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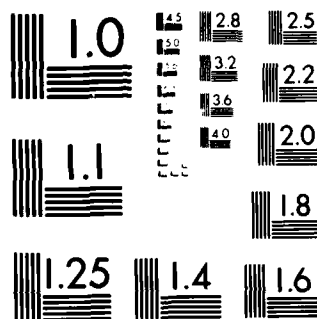
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THESIS

A SIMULATION STUDY OF MODELS
FOR
COMBINATIONS OF RANDOM LOADS

by
Moh, Jang Kab

September 1984

Thesis Advisor:

P.A. Jacobs

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ABSTRACT

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A Simulation Study of Models
for
Combinations of Random Loads

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I. DESCRIPTION OF A STOCHASTIC MODEL FOR COMBINATIONS OF RANDOM LOADS

Many physical structures are threatened by combinations of loads of varying magnitudes from various sources. For instance, bridges, piers, dams, and buildings can experience loads from wind, snow, ice, tides, earthquakes, and so forth. In many instances the total load or stress experienced by a structure varies in time in an apparently random fashion. Certain components of the loads vary rather slowly; others occur more nearly as impulses, such as those associated with winds or earthquakes. The problem is to design a structure to withstand the superposition of random loads from many sources with at least an approximately understood probability.

The purpose of this thesis is to describe and investigate certain simple but somewhat realistic probabilistic load models for use in design, and perhaps safety assessment of structures. In this thesis we confine attention to the superposition of just two load types; shock loads, and constant loads. For example, winds gusts, flash floods, and earthquakes have varying magnitudes and have relatively short durations in comparison to the times between their occurrences. These will be modelled as instantaneous shock loads. On the other hand, snow, ice, or water accumulation, or even the presence of slowly moving vehicles present loads that remain nearly constant in time, occasionally changing to new levels. These will be modeled as constant loads that change infrequently. Throughout this investigation it will be assumed that the effective stress exerted by several types of loads acting simultaneously can be expressed as a linear combination of the component loads, the load components being treated as stochastic processes.

The times between changes in magnitude of the constant load process are independently and identically distributed with distribution function H . The successive magnitudes of the constant load are independently and identically distributed with distribution function F . Given the constant load process, the shock load process is a compound Poisson process. The successive shock load magnitudes are independently and identically distributed with distribution function G . The conditional rate of arrival of shocks, given that the magnitude of the constant load process is x , is $\mu(x)$.

Let $X(t)$ be the magnitude of constant load process at time t ; and $Y(t)$ be the magnitude of shock load process at time t ; $Z(t) = X(t) + Y(t)$ is the superposition of the two loads at time t , and $M(t) = \sup_{s \leq t} Z(s)$, the maximum load combination in $[0, t]$, see figure 1.1.

Let $T_x = \{\inf t \geq 0: Z(t) > x\}$ ie, the first-passage time for the load combination to exceed a stress level x . T_x will represent the time to failure of a structure whose strength is x , and is subjected to a stress history $\{Z(t); t \geq 0\}$.

Gaver and Jacobs (1981) studied the above model for the case in which the distribution H is exponential and the shock load rate μ is a constant independent of the shock load process. Related load combination models that have been studied include Peir and Cornell (1973) [Ref. 1] Wen (1977) [Ref. 2] Pearce and Wen (1983) [Ref. 3].

Asymptotic results which appear in Gaver and Jacobs (1984) and Jacobs (1984) will be described in chapter 2 for the distribution of T_x as $x \rightarrow \infty$ and for the tail of the distribution of T_x for finite x .

In chapter 3, a simulation program for the load combination model will be described.

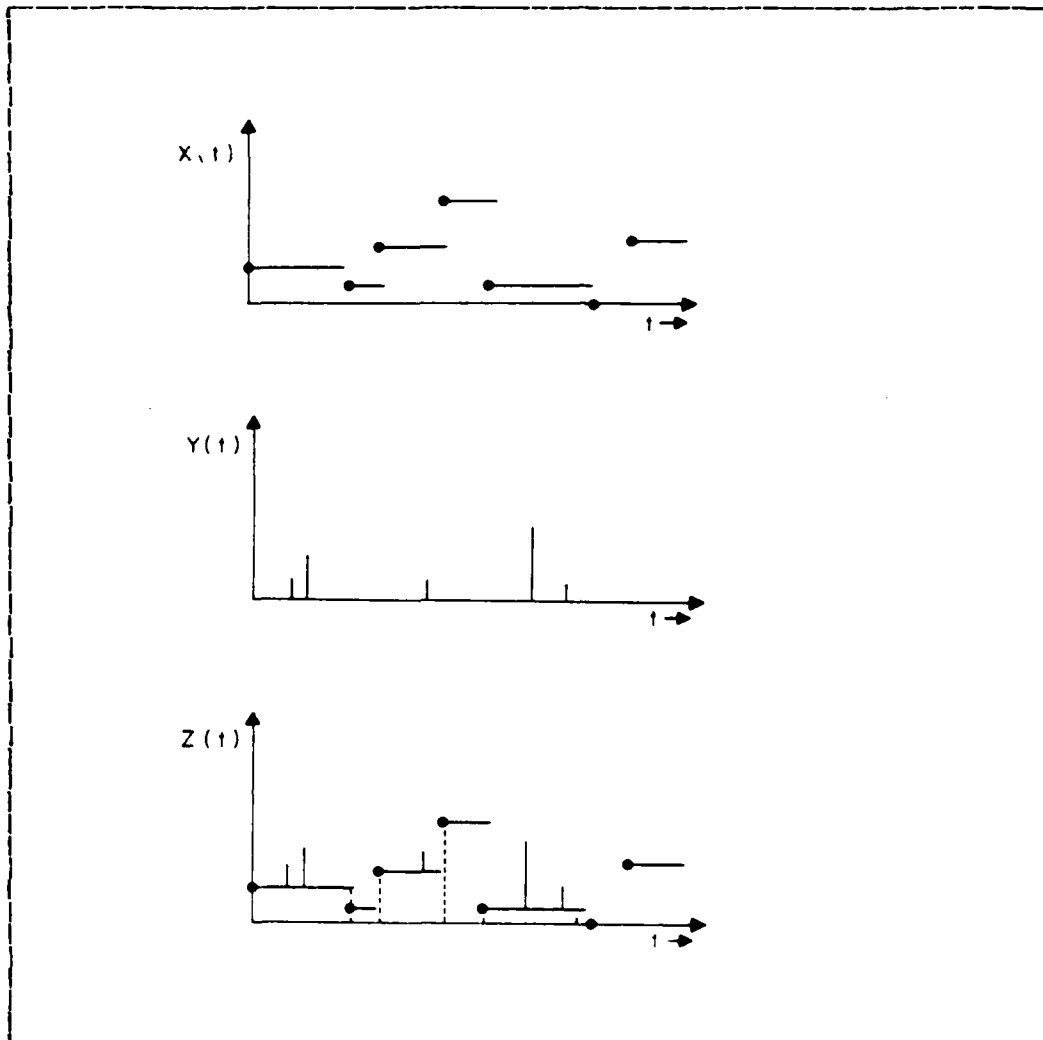


Figure 1.1 The Load Combination Process.

In chapter 4, a simulation study of the use of the asymptotic results of chapter 2 to approximate the distribution of T_x will be described.

II. ASYMPTOTIC RESULTS FOR THE DISTRIBUTION OF THE FIRST-PASSAGE TIME T_x

A. THE DISTRIBUTION OF T_x FOR LARGE x

This section summarizes some of the results found in Gaver and Jacobs (1981) [Ref. 4] for the model in which the distribution H is exponential(λ) and the shock load arrival rate μ is constant independent of the constant load process.

First, we consider the distribution function of the maximum load combination to occur during $[0, t]$, $M(t)$.

$$\begin{aligned} H(t) &= P\{M(t) \leq x\} \\ &= P\{M(t) \leq x, T_1 > t\} + P\{M(t) \leq x, T_1 < t\} \end{aligned} \quad (2-1)$$

where T_1 is the arrival time of the first shock.

A renewal argument yields;

$$\begin{aligned} P\{M(t) \leq x, T_1 > t\} &= e^{-\lambda t} \int_0^x \exp(-\mu t \bar{G}(x-y)) F(dy) \\ P\{M(t) \leq x, T_1 < t\} &= \int_0^t \lambda e^{-\lambda v} dv \int_0^x \exp(-\mu v \bar{G}(x-y)) H_x(t-v) F(dy) \end{aligned}$$

where $\bar{G}(x) = 1 - G(x)$.

Next take Laplace transforms with respect to t of (2-1).

$$\begin{aligned} \hat{H}_x(\xi) &= \int_0^\infty e^{-\xi t} H_x(t) dt \\ &= M_x(\xi) + \lambda M_x(\xi) \hat{H}_x(\xi) \end{aligned} \quad (2-2)$$

where

$$M_x(\xi) = \int_0^\infty [\xi + \lambda + \mu \bar{G}(x-y)]^{-1} F(dy) \quad (2-3)$$

Hence

$$\hat{H}_x(\xi) = \frac{M_x(\xi)}{1 - \lambda M_x(\xi)} \quad (2-4)$$

It seems to be difficult to invert $\hat{H}_x(\xi)$ for any interesting choice of the distributions $F(x)$ and $G(x)$. However, useful information can still be gleaned from equation (2-4). First, from the definition of T_x ,

$$P\{M(t) \leq x\} = P(T_x > t) \quad (2-5)$$

$$\text{Thus, } \hat{H}_x(\xi) = \int_0^\infty e^{-\xi t} P(T_x > t) dt \quad (2-6)$$

Let $\xi \rightarrow 0$, in equation (2-6) to find that

$$m(x) = E(T_x) = \hat{H}_x(0) = \frac{M_x(0)}{-\lambda M_x(0)} \quad \text{----- (2-7)}$$

The next result is the limiting property for the first time the load combination process exceeds a given level x . The limiting distribution of T_x is exponential in the sense that

$$\lim_{x \rightarrow x_0} P\{m(x)^{-1} T_x > t\} = e^{-t} \quad \text{----- (2-8)}$$

where $x_0 = \inf\{t; F * G(t) = 1\}$

The proof of this result for a more general model that includes that of chapter 2 can be found in Jacobs (1984) [Ref. 5].

B. THE TAIL OF THE DISTRIBUTION OF T_x FOR FINITE x

In this section we will summarize results for the asymptotic behavior of $P(T_x > t)$ for large t and fixed x as $t \rightarrow \infty$ for the model in which the distribution H is exponential and the shock load arrival rate is constant. More details and results for a more general model that includes that of chapter 2 can be found in Jacobs (1984).

From the equations (2-1) and (2-5)

$$P(T_x > t) = e^{-\lambda t} \int_0^x F(dy) e^{-\mu \bar{G}(x-y)t} + \lambda \int_0^t e^{-\lambda s} ds \int_0^x F(dy) e^{-\mu \bar{G}(x-y)s} P(T_x > t-s) \quad \text{----- (2-9)}$$

let

$$L(ds) = \int_0^x F(dy) \lambda e^{-(\lambda + \mu \bar{G}(x-y))s} ds \quad \text{----- (2-10)}$$

$$L(t) = \int_0^x F(dy) \frac{\lambda}{\lambda + \mu \bar{G}(x-y)} \{1 - e^{-(\lambda + \mu \bar{G}(x-y))t}\} \quad \text{----- (2-11)}$$

$$L(\infty) = \int_0^x F(dy) \frac{\lambda}{\lambda + \mu \bar{G}(x-y)} \quad \text{----- (2-12)}$$

The renewal equation (2-9) gives;

$$P(T_x > t) = g(t) + \int_0^t L(ds) H_x(t-s) \quad \text{--- --- (2-13)}$$

$$\text{where } g(t) = \int_0^t F(dy) e^{-(\lambda + \mu \bar{G}(x-y))t} \quad \text{----- (2-14)}$$

Following the development in Feller (1971; page 376) [Ref. 6] it will be assumed that there exists a number k such that

$$1 = \int_0^\infty e^{k(x)y} L(dy) \quad \text{----- (2-15)}$$

This root is unique and since the distribution of I is defective, is positive.

Let

$$L^*(ds) = e^{K(x)s} L(ds) \quad \text{----- (2-16)}$$

$$g^*(t) = e^{K(x)t} g(t) \quad \text{----- (2-17)}$$

$$F^*(T_x > t) = e^{K(x)t} H_x(t) \quad \text{----- (2-18)}$$

Then

$$F^*(T_x > t) = g^*(t) + \int_0^t F^*\{T_x > (t-s)\} L^*(ds) \quad \text{----- (2-19)}$$

It is assumed that the assumption for the key renewal theorem holds. Application of the key renewal theorem yields

$$\lim_{t \rightarrow \infty} e^{K(x)t} P(T_x > t) = \frac{1}{\delta^*} \int_0^\infty g^*(t) dt \quad \text{----- (2-20)}$$

$$\begin{aligned} \int_0^\infty g^*(t) dt &= \int_0^x F(dy) \int_0^\infty e^{K(x)t} e^{-[\lambda + \mu \bar{G}(x-y)]t} dt \\ &= \int_0^x F(dy) \frac{1}{\lambda + \mu \bar{G}(x-y) - K(x)} \quad \text{----- (2-21)} \end{aligned}$$

Rewriting the equation (2-15), the equation determining

K is

$$\begin{aligned} 1 &= \int_0^\infty e^{K(x)s} \int_0^x F(dy) \lambda e^{-[\lambda + \mu \bar{G}(x-y)]s} ds \\ &= \int_0^x F(dy) \frac{\lambda}{\lambda + \mu \bar{G}(x-y) - K(x)} \quad \text{----- (2-22)} \end{aligned}$$

and

$$\begin{aligned} \delta^* &= \int_0^\infty t \int_0^x F(dy) e^{K(x)t} \lambda e^{-[\lambda + \mu \bar{G}(x-y)]t} dt \\ &= \int_0^x F(dy) \frac{\lambda}{[\lambda + \mu \bar{G}(x-y) - K(x)]^2} \quad \text{----- (2-23)} \end{aligned}$$

The key renewal theorem implies;

$$\lim_{t \rightarrow \infty} e^{K(x)t} P(T_x > t) = \frac{1}{\lambda \int_0^x F(dy) \frac{\lambda}{[\lambda + \mu \bar{G}(x-y) - K(x)]^2}} \quad \text{----- (2-24)}$$

Examples:

It appears to be difficult to solve for K analytically in general. We will discuss the numerical computation of K for two examples. In both examples, $\lambda = \mu = 1$ and the distribution of the shock and constant load magnitudes F and G are of the form;

$$\bar{F}(x) = e^{-ax}$$

$$\bar{G}(x) = e^{-bx}$$

In example 1, $a=b=1$ and in example 2, $a/b=2$.
To compute the k for $\bar{F}(x) = e^{-ax}$, $\bar{G}(x) = e^{-bx}$,

let

$$f(x) = \int_0^x F(dy) \frac{\lambda}{\lambda + \mu \bar{G}(x-y) - x(x)} \quad \text{----- (2-25)}$$

where $\lambda = \mu = 1$.

For model 1,

$$f(x) = \frac{1}{(1-x)^2} \left\{ (1-x)(1-e^{-x}) - e^{-x} [\ln(1-x+e^{-x}) - \ln((1-x)e^{-x} + e^{-x})] \right\} \quad \text{-- (2-26)}$$

for $x < 1+e^{-x}$ and $x \neq 1$.

For $x=1$, $f(x)$ has a singular point, so by using the L'HÔPITAL RULE [Ref. 7]

$$\lim_{x \rightarrow 1} f(x) = \lim_{x \rightarrow 1} g'(x)/h'(x)$$

where $g(x)$ is the numerator and $h(x)$ is the denominator of the function $f(x)$.

For $x=1$,

$$f(1) = \frac{1}{2} (e^x - e^{-x}) \quad \text{---- (2-27)}$$

For model 2,

$$f(x) = \frac{2}{(1-x)^2} \left\{ \frac{(1-x+e^{-bx})^2}{2} - \frac{((1-x)e^{-bx} + e^{-bx})^2}{2} - ze^{-bx}((1-x)(1-e^{-bx})) \right. \\ \left. + e^{-2bx} [\ln(1-x+e^{-bx}) - \ln((1-x)e^{-bx} + e^{-bx})] \right\} \quad \text{----- (2-28)}$$

for $x < 1+e^{-bx}$ and $x \neq 1$.

For $x=1$ $f(x)$ has also singular point, so by using the same method as used in model 1;

for $x=1$

$$f(1) = \frac{a}{a+b} (e^{bx} - e^{-ax}) \quad \text{---- (2-29)}$$

Differentiation of the equation (2-25) gives

$$f'(x) = \int_0^x e^{-ay} \frac{\lambda}{[\lambda + \mu e^{-b(x-y)} - x]^2} \quad \text{----- (2-30)}$$

Equation (2-30) is greater than 0 for $0 \leq x < 1+e^{-bx}$ and $x \neq 1$. So the function $f(x)$ is a monotone increasing function in the interval $0 \leq x < 1+e^{-bx}$. Using this result, we compute the k from the equation (2-22) by applying the bisection method [Ref. 8].

Using the computed k we compute the constant γ^* such that;

$$\gamma^* = \lambda \int_0^x F(dy) \frac{\lambda}{[\lambda + \mu \bar{G}(x-y) - x]^2}$$

is the constant of equation (2-24).

For case $a=b=1$,

$$\gamma^{\#} = \frac{1}{(1-x)^3} \left\{ \left[(1-x+e^{-x}) - 2e^{-x} \ln(1-x+e^{-x}) - \frac{e^{-2x}}{1-x+e^{-x}} \right] \right. \\ \left. - \left[e^{-x}(2-x) - 2e^{-x} \ln(e^{-x}(2-x)) - \frac{e^{-2x}}{e^{-x}(2-x)} \right] \right\} \quad \text{-- (2-31)}$$

For case $a/b=2$,

$$\gamma^{\#} = \frac{2}{(1-x)^4} \left\{ \frac{(1-x+e^{-bx})^2}{2} - \frac{((1-x)e^{-bx}+e^{-bx})^2}{2} - 3e^{-bx}((1-x)(1-e^{-bx})) \right. \\ \left. + 3e^{-2bx} [\ln(1-x+e^{-bx}) - \ln((1-x)e^{-bx}+e^{-bx})] - e^{-3bx} \left[\frac{1}{(1-x)e^{-bx}+e^{-bx}} - \frac{1}{1-x+e^{-bx}} \right] \right\} \quad (2-32)$$

III. COMPUTER SIMULATION

This section describes the computer simulation model. We denote the constant load arrival rate by λ , the shock load arrival rate by μ , the distribution of constant load magnitude by $F(x)$, and the distribution of shock load magnitude by $G(x)$. The simulation model will be used to study the quality of the approximations of the distribution of T_x by the asymptotic results described in chapter 2.

The following describes the simulation algorithm. The stress levels are $x_1 < x_2 < x_3 < \dots < x_n$.

- 1, Set $T=0$, $N_0=0$.
- 2, Set $V=0$, generate a constant load magnitude, C , and duration interval T_c . Find the largest index $i > N_0$ such that $x_i \leq C$.
Put N_1 equal to that index and set
 $T_{x_i} = T$ for $N_0 < i \leq N_1$,
set $N_0 = N_1$.
- 3, Generate a shock load magnitude, S , and a time until the shock occurs, T_s .
- 4, If the shock interval does not exceed the excess life of the constant load duration, T_c , find the largest index $i > N_0$ such that $x_i \leq C+S$.
Let N_1 be that index.
Set $T_{x_i} = T+V+T_s$ for $N_0 < i \leq N_1$.
Set $N_0 = N_1$, $V=V+T_s$, $T_c=T_c-T_s$. Go to step 3.
- 5, If the shock interval exceeds the excess life time of the constant load duration,
set $T=T+T_c$, go to step 2.

One realization of the simulation is completed when one T_x is computed for each level $x_i, 1 \leq i \leq n$. Each simulation consisted of 5000 replications. Further details are

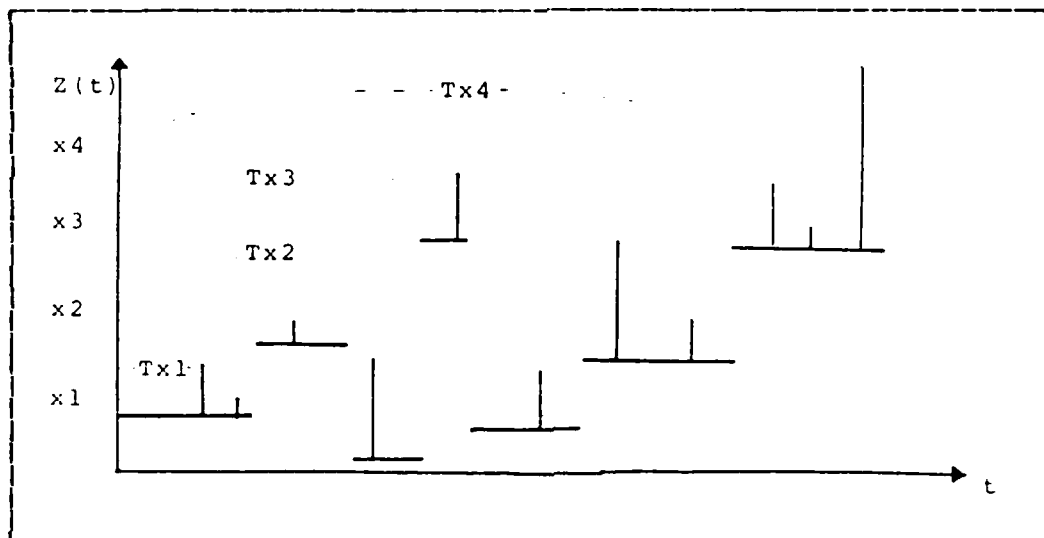


Figure 3.1 Generating T.

included in chapter 5.

The key subprograms are following. Other programs will be explained by reading from the output results.

FANDC ; generate the T_x .

COMPUT ; compute the $P(T_x=0)$, true $P(T_x=0)$, sample mean, true mean, standard deviation, coefficient of variance.

Sample probability

$$F(T_x=0) = \frac{\text{number of } T_x=0}{\text{sample size}}$$
 is obtained from the sample.

True probability

$$F(T_x=0) = 1 - P(T_x > 0) = 1 - P(M(0) \leq x) = 1 - \int_0^t F(dy) = \bar{F}(t)$$
 is obtained from the equation (2-5).

The sample mean

$$\bar{x} = \frac{1}{n} \sum_{i=1}^n T_{x_i} \quad \text{and}$$

the true mean $E(T_X)$ is obtained in special cases from the equation (2-7).

See details in Gaver and Jacobs (1981).

Sample standard deviation

$\hat{\sigma} = \frac{1}{n-1} \sum_{i=1}^n (T_{X_i} - \bar{X})$ is obtained from the sample, and

Sample coefficient of variance

$C(v) = \frac{\hat{\sigma}}{\bar{X}}$ is obtained from the sample mean and sample standard deviation.

CCMP ; compute the sample quantiles, and the quantiles of an exponential distribution with mean $E(T_X)$.

A sample quantile is

$\hat{g}_\alpha = (\alpha \times \text{sample size})$ th order statistic of sample and $g_\alpha = -E(T_X) \ln(1-\alpha)$ is the quantile of exponential distribution with mean $E(T_X)$.

CCM ; compute the quantile of the sample conditional distribution given $T_X > 0$ and the exponential distribution with mean $E(T_X)/P(T_X > 0)$.

The sample quantile of the conditional distribution

$\hat{g}_{\alpha_c} = [\alpha \times (\text{number of positive } T_X)]$ th order statistic is obtained for sample of positive T_X .

The conditional quantile of the exponential distribution with mean $E(T_X)/P(T_X > 0)$ is

$$g_{\alpha_c} = -\frac{E(T_X)}{P(T_X > 0)} \ln(1-\alpha_c).$$

SK ; compute the K such that;

$K = \{ K(x) : \int_0^x F(dy) \frac{\lambda}{\lambda + \mu \hat{g}(x-y) - K} \}$ by applying the bisection method.

INT ; compute the constant γ^* such that ;

$$\gamma^* = \sqrt{\int_0^x F(dy) \frac{\lambda}{[\lambda + \mu \hat{g}(x-y) - K]^2}}$$

by substituting the K found in SK.

The following cases were simulated.

<u>distribution</u>	<u>case</u>	<u>λ</u>	<u>$\mu(c)$</u>	<u>$F(x)$</u>	<u>$G(x)$</u>
a, exponential (a=b=1)	1,	1	1	$1-e^{-x}$	$1-e^{-x}$
	2,	1	2	"	"
	3,	2	1	"	"
b, exponential (a=2, b=1)	1,	1	1	$1-e^{-2x}$	$1-e^{-2x}$
	2,	1	2	"	"
	3,	2	1	"	"
c, exponential with varying arrival rate	1,	1	$1/c$	$1-e^{-x}$	$1-e^{-x}$
	2,	1	e^{-c}	"	"
	3,	1	e^c	"	"
d, Pareto case	1,	1	2	$1-\frac{1}{1+x}$	$1-e^{-x}$
	2,	1	2	$1-\frac{1}{(1+x)^2}$	$1-e^{-x}$
	3,	1	2	$1-\frac{1}{1+x}$	$1-\frac{1}{1+x}$

These different settings will be referred to as case (A-1), (A-2), (A-3), (B-1), (B-2), (B-3), (C-1), (C-2), (C-3), (D-1), (D-2), (D-3) respectively throughout this paper. For each above case, we compare the asymptotic result (2-8) and the experimental results.

The next setting is to examine the exponential approximation of the tail of the distribution of T_x for finite x . The cases of identical exponential distributions,

$\bar{F}(x) = \bar{G}(x) = e^{-x}$ and different exponential distributions,

$\bar{F}(x) = e^{-ax}$ $\bar{G}(x) = e^{-bx}$ where $a/b=2$

were considered.

The following conditions are considered.

e, $\lambda = \mu = 1$ $\bar{F}(x) = \bar{G}(x) = e^{-x}$

f, $\lambda = \mu = 1$ $\bar{F}(x) = e^{-2x}$, $\bar{G}(x) = e^{-x}$

These different settings will be referred to as case (E) and case (F) respectively. For each case, compute the $\mathcal{K}$, and constant δ^2 and compare the quantiles of the asymptotic results (2-24) to the simulated data for each level of x .

IV. ANALYSIS OF THE SIMULATION RESULTS

As indicated in chapter 2 the exact distribution of T_x is difficult to obtain in general. In this chapter simulation will be used to study the accuracy of the approximations of the distribution of T_x by the exponential distributions suggested by the asymptotic results of chapter 2.

A. THE DISTRIBUTION OF T_x FOR INCREASING x

The limiting result (2-8) indicated that the distribution of T_x approaches the exponential as level x increases. This result suggests that the distribution of T_x can be approximated by an exponential distribution at least for large x . In this section this exponential approximation will be studied via a simulation.

1. Constant and Shock Load Magnitudes Have Identical Exponential Distributions.

In the first collection of three models to be considered $\bar{F}(x) = \bar{G}(x) = e^{-x}$. The times between constant load changes are exponential with parameter λ and the shock load arrival rate μ is a constant independent of the constant load process.

For these cases it is possible to derive an analytical expression for $F(T_x)$.

$$F(T_x) = \frac{e^x}{\lambda^2} \left[\frac{\lambda(1-e^{-x}) - \mu x e^{-x} + \mu e^{-x} \ln\left(\frac{\lambda + \mu}{\lambda + \mu e^{-x}}\right)}{1 + \frac{\mu}{\lambda} x - \frac{\mu}{\lambda} \ln\left(\frac{\lambda + \mu}{\lambda + \mu e^{-x}}\right)} \right] \quad \text{----- (4-1)}$$

In the Tables, $P(T_x = 0)$ refers to the simulated sample probability that $T_x = 0$; X-BAS gives the simulated

sample mean of T_x ; ST-DEV is the simulated standard deviation of T_x ; VAR(X-B) is the variance of the sample mean; COEF-V is the sample standard deviation divided by the sample mean. $P(T_x=0)$ is the theoretical probability that $T_x=0$, $\bar{F}(x)$; T_x -bar is the theoretical mean of T_x computed from the equation (4-1).

The simulated coefficient of variation in all three cases decreases as the level x increases becoming quite close to the theoretical exponential value of 1 when $x=5$.

To compare the simulated distribution of T_x to the approximating exponential distribution, quantiles from the simulated data were computed and compared to the corresponding quantiles of an exponential distribution having theoretical mean (4-1). The simulated quantiles were computed as described in chapter 3. The exponential α -quantile is given by

$$q_\alpha = -a \ln(1-\alpha)$$

where a is the mean of the exponential.

For each level of x , the first row in the tables (A.1), (A.2), (A.3) for the quantiles of the distribution of T_x gives the quantiles of the simulated data and the second row gives the corresponding exponential quantiles for an exponential distribution having theoretical mean (4-1).

One way the distribution of T_x differs from an exponential is that it has an atom at 0. In particular,

$$P(T_x=0) = \bar{F}(x) = e^{-x}$$

for the case considered here.

Quantiles were used to compare the simulated conditional distribution of T_x given $T_x > 0$ to an exponential distribution having mean $E(T_x)/P(T_x > 0)$. The simulated quantiles for the conditional distribution were computed as described in chapter 3. These quantiles were compared to those of an exponential distribution having theoretical mean $E(T_x)/P(T_x > 0)$.

In the tables (A.1), (A.2), and (A.3) for the quantiles of the conditional distribution for each level x , the first row shows the simulated quantile and the second row the corresponding approximating exponential quantile.

The quantiles of the simulated conditional distribution are much closer to their exponential approximation than the quantiles for the unconditional distribution to their exponential approximation. However the quantiles for the unconditional distribution get closer to those of their approximating exponential as x increases.

Comparing tables (A.1), (A.2), and (A.3), it appears that increasing the arrival rate of shock or the rate of change of the constant load magnitude decreases the quantiles. The change of arrival rate of shocks appears to have the greater effect.

2. Constant and Shock Load Magnitudes Have Different Exponential Distributions.

In the next three cases, the distribution of the shock load magnitudes is exponential with parameter 1 and the distribution of the constant load magnitudes is exponential with mean 0.5. The other model assumptions in tables (B.1), (B.2), (B.3) correspond to those in tables (A.1), (A.2), and (A.3).

As before, for each level x , the first row in the quantile table gives the simulated quantile and the second row gives the approximating exponential quantile using the exponential distribution with theoretical mean. Comparing the quantiles of T_x in table (B.1) with (A.1) (respectively (B.2) with (A.2) and (B.3) with (A.3)), it is seen that decreasing the mean constant load magnitudes has increased the quantiles. Further, it appears that the convergence of the distribution of T_x to an exponential is faster in the case of smaller mean constant load magnitude.

The exponential approximation to the conditional distribution of T_x given $T_x > 0$ has theoretical mean $E(T_x)/P(T_x > 0)$. Once again for small levels x , it appears that the exponential approximation to the conditional distribution of T_x is better than that for the unconditional one.

3. Arrival Rate of Shock Loads Depends on the Constant Load Magnitude

In the next 3 cases, constant load magnitudes and shock load magnitudes are exponential with mean 1. Constant load magnitudes change according to a Poisson process with rate 1. Given the constant load magnitude at time t is C , the probability a shock load will arrive in the time interval $[t, t+h]$ is $\mu(C)h + o(h)$.

In case (C-1) the conditional shock load arrival rate is $\mu(C)^+ = C$. Comparison with table (A.1) indicates that conditional shock arrival rate $\mu(C)^+ = C$ has the effect of decreasing the quantiles of T_x . Further the exponential approximation to the quantiles of T_x is not as good as in case (A-1).

The approximating exponential distribution to the conditional distribution of T_x given $T_x > 0$ has a mean of $E(T_x)/P(T_x > 0)$. The quantiles of the exponential approximation to the quantiles of the conditional distribution appear to be closer than those for the unconditional distribution.

In case (C-2) shock loads arrive with conditional arrival rate $\mu(C)^+ = \exp(C)$. Comparing the table (C.1) and (C.2), it is seen that the quantiles of the case $\mu(C)^+ = C$ are less than those for the case $\mu(C)^+ = \exp(C)$. This is because interarrival times tends to be larger for the case $\mu(C)^+ = \exp(C)$. Comparing tables (A.1) and (C.2) indicates that the exponential approximation to the quantiles of T_x is not as good as in the case $\mu=1$.

In case (C-3) the conditional shock arrival rate is $\mu(C)^+ = \exp(-C)$. Comparing tables (C.2) and (C.3) indicates that the quantiles for the case $\mu(C)^+ = \exp(-C)$ are less than those for the case $\mu(C)^+ = \exp(C)$ and furthermore the exponential approximation of T_x appears to be close for the case (C-3) than that of for the case (C-2). In all of the cases the simulated mean was used as the parameter in the exponential approximations.

4. Load Magnitudes Have a Pareto Distribution

In the next three cases, the times between constant load changes of magnitude are exponential with mean 1. Shock loads arrive according to a Poisson process with rate 2. In case (D-1) the constant load magnitudes have a Pareto distribution with parameter $\alpha=1$ and the shock load magnitudes are exponentially distributed with mean 1. Comparison with table (A.2) indicates that the Pareto constant load magnitude has the effect of decreasing the quantiles of T_x . Further the exponential approximation to the quantiles of T_x is not good in the Pareto case; (the approximating exponential has a mean of the simulated sample mean).

The approximating exponential distribution of T_x given $T_x > 0$ to the conditional distribution has a mean of $E(T_x)/P(T_x > 0)$. Once again the quantiles of exponential approximation to the quantiles of the conditional distribution appear to be closer than those for the unconditional distribution.

In case (D-2) the constant load magnitudes have a Pareto distribution with parameter $\alpha=2$. Comparing tables (D.1) and (D.2) it is seen that the quantiles of the simulated distribution of T_x for the case $\alpha=2$ are larger than those for the case $\alpha=1$. Comparing tables (D.2) and (A.2) indicate that the exponential approximation to the quantiles of T_x is not as good for the Pareto case as for the exponential case.

In case (D-3) the constant load magnitudes have Pareto distribution with parameter $\alpha=1$ and the shock load magnitudes have Pareto distribution with $\alpha=1$. In all three cases the simulated mean was used in the approximating quantiles.

B. THE EXPONENTIAL TAIL OF THE DISTRIBUTION OF T_x FOR FINITE x

In this section simulation will be used to study the exponential approximation suggested by the asymptotic result

$$\lim_{t \rightarrow \infty} \frac{P(T_x > t)}{e^{-\lambda t}} = \frac{1}{\gamma^*}.$$

This is an approximation for $P(T_x > t)$ for fixed finite x ; it should become more accurate as t becomes large.

Two cases are considered. In both cases shock loads arrive according to a Poisson process with rate 1 and constant load magnitudes change at the times of arrival of a Poisson process with rate 1. The shock load magnitudes have exponential distribution with mean 1. In case (E), the constant load magnitude distribution is exponential with mean 1; In case (F) it is exponential with mean 0.5.

Description of the computation of λ and γ^* for these two cases can be found in chapters 2,3. It can be seen from the computed value of $\lambda(x)$ and $\gamma^*(x)$ appearing in tables, (E.1) and (F.1) that $\lambda(x)$ is approaching the value $1/E(T_x)$ (where $E(T_x)$ is the theoretical mean) as x becomes larger and for the cases computed $\lambda(x) \leq \frac{1}{E(T_x)}$. Further, $\gamma^*(x)$ is approaching 1 as x becomes larger.

To study the exponential approximation to the distribution of T , quantiles from the simulated data were computed. These can be found in tables (E.2) and (F.2). For each level x , the first row presents the simulated quantile; the second row gives the approximating α -quantile computed by

$$t_\alpha = \frac{1}{\lambda(x)} [-\ln\{\gamma^*(1-\alpha)\}] \quad \text{----- (I)}$$

The third row shows the approximating α -quantile computed by

$$q_\alpha = -E(T_x) \ln(1-\alpha) \quad \text{-----} \quad (\text{II})$$

where the theoretical mean is used for $E(T_x)$.

Comparing the quantiles in tables (E.2) and (F.2) it can be seen that the approximating quantiles (I) are close to the simulated ones even for $\alpha \approx .40$. The approximating quantiles computed in (II) do less well but improve as x becomes larger. The two approximating quantiles approach one another as x becomes larger, as expected. Approximating quantile (II) has the advantage, however, of being easier to compute.

Table (G.1) presents 90 % confidence intervals for α -quantiles with $\alpha \geq 0.9$ for case (E). These confidence intervals were computed using a large sample approximation, see Conover (1980; page 111-114) [Ref. 9].

Comparison with table (E.2) suggests that the approximating quantiles approximates the simulated ones well for these values.

C. CONCLUSIONS

Approximating quantile I is always better than approximating quantile II. Approximation I approximates q_α well for α as small as 0.40. However I is more difficult to compute than II.

The performance of approximating quantile II can be improved by changing it to

$$q_\alpha^* = - \frac{E(T_x)}{P(T_x > 0)} \ln(1-\alpha)$$

and using it to approximate the quantiles of the conditional distribution of T_x given $T_x > 0$; for the models considered $P(T_x = 0)$ is easy to compute being equal to $\bar{F}(x)$.

TABLE A.1

.. INPLT PARAMETERS ..

TABLE A.2
Identical Exponential (case A-2)

X-LEV	PITX=0	T PITX=0	X-BAR	T	X-BAR	ST-DEV	VAR(X-B)	COEF-V	** INPUT PARAMETERS **				1-001POLSSOMJ 1-001EXPONENTIALJ 2-001EXPONENTIALJ 1-001EXPONENTIALJ
									CONSTANT ARRIVAL RATE	SHOCK ARRIVAL RATE	CONSTANT-LCAL MAGNITUDE	SHOCK-LCAL MAGNITUDE	
0.50	0.4708	0.4045	0.1755	0.1848	0.3127	0.620	0.0000	2.0765	0.559	0.559	0.559	0.559	0.99
1.00	0.4708	0.4045	0.1755	0.1848	0.620	0.0000	0.0000	1.4934	0.559	0.559	0.559	0.559	0.98
2.00	0.4708	0.4045	0.1755	0.1848	0.9565	0.0000	0.0000	1.2534	0.559	0.559	0.559	0.559	0.97
3.00	0.4708	0.4045	0.1755	0.1848	1.4038	0.0000	0.0000	1.1823	0.559	0.559	0.559	0.559	0.96
4.00	0.4708	0.4045	0.1755	0.1848	2.1790	0.0000	0.0000	1.0984	0.559	0.559	0.559	0.559	0.95
5.00	0.4708	0.4045	0.1755	0.1848	3.2211	0.0000	0.0000	1.0231	0.559	0.559	0.559	0.559	0.94
					4.5938	0.0000	0.0000	1.0034	0.559	0.559	0.559	0.559	0.93
					7.6883	0.0000	0.0000	1.0135	0.559	0.559	0.559	0.559	0.92
					10.7705	0.0000	0.0000	1.0255	0.559	0.559	0.559	0.559	0.91
					13.8312	0.0000	0.0000	1.0356	0.559	0.559	0.559	0.559	0.90
					16.8687	0.0000	0.0000	1.0435	0.559	0.559	0.559	0.559	0.89
					19.8887	0.0000	0.0000	1.0495	0.559	0.559	0.559	0.559	0.88
					22.8887	0.0000	0.0000	1.0538	0.559	0.559	0.559	0.559	0.87
					25.8687	0.0000	0.0000	1.0565	0.559	0.559	0.559	0.559	0.86
					28.8287	0.0000	0.0000	1.0578	0.559	0.559	0.559	0.559	0.85
					31.7687	0.0000	0.0000	1.0578	0.559	0.559	0.559	0.559	0.84
					34.6887	0.0000	0.0000	1.0565	0.559	0.559	0.559	0.559	0.83
					37.5887	0.0000	0.0000	1.0538	0.559	0.559	0.559	0.559	0.82
					40.4687	0.0000	0.0000	1.0495	0.559	0.559	0.559	0.559	0.81
					43.3287	0.0000	0.0000	1.0435	0.559	0.559	0.559	0.559	0.80
					46.1687	0.0000	0.0000	1.0356	0.559	0.559	0.559	0.559	0.79
					48.9887	0.0000	0.0000	1.0255	0.559	0.559	0.559	0.559	0.78
					51.7887	0.0000	0.0000	1.0135	0.559	0.559	0.559	0.559	0.77
					54.5687	0.0000	0.0000	1.0034	0.559	0.559	0.559	0.559	0.76
					57.3287	0.0000	0.0000	1.0034	0.559	0.559	0.559	0.559	0.75
					60.0687	0.0000	0.0000	1.0034	0.559	0.559	0.559	0.559	0.74
					62.7887	0.0000	0.0000	1.0034	0.559	0.559	0.559	0.559	0.73
					65.4887	0.0000	0.0000	1.0034	0.559	0.559	0.559	0.559	0.72
					68.1687	0.0000	0.0000	1.0034	0.559	0.559	0.559	0.559	0.71
					70.8287	0.0000	0.0000	1.0034	0.559	0.559	0.559	0.559	0.70
					73.4687	0.0000	0.0000	1.0034	0.559	0.559	0.559	0.559	0.69
					76.0887	0.0000	0.0000	1.0034	0.559	0.559	0.559	0.559	0.68
					78.6887	0.0000	0.0000	1.0034	0.559	0.559	0.559	0.559	0.67
					81.2687	0.0000	0.0000	1.0034	0.559	0.559	0.559	0.559	0.66
					83.8287	0.0000	0.0000	1.0034	0.559	0.559	0.559	0.559	0.65
					86.3687	0.0000	0.0000	1.0034	0.559	0.559	0.559	0.559	0.64
					88.8887	0.0000	0.0000	1.0034	0.559	0.559	0.559	0.559	0.63
					91.3887	0.0000	0.0000	1.0034	0.559	0.559	0.559	0.559	0.62
					93.8687	0.0000	0.0000	1.0034	0.559	0.559	0.559	0.559	0.61
					96.3287	0.0000	0.0000	1.0034	0.559	0.559	0.559	0.559	0.60
					98.7687	0.0000	0.0000	1.0034	0.559	0.559	0.559	0.559	0.59
					101.1887	0.0000	0.0000	1.0034	0.559	0.559	0.559	0.559	0.58
					103.5887	0.0000	0.0000	1.0034	0.559	0.559	0.559	0.559	0.57
					105.9687	0.0000	0.0000	1.0034	0.559	0.559	0.559	0.559	0.56
					108.3287	0.0000	0.0000	1.0034	0.559	0.559	0.559	0.559	0.55
					110.6687	0.0000	0.0000	1.0034	0.559	0.559	0.559	0.559	0.54
					112.9887	0.0000	0.0000	1.0034	0.559	0.559	0.559	0.559	0.53
					115.2887	0.0000	0.0000	1.0034	0.559	0.559	0.559	0.559	0.52
					117.5687	0.0000	0.0000	1.0034	0.559	0.559	0.559	0.559	0.51
					119.8287	0.0000	0.0000	1.0034	0.559	0.559	0.559	0.559	0.50
					122.0687	0.0000	0.0000	1.0034	0.559	0.559	0.559	0.559	0.49
					124.2887	0.0000	0.0000	1.0034	0.559	0.559	0.559	0.559	0.48
					126.4887	0.0000	0.0000	1.0034	0.559	0.559	0.559	0.559	0.47
					128.6687	0.0000	0.0000	1.0034	0.559	0.559	0.559	0.559	0.46
					130.8287	0.0000	0.0000	1.0034	0.559	0.559	0.559	0.559	0.45
					132.9687	0.0000	0.0000	1.0034	0.559	0.559	0.559	0.559	0.44
					135.0887	0.0000	0.0000	1.0034	0.559	0.559	0.559	0.559	0.43
					137.1887	0.0000	0.0000	1.0034	0.559	0.559	0.559	0.559	0.42
					139.2687	0.0000	0.0000	1.0034	0.559	0.559	0.559	0.559	0.41
					141.3287	0.0000	0.0000	1.0034	0.559	0.559	0.559	0.559	0.40
					143.3687	0.0000	0.0000	1.0034	0.559	0.559	0.559	0.559	0.39
					145.3887	0.0000	0.0000	1.0034	0.559	0.559	0.559	0.559	0.38
					147.3887	0.0000	0.0000	1.0034	0.559	0.559	0.559	0.559	0.37
					149.3687	0.0000	0.0000	1.0034	0.559	0.559	0.559	0.559	0.36
					151.3287	0.0000	0.0000	1.0034	0.559	0.559	0.559	0.559	0.35
					153.2687	0.0000	0.0000	1.0034	0.559	0.559	0.559	0.559	0.34
					155.1887	0.0000	0.0000	1.0034	0.559	0.559	0.559	0.559	0.33
					157.0887	0.0000	0.0000	1.0034	0.559	0.559	0.559	0.559	0.32
					158.9687	0.0000	0.0000	1.0034	0.559	0.559	0.559	0.559	0.31
					160.8287	0.0000	0.0000	1.0034	0.559	0.559	0.559	0.559	0.30
					162.6687	0.0000	0.0000	1.0034	0.559	0.559	0.559	0.559	0.29
					164.4887	0.0000	0.0000	1.0034	0.559	0.559	0.559	0.559	0.28
					166.2887	0.0000	0.0000	1.0034	0.559	0.559	0.559	0.559	0.27
					168.0687	0.0000	0.0000	1.0034	0.559	0.559	0.559	0.559	0.26
					169.8287	0.0000	0.0000	1.0034	0.559	0.559	0.559	0.559	0.25
					171.5687	0.0000	0.0000	1.0034	0.559	0.559	0.559	0.559	0.24
					173.2887	0.0000	0.0000	1.0034	0.559	0.559	0.559	0.559	0.23
					175.0	0.0000	0.0000	1.0034	0.559	0.559	0.559	0.559	0.22

TABLE A.3

Identical Exponential (case A-3)

X-LEV	P(T=0)	T	P(T=0)	X-ERR	T	X-BAR	ST-DEV	VAR(X-9)	CEEV-V	** INPLT PARAMETERS **				X-LEV
										CONSTANT ARRIVAL RATE =	CONSTANT LCL MAGNITUDE =	SHOCK ARRIVAL RATE =	SHOCK LCL MAGNITUDE =	
0.50	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	2.0995	0.0000	0.0000	0.0000	0.0000	0.50
1.00	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000	0.0000	0.0000	0.0000	0.0000	1.00
2.00	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.5000	0.0000	0.0000	0.0000	0.0000	2.00
3.00	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.3333	0.0000	0.0000	0.0000	0.0000	3.00
4.00	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.2500	0.0000	0.0000	0.0000	0.0000	4.00
5.00	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.2000	0.0000	0.0000	0.0000	0.0000	5.00
** QUANTILE OF TX **														
X-LEV	0.1	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.99
	0.2	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.95
	0.3	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.90
	0.4	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.85
	0.5	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.80
	0.6	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.75
	0.7	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.70
	0.8	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.65
	0.9	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.60
	0.95	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.55
	0.99	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.50
X-LEV	0.1	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.99
	0.2	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.95
	0.3	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.90
	0.4	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.85
	0.5	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.80
	0.6	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.75
	0.7	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.70
	0.8	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.65
	0.9	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.60
	0.95	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.55
	0.99	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.50

TABLE B.1

●● INPUT PARAMETERS ●●

TABLE B.2

Different Exponential (case B-2)

X-LEV		PITX=0	T PITX=0	X-BAR	T X-BAR	ST-DEV	VAR(X-B)	COEF-V	** INPUT PARAMETERS **				QUANTILE OF TX **																CONDITIONAL QUANTILE OF TX **															
									CONSTANT ARRIVAL RATE =	CONSTANT-LOAD MAGNITUDE =	SHOCK ARRIVAL RATE =	SHOCK-LOAD MAGNITUDE =	Q-99	Q-98	Q-95	Q-9	Q-8	Q-75	Q-7	Q-6	Q-5	Q-4	Q-3	Q-25	Q-2	Q-1	1.00POISSON	2.00EXPONENTIAL	1.00EXPONENTIAL	2.00EXPONENTIAL														
0.50	0.000																																											

TABLE B.3

[illegible]

TABLE C.1

Varying Arrival Rate(case C-1)

** INPUT PARAMETERS **									
CONSTANT ARRIVAL RATE = 1.00(POLISSON)		CONSTANT LEAD MAGNITUDE = 1.00(EXPONENTIAL)		CONSTANT LEAD MAGNITUDE = 1.00(EXPONENTIAL)		CONSTANT LEAD MAGNITUDE = 1.00(EXPONENTIAL)		CONSTANT LEAD MAGNITUDE = 1.00(EXPONENTIAL)	
X-LEV	P(TX=0)	T P(TX=0)	X-BUR	ST-CEV	VAR(X-B)	COEF-Y	Q-75	Q-8	Q-9
0.50	0.618	0.603	0.518	0.1955	0.0000	2.2778	0.071	0.138	0.258
1.00	0.3556	0.3421	0.3199	0.5222	0.0001	1.3382	0.019	0.0548	0.138
1.50	0.2460	0.2321	0.2155	0.9222	0.0002	1.1611	0.007	0.0238	0.0548
2.00	0.1866	0.1721	0.1580	1.5330	0.0005	1.0886	0.003	0.0128	0.0238
2.50	0.1466	0.1321	0.1180	2.5371	0.0012	1.0886	0.001	0.0068	0.0128
3.00	0.1166	0.1021	0.0880	3.7638	0.0028	1.0886	0.000	0.0038	0.0068
3.50	0.0966	0.0821	0.0680	5.1291	0.0070	1.0886	0.000	0.0018	0.0038
4.00	0.0866	0.0721	0.0580	7.1291	0.0197	1.0886	0.000	0.0008	0.0018
4.50	0.0766	0.0621	0.0480	10.1291	0.0523	1.0886	0.000	0.0003	0.0008
5.00	0.0666	0.0521	0.0380	14.1291	0.0923	1.0886	0.000	0.0001	0.0003
** CONDITIONAL QUANTILE CF TX **									
X-LEV	Q-1	Q-2	Q-25	Q-5	Q-4	Q-3	Q-2	Q-1	Q-99
0.50	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
1.00	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
1.50	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
2.00	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
2.50	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
3.00	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
3.50	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
4.00	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
4.50	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
5.00	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

TABLE C.2
Varying Arrival Rate(case C-2)

		** INPLT PARAMETERS **																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																													
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TABLE C.3
Varying Arrival Rate(case C-3)

X-LEV	P(TX=0)	T P(TX=0)	X-BAR	ST-DEV	VAR(X-B)	CCF-V	** INPLT PARAMETERS **									
							CONSTANT ARRIVAL RATE	EXP-CONS	1.00(POISSON)	1.00(POISSON)	1.00(POISSON)	1.00(POISSON)	1.00(POISSON)	1.00(POISSON)	1.00(POISSON)	1.00(POISSON)
QUANTILE OF TX **																
X-LEV	Q-1	Q-2	Q-25	Q-3	Q-4	Q-5	Q-6	Q-7	Q-75	Q-8	Q-9	Q-95	Q-98	Q-99		
0.50	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
1.50	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
2.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
2.50	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
3.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
3.50	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
4.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
4.50	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
5.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
CONCITGAL QUANTILE CF TX **																
X-LEV	Q-1	Q-2	Q-25	Q-3	Q-4	Q-5	Q-6	Q-7	Q-75	Q-8	Q-9	Q-95	Q-98	Q-99		
0.50	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
1.50	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
2.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
2.50	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
3.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
3.50	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
4.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
4.50	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
5.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

TABLE D.2

Pareto Distribution (case D-2)

** INPLT PARAMETERS **									
CONSTANT ARRIVAL RATE = 1.00 (POISSON)									
CONSTANT LOAD MAGNITUDE = 2.00 (POISSON)									
SHOCK ARRIVAL RATE = 1.00 (POISSON)									
SHOCK LOAD MAGNITUDE = 1.00 (EXPONENTIAL)									
X-LEV	P(T=0)	T P(T=0)	X-BAR	ST-DEV	VAR(X-B)	CCFF-V			
0.50	0.442	0.444	0.4813	0.4813	0.0003	1.4123	0.99	0.99	0.99
1.00	0.770	0.760	0.7701	0.7701	0.0003	1.4123	0.98	0.98	0.98
2.00	0.914	0.911	0.9143	0.9143	0.0003	1.4123	0.95	0.95	0.95
3.00	0.960	0.954	0.9601	0.9601	0.0003	1.4123	0.9	0.9	0.9
4.00	0.980	0.974	0.9801	0.9801	0.0003	1.4123	0.8	0.8	0.8
5.00	0.990	0.984	0.9901	0.9901	0.0003	1.4123	0.75	0.75	0.75
** QUANTILE OF TX **									
X-LEV	0.1	0.2	0.25	0.3	0.4	0.6	0.7	0.75	0.8
0.50	0.001	0.007	0.086	0.09	0.12	0.173	0.219	0.249	0.273
1.00	0.003	0.013	0.116	0.12	0.17	0.249	0.337	0.399	0.450
1.50	0.004	0.020	0.182	0.19	0.27	0.377	0.489	0.579	0.650
2.00	0.006	0.030	0.260	0.27	0.39	0.541	0.689	0.819	0.919
2.50	0.008	0.040	0.350	0.36	0.50	0.749	0.919	1.079	1.250
3.00	0.010	0.050	0.450	0.46	0.60	0.989	1.189	1.389	1.600
3.50	0.012	0.060	0.560	0.57	0.70	1.289	1.489	1.719	1.950
4.00	0.014	0.070	0.680	0.69	0.80	1.419	1.619	1.869	2.100
4.50	0.016	0.080	0.810	0.82	0.90	1.569	1.769	2.039	2.250
5.00	0.018	0.090	0.950	0.96	1.00	1.739	1.939	2.219	2.450
** CRITICAL QUANTILE OF TX **									
X-LEV	0.1	0.2	0.25	0.3	0.4	0.6	0.7	0.75	0.8
0.50	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
1.00	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
1.50	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
2.00	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
2.50	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
3.00	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
3.50	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
4.00	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
4.50	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
5.00	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

TABLE D.3

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TABLE E.1
 $\chi(x)$ and $\chi^*(x)$ (case E)

** INPUT DATA **
 CONSTANT - C ARRIVAL = 1.00 (FCISSCH)
 CONSTANT - L PAC = 1.00 (EXP)
 SHOCK ARRIVAL RATE = 1.00 (FCISSCH)
 SHOCK - L PAC = 1.00 (EXP)

X-LEV	P(TX=0)	TRUE PR	X-BAR	T X-BAR	ST-CEVI	VAR(X-BAR)	COEF-V
0.20	0.83350	0.81813	0.10210	0.10535	0.33369	0.0001	2.23648
0.40	0.67150	0.67032	0.21618	0.22244	0.49075	0.0004	2.27015
0.60	0.54720	0.54881	0.33313	0.35540	0.65996	0.0007	1.89720
0.80	0.44330	0.44933	0.45650	0.50763	0.83743	0.0010	1.95229
1.00	0.36300	0.36768	0.57670	0.67563	1.01630	0.0015	1.90463
1.20	0.29600	0.30115	0.69652	0.81563	1.20322	0.0021	1.85199
1.40	0.23900	0.24490	0.81645	0.94412	1.40063	0.0028	1.79218
1.60	0.18900	0.19590	0.93619	1.07103	1.60794	0.0036	1.74491
1.80	0.14500	0.15334	1.05753	1.20304	1.82529	0.0045	1.70521
2.00	0.10700	0.11040	1.17330	1.34074	2.05274	0.0056	1.67331
2.20	0.07500	0.09072	1.29462	1.48374	2.28992	0.0068	1.64849
2.40	0.04900	0.07447	1.41662	1.63231	2.53625	0.0081	1.63053
2.60	0.03200	0.06066	1.54010	1.78632	2.79179	0.0104	1.61869
3.00	0.00660	0.00996	1.85510	2.46555	5.32632	0.0284	1.06701

** $\chi(x)$, $1/E(Tx)$, $\chi^*(x)$ **

X-LEV	0.200	0.400	0.600	0.800	1.000	1.200	1.400	1.600	1.800	2.000	2.200	2.400	2.600	2.800	3.000
$\chi(x)$	1.768	1.459	1.247	1.066	0.912	0.780	0.667	0.570	0.487	0.417	0.355	0.303	0.259	0.221	0.188
$1/E(Tx)$	9.452	4.486	2.814	1.577	1.475	1.142	0.906	0.730	0.556	0.491	0.407	0.335	0.284	0.238	0.201
$\chi^*(x)$	5.542	3.254	2.264	1.523	1.670	1.502	1.386	1.303	1.238	1.193	1.155	1.125	1.102	1.085	1.068

TABLE E.2
Quantiles (case E)

***	QUANTILE OF 1 X ***															
X-LEV	Q.1	Q.2	Q.25	Q.3	Q.4	Q.5	Q.6	Q.7	Q.75	Q.8	Q.9	Q.95	Q.98	Q.99		
0.20	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.309	0.777	1.234	0.309	1.633	
	0.011	0.024	0.030	0.038	0.054	0.073	0.097	0.127	0.166	0.210	0.243	0.316	0.412	0.316	0.483	
0.40	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.019	0.178	0.327	0.780	1.234	1.812	0.780	3.048	
	0.023	0.050	0.064	0.080	0.114	0.155	0.204	0.248	0.309	0.359	0.513	0.668	0.812	0.513	1.021	
0.60	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.311	0.449	0.619	1.121	1.744	2.467	1.121	3.069	
	0.027	0.079	0.102	0.127	0.182	0.246	0.326	0.428	0.493	0.572	0.818	1.073	1.350	0.818	1.657	
0.80	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.545	0.710	0.922	1.540	2.111	2.907	1.540	3.980	
	0.053	0.113	0.146	0.180	0.258	0.351	0.463	0.609	0.771	0.914	1.143	1.413	1.729	1.143	2.329	
1.00	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.749	0.971	1.208	1.599	2.111	2.907	1.599	3.509	
	0.071	0.151	0.195	0.242	0.346	0.470	0.621	0.816	0.990	1.093	1.361	1.641	2.004	1.361	2.122	
1.20	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.977	1.245	1.543	1.947	2.303	2.720	1.947	3.572	
	0.092	0.195	0.252	0.312	0.447	0.607	0.802	1.014	1.216	1.369	1.616	1.873	2.229	1.616	2.632	
1.40	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.310	1.590	1.944	2.403	2.794	3.219	2.403	4.190	
	0.116	0.246	0.318	0.394	0.534	0.729	1.012	1.329	1.531	1.777	2.142	2.518	2.919	2.142	3.085	
1.60	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.648	1.994	2.382	2.838	3.269	3.653	2.838	4.623	
	0.144	0.305	0.394	0.488	0.659	0.912	1.234	1.648	1.998	2.303	2.712	3.101	3.503	2.712	3.943	
1.80	0.0	0.0	0.0	0.0	0.0	0.0	0.0	2.019	2.447	2.884	3.292	3.674	4.021	3.292	5.023	
	0.177	0.374	0.482	0.598	0.811	1.072	1.437	2.019	2.439	2.869	3.262	3.614	3.941	3.262	5.123	
2.00	0.0	0.0	0.0	0.0	0.0	0.0	0.0	2.346	2.911	3.477	3.815	4.169	4.461	3.815	5.893	
	0.215	0.454	0.586	0.726	1.040	1.411	1.866	2.451	2.893	3.377	3.688	4.019	4.265	3.688	5.576	
2.20	0.0	0.0	0.0	0.0	0.0	0.0	0.0	2.673	3.473	4.123	4.370	4.633	4.805	4.370	6.350	
	0.255	0.523	0.676	0.846	1.234	1.702	2.230	2.958	3.464	3.952	4.244	4.477	4.593	4.244	6.376	
2.40	0.0	0.0	0.0	0.0	0.0	0.0	0.0	3.000	3.943	4.633	4.830	5.044	5.178	4.830	7.050	
	0.310	0.659	0.847	1.051	1.503	2.002	2.569	3.357	3.904	4.332	4.544	4.693	4.741	4.544	6.576	
2.60	0.0	0.0	0.0	0.0	0.0	0.0	0.0	3.327	4.411	5.093	5.193	5.347	5.403	5.193	7.346	
	0.371	0.786	1.013	1.256	1.758	2.440	3.226	4.247	4.907	5.344	5.483	5.595	5.612	5.483	7.516	
2.80	0.0	0.0	0.0	0.0	0.0	0.0	0.0	3.654	4.902	5.573	5.609	5.720	5.742	5.609	8.089	
	0.435	0.910	1.207	1.496	2.043	2.793	3.676	4.727	5.397	5.766	5.789	5.848	5.858	5.766	8.249	
3.00	0.0	0.0	0.0	0.0	0.0	0.0	0.0	3.981	5.371	6.032	6.032	6.114	6.114	6.032	8.821	
	0.505	1.091	1.446	1.778	2.380	3.247	4.246	5.395	5.892	6.200	6.193	6.235	6.235	6.193	9.394	
	0.572	1.195	1.446	1.778	2.380	3.247	4.246	5.395	5.892	6.200	6.193	6.235	6.235	6.193	9.394	

TABLE F.1
 $\chi(x)$ and $\chi^*(x)$ (case F)

** INPUT DATA **
 CONSTANT-L ARRIVAL= 1.00(FCISSCH)
 CONSTANT-L MAG= 2.00(EP)
 SHOCK ARRIVAL RATE= 1.00(FCISSCH)
 SHOCK-L MAG= 1.00(EP)

X-LEV	P(TX=0)	TRUE PR	X-BAF	T X-BAR	ST-CEVI	VAR(X-BAR)	COEF-V
0.20	0.66570	0.67032	0.21047	0.21009	0.46512	0.00002	2.20996
0.40	0.44510	0.44932	0.43515	0.44145	0.70431	0.00005	1.61840
0.60	0.30100	0.30119	0.65278	0.65778	0.94643	0.00009	1.36612
0.80	0.20110	0.20150	0.96280	0.98504	1.21559	0.00015	1.23687
1.00	0.13270	0.13534	1.30511	1.31172	1.51946	0.00023	1.16068
1.20	0.08650	0.09072	1.65664	1.68860	1.87235	0.00035	1.11010
1.40	0.05570	0.06081	2.12442	2.12882	2.27115	0.00052	1.06907
1.60	0.04060	0.04076	2.62433	2.64815	2.78008	0.00077	1.05532
1.80	0.02700	0.02732	3.24255	3.26570	3.32625	0.00111	1.02568
2.00	0.01600	0.01632	4.01151	4.00415	4.08579	0.00167	1.01841
2.20	0.01250	0.01228	4.51615	4.89106	4.99251	0.00245	1.01553
2.40	0.00840	0.00823	5.54709	5.55976	5.97537	0.00357	1.00475
2.60	0.00600	0.00552	7.25628	7.25071	7.34602	0.00540	1.01237
2.80	0.00410	0.00370	8.91776	8.81314	9.07498	0.00824	1.01763
3.00	0.00220	0.00248	10.76557	10.70656	10.97682	0.01205	1.01921

** $\chi(x)$, $1/E(Tx)$, $\chi^*(x)$ **

X-LEV	0.200	0.400	0.600	0.800	1.000	1.200	1.400	1.600	1.800	2.000	2.200	2.400	2.600	2.800	3.000
$\chi(x)$	1.562	1.237	0.975	0.801	0.650	0.521	0.435	0.358	0.276	0.244	0.199	0.166	0.136	0.112	0.092
$1/E(T)$	4.760	2.265	1.432	1.015	0.762	0.592	0.470	0.378	0.306	0.250	0.204	0.168	0.138	0.113	0.093
$\chi^*(x)$	3.105	1.860	1.400	1.286	1.183	1.122	1.084	1.059	1.044	1.032	1.019	1.017	1.011	1.007	1.005

TABLE F.2
Quantiles (case F)

***	QUANTILE OF T X ***															
	Q.1	Q.2	Q.25	Q.3	Q.4	Q.5	Q.6	Q.7	Q.75	Q.8	Q.9	Q.95	C.50	C.99		
-LEV	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0		
0.20	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0		
0.40	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0		
0.60	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0		
0.80	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0		
1.00	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0		
1.20	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0		
1.40	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0		
1.60	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0		
1.80	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0		
2.00	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0		
2.20	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0		
2.40	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0		
2.60	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0		
2.80	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0		
3.00	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0		

TABLE G.1
Confidence Interval in case E

0.9-CONFIDENCE INTERVAL FOR QUANTILE

X-LEV	Q.5	CL	CU	Q.95	CL	CU	Q.98	CL	CU	Q.99	CL	CU
0.20	0.235	0.307	0.357	0.722	0.680	0.769	1.234	1.171	1.311	1.639	1.559	1.792
0.40	0.786	0.745	0.814	1.231	1.185	1.279	1.912	1.822	1.974	2.304	2.210	2.422
0.60	1.151	1.116	1.200	1.744	1.679	1.802	2.484	2.355	2.568	3.086	2.949	3.261
0.80	1.560	1.503	1.618	2.226	2.147	2.283	3.117	2.956	3.230	3.680	3.545	3.809
1.00	1.957	1.945	2.047	2.779	2.701	2.820	3.669	3.543	3.809	4.506	4.335	4.661
1.20	2.447	2.380	2.524	3.340	3.268	3.424	4.478	4.301	4.589	5.394	5.162	5.666
1.40	3.005	2.926	3.073	3.956	3.879	4.100	5.335	5.129	5.544	6.396	6.165	6.660
1.60	3.558	3.485	3.647	4.766	4.670	4.896	6.353	6.218	6.556	7.623	7.206	8.055
1.80	4.256	4.164	4.343	5.665	5.552	5.825	7.609	7.262	7.949	8.943	8.576	9.172
2.00	5.113	4.996	5.217	6.813	6.634	6.988	9.104	8.855	9.377	10.895	10.429	11.401
2.20	6.067	5.915	6.224	8.084	7.879	8.345	10.878	10.362	11.305	13.075	12.599	13.599
2.40	7.155	6.937	7.326	9.477	9.294	9.739	12.913	12.518	13.203	15.031	14.516	15.653
2.60	8.502	8.352	8.734	11.420	11.124	11.741	15.224	14.853	15.879	17.831	17.138	18.528
2.80	10.122	9.922	10.416	13.668	13.326	14.168	17.805	17.366	18.441	21.155	20.597	21.892
3.00	12.063	11.765	12.431	15.933	15.505	16.269	20.633	20.073	21.204	23.946	23.086	25.077

V. SIMULATION ALGORITHM

A. VARIABLE DEFINITION

$T_{X_i} = \{ \inf t \geq 0 ; Z(t) \geq X_i \}$

where $Z(t)$ = superposition of constant load and shock load

N_{row} = maximum number of level X_i ; $X_1 < X_2 < X_3 < \dots < X_{N_{row}}$

N_{col} = maximum number of repetition of generating T_x

L_c = constant load magnitude

L_s = shock load magnitude

T_c = constant load interval

T_s = shock load interval

i_{pc} = counting the number of repetition

L = counting the maximum level of X_i

B. ALGORITHM

Input; parameters (λ, μ, a, b , seed numbers)

Initialize the stress level X_i ; $X_1 < X_2 < X_3 < \dots < X_{N_{row}}$

$i_{pc} = 0$

Repeat

 set $T_0 = 0$

$M = 0$

$TTs = 0$

 Repeat

 generate constant load; L_c

 generate constant interval; T_c

 Find L such that $L = \{ \sup i ; L_c \geq x_i \}$

 Set $T_{X_i} = T_0$ for $1 \leq i \leq L$

$M = L$

$C_i = T_c$

 A, generate the shock load; L_s

 generate the shock interval; T_s

 B, If $T_s > C_i$, then

```

T0=T0+Tc
Ts=Ts-Ci
TTs=0
generate constant load;Lc
generate constant interval;Tc
Find I such that  $L=\{\sup_{\geq M} i; Lc \geq Xi\}$ 
If  $L > M$ , then
    Set  $T_{xi} = T$  for  $M < i \leq L$ 
     $M=L$ 
Else continue
Ci=Tc
Go to E
Else, Find I such that  $L=\{\sup_{\geq M} i; Ls+Lc \geq Xi\}$ 
TTs=TTs+Ts
Set  $T_{xi} = T_0 + TTs$  for  $M < i \leq L$ 
Ci=Ci-Ts
Go to A
Until (L=Nrow)
Ipc=Ipc+1
Until (Ipc=Ncol)
End Algorithm

```

APPENDIX A

COMPUTER PROGRAM

```

*****
***** VARIABLE DEFINITION *****
TX=TIME OF COMBINATION LOAD EXCEEDS STRESS LEVEL X
X=STRESS LEVEL
NROW=MAXIMUM NUMBER OF REPLICATIONS OF GENERATING TX
NCOL=MAXIMUM NUMBER OF REPLICATIONS OF GENERATING TX
CL=CONSTANT LOAD MAGNITUDE
CI=CONSTANT LOAD INTERVAL
SI=SHOCK LOAD MAGNITUDE
SII=SHOCK LOAD INTERVAL
TIX=TRUE MEAN OF TX
TIX=TRUE MEAN OF TX
SK=VALUE OF KAP A
DENO=VALUE OF CONSTANT OF TAIL DISTRIBUTION TX
APRB=VALUE OF PROBABILITY OF TX=0
TTPR=TRUE PROBABILITY OF TX=0
IPC=COUNTING NUMBER OF REPETITION
RA1=CONSTANT LOAD ARRIVAL RATE
RA2=CONSTANT LOAD MAGNITUDE PARAMETER; EXPONENTIAL(RA2)
RA3=SHOCK LOAD ARRIVAL RATE
RA3=SHOCK LOAD MAGNITUDE PARAMETER; EXPONENTIAL(RA4)
*****
***** MAIN PROGRAM *****
REAL*4 TX(25,5000),CL,CJ,SL,SI,F1,F2,RA,XL,X(100)
*,RA1,RA2,RA3,RA4,TIX(100),ETX(100),APRB(100),SK(100),F,DENU(100)
*,TTPR(100),CC(100)
INTEGER M,I,IPC,I FILE,NROW,NCOL
RA1=1.0
RA2=2.0
RA3=1.0
RA4=1.0
F1=1.0/RA1
F2=1.0/RA2
F3=1.0/RA3
F4=1.0/RA4
IX1=27543
IX2=25737
IX3=27327
IX4=23777
DEL=0.5
XL=0.0
LC=1
TD=0.0

```



```

      K=K-1
      IF(K.EQ.0) GO TO 101
      GO TO 11
101  II=K
      RETURN
      END

```

C C

```

      SUBROUTINE STOREA(LC,II,IPC,TX,TO,NROW,NCOL)
      INTEGER N,IPC,II,NROW,NCOL
      REAL*4 TX(25,5000)
      IF(IPC.GE.(NCOL+1)) GO TO 901
      N=II
      DO 110 I=LC,N

```

```

          TX(I,IPC)=TO
          IF(I.EQ.NROW) GO TO 111

```

```

110  CONTINUE

```

```

      LC=II+1
      GO TO 901

```

```

111  IPC=IPC+1

```

```

      TO=0.0

```

```

      LC=1

```

```

901  RETURN
      END

```

F 9

----- GENERATE THE TX -----

C C

```

      SUBROUTINE RANDO(X,TX,LC,IPC,IX1,IX2,IX3,IX4,CL,C1,NROW,NCOL,TO
      *,F1,F2,F3,F4)
      CL=CL-TX(25,5000),C(1),D(1),X(100),TO,F1,F2,F3,F4,SD,S
      INTEGER IPC,LC,NROW,NCOL,IX1,IX2,IX3,IX4
      CALL CONST(IX1,IX2,CL,C1,F1,F2)
      II=0
      SD=C.0
      LC=1

```

```

      CDIST=C1
      IF(IPC.EQ.(NCOL+1)) GO TO 909
      IF(CL.LT.X(LC)) GO TO 10

```

```

      CALL FINDA(X,CL,II,NROW)
      CALL STOREA(LC,II,IPC,TX,TO,NROW,NCOL)
      IF(II.EQ.NROW) GO TO 909

```

```

      IF(IPC.EQ.(NCOL+1)) GO TO 909

```

```

      IF(LC.GT.NROW) GO TO 901

```

```

      CALL SEXP3

```

```

      CALL SEXP4(IX3,C,1,1,0)

```

```

      SL=D(1)*F4

```

10

```

209 IF(SI.GT.CDIST)GO TO 902
    CCL=SL+CL
    SD=SD+SI
    IF(CCL.LT.X(LC))GO TO 201
    CALL FINDA(X,CCL,I1,NROW)
    DO 110 K=L,C,I1
    TX(K,IPC)=TO+SD
    CONTINUE
    LC=I1+1
    IF(LC.GT.NROW)GO TO 901
    CDIST=CDIST-SI
    GO TO 10
902 SD=0.0
    TO=TO+CI
    SI=SI-CDIST
    CALL CONST(I,X1,IX2,CL,CI,F1,F2)
    CDIST=CI
    IF(CCL.LT.X(LC))GO TO 209
    CALL FINDA(X,CL,I1,NROW)
    CALL STOREALC,I1,IPC,IX,TU,NROW,NCOL)
    IF(I1.EQ.NROW)GO TO 909
    GO TO 209
901 TO=0.0
    LC=1
    SD=0.0
    SI=0.0
    CDIST=0.0
    CI=0.0
    CCL=0.0
    CL=0.0
    SL=0.0
    IPC=IPC+1
    RETURN
909 END
C ----- COMPLETE THE SAMPLE STATISTICS -----
C
    SUBROUTINE COMPUT(X,TX,NROW,NCOL,XB,SC,PRB,VXB,VSD
    *,RA1,RA2,RA3,RA4,ETX,ITX,APRB,ITPR,STX)
    REAL*4 X(100),TX(25,5000),XB,SD,PRB,VXB,VSD,SUM,S2,S4
    *,TS1,TS4,SM4,U4,SM2,T,TM,TMA,RA1,RA2,RA3,RA4,ETX(100)
    *,ITX(100),STX(25,5000),CL,CU,APRB(100),ITPR(100),XN
    INTEGER NROW,NCOL,I,J,K,L,N
    WRITE(6,600)
    FORMAT(IX,/,/,1X,'X-LEV P(TX=0) TRUE PR',
    *4X,'X-BAR ST-DEVI VAK(X-BAR)',
    *,COEF-V',/)
    DO 100 I=1,NROW
600

```

```

SUM=0.0
DO 200 J=1,NCOL
  SUM=SUM+TX(I,J)
CONTINUE
XB=SUM/FLOAT(NCOL)
ETX(I)=XB
S2=0.0
S4=0.0
DO 300 K=1,NCOL
  STX(I,K)=TX(I,K)
  TS2=(TX(I,K)-XB)**2
  S2=S2+TS2
  TS4=(TX(I,K)-XB)**4
  S4=S4+TS4
CONTINUE
A=NCOL
SD=(S2/FLOAT(N-1))**0.5
SM2=S2/FLOAT(N)
SM4=S4/FLOAT(N)
U4=((SM4*FLOAT(N**2-2*N+3))-(SM2**2*FLOAT(3*N*(2*N-3))))
VSD=(U4-(SD**4))/(4*FLOAT(N-1)/SD**2)
VXB=(SD**2)/N
II=0
DO 400 L=1,NCOL
  STX=TX(I,L)
  IF (TX(I,L).EQ.0.0)GO TO 401
  GO TO 400
  II=II+1
CONTINUE
PRB=FLOAT(II)/FLOAT(NCOL)
APRB(I)=PRB
VPRB=(PRB*(1.0-PRB))/FLOAT(NCOL)
CL1=PRB-1.645*SQR(VPRB)
CU1=PRB+1.645*SQR(VPRB)
TPR=EXP(-RA2*X(I))
TTPR(I)=TPR
----- COMPUTE THE TRUE MEAN -----
* * F(X) AND G(X) ARE IDENTICAL EXPONENTIAL (A=8=1) * *
T=ALOG((RA1+RA3)/(RA1+RA3*EXP(-X(I))))
TMA=RA1*(1-EXP(-X(I)))-RA3*X(I)*EXP(-X(I))*T
TMB=1+(RA3/RA1)*X(I)-(RA3/RA1)*T
TM=(EXP(X(I)))/(RA1**2))*TMA/TMB
* * F(X) AND G(X) ARE DIFFERENT EXPONENTIAL (A/B=2) * *
XN=RA1*(-1.0+EXP(2.0*RA4*X(I)))+2.0*RA3*(1.0-EXP(RA4*X(I)))
**2.0*((RA3**2)/RA1)*(RA4*X(I)-ALOG((RA1+RA3)/(RA1+RA3*EXP(-RA4
**X(I))))
TM=(1.0/RA1)*XN*(1.0/(RA1*EXP(2.0*RA4*X(I))-XN))

```

```

C
      ITX(1)=XB
      ITX(1)=TM
      CL=XB-1.645*(VXB**0.5)
      CU=XB+1.645*(VXB**0.5)
      COV=SD/XB
      WRITE(6,73 9)X(1),PR8,TPR,XB,TM,SD,VXB,CCV
      FORMAT(IX,F4.2,7F10.5,/)
709  CONTINUE
100  RETURN
      END

C----- COMPUTE THE QUANTILE -----
C
      SUBROUTINE COMPU(X,TX,NROW,NCOL,ETX,ITX,XTX)
      REAL*4 X(100),TX(25,5000),ETX(100),ITX(100),SK(100),DEND(100)
      *P(15),C(15),TQ(15),Q25,TQ25,Q75,TQ75,Q95
      *TQ95,Q98,TQ98,Q99,TQ99,F1,XTX(50,10)
      INTEGER NROW,NCOL,I,J,L,NQ(15),N25,N75,N95,N98,N99
      WRITE(6,12)
      FORMAT(IX,/,TX,*,** QUANTILE OF TX **,
      *//,IX,*,X-LEV,Q.1,Q.2,Q.7,Q.25,Q.75,Q.8,Q.9,Q.95,
      *,5X,*,Q.5,Q.6,Q.99,/,)
      DO 101 I=1,9
      P(I)=.1*FLOAT(I)
      CN=P(I)*FLOAT(NCOL)
      NQ(I)=1*FIX(CN)
101  CONTINUE
      DO 103 J=1,NROW
      DO 104 L=1,9
      Q(L)=TX(J,NQ(L))
      TQ(L)=-TX(J)*ALOG(1.0-P(L))
      CONTINUE
      F1=FLOAT(NCOL)
      N25=FIX(F1*(0.25*F1))
      N75=FIX(F1*(0.75*F1))
      N95=FIX(F1*(0.95*F1))
      N98=FIX(F1*(0.98*F1))
      N99=FIX(F1*(0.99*F1))
      Q25=TX(J,N25)
      TQ25=-TX(J)*ALOG(1.0-0.25)
      Q75=TX(J,N75)
      TQ75=-TX(J)*ALOG(1.0-0.75)
      Q95=TX(J,N95)
      TQ95=-TX(J)*ALOG(1.0-0.95)
      Q98=TX(J,N98)
      TQ98=-TX(J)*ALOG(1.0-0.98)
      Q99=TX(J,N99)

```

```

      IQ99=-TTX(J)*ALOG(1.0-0.99)
      TTX(J,1)=X(J)
      TTX(J,2)=ETX(J)
      TTX(J,3)=Q(9)
      TTX(J,4)=Q55
      TTX(J,5)=Q98
      TTX(J,6)=Q59
      TTX(J,7)=TQ(9)
      TTX(J,8)=TQ95
      TTX(J,9)=TQ98
      TTX(J,10)=TQ99
      WRITE(6,88)X(J),Q(1),Q(2),Q(3),Q(4),Q(5),Q(6),Q(7),Q(75)
88      *Q(8),Q(9),Q95,Q98,Q99
      FORMAT(1X,F5.2,14F8.3)
      WRITE(6,99)TQ(1),TQ(2),TQ(3),TQ(4),TQ(5),TQ(6),TQ(7)
99      *TQ75,TQ(8),TQ(9),TQ95,TQ98,TQ99
      FORMAT(16X,14F8.3)
103     CONTINUE
      RETURN
      END
C----- COMPUTE THE CONDITIONAL QUANTILE OF TX -----
C
      SUBROUTINE CQMP(X,TX,NROW,NCOL,EIX,TTX,APRB,STX,XXI)
      REAL*4 TX(25),X(100),ETX(100),TTX(100),APRB(100),AP(15)
      *AQ(15),ATQ(15),AQ25,ATQ25,ATQ75,ATQ75,AC95,ATQ95,AQ98
      *ATQ98,AQ99,ATQ99,F,F1,XX(100),XXI(50,10)
      INTEGER NROW,NCOL,I,J,L,ANQ(15),AN25,AN75,AN95,AN98,AN99
      WRITE(6,12)
      FORMAT(1X,/,1X, ** CONDITIONAL QUANTILE OF TX **,
      *5X,/,X-LEV Q.1 Q.2 Q.25 Q.75 Q.8 Q.9 Q.95,
      *,/,C.98 Q.99,/,)
      F=FLOAT(NCOL)
      DO 103 J=1,NROW
      NP1=FIX(APRB(J)*F)
      NP2=NCOL-NP1
      DO 101 I=1,10
      AP(I)=0.1*FLOAT(I)
      AQN=AP(I)*FLOAT(NP2)
      ANQ(I)=NP1+FIX(AQN)
      CCNTINUE
      XX(J)=ETX(J)/(1.0-APRB(J))
      DO 102 L=1,9
      AQ(L)=TX(J,ANQ(L))
      ATQ(L)=(-XX(J))*ALOG(1.0-AP(L))
      CCNTINUE
      F1=FLOAT(NP2)
101
102

```

```

AN25=NPI+IFIX(0.25*F1)
AN75=NPI+IFIX(0.75*F1)
AN55=NPI+IFIX(0.95*F1)
AN58=NPI+IFIX(0.98*F1)
AN59=NPI+IFIX(0.99*F1)
AQ25=TX(J,AN25)
ATC25=(-X(X(J))*ALOG(1.0-0.25))
AC75=TX(J,AN75)
ATG75=(-X(X(J))*ALOG(1.0-0.75))
AQ55=TX(J,AN55)
ATC95=(-X(X(J))*ALOG(1.0-0.95))
AC58=TX(J,AN58)
ATQ98=(-X(X(J))*ALOG(1.0-0.98))
AQ59=TX(J,AN59)
ATC99=(-X(X(J))*ALOG(1.0-0.99))
XXT(J,1)=X(J)
XXT(J,2)=ETX(J)
XXT(J,3)=AQ(9)
XXT(J,4)=AQ95
XXT(J,5)=AQ98
XXT(J,6)=AQ99
XXT(J,7)=ATQ(9)
XXT(J,8)=ATQ95
XXT(J,9)=ATQ98
XXT(J,10)=ATQ99
WRITE(6,990)X(J),AQ(1),AQ(2),AQ(3),AQ(4),
990  *,AQ(5),AQ(6),AQ(7),AQ(8),AQ(9),AQ95,AQ98,AQ99
      FCRMAT(1X,F5.2,14F8.3)
      WRITE(6,999)ATQ(1),ATQ(2),ATQ(3),ATQ(4),
999  *,ATQ(5),ATQ(6)
103  *,ATQ(7),ATQ(8),ATQ(9),ATQ95,ATQ98,ATQ99
      FCRMAT(6X,14F8.3)
      CONTINUE
      RETURN
      END

```

```

C ----- COMPLETE THE QUANTILE OF TAIL DISTRIBUTION -----
C
SUBROUTINE DCOMPU(X,TX,NROW,NCOL,ETX,TTX,SK,DEND)
*,CC,RA2,RA4)
*,CC,RA2,RA4)
*,P(15),C(15),TQ(15),Q25,TQ25,STQ25,C75,TQ75
*,TQ95,STQ95,Q98,TQ98,STQ98,Q99,TQ99,STQ99,F1,SL(100)
*,RA4,ATC(15),ATQ25,ATQ75,ATQ95,ATQ98,ATQ99
*,SK(100),DEND(100),STQ75,Q95,CC(100),RA2
INTEGER NROW,NCOL,I,J,L,NQ(15),N25,N75,N55,N98,N99
WRITE(6,12)
FORMAT(1X,/,1X,' *** QUANTILE (COMPARING TAIL'
12

```

```

*, DIST CF TX) **, //, 1X, 'X-LEV      C.1      Q.2:
*, 5X, 'N.25      Q.3      Q.4      Q.5      Q.6      Q.7:
*,      C.75      Q.8      Q.9      Q.95      Q.98      Q.99, //)
DO 101 I=1, 9
  F(I)=0.1*FLOAT(I)
  CN=P(I)*FLOAT(NCOL)
  NJ(I)=FIX(QN)
  CONTINUE
DO 103 J=1, NROW
  DO 104 L=1, 9
    Q(L)=TX(J,NQ(L))
    IQ(L)=-TX(J)*ALOG(1.0-P(L))
    ATC(L)=(-1.0/SK(J))*ALOG(1.0-P(L))
    STC(L)=(-1.0/SK(J))*ALOG(1.0-P(L))*DENO(J)
    IF(STQ(L).GT.0.0)GO TO 104
    STQ(L)=0.0
    CONTINUE
    F1=FLOAT(NCOL)
    N25=FIX(C.25*F1)
    N75=FIX(C.75*F1)
    N95=FIX(C.95*F1)
    N98=FIX(C.98*F1)
    N99=FIX(C.99*F1)
    C25=TX(J,N25)
    IQ25=-TX(J)*ALOG(1.0-C.25)
    ATC25=(-1.0/SK(J))*ALOG(1.0-C.25)
    STC25=(-1.0/SK(J))*ALOG(1.0-C.25)*DENO(J)
    IF(STQ25.GT.0.0)GO TO 75
    STQ25=0.0
    STQ25=TX(J,N75)
    IQ75=-TX(J)*ALOG(1.0-C.75)
    ATC75=(-1.0/SK(J))*ALOG(1.0-C.75)
    STC75=(-1.0/SK(J))*ALOG(1.0-C.75)*DENO(J)
    IF(STQ75.GT.0.0)GO TO 95
    STQ75=0.0
    STQ75=TX(J,N95)
    C95=TX(J,N95)
    IQ95=-TX(J)*ALOG(1.0-C.95)
    ATC95=(-1.0/SK(J))*ALOG(1.0-C.95)
    STC95=(-1.0/SK(J))*ALOG(1.0-C.95)*DENO(J)
    Q98=TX(J,N98)
    IQ98=-TX(J)*ALOG(1.0-C.98)
    ATC98=(-1.0/SK(J))*ALOG(1.0-C.98)
    STC98=(-1.0/SK(J))*ALOG(1.0-C.98)*DENO(J)
    C99=TX(J,N99)
    IQ99=-TX(J)*ALOG(1.0-C.99)
    ATC99=(-1.0/SK(J))*ALOG(1.0-C.99)
    STC99=(-1.0/SK(J))*ALOG(1.0-C.99)*DENO(J)
    WRITE(6,88)(J),Q(1),Q(2),Q(3),Q(4),Q(5)

```

```

88      *,Q(6),C(7),C75,C(8),C(9),Q95,Q98,Q99
        FORMAT(1X,F5.2,14F8.3)
        WRITE(6,109)STQ(1),STQ(2),STQ25,STQ(3),STQ(4)
109      *,STQ(5),STQ(6)
        *,STQ(7),STQ75,STQ(8),STQ(9),STQ95,STQ98,STQ99
        FCMAT(6X,14F8.3)
        WRITE(6,99)TQ(1),TQ(2),TQ25,TQ(3),TQ(4),TQ(5)
99      *,TQ(6),TQ(7),TQ75,TQ(8),TQ(9),TQ95,TQ98,TQ99
103     FORMAT(6X,14F8.3,/)
        CCNT INUE
        RETURN
        END

```

```

C      SUBROUTINE SURT(AA,NROW,NCOL)
        REAL*4 AA(25,5000),A(5000),B(5000)
        INTEGER NROW,NCOL
        DO 30 I = 1,NROW

```

```

40      DO 40 J = 1,NCOL
            A(J) = AA(I,J)
            B(J) = AA(I,J)
            CCNT INUE
            CALL SHSORT(A,B,NCOL)
            DO 50 K = 1,NCOL
                AA(I,K) = A(K)

```

```

50      CCNT INUE
30      CONTINUE
        RETURN
        END

```

```

C      --- COMPUTE THE KAPA -----

```

```

C      SUBROUTINE SKAPA(X,SK,NROW,RA2,RA4)
        REAL*4 X(100),SK(100),R,TL,TR,P,TNEW,CHECK,RA2,RA4
        INTEGER I,IC,NROW
        IC=0

```

```

        DO 100 I=1,NROW
            IC=IC+1
            R=X(I)
            TL=C.0
            TR=1.0+EXP(-RA4*R)-0.001
            TNEW=(TL+TR)/2.0
            P=CCM(TNEW,R,RA2,RA4)
            CHECK=ABS(P-1.0)
            IF (CHECK.LT.0.001)GO TO 90
            IF (IC.GT.100)GO TO 80
            IF (P.GT.1.0)GO TO 50
            TL=TNEW
            TR=TR

```

```

50      GC TO 5
      TL=TL
      TR=TNEW
      GO TO 5
80      WRITE(6,21)
21      FORMAT(1X,**)
90      SK(1)=TNEW
100     CONTINUE
      RETURN
      END

```

```

C----- COMPUTE THE FUNCTION F(K)-----
C
      FUNCTION COM(TNEW,R,RA2,RA4)
      REAL*4 R,TNEW
      IF(TNEW.EQ.1.0)GO TO 23
      COM=(2.0/(1.0-TNEW)**3)*((0.5*(1.0-TNEW)+EXP
      *(-RA4*R))**2)
      *-((0.5*(1.0-TNEW)*EXP(-RA4*R)+EXP(-RA4*R))**2)
      *-2.0*EXP(-RA4*R)*(1.0-TNEW)*(1.0-EXP(-RA4*R))
      *EXP(-2.0*R*RA4*R)*(ALOG(1.0-TNEW
      *EXP(-RA4*R))-ALOG((2.0-TNEW)*EXP(-RA4*R)))
      GO TO 45
23      COM=((RA2)/(RA2+RA4))*(EXP(RA4*R)-EXP(-RA2*R))
45      RETURN
      END

```

```

C----- COMPUTE THE CONSTANT OF TAIL DISTRIBUTION -----
C
      SUBROUTINE INT(X,SK,NROW,DENO,RA2,RA4,TTX)
      REAL*4 X(100),SK(100),DENO(100),RA2,RA4,TTX(100)
      *R,RR,RRR,ATTX(100)
      REAL*8 SA,B,AA,BB,DRA4,W
      INTEGER IER,IC,NROW
      DO 123 I=1,NROW
      DRA4=DBLE(RA4)
      SA=CBLE(SK(I))
      B=CBLE(X(I))
      AA=1.00-SA+DEXP(-DRA4*B)
      BB=DEXP(-DRA4*B)*(2.00-SA)
      W=(2.00/(1.00-SA)**4))*((AA**2)/2.00)-((BB**2)/2.00)-3.00*
      *DEXP(-CRA4*B)*(1.00-SA)*(1.00-DEXP(-DRA4*B))+3.00*XP(-2.00*
      *DRA4*B)*(DLOG(AA)-DLOG(BB))-DEXP(-3.00*DRA4*B))*((1.00/BB)-(1.00
      */AA))
      DENC(I)=SNGLE(W)
      ATTX(I)=1.0/TTX(I)
123     CONTINUE

```

***,//,IX,X-LEV.

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