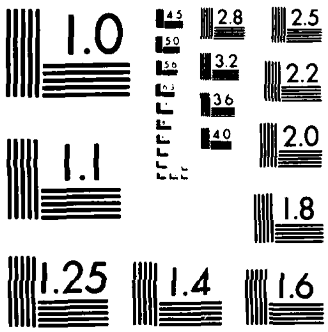
AD-A152 377 STATISTICAL EFFECTS OF IMPERFECT INSPECTION SAMPLING II 1/1
DOUBLE SAMPLING L..(U) MARYLAND UNIV COLLEGE PARK DEPT
OF MANAGEMENT SCIENCES AND ST.. N L JOHNSON ET AL.
UNCLASSIFIED MAR 85 UMD-DMSS-85-5 N00014-84-K-0301 F/G 12/1 NL

END



MICROCOPY RESOLUTION TEST CHART

NATIONAL BUREAU OF STANDARDS 1963 A

AD-A152 377

(2)

Statistical Effects of Imperfect Inspection

Sampling: II. Double Sampling, Link Sampling and Grading

Norman L. Johnson, Samuel Kotz and Robert N. Rodriguez

March 1985

Technical Report 85-03 (Final Version)

Department of Management Science and Statistics

College of Business and Management

University of Maryland

College Park

STATISTICAL EFFECTS OF IMPERFECT INSPECTION SAMPLING:

II. DOUBLE SAMPLING, LINK SAMPLING AND GRADING

by

Norman L. Johnson
University of North Carolina at Chapel Hill

Samuel Kotz
University of Maryland, College Park

and

Robert N. Rodriguez
SAS Institute, Cary, North Carolina

ABSTRACT: This paper, the second in a series of three, utilizes the basic distributions established in the first paper to assure the effect of inspection errors in double sampling, link and partial link sampling, and in grading acceptance procedures.

The numbering of sections, equations and tables follows from that of the previous paper in this series (Johnson, Kotz and Rodriguez (JKR) (1985)).

KEY WORDS: Acceptance Sampling Plans, Binomial Distribution, Compound Distribution, Double Sampling, Link Sampling, Hypergeometric Distribution, Inspection Error Tables.

4. Double Sampling

A double sampling procedure is defined by the values of five parameters:

n_1, n_2 - sizes of first and second stage random samples, respectively

a_1, a_2 - acceptance numbers at first and second stage, respectively

$a_1' + 1$ - rejection number at first-stage.

Letting Z_1, Z_2 denote the numbers of items classified (rightly or wrongly) as 'nonconforming' in the first and second stage samples, respectively, the procedure is as follows; -

(i) Take a random sample of size n, and record the (apparent) number, Z_1, of nonconforming items.

(ii) If $Z_1 \leq a_1$, accept. If $Z_1 > a_1'$, reject.

(iii) If $a_1 < Z_1 \leq a_1'$, takes a further random sample of size n_2, and record the (apparent) number, Z_2, of nonconforming items.

(iv) If $Z_1 + Z_2 < a_2$, accept. If $Z_1 + Z_2 > a_2$, reject.

(Commonly, though not necessarily, $n_2 = 2n_1$ and $a_2 = a_1'$.)

The probability of acceptance is

$$\Pr[Z_1 \leq a_1] + \Pr[(a_1 < Z_1 \leq a_1') \cap (Z_1 + Z_2 \leq a_2)] \tag{15}$$

We will suppose (as in JKR (1985)) that sampling is without replacement. The probabilities in (15) depend on p (probability of detecting a nonconforming item) and p' (probability of incorrectly describing a conforming item as 'nonconforming') as well as the parameters of the sampling procedure, N (the size of the lot) and D (the number of nonconforming items in the lot). The first term in (15) is evaluated by summing (4)' (with suffices '1' for h and z) over $0 \leq z \leq a_1$; the second is evaluated by summing (8)' (with k=2) over $(a_1 < z_1 \leq a_1')$ $(z_1 + z_2 \leq a_2)$.

Table 3 was constructed in this way. It gives acceptance probabilities for some procedures defined in Table III-A of Sampling Procedures and Tables for Inspection by Attributes (1981), for N = 100, 200; D/N = 0.05, 0.10, 0.20; p = 0.75, 0.90, 0.95, 0.98, 1.00; p' = 0, 0.01, 0.02, 0.05, 0.10. Figures 3a-c, 4a-c, 5a-c and 6a-c provide graphic representation of these values. In each set of these figures there are presented results from those double sampling schemes corresponding to average quality levels (AQL) 1.5%, 4% and 10% nonconforming items respectively.

5. Link Sampling

As an alternative to double sampling, it has been proposed to use results of routine samples from neighboring lots in production sequence for the second sample, when needed. Harishchandra and Srivenkataramana (HS) (1982) describe the following 'link sampling' procedure, based on random samples (without replacement) of size n each from out of a sequence of lots of size N. We will denote the actual number of nonconforming items in the i-th lot by D_i, the actual number in the sample of size n from this lot by Y_i, the actual number in the sample of size n from this lot by Y_i, and the number classified as nonconforming in this sample by Z_i. The link sampling division rules for the i-th lot are:

(a) If $Z_i \le a_1$ the lot is accepted

(b) If $Z_i > a_2$ the lot is rejected

(c) If $a_1 < Z_i \le a_2$ and $Z_{i-1} + Z_i + Z_{i+1} \le a'_2$ the lot is accepted

(d) If $a_1 < Z_i \le a_2$ and $Z_{i-1} + Z_i + Z_{i+1} > a'_2$ the lot is rejected

The Z_i's are mutually independent; Z_i has a distribution of form (1) with D replaced by D_i. The probability of acceptance for the i-th lot is:

$$\Pr[Z_i \le a_1 | D_i] + \sum_{z=a_1+1}^{a_2} \Pr[Z_i = z | D_i] \Pr[Z_{i-1} + Z_{i+1} \le a'_2 - z | D_{i-1}, D_{i+1}]$$

$$= \sum_{z_i=0}^{a_1} P(z_i|D_i) + \sum_{z_i=a_1+1}^{a_2} P(z_i|D_i) \sum_{z_{i+1}=0}^{a'_2-z_i} \sum_{z_{i-1}=0}^{a'_2-z_i-z_{i-1}} P(z_{i-1}|D_{i-1}) P(z_{i+1}|D_{i+1})$$

where $P(z_j|D_j) = \Pr[Z_j = z_j]$ (16)

$$= \binom{N}{n}^{-1} \sum_y \binom{D_j}{y}\binom{N-D_j}{y} \sum_{u=0}^{y} \binom{y}{u}\binom{n-y}{z_j-u} p^u p'^{z_j-u} (1-p)^{y-u} (1-p')^{n-y-z_j+u}$$

(cf (2))

In order to reduce the time needed to reach a decision, two alternative methods are suggested (in HS (1982) as possible alternatives.

(1) In (c) (and (d)) replace $Z_{i-1} + Z_i + Z_{i+1} \leq (>) \ a_2'$ by

$$Z_{i-1} + Z_i \leq (>) \ a_2''$$

or (2) If $a_1 < Z_i \leq a_2$ then take a second sample of size n (<u>not</u> 2n) from the i-th lot, and replace Z_{i+1} (in (a) and (d)) by Z_i', the number of items classified as nonconforming in this second sample.

Method (2) is termed 'partial link sampling' - it saves some sampling effort as compared with regular double sampling, and avoids the need to wait for results of sampling the next ((i+1)-th) lot. The analysis is a bit more complicated than for link sampling because Z_i and Z_i' are not independent (though they would be for sampling with replacement, or for N infinite). The joint distribution of Z_i and Z_i' is, symbolically

$$\begin{Bmatrix} Z_i \\ Z_i' \end{Bmatrix} \cap \begin{Bmatrix} \text{Bin}(Y_i, p) * \text{Bin}(n-Y_i, p') \\ \text{Bin}(Y_i', p) * \text{Bin}(n-Y_i', p') \end{Bmatrix} \bigwedge_{Y_i, Y_i'} \text{Mult Hypg}(n,n;D_i;N) \qquad (17)$$

(Y_i' denotes the actual number of nonconforming items in the second sample from the i-th lot, and the joint distribution of Y_i, Y_i' is given by (8)' with k=2, $n_1=n_2=n$.)

The expected number of items inspected in the i-th lot is $n\{1+\Pr[a_1 < Z_i \leq a_2']\}$, while with regular double sampling (with $n_1=n$, $n_2=2n$) it is $n\{1+2\Pr[a_1 < Z_i \leq a_2']\}$.

Link sampling decision procedures are useful only if D_i does not change too abruptly from one lot to the next. For $N=\infty$ (or sampling with replacement), if $D_{i-1}=D_i=D_{i+1}(=D)$ the acceptance probabilities for both link and partial link sampling are the <u>same</u> as for regular double sampling. Although this is nearly so for finite N (and sampling without replacement) it is not precisely so, because the convolution Hypg(n;D;N) * Hypg(n;D;N) is not the same as Hypg(2n;D;N) (or Hypg(2n;2D;2N)), Tables 4 and 5 contain values of acceptance probabilities for link sampling and partial link sampling, respectively, for the same values of p and p' as in Table 3, for a few sampling schemes chosen for illustrative purposes.

Tables 4-1 and 4-2 are comparable with 5-1, and Table 4.3 with 5.2. Table 4-4 can be compared with 5-3, and Tables 4-5, 4-6 and 4-7 with 5-4.

Figures 7a-c give some link sampling acceptance probability distributions. They can be compared with Figure 7d which gives the distributions for double sampling with the same sample sizes and with the same number of nonconforming items (D=20) in the lot under inspection. For this lot size (N=100) the differences are minor, but for lot size 70 (see Figures 8a and 8b) the discrepancies between double sampling and link sampling with the same (constant) D values become much more marked. (Note that if the second sample is used then 60 out of the 70 items in the lot are examined, so this is a rather extreme case.

6. Grading

Sometimes inspection does not lead to the simple all-or-none decision 'conforming' or 'nonconforming' but rather to assignment to one of k possible categories $c_1, \ldots c_k$. It is convenient to think of these categories as corresponding to 'quality' and the decision process as 'grading'.

We will denote the probability that an item which really belongs to c_j will be classified as b_i by P_{ij}. Of course,

$$\sum_{i=1}^{k} P_{ij} = 1 \quad \text{for all } j.$$

In this section Y_j will denote the number of items in c_j in a random sample of size n from a lot of size N containing N_j items in category c_j, and Z_{ij} will denote the number, among these Y_j, classified as $c_i(i,j=1,\ldots k)$.

Clearly, $\sum_{j=1}^{k} N_j=N$; $\sum_{j=1}^{k} Y_j=n$; $\sum_{i=1}^{k} Z_{ij}=Y_j$; $\underset{\sim}{Y} = (Y_1, \ldots, Y_k)$ has a multivariate hypergeometric distribution with parameters $(n;\underset{\sim}{N};N)$, so that

$$\Pr[\underset{\sim}{Y}=\underset{\sim}{y}] = \{\prod_{j=1}^{k} \binom{N_j}{y_j}\}/\binom{N}{n}$$

$$= \binom{N}{n}^{-1} \prod_{j=1}^{k} (N_j^{(y_j)}/y_j!) \qquad (18)$$

where $N_j^{(y_j)} = N_j(N_j-1)\ldots(N_j-y_j+1)$.

Given $\underset{\sim}{Y}$, the vector $\underset{\sim}{Z}_j = (Z_{1j}, \ldots, Z_{kj})$ of numbers of c_j grade items in the sample classified as grades $c_1, \ldots, c_k$ respectively, has a multinomial distribution with parameters $(y_j, \underset{\sim}{P}_j)$ where $\underset{\sim}{P}_j = (P_{1j}, \ldots, P_{kj})$, so that

$$\Pr[\underset{\sim}{Z}_j=\underset{\sim}{z}_j|\underset{\sim}{y}_j] = y_j! \prod_{i=1}^{k} (P_{ij}^{z_{ij}}/z_{ij}!) \qquad (\sum_{i=1}^{k} z_{ij} = y_j)$$

$\underset{\sim}{Z}_1, \ldots, \underset{\sim}{Z}_k$ are mutually independent so that

$$\Pr[\underset{\sim}{Z}=\underset{\sim}{z}] = \Pr[\underset{\sim}{Z}=\underset{\sim}{z}|\underset{\sim}{y}] \Pr[\underset{\sim}{Y}=\underset{\sim}{y}] = [\prod_{j=1}^{k} \Pr[\underset{\sim}{Z}_j=\underset{\sim}{z}_j|\underset{\sim}{y}_j]] \Pr[\underset{\sim}{Y}=\underset{\sim}{y}]$$

$$= \binom{N}{n}^{-1} \prod_{j=1}^{k} \{N_j^{(z_{\cdot j})} \prod_{i=1}^{k} (P_{ij}^{z_{ij}}/z_{ij}!)\} \tag{19}$$

where $z_{\cdot j} = \sum_{i=1}^{k} z_{ij}$.

(Note that since the values of $\underset{\sim}{y}$ are determined by the values of $\underset{\sim}{z}$, there is no need to take expected values with respect to $\underset{\sim}{Y}$.)

Symbolically

$$\underset{\sim}{Z} \cap \overset{k}{\underset{j=1}{*}} \text{ Multinomial } (Y_j, \underset{\sim}{P}_j) \bigwedge_{Y_j} \text{ Mult Hypg}(n;\underset{\sim}{N};N)$$

The mixed factorial moment $\mu_{(\underset{\sim}{r})}(\underset{\sim}{Z}) = E[\prod_{i=1}^{k} \prod_{j=1}^{k} Z_{ij}^{(Y_{ij})}]$

$$= E_{\underset{\sim}{Y}}[\prod_{j=1}^{k} \prod_{j=1}^{k} Z_{ij}^{(r_{ij})}|\underset{\sim}{Y}]]$$

$$= E_{\underset{\sim}{Y}}[\prod_{j=1}^{k} Y_j^{(r_{\cdot j})} \prod_{i=1}^{k} P_{ij}^{r_{ij}}]$$

$$= \frac{n^{(r_{\cdot\cdot})}}{N^{(r_{\cdot\cdot})}} \prod_{j=1}^{k} \left\{N_j^{(r_{\cdot j})} \prod_{i=1}^{k} P_{ij}^{r_{ij}}\right\} \tag{20}$$

where $r_{\cdot j} = \sum_{i=1}^{k} r_{ij}$; $r_{\cdot\cdot} = \sum_{i=1}^{k} \sum_{j=1}^{k} r_{ij}$.

Some further topics in grading will be discussed in the next paper in this series.

ACKNOWLEDGMENT

Samuel Kotz' work was supported by the Office of Naval Research under contract ONR-N-00014-84-K-301.

References

Johnson, N.L., Kotz, S. and Rodriguez, R.N. (1985) "Statistical Effects of Imperfect Inspection Sampling, I" Journal of Quality Technology, 17, 1-31.

Sampling Procedures and Tables for Inspection by Attributes (1981) ANSI/ASQC Z1.4, American Society for Quality Control, Milwaukee, Wisconsin.

Harishchandra, K. and Srivenkataramana, T. (1982) "Link Sampling for Attributes," Communications in Statistics - Theory and Methods, 11, 1855-1868.

Unclassified
SECURITY CLASSIFICATION OF THIS PAGE

REPORT DOCUMENTATION PAGE

1a REPORT SECURITY CLASSIFICATION Unclassified	1b RESTRICTIVE MARKINGS
2a SECURITY CLASSIFICATION AUTHORITY	3 DISTRIBUTION/AVAILABILITY OF REPORT Unlimited
2b DECLASSIFICATION/DOWNGRADING SCHEDULE	
4 PERFORMING ORGANIZATION REPORT NUMBER(S) UMD DMSS-85 5	5 MONITORING ORGANIZATION REPORT NUMBER(S)
6a NAME OF PERFORMING ORGANIZATION Dep't. of Mgmt. & Stat. 6b OFFICE SYMBOL (If applicable)	7a NAME OF MONITORING ORGANIZATION
6c ADDRESS (City, State, and ZIP Code) University of Maryland College Park, Md. 20742	7b ADDRESS (City, State, and ZIP Code)
8a NAME OF FUNDING/SPONSORING ORGANIZATION U.S. Office of Naval Research 8b OFFICE SYMBOL (If applicable)	9 PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER ONR N00014-84-K-0301
8c ADDRESS (City, State, and ZIP Code)	10 SOURCE OF FUNDING NUMBERS PROGRAM ELEMENT NO | PROJECT NO | TASK NO | WORK UNIT ACCESSION NO

11 TITLE (Include Security Classification)
Statistical Effects of Imperfect Sampling:II Double Sampling

12 PERSONAL AUTHOR(S)
Norman L. Johnson, Samuel Kotz and Robert N. Rodriguez

13a TYPE OF REPORT	13b TIME COVERED	14 DATE OF REPORT (Year, Month, Day)	15 PAGE COUNT
Technical (final)	FROM ______ TO ______	March 1985	8

16 SUPPLEMENTARY NOTATION

17 COSATI CODES			18 SUBJECT TERMS (Continue on reverse if necessary and identify by block number)
FIELD	GROUP	SUB-GROUP	Acceptance Sampling Plans, Binomial Distribution, Compound Distribution, Double Sampling, Link Sampling, Hypergeometric Distr., Inspection Error Tables.

19 ABSTRACT (Continue on reverse if necessary and identify by block number)

This paper, the second in a series of three, utilizes the basic distributions established in the first paper to assure the effect of inspection errors in double sampling, link and partial link sampling and in grading acceptance procedures.

20 DISTRIBUTION/AVAILABILITY OF ABSTRACT ☐ UNCLASSIFIED/UNLIMITED ☒ SAME AS RPT ☐ DTIC USERS	21 ABSTRACT SECURITY CLASSIFICATION Unclassified
22a NAME OF RESPONSIBLE INDIVIDUAL Samuel Kotz	22b TELEPHONE (Include Area Code) 301 454 6805 22c OFFICE SYMBOL

DD FORM 1473, 84 MAR — 83 APR edition may be used until exhausted. All other editions are obsolete.

SECURITY CLASSIFICATION OF THIS PAGE

END

FILMED

5-85

DTIC