625 TR
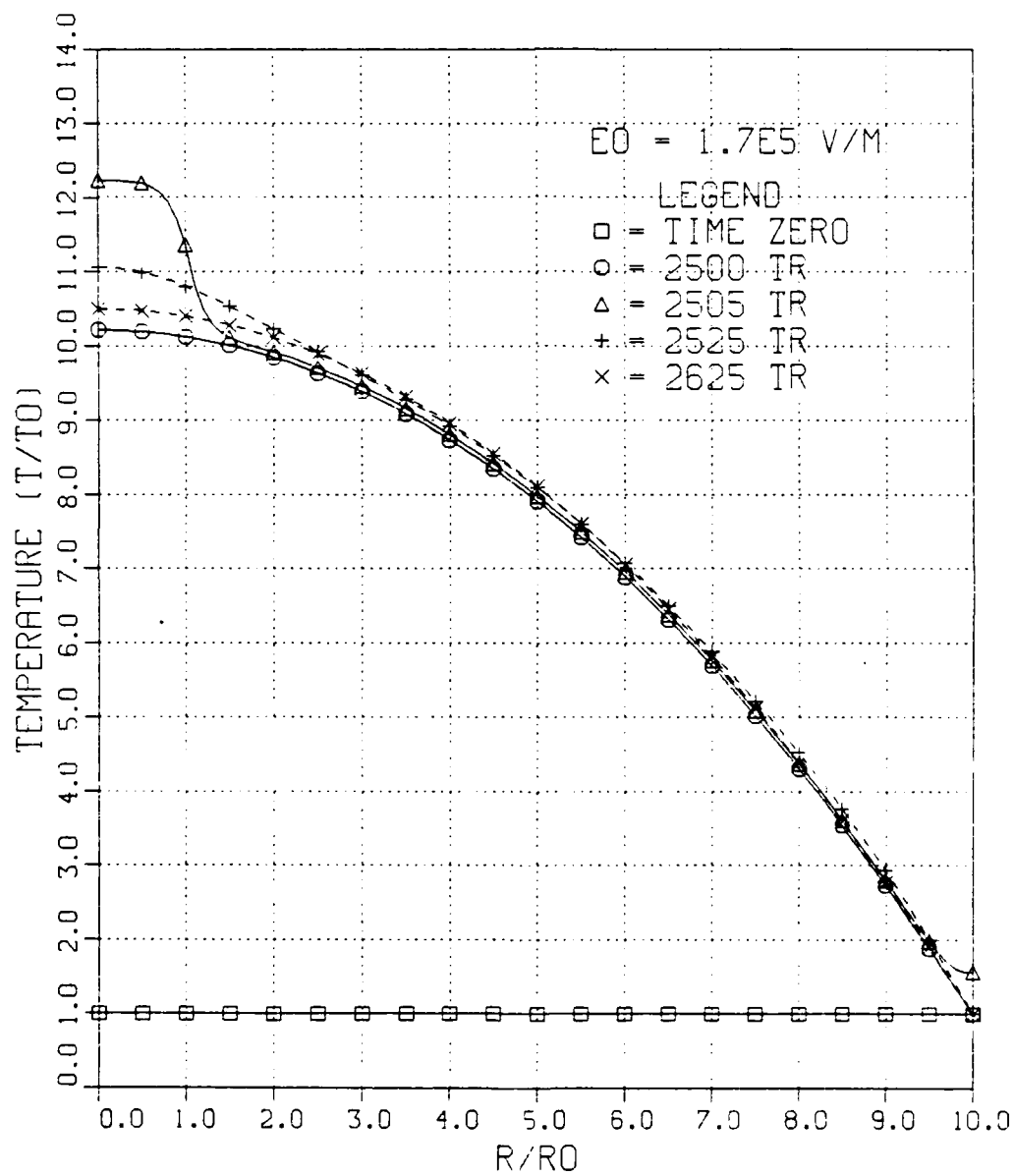
FIGURE 3.29

TEMPERATURE PROFILE

WITH STREAMER ON 9 TR

FIGURE 3.30

71
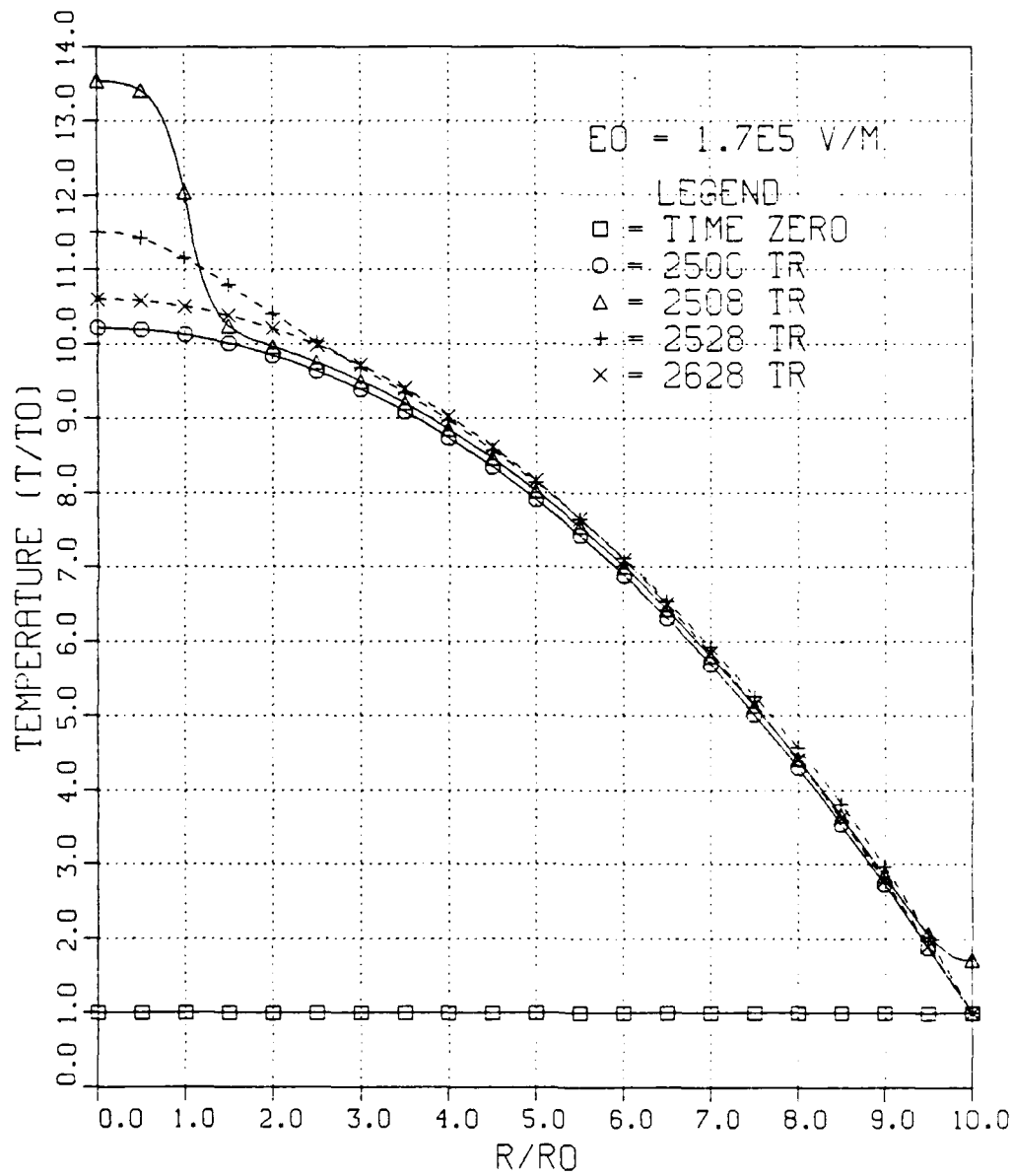
IV. CONCLUSIONS AND RECOMMENDATIONS

This thesis presents a mathematical model for describing the effects of varying levels of turbulence on glow discharge phenomena. The development is concerned only with the aftereffects of the streamer, rather than its propagation. The program that is presented in Appendix A is intended to allow the investigation of a large variety of cases of interest relating to the glow discharge. The cases discussed in the results chapter are those that appeared to develop a logical sequence of thought in attempting to answer the question of how turbulent gas flows can and do affect the stability of a glow discharge and the problem of delaying the glow-to-arc transition.

The results clearly show that the modelled turbulent flow does indeed have strong beneficial effects on the stability of the system. Additionally, the mathematical model successfully predicts within a reasonable level of uncertainty a region of stability that is consistent with the normal operating ranges of, say, an electrically pumped gas laser [Ref. 10]. The model suggests that the region for which the system maintains thermal stability is critically dependent on E/N. The research showed that a steady state temperature increase from 1200°K to 3000°K results from an increase in E/N of only 5.0E-17 V-cm^2 to 7.0E-17 V-cm^2.

One important question still remains however. That is; is there a particular level of turbulence which would have the most beneficial effect on the streamer-initiated break-down problem? This work suggests that the best situation is simply that the most desirable level is the highest level that can be maintained. However, the non-linear nature of the equations suggests that some optimal level may exist [Ref. 2]. Experimental work is probably the best means of answering this question.

The results presented here by no means exhaust the aspects of this problem that could be investigated. The intent of the program is that it can be used as a tool to analyze any desired case. Some logical next steps might be to expand on the streamer modeling aspects to investigate what happens when a follow-on streamer appears *before* the initial one has fully decayed back to the steady state condition. The "Z" and "Z_A" arrays could be set up to have two or more streamers appear at any desired location. Also, the values of "Z" and "Z_A" used in this model are somewhat arbitrary and probably have to be specified for each individual E/N case according to experimental observation. More research into the actual magnitude of the electric field perturbation may allow the model to more accurately predict the areas of instability with streamer on times that are more in agreement with experiment [Ref. 5].

Mathematically, the analysis of this problem consists of solving the continuity equation for three cases. The first

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is for the RHS of Equation (3.4) equal to less than zero. This is the non-ionized environment case. The second is for the RHS equal to zero. This is the steady state case which is what was applied to the ionized environment cases discussed. The third case, which was not applied in this thesis, is for the RHS of the continuity equation to be greater than zero. Investigation into this third case could possibly clarify some aspects of the problem. And finally, the energy equation may have eigenvalue solutions which might produce results of interest such as additional regions of stability.

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APPENDIX A

THE CHARGE PROGRAM

The Fortran program "CHARGE" is intended to be a working
tool for analyzing the glow discharge problem. It evolved
from a previously written program which solved the non-linear
continuity equation (2.1) [Ref. 2]. It is now arranged in a
somewhat different form. The program consists of a main
program section which calls up any needed combination of the
seven subroutines which perform the desired tasks. As cur-
rently written, it can compare four separate cases plus it
will display the initial conditions of either the charged
particle distribution or the temperature profile. Individual
test cases are set up by commenting out (placing a "C" in
column 1) those commands not desired. After the main program,
there are two sections. The first provides commands to print
out the data and the second provides commands to plot the
results by utilizing the "DISSPLA" system developed by the
Integrated Software Systems Corporation, San Diego, California.

The program as it appears following this discussion is
set up to produce the plot displayed in Figure 3.28. The
main program begins by defining all arrays. For this exam-
ple the arrays are set to 101 in the "i" direction which
specifies the size of the array with respect to radial posi-
tion. Since one DR, one radial step, is equal to $r_o/10$,

position "101" specifies a distance of 10 r_o perpendicular
from the centerline of the streamer. The "j" size is always
kept at 2 due to the computing scheme used in the subroutines.
After setting the array size, set the variable named
"RADIUS" equal to the same value as the "i" coordinate above.
In this case, RADIUS = 101. This determines how far in the
radial direction that the differentiation is to procede.
Next one must assign the desired values for BETA and BETAA
depending on the case at hand. The BMIN, BMAX, BAMIN and
BAMAX parameters apply when it is desired to transition the
calculation from a laminar to a turbulent environment. This
feature is designed to attempt to ensure that the calculation
accounts for the fact that the streamer initially appears so
quickly that the flow looks laminar to the streamer no matter
what the level of turbulence is. Then, after a period of
time, the charged particle distribution left behind by the
streamer has existed for sufficient time to now "see" the
turbulent effects. The time required for this transition is
estimated by noting that the characteristic diffusion time
for laminar flow is approximately 0.1 msec [Ref. 2]. Depend-
ing on the case at hand, one DT is on the order of a few
microseconds. Then, 100 DT (10TR) is on the order of a few
tenths of a millisecond. Therefore, for most cases, a
transition of 100 DT is used.

Returning to the example, one procedes by selecting the
desired initial conditions. In this example, all of the

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"N1" values are commented out since they apply to the Bessel
function initial condition profile. Four series of subrou-
tine calls are provided then to allow running of four
separate cases. Each case assigns its respective results to
the "VALX" (X = 0,1,2,3,4...) array for plotting later.

The initial conditions are assigned to VAL0 or VALT0.
Then the first subroutine call will see the initial distribu-
tion values of n while each successive call will see the
results of the previous call. If it is desired to recover
the initial conditions, call either "NZERO" or "TZERO" as
desired and the current values will then be reset to time
zero.

For this example now, "VAL1" is the result of two calls.
The first is for "TRNSTY" to transition from laminar to
turbulent conditions with time set to 101, then "DENSTY" is
called and runs for 23,900 DT. The result is the steady
state temperature distribution in turbulent flow at 2500TR.
VAL2 is then the result of calling up the subroutine "STREMR"
and running it for 500 DT with BETA and BETAA set to the
laminar values. VAL3 then results from calling up "TRNSTY"
for 100 DT and "DENSTY" for an additional 100 DT at the
turbulent values. Finally, VAL4 simply runs "DENSTY" for
another 1000 DT in turbulent conditions to check the decay-
ing temperature profile. Of course, any combination of these
subroutine calls and time values can be selected to view the
situation as desired.

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The results can then be printed out and/or plotted. To become familiar with the "DISSPLA" plot system, refer to DISSPLA Pocket Guide (current with version 9.0) by ISSCO Graphics, 10505 Sorrento Valley Rd, San Diego, California.

Some pointers of potential errors should be clarified. When selecting the desired heading lines, ensure that the last entry in the call to "HEADIN" correctly indicates the total number of heading lines. If no heading appears, this is probably the reason. Secondly, ensure that the third entry in the call to "CURVE" is equal to the "i" value on the arrays. This is the number of points (101 here) to be plotted. And finally, the second entry in the call to "LEGEND" is the number of lines of legend needed, 5, in this example.

The subroutines "DENSTY," "STREMR" and "TRNSTY" all contain essentially the same algorithm with modifications to perform their particular function. The basic idea here is that the calculation procedes from the first row of data and fills the second row with updated values. Then the first row of old data is replaced by the second row of new data by "SWITCH" and the loop increments one DT and procedes. With this scheme the program can be run as long as desired without using large values of memory storage.

The boundary conditions for the problems are met by the routine. The left boundary condition assumes that the slope is zero across the streamer centerline. So here, the new

value at $\hat{n}(2,2)$ is assigned to $\hat{n}(1,2)$ before the switch. The right boundary assumes that n is zero at the wall, that is station 101. This is met by limiting the maximum radius value to be one less than "RADIUS" (called "TEMRAD"). The routine procedes to "TEMRAD," then the last point seen by the calculation algorithm is zero. Similarly, the temperature of the right boundary is set to 1.0.

To compute the conductance parameter, gamma, that would result from any particular $\hat{n}$ distribution, call "CNDUCT" after assigning the desired $\hat{n}$ distribution to some "VALX." The single value "NN" will be returned, which is gamma.

It is hoped that the program is sufficiently self-explanatory. All variable, constant and subroutine names are defined and documentation comments are provided.

```
C     THIS PROGRAM MODELS THE EFFECTS OF TURBULENT GAS FLOW ON THE          CHA00010
C     STABILITY OF A GLOW DISCHARGE BREAKDOWN STREAMER.  TO ACCOMPLISH       CHA00020
C     THIS, THE PROGRAM CALCULATES THE CHARGED PARTICLE DENSITY AND          CHA00030
C     TEMPERATURE PROFILES FOR AN ELECTRIC DISCHARGE IN A FLOWING GAS        CHA00040
C     ENVIRONMENT. THE PROGRAM USES A FINITE DIFFERENCE SCHEME TO            CHA00050
C     NUMERICALLY SOLVE A CONTINUITY EQUATION TO DETERMINE THE CHARGED       CHA00060
C     PARTICLE DENSITY DISTRIBUTION THEN USES A TRAPIZOIDAL NUMERICAL        CHA00070
C     INTEGRATION SCHEME TO DETERMINE THE CONDUCTANCE ADDITIONALLY,          CHA00080
C     THE PROGRAM SIMULTANEOUSLY SOLVES AN ENERGY EQUATION UTILIZING         CHA00090
C     THE CALCULATED DENSITY VALUES TO DETERMINE THE TEMPERATURE             CHA00100
C     PROFILE IN THE GAS. IN THE IMMEDIATE REGION OF THE ELECTRIC            CHA00110
C     DISCHARGE STREAMER.  AND FINALLY, TO MODEL THE STREAMER, A             CHA00120
C     MOMENTARY PERTURBATION OF THE ELECTRIC FIELD OVER A RADIAL             CHA00130
C     DISTANCE FROM THE STREAMER CENTERLINE IS INTRODUCED AND THEN           CHA00140
C     THE RESULTING REACTION OF THE SYSTEM IS OBSERVED.                      CHA00150
C     BOTH THE CONTINUITY AND THE ENERGY EQUATIONS HAVE BEEN NON-            CHA00160
C     DIMENSIONALIZED.                                                       CHA00170
C                                                                            CHA00180
C     W.R.OKER - APRIL 1984                                                  CHA00190
C                                                                            CHA00200
C     NAVAL POSTGRADUATE SCHOOL, MONTEREY, CALIFORNIA                        CHA00210
C                                                                            CHA00220
C     ***************************************                                CHA00230
C     ***************************************                                CHA00240
C     ** DEFINITIONS OF VARIABLE AND CONSTANT NAMES **                       CHA00250
C     ***************************************                                CHA00260
C     ***************************************                                CHA00270
C                                                                            CHA00280
C     N  = NORMALIZED CHARGED PARTICLE DENSITY (N = N/NO)                     CHA00290
C     NO = N AT TIME ZERO                                                    CHA00300
C     NN = RESULT OF INTEGRATION, EQUALS THE CONDUCTANCE WHEN                 CHA00310
C          MULTIPLIED BY A CONSTANT, EQUAL TO GAMMA IN THE TEXT.             CHA00320
C     N1 = BESSEL FUNCTION INITIAL CONDITION FOR N/NO                        CHA00330
C     T  = NORMALIZED TEMPERATURE (T = T/TO), TO = 300 DEGREES KELVIN        CHA00340
C     DA = AMBIPOLAR DIFFUSION COEFFICIENT                                   CHA00350
C     RADIUS = SPECIFIES RADIAL DISTANCE FROM STREAMER CENTERLINE            CHA00360
C              FOR DIFFERENTIATION LIMIT                                     CHA00370
C     R  = RADIUS VALUE FOR EACH CALCULATION STEP                            CHA00380
C     RO = RADIUS OF ELECTRIC STREAMER AT TIME ZERO                          CHA00390
C     RHATO = RI COORDINATE OF RO                                           CHA00400
C     RHAT1 = RHATO + 1                                                     CHA00410
C     R1 = RADIUS VALUE FOR EACH CALCULATION STEP USED BY CONDUCTANCE        CHA00420
C          SUBROUTINE                                                       CHA00430
C     CONRAD = SPECIFIES RADIAL DISTANCE FROM STREAMER CENTERLINE FOR        CHA00440
C              CALCULATING THE CONDUCTANCE OUT TO A SPECIFIED RADIUS         CHA00450
C     TEMRAD = (RADIUS - 1) USED FOR TEMPERATURE PROFILE CALCULATION         CHA00460
C              TO INSURE PROPER BOUNDARY CONDITION                           CHA00470
C     A2 = TWO-BODY RECOMBINATION COEFFICIENT                                CHA00480
C     A3 = THREE-BODY RECOMBINATION COEFFICIENT
C     TIME = DETERMINES NUMBER OF TIME STEPS TO BE USED FOR CALCULATION
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```
         DO 10 I = 1,RHATO                                           CHA00570
         N(I,1) = 1.0                                                CHA00580
         N(I,2) = 1.0                                                CHA00590
 10   CONTINUE                                                       CHA01000
         DO 20 I = RHAT1,RADIUS                                      CHA01020
         N(I,1) = 0.0                                                CHA01030
         N(I,2) = 0.0                                                CHA01040
 20   CONTINUE                                                       CHA01050
      RETURN                                                         CHA01060
      END                                                            CHA01080
C **************************************************************     CHA01090
C * SET UP SUBROUTINE TO DEFINE INITIAL CONDITIONS FOR TEMPERATURE * CHA01100
C * PROFILE                                                       *  CHA01110
C **************************************************************     CHA01120
      SUBROUTINE TZERO(T,RADIUS)                                     CHA01130
      DOUBLE PRECISION T(RADIUS,2)                                   CHA01140
      INTEGER I,RADIUS                                               CHA01150
         DO 25 I = 1,RADIUS                                          CHA01160
         T(I,1) = 1.0                                                CHA01170
         T(I,2) = 1.0                                                CHA01180
 25   CONTINUE                                                       CHA01190
      RETURN                                                         CHA01200
      END                                                            CHA01210
C **************************************************************     CHA01220
C * SET UP SUBROUTINE TO SWITCH COMPUTED DENSITY AND TEMPERATURE *   CHA01240
C * VALUES FROM SECOND ROW BACK TO FIRST ROW FOR NEXT CALCULATION *  CHA01250
C * SEQUENCE                                                      *  CHA01260
C **************************************************************     CHA01270
      SUBROUTINE SWITCH(N,T,RADIUS)                                  CHA01280
      DOUBLE PRECISION N(RADIUS,2),T(RADIUS,2)                       CHA01290
      INTEGER I,RADIUS                                               CHA01300
         DO 30 I = 1,RADIUS                                          CHA01310
         N(I,1) = N(I,2)                                             CHA01320
         T(I,1) = T(I,2)                                             CHA01330
 30   CONTINUE                                                       CHA01340
      RETURN                                                         CHA01350
      END                                                            CHA01360
C **************************************************************     CHA01370
C * SET UP SUBROUTINE TO CALCULATE CHARGED PARTICLE DENSITY VALUES * CHA01380
C * AND TEMPERATURE PROFILE FOR ANY ARBITRARY BETA AND/OR BETAA   *  CHA01400
C * VALUE                                                         *  CHA01410
C **************************************************************     CHA01420
                                                                     CHA01430
                                                                     CHA01440
```

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      SUBROUTINE CENSTY(TIME,RADIUS,T,N,EETA,BETAA,N1)             CHA01450
      DOUBLE PRECISION A,B,C,N(RADIUS,2),NO,A2,A3,DT,DR,ALPHA,BETA,K,  CHA01460
     *T(RADIUS,2),BETAA,L,E,ETA,N1(RADIUS,2)                      CHA01470
      INTEGER I,J,K,TIME,RADIUS                                   CHA01480
      TEMRAD = RADIUS - 1                                         CHA01490
      NO = 8.02CE16                                               CHA01500
      A2 = 1.0CE-13                                               CHA01510
      A3 = 1.0E-35                                                CHA01520
      DTA = 0.1C57E-3                                             CHA01530
      DR = 0.1                                                    CHA01540
      ALPHA = (A3*NO)/A2                                          CHA01550
      DO 40 K = 1,TIME                                            CHA01560
      CALL SWITCH(N,I,RADIUS)                                     CHA01570
      DO 5C I = 2,TEMRAD                                          CHA01580
      R = (FLOAT(I)-1)/10.                                        CHA01590
      A = (1./(DR*CR))*(N((I+1),J)-(2.*N(I,J))+N((I-1),J))        CHA01600
      E = (1./(2.*R*DR))*(N((I+1),J)-N((I-1),J))                  CHA01610
      C = N(I,J)+ALPHA*N(I,J)**2.                                 CHA01620
C *********************************************                    CHA01630
C * LSE FOLLCWING VALUE FCR 'C' IF IGNIZATION TERM DESIRED *      CHA01640
C *********************************************                    CHA01650
      C = N(I,J)*N(I,J)+ALPHA*N(I,J)**2.-N(I,J)                   CHA01660
      E = (1./(2.*R*DR))*(T((I+1),J)-(2.*T(I,J))+T((I-1),J))      CHA01670
      N(I,(J+1)) = (DTA*((1./BETAA)*(A+E))-C)+N(I,J)              CHA01680
      T(I,(J+1)) = (DTA*((D+E)+N(I,J))+T(I,J)                     CHA01690
      IF(N(I,(J+1)).GT.1.E-14) GO TO 54                          CHA01700
      N(I,(J+1)) = 0.0                                           CHA01710
   54 CONTINUE                                                    CHA01720
      IF(I.NE.2) GC TO 55                                         CHA01730
      N((I-1),(J+1)) = N(I,(J+1))                                CHA01740
      T((I-1),(J+1)) = T(I,(J+1))                                CHA01750
   55 CONTINUE                                                    CHA01760
      IF(I.NE.TEMRAD) GU TO 56                                   CHA01770
      T((I+1),(J+1)) = 1.0                                        CHA01780
   56 CONTINUE                                                    CHA01790
   50 CONTINUE                                                    CHA01800
   40 CONTINUE                                                    CHA01810
      RETURN                                                      CHA01820
      END                                                         CHA01830
C *********************************************                    CHA01840
C ** SET LP SLEFGOTINE TO CALCULATE CHARGEC PARTICLE DENSITY VALUES **  CHA01850
C ** ANC TEMPEFATUFE PRCFILE FCR ANY ARBITRARY BETA AND/UR BETAA **     CHA01860
C *********************************************                    CHA01870
```

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C **********************************************************
C * VALUE DUE TO PERTURBATION OF ELECTRIC FIELD BY STREAMER *                          CHA01530
C **********************************************************                            CHA01540
C                                                                                      CHA01550
      SUBROUTINE STREMR(TIME,RADIUS,T,N,BETA,EETA,N1,Z,ZA)                              CHA01560
      DOUBLE PRECISION A,B,C,N(RADIUS,2),NO,A2,A3,DT,DR,ALPHA,BETA,R,                   CHA01970
     *T(RADIUS,2),BETAA,C,E,DTA,N1(RADIUS,2),Z(RADIUS),ZA(RADIUS)                       CHA01980
      INTEGER I,J,K,TIME,RADIUS,TEMRAD                                                  CHA01590
      TEMRAD = RADIUS - 1                                                               CHA02000
      NO = 8.520E16                                                                     CHA02010
      A2 = 1.E-13                                                                       CHA02020
      A3 = 1.E-35                                                                       CHA02030
      DT = DTA                                                                          CHA02040
      DTA = 0.1                                                                         CHA02050
      DR = 0.1                                                                          CHA02060
      CT = 1.057E-3                                                                     CHA02070
      ALPHA = (A3*NO)/A2                                                                CHA02080
      DO 60 K = 1,TIME                                                                  CHA02090
      CALL SWITCH(N,T,TEMRAD)                                                           CHA02100
      DO 70 I = 2,TEMRAD                                                                CHA02110
      R = (FLOAT(I)-1)/10.                                                              CHA02120
      A = (1./(DR*DR))*(N((I+1),J))-(2.*N(I,J))+N((I-1),J))                             CHA02130
      E = (1./(2.*R*DR))*(N((I+1),J)-N((I-1),J))                                        CHA02143
      C = N(I,J)*ALPHA*N(I,J)**3.                                                       CHA02150
C **********************************************************                            CHA02160
C * USE FOLLOWING VALUE FOR "C" IF IONIZATION TERM DESIRED *                            CHA02180
C **********************************************************                            CHA02190
      C = N(I,J)*N(I,J)+ALPHA*N(I,J)**3.-Z(I)*N(I,J)                                    CHA02200
      E = (1./(DR*DR))*(T((I+1),J)-(2.*T(I,J))+T((I-1),J))                              CHA02213
      E = (1./(2.*R*DR))*(T((I+1),J)-T((I-1),J))                                        CHA02220
      N(I,(J+1)) = (DTA*((1./BETAA)*(A+E))-C))+N(I,J)                                   CHA02230
      T(I,(J+1)) = CTA*((1./BETAA)*(D+E)+ZA(I)*N(I,J))+T(I,J)                           CHA02240
      IF (N(I,(J+1)).GT.1.E-14) GO TO 74                                               CHA02250
      N(I,(J+1)) = 0.0                                                                  CHA02260
      CONTINUE                                                                          CHA02270
   74 IF (I.NE.2) GO TO 75                                                             CHA02280
      N((I-1),(J+1)) = N(I,(J+1))                                                       CHA02200
      T((I-1),(J+1)) = T(I,(J+1))                                                       CHA02310
   75 CONTINUE                                                                          CHA02320
      IF (I.NE.TEMRAD) GO TO 76                                                        CHA02330
      T((I+1),(J+1)) = T(I,(J+1))                                                       CHA02340
   76 CONTINUE                                                                          CHA02350
   70 CONTINUE                                                                          CHA02360
   60 CONTINUE                                                                          CHA02370
      RETURN                                                                            CHA02380
      END                                                                              CHA02390
C                                                                                      CHA02400
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C ***********************************************************
C ** SET UP SUBROUTINE TO COMPUTE CHARGED PARTICLE DENSITY VALUES *
C *. AND/OR TEMPERATURE PROFILES FOR TRANSITION PHASE          *
C ***********************************************************
      SUBROUTINE TRNSTY(TIME,RADIUS,T,N,EMIN,EMAX,BAMIN,BAMAX,N1)       CHA02410
      DOUBLE PRECISION A,E,N(RADIUS,2),BETAA,D,E,ETA,A3,DT,DR,ALPHA,BMIN,CHA02420
     *BMAX,BETA,R,T(RADIUS,2),BETAA,D,E,ETA,EMIN,BAMAX,N1(RADIUS,2)      CHA02430
      INTEGER I,J,K,TIME,RADIUS,TEMRAD                                   CHA02440
      TEMRAD = RADIUS - 1                                               CHA02450
      NO = 8.12CE16                                                     CHA02460
      A2 = 1.CE-11                                                      CHA02470
      A3 = 1.E-35                                                       CHA02480
      DTA = 0.1.C57E-3                                                  CHA02490
      DR = 0.1                                                          CHA02500
      ALPHA = (A3*NO)/A2                                                CHA02510
      DO 80 K = 1,TIME                                                  CHA02520
      CALL SWITCH(N,T,RADIUS)                                           CHA02530
      DO 81 I = 2,TEMRAD                                                CHA02540
      J = 1                                                             CHA02550
      EETA = BMAX-(BMAX-BMIN)*(FLOAT(K)-1.)/(FLOAT(TIME)-1.)            CHA02560
      BETAA = BAMAX-(BAMAX-BAMIN)*(FLOAT(K)-1.)/(FLOAT(TIME)-1.)        CHA02570
      R = (FLOAT(I)-1.)/10.                                             CHA02590
      A = (1./(CR*CR))*(N(I+1,J)-(2.*N(I,J))+N((I-1),J))                CHA02600
      B = (1./(2.*R*DR))*(N(I+1,J)-N((I-1),J))**2.                      CHA02610
      C = N(I,J)*N(I,J)+ALPHA*N(I,J)                                    CHA02620
C ***********************************************************
C * USE FOLLOWING VALUE FOR "C" IF IGNIZATION TERM DESIRED  *
C ***********************************************************
      C = N(I,J)*N(I,J)+ALPHA*N(I,J)-N(I,J)                             CHA02650
      D = (1./(DR*DR))*(T(I+1,J)-(2.*T(I,J))+T((I-1),J))               CHA02660
      E = (1./(2.*R*DR))*(T(I+1,J)-T((I-1),J))                          CHA02670
      N1(I,J+1) = (DT*((1./BETAA)*(A+E))-C)+N(I,J)                      CHA02680
      T(I,J+1) = (DT*((1./BETAA)*(C+E))+N(I,J))+T(I,J)                  CHA02690
      IF (N(I,J+1).GT.1.E-14) GO TO 84                                  CHA02700
      N(I,J+1) = 0.0                                                    CHA02710
84    CONTINUE                                                          CHA02720
      IF (I.NE.2) GO TO 85                                              CHA02730
      N((I-1),(J+1)) = N(I,(J+1))                                       CHA02740
      T((I-1),(J+1)) = T(I,(J+1))                                       CHA02750
85    CONTINUE                                                          CHA02760
      N(I,(J+1)) = (DT*((1./BETAA)*(A+E))-C)+N(I,(J+1))                 CHA02770
      T(I,(J+1)) = T(I,(J+1))                                           CHA02780
      IF (I.NE.TEMRAD) GO TO 86                                         CHA02790
      N(I+1,(J+1)) = N(I,(J+1))                                         CHA02800
      T((I+1),(J+1)) = T(I,(J+1))                                       CHA02810
86    CONTINUE                                                          CHA02820
81    CONTINUE                                                          CHA02830
```

```
      80 CONTINUE                                                         CHA02890
         RETURN                                                          CHA02900
         END                                                             CHA02910
   C                                                                     CHA02920
   C     **************************************************              CHA02930
   C     * SET UP SUBROUTINE TO COMPUTE THE CONDUCTANCE PARAMETER *      CHA02940
   C     * GAMMA BY INTEGRATION                                   *      CHA02950
   C     **************************************************              CHA02960
                                                                         CHA02970
         SUBROUTINE CNDUCT(RADIUS,NN,VALX,R1)                            CHA02980
         REAL NA,VALX(RADIUS),R1(RADIUS),SUM,DR,FI                       CHA02990
         INTEGER J,RADIUS                                                CHA03000
         DR = 0.1                                                        CHA03010
         NN = 0.0                                                        CHA03020
         PI = 3.141592654                                                CHA03030
         DO 90 J = 2,RADIUS                                              CHA03040
            SUM = PI*((R1(J)*VALX(J))+(R1(J-1)*VALX(J-1)))*DR            CHA03050
            NN = NN + SUM                                                CHA03060
      90 CONTINUE                                                         CHA03070
         RETURN                                                          CHA03080
         END                                                             CHA03090
   C                                                                     CHA03100
   C     **************************************************              CHA03110
   C     * MAIN PROGRAM BEGINS *                                         CHA03120
   C     **************************************************              CHA03130
                                                                         CHA03140
         REAL RR(101),VAL0(101),VAL1(101),VAL2(101),VAL3(101),VAL4(101), CHA03150
        *VALT0(101),VALT1(101),VALT2(101),BETA,BMIN,BMAX,T(101,2),BETAA, CHA03160
         DOUBLE PRECISION N(101,2),BAMIN,Z(101),ZA(101)                  CHA03170
        *N1(101,2),BAMIN,NN1,NN2,NN3,NN4,E0                              CHA03180
         REAL NA,HI(101),NN1(101),ANI,NN2,NN3,NN4,E0                     CHA03190
         INTEGER I,TIME,RADIUS,CONRAD,RHAT0,RHAT1,RSTRM0,RSTRM1          CHA03200
   C                                                                     CHA03210
   C     **************************************************              CHA03220
   C     *. INITIALIZE VALUES. *                                         CHA03230
   C     **************************************************              CHA03240
                                                                         CHA03250
         BETA = 66.560000                                                CHA03260
         BMIN = 23..6300                                                 CHA03270
         BMAX = 66.5600                                                  CHA03280
         BETAA = 0.62520                                                 CHA03290
         BAMIN = 0.22190                                                 CHA03300
         BAMAX = 0.62520                                                 CHA03310
         E0 = 1.46EE5                                                    CHA03320
         RADIUS = 101                                                    CHA03330
         RHAT0 = 11                                                      CHA03340
         RHAT1 = RHAT0 + 1                                               CHA03350
         RSTRM0 = 11                                                     CHA03360
```

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```fortran
      RSTRM1 = RSTRMO + 1
      CONFAC = 11
      DO 50   I = 1,RSTRMO
         Z(I) = 10.0
         ZA(I) = 2.5
   5C CONTINUE
      DO 60   I = RSTRM1,RADIUS
         Z(I) = 1.0
         ZA(I) = 1.0
   6C CONTINUE
      DO 100  I = 1,RHATO
         N(I,1) = 1.0
         N(I,2) = 1.0
  10C CONTINUE
      DO 110  I = RHAT1,RADIUS
         N(I,1) = 0.0
         N(I,2) = 0.0
  11C CONTINUE
      DO 12C  I = 1,RADIUS
         T(I,1) = 1.0
         T(I,2) = 1.0
  12C CONTINUE
      NI(1,1)  = 1.000000
      NI(2,1)  = 0.99750
      NI(3,1)  = 0.99376
      NI(4,1)  = 0.99002
      NI(5,1)  = 0.98382
      NI(6,1)  = 0.97762
      NI(7,1)  = 0.96901
      NI(8,1)  = 0.96039
      NI(9,1)  = 0.94943
      NI(10,1) = 0.93846
      NI(11,1) = 0.92523
      NI(12,1) = 0.91200
      NI(13,1) = 0.85660
      NI(14,1) = 0.88120
      NI(15,1) = 0.86374
      NI(16,1) = 0.84628
      NI(17,1) = 0.82690
      NI(18,1) = 0.80752
      NI(19,1) = 0.78636
      NI(20,1) = 0.76519
      NI(21,1) = 0.74241
      NI(22,1) = 0.71962
      NI(23,1) = 0.65538
      NI(24,1) = 0.67113
      NI(25,1) = 0.64561
      NI(26,1) = 0.62008
```

```
CHA00370
CHA00380
CHA00390
CHA00400
CHA00410
CHA00420
CHA00430
CHA00440
CHA00450
CHA00460
CHA00470
CHA00480
CHA00490
CHA00500
CHA00510
CHA00520
CHA00530
CHA00540
CHA00550
CHA00560
CHA00570
CHA00590
CHA00600
CHA00610
CHA00620
CHA00630
CHA00640
CHA00650
CHA00660
CHA00670
CHA00680
CHA00690
CHA00700
CHA00710
CHA00720
CHA00730
CHA00740
CHA00750
CHA00760
CHA00770
CHA00780
CHA00790
CHA00800
CHA00810
CHA00830
CHA00840
```

```
            NI(27,1) = C.59347                                    CHA03650
            NI(28,1) = C.56685                                    CHA03660
            NI(29,1) = C.53934                                    CHA03670
            NI(30,1) = C.51182                                    CHA03680
            NI(31,1) = C.48361                                    CHA03690
            NI(32,1) = C.45540                                    CHA03500
            NI(33,1) = C.42669                                    CHA03510
            NI(34,1) = C.39758                                    CHA03520
            NI(35,1) = C.36858                                    CHA03530
            NI(36,1) = C.33958                                    CHA03540
            NI(37,1) = C.31090                                    CHA03550
            NI(38,1) = C.28181                                    CHA03560
            NI(39,1) = C.25285                                    CHA03570
            NI(40,1) = C.22389                                    CHA03580
            NI(41,1) = C.19525                                    CHA03590
            NI(42,1) = C.16660                                    CHA04600
            NI(43,1) = C.13848                                    CHA04C10
            NI(44,1) = C.11036                                    CHA04C20
            NI(45,1) = C.08295                                    CHA04C30
            NI(46,1) = C.05553                                    CHA04C40
            NI(47,1) = C.02902                                    CHA04C50
            NI(48,1) = C.00250                                    CHA04C60
            DO 210 I = 1,48                                       CHA04C70
              NI(I,2) = NI(I,1)                                   CHA04C80
        21C CONTINUE                                              CHA04C90
            DO 220 I = 49,RADIUS                                  CHA04100
              NI(I,1) = 0.0                                       CHA04110
              NI(I,2) = 0.0                                       CHA04120
        22C CONTINUE                                              CHA04130
      C *********************************************************  CHA04140
      C ** COMPUTE VARIOUS CHARGED PARTICLE DENSITY AND TEMPERATURE ** CHA04150
      C ** PROFILES AS NEEDED **                                  CHA04160
      C *********************************************************  CHA04170
      C                                                           CHA04180
            DO 300 I = 1,RADIUS                                   CHA04190
              VALC(I) = N(I,1) + 1.E-70                           CHA04200
        30C CONTINUE                                              CHA04210
            DO 310 I = 1,RADIUS                                   CHA04220
              VALC(I) = NI(I,1) + 1.E-70                          CHA04230
              NI(I,1) = NI(I,1)                                   CHA04240
        31C CONTINUE                                              CHA04250
            DO 350 I = 1,RADIUS                                   CHA04260
              VALT(I) = T(I,1)                                    CHA04270
        35C CONTINUE                                              CHA04280
            CALL DENSTY(1000100 1,RADIUS,T,N,BETA,BETAA,NI1,Z,ZA) CHA04290
      &     CALL SITEMR(3000000,RADIUS,T,N,BETA,BETAA,NI1,Z,ZA)   CHA04310
                                                                  CHA04320
```

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```
      CALL TRASTY(0000101,RADIUS,T,N,BMIN,EMAX,BAMIN,BAMAX,N1)
      CALL DENSTY(0024900,RADIUS,T,N,BMIN,EAMIN,N1)
      DO 400 I = 1,RADIUS
      VAL1(I) = N(I,1) + 1.E-70
      VALT1(I) = T(I,1) + 1.E-70
  40C CONTINUE
C
CCC   CALL NZERG(N,RADIUS,RHATO)
CC    CALL TZERG(T,RADIUS,J
CC    CALL DENSTY(0005001,RADIUS,T,N,BMIN,EAMIN,N1)
      CALL STFEMR(0000500,RADIUS,T,N,BETA,EETAA,N1,Z,ZA)
      CALL TRASTY(0009000,RADIUS,T,N,5000.,BMAX,BAMIN,BAMAX,N1)
      CALL DENSTY(0010000,RADIUS,T,N,5000.,BAMIN,N1)
      DO 410 I = 1,RADIUS
      VAL2(I) = N(I,1) + 1.E-70
      VALT2(I) = T(I,1) + 1.E-70
  41C CONTINUE
CCC   CALL NZERG(N,RADIUS,RHATO)
      CALL TZERG(T,RADIUS,J
      CALL DENSIY(0005001,RADIUS,T,N,BMIN,EAMIN,N1)
      CALL STFEMR(0000100,RADIUS,T,N,BETA,EETAA,N1,Z,ZA)
      CALL TRASTY(0009000,RADIUS,T,N,BMIN,EMAX,BAMIN,BAMAX,N1)
      CALL DENSTY(0010000,RADIUS,T,N,BMIN,EAMIN,N1)
      DO 420 I = 1,RADIUS
      VAL3(I) = N(I,1) + 1.E-70
      VALT3(I) = T(I,1) + 1.E-70
  42C CONTINUE
CCC   CALL NZERG(N,RADIUS,RHATO)
      CALL TZERG(T,RADIUS,J
      CALL DENSIY(0001000,RADIUS,T,N,BMIN,EAMIN,N1)
      CALL STFEMR(0000100,RADIUS,T,N,BETA,EETAA,N1,Z,ZA)
      CALL TRASTY(0009000,RADIUS,T,N,50.0,EMAX,BAMIN,BAMAX,N1)
      CALL DENSTY(0010000,RADIUS,T,N,50.0,EETAA,N1)
      DO 430 I = 1,RADIUS
      VAL4(I) = N(I,1) + 1.E-70
      VALT4(I) = T(I,1) + 1.E-70
  43C CONTINUE
CC******************************************************
C*  PERFCRM INTEGRATIONS FOR CONDUCTANCE CALCULATIONS *
CC******************************************************
C
      DO 499 I = 1,RADIUS
      RI(I) = (FLCAT(I)-1.)/10.
  495 CONTINUE
      CALL CNLUCT(RADIUS,NN,VAL1,R1)
      NN1 = NN
      CALL CNLCT(RADIUS,NN,VAL2,R1)
      NN2 = NN
```

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```fortran
      CALL CALCLT(RADIUS,NN,VAL3,R1)
      NN3 = NN
      CALL CALCLT(RADIUS,NN,VAL4,R1)
      NN4 = NN
C *******
C * PRINT DATA *
C *******
50C   WRITE(6,500)BETA
500   FORMAT(2X,'FOR 100TR AND BETA =',1X,F8.1)
51C   WRITE(6,510)
51C   FORMAT(2X,'FOR 400QTR AND')
      WRITE(6,551)NN1
      WRITE(6,552)NN2
      WRITE(6,553)NN3
      WRITE(6,553)NN4
550   FORMAT(2X,'EETA    =   50; CONDUCTANCE =',1X,E15.6)
551   FORMAT(2X,'EETA   =  500; CONDUCTANCE =',1X,E15.6)
554   FORMAT(2X,'EETA   = 5000; CONDUCTANCE = ',1X,E15.6)
553   FORMAT(2X,'EETA   = 50000; CONDUCTANCE = ',1X,E15.6)
56C   WRITE(6,560)N1
56C   FORMAT(6(1X,F10.6))
      WRITE(6,600)VALL0
      WRITE(6,600)VALL1
      WRITE(6,600)VALL2
      WRITE(6,600)VALL3
      WRITE(6,600)VALL4
      WRITE(6,600)VALLT0
      WRITE(6,600)VALLT1
      WRITE(6,600)VALLT2
      WRITE(6,600)VALLT3
      WRITE(6,600)VALLT4
60C   FORMAT(6(1X,F15.6))
C *****
C * FLCT RESULTS *
C *****
      DO 70C I = 1,RADIUS
      RR(I) = (FLOAT(I)-1.0)/10.0
70C   CONTINUE
C     CALL CCMPRS
      CALL TEKCLK
      CALL NCGREK
      CALL PAGE(8.5,11.)
      CALL PHYSCR(1.,75.,2.5)
      CALL AFEA2C(5.,6.)
```

CHAO4E10
CHAO4E20
CHAO4E30
CHAO4E40
CHAO4E50
CHAO4E70
CHAO4E80
CHAO4E90
CHAO4500
CHAO4510
CHAO4520
CHAO4530
CHAO4540
CHAO4550
CHAO4560
CHAO4570
CHAO4580
CHAO4590
CHAO4500
CHAO4C10
CHAO4C20
CHAO4C30
CHAO4C40
CHAO4C50
CHAO4C60
CHAO4C70
CHAO4C80
CHAO4C90
CHAO4100
CHAO4110
CHAO4120
CHAO4130
CHAO4140
CHAO4150
CHAO4160
CHAO4170
CHAO4180
CHAO4190
CHAO4200
CHAO4210
CHAO4220
CHAO4230
CHAO4240
CHAO4250
CHAO4260
CHAO4270
CHAO4280

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```
C
      CALL HEADIN("CHARGED PARTICLE DENSITY PROFILES",100,-2,2)      CHA0E290
C                                                                    CHA0E300
      CALL HEADIN("TEMPERATURE PROFILES",100,-2,2)                   CHA0E310
CCCC                                                                 CHA0E320
      CALL HEADIN("WITH STREAMER FLOWS",100,-2,2)                    CHA0E330
      CALL HEADIN("IN TURBULENT FLOWS",100,-2,2)                     CHA0E340
      CALL HEADIN("IN LAMINAR FLOWS",100,-2,2)                       CHA0E350
      CALL HEADIN("FOR BAMIN = 8COS",100,-2,2)                       CHA0E360
C                                                                    CHA0E370
      CALL HEADIN("BESSEL FUNCTION INITIAL CONDITIONS",100,-2,3)     CHA0E380
      CALL HEADIN("WITH IONIZATION TERMS",100,-2,2)                  CHA0E390
      CALL XNAME("R/RO$",100)                                        CHA0E400
      CALL YNAME("CHARGE DENSITY (N/NO)$",100)                       CHA0E410
      CALL YNAME("TEMPERATURE (T/TO)$",100)                          CHA0E420
      CALL GRAF(0.0,1.00,10.0,0.000,01.0,14.00)                      CHA0E430
C                                                                    CHA0E440
      CALL GRID(1,1)                                                 CHA0E450
      CALL RESET("DOT")                                              CHA0E460
      CALL YLOG(0.0,2.000,1.E-05,0.600)                             CHA0E470
      CALL MESSAG("BESSEL FUNCTIONS",100,2.8,5.6)                    CHA0E480
      CALL MESSAG("INITIAL CONDITIONS",100,2.8,5.4)                  CHA0E490
      CALL MESSAG("EO = 1.47E5 V/M$",100,3.5,5.8)                    CHA0E500
      CALL REALNO(BETA,2,4.0,5.8)                                    CHA0E510
      CALL MESSAG("BETA =$",100,3.5,5.6)                             CHA0E520
      CALL REALNO(BETAA,1,4.8,5.6)                                   CHA0E530
      CALL MESSAG("BETAA =$",100,4.5,5.6)                            CHA0E540
      CALL REALNO(EO,-2,5.0,5.4)                                     CHA0E550
      CALL MESSAG("EO =$",100,4.5,5.4)                               CHA0E560
      CALL REALNO(Z,2,4.5,5.2)                                       CHA0E570
      CALL MESSAG("Z =$",100,4.5,5.2)                                CHA0E580
      CALL REALNO(ZA,2,4.E5,5.0)                                     CHA0E590
      CALL MESSAG("ZA =$",100,4.5,5.0)                               CHA0E600
      CALL MESSAG("FIGURE 3.28$",100,2.0,-1.0)                       CHA0E610
      CALL GRACE(0)                                                  CHA0E620
      CALL CURVE(RR,VALO,101,5)                                      CHA0E630
      CALL CURVE(RR,VAL1,101,5)                                      CHA0E640
      CALL RESET("DASH")                                             CHA0E650
      CALL CURVE(RR,VAL2,101,5)                                      CHA0E660
      CALL RESET("DASH")                                             CHA0E670
      CALL CURVE(RR,VAL3,101,5)                                      CHA0E680
      CALL RESET("DASH")                                             CHA0E690
      CALL CURVE(RR,VAL4,101,5)                                      CHA0E700
      CALL RESET("DASH")                                             CHA0E710
      CALL CURVE(RR,VALTO,101,5)                                     CHA0E720
      CALL CURVE(PR,VALT1,101,5)                                     CHA0E740
      CALL RESET("DASH")                                             CHA0E760
```

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```fortran
      CALL DASH
      CALL CLRVE(RR,VALT2,101,5)
      CALL RESET("DASH")                                          CHAO5770
      CALL CLRVE(RR,VALT3,101,5)                                  CHAO5780
      CALL RESET("DASH")                                          CHAO5790
      CALL DASH                                                   CHAO5800
      CALL CLRVE(RR,VALT4,101,5)                                  CHAO5810
      CALL RESET("DASH")                                          CHAO5820
      CALL LINES(."TIME ZEROS",LP,1)                              CHAO5830
      CALL LINES(."2500 TR$",LP,2)                                CHAO5840
      CALL LINES(."2550 TR$",LP,3)                                CHAO5850
      CALL LINES(."2570 TR$",LP,4)                                CHAO5860
      CALL LINES(."2670 TR$",LP,5)                                CHAO5870
      CALL LEGEND(LP,5,2.8,4.0)                                   CHAO5880
      CALL ENDFL(0)                                               CHAO5890
      CALL DCAEPL                                                 CHAO5900
      STOP                                                        CHAO5910
      END                                                         CHAO5920
C                                                                 CHAO5930
C                                                                 CHAO5940
C                                                                 CHAO5950
```

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APPENDIX B

HP-41 PROGRAM

In the course of developing the results of Chapter III, it is necessary to perform a large number of simple but tedious calculations. The following HP-41CV program proved helpful. It should be easily adaptable to any small programmable calculator.

The program prompts the user for E/N (V-cm^2), ν_i/N (cm^3/s), N (cm^{-3}), α_2 (m^3/s) and v_d (cm/s). These data are available from Figure 2.2.

The program then displays E (V/m), N_o (m^{-3}), α_2 (m^3/s), DTA, BETAL (laminar), BETAT (turbulent), BETAAL (laminar), BETAAT (turbulent) and DT in true time in microsecs. The minimum BETA values computed are consistent with the stability criterion, Equation (2.17). It is good practice to add about 1% to these values to ensure a smooth running calculation.

| 01	LBL [Name Program]	37	STO08	73	3.199E-8
02	E/N?	38	RCL04	74	*
03	PROMPT	39	ENTER	75	STO13
04	STO01	40	RCL07	76	RCL10
05	V/N?	41	*	77	ENTER
06	PROMPT	42	1/X	78	RCL12
07	STO02	43	STO09	79	/
08	N?	44	RCL09	80	STO14
09	PROMPT	45	ENTER	81	200
10	STO03	46	10	82	ENTER
11	ALPHA2?	47	/	83	RCL14
12	PROMPT	48	STO10	84	*
13	STO04	49	1.6E-19	85	STO15
14	Vd?	50	ENTER	86	RCL08
15	PROMPT	51	RCL07	87	ENTER
16	STO05	52	*	88	RCL15
17	RCL01	53	RCL05	89	*
18	ENTER	54	*	90	RCL13
19	RCL03	55	RCL06	91	/
20	*	56	/	92	STO16
21	100	57	100	93	RCL06
22	*	58	/	94	E =
23	STO06	59	STO11	95	ARCL X
24	RCL02	60	2.223E5	96	AVIEW
25	ENTER	61	ENTER	97	STOP
26	RCL03	62	RCL11	98	RCL07
27	*	63	/	99	N0 =
28	RCL04	64	RCL06	100	ARCL X
29	/	65	X↗2	101	AVIEW
30	STO07	66	/	102	STOP
31	RCL04	67	STO12	103	RCL04
32	ENTER	68	RCL11	104	ALPHA2 =
33	RCL07	69	ENTER	105	ARCL X
34	*	70	RCL06	106	AVIEW
35	8E-3	71	X↗2	107	STOP
36	*	72	*	108	RCL14

END

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DTIC
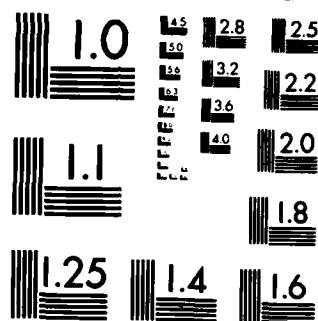
MICROCOPY RESOLUTION TEST CHART

NATIONAL BUREAU OF STANDARDS-1963-A

```
109   DTA =
110   ARCL X
111   AVIEW
112   STOP
113   RCL08
114   BETAL =
115   ARCL X
116   AVIEW
117   STOP
118   RCL16
119   BETAT =
120   ARCL X
121   AVIEW
122   STOP
123   RCL13
124   BETAAL =
125   ARCL X
126   AVIEW
127   STOP
128   RCL15
129   BETAAT =
130   ARCL X
131   AVIEW
132   STOP
133   RCL10
134   ENTER
135   1E-6
136   /
137   DTT =
138   ARCL X
139   AVIEW
140   END
```

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