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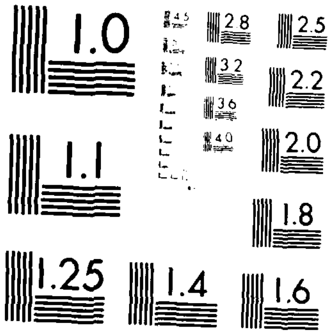

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TECHNICAL REPORT RG-84-13

UTM DETERMINATIONS FROM GEOGRAPHICAL DATA

Delina Rush
Guidance and Control Directorate
Army Missile Laboratory

JUNE 1984

U.S. ARMY MISSILE COMMAND
Redstone Arsenal, Alabama 35898-5000

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

The purpose of this study was to determine an accurate closed loop solution for calculating northing values along the central meridian. This report derives and evaluates three different techniques for determining this distance.

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The author would like to express her sincere appreciation to Mr. James Thacker and Dr. Henry Stern for their contributions, patience, and understanding which made this report possible.

TABLE OF CONTENTS

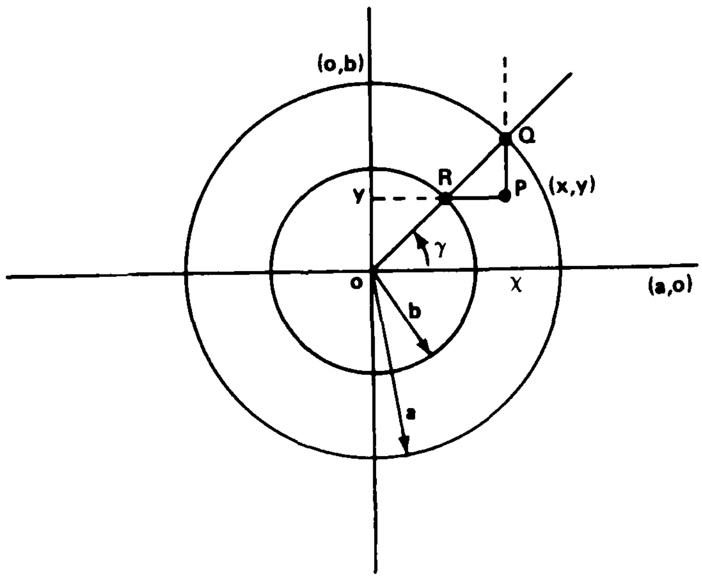
I. INTRODUCTION

This study is the first step in determining northing values in the Universal Transverse Mercator (UTM) Military Grid System. The UTM coordinate system is obtained by projecting the earth's surface onto an oblate cylinder slightly smaller than the earth's radii. The northing value is a measure of the distance from the equator to any point of latitude on the earth's surface. The first step in determining the northing value requires a very accurate calculation of the distance along an ellipse, which is analogous to calculating the UTM northing value along the UTM zone central meridian.

The purpose of this study was to determine an accurate closed loop solution for calculating northing values along the central meridian. This report derives and evaluates three different techniques for determining this distance. They are: (1) the Gaussian integration of a second order elliptical equation using parametric latitude, (2) the closed form approximation of the expanded and integrated elliptical equation, and (3) the summation of average radii of curvature as a function of the geodetic latitude angle.

A. Derivation of the Equation for an Ellipse

To begin this study, the equation of an ellipse was derived. In the following drawing, two concentric circles of radii a, b with centers at the origin were drawn. Next, a ray was drawn from the origin cutting both circles at the points Q and R and forming the angle γ. P is then the intersection of the line parallel to the Y-axis through Q and the line parallel to the X-axis through R. For each angle γ there is determined point P, whose coordinates (X, Y) are dependent upon γ .



Since OQ = a and OR = b, X and Y are expressed as functions of γ by,

$$X = a \cos\gamma$$
$$Y = b \sin \gamma \qquad (1)$$

From the equations above, it can be seen that

$$\frac{X}{a} = \cos\gamma \quad \text{and} \quad \frac{Y}{b} = \sin\gamma \tag{2}$$

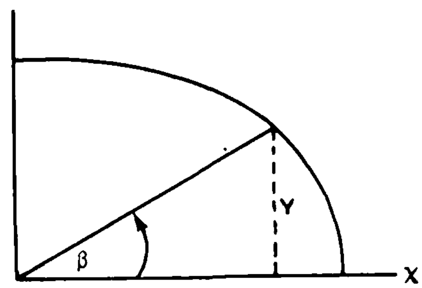
Then by squaring and adding these equations together and using the trigonometric identity $\cos^2\gamma + \sin^2\gamma = 1$, the general formula for an ellipse was derived. Hence,

$$\frac{X^2}{a^2} + \frac{Y^2}{b^2} = 1 \tag{3}$$

The complete ellipse is traced out as γ (the parametric latitude) goes from 0 to 2π. Also from equation (2) and the same trigonometric identity it was found that

$$\frac{Y}{X} = \frac{b}{a} \operatorname{Tan} \gamma \quad . \tag{4}$$

Next, by looking from a geocentric perspective the tan β also equalled $\frac{Y}{X}$.



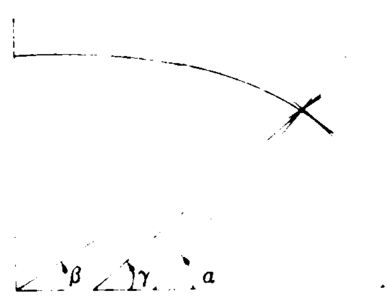
(5)

Therefore a relationship between the parametric and geocentric angles was established by combining equations (4) and (5).

$$\operatorname{Tan} \beta = \frac{b}{a} \operatorname{Tan} \gamma \ . \tag{6}$$

B. The Relationship of Beta and Gamma to Alpha

Since primary emphasis generated around geodetic data, a relationship between geodetic, geocentric, and parametric values was needed. The drawing below (the relationship of β and γ to α) shows an ellipse and the geodetic, geocentric and parametric angles.



The geodetic angle, α, is formed when taking a perpendicular to a tangent at any point on the ellipse. In Equation (3), the general formula for an ellipse was differentiated in order to find a relationship between alpha and the beta and gamma angles. This procedure is shown below:

$$\frac{X^2}{a^2} + \frac{Y^2}{b^2} = 1. \tag{7}$$

Rearranging Equation (3) into differentiable form gives,

$$\frac{X^2b^2}{a^2b^2} + \frac{Y^2a^2}{a^2b^2} = 1$$

and (8)

$$Y^2b^2 + Y^2a^2 = a^2b^2 \text{ (constant).}$$

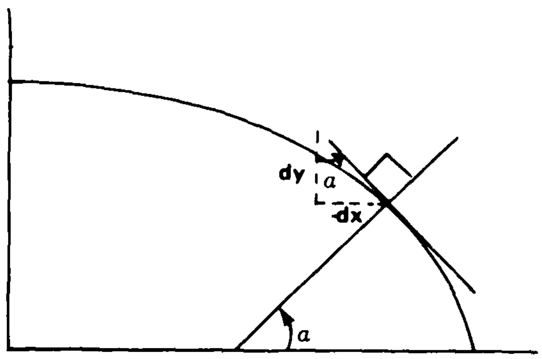
Differentiation gives,

$$2x\ dx\ b^2 + 2y\ dy\ a^2 = 0$$

and (9)

$$2y\ dy\ a^2 = -2x\ dx\ b^2$$

In order to determine the tangent of alpha, the diagram below was drawn.



By taking the tangent line as it follows the arrow, it was seen that dy was positive and dx was negative, thus giving a negative slope. The tangent of alpha was found by rotating α 90° and seeing that $\text{Tan}\alpha = \frac{-dx}{dy}$. So, by solving Equation (9) for $\frac{-dx}{dy}$ it was determined that

$$\text{Tan}\alpha = \frac{-dx}{dy} = \left(\frac{a^2}{b^2}\right)\left(\frac{Y}{X}\right). \tag{10}$$

With the relationships in Equations (6), (9), and (10), conversion equations for α were developed. Substituting the equation $\text{Tan}\beta = y/x$ into Equation (10) gives

$$\beta = \text{Tan}^{-1}\left(\frac{b^2}{a^2}\,\text{Tan}\alpha\right), \tag{11}$$

and by substituting $\text{Tan}\beta = b/a\,\text{Tan}\gamma$ into Equation (10) gives

$$\gamma = \text{Tan}^{-1}\left(\frac{b}{a}\,\text{Tan}\alpha\right) \tag{12}$$

Equations (11) and (12) were used in converting geodetic alpha to its geocentric and parametric values.

II. GAUSSIAN INTEGRATION OF A SECOND ORDER ELLIPTICAL EQUATION

Finally, the arc-length formula was derived from the parametric equations of the ellipse. The procedure is shown as follows:

$$X = a\cos\gamma$$
$$Y = b\sin\gamma\ , \text{ where } a > b.$$

$$\begin{aligned} ds^2 &= dx^2 + dy^2 \\ &= \left(a^2\sin^2\gamma + b^2\cos^2\gamma\right)d\gamma^2 \\ &= \left[a^2\left(1-\cos^2\gamma\right) + b^2\cos^2\gamma\right]d\gamma^2 \\ ds &= a\left[1 - \frac{a^2-b^2}{a^2}\cos^2\gamma\right]^{1/2} d\gamma \\ \int ds &= a\int\left[1 - \frac{a^2-b^2}{a^2}\cos^2\gamma\right]^{1/2} d\gamma \end{aligned}$$

This is an elliptical integral of the second kind where $e^2 = \frac{a^2-b^2}{a^2}$, where e is the eccentricity of the ellipse. Therefore,

$$\int ds = a\int\left(1-e^2\cos^2\gamma\right)^{1/2} d\gamma \tag{14}$$

The first of the three methods, the Gaussian numerical integration, using parametric angles, involved using the integral from Equation (14). This integral was evaluated from lower to upper gamma in steps of .5° and totaled for a cumulative sum of arc-distance.

III. CLOSED FORM APPROXIMATION OF EXPANDED AND INTEGRATED ELLIPTICAL EQUATION - METHOD 2

Since Equation (14) cannot be solved in closed form, the binomial theorem was used to expand the integrand into an integrable closed form solution. By using the definition of the binomial theorem

$$\left(1\pm\chi\right)^n = 1\pm n\chi + \frac{n(n-1)\chi^2}{2!} + \ldots \text{ etc. } \left(\chi^2 < 1\right), \tag{15}$$

and setting

$$\gamma = e^2\cos^2\gamma$$
$$n = 1/2, \tag{16}$$

the final solution became

$$1 - \frac{e^2\cos^2\gamma}{2} - \frac{e^4\cos^4\gamma}{8} + \ . \ . \ . \tag{17}$$

Therefore,

$$\int ds = a\int 1 - \frac{e^2\cos^2\gamma}{2} - \frac{e^4\cos^4\gamma}{8}\, d\gamma + \ . \ . \ . \tag{18}$$

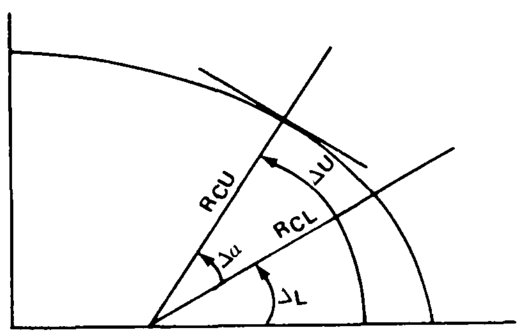
and

$$s = a\left[\left(1 - \frac{e^2}{4} - \frac{3e^4}{64}\right)\gamma - \left(\frac{e^2}{8} + \frac{e^4}{32}\right)\sin 2\gamma - \frac{e^4}{256}\sin 4\gamma + \ . \ . \ . \right. \tag{19}$$

The second method used to calculate arc distance was obtained from Equation (19) which was evaluated between two proper limits. Since for the case under study, one of these limits is $\gamma = 0°$, Equation (19) can be evaluated directly as a function of γ (parametric latitude).

IV. SUMMATION OF AVERAGE RADIUS OF CURVATURE - METHOD 3

The third and last method, independently derived using geodetic latitudes, involved taking the radii of curvature of the lower and upper limits (see drawing below), adding them, taking their average and multiplying them by $\Delta\alpha$, where $\Delta\alpha$ is the change in geodetic latitude.



In equation form it is,

$$S = \sum_{0}^{\alpha} \left(\frac{R_{CL} + R_{CU}}{2}\right) \Delta\alpha \ , \tag{20}$$

where

$$R_C = \frac{a(1-e^2)}{(1-e^2\sin^2\alpha)^{3/2}} \tag{21}$$

The derivation of Equation (21) is shown in the Appendix.

In order for these three methods to be calculated smoothly and efficiently, a Fortran program called ELIPSE was developed. This program, as listed below, carries out all the aforenamed integration and generates a table of all the geodetic, geocentric, and parametric angles along with the three calculated arc distances from 0° to 90° in steps of .5° (see Table 1).

This study was the first step in determining northing values along the Central Meridian. Three methods analogous to those in UTM were used to determine these values. The first method was a numerical integration evaluation of a closed form integral. The second method was a series approximation of the integral. The third method used average values of the curvature technique.

The results from comparisons between the three methods show differences of less than a meter. The conclusion is that any of the three methods could be used for the determination of UTM northing values from geographical data without accuracy penalty.

FORTRAN PROGRAM

```
C PROGRAM ELIPSE
C
C CALCULATES THE ARCLENGTH BETWEEN TWO LIMITS EXPRESSED IN ANGLES BY USE
C OF THE GAUSSIAN NUMERICAL INTEGRATION ROUTINE,APPROXIMATE CLOSED FORM
C SOLUTION,AND THE AVERAGE SUM OF THE EQUATORIAL RADII.
C
      IMPLICIT REAL*8 (A-H,O-Z)
      EXTERNAL EQUAT1
      COMMON/AA/RADC,E2
      CHARACTER*1 ANS
C
C      DATA FOR CALCULATIONS
C
      DATA A/6378206.4D0/
      DATA B/6356583.8D0/
      DATA E2/.006768658D0/
      DATA RADC/57.29577951310D0/
      DATA PI2/1.5707963267949D0/
      ADIF=0.5/RADC
      EP2=(A**2-B**2)/B**2
      E4=E2**2
      FAC1=(B/A)**2
      FAC2=B/A
      REWIND 3
      WRITE(2,100)
  100 FORMAT(' ',T7,'GEODETIC',T21,'GEOCENTRIC',T38,'PARAMETRIC',
     *T61,'NUMERICAL',T75,'APPROXIMATION',T93,'AVERAGE RADII',T110,
     *'NITER',/,/,/)
      GO TO 16
```

```
 10   CONTINUE
C
C               DATA CAN BE READ IN USING FOR003.DAT OR
C               BY TERMINAL INPUT
C
C       WRITE(6,*)'DO YOU WISH TO CONTINUE?(Y/N)'
C       READ(3,101) ANS
C 101   FORMAT(A)
C       IF (ANS.EQ.'Y') THEN
C        WRITE(6,*)'DO YOU WISH TO CONTINUE?(1/2)'
        READ(3,*)IANS
        IF(IANS.EQ.1) THEN
        GO TO 16
       ELSE
         GO TO 999
       END IF
  16   CONTINUE
C  WRITE(6,*)'INPUT LOWER LIMIT VALUES FOR GEODEDTIC ALPHA IN'
C       WRITE(6,*)'DEGREES,MINUTES,AND SECONDS.'
       READ(3,*,END=550)AIDL,AIML,AISL
C       WRITE(6,*)'POINTS?'
       READ(3,*)M
C       WRITE(6,*)'SUBINTERVALS?'
       READ(3,*)NINT
C       WRITE(6,*)'ACCURACY FACTOR?'
       READ(3,*)DEL
C       WRITE(6,*)'MAX INTEGRATIONS?'
       READ(3,*)MAX
C
C            CHANGING DEGREES,MINUTES,SECONDS TO DECIMAL DEGREES
C            AND CHANGING DEGREES TO RADIANS.
C
      CALL DEGREE(DEGRE,AIDL,AIML,AISL)
      CALL RADIAN(RAD,DEGRE)
      ALOW=RAD
C
C            INCREMENTING UPPER LIMIT IN STEPS OF .5 DEGREES
C
      AUP=ALOW+ADIF
C
C            CORRESPONDING GEOCENTRIC AND PARAMETRIC LIMITS
C            B( )=GEOCENTRIC   G( )=PARAMETRIC
C
      BLOW=DATAN(FAC1*DTAN(ALOW))
      GLOW=DATAN(FAC2*DTAN(ALOW))
C
C            TEST CASE FOR 90 DEGREES
C
      IF (ALOW.GE.1.555D0) THEN
         BUP=PI2
         GUP=PI2
      ELSE
         BUP=DATAN(FAC1*DTAN(AUP))
         GUP=DATAN(FAC2*DTAN(AUP))
      END IF
C
```

```
C              GAUSSIAN INTEGRATION ,STEP ONE.
C
      CALL GAUSA(G,GDIF,EQUAT1,GLOW,GUP,M,NINT,DEL,MAX,NITER)
      TSUM=G*A
      SUM1=SUM1+TSUM
C
C              APPROXIMATE CLOSED FORM SOLUTION ,STEP TWO.
C
      EQUAT2=(1.D0-.25D0*E2-3.D0/64.D0*E4)*GUP-(1.D0/8.D0*E2+1.D0/32.D0
     **E4)*DSIN(2*GUP)-(1.D0/256.D0*E4*DSIN(4*GUP))
      SUM2=EQUAT2*A
C
C              AVERAGE SUM EQUATORIAL RADII ,STEP THREE.
C
      RL=(A*(1.D0-E2))/
     *(DSQRT(1.D0-(E2*(DSIN(ALOW))**2))*(1.D0-(E2*(DSIN(ALOW))**2)))
      RU=(A*(1.D0-E2))/
     *(DSQRT(1.D0-(E2*(DSIN(AUP))**2))*(1.D0-(E2*(DSIN(AUP))**2)))
      DELTAA=AUP-ALOW
      EQUAT3=((RL+RU)/2.D0)*DELTAA
      SUM3=SUM3+EQUAT3
C
C              RADIANS BACK TO DEGREES,MINUTES,SECONDS
C
      CALL  INVRAD(AID,AUP)
      CALL  INVRAD(BID,BUP)
      CALL  INVRAD(GID,GUP)
      CALL  INVDEG(IAD,IAM,AS,AID)
      CALL  INVDEG(IBD,IBM,BS,BID)
      CALL  INVDEG(IGD,IGM,GS,GID)
C
C              OUTPUT
C
C
      WRITE(2,200)IAD,IAM,AS,IBD,IBM,BS,IGD,IGM,GS,SUM1,SUM2,SUM3,NITER
  200 FORMAT(' ',T2,I3,T7,I2,T10,F7.4,T19,I3,T24,I2,T27,F7.4,T36,I3,T41,
     *I2,T44,F7.4,T53,F17.0,T71,F17.0,T89,F17.0,T107,I5,/,/)
      GO TO 10
  550 CONTINUE
  999 CONTINUE
      STOP
      END
C
C
C              SUBROUTINES AND FUNCTIONS
C
C              DEGREE,RADIAN,EQUAT1,INVDEG,INVRAD
C
      SUBROUTINE DEGREE(DEGRE,DEG,RMIN,SEC)
      IMPLICIT REAL*8 (A-H,O-Z)
      DEGRE=DABS(DEG)+RMIN/60.D0+SEC/3600.D0
      DEGRE=DSIGN(DEGRE,DEG)
      RETURN
      END
C
```

```
C
      SUBROUTINE RADIAN(RAD,DEGRE)
      COMMON/AA/RADC,E2
      RAD=DEGRE/RADC
      RETURN
      END
C
C
      FUNCTION EQUAT1(GAMMA)
      IMPLICIT REAL*8 (A-H,O-Z)
      E2=.00676866D0
      EQUAT1=DSQRT(1.D0-(E2*DCOS(GAMMA)**2))
      RETURN
      END
C
C
      SUBROUTINE INVRAD(DEGRE,RAD)
      IMPLICIT REAL*8 (A-H,O-Z)
      COMMON/AA/RADC,E2
      DEGRE=RAD*RADC
      RETURN
      END
C
C
      SUBROUTINE INVDEG(INT,MIN,SEC,DEGRE)
      IMPLICIT REAL*8 (A-H,O-Z)
      INT=IIDINT(DEGRE)
      REAL=DFLOAT(INT)
      FRAC=DEGRE-REAL
      RSLT=FRAC*60.D0
      M=IIDINT(RSLT)
      REAL=DFLOAT(M)
      MIN=REAL
      FRAC=RSLT-REAL
      SEC=FRAC*60.D0
      RETURN
      END
```

TABLE 1. Angles and Calculated Arc Distances

GEODETIC	GEOCENTRIC	PARAMETRIC	NUMERICAL GAUSSIAN INTEGRATION OF $S=a\int(1-e^2\cos^2\gamma)d\gamma$ γ=parametric latitude	APPROXIMATION CLOSED FORM USING γ-PARAMETRIC	AVERAGE RADII OF CURVATURE $\sum\left(\frac{r_{C1}+r_{C2}}{2}\right)d\alpha$ α = geodetic latitude	NITER NO. OF ITERATIONS IN INTEGRAL EVALUATION
0 30 0.0000	0 29 47.8170	0 29 53.8982	55284	55284	55284	2
1 0 0.0002	0 59 35.6379	0 59 47.7984	110567	110567	110567	2
1 30 0.0004	1 29 23.4661	1 29 41.7023	165851	165851	165851	2
2 0 0.0002	1 59 11.3050	1 59 35.6113	221135	221135	221135	2
2 30 0.0007	2 28 59.1593	2 29 29.5285	276420	276420	276420	2
3 0 0.0005	2 58 47.0312	2 59 23.4542	331705	331705	331705	2
3 30 0.0003	3 28 34.9251	3 29 17.3910	386990	386990	386990	2
4 0 0.0009	3 58 22.8454	3 59 11.3414	442276	442276	442276	2
4 30 0.0014	4 28 10.7950	4 29 5.3066	497563	497563	497563	2
5 0 0.0005	4 57 58.7760	4 58 59.2869	552850	552850	552850	2
5 30 0.0010	5 27 46.7951	5 28 53.2870	608139	608139	608139	2
6 0 0.0016	5 57 34.8544	5 58 47.3074	663428	663428	663428	2
6 30 0.0006	6 27 22.9559	6 28 41.3483	718718	718718	718718	2
7 0 0.0012	6 57 11.1063	6 58 35.4146	774009	774010	774010	2
7 30 0.0017	7 26 59.3076	7 28 29.5065	829302	829302	829302	2
8 0 0.0008	7 56 47.5620	7 58 23.6243	884596	884596	884596	2
8 30 0.0029	8 26 35.8774	8 28 17.7743	939891	939891	939891	2
9 0 0.0019	8 56 24.2513	8 58.11.9524	995188	995188	995188	2

APPENDIX

From a standard calculus book* the radius of curvature is defined by the formula below.

$$R_C = \frac{[(f')^2 + (g')^2]^{3/2}}{[f'g'' - g'f'']} \qquad \text{(A-1)}$$

Using the equations from the ellipse the f and g functions are defined.

$$X = f(\gamma) = a \cos \gamma$$
$$Y = g(\gamma) = b \sin \gamma \qquad \text{(A-2)}$$

where γ is the parametric latitude defined earlier.

Then by differentiating these equations they can be used in the formula for the radius of curvature.

$$f' = -a \sin \gamma \qquad f'' = -a \cos \gamma$$
$$g' = b \cos \gamma \qquad g'' = -b \sin \gamma \qquad \text{(A-3)}$$

$$R_C = \frac{(a^2 \sin^2\gamma + b^2 \cos^2\gamma)^{3/2}}{(ab \sin^2\gamma + ab \cos^2\gamma)}$$

$$= \frac{1}{ab} [a^2(1-\cos^2\gamma) + b^2\cos^2\gamma]^{3/2}$$

$$= \frac{1}{ab} [a^2 - (a^2-b^2)\cos^2\gamma]^{3/2}$$

$$R_C = \frac{1}{ab} \left[a^2(1 - \frac{a^2-b^2}{a^2} \cos^2\gamma\right]^{3/2}$$

As seen earlier in this report $e^2 = \frac{a^2-b^2}{a^2}$. By transforming this equation the relationships below are obtained.

$$\frac{b^2}{a^2} = 1 - e^2 \qquad \frac{a}{b} = \frac{1}{(1-e^2)^{1/2}} \qquad \text{(A-4)}$$

Therefore e^2 and $\frac{a}{b}$ can be substituted into the R_C equation.

$$R_C = \frac{a^3}{ab} \; (1-e^2\cos^2\gamma)^{3/2}$$

$$= \left(\frac{a}{b}\right) a(1-e^2\cos^2\gamma)^{3/2}$$

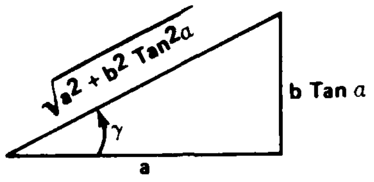
*<u>Calculus with Analytical Geometry</u> by Earl W. Swokowski

$$R_C = \frac{a(1-e^2\cos^2\gamma)^{3/2}}{(1-e^2)^{1/2}} \quad . \tag{A-5}$$

Since the radius of curvature equation is needed in terms of α (the geodetic latitude) the relationship between γ and α is needed.

$$\mathrm{Tan}\,\gamma = \frac{b}{a}\,\mathrm{Tan}\ \alpha$$

If a triangle like the one shown below is drawn, then the $\cos^2\gamma$ can be obtained.



$$\cos^2\gamma = \frac{a^2}{a^2+b^2\mathrm{Tan}^2\alpha}$$

$$= \frac{1}{1 + \frac{b^2\mathrm{Tan}^2\alpha}{a^2}} \quad . \tag{A-6}$$

Since $\frac{b^2}{a^2} = (1-e^2)$ then

$$\cos^2\gamma = \frac{1}{1+(1-e^2)\mathrm{Tan}^2\alpha} \tag{A-7}$$

By expanding Equation (A-7) and substituting it into Equation (A-5) then Equation (21) can be derived.

$$\cos^2\gamma = \frac{1}{1+\mathrm{Tan}^2\alpha-e^2\mathrm{Tan}^2\alpha}$$

$$R_C = \frac{a\left(1-e^2\left(\frac{1}{1+\mathrm{Tan}^2\alpha-e^2\mathrm{Tan}^2\alpha}\right)\right)^{3/2}}{(1-e^2)^{1/2}}$$

$$= \frac{a\left(1-\frac{e^2}{\sec^2\alpha-e^2\mathrm{Tan}^2\alpha}\right)^{3/2}}{(1-e^2)^{1/2}}$$

$$= a \frac{\left(1 - \frac{e^2}{\frac{1-e^2\sin^2\alpha}{\cos^2\alpha}}\right)^{3/2}}{(1-e^2)^{1/2}}$$

$$= a \left(\frac{1-e^2\sin^2\alpha - e^2\cos^2\alpha}{1-e^2\sin^2\alpha}\right)^{3/2} \cdot \frac{1}{(1-e^2)^{1/2}}$$

$$R_C = \frac{a(1-e^2)}{(1-e^2\sin^2\alpha)^{3/2}}$$

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