AD-A152 376 EFFECTS OF INSPECTION ERRORS ON CURTAILED DORFMAN-TYPE PROCEDURES(U) MARYLAND UNIV COLLEGE PARK DEPT OF MANAGEMENT SCIENCES AND ST.. S KOTZ ET AL. MAR 85 1/1

UNCLASSIFIED UMD-DMSS-85-4 N00014-84-K-0301 F/G 12/1 NL

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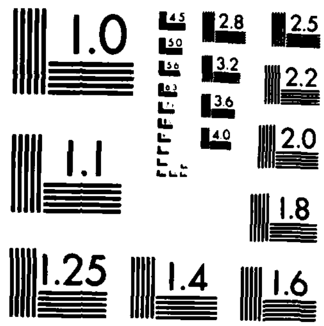


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# EFFECTS OF INSPECTION ERRORS ON CURTAILED DORFMAN-TYPE PROCEDURES

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March 1985

Technical Report 85-04

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# EFFECTS OF INSPECTION ERRORS ON CURTAILED DORFMAN-TYPE PROCEDURES

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*Key Words and Phrases: attribute inspection; binomial distribution; hypergeometric distribution; mixtures; sequential methods*

## ABSTRACT

The effects of inspection error on a two-stage procedure for identification of defective units is studied. The first stage is intended to provide the number of defective units in a group of n units; the second stage consists of individual inspection until the status of all units is (apparently) established.

## 1. INTRODUCTION

Dorfman (1943) described a method known as 'group screening' for defective units, in which random samples of size n (from a lot of size N) are tested as a group for presence of one or more defective units. If the result of this test is positive, then each unit in the group (sample) is inspected individually, in order to ascertain the units which are defective.

This procedure will sometimes reduce the expected number of inspections below n, the number required if each unit is tested immediately. This will be so if there is a sizeable chance that none of the units is defective, and so there is a low probability of proceeding to individual testing. In fact, the expected number of inspection is $1 + n\{1 - \Pr(\text{no defective units in random sample of size } n)\} = 1 + n(1 - P_o)$ which is less than n if $P_o > n^{-1}$.

Further reduction in average number of inspections can be effected by noting that if there is at least one defective unit, and none is observed in the first (n-1) units inspected individually, then the last one must be defective, and need not be inspected. The procedure so modified is a curtailed Dorfman procedure.

Pfeifer and Enis (1978) consider situations in which the result of the first (group) test is measurable, as the total of responses for individual units. It is supposed that a nondefective unit gives zero response, while a defective unit gives a positive response (not necessarily the same for all units, but the same for a given unit at each inspection). When individual testing is needed, it is not continued once the total of individual responses equals that observed from the group test. The remaining (untested) units are then known to be nondefective.

Even further reduction is possible when the response from any defective item is a constant which can be taken, without loss of generality, to be 1. Then the total response from the group test, $Z_1$ say, is the number of defective units (Y) among the n units in the group - for example, as a reading in an automatic counter - and individual testing simply identifies which are the defective units. (This is Sobel's (1968) "BH" problem. That paper is mainly concerned with construction of optimal sampling plans, analogous to weighing designs.) In this situation not only can testing be stopped as soon as $Z_1$ 'defective' units have been identified (the remaining, untested units being classed as nondefective), but if only $(Z_1-W)$ 'defectives' have been identified among the first $(n-W)$ units tested, the remaining W units 'must' be defective and do not need to be tested.

In this paper we will consider the effects of inspection errors on this last procedure. We suppose that for the group test: - $\Pr[\text{judge def.}|\text{def.}] = p_1$,
$\Pr[\text{judge def.}|\text{nondef.}] = p_1'$ ;
and for a unit test: - $\Pr[\text{judge def.}|\text{def.}] = p$,
$\Pr[\text{judge def.}|\text{nondef.}] = p'$.
In this case, of course, it will not always be true that

$$Z_1 = Y.$$

In fact given Y, $Z_1$ will be distributed as the convolution of two binomial distributions

$$Z_1|Y \frown \mathrm{Bin}(Y,p_1)*\mathrm{Bin}(n-Y,p_1'). \tag{1}$$

so that

$$P(z|n,Y,p_1,p_1') = \Pr[Z=z|n,Y,p_1,p_1')$$

$$= \sum_{w=0}^{z} \binom{Y}{w}\binom{n-Y}{z-w} p_1{}^{w} p_1'{}^{z-w}(1-p_1)^{Y-w}(1-p_1')^{n-Y-z+w} \tag{2}$$

We suppose that the procedure goes on <u>as if</u> there were no inspection errors, so that if $Z_1 > 0$ individual testing commences and continues until either $Z_1$ units are identified as 'defective' or $(Z_1-W)$ 'defectives' are identified among the first $(n-W)$ unit inspected (for any $W=0,1,\ldots,n-1$)).

There would be difficulty in dealing with the more general case of responses of different sizes for different units, because this would involve introduction of some hypotheses about the size of the response which might be obtained from a nondefective unit as a result of inspection error.

## 2. SINGLE STAGE SCREENING

2a *Distribution of Number of Tests*

The number of defective units ($Z_2$) which would be obtained if all units were tested individually has the same distribution as $Z_1$ (see (1)) but with $p_1, p_1'$ replaced by $p, p'$. Furthermore, $Z_1$ and $Z_2$ are independent given Y.

Denote by M the number of tests actually carried out with the curtailed modified Dorfman procedure. Clearly $M=0$ if $Z_1=0$ or $Z_1=n$. [If $Z_1=n$ we 'know' that all n units are defective.]

If $m=1,2,\ldots(n-1)$, then $M=m$ if either:

(a) the m-th unit tested is judged 'defective', and exactly $(Z_1-1)$ among the previous $(m-1)$ units were judged 'defective'.

or (b) the m-th unit tested is judged 'nondefective' and exactly $(n-Z_1-1)$ among the previous $(m-1)$ units were judged 'defective'.

Hence, for $m=1,2,\ldots,n-1$

$$\Pr[M=m|Z_1=z_1,Z_2=z_2] = \begin{cases} \binom{m-1}{z_1-1}\binom{n-m}{z_2-z_1}/\binom{n}{z_2} & \text{if } z_2 > z_1 \\ \{\binom{m-1}{z_1-1}+\binom{m-1}{n-z_1-1}\}/\binom{n}{z_1} & \text{if } z_2 = z_1 \\ \binom{m-1}{n-z_1-1}\binom{n-m}{z_1-z_2}/\binom{m}{z_2} & \text{if } z_2 < z_1, \end{cases} \tag{3.1}$$

while

$$\Pr[M=0|\,0,\ z_2] = \Pr[M=0|n,z_2] = 1 \qquad \text{for all } z_2 \tag{3.2}$$

and $\Pr[M=0|z_1,z_2]=0$ for all $z_1 \neq 0,n$.

(Note that $\binom{a}{b}=0$ if $a < 0$, $b < 0$ or $b > a$. We will also use the relation $\sum_{a=c_1}^{n-c_2} \binom{a}{c_1}\binom{n-a}{c_2} = \binom{n+1}{c_1+c_2+1}$.)

From (3), or by direct analysis: -

$$\underline{\text{if } z_2 > z_1} \qquad E[M|z_1,z_2] = \sum_{m=z_1}^{n-z_2+z_1} m\binom{m-1}{z_1-1}\binom{n-m}{z_2-z_1}\Big/\binom{n}{z_2}$$

$$= z_1 \sum_{m=z_1}^{n-z_2+z_1} \binom{m}{z_1}\binom{n-m}{z_2-z_1}\Big/\binom{n}{z_2}$$

$$= z_1 \binom{n+1}{z_2+1}\Big/\binom{n}{z_2} = (n+1)z_1/(z_2+1) \qquad (4.1)$$

$$\underline{\text{if } z_2 = z_1} \qquad E[M|z_1,z_1] = z_1(n-z_1)\{(z_1+1)^{-1}+(n-z_1+1)^{-1}\} \qquad (4.2)$$

$$\underline{\text{if } z_2 < z_1} \qquad E[M|z_1,z_2] = (n+1)(n-z_1)/(n-z_2+1) \qquad (4.3)$$

Since if $z_1=0$ (n) we cannot have $z_2<(>)z_1$, formulae (4) include the cases when $z_1=0$ or $z_1=n$ (see (3.2)).

The unconditional expected value of M is

$$E[M] = E_Y[E_{Z_1,Z_2}[E[M|Z_1,Z_2]]|Y]$$

$$= \sum_{y=o}^{n} \Pr[Y=y] \sum_{z_1=o}^{n} \sum_{z_2=o}^{n} P(z_1|n,Y,p_1,p_1')P(z_2|n,Y,p,p')E[M|z_1,z_2] \qquad (5)$$

If lot size is infinite with a proportion $\theta$ of defective units

$$\Pr[Y=y] = \binom{n}{y}\theta^y(1-\theta)^{n-y} \quad (y=0,1,\ldots,n) \qquad (6.1)$$

If lot size is N, with D defective units

$$\Pr[Y=y] = \binom{D}{y}\binom{N-D}{n-y}\Big/\binom{N}{n} \quad (\max(0,n-N+D) \le y \le \min(D,n)) \qquad (6.2)$$

If inspection is error-free then $Z_1 = Z_2 = Y$ and

$$E[M|Y] = n+2 - (n+1)\{(Y+1)^{-1}+(n-Y+1)^{-1}\} \qquad (7)$$

If lot size is infinite, with proportion of defective units equal to $\theta$ then

$$E[(Y+1)^{-1}] = \frac{1}{(n+1)\theta}\{1 - (1-\theta)^{n+1}\}$$

$$E[(n-Y+1)^{-1}] = \frac{1}{(n+1)(1-\theta)} \quad (1 - \theta^{n+1})$$

and so

$$E[M] = n+2 - \{\theta(1-\theta)\}^{-1}\{1 - \theta^{n+2} - (1-\theta)^{n+2}\} \tag{8.1}$$

The total expected number of inspections is

$$1 + E[M] = n+3 - \{\theta(1-\theta\ \}^{-1}\{1-\theta^{n+2} - (1-\theta)^{n+2}\} \tag{8.2}$$

If the lot size is N and there are D defective units in the lot then

$$E[(Y+1)^{-1}] = \begin{cases} \dfrac{N+1}{(n+1)(D+1)} \quad \{1 - \dfrac{(N-n)!(N-D)!}{(N+1)!(N-n-D-1)!}\} & \text{if } N > n+D \\ \dfrac{N+1}{(n+1)(D+1)} & \text{if } N \leq n+D \end{cases}$$

(Johnson and Kotz (1969, p. 145)).

and

$$E[(n-Y+1)^{-1}] = \begin{cases} \dfrac{N+1}{(n+1)(N-D+1)} \quad \{1 - \dfrac{(N-n)!D!}{(N+1)!(D-n-1)!}\} & \text{if } D > n \\ \dfrac{N+1}{(n+1)(N-D+1)} & \text{if } n \geq D\ . \end{cases}$$

Using the second formula in each case as an approximation,

$$\begin{aligned} E[M] &\doteqdot n+2 - (N+1)(\frac{1}{N-D+1} + \frac{1}{D+1}) \\ &\doteqdot n+2 - \frac{(N+1)(N+2)}{(N-D+1)(D+1)} \end{aligned} \tag{9.1}$$

and, of course, the total number of inspections is approximately

$$1 + E[M] \doteqdot n+3 - \frac{(N+1)(N+2)}{(N-D+1)(D+1)} \tag{9.2}$$

2b *Probability of Correct Classification*

A) A defective unit is classified correctly if

(i) $Z_1 > 0$

and (ii) it is classified 'defective' on individual testing, and is among the first $Z_1$ units so classified.

or (iii) it is classified 'nondefective' on individual testing, and there are already at least $(n-Z_1)$ units so classified.

Let $Z_2'$ denote the number of units classified as 'defective' on individual testing, among the remaining (n-1) units. Then $\Pr'[Z_2'=z] = P(z|n-1,Y-1,p,p')$.

Conditionally on $Z_2'$ and $Z_1$, the probability of correct classification is

$$PC(Def.|Z_1,Z_2') = p \min(\frac{Z_1}{Z_2'+1}),1) + (1-p) \max(\frac{Z_1-Z_2'}{n-Z_2'},0) \tag{10}$$

Note that this is zero if $Z_1=0$, so that condition (i) is allowed for automatically. The unconditional probability of corect classification is

$$PC(Def.) = E_Y^*[E_{Z_1,Z_2'}[p \min(\frac{Z_1}{Z_2'+1},1) + (1-p) \max(\frac{Z_1-Z_2'}{n-Z_2'},0)|Y]] \tag{11}$$

where

$E_Y^*$ refers to the conditional distribution of Y, given that $Y \geq 1$.

This probability can be calculated from the formula

$$PC(Def.) = \frac{1}{1-\Pr[Y=0]} \sum_{y=1}^{n} \Pr[Y=y] \sum_{z_1=1}^{n} \sum_{z_2'=0}^{n=1} P(z_1|n,y,p_1,p_1') \times P(z_2'|n-1,y-1,p,p')PC(Def.|z_1,z_2') \tag{12}$$

B) A nondefective unit is classified correctly if

(i) $Z_1=0$

or $Z_1 > 0$ and (ii) it is classified 'nondefective' on individual testing, and is among the first $(n-Z_1)$ units so classified,

or (iii) it is classified 'defective' on individual testing and there are already at least $Z_1$ units so classified.

We now have

$$\Pr[Z_2'=z] = P(z|n-1,Y,p,p']$$

and conditionally on $Z_2'$ and $Z_1$, the probability of correct classification is

$$P(\text{Nondef.}|Z_1,Z_2') = (1-p')\min\left(\frac{n-Z_1}{n-Z_2'},1\right) + p'\max\left(\frac{Z_2'-Z_1+1}{Z_2'+1},0\right) \tag{13}$$

Note that this equals 1 if $Z_1=0$, so that condition (i) is allowed for automatically.

The unconditional probability of correct classification is

$$= \frac{1}{1-\Pr[Y=n]}\sum_{y=0}^{n=1} \Pr[Y=y]\sum_{z_1=0}^{n}\sum_{z_2'=0}^{n-1} P(z_1|n,y,p_1,p_1') \times P(z_2'|n-1,y,p,p')PC(\text{Nondef.}|z_1,z_2') \tag{14}$$

The overall probability of correct classification is

$$\theta\, PC(\text{Def.}) + (1-\theta)PC(\text{Nondef.})$$

if lot size is infinite with properties defective unit ,

$$N^{-1}\{D\; PC(\text{def.}) + (N-D)PC(\text{Nondef.})\}$$

if lot size is N, with D defective units.

Tables of E[M], PC(def.) and PC(nondef.) for comparison with the tables (for standard Dorfman procedures) in Kotz and Johnson (1982), are in preparation.

## ACKNOWLEDGEMENTS

Dr. Samuel Kotz's work was supported by the U.S. Office of Naval Research under Contract N 00014-84-K-301.

Professor Mao Shisong's work was carried out during a visit to the University of Maryland, College Park in 1985.

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| RESS (City, State, and ZIP Code)<br>lington, Va. 22217 | 10. SOURCE OF FUNDING NUMBERS<br>PROGRAM ELEMENT NO. / PROJECT NO. / TASK NO. / WORK UNIT ACCESSION NO |

E (Include Security Classification)
ects of Inspection Errors on Curtailed Dorfman-Type Procedures

SONAL AUTHOR(S)
uel Kotz, Mao Shisong and Norman L. Johnson

| PE OF REPORT | 13b. TIME COVERED | 14. DATE OF REPORT (Year, Month, Day) | 15. PAGE COUNT |
|---|---|---|---|
| chnical | FROM ____ TO ____ | March 1985 | nine (9) |

PLEMENTARY NOTATION

| COSATI CODES | | | 18. SUBJECT TERMS (Continue on reverse if necessary and identify by block number) |
|---|---|---|---|
| LD | GROUP | SUB-GROUP | attribute inspection, binomial distribution, hypergeometric distribution, mixtures, sequential methods. |

TRACT (Continue on reverse if necessary and identify by block number)

e effects of inspection error on a two-stage procedure for identification
defective units is studied. The first stage is intended to provide
e number of defective units in a group of n units; the second stage
nsists of individual inspection until the status of all units is
pparently) established. [illegible]

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