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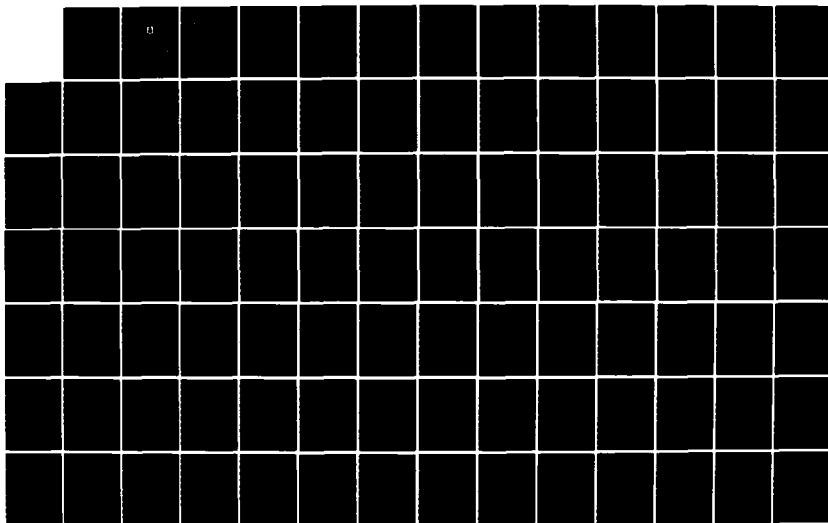
DIGITAL METHODS OF THE OPTIMUM PROCESSING OF RADAR
SIGNALS(U) FOREIGN TECHNOLOGY DIV WRIGHT-PATTERSON AFB
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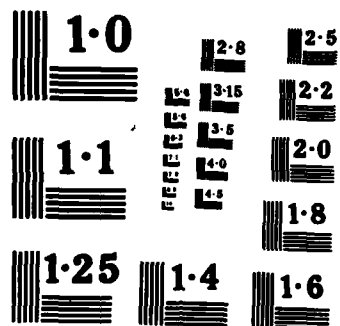
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DIGITAL METHODS OF THE OPTIMUM PROCESSING
OF RADAR SIGNALS

by

S. V. Samsonenko

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OF RADAR SIGNALS

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WP-AFB, OHIO.

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U. S. BOARD ON GEOGRAPHIC NAMES TRANSLITERATION SYSTEM

Block	Italic	Transliteration	Block	Italic	Transliteration
А а	<i>А а</i>	A, a	Р р	<i>Р р</i>	R, r
Б б	<i>Б б</i>	B, b	С с	<i>С с</i>	S, s
В в	<i>В в</i>	V, v	Т т	<i>Т т</i>	T, t
Г г	<i>Г г</i>	G, g	У у	<i>У у</i>	U, u
Д д	<i>Д д</i>	D, d	Ф ф	<i>Ф ф</i>	F, f
Е е	<i>Е е</i>	Ye, ye; E, e*	Х х	<i>Х х</i>	Kh, kh
Ж ж	<i>Ж ж</i>	Zh, zh	Ц ц	<i>Ц ц</i>	Ts, ts
З з	<i>З з</i>	Z, z	Ч ч	<i>Ч ч</i>	Ch, ch
И и	<i>И и</i>	I, i	Ш ш	<i>Ш ш</i>	Sh, sh
Й й	<i>Й й</i>	Y, y	Щ щ	<i>Щ щ</i>	Shch, shch
К к	<i>К к</i>	K, k	Ъ ъ	<i>Ъ ъ</i>	"
Л л	<i>Л л</i>	L, l	Ы ы	<i>Ы ы</i>	Y, y
М м	<i>М м</i>	M, m	Ь ь	<i>Ь ь</i>	'
Н н	<i>Н н</i>	N, n	Э э	<i>Э э</i>	E, e
О о	<i>О о</i>	O, o	Ю ю	<i>Ю ю</i>	Yu, yu
П п	<i>П п</i>	P, p	Я я	<i>Я я</i>	Ya, ya

*ye initially, after vowels, and after ъ, ь; e elsewhere.
When written as ë in Russian, transliterate as yë or ë.

RUSSIAN AND ENGLISH TRIGONOMETRIC FUNCTIONS

Russian	English	Russian	English	Russian	English
sin	sin	sh	sinh	arc sh	sinh ⁻¹
cos	cos	ch	cosh	arc ch	cosh ⁻¹
tg	tan	th	tanh	arc th	tanh ⁻¹
ctg	cot	cth	coth	arc cth	coth ⁻¹
sec	sec	sch	sech	arc sch	sech ⁻¹
cosec	csc	csch	csch	arc csch	csch ⁻¹

Russian	English
rot	curl
lg	log

GRAPHICS DISCLAIMER

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PAGE 1

Page 1.

DIGITAL METHODS OF THE OPTIMUM PROCESSING OF RADAR SIGNALS.

S. V. Samsonenko.

Page 2.

In book questions of use/application of digital computers for optimum processing of radar signals are examined. Primary attention is given to the detection of signals from the targets, hidden by interferences, and to the determination of the target coordinates. Is described the work of the simplest diagrams of working/treatment, their operating principle, and also work of some nodes of digital computers. Subsequently are given the more complicated cases, special theory and algorithms of working/treatment. Shows how it is realized the synthesis of the structures of machines and their algorithms on the base of the theory of statistical solutions. The necessary information according to the theory of statistical solutions is given on the course of presentation.

Book is intended for cadets of military colleges, students of military academies, military engineers and can be used by civil/civilian specialists, who work in region of radar.

Page 3.

Preface.

In recent years methods of mathematical statistics and information theory extensively were used for investigation of radar systems. The consecutive and fruitful use/application of these methods led to molding of the ordered theory of radar surveillance. This theory makes it possible to determine the structure of the optimum processing of radar signals for the different cases, to rate/estimate the potential possibilities of radar systems and to outline the advisable ways of their development.

Structure of optimum working/treatment, as a rule, escapes/ensues from algorithms of solution of problem about optimum detection of signal or measurement of its parameters under conditions of presence of interferences. Virtually the realization of processing signal on the algorithms of the statistical theory of detection and measurement, perhaps, most simply is accomplished by the means of electronic-computing technology, and in particular by digital computers. This led to the creation of the specialized electronic digital computers (computer(s)) of the discrete/digital processing of the radar signals, frequently called the machines of primary

processing, the radar input units, automated by radar observers, etc.

This relatively new branch of electronic engineering is located on joint of electronic digital computers and of technique of radar technique. At present questions of this direction are comparatively little illuminated in the literature. The proposed book to a certain degree must complete this gap/spacing in the literature.

Page 4.

In the book in essence questions of detection and measurement of angular coordinate are treated. To ranging is given smaller attention, since frequently reaching/achievement of the required accuracy of measurement virtually does not cause essential technical difficulties.

Presentation of questions indicated is systematic carried out in common plan on base of consecutive use/application of theory of statistical solutions. In the book initial relationships/ratios for the algorithms of processing signal and possible block diagrams of algorithms are given. Evaluation/estimate of the potential accuracy of the measurement of angle in the presence of interferences depending on a number of impulses/momenta/pulses in the packet with the different methods of amplitude direction finding is given. The

study of these problems is carried out for different types of the targets, the echo signal from which in the presence of interferences is characterized by different distribution functions.

In book some elements/cells and nodes of electronic digital computers, necessary for understanding of subsequent special sections, are described also.

For understanding of sections of a book, connected with statistical theory of detection and measurement, is necessary knowledge of probability theory and theory of random processes. However, for expanding the contingent of the readers some not very compound circuits of working/treatment are specially brought in chapter 4, where their qualitative description is given almost without the enlistment of mathematical apparatus. The content of this chapter is available to the wide circle of the readers. In this case the material of the chapter indicated nevertheless gives representation about the fundamental tasks of treating the signal and appears as introduction to the subsequent chapters, in which special theory is set forth and the algorithms of treatment are derived/concluded.

Book is intended for students (students) of old years of Vuzes (Higher Educational institutions) of radio engineering

specializations. It will be useful also for the officers - the engineers of radar profile, who desire become acquainted with questions of automatic removal/output and working/treatment by the radio: locating data with the aid of the electronic computers.

Functional diagrams of systems, description of units and their parameters are undertaken from open foreign literature.

Page 5.

Chapter 1.

COMMAND OF TROOPS AND THE TASKS OF PROCESSING RADAR SIGNALS.

Sec. 1.1. General problems of the automation of the command of troops.

Command of troops is realized by commanders and staffs with wide application of different type of technical operators. In the course of time control technique and its use by staff officers continuously are improved with a simultaneous increase in the effectiveness of making decisions. The development of new technical equipment and methods of control on the base of the contemporary achievements of mathematics, physics and cybernetics contributes to this.

Furthermore, an increase in the weapon power and speed of its delivery to the targets in turn continuously raises requirements for the effectiveness of the command of troops, since a deficiency/lack in the time between the moment/torque of target detection and the use of a weapon for their interception and damage/defeat distinctly is perceived.

Control process consists of continuous extraction, treatment/processing and mutual exchange of information between all command components/links and objects of control, entering system in question. To a certain extent the separate units, which participate in the armed fight, can be considered as the elements of the single system, which depends on a large quantity of parameters, determined by the factors of strategic, operational, tactical, technical and moral and political character.

Page 6.

In this "system" is performed collection, the working/treatment and the transmission of information in the broad sense of this word, i.e., signals, communications/reports, radio engineering signals or the communication signals, orders, reports, etc.

By fundamental requirement, presented to control system, is opportuneness of making decision for guidance actions of its forces and transmission of commands/crews to executors/performers. The time τ_0 , spent on control and execution of commands, is convenient to represent in the form of the sum of the following components:

$$\tau_0 = \tau_1 + \tau_2 + \tau_3 + \tau_4,$$

where τ_1 - time of working/treatment and transmission of the information of state (processing the signals of radars);

τ_1 - time of understanding situation and making of decision;

τ_2 - time of formulation and transmission of director information to executors/performers;

τ_3 - readiness time of forces and fulfillment of actions by executors/performers.

First three term they compose time of action of control system, and the fourth - time of action of forces. It is obvious that with the the constant/invariable τ_4 , an increase in the time of action of system decreases preparation time and action of forces, which in a number of cases is inadmissible. Hence ensues/escapes/flows out the need for raising the operating speed of the control system, which is reached, in particular, by the automation of the control system.

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Consequently, units of automated control of forces and of means must be they are capable of solving problems of automation:

a) during τ_1 :

- process of collection and preliminary information processing;
- transmission of information to control device (staff) and mutual exchange of information (between staffs);
- reception/procedure and distribution of information, which comes control device from different sources of its processing;
- representation/transformation and documentation of information;

b) during τ_1 :

- analysis of situation;
- operationally tactical calculations, necessary for making of decision;
- preparation and substantiation of solution;

c) during τ_2 :

- formulation of solution and transmission of commands/crews to appropriate channels;
- obtaining receipts about reception of transmitted information;
- monitoring of actions of controlled forces and means.

Time τ , is determined directly by use of forces and means taking into account special features/peculiarities of concrete/specific/actual form of armament.

Tasks of automation can be solved with the aid of different type of diagrams and devices/equipment. however, the means of electronic computational engineering (EVT) are most promising.

In this book greatest attention will be given automation of collection of information from radio equipment, i.e., to solution of problems of first stage. Here electronic computers are applied for the isolation/liberation of useful signal against the background of interferences, determination of the coordinates of the discovered object and parameters of its motion, for coding of these data, their automatic introduction/input into the channel of communication and transmission with the assigned address.

Let us examine some special features/peculiarities of use/application of computer(s) in automated systems of control and extraction of information.

Sec. 1.2. Special features/peculiarities of the automated control systems.

By characteristic feature of process of organizing control of forces and of means is need for exchange and processing of enormous volume of information about its troops, and also about hostile troops. In this case the fast response for the evaluation of situation and making of decisions is important.

Page 8.

Abilities of man from point of view of reaction time set specific limitations on functions, which it can perform. However, by of special trial and error of the frames/personnel of high qualification and their prolonged aging/training both under the conditions for daily combat training and during the special studies these limitations can be brought to the minimum, and nevertheless in this case the speed of reaction proves to be insufficient.

As basic means of automation of control it is expedient to

employ specialized electronic computers with flexible structure of control of commands/crews, which makes it possible to change program in accordance with changing structure of command of troops. In fact, tasks and methods of the command of troops under the contemporary conditions, apparently, it is difficult to describe with the aid of the adjustable for a prolonged time rules and the positions. Methods will continuously change in proportion to a change in the tactical situation, and also in proportion to new discoveries/openings in different areas of science and technology of armament. Furthermore, the specific changes will occur in connection with the possible changes in world policy and military science. In accordance with this must change the structure of the command of troops.

It must be noted that operators, created on base of small automation, rapidly grow antiquated, since they do not provide for within the framework of their structure of possibilities of solution of new problems. Actually/really, if these changes were the consequence of an increase in the quantitative indices, for example speed, range, etc., then such systems could service/maintain troops an even longer time.

However, experiment of recent past shows that in the course of time very essence of threat can change, as this was obtained with the advent of intercontinental ballistic missiles. The advisability of

applying the special purpose computers with the flexible structure as the basis of the complex of the means of the automated control hence ensues/escapes/flows out. The description of such complexes widely is given in the literature [21, 36-39, 65, 71, 72, 73, etc.].

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Without dwelling in detail on their characteristics, let us point out only that forming part of this complex radars transmit their data automatically without the participation of operator, but they transmit them not directly, but through the special device/equipment - the computer(s) of processing radar signals. These computers of working/treatment are in a sense the device/equipment of coupling radar with the nodes of the automated complex and must fulfill the following functions:

1. To screen interferences and to automatically detect useful signal.
2. To measure coordinates of targets detected.
3. To transform information about coordinates into standard communication/report in connection with computer(s) of further processing.

4. To change criterion of detection for guaranteeing necessary injection speed of information into dependence on interferences.

Such type problems are solved with the aid of computer(s) of primary processing of radar signals, called sometimes by automatic radar observer or device/equipment of removal/output of radar data. This type of computer(s) is directly connected with the very radar and is located, as a rule, in immediate proximity of it. Output data of this machine will be brought in into the special memory unit, which forms part of the computer(s) of primary processing.

Examination of some properties and structure of such machines composes contents of the book. They are based on the elements/cells of digital computer technology, but have the specific algorithms of the computations, which determine the structure of processing signals. However, before passing directly to the machines of working/treatment, let us examine briefly actions with the codes of binary numbers, and also some schematics of the nodes of electronic digital computers.

Page 10.

Chapter 2

ARITHMETIC OPERATIONS WITH THE BINARY NUMBERS.

Especially wide acceptance in computers received binary, position numeration system. This is explained mainly by simplicity of the realization of the elements/cells of machines in this case. Are actual/real, in nature many of the widespread systems and the instruments, such as, for example, ferrites, flip-flops, relays, varicaps and other elements/cells, they can have two sharply differing states: "it is connected", "it is switched off". If we now agree, that one of the states of this element/cell to designate by one, and another zero, then convenience in coding numbers of binary system becomes obvious.

Let us examine some arithmetic operations with binary numbers, which are represented in natural form, i.e., with fixed point, in more detail, and they are encountered in devices/equipment of processing signal.

According to operating principle electronic computer can perform

only operation/process of addition. The subtraction directly of computer(s) produce cannot. Therefore in order to nevertheless fulfill the operation/process of subtraction and respectively division, it is necessary with the aid of the appropriate mathematical devices to bring together the operation/process of subtraction to the operation/process of addition. This can be made, if subtrahend (negative number) is represented in the special code: reverse/inverse or supplementary.

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Furthermore, there are different modifications of these codes, from which subsequently we will examine only the modified inverse code and will show that with its aid considerably are simplified the machine signs/criteria of different operations/processes, in particular the overflow attribute of calculation grid.

Sec. 2.1. Arithmetic operations in the inverse code ¹.

Inverse code of binary number is determined by following expression [25]:

$$(x)_{\text{обр.}} = \begin{cases} x & \text{для } x > 0 \\ 2 - 2^{-n} - x & \text{для } x < 0. \end{cases} \quad \begin{matrix} (1) \\ \text{для } x > 0 \\ \text{для } x < 0 \end{matrix}$$

Key: (1). for.

Hence it follows that inverse code of positive number coincides with number itself, and negative differs from initial. On the basis of this formula it is possible to formulate the following rule for the recording of the inverse code of the negative numbers [25]: in order to write a negative number in the inverse code, it is necessary instead of minus sign to write one, to supply it on the place of zero wholes, and to the fractional part to use base minus one complement (inversion).

For example, let there be n-bit negative number:

$$-x = -0, \alpha_1, \alpha_2, \alpha_3, \dots, \alpha_n,$$

where $\alpha_1, \alpha_2, \dots, \alpha_i, \dots, \alpha_n$ numerals of corresponding bits in numeration system with foundation β , each of numerals can take value:

$$\alpha_i = 0, 1, 2, \dots, (\beta - 1).$$

Then according to rule image of this number in inverse code will be:

$$(-x)_{\text{opp}} = 1, \bar{\alpha}_1, \bar{\alpha}_2, \dots, \bar{\alpha}_i, \dots, \bar{\alpha}_n,$$

where numerals $\alpha_1, \alpha_2, \dots, \alpha_i, \dots, \alpha_n$ are base minus one complement, i.e., $\alpha_i = (\beta - 1) - \alpha_i$.

Simplest is reverse/inverse supplement for numerals of binary

number system ($\beta=2$).

FOOTNOTE ¹. In view of the limited size of the book initial of the positions of the arithmetic of binary numbers are here omitted. To become acquainted with them it is possible on the literature [10, 11, 12, 26, etc.]. Only actions with the codes of numbers and rule of signs are here examined. ENDFOOTNOTE.

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In this case the greatest numeral in each bit is equal to 1, and therefore in base minus one complement it is necessary instead of 1 to supply 0, and instead of 0 to supply 1.

For image of sign of number are accepted following designations: "+" (plus) is depicted as zero, and "-" (minus) by one. Convenience in this designation is connected with simplicity of the determination of the sign of product and quotient, which is here obtained by the addition of the codes of the signs of factors.

In detail arithmetic operations in inverse code are not examined in view of relatively small abundance of this code. As an example is given only one action - subtraction of numbers in the inverse code, and primary attention is paid to the modified inverse code as most

frequently utilized in the machines.

Subtraction of numbers in an inverse code.

During subtraction $X-Y$ are possible two cases, that correspond to positive and negative difference. In the first case the minuend is lower than the subtrahend and difference is negative: $x < y$ and $x - y < 0$.

In second case, on the contrary, subtrahend is less than minuend and difference is positive: $x > y$ and $x - y > 0$.

Let us examine both these of case and let us point out machine signs/criteria of each of operations/processes.

A). Case of negative difference ($x - y < 0$ with $x < y$).

Rule. During the subtraction, when subtrahend is more than minuend, it is necessary to present the subtrahend in the inverse code and to accumulate with the minuend. The obtained result is negative and is represented in the inverse code. For obtaining the result in the direct code it is necessary to make rotation/access (inversion) of the fractional part of the number, and before the result to supply minus sign.

Let us examine numerical example, utilizing so-called modernized decimal system (binarily decimal).

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In this system the signs of a number are depicted as in the reverse/inverse binary code, and the significant part of the number is represented in the decimal system.

Assume it is necessary to find difference in two numbers:

$$0,52 - 0,67.$$

Presenting subtrahend in the inverse code, we will obtain:

$$(-0,67)_{\text{inv.}} = 1,32.$$

Producing subtraction in the inverse code, i.e., by storing/adding up number 0.52 with the inverse code of number -0.67, we will obtain:

$$0,52 + (-0,67)_{\text{inv.}} = 0,52 + 1,32 = 01,84^*.$$

Unity in sign position means that number is negative and therefore it is given in inverse code. For the translation/conversion of result into the direct code it is necessary in the fractional part of the number to make inversion of numerals, i.e., to write base minus one complement in the obtained result. After fulfilling the

inversion of number 0.84 and taking into account that one in sign position corresponds to minus of a number, we will obtain final result in direct code -0.15.

Let us examine now example in binary number system. Assume it is necessary to accumulate two numbers of different signs:

$$0,01011 = \frac{11}{32} \text{ and } -0,10011 = -\frac{19}{32}.$$

Converting a negative number into the inverse code and storing/adding up, we will obtain:

$$\begin{array}{r} 0,01011 \\ + 1,01100 \\ \hline 1,10111. \end{array}$$

The obtained unity in sign position means that a number is negative and therefore it is represented in the inverse code. Inverting the inverse code of a number, we obtain final result in the direct code, which taking into account sign can be represented in the form -0.010000, i.e., -0.25.

It should be noted that frequently in machine inverse code of negative number they do not invert, since, if it is necessary to perform subsequently any operations/processes with this number, it to more conveniently leave in inverse code in order then again to not

convert number of direct code into reverse/inverse.

FOOTNOTE ¹. In this expression and it is further accepted that the inverse code of positive number is number itself, i.e.,

ENDFOOTNOTE.

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B). Case of positive difference ($x-y>0$ with $x>y$).

Rule. During the subtraction, when minuend is more than subtrahend, it is necessary to present the subtrahend in the inverse code and to accumulate with the minuend. In the obtained sum it is necessary to make an end-around carry, after which the result will be represented in the direct code by positive number.

End-around carry consists in the fact that 1 of extreme left (sign) bit remove and they add it into extreme right (low-order) digit of number.

Let us preliminarily examine numerical example in modernized decimal system.

Assume it is necessary to find difference in two numbers:

$$0,86 - 0,67.$$

Presenting subtrahend in the inverse code, we will obtain:

$$(-0,67)_{\text{оп.}} = 1,32.$$

Producing subtraction, i.e., by storing/adding up number 0.86 with the inverse code of number -0.67, we will obtain:

$$0,86 + (-0,67)_{\text{оп.}} = 0,86 + 1,32 = 10,18.$$

In this result one in bit, adjacent with sign, he indicates need for execution of operation/process of end-around carry, and presence of zero in sign position - to positive sign of number. After producing end-around carry we will obtain the final result:

$$\begin{array}{r} 10,18 \\ \hline 0,19 \end{array} \quad \begin{array}{l} (1) \\ \text{ц. п. (циклический перенос).} \end{array}$$

Key: (1). Ts. P. (ц.п.) (end-around carry).

Let us examine now example in binary number system. assume it is necessary to accumulate two numbers:

$$-\frac{11}{32} = -0,01011 \quad \text{and} \quad \frac{19}{32} = 0,10011.$$

Converting a negative number into the inverse code and storing/adding up, we will obtain:

$$\begin{array}{r} 1,10100 \\ 0,10011 \\ \hline 10,00111 \end{array}$$

As is evident, in this case numeral 1 in extreme left bit appeared. According to the rule established/installed earlier in this case it is necessary to produce end-around carry, i.e., one from the extreme left bit to transfer into the extreme rightist. After producing end-around carry, we obtain final result in the form:

$$\begin{array}{r} 10,00111 \\ \text{u. n. } \frac{1}{0,01000} \end{array}$$

Key: (1). Ts. P. (u.n.).

Important special feature/peculiarity of code, utilized in machine, is rule of signs, which characterizes signs/criteria of arithmetic operations, and also overfillings of calculation grid of machine. The latter appears when the sum of two numbers on the modulus/module of more than machine unit $M(i) = 1 - 2^{-n}$. In the process of computations the machine must automatically determine the sign of result, while in the corresponding cases and the overfilling of calculation grid, developing in this case the signal of the automatic stop of machine. The rules of signs for the operations/processes in the inverse code are given in table 2.1. From the table it is evident

that the overfilling of calculation grid will occur, when the signs of numbers are identical, and the sign numerals of result different. The sign/criterion of operation/process in other cases is easily determined from the given table via the account of the sign numerals of result and signs of base figures. As is evident, here necessary to consider the signs of components/terms/addends, and also the sign of the obtained result. However, since result of operation is explained later than the admission of initial numbers, for determining the sign/criterion of operation/process it is necessary in any manner to provide the memorization of the signs of initial numbers. This is undesirable, since it complicates the schematic of machine.

More simply sign/criterion of operation/process is determined during use of modified inverse code.

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Table 2.1. Rule of signs in the inverse code.

(1) Знаки слагаемых		(3) Код знаков слагаемых		(4) Знак результата	(5) Код знака результата	(6) Примечание
(2) 1-е число	(2) 2-е число	(2) 1-е число	(2) 2-е число			
+	+	0	0	+	00	(7) Результат положительный
+	+	0	0		01	(8) Переполнение разрядной сетки
+	-	0	1	+	10	(9) Циклический перенос. Результат положительный
-	+	1	0	-	01	(10) Результат отрицательный
-	-	1	1	-	11	(11) Циклический перенос. Результат отрицательный
-	-	1	1		10	(12) Переполнение разрядной сетки

Key: (1). Signs of components/terms/addends. (2). 1st number. (3). Code of signs of components/terms/addends. (4). Sign of result. (5). Code of sign of result. (6). Note. (7). Result, positive. (8). Overfilling of calculation grid. (9). End-around carry. Result is positive. (10). Result, negative. (11). End-around carry. Result is negative. (12). Overfilling of calculation grid.

Sec. 2.2. Modified reverse/inverse code.

Representation of numbers in modified inverse code differs from image examined earlier in inverse code only in terms of fact that for image of sign of number is utilized not one numeral, and two adjacent numerals, called sign numerals of number, in this case unit of end-around carry appears no longer the secondly, but in third digit of whole numbers. Thus, for instance, positive number $x=0.1011001$ in the modified inverse code will be recorded in the form:

$$(x)_{\text{m. odp.}} = 00,1011001.$$

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The same number, being negative, is written/recorded in reverse/inverse modified code just as in usual code, with the only difference that the instead of one unit in sign position it is necessary to supply two units in two adjacent low-order digits of whole numbers, i.e.

$$(-x)_{\text{m. odp.}} = 11,0100110.$$

Rule of recording of number in modified inverse code can be formulated in the following form.

Rule. In order to write a negative number in the modified inverse code, it is necessary instead of minus sign to write two units in the low-order digits of whole numbers, and in the fractional

part of the number to use base minus one complement.

Algebraic form of recording of numbers in this case, obviously, takes form:

$$(x)_{\text{mod}} = \begin{cases} x & \text{для } x \geq 0 \\ 4 - 2^{-n} - x & \text{для } x < 0 \end{cases} \quad (2.1)$$

Key: (1). for.

1. Arithmetic operations in the modified inverse code.

For numerical examples we will use modernized binary decimal numeration system. In this system the sign of a number is depicted in the modified inverse code (i.e. "+" it is depicted as 00, ..., and "-" it is depicted as 11, ...), and its significant part - in the decimal system.

2. Addition of two positive numbers.

It is here necessary to distinguish two cases, when sum of two numbers is less and more than machine unit. In the second case, as is known, occurs the overfilling of calculation grid and computation they cease. Let us examine each of these cases individually.

A). Sum of the numbers of less than the machine unit.

Rule. Two positive numbers, whose sum is lower than the machine unit, store/add up according to the rules of the arithmetic of decimal numbers. The obtained sum is positive and is represented in the direct code.

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Example. Assume it is necessary to accumulate two positive numbers, whose sum is lower than the machine unit:

$$00,756 + 00,121 = 00,877.$$

Presence of numerals 00 in sign positions of number is distinctive machine sign/criterion of this operation/process. It indicates this that the obtained result is positive. Analogously is conducted the addition of numbers, also, in the binary number system. In view of obvious simplicity of this question further examples are omitted.

B). Sum of the numbers of more than machine unit.

Let us show that during addition of two numbers, whose sum is more than machine unit, numbers in sign positions are obtained

different. Assume we should accumulate two numbers 0.756 and 0.542. Storing/adding up, we obtain $00.756+00.542=01.298$. As is evident, one in the low-order digit of whole numbers appeared. Thus, the distinguishing feature of this operation/process is the presence of numerals 01 in sign positions of a number.

3. Subtraction (addition of numbers of different signs (x-y)).

During subtraction should be distinguished two cases, that correspond to positive and negative difference.

A). A difference in the numbers (x-y) is negative: $x < y$ and $x-y < 0$.

Rule. During the subtraction, when subtrahend is more than minuend, it is necessary to present the subtrahend in the modified inverse code and to accumulate with the minuend. The obtained result is negative and is represented in the modified inverse code. In order to present result in the direct code, it is necessary to make rotation/access of the fractional part of the number, and before the result to supply minus sign.

Example. Assume we should find a difference in two numbers:

$$0.52-0.67.$$

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After presenting subtrahend in modified inverse code and prevailing it with minuend, we will obtain:

$$\begin{aligned}(0,52 - 0,67)_{\text{м.о.п.}} &= 00,52 + (-0,67)_{\text{м.о.п.}} = \\ &= 00,52 + 11,32 = 11,84.\end{aligned}$$

Two units, which were obtained in sign positions, mean that number is negative and is given in modified inverse code. For the translation/conversion of result into the direct code it is necessary to make inversion.

After producing inversion and after considering that two units in sign positions correspond to minus of number, we will obtain final result in the form -0.15.

Presence of two units in sign positions of obtained difference is machine sign/criterion of this operation/process. Let us show this.

Let there be difference $x - y < 0$. After presenting subtrahend $-y$ according to formula (2.1) in the modified inverse code and

prevailing with the minuend, we will obtain:

$$(x-y)_{\text{м.одр.}} = 4 - 2^{-n} + (x-y) = \Delta.$$

After rewriting this taking into account the fact that

$4 - 2^{-n} = \underbrace{11,111\dots11}_{(1) \text{ n разрядов}}$, we will obtain:

$$\Delta = \underbrace{11,111\dots11}_{\text{n разрядов (1)}} + (x-y).$$

Key: (1). digits.

Since second term on condition is negative $(x-y) < 0$ and on modulus/module of less than machine unit, i.e., $|x-y| < 0.1111 \dots 11$, then, obviously, in obtained difference sign positions will always remain without changes and they will be always equal to 11, ..., QED.

B). A difference in the numbers $x-y$ is positive: $x > y$ and the $x-y > 0$.

Rule. During the subtraction, when minuend is more than subtrahend, it is necessary to present the subtrahend in the modified inverse code and to accumulate with the minuend.

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In the obtained sum it is necessary to make an end-around carry from the third (old) digit of whole numbers. After end-around carry the

result will be represented in the direct code and be positive number.

Example. Assume it is necessary to find a difference in two numbers:

$$00,86 - 0,67.$$

After presenting subtrahend in the inverse code and prevailing with the minuend, we will obtain:

$$(0,86 - 0,67)_{\text{ap.}} = 00,86 + (-0,67)_{\text{ap.}} =$$

$$= 00,86 + 11,32 = 100,18.$$

As is evident, one in extreme left (third) digit of whole numbers appeared. One in this digit indicates the need for the execution of end-around carry. Two zeros in the next decades (00, ...) indicate that the obtained result is positive and is represented in the direct code. After producing end-around carry, we will obtain final result in the form:

$$\begin{array}{r} (1) \quad 100,18 \\ \text{и. п.} \quad \xrightarrow{+1} \\ \hline 00,19 \end{array}$$

Key: (1). Ts. P. (и.п.).

Presence of numerals 00 in sign positions of result is machine sign/criterion of this operation/process.

4. Addition of two negative numbers.

Here should be examined two cases, when modulus/module of

numbers is less and more than machine unit.

A). Sum of the numbers on the modulus/module of less than the machine unit.

Rule. During the addition of two negative numbers, the modulus/module of sum of which is lower than the machine unit, it is necessary to accumulate the inverse codes of these numbers, and in the obtained sum to produce the end-around carry. Result is negative and is represented in the inverse code.

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If it is necessary to present result in direct code, then it is necessary to make inversion of number, and before obtained result to supply minus sign.

Example. Assume it is necessary to accumulate two numbers -0.32 and -0.51. After representing numbers in the modified inverse code and prevailing, we will obtain:

$$\begin{aligned} (-0,32 - 0,51)_{\text{м.о.б.р.}} &= (-0,32)_{\text{м.о.б.р.}} + (-0,51)_{\text{м.о.б.р.}} = \\ &= 11,67 + 11,48 = 111,15. \end{aligned}$$

As is evident, in obtained sum of inverse codes are three units

in digit of whole numbers 111, ... Ones in two low-order digits indicate the negative sign of result, and one in the extreme left digit indicates the need for end-around carry. After producing end-around carry, we will obtain result in the modified inverse code:

$$\begin{array}{r} 111,15 \\ \text{ц. п.} \xrightarrow{+1} \\ \hline 11,16 \end{array}$$

Key: (1). Ts. P. (ц.п.).

After fulfilling inversion, we will obtain final result in direct code -0.83.

Machine sign/criterion of operation/process in question is presence of two units in sign positions of number (11, ...). Let us show this.

As is known, sum of two negative binary numbers in reverse/inverse modified code will take form:

$$(-x - y) = 4 - 2^{-n} - (x + y) = \Delta.$$

Taking into account that $4 - 2^{-n} = 11,11 \dots 11$, it is possible to rewrite the sum indicated in the form:

$$\Delta = 11,111 \dots 11 - (x + y).$$

Since on condition $(x+y)$ it is less than the machine unit, i.e. $(x+y)$

$\langle M(1) = 0.1 \dots 11$, then, obviously, numerals in sign positions will remain without the change (i.e. they will be equal to 11).

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B). Sum of the numbers on the modulus/module of more than machine unit.

This case corresponds to overfilling of calculation grid. Machine loses the high-order digits of a number, and computations cease via its automatic stop.

Appearance of different numerals in sign positions (01, ...) is machine sign/criterion of this case. Actually/really, from formula (2.1) it follows that if $x+y$ is more than machine unit, i.e.

$$x+y > 0,111\dots 11,$$

then, obviously, during the subtraction always it is necessary to occupy one from the digit of whole numbers (from low-order sign position) and, therefore, always sign position, on which is conducted the determination of the machine signs/criteria of operation/process, will be equal to (10, ...).

5. Rules of signs with the arithmetic operations in the reverse/inverse modified code.

Examining obtained above machine signs/criteria of operation/process, it is not difficult to formulate rules of signs during arithmetic operations in modified inverse code. The machine signs/criteria of the corresponding operations/processes are given in Table 2.2.

As can be seen from Table 2.2, rules of signs are obtained considerably simpler than in simple inverse code. Actually/really, here the sign/criterion of operation/process is determined exclusively on the numerals of sign positions of result and does not depend on the signs of components/terms/addends. This fact substantially simplifies algorithm and, consequently, also the diagram of the determination of the sign of result. Actually/really, in this case drops off the need for the "memorization" of the sign of the component/term/addend to the period of operation/process for the definition of overfilling as this was in the simple inverse code. For these reasons the modified inverse code of numbers has larger propagation in the electronic digital computers.

Table 2.2. Rule of signs in the modified inverse code.

(1) Знаки чисел		(3) Знаковые цифры результата	(4) Знак результата	(5) Примечание
(2) 1-е число	(2) 2-е число			
+	+	00	+	(6) Результат положительный
+	+	10		(7) Переполнение разрядной сетки
+	-	00	+	(8) Циклический перенос Результат положительный
+	-	11	-	(9) Результат отрицательный
-	-	11	-	(10) Циклический перенос Результат отрицательный
-	-	01		(7) Переполнение разрядной сетки

Key: (1). Signs of numbers. (2). 1st number. (3). Sign numerals of result. (4). Sign of result. (5). Note. (6). Result, positive. (7). Overfilling of calculation grid. (8). Cyclic over/re. Result is positive. (9). Result, negative. (10). Cyclic over/re. Result is negative.

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Sec. 2.3. Supplementary code of numbers.

Two's complement of number is designated by symbol $(x)_{\text{доп.}}$. For the positive numbers it coincides with number itself: $(x)_{\text{доп.}} = x$. The two's

complement of a negative number is obtained by the addition of unit of low-order digit to the inverse code of a number, i.e.

$$(x)_{\text{inv}} = (x)_{\text{op}} + 1 \cdot \beta^{-n}.$$

Presence of this supplementary unit eliminates need for during addition of negative numbers performing end-around carry, what is definite advantage of two's complement.

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However, if one considers that the end-around carry (addition of one into the low-order digit) appears not during each addition of negative numbers, but the addition of one into the two's complements occurs in each negative number, then it appears that the number of additions (with one in the low-order digit) is more in the two's complement, than in the reverse/inverse.

Let us examine addition of numbers in two's complement and machine signs/criteria of operation/process based on examples. In more detail to the two's complement it is possible to be introduced in the specialized literature on computer technology [12].

1. Difference in two numbers is positive:

$$0,67 - 0,15 = 0,52.$$

Let us fulfill subtraction of these numbers in two's complement:

$$\begin{aligned}(0,67 - 0,15)_{\text{non.}} &= 0,67 + (-0,15)_{\text{non.}} = \\ &= 0,67 + 0,85 = 10,52.\end{aligned}$$

Unity in second sign position is not considered, since it proves to be beyond limits of calculation grid. The remaining part gives the required result.

2. Difference in two numbers is negative:

$$0,15 - 0,67 = -0,52.$$

After fulfilling this subtraction in two's complement, we will obtain:

$$(0,15 - 0,67)_{\text{non.}} = 0,15 + (-0,67)_{\text{non.}} = 0,15 + 1,33 = 1,48.$$

Obtained result is negative and is represented in two's complement. For its translation/conversion into the direct code it is necessary to make turning a number, i.e., instead of one, which is located in the digit of whole numbers, to supply minus sign, and to fractional part to use inversion with the addition of one:

$$1,48 \rightarrow -(0,51 + 1) = -0,52.$$

3. Sum of two negative numbers (on modulus/module of less than one):

$$-0,25 - 0,17 = -0,42.$$

Addition in two's complement takes form:

$$\begin{aligned} (-0,25 - 0,17)_{\text{non.}} &= (-0,25)_{\text{non.}} + (-0,17)_{\text{non.}} = \\ &= 1,75 + 1,83 = 1) 1,58. \end{aligned}$$

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One in high-order sign position is not considered - it drops out for calculation grid. The remaining part gives result in the form of the negative number, represented in the inverse code. After producing rotation/access, we will obtain:

$$1,58 \rightarrow -(0,41 + 1) = -0,42.$$

4. Sum of two negative numbers (on modulus/module of more than one):

$$-0,83 - 0,68 = -1,51.$$

Addition in two's complement it gives:

$$(-0,83 - 0,68)_{\text{non.}} = 1,17 + 1,32 = 1) 0,49.$$

In this case overfilling of calculation grid occurs. One in sign

positions of the components/terms/addends and zero in sign position of result is the distinguishing feature of this.

Addition of positive numbers in two's complement and distinguishing features of these operations/processes are accurately the same as in inverse code (see above).

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Chapter 3.

ELEMENTS AND UNITS OF ELECTRONIC DIGITAL COMPUTERS.

3.1. Registers.

Device/equipment, intended for short-term memorization of code of number, is called register. Besides this, on the registers can be realized sign change, order and code of this number.

Registers are utilized in arithmetical units, and also in units of control/checking, control and in memory units. In the arithmetic unit the registers serve for the storage of components/terms/addends or factors, selected from the storage unit. In the memory unit the registers are utilized for the short-term storage of the number selected from the memory before its dispatch into the arithmetical or control system.

In registers can be realized translation of number of

consecutive form into parallel and back, shift/shear of number to the right or to the left, transformation of code of number, etc.

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Subsequently we will consider that the register is the totality of trigger circuits [1, 3, 7, 8, 9, etc.], whose number corresponds to a quantity of digits in a number, and the auxiliary diagrams, which ensure the fulfillment of the following operations/processes:

- (• - reception/procedure of a number (from the memory unit or from another register);
- delivery of a number (into the summator, memory unit or into another register);
- change in the code of a number (delivery of numbers in the straight/direct or inverse code);
- shift/shear of numbers to the right or to the left to the necessary numerical length (change in the exponent);
- transformation of the consecutive code of a number into the parallel code and other functions.

Together with trigger registers, carried out on ferrite cores (see below) are propagated. They are also equipped with auxiliary diagrams for the input, the output, the shift/shear and changing the code of a number.

For reception/procedure and delivering numbers to input and output circuits of register corresponding circuits of input and output are connected up. The shift/shear of numbers in the register is realized by means of the special shift circuit, which connects between themselves two adjacent digits. Therefore for explaining the work of the entire diagram of register as a whole it is necessary to examine the operating principle of its separate elements/cells. Let us switch over to the description of the fundamental nodes of register.

1. Diagram of the input of a number into the register and his output from the register.

Diagrams of input and output of number make it possible to transmit number of one register in another. The transmission of a number can be conducted both in the straight line and in the inverse code, in this case numbers can be represented in the consecutive and

parallel code.

Diagrams of input and output have much in common in their work; therefore let us in detail examine only one of them, in particular work of diagram of introduction/input of number. Let us assume that it is necessary to transmit the parallel code of a number of the register A into the register B (Fig. 3.1). Let us assume register A corresponds to intermediate register, which is located after memory unit. In this register the numbers, selected from the memory before their impulse/transmission into the register of arithmetic unit or into other nodes of machine, are stored. Register B corresponds to the register of arithmetic unit. In this register before the introduction/input of a number all flip-flops are established/installed into the state by 0 supply of signal to the busbar of installation 0. For simplicity of reasonings are here shown only two adjacent digits of the i -th and $(i+1)$. In other digits of register the transmission of numbers is conducted completely analogously. Let us assume that in the i digit is transmitted one, and in $(i+1)$ - zero. Let us examine the transmission of one.

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As can be seen from Fig. 3.1, to calculating input of flip-flops of register B signal is supplied through circuit of coincidence AND.

This diagram is controlled on three inputs, and the presence of signal at the output depends on the presence of signals for each of its inputs.

AND circuit can be made on diodes or on other elements/cells.

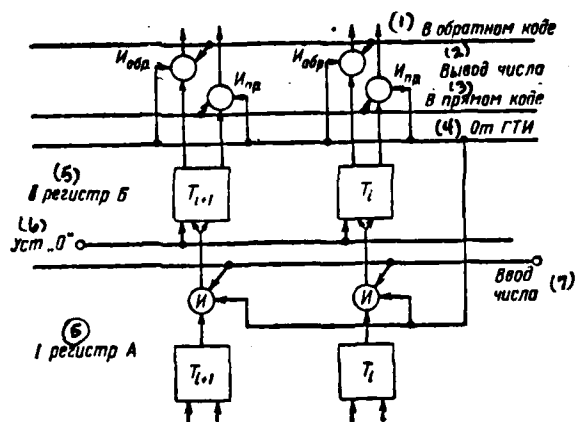


Fig. 3.1.

Fig. 3.1. Introduction/input of number into register.

Key: (1). In the inverse code. (2). Conclusion/output of number. (3). In direct code. (4). From GTI. (5). register. (6). Obsolete "0". (7). Introduction/input of number.

Fig. 3.2. AND circuit on pentode.

Key: (1). Clock impulses/momenta/pulses.

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Let us assume for simplicity that the diagram is carried out on the pentode (Fig. 3.2). The input signals U_1 , U_2 , and U_3 are supplied

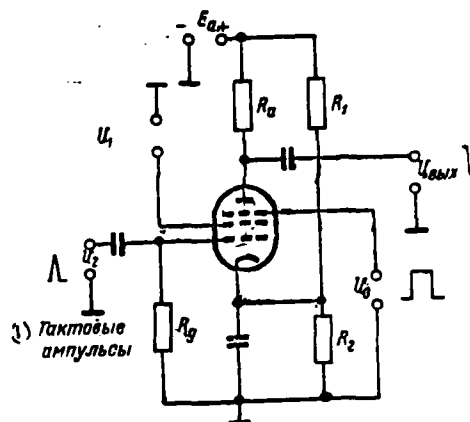


Fig. 3.2.

respectively to screen, controlling and suppressor grids of pentode. Screen grid is controlled with the output voltage/stress of the flip-flop, from which is read a number. If in the flip-flop one is recorded, then on its output there will be high voltage (line 1, Fig. 3.3a), while if zero, then low (line 0). The latter corresponds to the open state of right flip-flop tube of register A.

To second input of this diagram clock impulses/momenta/pulses of computer(s) from assigning pulse generator of machine (Fig. 3.3b) are supplied. To the third input is supplied enabling pulse the "introduction/input of a number", which synchronizes with one of the clock impulses/momenta/pulses and is determined the moment/torque of the admission of a number into the register. This moment/torque is characterized by the conditions for the work of machine as a whole. The mode/conditions of pentode (Fig. 3.2) is determined by the values of voltages/stresses E_a , U_1 , U_2 and U_3 , and also by the values of resistors/resistances of R_1 , R_2 and R_a .

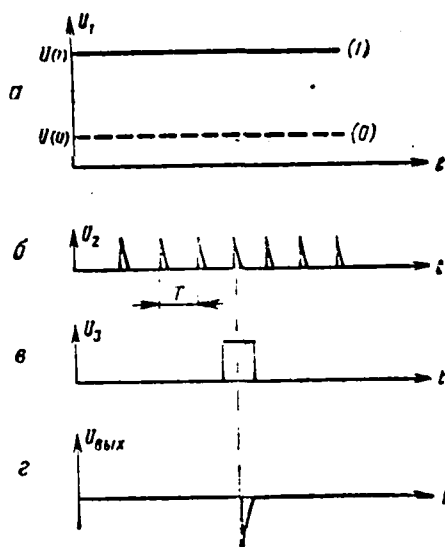


Fig. 3.3. Diagrams of voltages/stresses in the AND circuit on the pentode.

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It is selected in such a way that the signal at the output will appear only in such a case, when there will be signals at the second and third input, and on the screen grid appears high voltage, which corresponds to "recording" in the flip-flop of one.

But if on screen grid there will be low voltage, which corresponds to recording of zero (Fig. 3.3a), AND circuit will prove to be closed and output signal will be absent. Let us return to the diagram of the introduction/input of a number. Its work flows/occurs/lasts as follows. The output voltage/stress of

flip-flop, which corresponds to the "recording" of one, is supplied to the right input of the AND circuit of coincidence and triggers it on the screen grid. To the second input of AND circuit clock impulses/momenta/pulses in entire period of its work (Fig. 3.3b) are supplied. To the third input is supplied enabling pulse the "introduction/input of a number" (Fig. 3.3c). The moment of supplying this impulse/momentum/pulse corresponds to the moment/torque of the transmission of a number of one register in another. With the coincidence in the time of all three signals through the AND circuit one of the clock impulses/momenta/pulses is passed and in the form of the signal of negative polarity it is supplied to the calculating input of the flip-flop of register B. Entered thus impulse/momentum/pulse in the i digit of register B will cause functioning flip-flop, in this case they indicate that in this digit "is recorded" one.

During transmission of zero in $(i+1)$ digit of no signal calculating input of flip-flop of register B enters, since to AND circuit on first input is supplied low voltage (0) (Fig. 3.3a) and diagram proves to be closed. The absence of impulse/momentum/pulse corresponds to transmission of zero, in this case they indicate that in this digit "are transmitted zero". However, one should remember that actually here no signal is transmitted and the flip-flop of this digit remains in the same state, as before the transmission of a

number.

Let us examine work of diagram of conclusion/output of number. Let us assume that we should transmit a number of the register B to output terminals of the i -th and $(i+1)$ digit. As can be seen from Fig. 3.1, to the output busbars of flip-flops are connected the circuits of coincidence U_{np} and $U_{обн}$, which provide the conclusion/output of a number in the straight line or in the inverse code.

To each of diagrams are conducted/supplied accurately the same voltages/stresses, as to coincidence circuit, which is located in diagram of introduction/input of number.

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To the first input the voltage/stress from output terminal of flip-flop (in the straight/direct or inverse code) is supplied. To the second input (control electrode) are supplied the clock impulses/momenta/pulses of computer(s), also, on the third input (suppressor grid) - the resolving command/crew the "introduction/input of a number". Command/crew can be supplied separately - to diagrams U_{np} or $U_{обн}$. In the first case a number will be derived/concluded in the direct code, the secondly - in the

reverse/inverse. The passage of the signals through the diagrams of the conclusion/output of a number is accurate the same, as in the diagram of the introduction/input of a number into the register; therefore for explaining the passage of signals can be used the diagrams of voltage/stress, given earlier.

Diagrams examined above of input and output of number determine form of communication between registers, called frequently pulse-potential connection/communication.

2. Shift/shear of the code of a number in the register.

Shift/shear of code of number in register is propagated operation/process. It is encountered during multiplication and division of numbers, with the matching of exponents of numbers in the machines with floating point, during the introduction of scale factors in the case of the overfilling of the calculation grid of machine and in other cases, where a change in the exponent is required.

There are several types of diagrams of number in register. Let us examine diagram with the differentiating circuit (Fig. 3.4).

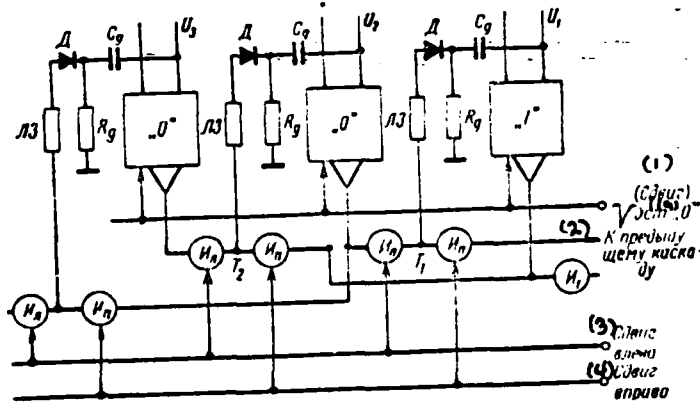


Fig. 3.4. Shift/shear of numbers in the register.

Key: (1). (Shift/shear). (1A). Obsolete "0". (2). To preceding/previous cascade/stage. (3). Shift-left. (4). Right-shift.

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This diagram can provide shift/shear of numbers both to the right and to the left to necessary numerical length. As is evident, it consists of trigger circuits, differentiating circuits $R_g C_g$ and valves/gates of the shift/shear of a number to the left H_n and to the right H_{n-1} . Depending on the triggering/opening of one or the other group of valves/gates a number will be shifted/sheared to the left or to the right.

Let us take shift/shear of concrete/specific/actual number 001

for example. This means that in the extreme right digit recorded one and we should move it into the extreme left digit. In the remaining digits zero are recorded. The diagrams/curves, which elucidate the work of diagram, are represented in Fig. 3.5. At zero times voltage/stress U_a on the anode of the first flip-flop was high, since one was there recorded. On the anodes of remaining flip-flops voltage U_a and U_a is low, since in these flip-flops zero are recorded.

At moment of time t , busbar of "shift/shear" (extinction) enters negative pulse (Fig. 3.5f), which establishes/install readings/indications of all flip-flops in zero position, which corresponds to "recording" of zero. At the same moment/torque on the output busbar of right flip-flop voltage surge, which, being it is differentiated with the aid of differentiating circuit $R_d C_d$ will create the impulse/momentum/pulse of negative polarity (Fig. 3.5b) ¹, will occur. This impulse/momentum/pulse is supplied by the delay line LZ (Fig. 3.4) and is delayed there to the period τ , (Fig. 3.5b), determined by the duration of the pulse of "shift/shear" (extinction) and by transient processes in the flip-flop. Delayed pulse is fed/conducted to the branch point T_1 (Fig. 3.4) and depending on the open state of one or the other AND circuit of coincidence will be transmitted to the calculating input of the flip-flop of left or right digit. In our case the shift/shear of a number occurs to the left, and therefore open circuit U_{π} (Fig. 3.4).

Impulse/momentum/pulse enters the calculating input of adjacent left flip-flop, are activated and increase in the voltage on its anode (Fig. 3.5c). The presence of high voltage on the output busbar of flip-flop testifies about the "recording" in it one. Thus, the time τ , after cycling of shift/shear a number proves to be that moved to one digit to the left.

FOOTNOTE ¹. The impulses/momenta/pulses of positive polarity (Fig. 3.5d) appearing during the differentiation are limited to diodes Δ .
ENDFOOTNOTE.

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With arrival of following shift pulse at moment/torque t , in second digit processes, analogous to processes in first digit (Fig. 3.5c, d, e), will occur and up to moment/torque of time $t + \tau$, number will prove to be that moved to two digits. Thus, for the shift/shear of a number to the left to two digits it is necessary to feed two shift pulses and on this period to open the group of the circuits of coincidence Π_2 (Fig. 3.4).

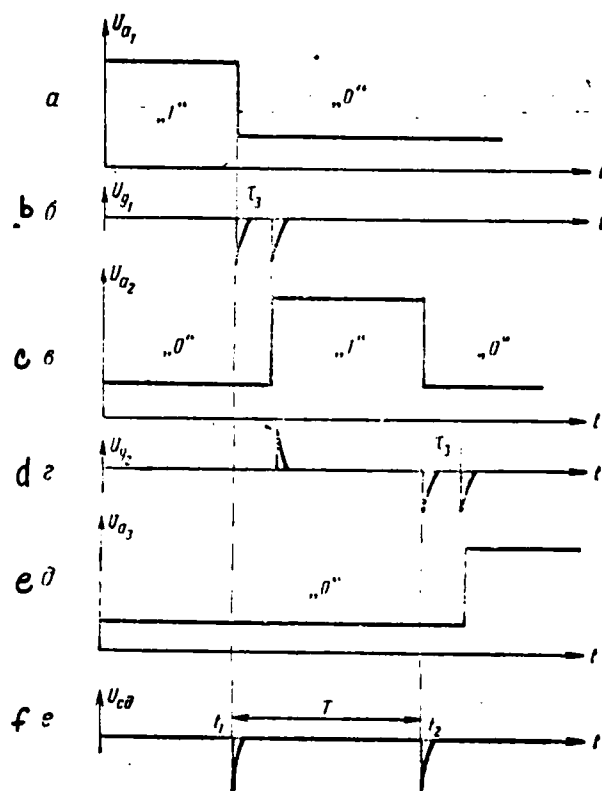


Fig. 3.5. Diagrams of voltages/stresses in the shift circuit.

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If we in this course of time open circuits U_n and U_n shut, then delayed pulse will enter a calculating input of adjacent right digits and a number it will be moved to the right. The simultaneous triggering/opening of valves/gates U_n and U_n leads to the loss of a number.

In general case for n -fold shift/shear of number to the right (to the left) to n digits it is necessary to feed n shift pulses and on the same period to open valves/gates of shift/shear of number to the right U_n (to the left U_n).

Triggering/opening of valves/gates U_n or U_n is conducted by supply on them of special pulse of voltage, whose duration is equal to

$$\tau = nT,$$

where n - numerical length;

T - pulse repetition period of shift/shear, determined usually by repetition frequency of clock impulses/momenta/pulses of computer(s).

Variety of diagram examined is circuit of shift of number with the aid of AND valves/gates (Fig. 3.6 and 3.7). Its work flows/occurs/lasts as follows. To the input terminal are supplied clock impulses/momenta/pulses U_{cl} (Fig. 3.7b), which "request" the state of the flip-flops of the register through the AND circuits.

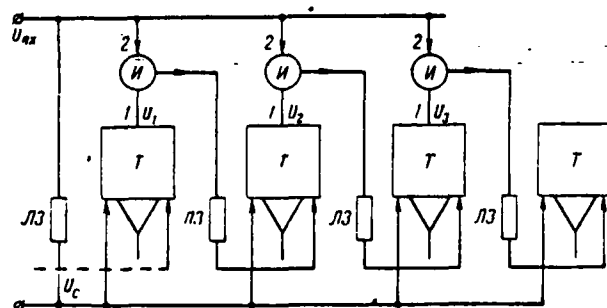


Fig. 3.6. Circuit of the shift of a number in the register.

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If in this digit is recorded 1 (Fig. 3.7a), then the input pulse α (Fig. 3.7b), which entered at moment/torque t_1 , passes through AND (Fig. 3.6) into the delay line LZ, and after time t_2 will enter the input "installation 1" (Fig. 3.7c). After time t_3 the impulse/momentum/pulse α will traverse the delay line LZ, (Fig. 3.7d) to the busbar "installation 0" and will dump readings/indications of flip-flops into zero. Then after time $t_4 - t_3$ the new shifted number will be established/installed in the digits of flip-flop.

If in digit of flip-flop will be recorded 0, then "interrogating" impulse/momentum/pulse will not traverse AND circuit. In this case in the next decade will not be the signal, which calls functioning trigger cascade/stage. Analogous processes will occur

with second clock-impulse/momentum/pulse U_{n+1} , which enters moment/torque t , (Fig. 3.7b, c, d, e, f).

Fig. 3.6 shows register with shift/shear of number to the right. However, if we change the direction of cycling α , feeding/conducting it to the flip-flops of adjacent left digits, it is possible to obtain the shift/shear of a number to the left.

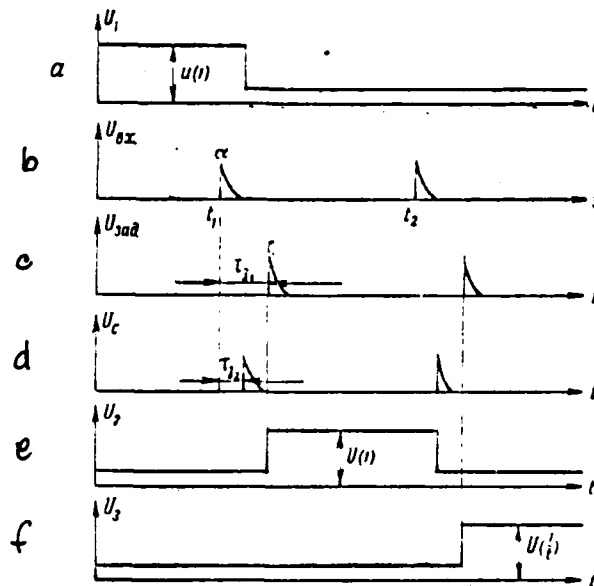


Fig. 3.7. Graphs/curves of voltage/stress in the shift circuit.

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For m-fold shift/shear it is necessary as in preceding/previous diagram, to supply m of shift pulses and on this period to open appropriate direction of transmission to carry pulses to the right or to the left.

3. Introduction/input and the conclusion/output of a consecutive number into the register.

Operation/process of introduction/input (conclusion/output) of

number, represented in consecutive code, into register is encountered in series machines.

Into this case introduction/input (conclusion/output) of M-discharge number is reduced to its m-fold shift/shear with the aid of diagrams examined above. In this case to the register it is necessary to conduct exactly as many shift pulses, as digits have the introduced number.

If number of shift pulses is more or less than quantity of digits, then during introduction/input extreme digits of number will be lost. Therefore the shift circuit it is necessary to supplement with the special device/equipment (Fig. 3.8), which feeds the fixed/recorded number of shift pulses, the equal to a quantity of digits of a number, and opening/disclosing to this period the valves/gates of shift/shear. Device/equipment consists of counter with the modulus/module of translation, equal to the necessary quantity of shifted/sheared digits, flip-flop tr AND valve/gate.

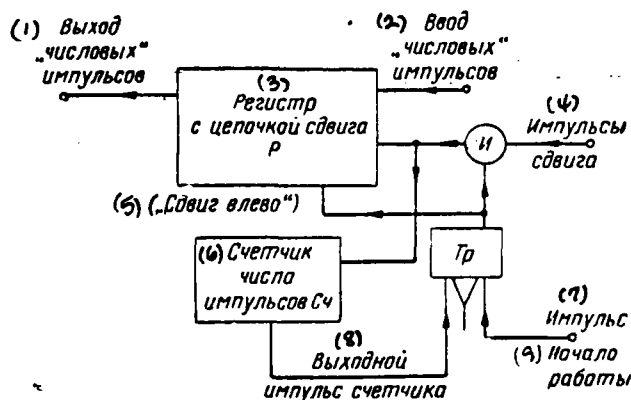


Fig. 3.8. Diagram of input and output of a consecutive number into the register.

Key: (1). Output of "numerical" impulses/momenta/pulses. (2). Introduction/input of "numerical" impulses/momenta/pulses. (3). Register with chain/network of shift/shear R. (4). Shift pulses. (5). (shift to the left"). (6). Counter of number of impulses/momenta/pulses of mf/SCh. (7). Impulse/momentum/pulse. (8). Output counter pulse. (9). Beginning of work.

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The totality of these elements/cells provides the supply of the required number of shift pulses. The beginning of the work of diagram corresponds to cycling the "beginning of work", synchronized with the numerical impulses/momenta/pulses. Termination is determined by the

counter of mf/SCh; which locks with the aid of the flip-flop tr the AND circuit of coincidence.

Introduction/input of number can be conducted both by low-order and high-order digits forward.

4. Circuit of the shift of a number with the aid of the decoder of nonconformity.

In some shift registers of number is realized with the aid of decoder of nonconformity (2, 10, 11, 12] (Fig. 3.9). This is observed in coincidence-types adder, where the schematic of the decoder of nonconformity already is in the register on the working conditions of summator. Then the schematic of the decoder of nonconformity obtains supplementary use for the shift/shear of the code of numbers here.

To output busbars of trigger cascades/stages is connected up decoder of nonconformity DN, control AND circuit, to which are fed/conducted shift pulses.

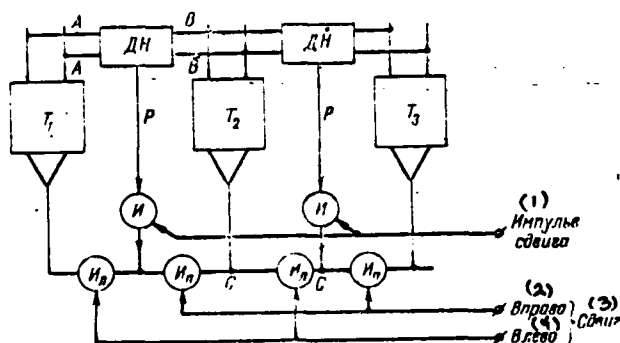


Fig. 3.9. Shift/shear of numbers with the aid of the decoder of nonconformity.

Key: (1). Pulse of shift/shear. (2). To the right. (3). Shift/shear. (4). To the left.

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If DN to the AND circuit is supplied high voltage, then through the AND circuit shift pulse is passed. As a result in the adjacent (low-order or average) digit the change-over of flip-flop will occur. The direction of the supply of this impulse/momentum/pulse depends on the state of the valves/gates of groups $И_n$ and $И_n$. If is opened the group of valves/gates $И_n$, then impulse/momentum/pulse goes to the high-order digits, and shift/shear - to the right; if are opened then a number is shifted/sheared to the left, in the direction of low-order digits.

Decoder of nonconformity is connected by similar/analogous terminals with output busbars of trigger cascades/stages, i.e., inputs, which correspond to values A and A, and also B and B, they are connected with analogous busbars of trigger cascades/stages. Output voltage/stress is defined, as is known, by the equality

$$P = A\bar{B} + B\bar{A}.$$

Consequently, output signal will be different from zero only in only case, when $A \neq B$, i.e., when in adjacent flip-flops will be "recorded" different numerals. But if the state of these flip-flops is equal, i.e., $A=B$, then $P=0$ and no carry pulse into the adjacent digits will enter.

Example. Let in flip-flop T_1 zero be recorded, and in T_2 - one, i.e., $A \neq B$. Consequently, high output potential DN will trigger AND circuit, and shift pulse, after traversing it, will enter the branch point C.

For n-fold shift/shear, just as in preceding/previous diagrams, it is necessary to feed n shift pulses and to open valves/gates of chosen direction of transmission of carry pulses.

§3.2. Summators.

Operation/process of addition most frequently as fundamental arithmetic operation of electronic digital computer serves. This means that all other arithmetic operations in the machine are reduced as the final result to the totality of the actions of addition. The operations/processes of addition are realized in the diagrams, called summators.

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During addition of two numerals of separate digit sum is formed and transfer into following high-order digit can arise. Therefore in the high-order digit can in the general case be summarized three numerals: two numerals of this digit and one - transfer of preceding/previous.

Devices/equipment, which realize addition of three numerals of digit, are called single-digits adder.

Together with single-column ones there are multidigit summators, which usually consist of identical single-digits adder, whose number is equal to quantity of digits in components/terms/addends.

Depending on character of numbers, supplied to summator, are distinguished two types of summators:

1. Parallel-action adders, intended for adding the numbers, represented in the parallel code.

2. Summators of sequential action, intended for adding numbers, represented in consecutive code.

In first case to summator numerals of all digits are fed/conducted simultaneously. This summator is multidigit and consists of parallel-connected single-digits adder in a quantity of digits in the components/terms/addends.

In second case to summator are fed/conducted components/terms/addends by step-by-step low-order digits forward. In proportion to the addition of low-orders digit to it high-orders digit etc., are fed/conducted.

1. Parallel-action adders.

Are distinguished two groups of parallel-action adders:

1. Counter-types adder, in which are utilized trigger

cascades/stages or ferrites as the accumulating elements/cells of single-digits adder.

2. Combination-logical type summators, in which sum is formed with the aid of logic elements of type AND, OR and so forth.

In first summator components/terms/addends enter one after others with interval of T , equal to repetition period of clock impulses/momenta/pulses in machine.

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In this case the sum at the output is obtained after certain command and loading time of summator τ_p after the supply of second term. The time of addition proves to be equal to

$$\tau = T + \tau_p.$$

In second summator both components/terms/addends enter simultaneously, and result at output is obtained as in first summator, after command and loading time τ_{pk} , inherent in this diagram. Time of addition $\tau = \tau_{pk}$.

As is evident, here τ is determined only by command and loading time and does not depend on T . The creation of coincidence-type adder

to a certain extent is connected with the tendency of further increase in the operating speed of adder circuits. However, in the real diagrams of combination-logical summators this is not always feasible, since its command and loading time r_{pk} , determined by transient processes in branched logical nets, is sufficiently large and in a number of cases can exceed the duration of addition in the accumulators. This fact diminishes the advantage of combination-logical summators in comparison with those accumulating.

Furthermore, positive in counter-types adder is the property, that result of addition, and also separate components/terms/addends here can be retained unlimitedly for long after reception of components/terms/addends and termination of operation/process of addition. Whereas in the combined summator result is retained only in the course of time when components/terms/addends are located on the input of summator that it is inconvenient, since in this case is eliminated the possibility of using the summator for the short-term memorization of the results of operation, and also separate components/terms/addends.

2. Multidigit counter-type adder with the consecutive transfer.

Summator of such type consists of single-digits adder, whose number is equal to quantity of digits in components/terms/addends

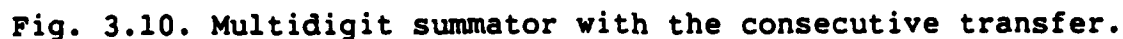
taking into account sign numerals. Fig. 3.10 shows the diagram of three adjacent digits $(i-1)$, i ($i+1$), each of which consists of trigger cascade/stage and chain/network of the formation/education of carry pulse (TsOIP). The latter consists of differentiating circuit $C_d R_d$ and diode Λ .

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Let us examine the work of diagram based on the example of the addition of numbers 011 and 001. At first summator "is cleaned", i.e., is supplied the negative pulse of "zero-setting". On all busbars of the direct code of flip-flops there will be low voltage (Fig. 3.11). Then at moment/torque $t=t_1$, to the calculating inputs of flip-flops first number 011 is supplied. The corresponding changes in the output voltage/stress (Fig. 3.11a, c) occur. Positive pulses appearing in this case at the output of the differentiating circuit (Fig. 3.11b) are limited to diodes Λ and do not enter further the calculating input of the following cascade/stage (Fig. 3.10).

Then at moment/torque $t=t_2$, calculating inputs of flip-flops impulses/momenta/pulses of second number enter, causing appropriate changes in output voltage/stress. The chain/network of the formation/education of carry pulses puts out signal into the next decade only when the flip-flop of the preceding/previous digit passes

from state 1 into state 0. In the example in question this corresponds to the appearance of a negative drop/jump on the output busbar of direct code or positive drop/jump on the busbar of inverse code.



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In low-order digit at moment/torque t , is formed carry pulse, which through gap/interval τ , is supplied into following cascade/stage. Since in the i cascade/stage one was recorded, the summator of this digit, passing into state 0, gives the transfer into the next decade. Obtained in this case impulse/momentum/pulse (Fig. 3.11d) through the interval 2τ , i.e., at moment/torque $t + 2\tau$, will install flip-flop of $(i+1)$ digit into state 1. The duration of delay τ , is determined by the duration of input pulses and by transit time in the flip-flop, which appear during the recording of numbers.

Delay is introduced in order to exclude joint action on the cascade/stage of numerical impulses/momenta/pulses and carry pulses. Its value is selected so that carry pulse would enter the calculating input after the termination of the effect of numerical impulse/momentum/pulse and transient processes connected with this in this digit.

Time of addition in multidigit summator of such type is determined by interval of T between supply of first and second number, and also by time, necessary to transmission of carry pulses.

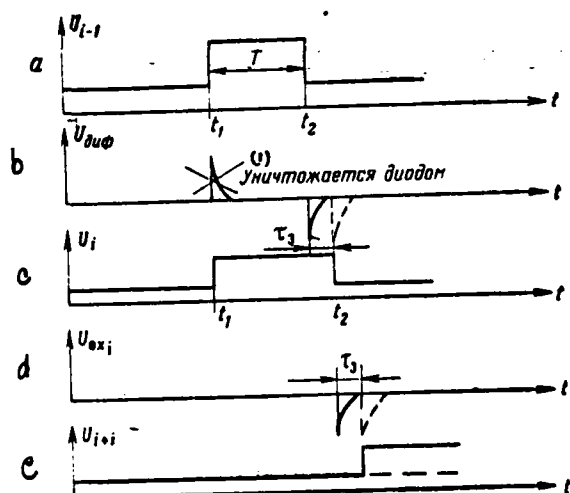


Fig. 3.11. Graphs/curves of voltage/stress in the adder circuit (with the consecutive transfer).

Key: (1). It is eliminated by diode.

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A quantity of transfers depends on the character of the stored/added up numbers. The worst case, when a quantity of transfers greatest, occurs during the addition of the numbers $\frac{111...1111}{n \text{ разрядов}}$ and $\frac{000...0001}{n \text{ разрядов}}$. In this case they will consecutively/serially appear $(n-1)$ the signals of transfer. During the determination of the time of addition it is necessary to proceed from the worst case, then the time of addition should be considered equal to

$$\tau = T + (n-1) \tau_3.$$

As is evident, time of addition is determined by quantity of digits n dependence of time of addition on quantity of digits it is deficiency/lack in this summator, since with multidigit number time of addition can reach noticeable value. In order to increase the operating speed of multidigit summator, summator with the parallel translation is applied.

3. Multidigit summator with the parallel (through) transfer.

For decreasing time of addition in accumulators it is necessary to shorten duration of transmission of signals of transfer between separate digits. In this case it is desirable to carry out not a consecutive, but parallel transmission of the units of transfer.

Fig. 3.12 depicts adder circuit with parallel translation, while to Fig. 3.13 - graphs/curves, which elucidate its work.

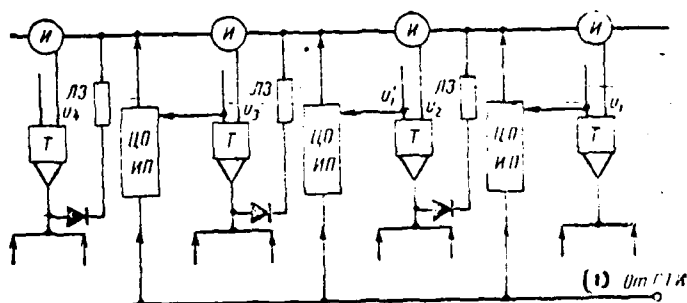


Fig. 3.12. Multidigit summator with the parallel translation.

Key: (1). From.

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Work of diagram consists in the fact that carry pulse emergent in low-order digit is supplied into high-order digits not consecutively/serially, as this was in preceding/previous diagram, but virtually simultaneously into all corresponding digits.

Distinctive special feature/peculiarity of diagram of this summator is presence of busbar of ripple-through carry, to which are supplied signals from output TsOIP. In each digit the busbar of the ripple-through carry passes through the AND circuit, which is controlled with the voltage/stress of the flip-flop of the corresponding digit.

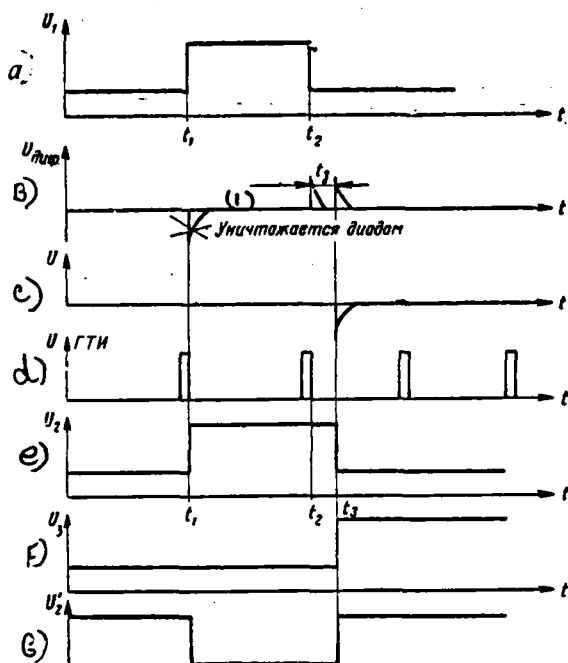


Fig. 3.13. Graphs/curves of voltage/stress at different points of adder circuit with the parallel translation.

Key: (1). Is eliminated by diode.

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If in the flip-flop is recorded 1, then AND circuit is opened/disclosed and passes carry pulse in the section of the busbar of the ripple-through carry of the following high-order digit, i.e., AND circuits possess unidirectional conductivity. Carry pulse, after falling in the section of the busbar of the ripple-through carry of

the corresponding digit, can be propagated further in two ways: the first path - through the delay line to the calculating input of the flip-flop of adjacent high-order digit (Fig. 3.12); the alternate path - through the AND circuit in the section of the busbar of the following high-order digit. The first path is opened always, and the second is opened/disclosed only if in this digit one is recorded. The signal of transfer is supplied to the busbar of ripple-through carry from the diagram TsOIP, connected to the busbar of the straight/direct or inverse code of trigger cascade/stage. Carry pulse here, just as is earlier, it is formed upon transfer of flip-flop from state 1 into state 0; however, essential difference is that as the "useful" carry pulse is considered only that, which is formed at the moment of supplying the second number, i.e., when the transition/junction of flip-flop from state 1 in 0 occurs under the effect of the impulses/momenta/pulses of the second number. Running in forward, let us point out that this transition/junction can appear also at other moments of time, which leads to the appearance of a false carry pulse, which leads to the errors. In order to exclude contact of spurious signals with the busbar of ripple-through carry, consecutively/serially in TsOIP (Fig. 3.14) circuit I_1 , controlled by clock impulses/momenta/pulses GTI (Fig. 3.13d) and which realizes time selection of output signals TsOIP at moment/torque t_1 , is introduced additionally.

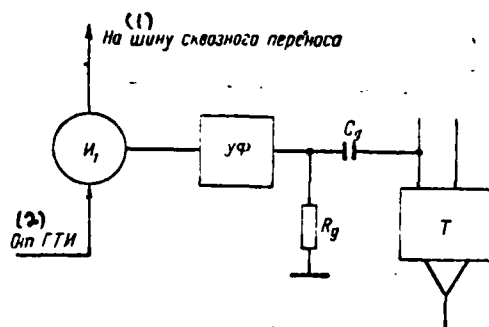


Fig. 3.14. Circuit of the formation/education of the carry pulses together with the AND circuit, which selects "false" impulses/momenta/pulses.

Key: (1). To the busbar of ripple-through carry. (2). From.

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It is assumed that the numerical impulses/momenta/pulses coincide with the clock ones (they have rigid synchronization), since in the computer(s) the first are formed from the latter. The presence of I_1 eliminates appearance to the output of the so-called "false" carry pulses, which can appear at moment/torque $t_1 + \tau$, (see below the description of the work of diagram). The given on Fig. 3.14 diagram TsOIP is analogous examined earlier and consists of differentiating circuit $R_g C_g$ and amplifier-shaper of UV which agrees polarity, and also the waveform, necessary for the correct work of AND valve/gate.

Signal on TsOIP can enter from the busbars of both the straight line and of the inverse codes of trigger cascade/stage.

Let us trace work of multidigit summator with ripple-through carry based on example of addition of numbers 011 and 001. At first summator is cleaned, and all flip-flops are set in position 0 (Fig. 3.12). Then at moment/torque t , the first number is supplied and on the busbars of the direct code of three digits high voltage (Fig. 3.13a, e, f, g), which corresponds to one in each digit, is established/installed, i.e., number 011 is written/recorded. After the repetition period of the clock impulses/momenta/pulses T second number 001 is supplied, activating of the flip-flops of those digits, where there are units of the second number. In this case if the flip-flop passes of 1 to 0, then in this digit the unit of transfer is formed. Thus, at moment/torque $t, = t+T$ are formed in the summator intermediate result (Table 3.1) and the units of the transfer in the appropriate digits.

Table 3.1.

(1) Состояние разрядов в момент t_1 . . . 0111		(2) Подается первое число
(3) Процессы в сумматоре в момент t_2	0001 0001 0110	(4) Подается второе число (5) Образуется единица переноса (6) Образуется промежуточный результат
(7) Состояние разрядов в момент t_3 . . . 1000		(8) Окончательный результат после сложения с единицами переноса

Key: (1). State of the digits at the moment/torque. (2). First number is supplied. (3). Processes in summator at moment/torque. (4). Second number is supplied. (5). Unit of transfer is formed. (6).

Intermediate result is formed. (7). State of digits at moment/torque t_3 . . . 1000. (8). Final result after addition with units of transfer.

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Carry pulse (Fig. 3.13b) enters virtually immediately into all adjacent subsequent digits, where one is recorded, after reaching the nearest digit, where it is recorded by 0 (Fig. 3.13f). Further carry pulse will not go, since in this digit on the busbar of direct code there will be the low voltage and AND circuit will be closed. The entered unit of transfer is added into all adjacent high-order digits, including the digit, where was recorded 0. After the delay time τ , i.e., at moment/torque $t, = t_1 + \tau$, the carry pulses will enter

the calculating input of flip-flops, they will cause their functioning and in the summator the final result of addition 1000 is formed.

It must be noted that at moment/torque t , series/row of flip-flops, in particular flip-flops of second and third digits, passes from state 1 in 0. This transition/junction causes the appearance of a unit of transfer; in this case at the output of UV the impulses/momenta/pulses are formed, but they are the so-called "false" carry pulses. Actually/really, the carry pulse, which arose at moment/torque t , because of the diagram of ripple-through carry already entered into the adjacent digits; therefore the addition one additional unit, i.e., impulse/momentum/pulse, at moment/torque t , it is possible to lead only to the erroneous results. The limitation of this impulse/momentum/pulse is achieved by the introduction to selection with the aid of the circuit Π_1 .

Time of addition in this diagram

$$\tau = T + \tau_3$$

virtually does not depend on quantity of digits. The duration of delay, as in the preceding case, it is determined by the duration of numerical pulses and by transient processes in the flip-flop. Delay is introduced into the cascade/stage in order to exclude joint action on the flip-flop of numerical impulses/momenta/pulses and carry

pulses.

Diodes $\bar{D}$ are untying and eliminate entry/incidence of numerical impulses/momenta/pulses on busbar of ripple-through carry.

4. Diagram of the formation/education of end-around carry.

End-around carry (TsP) is observed during addition of negative numbers in reverse/inverse and modified inverse codes. It appears in two cases:

- 1) during the addition of two negative numbers;
- 2) during the addition of numbers of the different signs, when their sum is positive.

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In these cases one in supplementary digit, adjacent with sign (more left than sign), appears, causing transition/junction of this digit from 0 to 1. The diagram of the formation/education of end-around carry (Fig. 3.15) consists of supplementary digit and pulse-shaping circuit TsP. In the supplementary digit there is a trigger cascade/stage DT, connected usually by the circuit of the

formation/education of carry pulse with fundamental sign position. At the output of this supplementary digit is a circuit of the formation/education of the impulse/momentum/pulse of end-around carry, which is sometimes the differentiating circuit, and the shaping amplifier, intended for the agreement of polarity and obtaining of the necessary form of output signal. This chain/network is connected constantly with the calculating input of the low-order digit of summator and puts out impulse/momentum/pulse only upon transfer of flip-flop DT from state 0 into 1. By supply of this impulse/momentum/pulse is realized end-around carry. The addition of one can cause the transfers in the summator, for which must be provided the special time, necessary for the realization of end-around carry.

In summators, which use modified inverse code, formation/education of end-around carry is conducted thus. However, it is necessary to consider that there there are two sign flip-flops, therefore, flip-flop DT corresponds to the third digit.

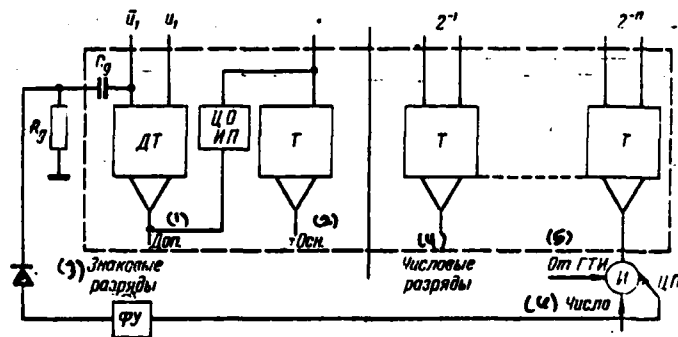


Fig. 3.15. Circuit of the formation/education of end-around carry.

Key: (1). Dopa. (2). Osn. (3). Sign positions. (4). Numerical digits.
 (5). From. (6). Number.

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5. Summator for the numbers, represented in the consecutive code.

For addition of numbers, represented in consecutive code, summators of combination and accumulating types can be used.

Coincidence-type adder (Fig. 3.16) consists of single digit combination-logical adder to three inputs OC-3 [10-12], registers of first and second number (P_1 and P_2), register of sum P_3 and also delay units Λ and UVZ. Components/terms/addends in advance are introduced into the registers P_1 and P_2 , whence then they enter on OC-3 (low-order digits forward). The register of sum preliminarily is cleaned, and all its digits are found in the zero state. As the register of sum one of the numerical registers (P_1 or P_2) can be used, where sum in proportion to the release of register from the number, supplied to the summator, will be brought in. However, for simplification in the reasoning systematic justified will be the examination of diagram with the chosen register of sum.

Work of diagram flows/occurs/lasts in this sequence. Entered the diagram at moment/torque t , clock impulse/momentum/pulse (α) (Fig. 3.17) establishes/installs all flip-flops OC-3 in the zero position, acting on busbar "Obsolete 0".

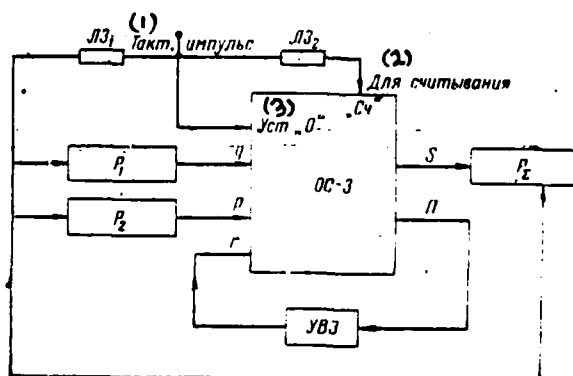


Fig. 3.16. a combination-logical summator for the numbers, represented in the consecutive code.

Key: (1). Stroke/cycle. impulse/momentum/pulse. (2). For reading.
(3). Obsolete "0".

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After the time τ_1 , the same impulse (β) produces the shift/shear of numbers in the registers, as a result of which of numeral of both numbers they enter summator. Through the interval τ , the same impulse/momentum/pulse (γ), after passing delay $\Pi 3$, enters the busbar of mf/SCh, occurs the reading of sum S and transfer Π with the aid of the diagram of pulse-potential connections/communications. The numeral of sum is written/recorded in the register P_2 and the signal of transfer enters delay unit UVZ and is delayed by the period $\tau_2(\delta)$. In the period τ , on the diagram clock impulses/momenta/pulses a_i and

signal of previous carry δ operate. Time τ_1 is determined by the duration of clock pulses and transient processes, connected with their effect.

The interval must be selected so that at output OC-3 sum and transfer would have time to be formed. After reading at moment t_1 the following clock impulse/momentum/pulse enters. It dumps the readings/indications OC-3 into zero, and through τ_1 is realized the introduction/input of the numerals of the next decade into the summator. The same time carry digit from the preceding/previous digit already will enter the input busbar of summator r . Thus, the time of addition here coincides with period T of clock impulses/momenta/pulses in the machine.

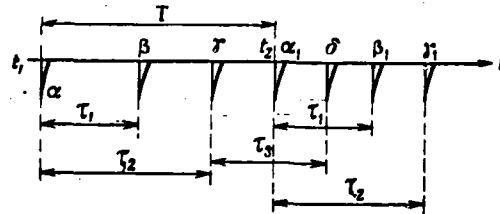


Fig. 3.17. The time diagram of a combination-logical summator: α - clock impulse/momentum/pulse; β - shift pulse of a number in the registers; γ - read pulse; δ - carry pulse into the next decade; T - period of clock impulses/momenta/pulses; α_1 ; β_1 ; γ_1 - analogous impulses/momenta/pulses of the following stroke/cycle (digit).

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However, the latter to expediently restrict by the relationship/ratio

$$T \geq \tau_2 + \frac{\tau_3}{2},$$

whence it follows that the command and loading time OC-3 is limited to the duration of transient processes with the recording, the reading of numbers, and also with the formation of sum in the logic circuits. The resulting time of the addition of one digit in this diagram is nevertheless small, which indicates the prospect of this diagram for adding the numbers in the consecutive code.

6. Counter-type adder.

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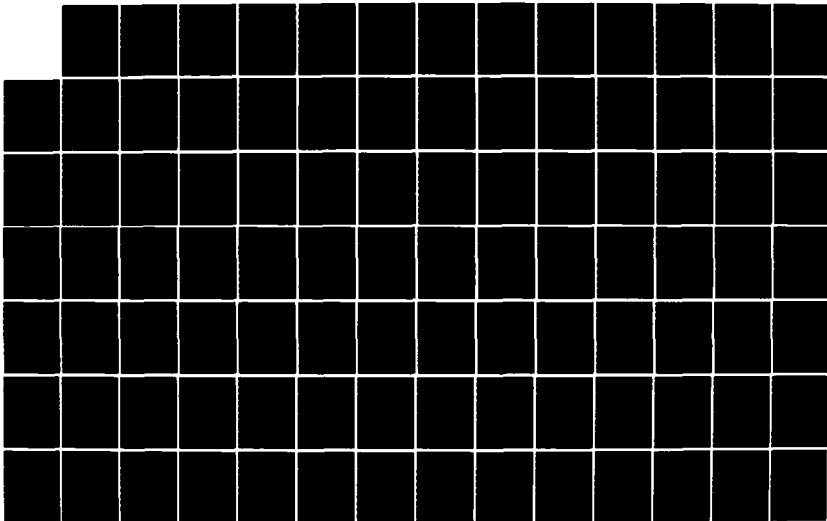
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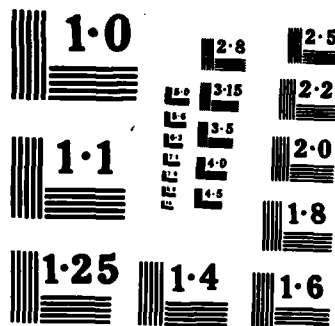


Diagram of accumulator (Fig. 3.18) for adding numbers, represented in consecutive code, consists, just as in the preceding case, of registers of numbers P_1 and P_2 , register of sum P_3 and strictly single-column accumulator (ONS), which is most specific node of this diagram. The work of this diagram flows/occurs/lasts as follows. To the diagram are supplied the clock impulses/momenta/pulses, under effect of which numbers enter from the registers P_1 and P_2 into the summator ONS by low-order digits forward. For each stroke/cycle into the summator enter numbers only of one digit, they store/add up, and the sum of this digit "is written" in register P_3 . In the register of sum the code of a number of sum also is shifted/sheared under the action of clock impulses/momenta/pulses on one digit from each impulse/momentum/pulse. Thus, the addition of entire n-bit number is conducted after n of the repetition periods of clock impulses/momenta/pulses.

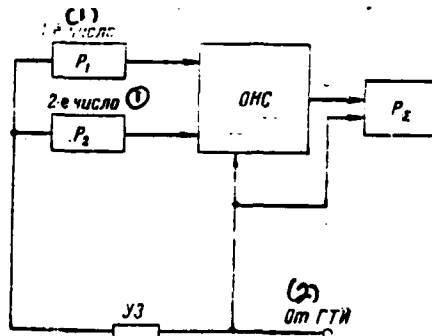


Fig. 3.18. Block diagram of the accumulator for adding the numbers, represented in the consecutive code.

Key: (1). a number. (2). From.

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Let us examine special features/peculiarities of work of single-column accumulator. Its electrical circuit is represented in Fig. 3.19, and time diagrams - in Fig. 3.20. Diagram consists of the summing flip-flop T_1 , which commutates T_1 , delay units $ЛЗ_1$, $ЛЗ$, and $ЛЗ$, logic circuits $И_1$, $И$, and differentiating circuit C_0R_0 with the diode $Д$. Into the diagram numerical and clock impulses/momenta/pulses enter, and one sum pulse S is output only; the signal of transfer is formed within the diagram and is delayed in the channel $ЛЗ$.

Time of addition of numerals of one digit occupies six

conditional strokes/cycles, noted in Fig. 3.20 by letters α ; β ; γ ; δ ; ϵ ; μ . Let at moment t , low-orders digit of the first and second number be supplied to the summator, entering the terminals 1 ν and 2 ν (Fig. 3.19). However, the calculating input of flip-flop T, they enter with the interval τ_1 , determined by delay time $\Pi 3_1$.

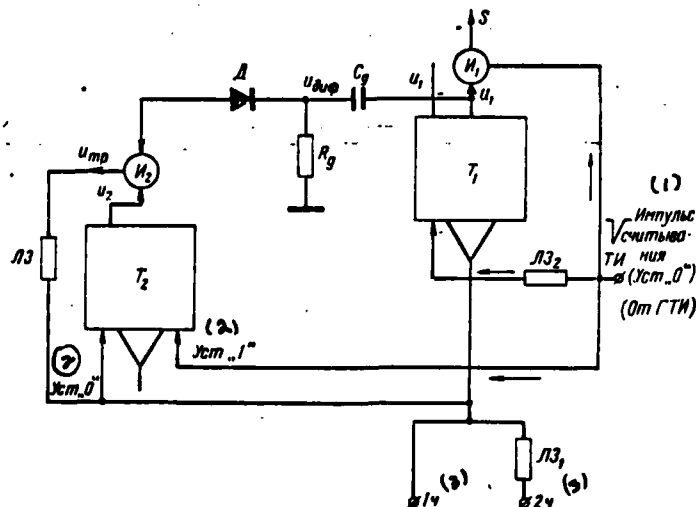


Fig. 3.19. Electrical circuit of the accumulator for adding the numbers in the consecutive code (ONS).

Key: (1). Impulse/momentum/pulse of the V reading (Absolute "0") (from GTI). (2). Set "0". (3). h.

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Let both numerals have a value of 1, then at moment/torque t_1 appears the transfer, which is formed at the output of the differentiating circuit (Fig. 3.19 and 3.20) as a result of differentiation of drops/jumps in voltage/stress u_1 (Fig. 3.20) on the output busbar of flip-flop T_1 (Fig. 3.19). Positive pulse is limited to diode D , and negative is passed through N_1 , and it is delayed in LZ by four conditional strokes/cycles. Diagram N_1 is controlled by flip-flop T_1 on the busbar of inverse code; it is triggered, when in T_1 it is recorded by 0, and it is closed, when it is recorded 1.

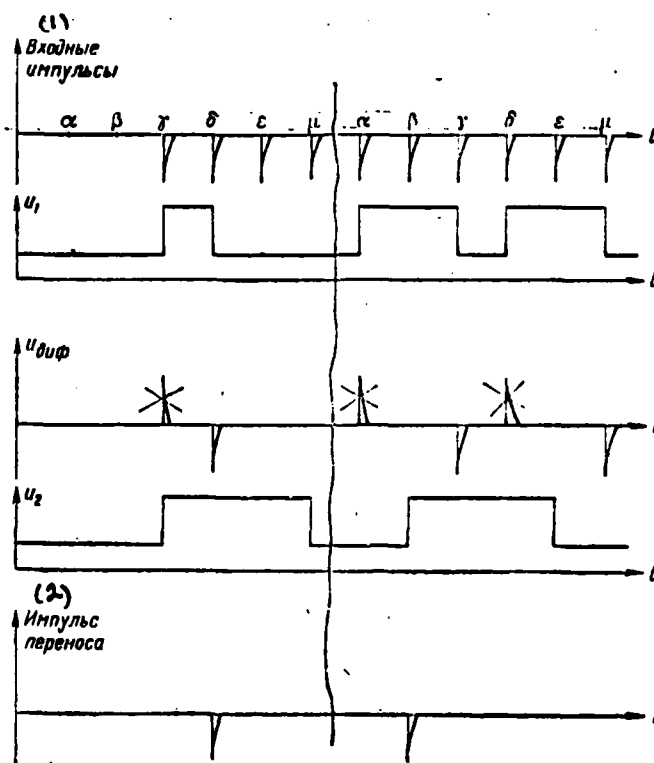


Fig. 3.20. Time performance record of the accumulator ONS.

Key: (1). Input pulses. (2). Carry pulse.

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At moment t , to the terminal TI enters clock impulse/momentum/pulse, being propagated hence in three directions: the first - through the diagram H_1 for the reading of the sum of this digit; the second - to flip-flop T_1 , converting it into state 1 and cutting off by that H_1 ;

the third - through the delay line J3, to the busbar "Obsolete 0" flip-flop T_1 . In the latter case this impulse/momentum/pulse will install T_1 to the zero position at moment/torque $t = t_p$. On this the stroke/cycle of the addition of numbers of this digit concludes (undulating line) and the addition of the numerals of the next decade begins.

To the first during addition of numerals of next decade at moment t_p (or t_n) enters carry pulse. Then at moments/torques t_r and t_s follow numerical impulses/momenta/pulses, causing the appropriate changes in the state of flip-flops. Let in both digits one store/add up again, then the sum of digit is newly equal to 1, which is read at moment/torque t_n and then when t_p flip-flop T_1 is readjusted to zero.

Upon transfer of T_1 from one state into zero in diagram carry pulse appears. However, if this transition appears not under the effect of numerical impulse/momentum/pulse (at moments/torques t_r or t_n), and under the effect of the "official" impulse/momentum/pulse "Obsolete 0" (at moment/torque t_p), then carry pulse is false. For eliminating the false carry pulse into the diagram is introduced the totality of elements T_1 and H_1 , which locks the channel of the transfer immediately after the reading of sum, i.e., at moment/torque t_n . Thus, at moment/torque t_p the channel of the transfer will be always closed and the appearing in the diagram "false" carry pulse

will not enter the input of calculating flip-flop T_1 .

On the contrary, effective impulses of transfer always can pass to calculating input T_1 . In fact, these impulses/momenta/pulses can appear only at the moment of supplying the first either second number, i.e., at moments/torques t_1 or t_2 . Since the delay time τ , in the channel UZ is constant, in the next decade carry pulse can appear only at moments/torques t_1 or t_2 . moreover diagram H, for these impulses will be always open.

Duration of conditional strokes/cycles can be different. With the arbitrary relationship/ratio of cycle duration it is desirable to have their value of minimum. However, the smallest duration of each stroke/cycle is limited to the appropriate transient processes in the diagram.

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Totality of six conditional strokes/cycles indicated determines duration of addition and respectively rate of supply of clock and numerical impulses/momenta/pulses into diagram. The addition of each digit begins from the supply of clock impulses/momenta/pulses and installation of calculating flip-flop to position 0, i.e., actually the beginning of addition coincides with moment/torque t_1 . Therefore

on the numerical registers clock impulses/momenta/pulses are supplied through the delay UZ (Fig. 3.18) after the time τ .

As delay units of UZ can be utilized depending on required duration of delay artificial lines or relaxation diagrams of type of half-wave multivibrator. The operating speed of the single-column accumulator, apparently, is less than combination-logical type summator. Therefore for the addition of the numbers, which enter in the consecutive code, it is more expedient to apply combination-logical summators.

§ 3.3. Ferrite diagrams.

In computers together with diode and transistor elements/cells diagrams, which use ferromagnetic elements/cells with right-angle hysteresis loop, extensively are used. Depending on the mode of operation of core these diagrams are divided into two groups. In the first the core intensely reverses magnetism on the section of curve of magnetization ab or cd (Fig. 3.21) in transmitting of energy from the source to the load. In the second case, the vice versa magnetic flux of core virtually does not change in transmitting of energy (section ad or bc). conditionally by analogy with the magnetic amplifiers they call these two modes/conditions respectively transformer and choking.

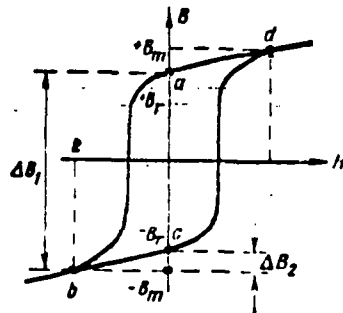


Fig. 3.21. Maximum hysteresis loop of ferrite.

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In the contemporary computers the widest use received the transformer diagrams, which are examined below.

Each ferrite can store numeral only of one binary digit. The process of storing the number is reduced to the recording of a number, strictly to storage and to reading. All this occurs for three separate strokes/cycles, components the cycle of the rotation/access of digital information in the ferrite. For the realization of cycle the ferrite usually has one or several input windings w_1 ; w_2 (Fig. 3.22), one output w_3 , and one clock winding w_4 , or the winding of reading. In the clock and input windings the pulse signals, which call the magnetic reversal of core, are supplied. The polarity of the reversing magnetism pulses is usually identical, and the necessary

direction of the magnetic field is provided by the direction of the turns of windings. However, in certain cases the supply also of heteropolar impulses/momenta/pulses is possible. Subsequently the beginning of windings in the figures is marked by points.

Let us examine processes in ferrite with reading and recordings of codes of numbers of binary number system, after using idealized hysteresis loop (Fig. 3.23). Let us assume that the core was given to state 1, i.e., had residual magnetization, equal to $+B_r$. It is necessary to produce the reading of this unit. For this to the clock winding the read pulse, which creates in the core the magnetic field of negative polarity, in the value which exceeds coercive force, is applied. Disregarding in the diagram transient processes, we assume that a change of the magnetic field in the ferrite takes the form of square pulse with an amplitude of H_r (Fig. 3.23a) and by the duration, which exceeds the switching time of samples/specimens. We assume that the pulse amplitude is proportional to current I_r in the reading winding:

$$H_r = \frac{0.4\pi w_r I_r}{l_{cp}}, \quad (3.3.1)$$

where l_{cp} - average/mean length of line of magnetic force.

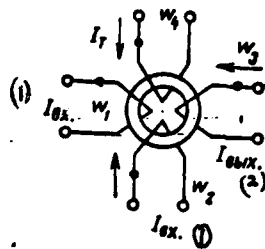


Fig. 3.22. Core with the windings.

Key: (1). Input. (2). Output.

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Under action of leading impulse front magnetic reversal of ferrite on section of curve 1, b, c will occur and magnetic induction will change to value

$$\Delta B(1) = -[2B_r + \Delta B'(1)].$$

As a result in windings will be directed emf, proportional to rate of change in flow of magnetic induction:

$$e(1) \approx -\frac{dB}{dt}.$$

However in order to consider qualitative character of obtained signals, let us replace differentials with finite increments: $db \approx \Delta B$; $dt \approx \Delta t$. In this case signal in the output winding ¹ (taking into account sign $\Delta B(1) < 0$)

$$e_c(1) \approx w_s \frac{\Delta B(1)}{\Delta t}. \quad (3.3.2)$$

FOOTNOTE ¹. The shapes of pulses given in Fig. 3.23 are approximate, intended for the qualitative characteristic of signals. ENDFOOTNOTE.

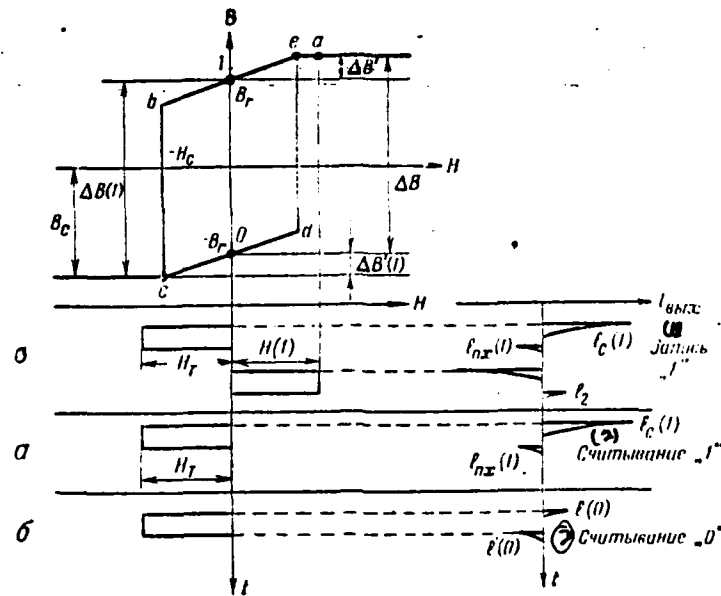


Fig. 3.23. Diagram of signals in the ferrite.

Key: (1). Recording. (2). Reading.

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Under action of trailing edge of pulse change in magnetic induction to value $\Delta B'(t) = -B_r + B_c$, will occur which will cause emf.

$$e_{nx}(t) \approx -w_s \frac{\Delta B'(t)}{\Delta t}. \quad (3.3.3)$$

Since

$$B_c > B_r, \text{ then } \Delta B'(t) < 0 \text{ and } e_{nx}(t) < 0.$$

Negative pulse $e_{nx}(l)$ is noise signal, while positive pulse $e_r(l)$ - by useful signal. The ratio of useful signal to the interference is determined by the squareness of hysteresis loop. Actually/really, from (3.3.2) and (3.3.3) it follows that

$$\left| \frac{e_c(l)}{e_{nx}(l)} \right| = \frac{B_c + B_r}{B_c - B_r}.$$

It is possible to consider that the value of induction B_c corresponds to saturation induction B_m . Then, taking into account that squareness ratio $\alpha = \frac{B_r}{B_m}$, we will obtain

$$\left| \frac{e_c(l)}{e_{nx}(l)} \right| = \frac{1 + \alpha}{1 - \alpha}. \quad (3.3.4)$$

Hence it follows that for increasing ratio of signal to interference in logic circuits it is desirable to apply ferrites with high squareness ratio α . In the ferrites, utilized in the contemporary computers, this value reaches value of 0.9-0.95, in this case the ratio of signal to the interference

$$\frac{e_c(l)}{e_{nx}(l)} = 19 \div 40.$$

Let us examine processes with reading of zero (Fig. 3.23b). If core is found in the zero state, then under the influence of clock impulse/momentum/pulse magnetic induction will vary within the limits $\Delta B'$, i.e., in the section OS of magnetization curve.

Under the action of leading edge is obtained change in induction $\Delta B_1 = B_r - B_c < 0$, while under action of trailing edge $\Delta B'_2 = B_c - B_r > 0$. Two small impulses of positive and negative polarity (Fig. 3.23b) respectively appear.

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Thus, with the reading of one they appear two different by the value of the impulse/momentum/pulse of different polarity, in this case positive approximately/exemplarily in $\frac{1+a}{1-a}$ - once is more than negative.

However, with reading of zero appear two heteropolar impulses/momenta/pulses of small amplitude, which are actually disturbing pulses. In ferrites with the high squareness ratio the value of these impulses/momenta/pulses is small. For their compensation apply clipping to the value of interference level or special diagrams with the supplementary compensative cores. The presence in the output winding high-amplitude of positive pulse attests to the fact that in the ferrite one was recorded. After reading the ferrite passes zero into state and the information recorded previously disappears, if there are no special transcribing

circuits.

Recording of one is realized by cycling of positive polarity $H(1)$ (Fig. 3.23c) on input winding ω_1 . Value $H(1)$ must exceed coercive force H_c . If ferrite was in state of zero, then the applied impulse/momentum/pulse will produce the magnetic reversal of ferrite. In this case under the action of leading edge magnetic induction will change to value $\Delta B = \Delta B(1)$, and under the action of rear - to value $\Delta B'$. Respectively in the output winding negative and positive pulses appear. From preceding/previous reasoning it follows that the negative pulse has large amplitude, and positive - small. These impulses/momenta/pulses are not the carriers of numerical data and therefore they are limited to the diodes or other diagrams.

REGISTERS ON FERRITE CORES.

Registers on ferrite cores are intended for short-term storage of code of number and shift/shear, necessary for changing exponent. With the shift/shear of numbers it is necessary to transmit information into the adjacent cores.

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However, direct drive cannot be realized, since a change in state of

one of the cores immediately produces change in the magnetic field of another. Therefore the intermediate coupling elements, which possess the following properties, are necessary: they must have the required time lag in the transmission of signal, pass numerical information only in one direction (to the right or left core), have low losses.

First property depends on method of supplying clock impulses/momenta/pulses to ferrite cores of register. Simultaneous and nonsimultaneous cycling on the adjacent digits of register is distinguished. Registers with cophasal and asynphasal clock windings [5, 14] respectively are distinguished.

Simplest cophasal register is shown in Fig. 3.24. Here ferrites are connect/joined together by the quadrupole, whose schematic is represented in Fig. 3.25. Shift/shear is realized into three stages:

1. The reading of information in the ferrites (it is realized simultaneously in all ferrites immediately - cophasal reading).

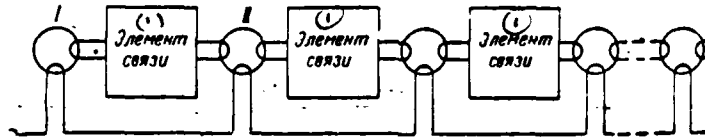


Fig. 3.24. Register on the ferrites.

Key: (1). Coupling element.

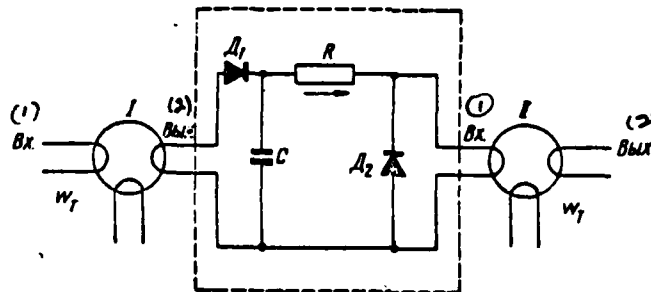


Fig. 3.25. Coupling element of register on ferrites.

Key: (1). Input. (2). Output.

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2. Intermediate storage of counted information (since to simultaneously produce reading and recording of information in ferrite is impossible; therefore it is necessary to termination of reading to memorize number).

Intermediate storage is realized with the aid of

capacity/capacitance C of quadrupole.

3. Recording of numerical information from device/equipment of intermediate storage into adjacent ferrite. Let us examine the work of diagram. assume that in ferrites I and II it was recorded 1. The entered clock impulse will set ferrite into state 0 ($-B_r$), in this case on the output winding the impulse/momentum/pulse of positive polarity will appear. If

$$\sqrt{R^2 + \omega_H^2 L^2} > \frac{1}{\omega_H C},$$

where ω_H - lowest frequency of the spectrum, and L - inductance of the winding of the following ferrite, then in essence pulse current will go to the charge of capacitor C. At the termination of read pulse the capacitor will be discharged through resistor/resistance of R and input core coil II, after transferring it into state 1. Diode Δ_1 serves for limiting the negative pulse, which appears during the recording in ferrite I, and also for preventing the digit of capacity/capacitance C through the input core coil I. Diode Δ_1 exerts by-passing on the input core coil II, limiting the negative pulses, which appear with the reading of one. This provides the one-sided motion of information in the register.

Register examined possesses deficiencies/lacks, connected with energy losses in quadrupole on resistor of R, and also somewhat increased time of shift/shear in view of slow response of discharge

of capacitance C.

From these deficiencies/lacks two-phase register (Fig. 3.26) proves to be free. Here clock impulses/momenta/pulses on any pair of adjacent ferrites are supplied nonsimultaneously, since they are supplied from the different busbars, in which clock impulses/momenta/pulses are shifted on the time the specific value.

For intermediate storage here are utilized not capacitances, as this was in the preceding case, but special ferrite cores.

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Thus, in this register must exist two forms of the ferrites: numerical - odd (1, 3, ...) and intermediate storage - even (2, 4, ...). Entire number of ferrite cores in the two-phase register must be two times more than a quantity of digits in a number. Shift/shear is realized into two stages. At first from the numerical ferrites information is converted into the intermediate ones - even, and then by the second impulse/momentum/pulse is converted into the following adjacent numerical digit.

Registers with split winding as coupling element between cores (Fig. 3.27) have special importance. In this case in the absence of

clock impulses/momenta/pulses it is possible to produce different changes in the state of one or several cores, without disrupting the selected states of other cores, that, in particular, makes it possible to carry out a parallel input and an output of a number of this register, the reversible shift/shear and other operations/processes.

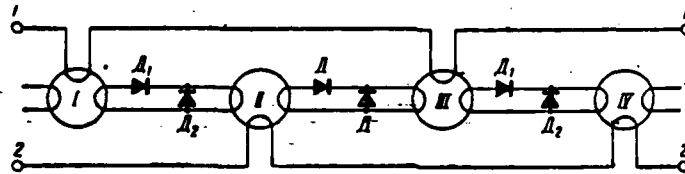


Fig. 3.26. Two-phase register.

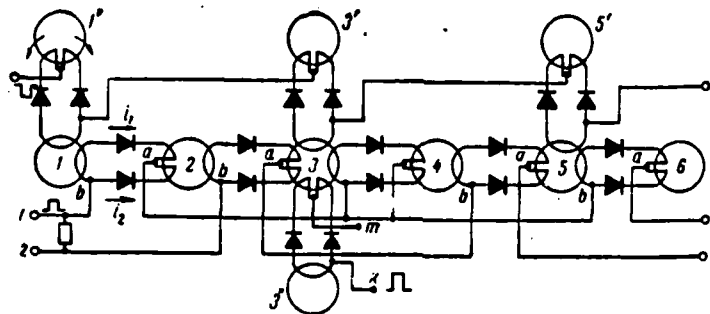


Fig. 3.27. Register with split winding.

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Transmission of information between ferrites with the aid of split winding is realized with use of clock impulse/momentum/pulse, applied to terminals ab. It is known that in the absence of transformer losses the applied voltage/stress because of a change in the magnetic fluxes will be always equal and it is opposite on the sign induced emf. This induced emf counterbalances the external applied voltage/stress. For example, if ferrite I it was in state 0, then under the influence of clock read pulse currents in both branches of split winding will be approximately/exemplarily identical

($i_1 = i_2$) and the resulting field in the second ferrite from the effect of clock impulse/momentum/pulse will be equal to zero. But if ferrite 1 is in state 1, then in this case currents in the branches of split winding are different ($i_1 > i_2$). Spill current creates the impulse/momentum/pulse of the magnetic field of positive polarity, which writes/records one in the second core, if the latter was found in the zero state. Thus, transmission 1 is realized into that core, on which is arranged/located split winding. In this case the bifilarity of split winding eliminates the transmission of information in the opposite direction.

Let us note that in multidigit register with split winding (Fig. 3.27) supply of clock impulses/momenta/pulses into adjacent ferrites must be conducted according to two-phase diagram. In this case here also there are two groups of the ferrites: numerical - odd (1, 3, 5, ...) and intermediate memorization - even (2, 4, 6, ...), necessary with the shift/shear numbers.

To numerical ferrites are connected diagrams of input and output, which use ferrite cascades/stages - special ferrite cores with split winding. For input and output of numbers is utilized the cophasal supply of the clock impulses/momenta/pulses, which are supplied consecutively/serially to the ferrites of the introduction/input (conclusion/output) of numbers.

Together with one-sided shift/shear in ferrite register shift/shear into both sides can be realized. For this it is necessary to supplement the ferrite cores of register with the second group of split windings (Fig. 3.28). One group of windings w_{n1}, w_{n2}, w_{n3} provides the shift/shear of numbers to the right, and another w_{n1}, w_{n2}, w_{n3} - to the left.

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Diagrams of connection of both groups of windings are identical and analogous to diagram in Fig. 3.27; therefore further description of diagram we omit. Diagram in Fig. 3.28 can be supplemented still by ferrites of input and output, which must be connected up to the numerical ferrites.

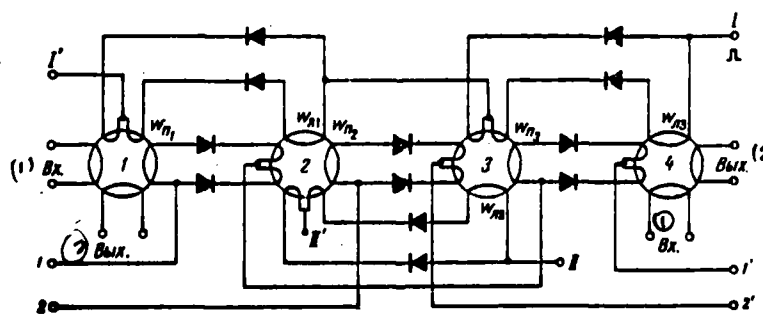


Fig. 3.28. Reversible register.

Key: (1). Input. (2). Output.

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Chapter 4.

SIMPLEST DIAGRAMS OF THE PRIMARY PROCESSING OF RADAR SIGNALS.

Device/equipment of primary information processing can be considered as terminal device of radar, which realizes post-detector processing of signal in period, equal to period of survey/coverage of radar or close to it.

Fundamental problem of primary working/treatment is detection of signal (target) in presence of interferences and measurement of coordinates of object detected.

Representation of target coordinates in the form, convenient for using these data in further nodes of processing radar information, is supplementary function of devices/equipment of primary information processing. Usually these data are represented in the form of the binary code.

It must be noted that measurement of target coordinates and their conversion into digital code can be realized by terminal devices of other types, for example with the aid of diagrams of automatic removal/output in the form of servo systems with subsequent data processing into discrete/digital code by converters of type "shaft - numeral" or "voltage/stress - numeral".

However, for work on many targets creation of this diagram causes essential difficulties.

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In any case in the presence of interferences, apparently, with one of the convenient methods of processing signal is the processing with the aid of the electronic digital computers. In this case the conversion into the digital code is here obtained naturally in the form of supplementary result and does not require the use/application of the special converting devices/equipment.

In contrast to conventional displays on cathode-ray tubes in machines of primary working/treatment appear even supplementary functions, connected with memorization of coordinates of separate marks and in number of cases with numbering of objects, discovered during one period of survey/coverage. The latter is caused by the

fact that into the machine of primary working/treatment for the time of one survey/coverage come data immediately from several targets, whose quantity can be changed from one survey/coverage to the next. For the discrimination of marks and account of the appearance of new targets, naturally, is necessary the preliminary numbering of objects. Diagrams and algorithms of the work of the devices/equipment of the numbering of targets depend on a quantity of predicted targets in the zone of the detection of station.

Terminal devices of radars in the form of indicators with cathode-ray tubes can be utilized simultaneously with devices/equipment of computer(s) of primary working/treatment, fulfilling functions of control of proper working order of work of radar channel and presentation of information about situation in zone of detection of RLS in their simplest, unfinished form.

Devices/equipment of primary processing of radar information in connection with creation of antimissile system (for example, in USA PLATO system) acquire especially important importance. The creation of such devices/equipment was caused by obvious need remove man as operator out of the channel of radar, which works in the circuit of automatic detection and guidance of the means of antimissile technology. Actually/really, under the actual conditions of men one cannot reveal/detect the objects, which move at a velocity, which

exceeds the speed of sound, and follow them. It cannot also operationally rate/estimate the manifold and rapidly changing situation, created by the rapidly flying targets, and make correct decision.

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Therefore it became necessary to create the special automatic devices/equipment of the detections, which in the complex with the universal computers could realize automatic control of the means of antimissile technology.

Furthermore, it is possible to indicate one more fact, which stimulates development of electronic computers of primary working/treatment (EVM of PO), this automatic flight control of high-speed aircraft in airports with large traffic volume. In fact, with an increase in the intensity of air flights the dispatcher of airfield is unable to productively make entire manifold coordinating work, while the general purpose computers can rapidly process large quantity of information, connected with traffic control in the airport.

For using general purpose computers in these conditions automatic target detection and transformation of information into

form acceptable for these machines is necessary.

Is necessary also corresponding speed of admission of data into machine, with which computer could simultaneously service/maintain several information tracks on real time. All these questions sufficiently easily can be solved by the use/application of the specialized machines of processing signals.

Finally, should be noted one additional series of questions, whose solution also stimulates development of means of primary processing of radar signals. These are maintenance/servicing and the operation of radars, which are found in the extreme operating conditions.

Obviously, transmission of unfinished radar data together with noises along radio relay lines or along other any channels is economically unfavorable in view of large width of spectrum of this signal. Furthermore, unfinished radar data are unsuitable, as is known, for the direct introduction/input into the digital computer they require at receiving end of the corresponding filtration and transformation into the discrete/digital form.

Apparently, transmission along lines of communications of radar data, that contain only useful information to maximum degree about discovered targets is most advantageous. In this case the relayed signal has a small frequency band and its transmission does not cause large difficulties.

Narrow-band signal of this form is obtained in output of specialized electronic computer of primary processing, given to radar or built in it. Here signal is purified from the interferences and contains only useful information in essence.

Computers of primary processing of radar signals in application to problems of detection with plan position indicator at first will examine. Diagram and device/equipment of machine will depend on a quantity of simultaneously recorded targets.

Structure and composition of machine of primary processing of radar signals significantly depend on whether pulse repetition frequency in radar is constant or variable. All diagrams of detection in the devices/equipment of primary working/treatment utilize building-up principle of signal in this or another form.

In this case accumulation must be conducted in such a way that signals, accepted in one and the same sections of range, would be

accumulated. Hence follows the dependence of the schematic of accumulator/storage on the constancy of the pulse repetition frequency of RLS.

If frequency in station is constant, then are applied synchronous accumulators/storage, which use these or other devices/equipment of constant delay (magnetic drums, ultrasonic lines, charge-storage tubes, etc.).

But if pulse frequency is changed, then character of storage device/equipment will be other. A change in the structure of storage device/equipment leads to a change in interaction between the schematics of the device/equipment of detection and ranging. Thus, for instance, in certain cases here can be conducted the quantization of range in the separate elementary sections and processing signal is performed in each section independently. Are possible other solutions.

Let us examine diagrams and composition of device/equipment of primary working/treatment separately for two cases: constant and staggered repetition rate of radar.

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S4.1. Diagrams of processing radar signals at the constant pulse repetition frequency.

Device/equipment of the detection of signal (UOS).

Let us examine simple device/equipment of primary processing of radar signals, after assuming that it services/maintains station, intended for measuring only two coordinates of individual target - range and azimuth. Let us assume that the survey/coverage of space is realized during the uniform rotation of antenna in the horizontal plane and the pulsed mode of radar.

During one turn of antenna device/equipment of primary working/treatment must with specific value of probability establish/install presence of target in zone of detection of station and measure distance of it and azimuth.

Simplified circuit of device/equipment of primary processing of radar signals (Fig. 4.1) encompasses device/equipment of detection of useful signal against the background of noises (UOS), ranging circuit to discovered target (SID), measuring circuit of angular coordinate

(SIUK), and also output memory unit (ZU), in which are written/recorded target coordinates, obtained during one period of survey/coverage of space.

Given diagram is to a certain degree of conditional, and its block composition, and also complexity can be changed depending on signal-to-noise ratio, accuracy measurements and special features/peculiarities of work of diagram. It is obvious, the accuracy of measurement depends on signal-to-noise ratio and with the high values of signal, which considerably exceed noise level. Detection, apparently, can be conducted together with the measurement of coordinates. This should be understood in the sense that many elements of networks of UOS and SIUK can be general/common/total, since the detection to a certain extent is always accompanied by measurement.

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However, the results of measurement, obtained in the process of detection, in the majority of the cases are not optimum for the accuracy. The latter is reached, generally speaking, by the special structure of processing signal. Therefore in the general case, are necessary the separate channels of detection and measurement, where processing signal is performed by different methods. In this case

into the channel of measurement only those signals, which preliminarily passed the procedure of detection, usually enter.

Let us examine at first operation of diagrams and devices/equipment in large signal-to-noise ratio, where fundamental functional connections can be sufficiently simply revealed, and designation/purpose of separate network elements.

Device/equipment of detection of signal has by its problem of establishing/installing presence of useful signal on output of receiving channel of station. In radars this signal is the result of the reflection of sounding pulses from the oriented target. Depending on the direction-finding method and survey/coverage of space accepted the character of the echo signal can be diverse. For the station of detection with the circular scan the echo signal, as is known, takes the form of the packet (packet) of impulses/momenta/pulses, in this case for the pinpoint target the envelope of impulses/momenta/pulses is determined by the form of radiation pattern.

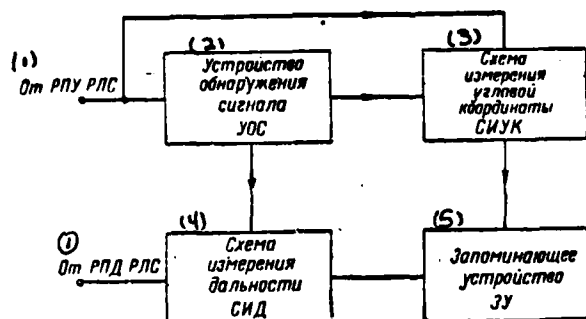


Fig. 4.1. Block diagram of the device/equipment of primary working/treatment.

Key: (1). From. (2). Device of detection of signal of UOS. (3). Measuring circuit of angular coordinate SIUK. (4). Ranging circuit SID. (5). Memory unit ZU.

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Recording presence of target in device/equipment of detection of signal (UOS) is accompanied by generation of special impulse/momentum/pulse "fact of detection", which is supplied to ranging circuit for reading of distance of target. As the signal the "fact of detection" can be used the impulse/momentum/pulse of the beginning of packet, whose appearance, as this will be evidently from the following, in certain cases it satisfies the criteria of target detection.

Besides problem of detection, UOS in specific cases can fulfill functions of measurement of angular coordinates, in particular determine beginning and end/lead of packet. The method of measurement of angular coordinates, based on the determination of the angular position of beginning and end/lead of the packet, is not optimum, but in the large signal-to-noise ratio it is possible to lead to the desired result. However, the optimum method, based on the theory of the estimation of the parameter (method of the maximum of plausibility), will be examined below.

Determination of beginning and end/lead of packet is supplementary problem, which it can solve device/equipment of detection of signal. It is naturally connected with the problem of the detection of packet itself and is obtained in these devices/equipment as the supplementary result without the supplementary complication of diagram. In this case at the moments/torques, which correspond to beginning and end/lead of the packet, in the diagram of UOS must be developed the impulses/momenta/pulses, supplied to the schematic of the removal/output (measurement) of angular coordinate. In this diagram is taken a reading of the angular positions of beginning and end/lead of the packets, whose averaging gives the azimuth of target. The

impulses/momenta/pulses of beginning and end/lead of the packet are supplied into the measuring circuit of azimuth, as a rule, separately on the independent busbars.

In further reasonings for purpose of simplification let us assume that antenna of RLS has characteristic of radiation pattern in the form of sector by width θ . For the signal, of the considerably exceeding interference, the problem detection does not have vital importance, and fundamental problem of UOS is reduced here only to recording of beginning and end/lead of this packet. As the impulse/momentum/pulse of the beginning of packet in the absence of interferences its first impulse/momentum/pulse can be accepted.

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However, for increasing the authenticity of data it is expedient to accept for the beginning of packet not one, but totality of several first impulses/momenta/pulses; otherwise any randomly emergent signal can be accepted as the beginning of packet, which, naturally, will lead to the errors.

In connection with this for beginning of packet they accept small series from n of in a row following impulses/momenta/pulses with constant repetition period T . The end/lead of the packet is

fixed/recorded with the absence of next impulse/momentum/pulse. These conditions determine the logic of work of UOS.

In small signal-to-noise ratio logic of work of device/equipment of detection of signal is more complicated. Here for the criterion of the beginning of packet is appropriate to require m of impulses/momenta/pulses on n adjacent positions, since some impulses/momenta/pulses of packet can disappear as a result of the interference effect. The criterion of the end/lead of the packet is here also determined by absence k impulses/momenta/pulses on l the adjacent positions. The logic of work of UOS indicated is frequently called logic of the type " m from n " (or " k from l ").

In the absence of interferences logic of work UOS is reduced to logic of type " n from n ".

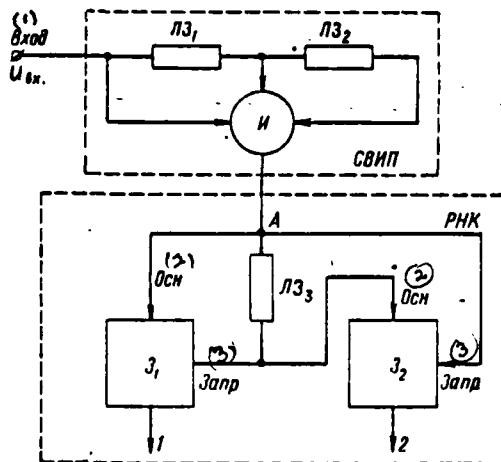


Fig. 4.2. Diagram of the isolation/liberation of the impulses/momenta/pulses of packet.

Key: (1). Input. (2). Main. (3). Inhibit.

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The schematic of this device/equipment with $n=3$ (Fig. 4.2) consists of two to a certain degree of the self-contained units: the diagram of the isolation/liberation of the impulses/momenta/pulses of packet (SVIP) and circuit of separation of the first and latter/last impulse/momentum/pulse of packet (RNK). The first diagram consists of two lines of delay $ЛЗ_1$ and $ЛЗ_2$, and AND circuit of coincidence to three inputs. Delay time in the devices/equipment $ЛЗ_1$ and $ЛЗ_2$, corresponds precisely to the pulse repetition period $\Delta\theta$. Then signal

at the output of AND circuit appears only when the diagram three impulses/momenta/pulses will enter in a row. The subsequent impulses/momenta/pulses will also give coincidence, and at the output the pulse train, which stretches to the latter/last impulse/momentum/pulse of packet (Fig. 4.3), will be obtained. It is obvious, if the criteria of the beginning of packet are n impulses/momenta/pulses in a row, then in the schematic (Fig. 4.2) must be $(n-1)$ of delay lines in question and AND circuit on n inputs. In this case if the input of AND circuit the packet from the n impulses/momenta/pulses entered, then at its output there will be packet from $h-n+1$ impulses/momenta/pulses.

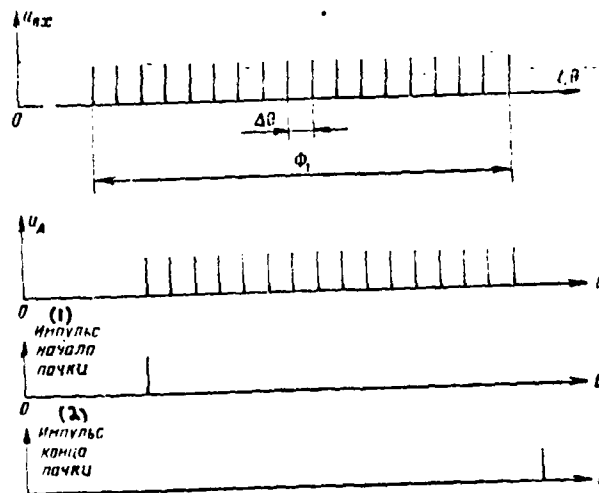


Fig. 4.3. Graphs/curves of impulses/momenta/pulses in the diagram of the isolation/liberation of the impulses/momenta/pulses of packet (Fig. 4.2).

Key: (1). Impulse/momentum/pulse of the beginning of packet. (2). Impulse/momentum/pulse of end/lead of packet.

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Element/cell AND on n inputs is relative to compound circuit, especially for the nonstandardized in the signal amplitude; therefore is desirable construction of SVIP with the less complicated elements/cells. Fig. 4.4 gives the diagram with the processing logic by 3/3, which uses two AND circuits to two inputs. Analogously can be

obtained diagram for the logic "k from k". It will contain (k-1) AND circuits to two inputs and (k-1) delay lines.

Signal from output of logic unit of diagram (Fig. 4.2) is supplied on circuit of separation of the first and latter of impulses/momenta/pulses of packet (RNK). One of the versions of this diagram is shown below in Fig. 4.2. This diagram consists of the line of delay $\Pi 3$, and two inhibit circuits 3_1 and 3_2 . The signals of beginning and end/lead of the packet (shackle) are formed respectively at the 1st and 2nd outputs of diagram. Actually this diagram divides beginning and end/lead of the series of pulse signals. The impulse/momentum/pulse of the beginning of packet is obtained at the output of inhibit circuit, to input of which are supplied the signals directly from the output of SVIP, and delayed pulses - to the forbidding input as this is done in inhibit circuit 3_1 . In this case to the input 3_1 only the first impulse/momentum/pulse of packet will pass, since at the forbidding input the signal still is absent, and to the second impulse/momentum/pulse after the time T signal to the forbidding input will approach and it will overlap the subsequent impulses/momenta/pulses.

With isolation/liberation of impulse/momentum/pulse of end/lead of packet, on the contrary, delayed pulse train is supplied to main

input, and signals directly from output of diagram SVIP - to forbidding input. In this case through the diagram 3, only the latter/last impulse/momentum/pulse of packet, delayed for the repetition period in the delay line ПЗ,, will pass, since all preceding/previous will be closed with the impulses/momenta/pulses of straight/direct channel, which enter the input of exclusion. This impulse/momentum/pulse is accepted subsequently as the signal of the end/lead of the packet.

In diagram examined signals of beginning and, end/lead of packet are separated along different channels.

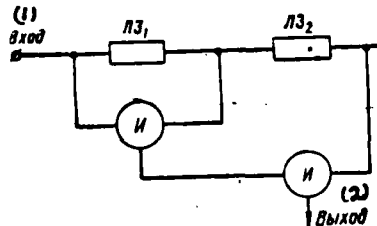


Fig. 4.4. Diagram with the processing logic by 3/3 on two AND circuits.

Key: (1). Input. (2). Output.

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This diagram of UVS proves to be convenient for the simultaneous determination of the coordinates of several targets, which are located on one direction (azimuth). However, if this is not required and is assumed work only on one target, then it is possible to obtain the impulses/momenta/pulses of beginning and end/lead of the packet along one channel. In this case the diagram RNK is simpler (Fig. 4.5). It consists of one anti-coincidence circuit and one delay unit ЛЗ₁. However, the diagram of the isolation/liberation strictly of the impulses/momenta/pulses of packet SVIP remains the same as in the preceding case.

Work of simplified circuit RNK flows/occurs/lasts as follows.

The first impulse/momentum/pulse of packet from the output of the diagram of the isolation/liberation of the impulses/momenta/pulses of packet (SVIP) enters the inhibit circuit (noncoincidence) and in view of the absence of signal on the second input are passed through the anti-coincidence circuit in the form of the impulse/momentum/pulse of the beginning of packet. Since the delay unit causes signal lag accurately on the pulse repetition period, all preceding/previous impulses/momenta/pulses of packet will not traverse the diagram 3, except the latter. The latter/last impulse/momentum/pulse of packet, after traversing the delay unit $\Pi 3_1$, will approach the second input of anti-coincidence circuit and will pass to its output, since at the first input the signals already are absent.

Now let us switch over to description of ranging circuits.

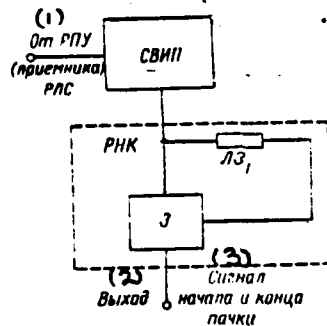


Fig. 4.5. Diagram of pulse separation of beginning and end/lead of the packet with the general/common/total output busbar.

Key: (1). From RPU (receiver) of RLS. (2). Output. (3). Signal of beginning and end/lead of packet.

4.2. Diagrams of discrete/digital ranging.

In pulse-modulated radar, as is known, ranging is reduced to measurement of time lag of echo signal, i.e., to interval measurement between sounding and echo pulses.

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The need for obtaining output data in the form, convenient for their use in further nodes of the computer(s) of processing signal is the distinctive special feature/peculiarity of the measuring circuits of

time, used in the devices/equipment of the primary processing of signal. The most convenient form is the signal, which is the code of the binary number, whose value corresponds to the range of target. Thus, in the diagrams in question not only is measured time (range), but also the measured value is converted into the discrete/digital digital code.

Therefore frequently ranging circuits, used in devices/equipment of primary working/treatment, are called diagrams of discrete/digital measurement of time or transformation circuits of time into digital code, which designate in abbreviated form "time - numeral".

1. Simplest diagrams of the discrete/digital measurement of time.

In majority of diagrams of discrete/digital measurement of time method of filling of measured momentum range of standard frequency with their subsequent calculation is utilized. The simplest schematic of this converter "time - the numeral" as, for example, represented in Fig. 4.6, makes it possible to measure the range to the individual target. The impulses/momenta/pulses of standard stable repetition frequency from the special generator of count pulses (GIS) are supplied through the AND gate to the trigger counter. Valve/gate is controlled by the flip-flop Tp_1 , to which are supplied the impulses/momenta/pulses "starting impulse/momentum/pulse" from the

transmitter of the radar of stations and "Stop" from the output of the device/equipment of the detection of signals. The detection of useful signal, i.e., the presence of target in the zone of the survey/coverage of radar, is characterized by the presence of latter/last impulse/momentum/pulse.

In gap/interval between starting and stop impulses/momenta/pulses the AND gate is opened/disclosed and gating pulses of standard generator GIS in quantity, proportional to this interval. Impulses/momenta/pulses are computed with the n-bit trigger counter, whose output gives a number in the binary code.

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This number is read by potential-pulse diagram of output to valves/gates $H_1, H_2, \dots, H_n$ into the special register of range RD^1 . Reading is conducted at the moment of supplying the special read pulse, obtained from the stop impulse/momentum/pulse via its delay on the period τ_1 in the delay line $ЛЗ_1$. Minimum time τ_1 is selected from the condition of the end of transient processes in the channel of calculation after the arrival of the echo (stop) signal. At the termination of the removal/output of the readings/indications to range the counter is installed in zero special impulse/momentum/pulse "Installation of 0", obtained from the read pulse by its delay on the

period τ , in the line J3,. Minimum value of τ , is selected from the account of the duration of transient processes in the diagram of data extraction.

Time is measured here discretely with an accuracy to repetition period of standard impulses/momenta/pulses τ_r . The same period determines the accuracy of ranging. If a number of intervals during measurement N , then target range

$$R = N\Delta R,$$

where

$$\Delta R = \frac{c\tau_r}{2}. \quad (4.2.1)$$

However, in view of random phase relationships/ratios between impulses/momenta/pulses of standard generator and starting (sounding) impulse/momentum/pulse of RLS in diagram (Fig. 4.6) even larger error is possible and range will not be determined by formula (4.2.1).

FOOTNOTE ¹. Input and output of a number of the register are examined into Chapter 3. ENDFOOTNOTE.

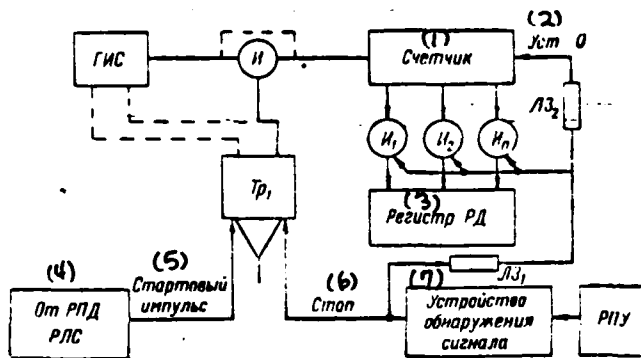


Fig. 4.6. Simplest ranging circuit.

Key: (1). Counter. (2). Obsolete 0. (3). Register RD. (4). From RPD of RLS. (5). Starting pulse. (6). Stop. (7). Device/equipment of detection of signal.

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Let us assume the phase relationships/ratios between the impulses/momenta/pulses indicated are arbitrary:

$$t_n = \tau_{\text{нач}} + (n-1)T_{\text{ог}} + \tau_{\text{ост}},$$

where t_n - measured interval;

T_{sr} - period of the generator of count pulses;

 $\tau_{\text{нач}}$ - error of reference point;

τ_{oct} - error of the end/lead of the reading;

n - number of counted off impulses/momenta/pulses.

An error of measurement:

$$\delta = \tau_{HAY} + (n-1) \Delta T_{gr} + \tau_{oct},$$

where ΔT_{gr} - instability of the period of standard generator.

Usually generators of count pulses have high stability. Thus, for instance, in the generators, stabilized by quartz, ΔT_{gr} very little and comprises $(10^{-6}-10^{-8}) T_{gr}$. Therefore with the usual values n value $(n-1) \Delta T_{gr}$ is so low, that it can be disregarded/neglected, and error will be determined by the sum of the errors of beginning and end/lead of readings:

$$\delta \approx \tau_{HAY} + \tau_{oct}.$$

It is natural that must be observed inequalities:

$$\tau_{HAY} < T_{gr} \text{ и } \tau_{oct} < T_{gr}.$$

If this condition is not satisfied, then with absences of synchronization error of reference point can reach in worst case of repetition period T_{gr} . In order to exclude it, it is necessary to rigidly synchronize starting, i.e., sounding, impulse/momentum/pulse with the oscillations/vibrations of GIS. The diagram, which considers

this synchronization, is represented in Fig. 4.6 by the dotted lines, where is shown connection of GIS directly with Tp_1 , i.e., the need for AND gate is eliminated. In this diagram it is assumed that pulsing GIS appears under the action of starting impulse/momentum/pulse and occurs only in the period, when to it voltage/stress from the trigger cascade/stage is supplied.

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In this case the first impulse/momentum/pulse GIS coincides with the leading edge of the signal, which comes from flip-flop, i.e., it coincides with the starting (sounding) impulse/momentum/pulse of RLS.

This work of GIS can be obtained, in particular, by use/application in oscillator circuit with duct/contour of collision excitation in small positive feedback. In this case it is possible to assume that the error of reference point is small and to virtually it is possible exclude it.

Then remains only error of end/lead of reading (Fig. 4.7), since echo signal can enter at arbitrary moment of time. Greatest this error will be when the echo signal will enter directly before the next impulse/momentum/pulse of GIS. Then it is possible to consider that the error will be approximately/exemplarily equal to

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the pulse repetition period of GIS; moreover error will be in the direction of the decrease of distance, i.e., in actuality distance will prove to be always more than measured, but not less

$$\delta R < \frac{c f_{sr}}{2}.$$

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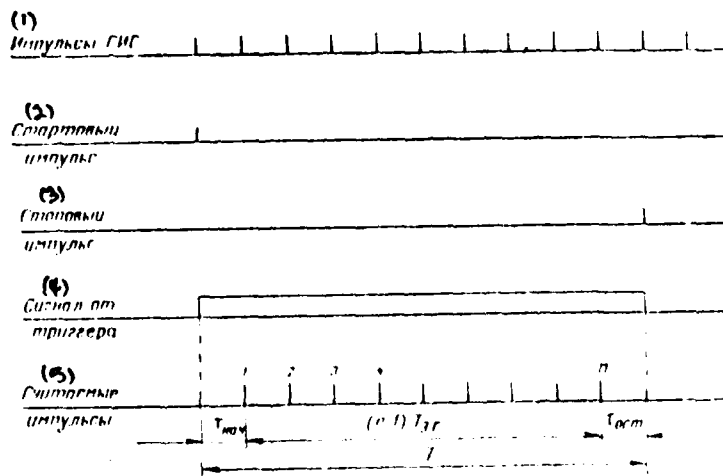


Fig. 4.7. Diagrams of impulses/momenta/pulses for the ranging circuit.

Key: (1). Impulses/momenta/pulses of GIS. (2). Starting pulse. (3). Stop impulse/momentum/pulse. (4). Signal from flip-flop. (5). Counted impulses/momenta/pulses.

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If we shift starting impulse/momentum/pulse with respect to standard impulses/momenta/pulses on half of period T_m , then error of reading

$$\delta R \approx \pm \frac{cT_m}{4}.$$

Let us examine error, which appears as a result of instability

of pulse repetition frequency of GIS. Obviously, the instability of standard generator will lead to a change in the scale graduation of range, in this case the resultant error will be greater, the greater the measured time interval, i.e., error is accumulated. Therefore during the analysis it is necessary to proceed from the greatest interval of time $t_{\max}$, which corresponds to maximum range. Quantity of repetition periods for time $t_{\max}$

$$N = \frac{t_{\max}}{T_{\text{gr}}}.$$

Let us assume that due to instability of standard generator period of its oscillations, after achieving steady state, increased on ΔT_{gr} and will be equal to $T_{\text{gr}} + \Delta T_{\text{gr}}$, then quantity of periods for time $t_{\max}$ will be

$$N > N' = \frac{t_{\max}}{T_{\text{gr}} + \Delta T_{\text{gr}}}.$$

In view of smallness of relative instability $\frac{\Delta T_{\text{gr}}}{T_{\text{gr}}} \ll 1$ preceding/previous relationship/ratio can be recorded in the form

$$\frac{t_{\max}}{T_{\text{gr}} \left(1 + \frac{\Delta T_{\text{gr}}}{T_{\text{gr}}}\right)} \approx \frac{t_{\max}}{T_{\text{gr}}} \left(1 - \frac{\Delta T_{\text{gr}}}{T_{\text{gr}}}\right).$$

Assuming that due to instability of GIS for time $t_{\max}$ frequency drift must not be more than for one period, we will obtain that $N = N' + 1$, and

$$\frac{t_{\max} \Delta T_{\text{gr}}}{T_{\text{gr}}^2} = 1,$$

whence

$$\Delta \dot{T}_{gr} = \frac{T_{gr}^2}{t_{max}}$$

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Then relative instability will be determined by relationship/ratio

$$\frac{\Delta \dot{T}_{gr}}{T_{gr}} = \frac{1}{N}.$$

2. Removal/output of counter readouts taking into account transient processes.

Diagram examined above of conclusion/output of range from counter into register (Fig. 4.6) can give incorrect results, if read pulse arrives at moment/torque, when in counter still they will pass transient processes, caused by transmission of units of transfer between discharges of counter, and also as mode switch in quite trigger cascades/stages.

For eliminating this it is necessary to use diagram (Fig. 4.8), in which removal/output of counter readouts into register would be conducted always at that moment/torque, when transient processes in counter knowingly are absent. For simplicity Fig. 4.8 gives the diagram of conclusion/output only for the valve/gate of IR one k

discharge; however, it can service/maintain, naturally, and the remaining valves/gates of the schematic of the conclusion/output of counter readouts.

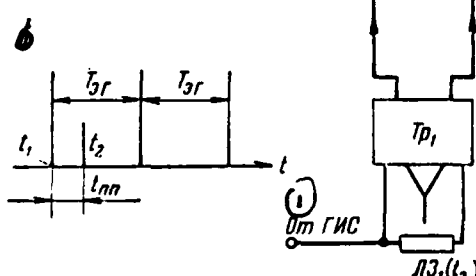
As can be seen from figure, to diagram of removal/output reading impulses/momenta/pulses u_{eq} and signals GIS are fed/conducted. The latter activate of the flip-flop Tp_1 , which controls valves/gates H_{II} and H_I . These valves/gates in turn switch the channels of read pulses (straight line and delayed).

Work of diagram flows/occurs/lasts as follows. Let moment/torque t_1 correspond to the appearance of an impulse/momentum/pulse of GIS (Fig. 4.8b). The transient processes in the counter, connected with the advent of carry pulses and switching of trigger cascades/stages immediately after the appearance of this impulse/momentum/pulse can arise. We assume that the duration of transient processes does not exceed interval $t_1 - t_2 = t_{nn}$, which must be established/installed previously, on the basis of the special features/peculiarities of trigger counter. Then it is obvious that the read pulse must not enter counter in the interval of time $t_1 - t_2$.

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If read pulse nevertheless enters at this moment, then it must

be delayed and after delay given to valves/gates of introduction/input of number. This process is realized in the diagram in Fig. 4.8a. Impulse/momentum/pulse of GIS with the aid of the flip-flop T_p , and valves/gates H_n and H_1 switches the channels of the admission of read pulses - A and B; with the latter the delay line Π_3 , is connected. Channel B is switched on immediately after each impulse/momentum/pulse GIS on the transit time in the counter. This switching is provided by the presence of delay line Π_3 , between the inputs of flip-flop "Installation of 0" and "Installation of 1". Delay time in the lines Π_3 , and Π_3 , is equal.



Key: (1). From. (2). Counter. (3). To RD. (4). Straight/direct channel. (5). Delayed channel.

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It must be noted that diagram indicated will satisfactorily work only when repetition period of standard impulses/momenta/pulses not less than twice exceeds duration of transient processes in counter,

i.e.

$$T_{er} \geq 2t_{nn}$$

otherwise delayed read pulse will fall in following period of oscillations of standard generator, which will lead to incorrect reading of range.

In given reasonings it was assumed also that duration of pulse of GIS is considered by duration of transient processes of counter. Analogously discussing, it is not difficult to consider the effect of the pulse duration on the resolution of the diagram in question.

3. Diagrams of discrete/digital ranging of the increased accuracy.

Simplest diagram examined earlier gives comparatively small accuracy during its use in radars. True, this diagram measures the range with an accuracy to discrete/digital interval $\Delta R = \frac{cT_{er}}{2}$, where s - velocity of propagation of electromagnetic waves.

If period T_{er} is equal to $1 \mu s$, what is already sufficiently small interval, then error of discreteness will be 150 m. This error is large and in a number of cases inadmissible. If we assume that stations have an accuracy of measurement of approximately 15 m, then for guaranteeing this accuracy in the diagrams examined it is necessary to have a generator of standard impulses/momenta/pulses

with a frequency of 15 MHz. Work with such impulses/momenta/pulses requires the large operating speed of the elements of network, especially input cascades/stages of counter, to realize which is sometimes difficultly. Therefore the creation of the diagrams of discrete/digital measurement of larger accuracy is desirable to reach not by increasing the repetition frequency of standard impulses/momenta/pulses of GIS, but with the aid of other circuit solutions, which do not require the use of a maximum operating speed (small inertial properties) of the network elements.

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A. Vernier method of ranging.

Actually vernier method makes it possible to accurately measure error, connected with remainder/residue τ_{OCT} , by introduction of supplementary, precision dial of reading.

Diagram of vernier method of measurement is represented in Fig. 4.9, and graph which elucidates its work, in Fig. 4.10. Diagram contains two counters - rough and fine reading. The designation/purpose of the counter of coarse reading the same as in the preceding/previous diagram, i.e., it is intended for the reading of the whole number of periods of standard generator. The counter of fine reading is intended for determination of τ_{OCT} and decrease thereby of the error of reading. This is conducted as follows.

Let us assume that at moment t , stop impulse/momentum/pulse (there arrived signal, reflected from target), appeared, then impulse/momentum/pulse of standard generator ϵ will be latter, which

will record counter of coarse reading. In this case is obtained the error, which must be "selected". For this stop impulse/momentum/pulse starts the vernier generator, the period of oscillations of which differs somewhat from the period of oscillations of standard T_n generator T_{gr} ; for the certainty let us assume that $T_n < T_{gr}$. The impulses/momenta/pulses of vernier generator are computed with the counter of fine reading.

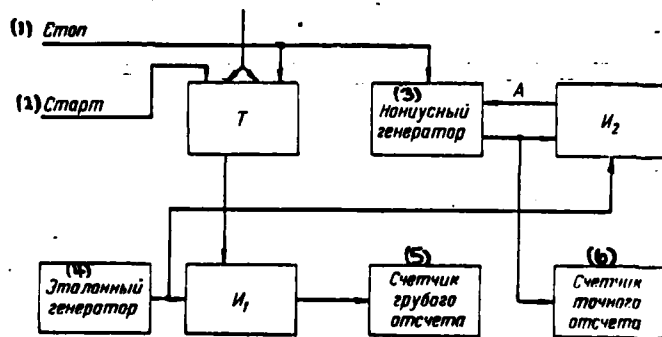


Fig. 4.9. Diagram of the vernier method of ranging.

Key: (1). Stop. (2). Start. (3). Vernier generator. (4). Standard generator. (5). Counter of coarse reading. (6). Counter of fine reading.

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Oscillations/vibrations in the vernier generator are stopped by the impulse/momentum/pulse, which comes from diagram И₁ (busbar A). This impulse/momentum/pulse is formed at the moment of coincidence in the valve/gate И₁ of the impulses/momenta/pulses of vernier and standard generators.

Coincidence of these impulses/momenta/pulses is formed not immediately, but through m of periods of vernier (or standard) generator. Actually/really, at first, i.e., at the moment of the

arrival of stop impulse/momentum/pulse, a difference in the temporary situation between the vernier and standard impulses/momenta/pulses is equal to τ_{corr} (Fig. 4.10). With each following period this difference in the temporary situation between the impulses/momenta/pulses decreases by value $T_{\text{or}} - T_{\text{u}}$ since $T_{\text{or}} - T_{\text{u}} > 0$. If difference $\Delta T = T_{\text{or}} - T_{\text{u}}$ is multiple to remainder/residue τ_{corr} with multiplicity m , then it is obvious that accurately through m of the pulse repetition periods of vernier generator will be made the equality

$$\tau_{\text{corr}} - m(T_{\text{or}} - T_{\text{u}}) \approx 0 \quad (4.2.2)$$

and will begin the coincidence of standard and vernier impulses/momenta/pulses.

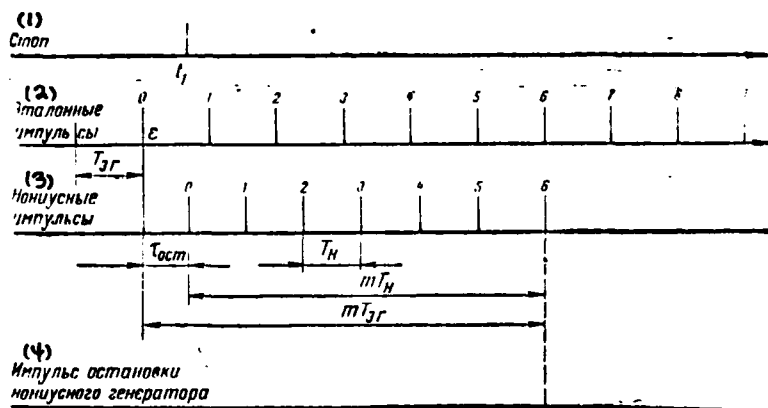


Fig. 4.10. Graph/curve of the impulses/momenta/pulses of vernier method.

Key: (1). Stop. (2). Standard impulses/momenta/pulses. (3). Vernier impulses/momenta/pulses. (4). Impulse/momentum/pulse of stop of vernier generator.

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And vice versa, the presence of impulse/momentum/pulse at the output of gate H, indicates the fulfillment of equality (4.2.2), whence the unknown error

$$\tau_{\text{ост}} = m\Delta T,$$

where a number of periods m is given by the counter of fine reading. It is natural that a difference in the periods of oscillations of both generators ΔT must be known previously.

As is evident, in this method value of remainder/residue τ_{oct} is measured with an accuracy to difference in ΔT . The less the difference, the more precise the measurement. However, with the decrease of difference requirements for the stability of vernier and standard generators are raised, furthermore, is lengthened the time of obtaining one reading, since value m increases.

If error τ_{oct} and difference in ΔT are not multiple between themselves, then precise pulse coincidence of vernier and standard generators will be not at what value of m . In this case only with certain $m=m_0$, difference (4.2.2) will be minimum and formula for this case can be rewritten in the form

$$\tau_{gr} - m_0 (T_{gr} - T_H) = \theta_{min}.$$

But this means that there is no precise pulse coincidence of vernier and standard impulses/momenta/pulses and they at best will approach each other to value θ_{min} after m_0 repetition periods of vernier impulses/momenta/pulses.

So that coincidence circuit could fix this case and give out coincidence pulse, it is necessary that duration of vernier pulse would be no less than θ_{min} , i.e. $\tau_H \approx \theta_{min}$. However, an increase in the duration

both of standard and vernier pulses in more than the value indicated worsens/impairs the accuracy of measurement, since in this case the coincidence can occur not in the m period, but earlier. Thus, the duration of the pulses of generators should be selected in the interval

$$\theta_{\min} < \tau_n < \theta_{\min} + \Delta T.$$

Besides errors indicated, accuracy of method in question is determined by stability of vernier generator. The permissible error in the repetition period of vernier generator ΔT_n can be determined from the expression

$$\Delta T_n m < \Delta T.$$

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B. Method of the delayed coincidences.

Method of delayed coincidences is also intended for measuring error τ_{OCT} . This method requires larger equipment in comparison with the vernier method, but, apparently, it can ensure high accuracy. The diagram, which realizes method (Fig. 4.11), actually supplements the simplest diagram of discrete/digital ranging (Fig. 4.6), measuring remainder/residue τ_{OCT} and raising thereby accuracy of reading as a whole.

Work of diagram, based on method of delayed coincidences, flows/occurs/lasts as follows. Impulses/momenta/pulses from the generator of standard (calculating) impulses/momenta/pulses (Fig. 4.11) come the delay line with the removals/outlets, which is the fundamental element of attachment. The duration of the delay of entire line is accurately equal to the pulse repetition period T , and the quantity of removals/outlets, made through the identical gaps/intervals, equal to n is determined by the required accuracy of the measurement of remainder/residue. The signals, taken from the removals/outlets of line, enter valves/gates $H_1 + H_n$, which on the second input are controlled by stop impulse/momentum/pulse.

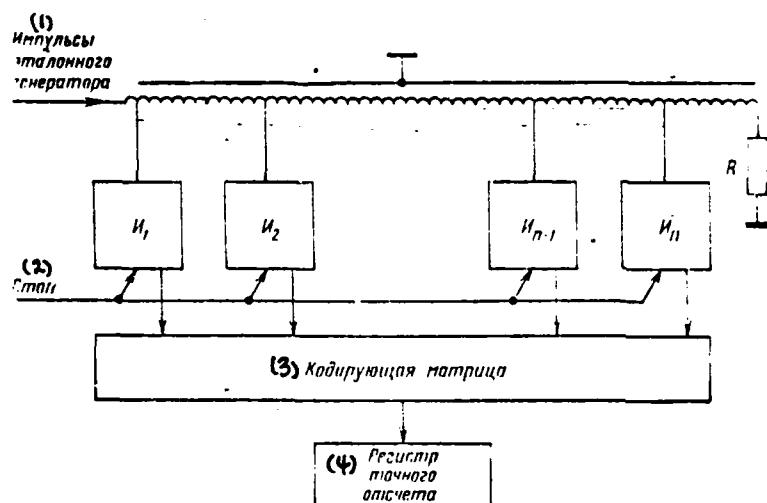


Fig. 4.11. Delayed-coincidence circuit.

Key: (1). Impulses/momenta/pulses of standard generator. (2). Stop.
 (3). Coding matrix/die. (4). Register of fine reading.

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Preceded stop impulse/momentum/pulse signal of standard generator, passing along delay line, consistently approaches removals/outlets of line ^{and} respectively inputs of gates $I_1 + I_n$. However, none of the pulses will traverse the valve/gate until stop pulse enters. The approached stop impulse/momentum/pulse will open up all coincidence circuits; however, the impulse/momentum/pulse of standard generator will be isolated only in one of them, namely in that, which is located from the beginning of line on delay factor,

equal to the value of remainder/residue τ_{occ} . Consequently, the number of the valve/gate, in which will occur the coincidence, will uniquely determine the value of remainder/residue.

From the output of gates pulses are supplied to diode decoding matrix/die, which gives number of gate circuit in binary code at output. The value of this number is proportional to remainder/residue with the proportionality factor, equal to the delay of line between two adjacent removals/outlets, i.e., $\tau_3 = \frac{T_{\text{or}}}{n}$. Thus, the value of remainder/residue is determined by the number of that cascade/stage, in which will occur coincidence.

Time diagrams, which illustrate work of diagram, are given in Fig. 4.12. There the values indicated have the following designations:

τ_1 - pulse duration "stop";

τ_2 - pulse delay between the removals/outlets;

τ_4 - duration of signal pulse on the removals/outlets of delay line;

τ_k - delay time of line from the beginning to the k

removal/outlet.

Accuracy of method is determined by duration of delay between removals/outlets of delay line, and also by duration of pulses, which realize coincidence in this diagram.

It is obvious, the less duration of delay between branches, the more precise measurement can be produced. The minimum duration of delay between the adjacent removals/outlets it is expedient to select the equal duration of delayed signal pulse τ_m , otherwise constant coincidence with partial impulse flashing over in all adjacent channels will occur.

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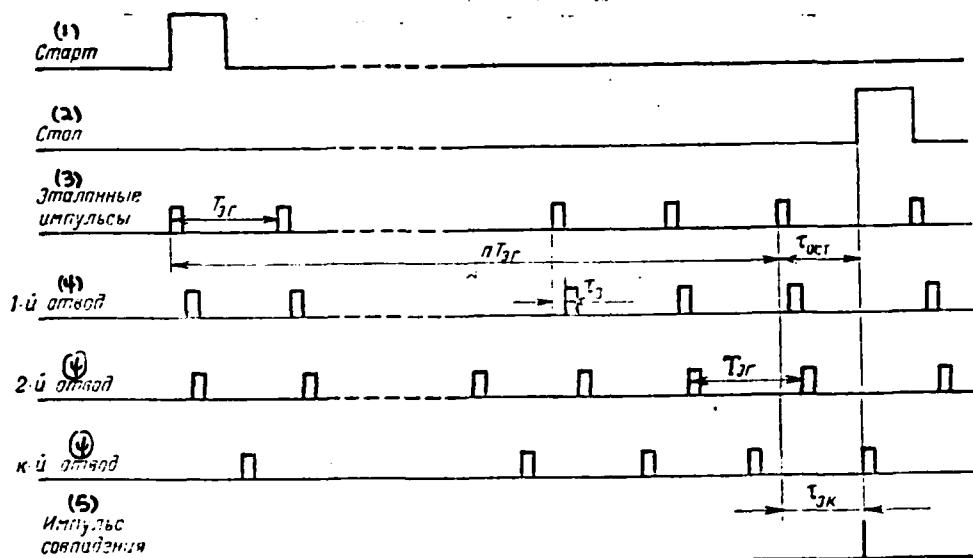


Fig. 4.12. Time diagram, which elucidates method of delayed coincidences..

Key: (1). Start. (2). Stop. (3). Standard pulses. (4). removal/outlet. (5). Coincidence pulse.

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Let us examine questions of accuracy of measurement by this method based on example of two adjacent channels in more detail. The graphs/curves, which elucidate the work of these devices/equipment, they are shown in Fig. 4.13.

Let there be ν th and $(\nu-1)$ th branches of delay line.

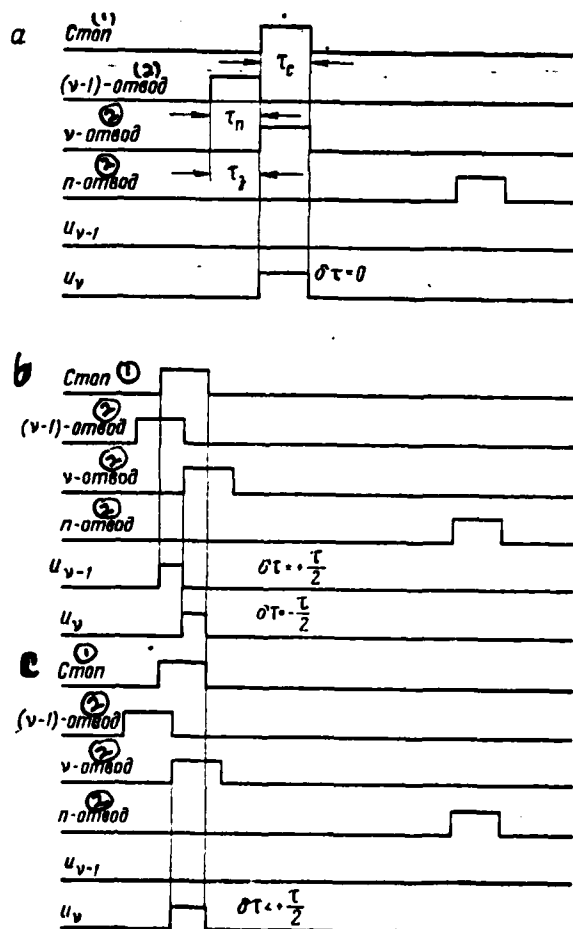


Fig. 4.13. Three possible forms of coincidences.

Key: (1). Stop. (2). Outlet.

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Let us examine the case, when $\tau_c = \tau_n = \tau_v = \tau$, in this case we will consider

that valves/gates H_i and H_{i+1} operate/wear only when impulses/momenta/pulses overlap each other in the course of time, equal and larger $\tau/2$. With the smaller inertness of functioning out-gates the simultaneous recording of signal in two adjacent channels can appear. In order to exclude this undesirable phenomenon, it is necessary to increase triggering time of valves/gates. For the duration of their functioning, equal to $\tau/2$, the simultaneity of functioning will be observed only if stop impulse/momentum/pulse overlaps half of the delayed signal (Fig. 4.13b). If we select the duration of functioning more $\tau/2$, then the case, indicated in Fig. 4.13b, when pulses half overlap one another, it will be not at all recorded, i.e., there will be the passage of signal, which is also undesirable. Thus, the selection of the duration of functioning is compromise between the passage of signal with the large inertness of valve/gate and appearance of false recordings (simultaneous recordings in adjacent channels) with the rapid response.

Fig. 4.13 show three different cases, with which stop pulse coincides with impulse/momentum/pulse of the ν th channel completely or it lags behind it to period, equal or is larger $\tau/2$.

In first case (Fig. 4.13a) output pulse appears accurately with moment/torque of onset of stop, since it completely coincides with impulse/momentum/pulse of the ν th channel of removal/outlet.

In second case (Fig. 4.13b) stop impulse/momentum/pulse overlaps half impulse/momentum/pulse of $(\nu-1)$ th and ν th outlets. Output signals in both channels here appear, which is undesirable, and error along the leading edge of stop impulse/momentum/pulse in each of the channels is equal to $\pm\tau/2$ respectively.

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In third case (Fig. 4.13c) stop impulse/momentum/pulse overlaps interval more $\tau/2$ of ν th channel, and error along leading edge of stop impulse/momentum/pulse will be less $\tau/2$. Thus, the greatest error inherent in the method of the delayed coincidences

$$\delta_{\max} = \pm \frac{\tau}{2}.$$

It is obvious that increase in duration of stop pulse does not affect accuracy of measurement, since moment/torque of reading is determined along leading edge by its pulse. However, from elucidating Figure 4.14 it follows that in this case can occur the coincidence not only of the ν th channel, but also in all subsequent channels. The expansion of the signal impulses/momenta/pulses, obtained from the outlets of lines of delay, so will cause error, since in this case can occur the coincidence not only of the ν th channel, but also earlier.

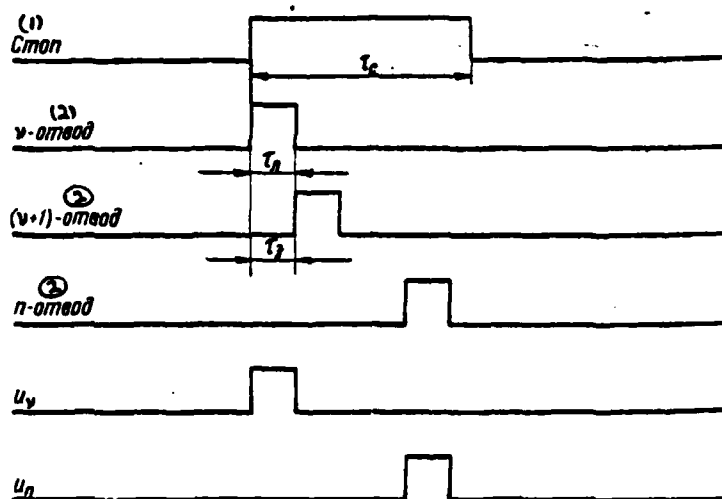


Fig. 4.14. Case of the expanded stop impulse/momentum/pulse.

Key: (1). Stop. (2). outlet.

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This position distinctly is explained in Fig. 4.15, where it is evident that recording moment/torque of arrival of stop pulse occurred not only of v th channel, which must correspond to true position of stop signal, but also in $(v-1)$ th and in $(v-2)$ th channels, which leads to erroneous results.

Virtually in the implementation of this system, let us allow on semiconductor devices, it is possible to obtain operating pulses on

the order of $0.1 \mu s$ and to select, therefore, $\tau = \tau_1 = \tau_2 = 0.1 \mu s$,
 $\tau_c > 0.1$ ns. In this case maximum error $\delta_{max} \approx \pm 0.05 \mu s$.

However, it is necessary to keep in mind that pulse edges are not ideal, but furthermore, is observed spread of threshold of functioning circuits of coincidence $H_1, H_2, \dots, H_n$ which in semiconductor devices can have significant magnitude. In connection with this error $\delta_{max} = \pm 0.05 \mu s$ is hardly realized, and, apparently, the accuracy of measurement in effect will be determined by the value of $0.1 \mu s$.

During use of vacuum-tube circuits accuracy of transformation can be increased, obviously, to $0.05-0.03 \mu s$.

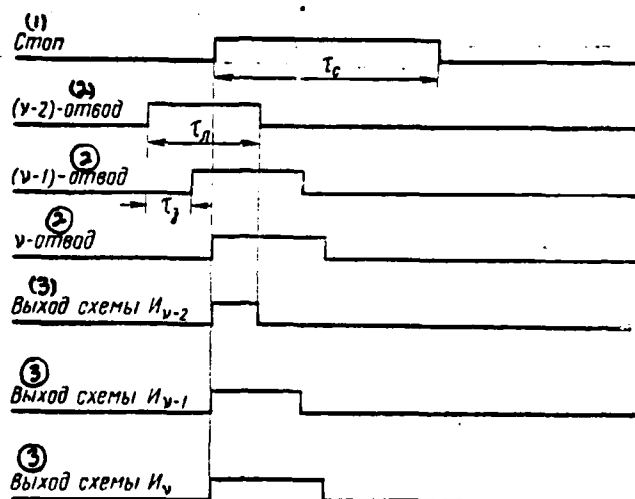


Fig. 4.15. Case of the expanded impulses/momenta/pulses on the removals/outlets of delay line.

Key: (1). Stop. (2). outlet. (3). Output of circuit.

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This improvement can be achieved/reached due to the decrease of the duration of operating pulses with the simultaneous decrease of the distance between the removals/outlets of delay line.

Let us examine now diode coding matrix/die, utilized at output of delayed-coincidence circuit. As noted, it is applied for the transformation of the number of channel into the binary number

corresponding to it. Here is utilized the property of the reversibility of diode matrices/dies, in particular rectangular type matrix/die. The schematic of this matrix/die to eight output busbars is shown in Fig. 4.16.

Property of reversibility of diode matrices/dies lies in the fact that if we supply signal (high voltage) to output, horizontal busbar, then at the input, vertical busbars it is possible to obtain binary code of number, which corresponds to number of that busbar, to which is given signal. In this case it is necessary that the vertical busbars would be connected with the horizontal diodes according to the rule, analogous for the decoder diagrams, with the only difference that the the polarity of the start of diodes is determined by signal polarity, taken from the outputs of gate circuits $H_1, H_2, \dots, H_n$.

Fig. 4.16 shows the connection of diodes for signals of positive polarity. Here each pair of vertical busbars corresponds to the specific binary digit; each busbar within the pair corresponds to the specific value of binary number, namely: left - zero, and right - one. With the vertical busbars are connected the resistors/resistances: $R_{11}, R_{10}, R_{11}, R_{10}, R_{11}, R_{10}$. The presence of voltage on these resistors/resistances indicates that in this digit is one, the absence of voltage/stress - zero. Then it is obtained,

that from the resistors/resistances, connected with the left busbars, is removed/taken the direct code of a number, and resistance, connected with the right busbars, inverse code. This position easily is checked on the equivalent diagrams for each horizontal busbar; therefore here it is not examined.

If on conditions of using diagram is required only straight line or only inverse code of binary number, then corresponding vertical busbars can be simply excluded and left only those, from which is removed/taken required code of number.

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Thus, for instance, in the diagram, shown in Fig. 4.16, if only direct code is required, should be left left vertical busbars, if reverse/inverse - right busbars.

If there is more than removals/outlets in delay line, then it is necessary to apply coding matrix/die with large number of inputs.

Subsequently, code of binary number taken from vertical busbars it is expedient to feed to trigger register of short-term memorization or for transmission to subsequent channels.

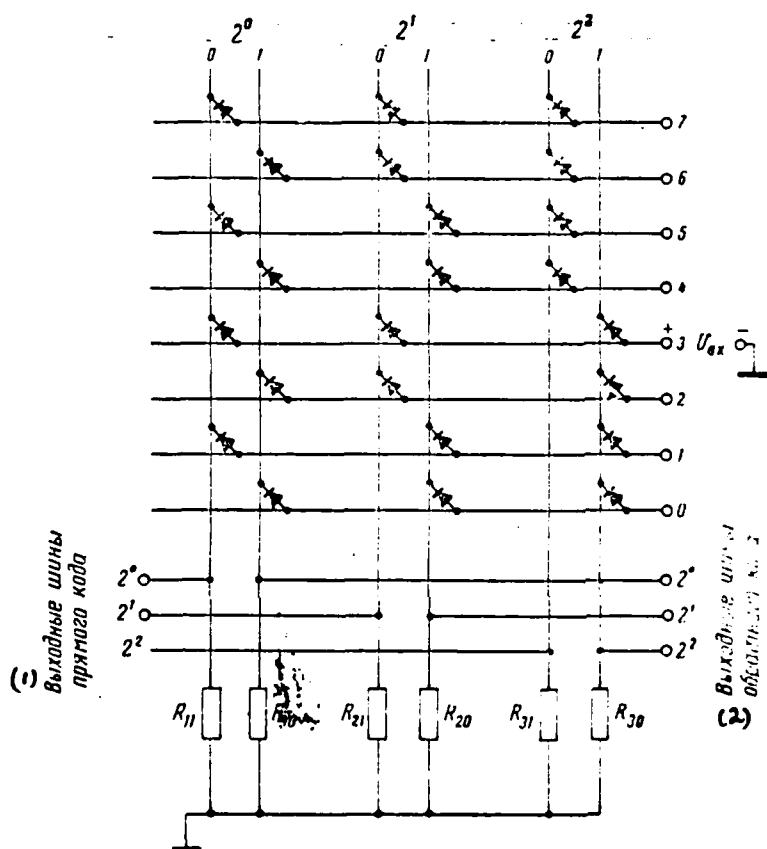


Fig. 4.16. Coding diode matrix/die.

Key: (1). Output busbars of direct code. (2). Output busbars of inverse code.

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§ 4.3. Work of the diagram of feeler in a small signal-to-noise

ratio.

If amplitude of received pulses is small and commensurated with fluctuating interferences, then change in amplitude of pulses of packet, which carries random character, becomes noticeable.

There is series/row of reasons, which cause fluctuations of received signal. These reasons can be developed in the totality or separately. The fact of the addition of the impulses/momenta/pulses of packet with the fluctuating voltages of receiving channel, which carry the random character (see Chapter 5) is one of the main reasons. However, in some cases modulation of the echo signal as a result of the glimmer of the diffusing surface of target (§ 5.2) is the supplementary reason for fluctuations. As a result of the combined action of these factors the burst of pulses can prove to be strongly distorted, i.e., some impulses/momenta/pulses can have large amplitude, others small, and some pulses can entirely drop out from the composition of packet. The character of a change in the impulses/momenta/pulses of packet will be even more complicated, if one takes into account the nonuniformity of radiation pattern. However, in further reasonings we will assume that the diagram of antenna takes the form of circular sector. The presence of fluctuations, naturally, impedes detection and measurement of the target coordinates.

Actually, with fluctuations of impulses/momenta/pulses we do not have confidence in the fact that in the beginning of packet compulsorily will appear several impulses/momenta/pulses in a row, which it would be possible to accept for beginning of packet. On the contrary, is here possible the disappearance of one or two impulses/momenta/pulses, which can lead to the errors in the establishment of presence or absence of target, and also to the error in determination of the moment of onset or termination of packet. Hence it follows that the criterion of beginning and end/lead of the packet with the fluctuating impulses/momenta/pulses must be selected by another, more advisable method, by which this was done in § 4.1.

Let us accept for criterion of beginning of packet presence on n adjacent positions of not less than m pulses, which have certain "average/mean" amplitude. The same criterion can be accepted, also, during the detection of useful signal.

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For the criterion of the end/lead of the packet it is possible to accept the appearance less than m impulses/momenta/pulses on n adjacent positions (with the certain "average/mean" amplitude).

Subsequently correctness of these criteria will be substantiated, will be indicated to selection of necessary relationship/ratio between m and n definite meaning of concept of "average/mean" amplitude. Let us now concentrate our attention in the qualitative description of processes and will examine some possible versions of the diagrams, which realize logic of the type " m from n ".

Establishment of presence m impulses/momenta/pulses on n adjacent positions is reduced in simplest case to addition, i.e., to accumulation of signal in specific sections of range after n of strokes/cycles of pulse repetition of radar (after n of range sweeps). Thus, the fundamental element/cell of the device/equipment of the isolation/liberation of signal is the schematic of accumulator/storage.

Character of schematic of accumulator/storage significantly depends on constancy of pulse repetition frequency of radar. Let us examine the schematics of some accumulators/storage of signal at the constant pulse repetition frequency.

Recording presence of signal on criterion " m from n " can be realized differently. The method of the so-called ideal integration

or the method of binary integration most frequently is utilized.

Detection during ideal integration occurs by addition of impulses/momenta/pulses of packet during adjacent scannings/sweeps with subsequent testing on threshold. Diagram of one of the versions of such a UOS (Fig. 4.17) consists of synchronous accumulator/storage on the delay lines $\Pi_3, -\Pi_3$,¹, which work together with summing amplifier (SU) of the threshold cascade/stage (PK), made on the tube Π , the generator of standard impulses/momenta/pulses GSI and circuit of separation of beginning and end/lead of the packet (RNA).

Ultrasonic lines have identical delay time, equal to pulse repetition period of radar. Delay lines are connected in series, in this case from the output of each line the signal is supplied to summing amplifier SU. Because of the constancy of the periodicity of sounding pulses in the summing cascade/stage store/add up the signals from the identical sections of range on five adjacent positions of scanning/sweep (Fig. 4.18a - 4.18e).

FOOTNOTE ¹. Frequently here are utilized ultrasonic lines delay. Analogous type diagrams are examined in [62]. ENDFOOTNOTE.

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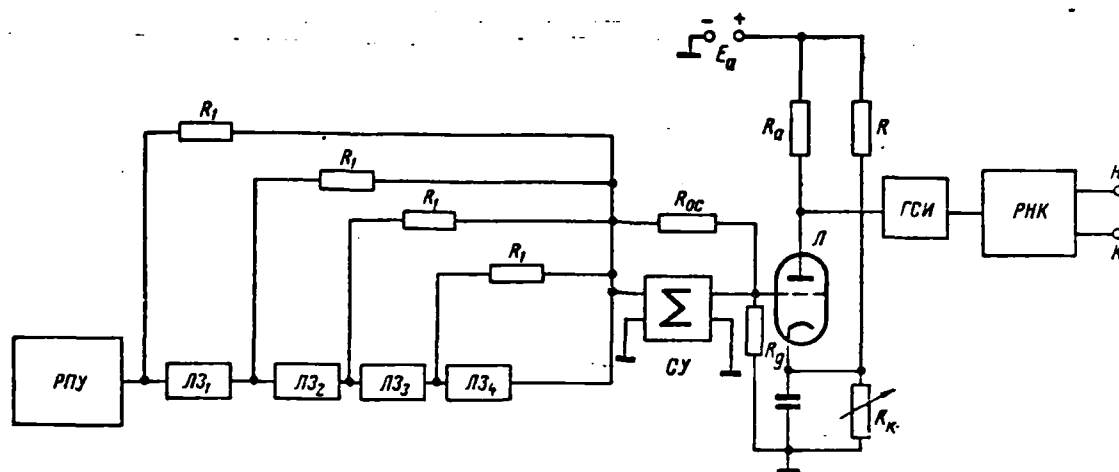


Fig. 4.17. UOS on delay lines.

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As a result of this at the output is formed the total random process (Fig. 4.18f), which is supplied to the following cascade/stage for the testing on the threshold. Normally threshold cascade/stage is closed by supply to the cathode of the tube of the positive voltage/stress

$$U_k = E_s \frac{R_k}{R_1 + R_k},$$

in this case the value of threshold can be varied, changing the relationship/ratio between the resistors/resistances of divider R_1 and R_k .

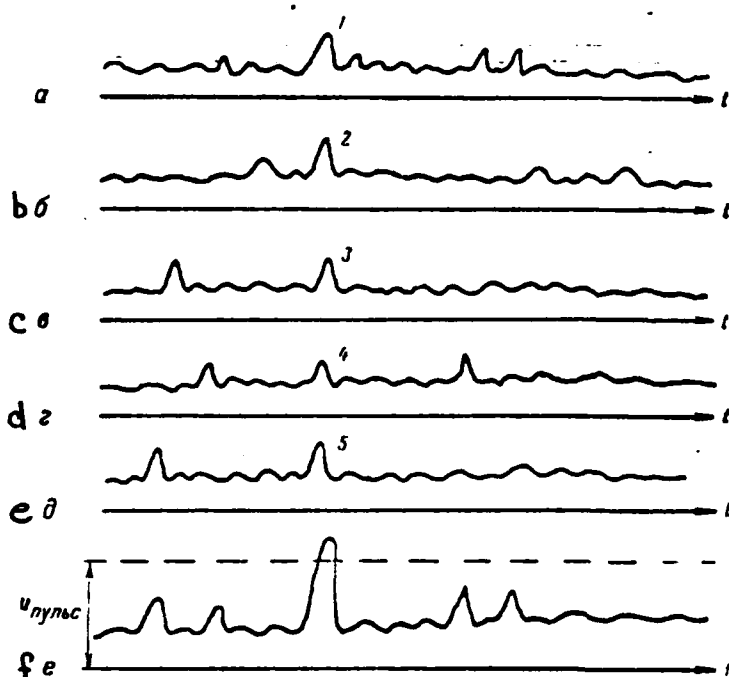


Fig. 4.18. Exemplary/approximate form of the realizations of random process.

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If the total voltage/stress of random process during five repetition periods exceeds at the given moment the threshold of the triggering/opening of tube $U_0 = -U_H + E_{c0}$, where E_{c0} - cutoff voltage of tube, then tube will be opened and at the output will appear the signal, supplied to the special cascade/stage - the generator of standard impulses/momenta/pulses (GSI). The latter develops the

impulse/momentum/pulse of standard form in this case.

This impulse/momentum/pulse appears each time, when on five (or n) adjacent positions total process at isolated points exceeds threshold level U_0 . The diagram UOS in question reacts to threshold crossing by total signal during five (or n) repetition periods, but generally the signal of necessary value can be formed not for five, but for a smaller number of periods; however, the probability of such cases will be less. But if we use the concept of average/mean, most probable amplitude U_{cp} of that section of the random process, which contains the echo signal, then for it can be written inequality $mU_{cp} > U_0$, which determines the logic of the work of UOS of diagram (Fig. 4.17) according to the type "m from n".

Signal, reflected from target, is periodic sequence with period T , appearing in one and the same sections of range. The overshoots of interferences hardly can form the sequence of impulses/momenta/pulses with the equal intervals, which imitate signal from the target. Furthermore, noise overshoots have another average amplitude. Difference at periodicity and amplitude of pulses of signal and overshoots of noise is the basis of the work of the diagram UOS in question. Let us note that in the devices/equipment of ideal addition (integration, Fig. 4.17) a detailed difference in the signal amplitudes and interference is considered. Actually, in this case the

signal from one and the same sections of range in the different repetition periods (Fig. 4.18) directly is summarized. Then total signal tests on the threshold for the establishment of presence or absence of target.

During binary integration processing signal is performed in another way. The signals, which come from radar receiver, pass through the threshold cascade/stage (PK), where they are quantized in the amplitude to two levels.

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If signal exceeds threshold level U_{th} , then from output of PK is supplied signal to generator of standard impulses/momenta/pulses (GSI), at output of which is formed pulse of standard form.

If signal is lower than level, then standard pulse is absent. Then is conducted the current memorization of the obtained signals during five (or n) adjacent periods of scanning/sweep with the simultaneous addition of these impulses/momenta/pulses, which appear in one and the same sections of range, with the aid of the special logic circuit, which totals a number of impulses/momenta/pulses, which exceeded threshold during five (or n) adjacent periods of scanning/sweep. If such impulses/momenta/pulses it proves to be more

than the given number m , then target is considered discovered. Otherwise they consider that the target is absent. As is evident, this logic circuit realizes working/treatment according to the type " m , from n ", but only for the impulses/momenta/pulses of standard form.

During pulse analysis with constant repetition frequency can be used the same synchronous accumulator/storage, such as is shown in Fig. 4.17, only to input of line of signal delay from output of RPU must be supplied not directly, but after quantizer with threshold and with subsequent testing on threshold of accumulation. The value of the threshold of accumulation depends on the amplitude of standard pulses and their number m . Actually accumulator/storage is here amplitude summator.

Together with amplitude summator logical addition, connected with use of logic elements together with ultrasonic delay lines, can be used. In particular, the diagram, which makes it possible to realize working/treatment according to the method "3 of 4" (in Fig. 4.19), contains three delay lines, four circuits of coincidence N to three inputs and one elements/cells NM . In this diagram the elements/cells N to three inputs can be replaced by the simpler diagrams N to two inputs, which work more stably (Fig. 4.20). Are possible the also further modifications of the logical part of this

diagram; in particular, the diagram with a minimum quantity of diagrams Π , which, as is known, are more complicated than diagram $\Pi\Pi\Pi$, can be constructed.

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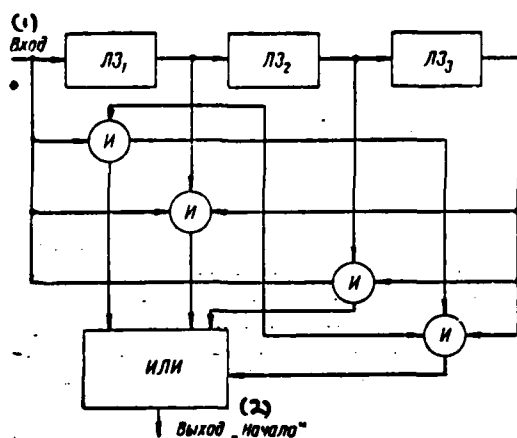


Fig. 4.19.

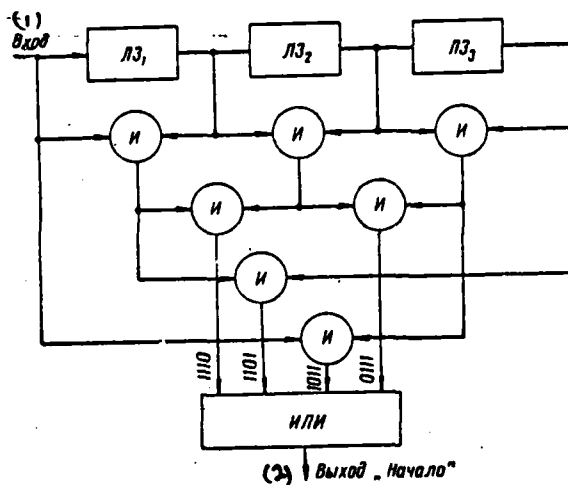


Fig. 4.20.

Fig. 4.19. Schematic of accumulator/storage on logic elements with logic of type 3/4.

Key: (1). Input. (2). Output "Beginning".

Fig. 4.20. Simplified circuit of accumulator/storage on logic elements.

Key: (1). Input. (2). Output "Beginning".

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For synthesis of structure of logic unit of schematic of this accumulator/storage apparatus of Boolean algebra, which uses these or other functional units, can extensively be used. Let us examine some positions of the synthesis of such diagrams. Subsequently we will utilize only elements/cells И and ИЛИ, although, generally speaking, here they can be used and other elements/cells (§ 3.1).

Diagram, which realizes working/treatment of type "m from m", is simply a constituent unit (see § 3.3) and consists of logic circuit И on m inputs, which can be represented in the form of totality m of diagrams И on two inputs.

Logic circuit, which realizes information processing according to logic "m from n" (m/n), is diagram on m inputs and one output. Let us assume that to the inputs of this diagram the signals of standard form are supplied from the memory units (on the delay lines). In this case for each of the inputs with numbers 1, 2, ..., n are supplied signals U_i , which subsequently for simplicity we will designate by value 1 or 0 (zero - if signal it is absent). Then for each of the inputs with numbers 1, 2, ..., n can be fixed one of two values of the random value

$$U_i = \begin{cases} 1 \\ 0 \end{cases} \text{ or}$$

where i - number of introduction/input. Signal is considered discovered, if on the n positions of input there will be not less than m ones.

Consequently, for synthesis of structure of logic unit it is necessary to find all logical products, which contain in different combinations on n of ones (1) into m factors, and to accumulate products indicated. If logical sum is equal to one, then detection is recorded.

Let us illustrate that presented based on example of working/treatment according to logic "2 of 3" (2/3). Here the products, which correspond to one, can be obtained during the following combinations of input values:

$$\begin{array}{ccc|ccc|ccc|ccc} U_1 & U_2 & U_3 & U_1 & U_2 & U_3 & U_1 & U_2 & U_3 & U_1 & U_2 & U_3 \\ 1 & 1 & 1 & 1 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 1 \end{array}$$

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Since logic circuit must fix any of these situations, then by producing addition of values, we will obtain logical formula, which determines structure of diagram of working/treatment according to logic "2 of 3" (2/3):

$$U_1 U_2 U_3 + U_1 U_2 + U_1 U_3 + U_2 U_3.$$

Producing the transformation of this expression with the aid of the known relationships/ratios of Boolean algebra, it is possible to obtain one or another the form of the diagram of working/treatment, optimum in structural/design or circuit sense.

Such type research on the schemes of structures in general case for logic of type "k from m" was carried out by Ya. S. Itskhoki, N. I. Ovchinnikov and L. P. Firsov, who obtained formulas of concrete/specific/actual structures and they were given to form, which contains minimum quantity of diagrams N on two inputs. Some of these formulas are indicated in Table 4.1.

Together with primitive logic circuits examined above of storage devices/equipment, which contain only logic elements, can be utilized structures of type of finite automats (finite automations), containing memory elements in logic unit of diagram UOS.

Table 4.1.

(1) Вид логики	(2) Логическая формула структуры
$2/3$	$U_1 (U_2 + U_3) + U_2 U_3$
$2/4$	$U_1 (U_2 + U_3 + U_4) + U_2 (U_3 + U_4) + U_3 U_4$
$3/4$	$U_1 (U_3 + U_4) U_2 + U_3 U_4 (U_1 + U_2)$
$3/6$	$U_1 U_2 (U_3 + U_4 + U_6) + U_4 U_6 (U_1 + U_2 + U_3) +$ $+ U_3 (U_1 + U_2) (U_4 + U_6)$
$4/6$	$U_1 U_2 [U_3 (U_4 + U_6) + U_4 U_6] + U_3 (U_4 U_6) (U_1 + U_2)$
$4/4$	$[(U_1 U_2) U_3] U_4$
$4/6$	$U_1 (U_2 U_3) (U_4 + U_6 + U_6) + U_4 (U_6 U_6) (U_1 + U_2 + U_3) +$ $+ [U_1 (U_2 + U_3) + U_2 U_3] [U_4 U_6 + U_6 (U_4 + U_6)]$

Key: (1). Form of logic. (2). Logical formula of structure.

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Examined above logic circuits of working/treatment, which use ultrasonic lines together with logic elements, hardly can be effective used in diagrams of ideal integration, since logic elements, in particular element/cell И, are virtually diagram of threshold action. More effectively they can be used in the diagrams of working/treatment according to the type of the binary integration, where the instability of the amplitude of standard pulses is usually small.

On the contrary, diagram, given in Fig. 4.17, can be utilized both with ideal and during binary integration. However, the effectiveness of working/treatment according to the method of binary

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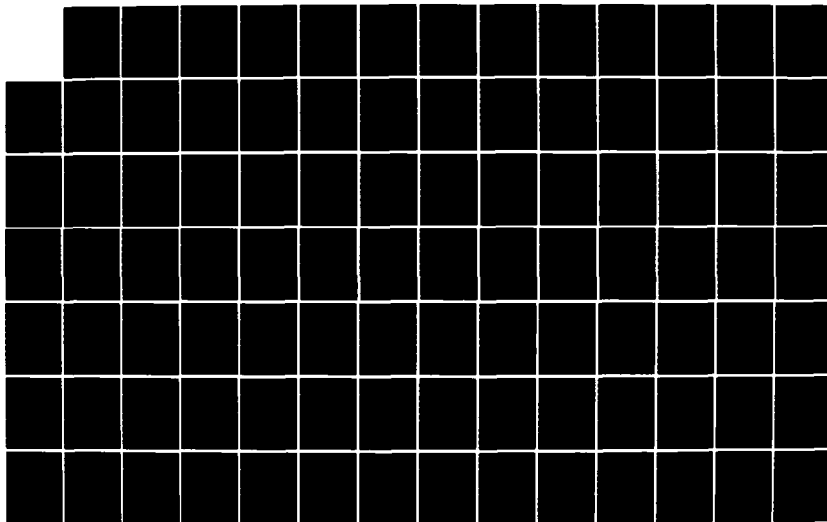
DIGITAL METHODS OF THE OPTIMUM PROCESSING OF RADAR
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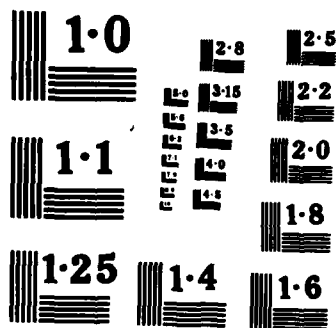
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integration in the principle is lower than for the method of ideal integration, since into latter/last more fully is utilized the difference between the signal and the interference.

Together with those examined there are other, more compound circuits and methods of working/treatment, connected with quantization of signal to several levels or considering phase and other parameters of signal. However, in view of the limited space in this book are not examined.

In given above diagrams of working/treatment was realized by fact or another method accumulation of signal on n adjacent positions of scanning/sweep. If on these positions total signal exceeded the assigned threshold, i.e., was made a criterion of the type " m from n ", then in the block GSI the standard pulse, which enters further the output of diagram, was generated. But if this condition was satisfied to following n scannings, then the second standard impulse/momentum/pulse, etc appears. In the presence of burst of pulses from the cascade/stage of GSI is obtained the series of standard impulses/momenta/pulses with the constant repetition period, which enters, in particular, the circuit of separation of beginning and end/lead of the packet (RNA). At the output of this diagram are obtained two impulses/momenta/pulses: one, which corresponds to the beginning of the packet, and another, that corresponds to the

end/lead of the burst of pulses (see § 4.1 and Fig. 4.2).

Impulses from output of UOS are utilized for further control of nodes of diagram of detection of signal. In particular, the impulse/momentum/pulse of the beginning of packet is utilized as the official signal, which states/establishes the fact of the detection of burst of pulses. However, the totality of the signals of beginning and end/lead of the packet can be used for determining the angular coordinates of target, if measurement is made by the method of linear scanning.

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However, it must be noted that if with the nonfluctuating packet the position of the impulse/momentum/pulse of the beginning of packet could be accurately fixed with respect to the impulses/momenta/pulses of packet itself, then in the case of the fluctuating impulses/momenta/pulses of this fixation cannot be made. Actually, threshold crossing and, therefore, the appearance of the first standard impulse/momentum/pulse can begin after any of m of the first impulses/momenta/pulses of packet, although, apparently, the appearance of a standard impulse of GSI at the end of the totality of m of the first scanings/sweeps is most probable. This is the fact, when the position of impulses/momenta/pulses of GSI within certain

limits is random with respect to the position of packet itself, it leads to the measuring error of the coordinates of angular coordinate. Furthermore, in the measurement of angle according to the method of fixing beginning and end/lead of the packet usually is not utilized an effective direction-finding characteristic of radiation pattern, since the marks of beginning and end/lead of the packet coincide with the edges of radiation pattern, where the slope/transconductance of direction-finding characteristic is small. This leads to a decrease in the accuracy of the measurement of angular coordinate. More precise prove to be other methods, in particular the methods, which escape/ensue from the method of the maximum of plausibility, which is given below.

In conclusion let us examine one additional schematic of synchronous accumulator/storage (Fig. 4.21), which uses magnetic drum as memory unit. Actually, it is analogous to the diagram, shown in Fig. 4.17, with the only difference that the instead of the memory unit on the ultrasonic lines the sections of magnetic drum are utilized. Diagram can be used in the mode/conditions both of ideal and binary integration. The work of device/equipment lies in the fact that the signals from the output of radio receiver are written/recorded on the circle/circumference of magnetic drum, in this case on each circle/circumference it is arranged/located not with one, but five range sweeps. The number of revolutions of drum is

accurate five times of less than the frequency of the impulses/transmissions of station, and the speed of rotation of drum is strictly constant. In the circle/circumference of drum on equidistances are arranged/located five caps - four reading and one recording.

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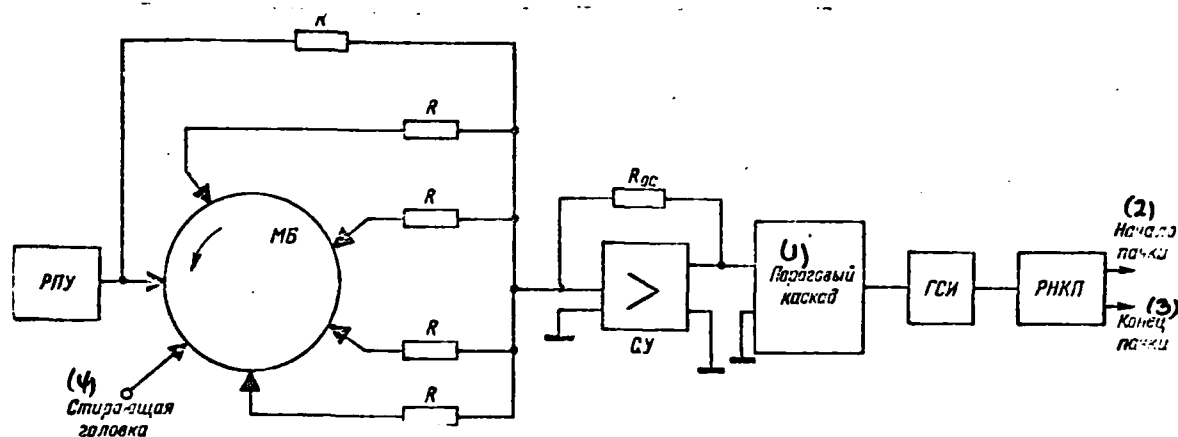


Fig. 4.21. Diagram of feeler on magnetic drum.

Key: (1). Threshold cascade/stage. (2). Beginning of packet. (3). End of packet. (4). Erasing head.

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Thus, input signal and signal from the output of reading heads in any angle of rotation of magnetic drum correspond to one and the same range in five periods of impulses/transmissions simultaneously. Signals from the identical sections of range store/add up in the adder and just as in the preceding/previous diagram, are supplied to the threshold cascade/stage, whose output is connected with the generator of standard impulses/momenta/pulses. From GSI the signals

come the circuit of separation of beginning and end/lead of the packet of the RNA.

Changing value of cutoff voltage in threshold cascade/stage, we change probability of its functioning from random signals of different level. Thus, for instance, decreasing the threshold, we improve the possibility of functioning the generator of standard impulses/momenta/pulses both from the useful signals and from the false ones - noise overshoots, increasing thereby the probability of false alarm. On the contrary, by making the circuit less sensitive, we raise the chance of the passage of useful signal, i.e., the passage of target, decreasing in this case the probability of false alarm.

Selection of optimum value of threshold voltage is important question during creation of this diagram of detection of signal of UOS. For its correct solution it is necessary to, first of all, formulate qualitative indices of the correct work of the device/equipment of the detection of the signals, on base of which and to select subsequently the optimum parameters of these circuits. Naturally, these criteria must have statistical character, since the workable signal has random structure.

§ 4.4. Removal of angular coordinates.

Character of device/equipment of removal/output of angular coordinates depends on method of their determination. In turn the selection of one or the other method depends on a number of factors: the required accuracy of measurement, signal-to-noise ratio, time of measurement, character of survey/coverage, etc.

In literature in recent years question about finding of optimum methods of measurement of angular coordinates repeatedly was examined. The solution of this question for the signal, taken against the background of interferences, is especially important. In this case deserve attention the methods, in which this problem is solved by the method of the maximum plausibility, known from the mathematical statistics.

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The use/application of this method especially conveniently is combined with the use of the electronic computers, which to a certain degree stimulate its use/application, since the structural/design embodiment of method and its different modifications is here especially simple and convenient.

In more detail these questions will be examined below. Let us now examine the simplest formulation of the problem of measuring the angle by fixing beginning and end/lead of the packet in connection with stations with the circular scan under the following assumptions: the pulse repetition frequency of station is constant, the value of received signal considerably exceeds interference level, the antenna radiation pattern has a form of sector and a rotation of antenna it occurs evenly with the angular velocity Ω . Then burst of pulses will consist of the signals of constant amplitude, distant on identical angular distance one from another. A quantity of impulses/momenta/pulses in the packet in the case of pinpoint target is determined by the pulse repetition period, by the antenna scan rate and by the width of its radiation pattern θ_a , calculated usually at the level of half power.

As is known, this packet on PPI scope creates mark in the form of shackle (Fig. 4.22), which occupies specific angular dimensions, and true direction to target is located as arithmetic mean two readings - beginning and end/lead of packet. If β_n - azimuth of beginning, and β_k - azimuth of the end/lead of the packet, then direction to the target is determined by the relationship/ratio:

$$\beta = \frac{\beta_n + \beta_k}{2}. \quad (4.4.1)$$

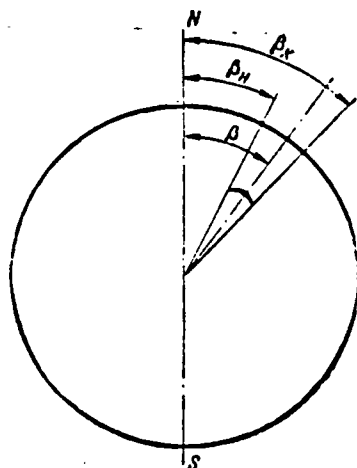


Fig. 4.22. Form of mark on the screen of PPI.

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Diagrams of discrete/digital removal/output of azimuth utilize this algorithm for determining angular coordinate. In accordance with this formula for the azimuth determination β are necessary: the sensor of the current azimuth (DTA, Fig. 4.23), salient the value of azimuth in the binary code; the diagram, which calculates angular coordinate according to formula (4.4.1), VUK (calculator of angular coordinate), and also device/equipment of the detection of signal (UOS) with the output pulses, which fix beginning and end/lead of the packet. The latter take readings of the DTA at the moment of beginning and end/lead of the packet, and they are also the signals, which synchronize work of the arithmetic unit, which forms part of VUK.

Schematic of converter of angle of rotation into binary code is the fundamental element of the DTA. Since the shaft of antenna usually has the large torsional moment and allows/assumes considerable load from the side of the schematic of converter, is here possible use/application of the most diverse schematics of

converters. Thus, for instance, can be utilized converters with the code disks, with the induction sensors, with the use of a magnetic drum and with other devices/equipment.

Fig. 4.24 gives one of possible versions of more detailed functional diagram for removal/output of angular coordinate of one target. Here as DTA magnetic drum together with the trigger counter, which gives the instantaneous value of angle in the binary code, is utilized.

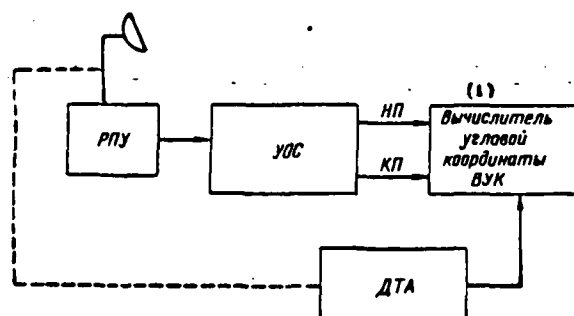


Fig. 4.23. Block diagram of the discrete/digital removal/output of angle.

Key: (1). Calculator of angular coordinate of VUK.

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The drum is rigidly connected with the shaft of antenna, and on its lateral surface along the circle/circumference magnetic lines with density δ are evenly plotted/applied.

During rotation of antenna in the reading head Cyl pulses transmitted to counter, are induced. A quantity of impulses/momenta/pulses is proportional to the angle of rotation of antenna. Furthermore, is an even more magnetic mark "0", located in that place of the magnetic drum, from which desirably to count off it began the angular-singing of coordinate.

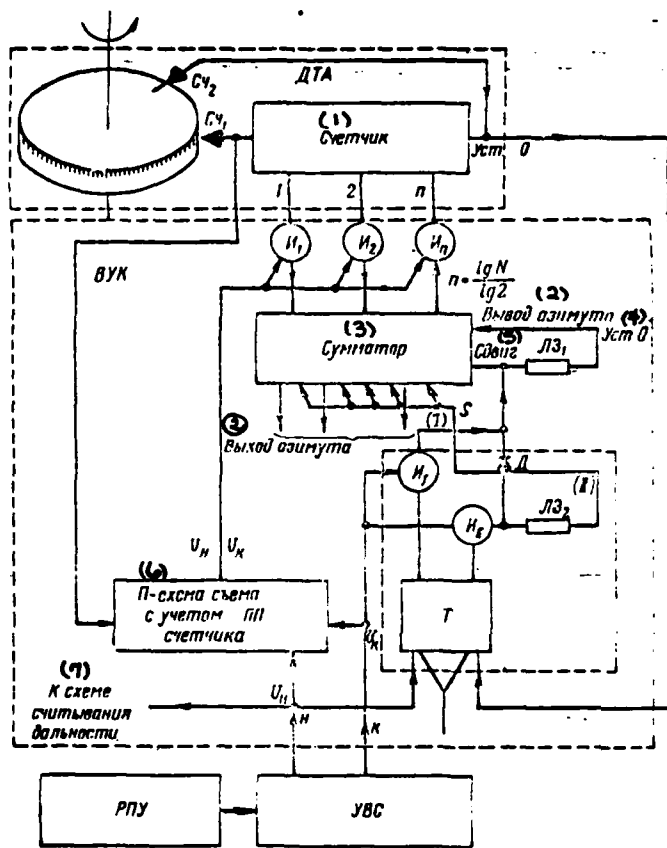


Fig. 4.24. Schematic of the removal/output of angular coordinate.

Key: (1). Counter. (2). Conclusion/output of azimuth. (3). Summator. (4). Mouths. (5). Shift/shear. (6). Parallelserial structure of removal/output taking into account PP of the counter. (7). To reading circuit of range.

Usually this place corresponds to the direction of the maximum of antenna radiation to the north or to the center-line plane of the object (ship, aircraft), on which is established/installed radar.

Head C₄,, which reads zero impulse/momentum/pulse, is connected with busbar of zero-setting counters, which begins reading at moment of passage of magnetic mark "0" under reading head C₄,. Let us note that initially after the inclusion of radar in this diagram the correct counter readouts will be only after the passage of zero mark under C₄,, since the value of counter readout is direct at the moment of the switching on of circuit and the true value of angle can not correspond one to another.

Sizes/dimensions of drum are determined by required accuracy of reading of angular coordinate, since minimum value of density writing is limited and comprises usually 8-10 impulses/momenta/pulses on 1 mm. Hence the diameter of the drum

$$D = \frac{N}{\pi b},$$

where N - quantity of impulses/momenta/pulses on the circle/circumference of drum. This quantity of impulses/momenta/pulses is determined by the accuracy of removal/output. Thus, for instance, N=360 gives error on the order of

1°, and N=3600 it makes it possible to count off with an accuracy to 0.1°, etc. The required accuracy of reading of DTA depends on operational requirements for the device/equipment.

As a whole measuring circuit of angular coordinate (Fig. 4.24) works as follows. The device/equipment of the isolation/liberation of signal of UOS puts out the impulses/momenta/pulses of beginning and end/lead of packets U_n and U_n which through the circuit Π enter n gate circuits M for the removal/output of the instantaneous value of azimuth. Circuit Π is intended for eliminating the effect of transient processes in the counter on the work of the valves/gates of removal of M . For this into circuit Π are pulses from the magnetic drum from head C_1 , are introduced. The work of analogous diagram is described in § 4.2 and therefore here it is not brought.

Number of elements/cells of type M is determined by quantity of bits of counter

$$n = \frac{\ln N}{\ln 2}.$$

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Taken/removed values of current azimuth β_n and β_n are introduced into summator Σ into stroke/cycle with impulses/momenta/pulses respectively U_n and U_n . In the summator they store/add up and sum is

divided by 2. Division is achieved by the shift of sum to one digit to the side of low-order digits. For the execution of shift the impulse/momentum/pulse of the end/lead of packet U_k is supplied to the busbar of the shift of the summator through the delay line LZ,. Delay time is determined by the duration of the transient processes in the summator, which appear during the addition after the supply there of the second number.

However, in work of summator VUK, besides described operations/processes, are other peculiarities associated with azimuth determination near zero values of azimuth, i.e., near marker of zero-setting. Actually/really, if the impulse/momentum/pulse of zero marker is located in the middle between beginning and end/lead of the packet, then azimuth determination from formula (4.4.1) leads to the errors. In this case the azimuth must be determined from the formula

$$\beta = \frac{\beta_n + \beta_k}{2} - 180^\circ. \quad (4.4.2)$$

Computation according to this formula to more conveniently produce in modified or simple inverse code with subsequent end-around carry. The binary notation of number (180)₁₀ = 10,110100 = x_1 , and in modified inverse code ($-x_1$) = 11,01001011.

Then formula (4.4.2) can be rewritten in the form

$$\beta = \frac{\beta_n + \beta_k}{2} + x_1 \quad \text{ц. п.} \quad (4.4.3)$$

end around-carry.

End-around carry (TsP)[III] is realized within summator by special connection/communication through differentiating circuit from output of corresponding sign position to input of low-order digit of summator.

Transition from formula (4.4.1) to formula (4.4.3) in process of computations is realized by schematic of calculator of angular coordinate (VUK) automatically with the aid of local control unit, which consists of flip-flop T, gates M₁ and M₂, of delay lines ЛЗ₁, ЛЗ₂. These elements of the circuit are controlled by signals "obsolete 0" and by impulse/momentum/pulse the "beginning of packet" U_n.

FOOTNOTE 1. The work of summator is examined in main Z. ENDFOOTNOTE.

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The latter switch the channels of the supply of signal the "end/lead of the packet" U_n depending on the position of impulse/momentum/pulse "obsolete 0" with respect to the interval of packet. If this signal is located out of the interval, then the pulse "end/lead of the packet" U_n enters along channel I, by information gate M₁ with the

aid of the flip-flop T. In this channel impulse/momentum/pulse $\bar{U}_n$ realizes a shift of a number in the summator (division into 2) and then a conclusion of result after the time τ_1 . Delay to the period τ_1 is realized by a delay line LZ₁.

But if signal "Ycr. 0" is located in pulse separation U_n and U_{n+1} , then in this case is triggered channel, also, with the aid of valve/gate V_n and flip-flop T, and channel I is cut off. In channel II impulse/momentum/pulse U_n makes a larger quantity of operations/processes, namely: in accordance with formula (4.4.3) it realizes a shift of a number in the register of summator (division into 2), then after the time τ_1 adds the inverse code of the binary equivalent of number 180, i.e., number 11.001011, and then after the time τ_1 , determined by transient processes in the summator, the conclusion of result is realized. Addition, i.e., the introduction/input of number 11.01001011, is realized by supply of pulse signal to the calculating inputs of the corresponding digits of summator. These inputs are commuted constantly and are connected with busbar S, to which is fed/conducted delayed pulse U_n . Pulse arrival on busbar S in view of the constancy of the commutation of its outputs indicates the introduction/input of a number x_1 into the summator.

Diode D protects entry of impulses/momenta/pulses from channel I into channel II.

Method examined can be technically realized with the aid of not only magnetic drum, but also code disks or other transformation circuits angles of rotation (continuous) into digital code (PND).

Besides method of measurement of azimuth examined according to formula (4.4.2), are possible other methods. For example, it is possible to create the diagram, in which at the moment of the arrival of the impulse/momentum/pulse of the beginning of packet is started the counter, which computes a quantity of discrete/digital units (impulses/momenta/pulses of magnetic drum) to the arrival of the impulse/momentum/pulse of the end/lead of the packet. Difference $\Delta\beta = \beta_n - \beta_m$ is recorded thus.

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However, at the moment of the arrival of the impulse/momentum/pulse of the end/lead of the packet azimuth β_n is recorded. From this value is deducted the half-difference $\Delta\beta/2$ i.e.

$$\beta = \beta_n - \frac{\Delta\beta}{2}.$$

As noted above, instantaneous value of azimuth can be determined by different methods. We examined the methods, based on fixing of the

points of beginning and end/lead of the packet. It should be noted that these methods of measurement are characterized by comparative simplicity of technical realization, but they are not optimum and do not always provide the best accuracy. Actually/really, the points of beginning and end/lead of the packet usually correspond to the edges of the antenna radiation pattern, where the slope/transconductance of direction-finding characteristic is small, and because of this accuracy is low. The method, based on the method of maximum plausibility, is more precise, since with this method are utilized, in essence, those impulses/momenta/pulses of the packets, which correspond to the sections of diagram with the greatest slope/transconductance of the direction-finding characteristic (see Chapter 7).

§ 4.5. Processing signal at the staggered repetition rate.

In radars with constant pulse repetition frequency detection of signals against the background of interferences was conducted, as is known, via accumulation of signals in one and the same elementary sections of range with the aid of diagrams of memorization, which use devices/equipment of constant delay.

In stagger rep-rate radar, whose frequency changes frequently randomly, to difficult carry out accumulation of signals with the aid

of devices/equipment of constant delay. Therefore other diagrams and methods here are applied. However, the fundamental task of these diagrams is reduced nevertheless in order by on or another manner to carry out accumulation of signals in the equidistant sections of range.

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As diagrams of automatic detection of signal in radars with variable/alternating repetition frequency are applied special devices/equipment of discrete/digital equipment, using elements/cells of digital computers. In such devices processing signal in the separate elementary sections of range is realized. For these purposes the examined/scanned detection range R is quantized (it is strobed) in the elementary sections with an extent of ΔR . Number of such sections

$$\frac{R_{max}}{\Delta R} = v$$

when $\Delta R = \frac{c\tau_0}{2}$, where τ_0 is the duration of the elementary section of range.

For each elementary section of range is its channel of analysis of radar information, which contains special device/equipment (UOS) of detection of useful signal, reflected from target. This device/equipment processes signal in the separate elementary section

for several pulse repetition periods of radar. As a result of processing at the output of UOS there appears standard signal - impulse/momentum/pulse, if in this elementary section of range is a signal, reflected from the target. In the opposite case standard impulse/momentum/pulse is absent.

Duration of elementary section τ , must be matched with passband of radar channel and pulse duration, somewhat exceeding it in value. Without examining all considerations, which determine the selection of the optimum duration of this section, let us assume that its extent composes 1.2-1.5 pulse durations.

Exemplary/approximate block diagram of digital unit, which realizes strobing/gating elementary sections of range (Fig. 4.25), consists of shift register R of circuits of coincidence $H_1 + H_2$ and device/equipment of detection of signal for each elementary section of range [41, 61, 63]. As shift register can be used different diagrams, in particular trigger registers, registers with the decoder and other diagrams.

Most high speed, apparently, is diagram of trigger register.

In this in the beginning is recorded a one, which is introduced into the low-order digit by cycling at the moment of radiating/emitting the sounding signal of radar. The generator of time, which is the synchronized pulse generator of the same type, such as it is utilized in radars for calibrated distance marks, is started by the same pulse. Impulses/momenta/pulses from this generator are utilized as the shift pulses in the register R for displacing one along the shift register. The displacement/movement of one from one discharge to the next is conducted by jumps into the stroke/cycle with cycling of shift. A quantity of digits in shift register R is equal to a quantity of elementary sections of range $v = \frac{R_{max}}{\Delta R}$.

Output voltages from each digit of shifting register control circuits of coincidence Π_0 to signal input of which is supplied information from output of receiving channel of radar. If in the i th digit of the flip-flop a one is located, then at the anode of this flip-flop high voltage appears and corresponding diagram 11 will be opened. Then signal from the output of radar enters through the valve/gate into the appropriate device/equipment of the detection of signal.

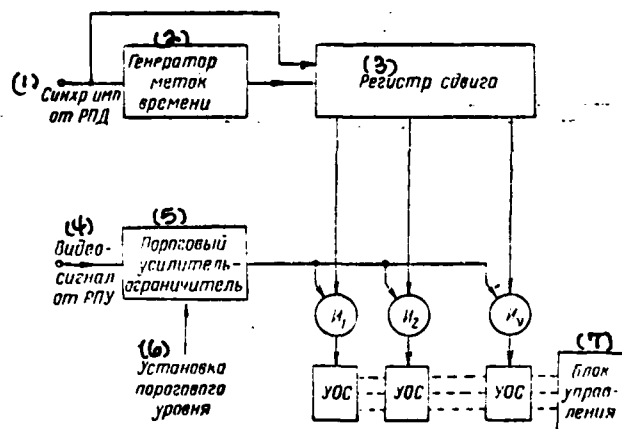


Fig. 4.25. Diagram of strobing/gating the sections of range.

Key: (1). Synchronization of pulse from RPD. (2). Time-marking generator. (3). Shift register. (4). Video signal from RPU. (5). Threshold amplifier-limiter. (6). Setting of threshold level. (7). Control unit.

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Signal from output of radar enters device/equipment of detection of separate range channels not directly, but through special block - threshold amplifier-limiter (BUO). The mode of this block depends on diagram and mode/conditions of the work of the device/equipment of the detection of signal (UOS), elementary sections of range utilized in the channels. With the work of UOS in the mode/conditions of ideal

accumulator/storage (integrator, see § 6.2) the circuit of BUO is linear amplifier with the low-resistance output in the form of cathode-loaded cascade/stage. In other cases, when UOS works with quantum data, the circuit of VUO is the amplifier-limiter, which quantizes in the amplitude the output signal of radio receiving equipment to two or several levels. In the latter case the circuit of VUO is actually the pulse-height analyzer.

Repetition period of synchronized generator corresponds to doubled time of emission of the signal along elementary section of range, i.e.

$$T_n = \frac{2\Delta R}{c}.$$

Therefore in each digit the shifted unit will be delayed only to period $T_n = \frac{2\Delta R}{c}$, and respectively valve/gate B_i of each digit will be opened only to this period T_n during which radar information enters device/equipment of detection of signal (UOS) of corresponding channel. In the following period the following valve/gate, etc is opened. As a result after ν the pulse repetition periods of mark generator will be switched all ν of the channels (UVS), which correspond ν to the elementary sections of range.

This commutation of signals at output of receiving channel of radar is realized each time during repetition period of sounding

pulses. After n repetition periods in the device/equipment of the liberation of the signals of each channel the specific information, which with that corresponding to processing will allow to make a conclusion about presence or absence of signal in this elementary section of range, will be accumulated.

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Each signal of UOS has its number and corresponds to the specific range. Distance R of the discovered target is determined by the number of that channel, in which the detection of the useful echo signal occurred, and it is equal

$$R = i\Delta R,$$

where i - number of channel;

ΔR - elementary section of range.

Schematic of digital unit, represented in Fig. 4.25, makes it possible for time of repetition period to detect useful signal in all elementary sections of range. Subsequently such type diagram we will call the diagram of parallel detection or the diagram of the parallel search.

Time of detection of useful signal in this diagram depends on parameters of radar channel, and also on value of received signal with respect to noise.

In connection with this it is obvious that device/equipment of detection of signal, which works in channel of section of maximum range, requires larger time for processing of signal, than the same device/equipment, which works in channel of section of short distance. In this case the required detection time in the diagram of the parallel search is selected, on the basis of the work of UOS under the worst conditions, which correspond to the location of target at maximum range, when relationship/ratio signal/noise is minimal. This time will be determined by the necessary quantity of selective values of radar information, required for the correct work of this UOS. Let us assume that for the correct work of circuit of UOS it is necessary to have n selective values of the radar information (selection n it is examined below during the analysis of the work of circuit UOS).

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Taking into account, that during one repetition period ^{into} ~~not~~ each circuit of UOS enter only one selective value of signal from the output of radar, the required detection time in the diagram of the

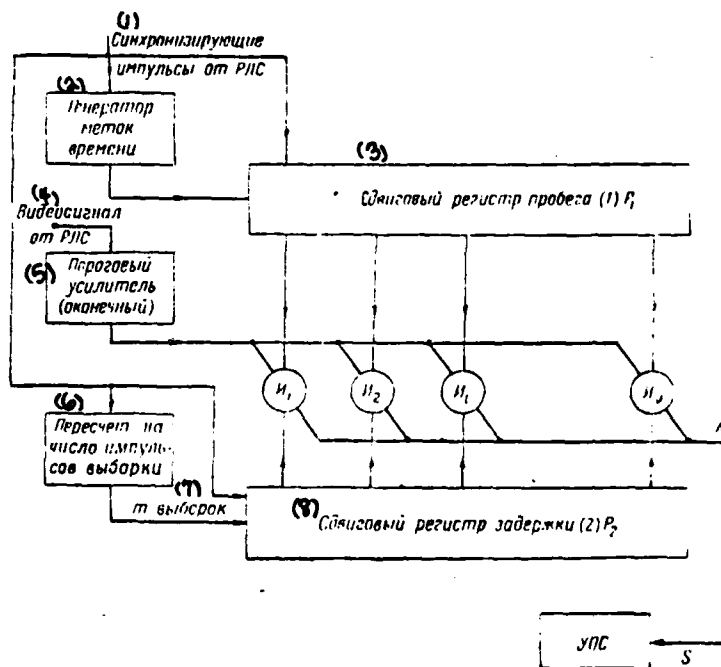


Fig. 4.26. Diagrams of sequential search.

Key: (1). Synchronizing pulses from radar; (2). Generator of time marks; (3). Shift register of path; (4). Video signal from radar; (5). Threshold amplifier (terminal) (6). Conversion into number of access pulses; (7). of (t) accesses; (8). Shift delay register.

parallel search

$$T_{\text{обн}} = \sum_{i=1}^n T_{i\text{п}}$$

where T_i — interval between the sounding pulses of the RLS.

Together with diagrams of parallel search there can be diagrams of consecutive and series-parallel search [70].

In diagram of consecutive search each elementary section of range is examined/scanned consecutively/serially with the aid of one device/equipment of isolation/liberation of signal. In this case the signal from each section of range is supplied on UOS during time $T_{\text{обн}}$, device/equipment of detection necessary for the correct work. Total detection time, necessary for the survey of the entire distance:

$$T_{\text{обс}} = \nu T_{\text{обн}}$$

where ν a quantity of elementary sections $\Delta R = \frac{R_{\text{max}}}{\nu}$.

Detection time during consecutive search in ν of times is more than with parallel; however, quantity of equipment many times less.

One of possible diagrams, which make it possible to carry out consecutive search along distance (Fig. 4.26), differs from diagram,

shown by Fig. 4.25, fact that into it are introduced one additional shift register P_1 and scaling circuit with modulus/module of translation, equal to number of signals in selection n , i.e., to quantity of impulses/momenta/pulses of packet. A difference in the described (Fig. 4.26) diagram from preceding/previous (Fig. 4.25) is the fact that here a quantity of UOS is considerably less than in preceding/previous, only one; furthermore, diagrams $H_1 + H_2$ have dual control: they are controlled by potentials from the registers P_1 and P_2 . Only during the simultaneous supply of signals from both registers the corresponding valve/gate will pass radar information to the input of UOS.

Work of register of delay P_2 occurs analogously with work of shift register of landing run P_1 with the only difference that the shift pulses on register P_2 enter considerably more rarely than register P_1 . After of each of n synchronizing pulses on the register P_2 will be given only one shift pulse. Initially into the extreme digit P_2 also is introduced one, which will be moved under the action of shift pulses along the register.

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In proportion to the displacement/movement of one will be prepared consecutively circuits $H_1, H_2, H_3, \dots, H_n$ each of which is supplied

from the appropriate digit of register during the n repetition periods.

Thus, at what specific moment to opening will be prepared only by one of $v = \frac{R_{\max}}{\Delta R}$ available circuits let us assume the i -th. However, this circuit will be opened only at that moment/torque, when to it trigger pulse is given by duration T_n from the register of transit P_i . It is obvious, the onset of this signal will be delayed relative to synchronizing pulse of radar on period T_{ni} , where i - quantity of pulse repetition periods of the time-marking generator.

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Thus to the busbar A_i give radar signal only from the i elementary section of range n of times in a row.

After admission of following shift pulse into register P , one from i th digit will move in $(i+1)$ th digit and high voltage it will arise on busbar, which opens gate B_{i+1} , i. e. $\wedge$ detection of signal will be conducted in $(i+1)$ th elementary section of range.

During the supplying of next shift pulse into register P , one will move into following $(i+2)$ th digit and detection will be conducted in $(i+2)$ th section of range, etc. Thus, in proportion to

the admission of shift pulses on P, will be looked over consecutively/serially all elementary sections of range.

Relatively small quantity of utilized equipment and long time of survey/coverage of distance, which ν once exceeds time of survey/coverage during parallel search, is distinctive special features of method of consecutive search in comparison with parallel. However, this deficiency/lack to a certain degree can be overcome, if we utilize a variable/alternating sample size during the detection along the distance, making with its small at the close distances, where signal level is high, and selecting large at the considerable removals/distances, where the signal-to-noise ratio proves to be small. By this it is possible to decrease the total volume of the selections, necessary for the survey of distance as a whole, and consequently, to decrease the time of the survey/coverage of the assigned section of space.

Laws governing change in quantity of selections along distance during detection of signal in elementary sections can be selected previously in accordance with operating conditions and special features/peculiarities of detection conditions. In this case it is assumed that the sample size in the separate elementary section will be constant, changing only from one section to the next or from one group of sections to the next.

For determining time of coverage during consecutive search sufficient to fruitful ones is use/application of method of sequential analysis of Wald, where quantity of selections for elementary section is random and depends on signal-to-noise ratio in each section.

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As a whole an average quantity of selections during the detection of signal along the entire distance proves to be small.

Method of series-parallel search occupies intermediate position according to its technical specifications. It requires with respect to other methods an average quantity of equipment and the average duration of the survey/coverage of distance.

With this method ν of elementary sections ranges are divided/marked off into groups - on μ elementary sections within each group. A number of sections within each group can be identical, but it can be different. Within each group the detection of signal in the elementary sections is conducted in parallel, and transition from one group to another is realized consecutively/serially. Let us assume

that a quantity of groups is equal to k with an identical number of sections in group $k = \nu/\mu$. The detection time with k parallel channels is equal

$$T_{\text{офн}} = kT = \frac{\nu}{\mu} T.$$

The diagram, which realizes the method of search (Fig. 4.27) indicated, by its composition is analogous to the diagram of consecutive search (Fig. 4.26), only here additionally there is a larger number of devices/equipment of the liberation of signal (UOS), equal to a number of elementary sections in the group. Each node of UOS is connected constantly to several diagrams H (to gates), which have identical number within each group: in each group all first diagrams H are connected with the diagram UOS_1 , the second - with UOS_2 , etc. All diagrams of H , just as in the diagram, shown in Fig. 4.26, have dual control - by registers P_1 and P_2 , and are opened only during the simultaneous supply of signals from both registers, in this case into the diagrams of UOS radar information enters.

Register P_1 in this diagram has numerical length equal to quantity of groups. Shift pulse is supplied to this register after n of the pulse repetition periods of radar, causing switching the following group of valves/gates. The voltage removed with P_1 , prepares the immediately whole group of valves/gates for the triggering/opening.

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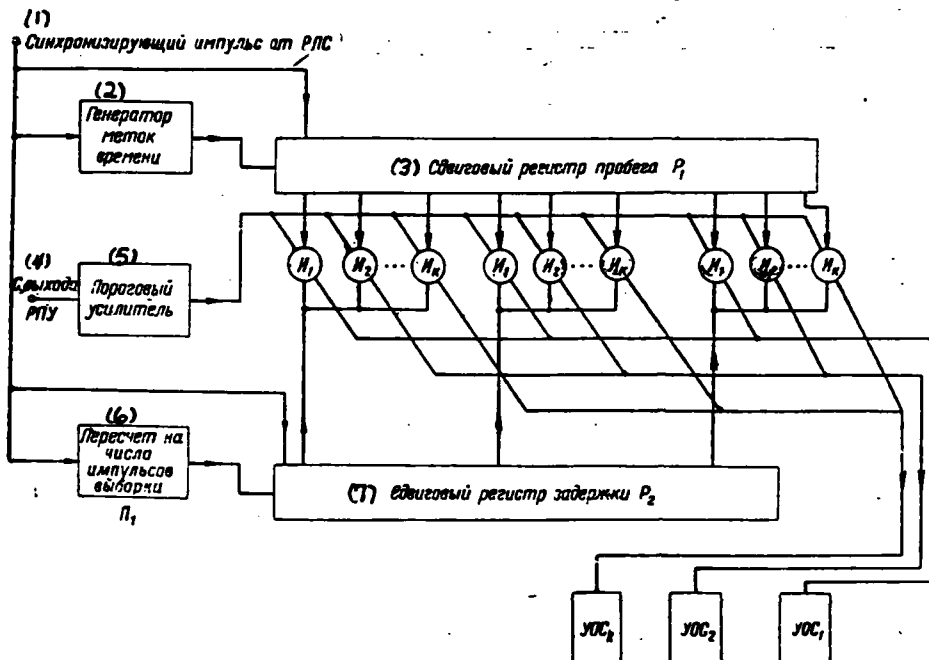


Fig. 4.27. Diagram of series-parallel search.

Key: (1). Synchronizing pulse from RLS. (2). Time-marking generator. (3). Shift register of landing run P_1 . (4). From output of RPU. (5). Threshold amplifier. (6). Translation to numbers of impulses/momenta/pulses of selection. (7). Shift register of delay P_2 .

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The character of processes during the control of gates of registers

P_1 and P_2 is here accurate the same, as in the case of consecutive search; therefore switching channels and shift of one in the registers here are not examined. Each valve/gate corresponds to the specific range, and the group of valves/gates gives the specific interval of ranges. Distance of the discovered target is determined in this case not only according to the number of the valve/gate, which coincides with the number of UOS, but it depends also on the number of the group of the valves/gates, which are connected at the given moment/torque to the diagrams of UOS, moreover the number of group must be fixed/recorded with special diagram.

In conclusion should be indicated one additional version of series-parallel search, during which within group is conducted consecutive search, fulfilled simultaneously into all h groups (Fig. 4.28). Here each diagram of UOS is connected up simultaneously to all valves/gates of this group, but radar information enters only through one gate, which is opened by shift register of delay P_1 . Each diagram of UOS corresponds to the agreed to group of valves/gates depending on the measured distance. For measuring the distance of the discovered target it is necessary to know not only the number of UOS (number of group), in which occurred the detection of signal, but also the number of the valve/gate, which was connected at the given moment to UOS, which fixed the presence of target.

Diagram of described version of series-parallel search by its composition is analogous to diagram, shown in Fig. 4.24, but it is characterized by method of joining gates, namely: connections of gates with UOS and register P_i interchanged the position themselves (compare it with diagram, given in Fig. 4.27). In this diagram the groups of valves/gates are constantly connected with the appropriate diagrams of UOS, and digits of the register of delay P_i - with several valves/gates (with one in each group). A quantity of digits in the register P_i is equal to a number of valves/gates in the group, but not groups as in the preceding/previous diagram, a quantity of diagrams of UOS is equal to a number of groups.

The space of selective values (number of impulses/momenta/pulses in the packet) is defined in this diagram, just as in preceding/previous, with the aid of the scaling circuit Π_1 , which has the scaling factor, equal to the size of sample n .

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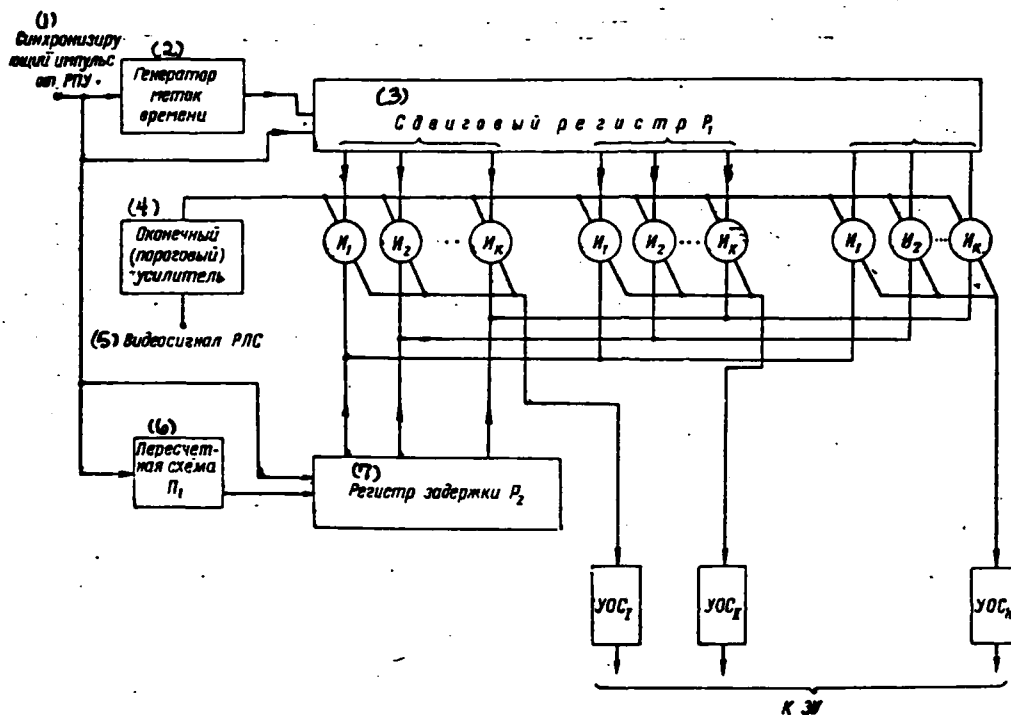


Fig. 4.28. Diagram of combined search.

Key: (1). Synchronizing pulse from RPU. (2). Time-marking generator. (3). Shift register P_1 . (4). Terminal (threshold) amplifier. (5). Video signal of RLS. (6). Scaling circuit. (7). Register of delay P_2 .

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Pulse from the output of scaling circuit is supplied to the register of delay P_2 , as the shift pulse. The detection time in this version of

the diagram of series-parallel search, which has k of elementary sections within the group, is determined by the expression

$$T_{061} = knT_n.$$

In diagrams examined device/equipment of detection performs processing signal in elementary sections of range after n of adjacent repetition periods.

Let us examine still some special features/peculiarities of device/equipment of detection of signals with quantized data, which have value in stations of precise ranging. Actually/really, in the stations of a precise range the value of elementary section must be small, and their quantity is great. It is natural that the system as a whole becomes complicated as a result of an increase in the number of channels and need to respectively accelerate the operating speed of the work of diagram within each channel. Thus, for instance, for the duration of the gates/strobes of the order of the portion of the microsecond of diagram actually they work in the maximum mode/conditions of operating speed.

As measure for simplification it is expedient to utilize quantization of output signal. In this case the device/equipment of detection the signals of simpler form enter, which simplifies its work. This procedure brings, as is known, to certain loss of

information; however, the schematics of the devices/equipment of detection (UOS) in this case are simplified and acquire the character of threshold counter.

Quantization to two levels realizes binary converter (Fig. 4.29), which enter signals from output of receiving channel of radar.

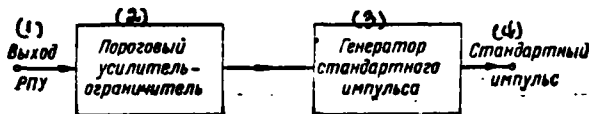


Fig. 4.29. Structure of binary converter (VP).

Key: (1). Output of RPU. (2). Threshold amplifier-limiter. (3). Generator of standard pulse. (4). Standard pulse.

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At the output of this converter signals appear, as is known, only in such a case when they exceed the certain threshold of quantization $U_{\cdot}$. If signal at the output of receiver does not exceed this level, then signal at the output of converter is absent. Sometimes the fact of excess (or nonexcess) is noted by the generation of auxiliary impulse/momentum/pulse by the special generator of standard impulses/momenta/pulses (Fig. 4.30).

Threshold limiter-amplifier, shown in Fig. 4.25, in which limitation on threshold, is realized is input computer of converter, it follows generator of auxiliary standard ones them pulses.

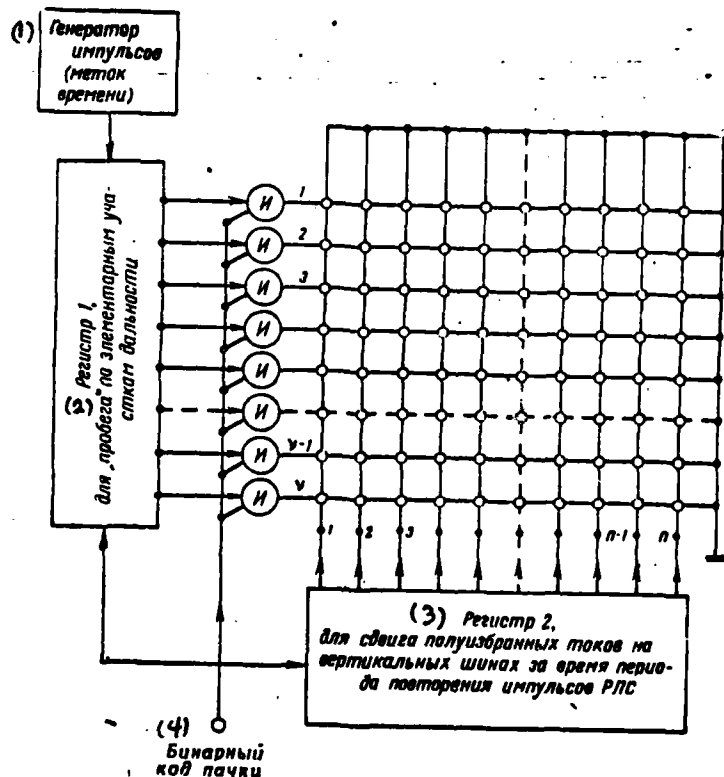


Fig. 4.30. UOS on ferrites for the quantized packet.

Key: (1). Pulse generator (oscillogram time marks). (2). Register 1, for "landing run" on elementary sections of range. (3). Register 2, for shift of half-selected currents on vertical busbars for time of pulse repetition period of the RLS. (4). Binary code of packet.

At the output of this device is obtained the so-called binary code of the analyzed packet, which is utilized subsequently. The subsequent use of this code is possible, as before in the devices/equipment of the detection of parallel and consecutive types.

In first case for each elementary section special channel with its device/equipment of detection of signal (UOS) is created.

In second case quantity of devices/equipment of detection proves to be less than number of channels (elementary sections). It depends on a quantity of discovered targets in the examined/scanned section of range.

Possibility of applying here memory unit (ZU) on ferrites (Fig. 4.30) is important special feature of use of quantized type signals. The process of detection in this case includes the following basic operations: the recording of the code of signal in matrices of ZU, analysis of the recorded signal.

For recording of binary code in device/equipment in question can be utilized by one or several matrices/dies on ferrite rings ¹. In this case for each elementary section of range is its ferrite. Let us assume that ferrites of all sections of the ranges, which correspond to one repetition period, are arranged/located along the column of

one matrix of ZU (Fig. 4.30), and a number of columns is equal to n - to number of analyzed repetition periods of radar. Value n depends on the available conditions of detection and is selected depending on signal-to-noise ratio, probability of false alarm, diagram of detection and from other factors. Consequently, the specific column contains information from each period, and the specific ferrite ring corresponds to each elementary section. As a result during the recording of the binary code in each row of ZU there is recorded n quantified values of signal taken through the repetition period of signal T . At the termination of recording analysis of information, recorded in ZU, begins. The target of analysis is the search for the rows, in which is fixed the state 1 more than established/installed threshold number n , among n of adjacent ferrites.

FOOTNOTE ¹. The work of ferrite elements/cells with transferring of binary numbers is examined in Chapter 3. ENDFOOTNOTE.

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During the analysis of the first matrix/die the recording into the second matrix/die of ZU of analogous form is conducted, after which this subsequent matrix/die, etc is analyzed. The diagram, which establishes/installs a quantity of recorded units into n adjacent ferrites, and composes together with the ferrite matrix/die the

device/equipment of the detection of signal in this case.

As is known, quantity of elementary sections

$$\frac{R_{max}}{\Delta R} = v.$$

It can reach high values. In this case ferrite rings from all elementary sections of range cannot be contained along one column of matrix/die, and then it is necessary to have several columns for entire range of range, which will require the appropriate complication of the commutation of storage elements in the device/equipment detection. However, in both cases the accumulation of signal in the elementary section is realized with the aid of the memory unit on the ferrites.

As another version of diagram of UOS for quantized signals can serve diagram, which uses storage not on ferrites, but on trigger counters. Its exemplary/approximate diagram is represented in Fig. 4.31. In it there are two trigger counters - null indicator C_q-0 and counters of the units of C_q-1, which provide the logic of work according to the type "n, from n". The moduli/modules of the translation of counters are respectively equal to n₀-n and n₀. To null indicator signal is supplied when in this elementary section signal from the output of RPU proved to be below the threshold of quantization. In this case impulse/momentum/pulse of zeros is formed

by means of the inhibit circuit, on which are supplied the signals from the output of binary converter and the delayed clock impulses/momenta/pulses. At the output of UOS there appears the signal, which indicates the presence of target in this section of range, only when the C_4-1 will compute accurately n , ones, and counter is the C_4-0 less than $n-n$, zeroes. Otherwise the gate M will be closed and signal at the output will be absent.

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In diagram there are cross couplings of setting 0 for bringing diagram into initial state upon reaching/achievement by counters of assigned modulus/module of translation. The corresponding impulses/momenta/pulses of connection/communication are formed by differentiation of drops/jumps in the counter by circuits C_0R_0 .

Is possible diagram of consecutive detection of signal with quantized data, which has quantity of channels, equal to number of elementary sections, it is constantly and serviced/maintained by one device/equipment of detection (UOS), changed over in process of detection from one elementary section to another. Are possible other diagrams, for example parallel-series search. Both versions of such devices/equipment are described above in connection with nonquantized type signal.

One should, however, note that with small quantity of targets at distance not all channels of elementary sections of range prove to be begun to operate. Therefore, from the point of view of the more effective use of equipment this form of the parallel-series search, during which a quantity of channels and devices/equipment of detection is equal to a number of targets, here proves to be advantageous, but not to a quantity of elementary sections of range.

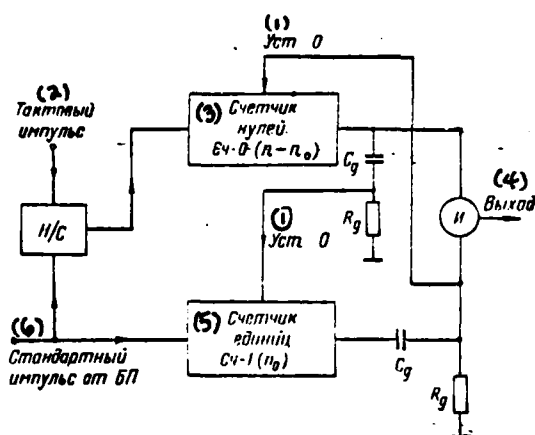


Fig. 4.31. The UOS with quantized data on counters.

Key: (1). Setting 0. (2). Clock impulse/momentum/pulse. (3). Null indicator. (4). Output. (5). Counter of ones. (6). Standard pulse from BP.

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In this case not all channels are marked, but are noted only those of them (elementary sections), on which occurred excess by output signal of threshold level of quantization. The work of the schematic of this device/equipment (Fig. 4.32) flows/occurs/lasts as follows. In the case of excess by the signal of the threshold level at the output of converter appears the standard impulse/momentum/pulse, which reads the present range of mark in the diagram of discrete/digital ranging (DID) and puts out result in the

form of binary number. The binary code of the read number through the buffer memory (BZU) is recorded in ZU, whence the codes of numbers, which characterize distance marks, enter detection unit, where they are compared between themselves after n of repetition periods and are divided/marked off on the groups, which have identical range. Then within each group a quantity of marks, which have identical range, is computed. If it exceeds the threshold number n , then this means that is fulfilled the criterion " n , from n " and is accepted the solution that at this range the echo signal (target) is discovered. Subsequently this distance mark is labeled and is accepted for the coordinate of the range of target.

Diagrams DID, special features of their work and methods of reading of range with increased accuracy are examined in § 4.2. It is necessary to only note that in the diagram, shown in Fig. 4.32, it is possible to utilize the devices/equipment, which make it possible to count off range not only for one, but also for many targets.

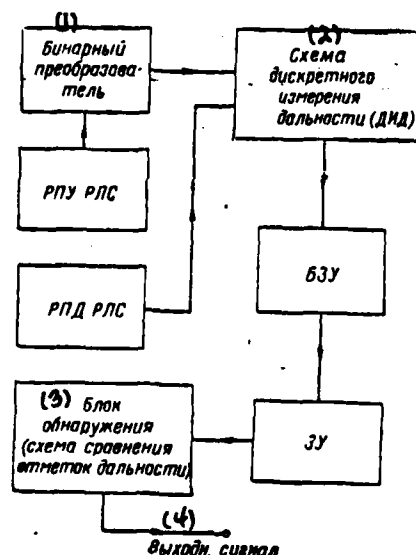


Fig. 4.32. Diagram of UOS with the quantity of channels, equal to a number of targets.

Key: (1). Binary converter. (2). Diagram of digital ranging (DID). (3). Detection unit (comparison circuit of range marks). (4). Output signal.

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The need of applying of BZU is one of the special features/peculiarities of the described diagram, which is caused by the nonuniformity of the admission of the readings of range from the diagram of DID. In the case of the closely spaced targets in the

range the readings can enter through a small time interval. Therefore BZU must possess large operating speed and relatively small capacitance, whose value is determined by a quantity of the predicted targets m_0 , situated simultaneously on one azimuth bearing. In the simplest case the BZU can be the totality of the trigger registers m_0 .

Quantity of binary digits, which characterizes range, is determined by accuracy of ranging ΔR and by interval of distances R , at which is made measurement. Taking into account that the maximum binary number, which can be recorded in γ_0 digits, will be $\underbrace{111 \dots 11}_{\gamma_0 \text{ раз}} = 2^{\gamma_0} - 1$, a quantity of digits for the designation of range can be determined from the condition

$$\frac{R}{\Delta R} < 2^{\gamma_0} - 1,$$

whence

$$\gamma_0 \geq \frac{\ln \left(\frac{R}{\Delta R} - 1 \right)}{\ln 2}.$$

Since to each digit one ferrite for memorization is required, for memorization of entire information, obtained during one pulse repetition period of RLS, is necessary $m_0 \gamma_0$ ferrites, and capacitance of one matrix of UOS $m_0 \gamma_0 = N_1$. On the basis of these reasonings it is possible to show that the necessary capacitance of one matrix/die of UOS in the diagram of parallel detection is equal to $\frac{R}{\Delta R} n = N_2$, then the relative value of capacity of ZU in both cases

$$\eta = \frac{N_1}{N_2} = \frac{m_0 \ln \left(\frac{R}{\Delta R} - 1 \right)}{\frac{R}{\Delta R}}.$$

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This relation, which shows how often capacity of ZU (quantity of ferrite cells) in parallel device/equipment of detection with constant number of channels more than with variable, depends on number m , and ratio $R/\Delta R$. With a large quantity of targets, which are located simultaneously on one azimuth, is more advantageous from the point of view of volume of ZU the diagram of detection with a constant number of channels, with small - vice versa. Number m_0 , with which becomes more advantageous the system with a variable number of channels, is found from condition $\eta \ll 1$, i.e., $N_1 < N_2$, whence

$$m_0 < \frac{\frac{R}{\Delta R}}{\ln \frac{R}{\Delta R}}.$$

Together with quantization to two levels systems with quantization by several levels are possible. Such systems more fully utilize the information, available in the echo signal; however, they are bulkier and more complicated.

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Chapter 5.

SIGNALS AND INTERFERENCES IN THEORY OF DETECTION.

Task of target detection with the aid of radio waves is reduced, as is known, to task of detection of that reflected by this target of signal against the background of different form of interferences. They solve this problem with the aid of the special devices/equipment, which realize the algorithms of the statistical theory of detection.

Starting point for determining algorithm of detection are functions of array of signal and interference, which characterize their statistical properties. It is natural that the form of these functions depends on the character of signal and interference, and also on the conditions for their interaction.

We will be restricted to examination of additive combination of signal and interference.

In general case signal aspect is complicated. Actually/really, the impulse/momentum/pulse of radar can have one or another the law of modulation, which changes its form and spectral composition. Furthermore, the value of impulse/momentum/pulse can change in the process of propagation because of the disturbance of propagation conditions in connection with appearance or change in the heterogeneity of tropospheric medium or in connection with other factors. Finally, with the reflection of impulse/momentum/pulse from the target in a number of cases the random changes in the intensity of signal, connected with the irregular oscillations/vibrations of echoing area of target, can appear.

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Reasons indicated, operating together or separately, lead to complication of structure of signal, causing respectively complication, and structure of formulas, which determine statistical characteristics of totality of signal and interferences.

Use of specific models of signal, which correspond to specific cases of radar detection, is here systematically justified. This model must reflect the most essential features of signal, the

characteristic and important from the point views of the work of the channel of the detection of one or the other radar device/equipment. Furthermore, model must so satisfy the requirements of known simplicity of its mathematical description, which simplifies the appropriate analysis.

Series/row of models of signal is given below and density function of distribution of their sum with fluctuating interferences are calculated.

S5.1. Harmonic signal with the constant amplitude.

This model of signal corresponds to simple detection of impulses/momenta/pulses, reflected from target with constant diffusing surface. Virtually this corresponds, for example, to the detection of surface targets in the calm sea surface, and also ground targets on the even surface in the absence of the fluctuating background of locality/terrain and in other cases of analogous character.

We will consider that echo pulse is section/segment of constant-amplitude sinusoid A and fixed/recorded phase ψ . Amplitude can be calculated on the basis of the equation of range (16] in the known parameters of RLS, target and radio-receiving channel.

Together with useful signal in channel of radar receiver are fluctuating interferences (noises).

These interferences appear in elements/cells of radio-receiving circuit and depend on different factors, for example by thermal electron motion in conductors and resistors/resistances, by shot effect in amplifier tubes, etc.

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Fluctuating interferences in the input computers of receiving circuit have the greatest value, since they are amplified by all subsequent cascades/stages.

Fluctuating type interferences can be induced also in antennas by electromagnetic vibrations, caused by cosmic radiation or motion of charges in surrounding terrestrial and outer space. The reasons indicated are nonremovable, that also explains the special position, which the fluctuating noises among the interferences of another type occupy.

Let us calculate probability density for amplitude and phases of

sum of two independent processes: harmonic signal $A \cos (\omega t + \psi)$ and normal noise.

Fluctuating noises relate to number of stationary type random processes, i.e., processes, which do not change their parameters in the course of time. This process, passing through the IF amplifier, causes random fluctuations in the band-pass amplifier. Actually/really, the separate jerks/impulses of noise process will swing the oscillatory system of amplifier. Since the jerks/impulses are irregular, signal at the output of UPCh will be oscillation/vibration with the random ones by amplitude and the phase, as a result of the fact that it is the result of the superposition of the reactions of the oscillatory system of amplifier for each of the jerks/impulses. Following V. I. Bunimovich [27], it is possible to represent the instantaneous value of noise output potential of narrow-band filter, i.e., narrow-band random process at the output of UPCh, in the form of the harmonic oscillation

$$u_m(t) = \xi \cos \omega t + \eta \sin \omega t, \quad (5.1.1)$$

where ξ and η - the independent random quantities, distributed according to the normal law with the zero averages $\bar{\xi} = \bar{\eta} = 0$ and by the dispersions

$$\sigma_{\xi} = \sigma_{\eta} = \sigma;$$

ω - the central frequency of tuning/adjusting of UPCh.

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Random noise indicated stores/adds up in channel of receiver to sine voltage of useful signal

$$u_c = A \cos(\omega t + \psi) = a \cos \omega t + b \sin \omega t,$$

where $a = A \cos \psi$; $b = A \sin \psi$ - nonrandom (constant) values; ψ - initial phase of received signal.

Let us examine case of imposition of narrow-band noise (5.1.1) on sine wave u_c and it is computed function of distribution of resulting signal.

Total oscillation/vibration takes form

$$(\xi + a) \cos \omega t + (\eta + b) \sin \omega t = x \cos \omega t + y \sin \omega t = \\ = \rho \cos(\omega t + \varphi),$$

where x and y have average/mean values, respectively equal to $\bar{x} = a$, $\bar{y} = b$. Since the random variables ξ and η are distributed according to the normal law

$$W(\xi) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{\xi^2}{2\sigma^2}}; \quad W(\eta) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{\eta^2}{2\sigma^2}}, \quad (5.1.2)$$

then the density function of the distribution of random vector with the independent components x and y in

$$W_1(x, y) = \frac{1}{2\pi\sigma^2} e^{-\frac{(x-a)^2 + (y-b)^2}{2\sigma^2}}. \quad (5.1.3)$$

Let us find now amplitude distribution (by envelope) ρ and phases φ :

$$\rho = \sqrt{x^2 + y^2}$$

and

$$\varphi = \operatorname{arctg} \frac{y}{x},$$

with this $x = \rho \cos \varphi$, and $y = \rho \sin \varphi$.

Conversion of random variables and transition/junction from X and u to ρ and φ can be considered as transition/junction from Cartesian to polar coordinate system.

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From theory of conversion of random processes and their functions of distribution [23] it follows that

$$W_2(\rho, \varphi) = W_1(x, y) I\left(\frac{x, y}{\rho, \varphi}\right), \quad (5.1.4)$$

where

$$I\left(\frac{x, y}{\rho, \varphi}\right) = \begin{vmatrix} \frac{\partial x}{\partial \rho} & \frac{\partial x}{\partial \varphi} \\ \frac{\partial y}{\partial \rho} & \frac{\partial y}{\partial \varphi} \end{vmatrix}$$

there is so-called jacobian of conversion, or Jacobian functional determinant. Differentiating, we will obtain

$$I\left(\frac{x, y}{\rho, \varphi}\right) = \begin{vmatrix} \cos \varphi & -\rho \sin \varphi \\ \sin \varphi & \rho \cos \varphi \end{vmatrix} = \rho \cos^2 \varphi + \rho \sin^2 \varphi = \rho. \quad (5.1.5)$$

Substituting (5.1.3) and (5.1.5) in (5.1.4), we will obtain

$$W_2(\rho, \varphi) = \frac{\rho}{2\pi\sigma^2} e^{-\frac{(x-a)^2 + (y-b)^2}{2\sigma^2}}$$

for $\rho > 0$ and $0 < \varphi < 2\pi$.

Substituting for x and y their values, we will obtain

$$W_2(\rho, \varphi) = \frac{\rho}{2\pi\sigma^2} e^{-\frac{(\rho \cos \varphi - a)^2 + (\rho \sin \varphi - b)^2}{2\sigma^2}} \quad (5.1.6)$$

Taking into account that $\sqrt{a^2 + b^2} = A$, where $a = S \cos \psi$, $b = A \sin \psi$ and $\varphi = \arctg b/a$ is respectively amplitudes phase of received signal, relationship/ratio (5.1.6) it is possible to reduce to following form:

$$W_2(\rho, \varphi) = \frac{\rho}{2\pi\sigma^2} e^{-\frac{\rho^2 + A^2}{2\sigma^2}} e^{\frac{\rho A}{\sigma^2} \cos(\varphi - \psi)} \quad (5.1.7)$$

Let us now move on to computation of one-dimensional distribution densities for ρ and φ :

$$\begin{aligned} W(\rho) &= \int_0^{2\pi} W_2(\rho, \varphi) d\varphi = \\ &= \frac{\rho}{2\pi\sigma^2} e^{-\frac{\rho^2 + A^2}{2\sigma^2}} \int_0^{2\pi} e^{-\frac{\rho A}{\sigma^2} \cos(\varphi - \psi)} d\varphi. \end{aligned} \quad (5.1.8)$$

FOOTNOTE ¹. For the computation of one-dimensional densities position known from the probability theory is used, which for obtaining the density of distribution of one of the values, entering the systems, is necessary the density of the distribution of system $[W, (\rho, \phi)]$ to integrate over all values of the argument, which corresponds to another random variable (see [47] §8.4). ENDFOOTNOTE.

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Integral in right side of equality (5.1.8) by replacement $\phi - \psi = u$ ($du = d\phi$) can be represented in the form of Bessel function zero order from imaginary argument:

$$\frac{1}{2\pi} \int_{-\psi}^{2\pi-\psi} e^{-j\left(\frac{JA\rho}{\sigma^2}\right) \cos u} du = I_0\left(\frac{JA\rho}{\sigma^2}\right) = I_0\left(\frac{A\rho}{\sigma^2}\right). \quad (5.1.9)$$

Substituting (5.1.8) in (5.1.7), we obtain unknown function of distribution of envelope:

$$W(\rho) = \frac{\rho}{\sigma^2} e^{-\frac{\rho^2 + A^2}{2\sigma^2}} I_0\left(\frac{A\rho}{\sigma^2}\right), \quad (5.1.10)$$

for $\rho > 0$

$$W(\rho) = 0; \quad \rho < 0.$$

This distribution is called generalized Rayleigh's law. Let us recall that here A and ρ make sense of amplitudes (envelopes) sinusoidal and respectively total (signal + noise) of signals, and σ

is rms (operating) value of noise. The curves of the functions of generalized distribution (5.1.10) for different values of λ/σ are represented in Fig. 5.1.

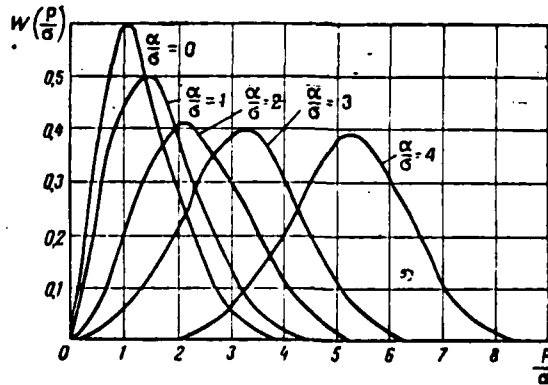


Fig. 5.1. Generalized function of Rayleigh distribution.

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It is important to note that case $A/\sigma=0$ corresponds to the law of distribution of pure/clean noise (Rayleigh distribution). As is evident, this curve ($A=0$) differs from distribution curves, which correspond to the mixture of signal with the noise. This difference increases with an increase in value A/σ . On a difference in these laws are based the contemporary methods of the detection of signal in the noises.

Let us find now density function of phase distribution:

$$W(\varphi) = \int_0^{\infty} W_2(\rho, \varphi) d\rho. \quad (5.1.11)$$

Substituting here (5.1.7) and introducing variable $x=\rho/\sigma$, we

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will obtain integral, which can be reduced to tabular:

$$W(\varphi) = \frac{1}{2\pi} \int_0^\infty \frac{\rho}{\sigma^2} e^{-\frac{\rho^2 + A^2}{2\sigma^2}} e^{\frac{\rho A}{\sigma^2} \cos(\varphi - \psi)} d\rho =$$

$$= \frac{1}{2\pi} e^{-\frac{A^2}{2\sigma^2}} \int_0^\infty x e^{-\mu x^2 - 2\nu x} dx,$$

where $\mu = 1/2$, $\nu = -A/2\sigma \cos(\varphi - \psi)$.

Utilizing tabular value of this integral (41, S3, 463, p. 5), finally we have

$$W(\varphi) =$$

$$= \frac{1}{2\pi} e^{-\frac{A^2}{2\sigma^2}} \left[1 + \frac{A}{\sigma} \sqrt{\frac{\pi}{2}} e^{\frac{A^2}{2\sigma^2} \cos^2(\varphi - \psi)} \cos(\varphi - \psi) \Phi(z_0) \right],$$

where probability integral

$$\Phi(z_0) = \frac{2}{\sqrt{\pi}} \int_0^{z_0} e^{-\frac{t^2}{2}} dt, \text{ a } z_0 = \frac{A}{\sigma \sqrt{2}} \cos(\varphi - \psi).$$

As is evident, with low signals, when $A/\sigma \approx 0$, phase distribution is uniform to

$$W(\varphi) \approx \frac{1}{2\pi}.$$

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§5.2. Harmonic signal with the random amplitude and the phase (fluctuating target).

If sizes/dimensions of target are great in comparison with wavelength of radar, then in number of cases fluctuations of echo signal as a result of change in target position and interference of waves, reflected from different sections of surface of target, appear.

This to a certain extent is characteristic, for example, for PT boat, rocket and aircraft, where sizes/dimensions of housing, fuselage and wings considerably exceed wavelengths of radar of centimeter band ¹. Therefore aircraft conditionally can be represented in the form of many separate reflectors, each of which gives reflection according to the type of the "bright points", the totality of reflections from which determines the resulting signal.

Since in process of flight aircraft always completes relatively rapid random fluctuations in three mutually perpendicular directions as a result of vibration of motors, yawings and pitch, separate aircraft components are moved, which causes chaotic changes in reciprocal location of "bright points" over surface of target. As a result the echo signal obtains the fluctuating (flickering) character.

If duration of these changes is less than pulse repetition period of radar, then echo pulses are independent random

fluctuations. For the separate impulse/momentum/pulse in the channel of IF amplifier there can be written following expression:

$$u_c = A \cos(\omega t + \epsilon) = \alpha \cos \omega t + \beta \sin \omega t, \quad (5.2.1)$$

where $\alpha = A \sin \epsilon$; $\beta = A \cos \epsilon$ - random variables, which we assume/set by those distributed normally with the average, equal to zero, and by dispersion $\sigma_\alpha = \sigma_\beta = \sigma_c$. This assumption can consider valid [75] during uniform phase distribution ϵ and by Rayleigh amplitude distribution A dispersion σ_c .

FOOTNOTE 1. The statistical characteristics of the radar signals, reflected from the aircraft, are given in [68]. These data are acquired on the basis of experiment. ENDFOOTNOTE.

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Probability densities of these values are distributed according to Gauss:

$$W(\alpha) = \frac{1}{\sqrt{2\pi} \sigma_c} e^{-\frac{\alpha^2}{2\sigma_c^2}}, \quad (5.2.2)$$

$$W(\beta) = \frac{1}{\sqrt{2\pi} \sigma_c} e^{-\frac{\beta^2}{2\sigma_c^2}}.$$

Fluctuating interference in channel of UPCh according to reasonings, given in preceding/previous paragraph, will take form,

analogous (5.1.1):

$$u_m(t) = \xi \cos \omega t + \eta \sin \omega t,$$

where ξ and η - random variables, distributed normally with dispersions σ and by averages, equal to zero.

Total oscillation/vibration for signal and interference in accordance with (5.2.1) and (5.1.1) will take form

$$(\xi + \alpha) \cos \omega t + (\eta + \beta) \sin \omega t = Z_1 \cos \omega t + Z_2 \sin \omega t.$$

Probability densities of total amplitudes Z_1 and Z_2 according to (5.1.3) and (5.2.2) will be:

$$\begin{aligned} W(Z_1) &= \frac{1}{\sqrt{2\pi(\sigma_c^2 + \sigma^2)}} e^{-\frac{Z_1^2}{2(\sigma_c^2 + \sigma^2)}}, \\ W(Z_2) &= \frac{1}{\sqrt{2\pi(\sigma_c^2 + \sigma^2)}} e^{-\frac{Z_2^2}{2(\sigma_c^2 + \sigma^2)}}. \end{aligned} \quad (5.2.3)$$

Then probability density of random vector with components Z_1 and Z_2 will be determined

$$W(Z_1, Z_2) = \frac{1}{2\pi(\sigma_c^2 + \sigma^2)} e^{-\frac{Z_1^2 + Z_2^2}{2(\sigma_c^2 + \sigma^2)}}. \quad (5.2.4)$$

Let us find now distribution for amplitude (by envelope) ρ and phases δ of resulting random process:

$$\begin{aligned}\rho &= \sqrt{Z_a^2 + Z_\beta^2}; \\ \delta &= \arctg \frac{Z_\beta}{Z_a}.\end{aligned}\quad (5.2.5)$$

In this case $Z_a = \rho \cos \delta$, $Z_\beta = \rho \sin \delta$.

Transition/junction to distribution functions for random variables ρ and δ will be carried out just as in §5.1:

$$W_2(\rho, \delta) = W(Z_a, Z_\beta) I\left(\frac{Z_a, Z_\beta}{\rho, \delta}\right). \quad (5.2.6)$$

Since the jacobian

$$I\left(\frac{Z_a, Z_\beta}{\rho, \delta}\right) = \begin{vmatrix} \cos \delta & -\rho \sin \delta \\ \sin \delta & \rho \cos \delta \end{vmatrix} = \rho, \quad (5.2.7)$$

then, substituting (5.2.4) (5.2.5) and (5.2.7) in (5.2.6), we will obtain

$$W_2(\rho, \delta) = \frac{\rho}{2\pi(\sigma_c^2 + \sigma^2)} e^{-\frac{\rho^2}{2(\sigma_c^2 + \sigma^2)}}. \quad (5.2.8)$$

Let us calculate now one-dimensional distributions

$$W(\rho) = \int_0^{2\pi} W_2(\rho, \delta) d\delta = \frac{\rho}{\sigma_c^2 + \sigma^2} e^{-\frac{\rho^2}{2(\sigma_c^2 + \sigma^2)}}. \quad (5.2.9)$$

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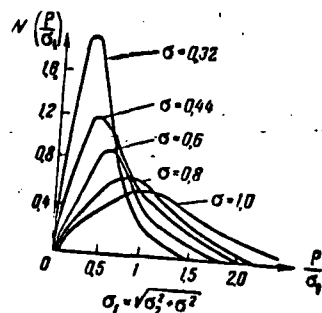


Fig. 5.2. Raleigh Distribution

This law is called law of Rayleigh distribution. With an accuracy to constant factor it coincides with "x²-distribution" with three degrees of freedom; therefore sometimes it they call also "x²-distribution" [23].

Graph/curve of distribution (5.2.9) is shown in Fig. 5.2.

Law of phase distribution taking into account (5.2.8)

$$W(\delta) = \int_0^{\infty} W_2(p, \delta) dp = \frac{1}{2\pi} \int_0^{\infty} x e^{-\frac{x^2}{2}} dx = \frac{1}{2\pi}, \quad (5.2.10)$$

where variable of integration

$$x = \sqrt{\frac{p}{\sigma_c^2 + \sigma^2}}.$$

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Chapter 6.

DETECTION OF SIGNAL IN THE PRESENCE OF NOISE.

S6.1. Basic condition/positions of the statistical theory of the detection of radar signals.

In preceding/previous sections qualitative description of work of diagrams of processing signals at output of receiving channel of RLS for their automated detection in presence of interferences was given. In this case the diagrams for the systems with the constant and variable/alternating repetition frequency were examined. In both cases the detection was conducted in essence as a result of the accumulation of signal in the separate elementary sections of range. This escaped/ensued from the special feature/peculiarity of radar detection, which consists in the fact that the useful signal has pulse character and short duration, whereas noise interference, on the contrary, operates at the output continuously, and therefore with the accumulation it is necessary to select the section, occupied by useful signal, in the time.

Depending on constancy of pulse repetition frequency of radar selection is realized by different methods. In the stations with the variable/alternating repetition frequency this is achieved by strobing/gating the elementary sections of range.

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In the stations with the constant frequency are utilized the devices/equipment of constant delay (memory units on the ultrasonic lines, the magnetic drums, etc.), with the aid of which is achieved sufficiently clear separation in the time of the results of accumulation, obtained in the individual sections of range.

Duration Δr of elementary section of range must be matched with passband of radio- locating channel Δf and duration of pulse τ . For the best separation in the time of signal and interference it is desirable to select the duration of elementary section small. However, it cannot be selected less than the duration of pulse τ . Hence it follows that value τ must be of the same order as as the pulse duration. Laying aside the presentation of full weight of considerations, that determine the selection of the optimum duration of the elementary section of range, we will assume that its extent composes 1.2-1.5 pulse durations. In this case it is assumed that the pulse duration is matched with the passband of radar channel, i.e.,

$$\tau = 1/\Delta f.$$

On this foundation, without detailing shape of pulse, it is possible to assume that fixation of presence of useful signal or its absence on interval $\Delta\tau$ can be conducted on one reading at any point of this interval. The use of reading values makes it possible to effectively use the developed discrete-selective apparatus of mathematical statistics and to uniform describe useful signals and interferences. In any case in radars both with the constant and with the variable/alternating repetition frequency methods indicated above make it possible to be concentrated in the elementary time interval, where useful signal can be located.

Task of radar detection can be formulated as follows. There is a gate/strobe, which fixes the elementary section of range (ring of range for surveillance radar). Within the section is observed the totality of signal and interference or only interference. It is necessary according to observations of the realization of these random signals to make a decision about presence or absence of useful signal in this section. For this the observational data must be processed optimally from the point of view of the selected criteria of the quality of detection.

The determination of the structure of optimum working/treatment, i.e., the determination of the optimum algorithms of the machines of primary working/treatment, is the object/subject of the following presentation of present chapter. The investigation of these questions is conducted on the base of the theory of the statistical solutions¹, which is occupied by investigation, comparison and finding of the best methods of accepting similar solutions.

However, before passing to determination of optimum algorithms, it is necessary to formulate figures of merit of radar detection, utilized in theory of statistical solutions.

A. Probabilistic quality coefficients of radar detection.

As a result of random character of interferences and signals analysis of work of device/equipment of detection is carried out by methods of statistics, and figures of merit of its work are statistical parameters.

As a result of processing signal in schematic of device/equipment of detection must be obtained solution about presence or absence of useful signal in this elementary section of

range. As the useful signal is understood one impulse/momentum/pulse or the burst of pulses, reflected from the target. This solution objectively is accepted under two mutually eliminating conditions, which characterize the state of the target:

1. "Is target" - H_1 ;
2. "No target" - H_0 . (6.1.1).

As a result of presence of interferences and fluctuations of signal solutions can be accepted with errors, since making decisions condition (6.1.1) remains unknown. In this case by them can be only expressed separate hypotheses (hypothesis, i.e., the assumed solutions, will designate by asterisk), namely:

1. "Is target" - H_1^* ;
2. "No target" - H_0^* .

FOOTNOTE ¹. By term "statistical solutions" are understood the solutions, taken on the base of the observation of certain totality of random variables or realizations of random process. ENDFOOTNOTE.

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Other hypotheses are not given, since solution of assumed plan/layout ("do not know", "apparently", "is possible" and the like) they are eliminated from examination.

As a result final diagram of solutions takes form:

(1) Состояние цели	(2) Возможные решения	(3) Наименование случая
(4) H_1 есть цель	(5) H_1^* (есть цель)	(6) Правильное обнаружение
	(7) H_0^* (нет цели)	(7) Пропуск цели
(8) H_0 нет цели	(9) H_1^* (есть цель)	(8) Ложная тревога
	(9) H_0^* (нет цели)	(9) Правильное необнаружение

Key: (1). State of target. (2). Possible solutions. (3). Designation of case. (4)-(5). there is a circuit. (6). Correct detection. (7). Passage of target. (8). False alarm. (8). Correct nondetection.

As is evident, are here possible erroneous and correct solutions.

Erroneous solutions appear in two cases - absences and presence of target respectively are called:

1. The false alarm, when accepted the solution about the presence of target, while actually target is absent.

2. Passage of target, when accepted solution about absence of target, while in actuality target occurred.

Two forms of correct solutions correspond also to presence and absence of target and respectively they are called:

3. The correct detection, when decision about the presence of target in the presence its actual is made.

4. Correct nondetection, when accepted solution about absence of target, while in actuality target was absent.

Each of these cases is characterized by unconditional probability, which in accordance with theorem of compound probability can be represented in the following form:

Unconditional probability of correct detection

$$p(H_1^*, H_1) = p(H_1) p(H_1^*/H_1). \quad (6.1.2a)$$

Unconditional probability of false alarm

$$p(H_1^*, H_0) = p(H_0) p(H_1^*/H_0). \quad (6.1.2b)$$

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Unconditional probability of correct nondetection

$$p(H_0^*, H_0) = p(H_0) p(H_0^*/H_0). \quad (6.1.2B)$$

Unconditional probability of passage of target

$$p(H_0^*, H_1) = p(H_1) p(H_0^*/H_1). \quad (6.1.2r)$$

Here $p(H_1)$ and $p(H_0)$ - the a priori probabilities of presence and absence of target (signal), and $p(H_1^*/H_1)$; $p(H_1^*/H_0)$ $p(H_0^*/H_0)$ and $p(H_0^*/H_1)$ - the corresponding conditional probabilities, calculated under the assumption of the actual presence or absence of signal. These probabilities are called a posteriori.

Determination of unconditional probabilities can be fulfilled, if a priori and a posteriori probabilities are known.

A priori probabilities $p(H_1)$ and $p(H_0)$ can be determined according to known preliminary data about location of target. If this is not possible to do, then the so-called a priori difficulty, which frequently occurs in the radar, appears; in this case it is possible, apparently, to count absence and presence of signal equally probable

$$p(H_1) = p(H_0) = 1/2.$$

A posteriori probabilities can be calculated for threshold acquisition systems, if are known to density function distributions of conditional probabilities of process being investigated.

Let $W_w(x)$ and $W_{cm}(x)$ be to density function of probability distribution respectively for noise and for mixture signal + noise, where X - argument of distribution function. As is known, this argument is connected with the realizations of random variable ξ with inequality $\xi \leq x$. Subsequently we will frequently apply one and the same designation for the random variable and for its realization, if this does not cause any misunderstanding¹.

Frequently by X is understood standardized/normalized value of random signal u_c (at output of receiving channel of radar) with respect to average noise σ , i.e., $X = \frac{u_c}{\sigma}$.

FOOTNOTE ¹. In the subsequent sections the realization of random variables and the argument of their distribution functions are frequently designated by one and the same letter ρ . ENDFOOTNOTE.

The usual form of curves for distribution functions indicated above is given in Fig. 6.1. Let x_0 be relative value of the threshold of the accumulation ($x_0 = \mu_0 / \sigma$, where μ_0 - absolute value of threshold), then the a posteriori probabilities (conditional probabilities), which characterize the quality of radar detection, they can be found from the following expressions.

1. Probability of correct detection is area, limited by section of curve $W_{cm}(x)$ and by axis of abscissas, arranged/located of more to the right threshold line x_0 , i.e.

$$p(H_1^*/H_1) = D = \int_{x_0}^{\infty} W_{cm}(x) dx. \quad (6.1.3)$$

2. Probability of passage of target $\bar{D}$ to eat area, which is located under curve $W_{cm}(x)$, arranged/located more left threshold line x_0 , i.e.

$$p(H_0^*/H_1) = \bar{D} = \int_0^{x_0} W_{cm}(x) dx. \quad (6.1.4)$$

3. Probability of false alarm F is characterized by area, arranged/located under section of curve $W_w(x)$, arranged/located more to the right of threshold x_0 :

$$p(H_1^*/H_0) = F = \int_{x_0}^{\infty} W_w(x) dx. \quad (6.1.5)$$

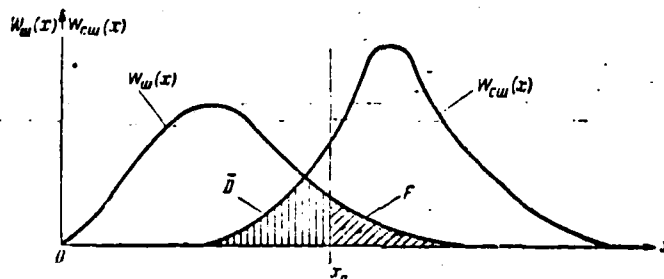


Fig. 6.1. Function of probability density.

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4. Probability of correct nondetection $\bar{F}$ is characterized by area, arranged/located under density function of distribution $W_w(x)$ it is more left than threshold of limitation:

$$p(H_0^*/H_0) = \bar{F} = \int_0^{x_0} W_w(x) dx. \quad (6.1.6)$$

Taking into account that area, limited by entire density function of distribution $W_w(x)$ or $W_{cw}(x)$ is equal to one, since it characterizes certain event, we have obvious relationships/ratios:

$$\begin{aligned} D + \bar{D} &= 1, \\ F + \bar{F} &= 1. \end{aligned} \quad (6.1.7)$$

As is evident, of four parameters, which characterize detecting properties UOS, independent are only two D and F ; $\bar{D}$ and $\bar{F}$ and the like. Any pair of these values can be accepted as the initial characteristic of the figures of merit of radar feeler.

B. Selection of the criteria of the quality of detection D and F (from the position of average/mean risk).

Selection of optimum figures of merit of detection D and F is contradictory task, which can be solved only on base of reasonable compromise. Actually/really, for the correct work of the channel of the device/equipment of the detection of signal (UOS) natural is the great probability of the correct detection D and a small probability of the false alarm F. However, to satisfy simultaneously both these requirements during the threshold testing is impossible, since for increase in D (6.1.3) it is necessary to decrease the threshold value, while for decreasing of F (6.1.5) the threshold must be increased.

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Thus, it is necessary to go to reasonable compromise, selecting optimum mode/conditions of detector from point of view of entire totality of possible conditions for working/treatment. It is natural that solution obtained in this case of the mode/conditions of working/treatment will be not necessarily best from the point of view of any particular condition, for example the maximum of the

probability of the correct detection or minimum of false alarm.

Solution adopted must be optimum on the average for all possible conditions for work of detector taking into account statistics of distribution, possible conditions of detection and importance of one or the other situation.

For determining character processings of signal in similar cases use method of average/mean risk. Let us examine the short description of this method, explaining by the examples, borrowed from the radar. In more detail these questions are illuminated both in the Soviet monographic and periodical literature [17, 28, 48-52, etc.] and in the foreign [15, 16, 25, 62, 70, etc.].

Entire/all totality of situations of radar detection has following cases: correct detection, passage of target, false alarm and correct nondetection.

Each of these situations from quantitative side is characterized by their unconditional probability (6.1.2).

For each situation is introduced concept of value and they set in conformity certain board z and error, which depends on importance or cost/value of permissible error in one or the other situation.

Thus, for instance, in certain cases board for the passage of target can be considerably higher than for the false alarm. Actually/really, the result of the first case can be air raid and the breakdown of object, while in the second case only the anxiety of personnel of the guarded object, which usually is not connected with the considerable material expenditures.

Determination and designation/purpose of cost/value of errors are critical stage of analysis and must consider entire totality of considerations about that, to what extent one or another form of error is undesirable. It is clear that the correct detection and correct nondetection will have the zero value of cost/value, i.e., the error-free solutions have zero board.

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In this case it is important that the value of the error of any incorrect solution would be more than correct, i.e., the high cost/value of error corresponds to large error. Then an average/mean risk of each situation or a quantity of cost/value, "paid out" in each case, it is proportional to the probability of the appearance of this situation p_i and cost/value of the error of this situation r_i :

$$q_i = r_i p_i.$$

Actually, this is the loss, paid out for the risk to make a decision in the i situation.

Average/mean risk of totality p of situations is calculated from rules of mathematical expectation and for discrete/digital cases is equal to sum of average/mean losses, obtained from separate situations:

$$R = \sum_{i=1}^n q_i = \sum_{i=1}^n r_i p_i.$$

During analysis of process of detection it suffices to assign altogether only two costs/values: cost/value of error of false alarm

$$r_F = r(H_1^*, H_0)$$

and cost/value of error of passage of target

$$r_{\bar{D}} = r(H_0^*, H_1).$$

since cost/value of "errors" of correct detection and cost/value of correct nondetection are equal to zero. Then in the expression for the average/mean risk will be present only the terms, which characterize the erroneous detection:

$$R = r_F(H_1^*, H_0)p(H_1^*, H_0) + r_{\bar{D}}(H_0^*, H_1)p(H_0^*, H_1).$$

Taking into account (6.1.2) (6.1.4) and (6.1.5) this equality can be rewritten in the form

$$R = r_F F p(H_0) + r_{\bar{D}} \bar{D} p(H_1). \quad (6.1.8)$$

FOOTNOTE ². When $r_i = 0$ (error-free situation), and also when $\dot{p}_i = 0$ (improbable situation) $q_i = 0$. ENDFOOTNOTE.

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Comparing different systems of working/treatment, should be given preference this, for which average/mean risk is less. Consequently, optimum conditions and the parameters of feeler should be found through the criterion of the minimum of average/mean risk, i.e., from the condition of the minimum of expression (6.1.8).

In particular, if we consider that board for situation of false alarm and passage of target is identical and equal to one, then average/mean risk will be equal to sum of probabilities of errors of detection:

$$R = Fp(H_0) + \bar{D}p(H_1). \quad (6.1.9)$$

Minimum of this probability is called criterion of ideal observer. It is obvious that the criterion of the minimization of average/mean risk is more general/more common/more total criterion in the comparison with the criterion of ideal observer, since it considers difference in the cost/value of the errors of false alarm and passage of target.

Substituting in (6.1.8) $\bar{D}=1-D$, we will obtain

$$R = r_{\bar{D}} p(H_1) - r_{\bar{D}} D p(H_1) + r_F F p(H_0).$$

By grouping latter/last two components/terms/addends and after carrying $r_{\bar{D}} p(H_1)$ for bracket, we will obtain

$$R = r_{\bar{D}} p(H_1) - r_{\bar{D}} p(H_1) [D - l_0 F], \quad (6.1.10)$$

where

$$l_0 = \frac{r_F p(H_0)}{r_{\bar{D}} p(H_1)}. \quad (6.1.10')$$

Since first term is positive, criterion of minimum of average/mean risk is reduced to criterion

$$D - l_0 F = \sup(\max), \quad (6.1.11)$$

which is called criterion of weighed combination. This criterion requires such improvement in the conditional probability of the correct detection D and the decrease in the conditional probability of the false alarm F , with which increases the weighed difference $D - l_0 F$. Factor l_0 is called weight factor.

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It depends on the a priori probabilities of presence or absence of target in the section of space and cost/value of the errors of each form being investigated. We consider all these values given.

If two systems of working/treatment of detection under identical conditions (identical l_0) are compared, then optimum of them there will be that, for which

$$D_{\text{opt}} - l_0 F_{\text{opt}} \geq (D' - l_0 F').$$

Then

$$D_{\text{opt}} \geq D' + l_0 (F_{\text{opt}} - F')$$

and when

$$\begin{aligned} F' &= F_{\text{opt}}, \\ D_{\text{opt}} &\geq D'. \end{aligned}$$

This means that during fixing of false alarm F optimum system there will be that, for which probability of correct detection D will be greatest. This condition in the literature [15-17, 25, etc.] is called Neumann-Pearson criterion on the name of two well-known mathematicians, specialists in the region of mathematical statistics.

For obtaining concrete/specific/actual values D and F it is necessary to assign value of average/mean risk R , and also value of l_0 and measure of cost of errors r_D and r_F .

In this case

$$D - l_0 F = \frac{r_D p(H_1) - R}{r_D p(H_1)}.$$

In radar values of a priori probabilities $p(H_0)$ and $p(H_1)$, as a

rule, are unknown. Their determination composes the so-called a priori difficulty, since in the majority of the cases there is no possibility to determine the probability of the appearance of a target (ship, aircraft) in the area of the detection of station at one or another moment of time. With the identical success it is possible to express itself in favor of one or the other hypothesis, i.e., in favor of presence or absence of target. Therefore it is possible to consider that both hypotheses are equally probable, and to suppose that $p(H_0) = p(H_1) = 0.5$.

So with difficulty sometimes it is to rate/estimate board for one or the other situation.

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Actual/real, as to rate/estimate the losses, connected with the false alarm and the passage of target? How to rate/estimate quantitatively the results of panic among the population, caused by false alarm, or blunting alertness in personnel of defense system as a result of the false alarm? Finally, in what units it is possible to measure human victims and the destruction, caused by the passage of target to the object? The absence in a number of cases of precise data about the cost/value of losses leads to the need for assuming/setting

$$r_D = 1 \quad \text{and} \quad r_F = 1.$$

Under these conditions $l_s = 1$ and then

$$D - F = \frac{0.5 - R}{0.5}.$$

This relationship/ratio between D and F can be applied for estimate calculations.

C. Likelihood ratio and the structure of optimum feeler.

Earlier in chapter 4 are examined some schematics of devices/equipment of threshold type detection. These were different kind of the diagrams, which realize accumulation of signals after n of adjacent repetition periods. In this case the criterion of the detection (frequently the very same it was the criterion of the beginning of packet) required the presence of not less than m signal-impulses/momenta/pulses on n adjacent positions.

It is necessary to note that diagrams examined above of detection and selected criteria were to a certain degree intuitive and they were accepted without any substantiation. In this case,

naturally, arises the question: they are the diagrams of processing signal examined best of the possible ones and do utilize they in this case full weight of information, available in the input signals. In connection with this the correctness of those accepted of above the criteria of detection can be subjected to doubt also.

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In fact, if criterion or diagram are selected insufficiently soundly, then always it is possible to expect that there are other optimum methods, which more fully utilize the information, available in the input signal, and will give the best result. In this case, naturally, the diagram of working/treatment is the consequence of the criterion, which must be obtained on the base of the very general/common/total reasonings, which decide the task of detection optimally from all, frequently contradictory ones, the points of view.

Number of such total criteria includes criterion of minimum of average/mean risk of losses, examined in preceding/previous section. As it follows from the reasonings given there, the criterion of the minimum of average/mean risk is reduced to the maximization of the criterion of weighed combination (6.1.11)

$$D - I_0 F = \sup (\max).$$

Substituting D and F from (6.1.3) and (6.1.5) and taking into account that integration for D and F is conducted in identical limits, we will obtain

$$\int_{x_0}^{\infty} [W_{cw}(x) - l_0 W_m(x)] dx = \max.$$

So that integral would be maximum, it is necessary to relate to range of integration only those values X, for which integrand was not negative, i.e.

$$W_{cw}(x) - l_0 W_m(x) > 0, \quad (6.1.12)$$

or

$$\frac{W_{cw}(x)}{W_m(x)} = \lambda > l_0.$$

Value λ is called likelihood ratio or coefficient of plausibility.

It must be noted that probabilities D and F in accordance with formulas (6.1.3) and (6.1.5) depend on threshold U. Hence it follows that D and F is also interdependent. The dependence between the probabilities of correct detection and false alarm is frequently called the working characteristic of feeler (performance characteristic of receiver RKhP). This characteristic is constructed in coordinates D and F and characterizes likelihood ratio.

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Since the derivative of integral on the lower limit is equal to the integrand, undertaken with the opposite sign, from (6.1.3) and (6.1.5) it follows that

$$W_{cu}(x_0) = - \frac{dD}{dx_0}$$

and

$$W_u(x_0) = - \frac{dF}{dx_0};$$

$$\left(\frac{dD}{dF} \right)_{x=x_0} = \frac{W_{cu}(x_0)}{W_u(x_0)} = \lambda_0,$$

where λ_0 - the threshold quantity of the coefficients of plausibility. Thus, at any point of the characteristic of feeler rate of change is numerically equal to the value of the threshold coefficient of plausibility.

Performance characteristic passes through two characteristic points ($D=0$; $F=0$) with $\lambda_0=\infty$ and ($D=1$; $F=1$) with $\lambda_0=0$. In the first case the value of threshold λ_0 is selected infinite and neither signal plus noise nor pure/clean noise can exceed it. In the second case the threshold is absent and any signal can be accepted as the useful.

Returning to (6.1.12), it is important to note that optimum detection is reduced to computation of likelihood ratio λ and its comparison with threshold λ_0 . Thus, into optimum detection enters

also the operation/process of comparison on the threshold.

For computing likelihood ratio it is necessary to have available expressions of functions of distribution of mixture adopted signal plus noise also of pure/clean noise, and also with selective values of realizations of received signal. The character of signals is very complicated and diverse; therefore the distribution functions to more conveniently calculate for the simplified statistical models of signal, which contain the most essential features of the behavior of impulse/momentum/pulse, reflected from one or the other targets. Some models of such signals with the distribution functions calculated for them are examined in chapter 5.

Structure of processing signal for making of decision about detection is reduced to computational operation/process in accordance with formula (6.1.12).

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This formula can be considered as initial expression for the composition of the algorithm of the computations, which must be conducted above the realizations of output signal in order to reveal/detect the echo pulse. Detector itself must be nothing else but specialized type computer, which fulfills computations in

accordance with the formula indicated.

Depending on character of model of radar signal and form of the function of distributions algorithms of computations, which subsequently we will call algorithms of working/treatment, can be different degree of complexity. The procedure of obtaining such algorithms for the concrete/specific/actual type of the models of signals and the determination of the possible structures of processing compose the subsequent presentation of present chapter.

D. Criteria of detection during the repeated tests.

Target detection by radars is conducted, as a rule, as a result of reception/procedure and processing of totality of several echo signals, which compose packet of radar impulses/momenta/pulses. Target detection on the single impulse/momentum/pulse in the limited signal-to-noise ratio the in practice scarcely probable case. A quantity of impulses/momenta/pulses in packet n corresponds to a number of observed values of random variable, obtained in the elementary sections of range after n of the adjacent positions of scanning/sweep, i.e., detection in this case is conducted according to observations n of values of random variable. Obtained as a result of these tests discrete/digital selection with space n is the multidimensional random variable, characterized by the

multidimensional distribution functions. The case of detection during the repeated tests is most general/most common/most total; therefore let us examine its fundamental, characteristic relationships/ratios.

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Let n -dimensional density of random variable X be

$$W_n(X) = W_n(x_1, x_2, \dots, x_n | \epsilon),$$

where $x_1, x_2, \dots, x_n$ - selective values of random variable X in specific interval of observation;

ϵ - parameter, which characterizes distribution (signal amplitude, phase, etc.).

We will consider that in presence of signal $\epsilon=c$, and in the absence $\epsilon=0$. The distributions corresponding to density for these cases will be:

$$W_{ncw}(x_1, x_2, \dots, x_n | \epsilon), \quad (6.1.13)$$

$$W_{nw}(x_1, x_2, \dots, x_n | 0).$$

Testing hypotheses about presence of signal (hypothesis H_1) and its absence (hypothesis H_0) requires, as before separations are now narrower than n -dimensional space of observations Γ to two regions Γ_1 and Γ_0 , the first of which corresponds to hypothesis H_1 , and the

second - to hypothesis H_0 .

Respectively figures of merit of detection F , D , $\bar{D}$, $\bar{F}$ in this case will take form:

probability of false alarm

$$F = \int \dots \int_{\Gamma_1} W_{nm}(x_1, x_2, \dots, x_n|_0) dx_1 dx_2 \dots dx_n \quad (6.1.14)$$

probability of correct detection

$$D = \int \dots \int_{\Gamma_1} W_{ncw}(x_1, x_2, \dots, x_n|_c) dx_1 dx_2 \dots dx_n \quad (6.1.15)$$

probability of passage of target

$$\bar{D} = \int \dots \int_{\Gamma_1} W_{ncw}(x_1, x_2, \dots, x_n|_c) dx_1 dx_2 \dots dx_n \quad (6.1.16)$$

probability of correct nondetection

$$\bar{F} = \int \dots \int_{\Gamma_1} W_{nm}(x_1, x_2, \dots, x_n|_0) dx_1 dx_2 \dots dx_n \quad (6.1.17)$$

If parameter ϵ is random variable, then it is necessary distribution function to average also from this parameter.

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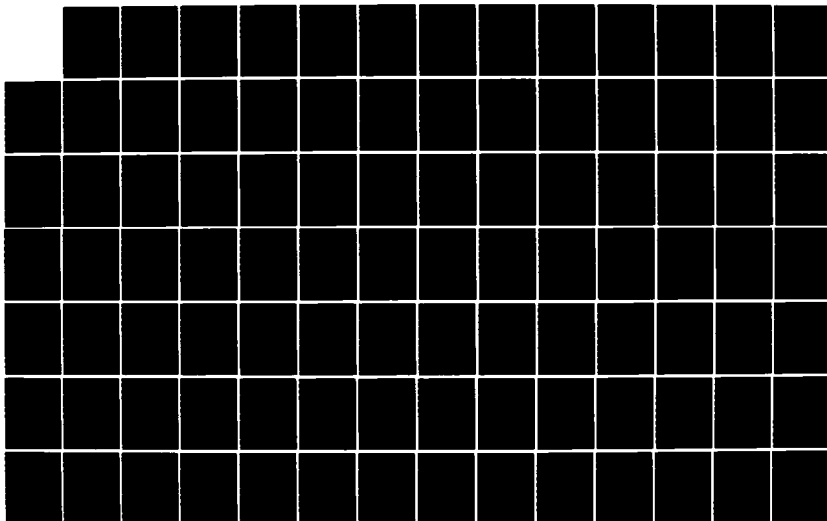
DIGITAL METHODS OF THE OPTIMUM PROCESSING OF RADAR
SIGNALS(U) FOREIGN TECHNOLOGY DIV WRIGHT-PATTERSON AFB
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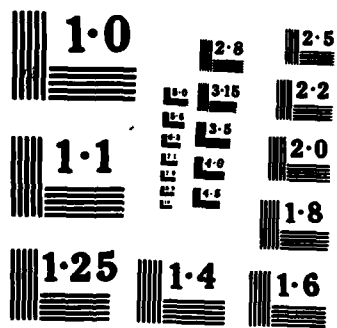
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During separation of region Γ on region Γ_1 and Γ_2 , it is necessary to use such rules, which provide minimum of average/mean risk of losses, obtained as a result of incorrect solution under assigned conditions. These rules are, as is known, the criteria: ideal observer, Neumann-Pearson and the most general/most common/most total criterion of weighed combination (6.1.11), which, as is known, leads to the criterion of likelihood ratio (6.1.12). In the case of the multidimensional distribution functions this criterion, obtained on the basis of analogous reasonings, can be recorded in the following form:

$$\frac{W_{ncw}(x_1, x_2, \dots, x_n|c)}{W_{nw}(x_1, x_2, \dots, x_n|0)} > l_0 \quad (6.1.18)$$

It is necessary to note that in number of practical cases there is no correlation between selective values of signal, obtained in elementary sections of range in different pulse repetition periods. This makes it possible to simplify distributions. Actually/really, in this case the function of distribution of a sample with space n , which consists of the mixture signal plus noise, it will be determined by the product of the distribution functions, which

characterize random variable in each elementary section:

$$W_{ncm} = \prod_{i=1}^H W_{cm_i}(x_i). \quad (6.1.19)$$

It is analogous for n-dimensional selection, which consists only of pure/clean noise:

$$W_{nm} = \prod_{i=1}^H W_{m_i}(x_i), \quad (6.1.20)$$

where x_i - selective values of signal in i -st elementary section.

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The probability densities of selections in the mathematical statistics are called the functions of the plausibility of the selections, which subsequently for the brevity we will call simply the functions of plausibility. The relation of the function of the plausibility of distribution $W_{ncm}(X)$ and $W_{nm}(X)$ is called likelihood ratio or coefficient of the plausibility

$$\lambda = \prod_{i=1}^n \lambda_i, \quad (6.1.21)$$

where

$$\lambda_i = \frac{W_{cm_i}(x_i)}{W_{m_i}(x_i)};$$

in this case the criterion of detection will be determined by the inequality

$$\lambda > l_0, \quad (6.1.22)$$

If the realizations of random signals in different selections are

obtained under the identical conditions, i.e., in the identical parameters of signal and interference, then their distribution functions are identical

$$W_{cw_1} = W_{cw_2} = \dots = W_{cw_n};$$

$$W_{w_1} = W_{w_2} = \dots = W_{w_n}.$$

In this case functions of plausibility for mixture signal and noise and for pure/clean noise will be recorded in the form

$$W_{ncw} = W_{cn}^n(x); \quad (6.1.23)$$

$$W_{nw} = W_{wn}^n(x),$$

coefficient of plausibility

$$\lambda = \left(\frac{W_{cn}^n(x)}{W_{wn}^n(x)} \right)^n, \quad (6.1.24)$$

and inequality for criterion of detection will coincide with (6.1.22).

Let us note that functions of plausibility, calculated for multidimensional case according to formulas (6.1.23) and (6.1.24), in comparison with one-dimensional case take form of curve with more sharply pronounced maximum, in this case sharpness of maximum will be higher, the greater size of sample n .

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Virtually this means that the inequality, which corresponds to the

criterion of the weighed combination, will be satisfied with the smaller thresholds of accumulation x . The decrease of threshold leads to an improvement in the sensitivity of receiver-amplifier circuit. This is reached due to an increase in the time of the detection of signal by using the totality of actions from n of the echo pulses.

Increase in sample size and change connected with it in form of the function of plausibility is especially important for feeler, which works according to Neumann-Pearson criterion, since in this case possibility to lower threshold of accumulation appears and - to raise sensitivity of circuit.

Let us examine briefly, how can be at least in principle realized automatic detection with the aid of computer(s), which works in accordance with criteria examined above of detection, and let us point out, electronic computer, which works in this feeler, must perform what operations/processes.

Let feeler totality n of realizations of random process from output of receiving channel of radar, undertaken in one and the same sections of range through intervals, equal to repetition period of impulses/momenta/pulses, enter. It is necessary to determine, belongs this selection to the noise or to the mixture signal plus noise. For

this the following operations/processes are fulfilled. Each selective value of signal x_i supplies on the converter of the type "voltage - the code", at output of which is formed the code of the number, which characterizes the analyzed parameter of the process in question: amplitude, phase, etc. The obtained number enters the electronic computer of feeler, which must fulfill the following operations/processes:

1. To calculate the values of functions $W_{cm}(x_i)$ and $W_{m}(x_i)$ for each x_i . The form of functions themselves is known or must be assigned.

2. To divide $W_{cm}(x_i)$ on $W_{m}(x_i)$ and to determine thereby coefficient of plausibility

$$\lambda_i = \frac{W_{cm}(x_i)}{W_{m}(x_i)}.$$

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3. To multiply n of times λ_i to calculate product

$$\lambda = \prod_{i=1}^n \lambda_i$$

which is relative to bulky operation/process for summing computers.

4. To compare obtained product with threshold in accordance with (6.1.22), as a result of which solution about presence of signal is

accepted. The value of threshold λ_0 is determined previously on the basis of relationship/ratio (6.1.10') on the assigned costs/values of erroneous solutions and the a priori probabilities.

From given enumeration it is evident that for acceptance by feeler of solution about presence of signal it is necessary to fulfill major cycle of computational works. Especially labor-consuming are the computations, indicated in paragraphs 2 and 3, i.e., division and multiplication, which are bulkier operations/processes than addition or subtraction.

By introduction of corresponding conversions, in particular logarithmic conversion of coefficient of plausibility, it is possible to decrease volume of computational works, after replacing repeated product with appropriate addition. Actually/really, taking the logarithm of both parts of inequality (6.1.22), we will obtain the criterion of detection in the form

where

$$\sum_{i=1}^n \ln \lambda_i > c_0 \quad (6.1.25)$$
$$c_0 = \ln l_0.$$

As is evident, here bulky operation/process of multiplication is replaced by addition, in this case only changes threshold value λ_0 for $\ln \lambda_0$.

Further conversions of inequality (6.1.25) for concrete/specific/actual distribution functions and respectively for concrete/specific/actual models radar of signal make it possible to conduct subsequent simplifications in computational operations/processes, after bringing together them to complex of actions, which has simplest technical realization. The latter is expedient to examine based on example of specific cases.

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§6.2. Optimum detection of incoherent impulses/momenta/pulses with the constant amplitude.

A. Structure of optimum feeler.

Structure optimum of feeler, as is known, must be selected on the basis of general/common/total criteria, which determine task of detection. As this criterion the criterion of the minimum of average/mean risk, examined in the preceding/previous paragraph, can be accepted. We will call the device/equipment of detection, which works on the base of the use of this criterion, subsequently optimum Alni device/equipment, since it determines the best positive result,

realized in the tasks of detection. The criterion indicated, as is known, is reduced to the comparison of likelihood ratio with the threshold l_0 . For the packet from n uncorrelated impulses/momenta/pulses this condition will be recorded in the form

$$\prod_{i=1}^n \frac{W_{cm}(\rho_i)}{W_w(\rho_i)} > l_0 \quad (6.2.1)$$

where $W_{cm}(\rho_i)$ and $W_w(\rho_i)$ - function of the distribution of the random processes of mixture signal + the noise also of pure/clean noise;

ρ_i - the arguments of the function of the distribution of the i signal of selection - selective values of output signal¹;

l_0 - threshold, depending on a priori probabilities and cost/value of erroneous solutions during the detection.

Relationship/ratio (6.2.1), being consequence of most general/most common/most total criterion, is initial for synthesis of optimum schematic of detector. Distributions $W_{cm}(\rho_i)$ and $W_w(\rho_i)$ necessary for determining the structure of function must be known.

FOOTNOTE ¹. See footnote on page 155. ENDFOOTNOTE.

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In particular, for the case of incoherent impulses/momenta/pulses in

question with the constant amplitude they are calculated in Chapter 5 (§ 5-1).

According to relationship/ratio (5.1.10) density function of distribution for additive mixture signal plus noise

$$W_{\text{cm}}(\rho_i) = \frac{\rho_i}{\sigma^2} e^{-\frac{\rho_i^2 + A_i^2}{2\sigma^2}} I_0\left(\frac{A_i \rho_i}{\sigma^2}\right),$$

where A_i - amplitude of i signal (impulse/momentum/pulse) of selection, and density function of distribution for pure/clean noise from (5.1.10) when $A_i=0$ is equal to

$$W_w(\rho_i) = \frac{\rho_i}{\sigma^2} e^{-\frac{\rho_i^2}{2\sigma^2}}.$$

Likelihood ratio for separate i signal of selection

$$\frac{W_{\text{cm}_i}(\rho_i)}{W_{w_i}(\rho_i)} = e^{-\frac{A_i^2}{2\sigma^2}} I_0\left(\frac{A_i \rho_i}{\sigma^2}\right). \quad (6.2.1)$$

Then criterion of plausibility for packet from p impulses/momenta/pulses in accordance with formula (6.1.22) will be determined by inequality

$$\prod_{i=1}^n e^{-\frac{A_i^2}{2\sigma^2}} I_0\left(\frac{A_i \rho_i}{\sigma^2}\right) > I_0. \quad (6.2.2)$$

This relationship/ratio should be considered as algorithm of computations, produced above selective values ρ_i in diagram of optimum feeler, which is actually computer.

For simplification in structure of this computer let us switch over to logarithmic threshold value, i.e., to value $\ln l_0$. In this case let us note that in view of the monotonicity of logarithmic function inequality (6.2.2) will not be destroyed, only numerical values of right and left parts will change.

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After producing logarithmic conversion, it is attained replacement in (6.2.2) the bulky operation/process of n-fold multiplication by a simpler operation/process of the addition:

$$-\sum_{i=1}^n \frac{A_i^2}{2\sigma^2} + \sum_{i=1}^n \ln l_0 \left(\frac{A_i \rho_i}{\sigma^2} \right) > \ln l_0 \quad (6.2.3)$$

Subsequently we assume that pulse amplitudes in separate values of selection are identical: $A_1=A_2=\dots=A_i=\dots=A_n=A$. Virtually this is possible with the motionless antenna or with the antenna, which rotates within the limits of the small sector, where the coefficient of directivity can be considered constant. Then relationship/ratio (6.2.3) can be rewritten in the following form:

$$\sum_{i=1}^n \ln l_0 \left(\frac{A \rho_i}{\sigma^2} \right) > \ln l_0 + n \frac{A^2}{2\sigma^2}. \quad (6.2.4)$$

This inequality determines structure of feeler. From it follows that the optimum feeler must summarize the analyzed samples/specimens of the selective values of process after the

functional conversion

$$y = \ln I_0 \left(\frac{A p_l}{\sigma^2} \right) \quad (6.2.5)$$

and then compare the obtained sum with the threshold, which depends on the size of sample n . The exemplary/approximate diagram of this feeler is shown in Fig. 6.2. Here the multiplication of the samples/specimens of selective values for constant $\frac{A}{\sigma^2} = K$ and their functional conversion (6.2.5) they are realized with the aid of the simulating circuits, and addition (accumulation) - by means of the diagrams of discrete/digital action. In the first case the signal is represented in the continuous form, and the second - in the discrete/digital. Conversion is realized by device/equipment PND, which convert analog quantity into the discrete/digital. The use here of discrete/digital equipment first of all is caused by the fact that the memorization and the accumulation of signals more easily are realized during their discrete/digital representation, than with the continuous. In fact, usually the signal at the output of converter is represented in the form of the binary number, for the memorization and computational conversions of which can be used different means of discrete/digital equipment.

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Use/application of means of digital computer (discrete/digital) technology in diagrams of detection has following advantages:

a) simplicity and uniformity of circuit solutions, connected with use of standard logic elements of digital computer technology;

b) relative simplicity of memorization of both intermediate, and final of results. In this case the numbers, memorized, for example, by ferrite cores, by registers and the like devices/equipment, can be retained in practice during the unlimited time interval;

c) the linearity of accumulation, attained due to the use/application of digital summators (or counters);

d) simplicity of obtaining diverse mathematical relationships/ratios for processing of signal. A change of the working/treatment in the majority of the cases is connected with a change in altogether only of the program of the computations of the arithmetic unit of feeler;

e) the stability of the results of processing and their small dependence on the stability of the mode/conditions of the network elements.

By virtue of special features/peculiarities of digital computer

technology indicated its use, apparently, is more preferably than simulating analogue computers.

For most important case - low signals - further possible simplification in diagram of feeler (Fig. 6.2), which escapes/ensues from asymptotic representation of formula (6.2.5). For this formula it is correct

$$\ln I_0\left(\frac{A\rho_l}{\sigma^2}\right) \approx \begin{cases} \frac{A^2\rho_l^2}{4\sigma^4} \text{ для } \frac{A\rho_l}{\sigma^2} \ll 1; \\ \frac{A\rho_l}{\sigma^2} \text{ для } \frac{A\rho_l}{\sigma^2} \gg 1. \end{cases} \quad (6.2.6)$$

Key: (1). for.

Case of low signal occurs, when $A \ll \sigma$.

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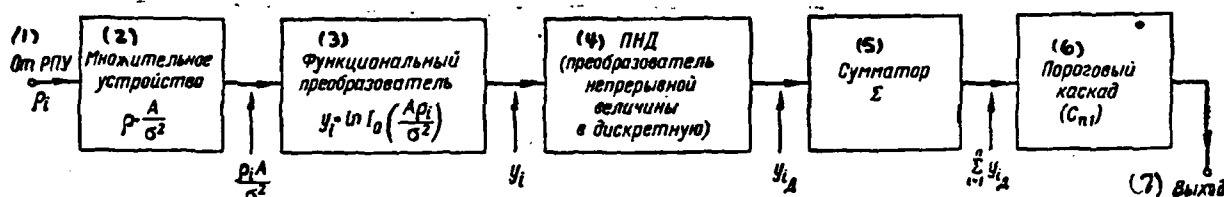


Fig. 6.2. Exemplary/approximate schematic of device/equipment of detection of signal (UOS) in elementary section of range.

Key: (1). From RPU. (2). Multiplier. (3). Function generator. (4). PND (converter of analog quantity into discrete/digital). (5). Summator. (6). Threshold cascade/stage. (7). output.

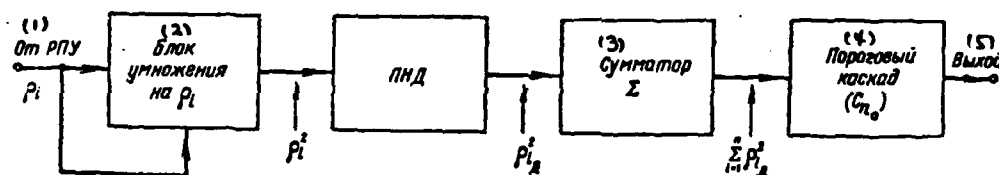


Fig. 6.3. Diagram of detection of weak signals (y_{iD} and P_{iD} - discrete/digital values of values y_i and P_i)

Key: (1). From RPU. (2). Block of multiplication on. (3). Summator. (4). Threshold cascade/stage. (5). output.

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Substituting (6.2.6) in (6.2.4), we will obtain the criterion of detection for the low signal:

$$\frac{A^2}{4\sigma^4} \sum_{l=1}^n p_l^2 > \ln l_0 + n \frac{A^2}{2\sigma^4}$$

or it is final

$$\sum_{l=1}^n p_l^2 > C_n \sigma^2, \quad (6.2.7)$$

where the dimensionless quantity of the threshold of the accumulation

$$C_n = \frac{4\sigma^4}{A^2} \ln l_0 + 2n. \quad (6.2.8)$$

Hence it is apparent that optimum processing of signal in this case consists of addition of squares of envelope (its separate selective values) with subsequent comparison of sum on threshold. In this case the value of threshold (6.2.8) depends on a signal-to-noise ratio A/σ and a quantity of impulses/momenta/pulses in packet n (sample size). The exemplary/approximate diagram of this feeler is shown in Fig. 6.3. In this diagram there are a block of multiplication, which realizes squaring of the samples/specimens of selective values, and a diagram of the addition (accumulation) of the squares of signal. Addition is realized with the aid of the diagrams of discrete/digital action; therefore into the diagram of addition enters, besides summator, even and the converter of analog quantity into discrete/digital (PND).

In the case of strong signal, when $A \gg \sigma$ and $\frac{A}{\sigma} \gg 1$ diagram in question can obtain supplementary simplifications, which escape/ensue also from asymptotic representation of formula (6.2.5). Substituting lower row (6.2.6) in (6.2.4), we will obtain the criterion of detection for the strong signal

$$\sum_{i=1}^n p_i > C_{n1} \sigma, \quad (6.2.9)$$

where the dimensionless quantity of the threshold

$$C_{n1} = \frac{\sigma}{A} \ln I_0 + \frac{nA}{2\sigma}. \quad (6.2.10)$$

From (6.2.9) it follows that optimum working/treatment in this case is reduced simply to addition of separate samples/specimens of selection with subsequent testing of sum on threshold $C_{n1}\sigma$. Therefore the diagram of this feeler consists of converter and summator, which realizes the accumulation of signal (Fig. 6.4).

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It must be noted that diode detector of radar receiver allows, in any case in principle, to carry out processing indicated above signal. Actually/really, on small signal levels this detector can be set in the mode/conditions of quadratic element/cell, and in the large ratios of signal to the interference - into the mode/conditions of linear detector. Consequently, it is possible to consider that

this detector fulfills conversion (6.2.6) for two extreme values of received signal. The possibility of the realization of this mode/conditions is determined by the value of total signal with respect to the range of the quadratic section of the characteristic of detector, and also by the parameters of the characteristics of diode and elements of the network of detector.

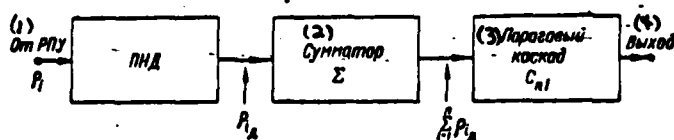


Fig. 6.4. Diagram of the detection of strong signals.

Key: (1). From RPU. (2). Summator. (3). Threshold cascade/stage. (4). output.

B. Evaluation/estimate of the figures of merit of detection.

Let us examine evaluation/estimate of figures of merit of detection F and D for most important case - low signals. As is known, in this case the working/treatment is reduced to the addition of the squares of envelope. For computing the evaluation/estimate of the figures of merit, it is necessary to know two forms of the function of the distribution of the sum of the squares of the envelope: when selections are taken from the pure/clean noise and from the mixture signal + noise. Let us designate the density function of the distribution of these selections respectively through $f_m(Z)$ and $f_{cm}(Z)$.

If they are known, then with the assigned threshold of accumulation C_n , the unknown figures of merit can be calculated with the aid of the obvious relationships/ratios:

$$F = \int_{C_{n_z}}^{\infty} f_w(z) dz; \quad (6.2.11)$$

$$D = \int_{C_{n_z}}^{\infty} f_{cm}(z) dz. \quad (6.2.12)$$

However, these distributions to us are unknown; therefore let us switch over to their computation. During the computation of the density function of the distribution of the sum of the squares of random variables ρ_i^2 , to the density of random number distribution $z = \sum_{i=1}^n \rho_i^2$, we will enter as follows: let us determine mathematical expectation and dispersion of each of the components/terms/addends $\rho_i^2 = \eta_i$, for two cases, when selections ρ_i are undertaken respectively from the noise and from the mixture signal + noise. Then, on the basis of the central limit theorem, we will assume that random variable $z = \sum_{i=1}^n \eta_i$ is subordinated to the normal distribution law. The latter, as is known, is correct with the large size of sample n ; however, in the case in question this occurs, since computations F and D are conducted for a small intensity of signal and a quantity of impulses/momenta/pulses in the packet with target detection must be large. Mathematical expectation and the dispersion of the resulting random process with the normal distribution find as the sums of respectively mathematical ones expectation and the dispersions of

composition from n of components/terms/addends.

As is known, distribution of selective value ρ_i of that belonging to noise, it is equal to:

$$W_m(\rho_i) = \frac{\rho_i}{\sigma^2} e^{-\frac{\rho_i^2}{2\sigma^2}}.$$

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Then distribution for random variable $\eta_i = \rho_i^2$ in accordance with rules of conversion of distribution functions for converted random variables will take form

$$U_m(\eta_i) = W_m(\rho_i) \frac{1}{\left| \frac{d\eta_i}{d\rho_i} \right|} = W_m(\sqrt{\eta_i}) \frac{1}{\left| \frac{d\eta_i}{d\rho_i} \right|} = \frac{1}{2\sigma^2} e^{-\frac{\eta_i}{2\sigma^2}}. \quad (6.2.13)$$

Let us find average and dispersion of this distribution. For this it is necessary to find the first and second moments/torques of distribution (6.2.13). For the generality of reasonings let us find the moment/torque of ν order:

$$\alpha_\nu = \int_0^\infty \eta_i^\nu U_m(\eta_i) d\eta_i. \quad (6.2.14)$$

Substituting here (6.2.13) and taking into account integral expression for gamma function

$$\int_0^\infty x^n e^{-ax} dx = \frac{1}{a^{n+1}} \Gamma(n+1).$$

we will obtain that

$$\alpha_1 = \int_0^\infty \frac{1}{2\sigma^2} \eta_i e^{-\frac{\eta_i}{2\sigma^2}} d\eta_i = \nu (2\sigma^2)^{-1}. \quad (6.2.15)$$

Hence the first and second moments/torques will be respectively equal to:

$$\alpha_1 = 2\sigma^2, \quad \alpha_2 = 8\sigma^4. \quad (6.2.16)$$

In this case dispersion, or second central moment, distribution (6.2.13)

$$\mu_2 = \alpha_2 - \alpha_1^2 = 4\sigma^4. \quad (6.2.17)$$

Let us switch over to parameters of distribution of composition from n of selective values, which belong to noise.

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In this case resulting distribution $f_w(z)$ should be considered the subordinate to normal law with the parameters

$$M\left(\sum_{i=1}^n p_i^2\right) = 2n\sigma^2 = B_w; \quad (6.2.18)$$

$$D^0\left(\sum_{i=1}^n p_i^2\right) = 4n\sigma^4 = D_w,$$

where $\sqrt{D_w} = \sigma$, - root-mean-square deviation.

In this case very density of distribution

$$f_w(z) = \frac{1}{\sqrt{2\pi D_w}} e^{-\frac{(z-B_w)^2}{2D_w}} \quad (6.2.19)$$

Knowing $f_w(z)$, it is possible to determine probability of false alarm F , i.e., probability of that, when sum of squares of selective values, which belong to pure/clean noise, exceeds threshold $C_{nz} = C_{nz}^?$, where C_n - dimensionless threshold value, determined by expression (6.2.8). Substituting (6.2.19) in (6.2.11), we will obtain

$$F = \frac{1}{\sqrt{2\pi D_w}} \int_{C_{nz}}^{\infty} \exp \left[-\frac{(z-B_w)^2}{2D_w} \right] dz.$$

After introducing new variable $t = \frac{z-B_w}{\sqrt{2D_w}}$; $dz = \sqrt{2D_w} dt$, this relationship/ratio can be rewritten in the form

$$F = \frac{\sqrt{2D_w}}{\sqrt{2\pi D_w}} \int_{t_1}^{\infty} e^{-t^2} dt = \frac{1}{\sqrt{\pi}} \int_{t_1}^{\infty} e^{-t^2} dt,$$

where

$$t_1 = \frac{C_{nz} - B_w}{\sqrt{2D_w}} = \sqrt{2} \frac{\frac{\sigma^2}{A^2} \ln I_0}{\sqrt{n}}. \quad (6.2.20)$$

Taking into account that

$$\int_{t_1}^{\infty} e^{-t^2} dt = \int_0^{\infty} e^{-t^2} dt - \int_0^{t_1} e^{-t^2} dt,$$

finally we have

$$F = \frac{1}{2} [1 - \Phi(t_1)], \quad (6.2.21)$$

where

$$\Phi(t_1) = \frac{2}{\sqrt{\pi}} \int_0^{t_1} e^{-t^2} dt \quad (6.2.22)$$

it is tabulated Laplace function.

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Let us switch over to computation of function of distribution $f_{cm}(z)$, which characterizes distribution $\sum p_i^2$, when tests/samples p_i are taken from mixture signal + noise. According to (5.1.10) the function of the distribution of the separate test/sample

$$W_{cm}(p_i) = \frac{p_i}{\sigma^2} e^{-\frac{p_i^2 + A^2}{2\sigma^2}} I_0\left(\frac{Ap_i}{\sigma^2}\right).$$

It is analogous with preceding/previous computations, carried out for function of distribution of tests/samples, that relate to realizations of interference, we find distribution for random variable $\eta_i = p_i^2$:

$$\begin{aligned} U_{cm}(\eta_i) &= W_{cm}(V\eta_i) \frac{1}{\left| \frac{d\eta_i}{dp_i} \right|} = \\ &= \frac{1}{2\sigma^2} e^{-\frac{\eta_i + A^2}{2\sigma^2}} I_0\left(\frac{A\sqrt{\eta_i}}{\sigma^2}\right). \end{aligned} \quad (6.2.23)$$

For determining average and dispersion of this distribution let us calculate first and second moments/torques of distribution (6.2.23). For the generality of reasonings let us find the moment/torque of the k order from the square of the random variable, distributed according to generalized Rayleigh's law (5.1.10);

$$\alpha_k = \int_0^\infty \eta_i^k U_{cm}(\eta_i) d\eta_i.$$

Substituting here (6.2.23) and replacing variable of integration $\eta_1 = x^2$; $d\eta_1 = 2x dx$, we will obtain, taking into account in this case known relationship/ratio for function of Bessel $J_n(x) = i^{-n} J_n(ix)$.

$$a_k = \frac{1}{\sigma^2} e^{-\frac{A^2}{2\sigma^2}} \int_0^\infty x^{2k+1} e^{-\frac{x^2}{2\sigma^2}} J_0\left(i \frac{A}{\sigma^2} x\right) dx.$$

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Integral, entering this formula, is tabular, its value is given, for example, in tables of integrals [42].

Utilizing this relationship/ratio, we will obtain:

$$a_k = \frac{1}{2\sigma^2} e^{-\frac{A^2}{2\sigma^2}} \frac{\Gamma(k+1)}{\left(\frac{1}{2\sigma^2}\right)^{k+1}} {}_1F_1\left(k+1; 1; -\frac{A^2}{2\sigma^2}\right),$$

where ${}_1F_1$ - degenerate hypergeometric function [24]. Utilizing a known relationship/ratio for the negative values of argument ${}_1F_1$, i.e.,

$${}_1F_1(x; \gamma; z) = e^z {}_1F_1(\gamma - x; \gamma; -z),$$

finally we obtain

$$a_k = (2\sigma^2)^k k! {}_1F_1\left(-k; 1; -\frac{A^2}{2\sigma^2}\right). \quad (6.2.24)$$

Hence first and second moments/torques can be expressed through degenerate hypergeometric function

$$a_1 = 2\sigma^2 {}_1F_1\left(-1; 1; -\frac{A^2}{2\sigma^2}\right) \quad (6.2.25)$$

and

$$a_2 = (2\sigma^2)^2 2 {}_1F_1\left(-2; 1; -\frac{A^2}{2\sigma^2}\right).$$

For determining the moments/torques let us represent ${}_1F_1$ in the expanded/scanned form for us interesting of the values of the parameters of this function.

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As is known, ${}_1F_1$ - hyper-geometric series, or hypergeometric function (called another function of Kummer), has form [42, 24]

$${}_1F_1(\alpha; \gamma; x) = \sum_{n=0}^{\infty} \frac{(a)_n}{(\gamma)_n} \cdot \frac{x^n}{n!},$$

where symbol $(\gamma)_n$ designates value

$$(\gamma)_n = \frac{\Gamma(\gamma + n)}{\Gamma(\gamma)} = \gamma(\gamma + 1) \dots (\gamma + n - 1).$$

If α - negative integer, then ${}_1F_1$ is reduced to polynomial

$${}_1F_1(-k; \gamma; x) = \frac{x^{\gamma-1} e^x}{k!} \frac{d^k}{dx^k} (x^{\gamma+k-1} e^{-x}).$$

Since in our case $\gamma=1$, this function with an accuracy to constant coincides with Laguerre's polynomials [24]

$${}_1F_1(-k; 1; x) = \frac{1}{k!} L_k(x), \quad (6.2.26)$$

where by Laguerre's polynomials is understood the relationship/ratio

$$L_k(x) = e^x \frac{d^k}{dx^k} (x^k e^{-x}),$$

in particular

$$\begin{aligned} L_0 &= 1; \quad L_1 = 1 - x; \quad L_2 = x^2 - 4x + 2; \\ L_3 &= 6 - 18x + 9x^2 - x^3 \dots \end{aligned} \quad (6.2.27)$$

Substituting (6.2.26) and (6.2.27) in (6.2.25) and taking into account that $x = -\frac{A^2}{2\sigma^2}$, we obtain resultant expressions for moments/torques of function of distribution of squares of random variable

$$\alpha_1 = 2\sigma^2 \left(1 + \frac{A^2}{2\sigma^2} \right) \quad (6.2.28)$$

and

$$\alpha_2 = 8\sigma^4 \left(1 + \frac{A^2}{\sigma^2} + \frac{A^4}{8\sigma^4} \right). \quad (6.2.29)$$

Dispersion

$$\mu_2 = \alpha_2 - \alpha_1^2 = 4\sigma^4 \left(1 + \frac{A^2}{\sigma^2} \right) \quad (6.2.30)$$

now can be calculated.

FOOTNOTE ¹. This relationship/ratio together with (6.2.24) gives relative to the simple method of computing the moments/torques of distribution (6.2.23). ENDFOOTNOTE.

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Parameters of distribution of resulting process $z = \sum_{i=1}^n p_i^2$ in the case, when tests/samples are taken from mixture signal + noise, will

be:

$$M\left(\sum_{i=1}^n p_i^2\right) = 2\sigma^2 n \left(1 + \frac{A^2}{2\sigma^2}\right) = B_{\text{cw}}; \quad (6.2.31)$$

$$D^0\left(\sum_{i=1}^n p_i^2\right) = 4\sigma^4 n \left(1 + \frac{A^2}{\sigma^2}\right) = D_{\text{cw}}.$$

In this case very function of distribution

$$f_{\text{cw}}(z) = \frac{1}{\sqrt{2\pi D_{\text{cw}}}} e^{-\frac{(z-B_{\text{cw}})^2}{2D_{\text{cw}}}}. \quad (6.2.32)$$

This distribution function according to (6.2.12) determines probability of correct detection D . The latter characterizes the probability of that event, that sum n of the squares of selective values p_i , $(\sum_{i=1}^n p_i^2)$, that belong to a process of the type signal + noise, will exceed threshold $C_{nz} = \sigma^2 C_n$, where C_n - dimensionless coefficient, determined by expression (6.2.8). Substituting (6.2.32) in (6.2.12), we will obtain

$$D = \frac{1}{\sqrt{2\pi D_{\text{cw}}}} \int_{C_{nz}}^{\infty} e^{-\frac{(z-B_{\text{cw}})^2}{2D_{\text{cw}}}} dz.$$

Introducing variable $t = \frac{z-B_{\text{cw}}}{\sqrt{2D_{\text{cw}}}}$; $dz = \sqrt{2D_{\text{cw}}} dt$, we convert this expression

$$D = \frac{1}{\sqrt{\pi}} \int_{t_0}^{\infty} e^{-t^2} dt,$$

where

$$t_0 = \frac{C_{nz} - B_{\text{cw}}}{\sqrt{2D_{\text{cw}}}}$$

or taking into account (6.2.31)

$$t_2 = \frac{2 \frac{\sigma^2}{A^2} \ln l_0 - n \frac{A^2}{2\sigma^2}}{\sqrt{2n \left(1 + \frac{A^2}{\sigma^4}\right)}}. \quad (6.2.33)$$

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Carrying out analogous conversions as during derivation of formulas (6.2.21), we will obtain that

$$D = \frac{1}{2} [1 - \Phi(t_2)]. \quad (6.2.34)$$

Relationships/ratios (6.2.21) and (6.2.34) are calculated and make it possible to determine F and D on known t_1 and t_2 . They can be used for determining the parameters of the device/equipment of the detection of signal from preset parameters of the quality of detection F and D, in particular, here can be determined a quantity of impulses/momenta/pulses in packet n or signal-to-noise ratios A/σ , at which attain the given values F or D. For this expression (6.2.21) and (6.2.34) are solved relative to t_1 and t_2 , from which by formulas (6.2.20) and (6.2.33) are found the required circuit parameters.

56.3. The binary system of the optimum of the detections of incoherent impulses/momenta/pulses with the constant amplitude.

Diagram examined in preceding/previous paragraph of detection of

incoherent impulses/momenta/pulses contains block of converting analog quantity into discrete/digital (PND). Diagram and construction/design of this block are complicated, in a number of cases by difficultly feasible; therefore is desirable the use/application of other methods and diagrams of detection, which do not contain this block. In particular, here it is possible to utilize the so-called binary diagram of detection.

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Binary system of detection is characterized by the fact that in each elementary section signal is quantized according to amplitude sign/criterion to two levels and analyzed random processes it is represented not by its instantaneous value, but only by two levels 0 and U_0 . Value U_0 is called the threshold of quantization (limitation). The signals, which enter from the output of radar receiver, are supplied to the limiter with threshold U_0 (Fig. 6.5), which quantizes output signals to two levels in the amplitude. If the instantaneous value of random signal exceeds the threshold level U_0 , then standard impulse/momentum/pulse is put out; if it does not exceed, then impulse/momentum/pulse is absent.

It should be noted that amplitude quantization to two levels leads to certain loss of information relative to waveform with

respect to initial nonquantum structure. However, simplifications in the diagram obtained here give the possibility to carry out the qualitatively subsequent working/treatment by comparatively simple technical equipment. This to a certain degree can compensate the losses of information in the quantum signal.

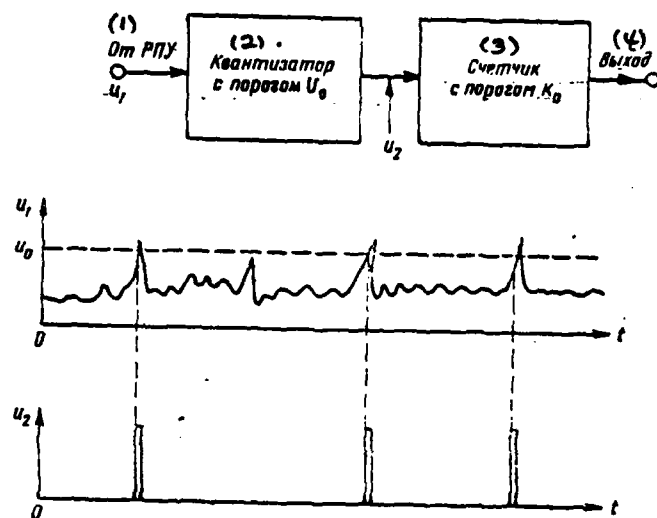


Fig. 6.5. Signals at input and output of quantized charge.

Key: (1). From RPU. (2). Quantizer with threshold U_0 . (3). Counter with threshold K_0 . (4). output.

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Subsequently is conducted accumulation - counting of standard pulses according to specific rules, as a result of which solution about presence or absence of signal is accepted. This processing of signal is called sometimes binary integration in contrast to the ideal, obtained in diagrams (§6.2), in which is analyzed the instantaneous value of random process at the output of the receiving channel of radar.

Subsequently we will consider that if instantaneous value of envelope of analyzed process exceeds U_0 , then function accepts state 1, if it does not exceed, then accepts state 0. It is obvious, the probability of state 1 in the presence and the absence signal; it is different. In the presence of signal the probability of this from the standing is more, while in the absence - it is less. On the principle of different probability state 1 and 0 in presence and absence of signal is constructed the binary system of detection in question.

Probability of excess of envelope of mixture signal + noise of threshold level U_0 is equal for the i -th and pulse

$$p_{l_{cm}} = \int_{U_0}^{\infty} W_{cm_i}(\rho_i) d\rho_i, \quad (6.3.1)$$

where $W_{cm_i}(\rho_i)$ — function on distribution density. For the case of incoherent ones in question the impulse/momentum/pulse with the constant amplitude, when function $W_{cm_i}(\rho_i)$ is determined by expression (5.1.10)

$$W_{cm_i}(\rho_i) = \frac{\rho_i}{\sigma^2} e^{-\frac{\rho_i^2 + A_i^2}{2\sigma^2}} I_0\left(\frac{A_i \rho_i}{\sigma^2}\right),$$

probability $p_{l_{cm}}$ is located by substitution (5.1.10) in (6.3.1).

As a result we will obtain

$$p_{icm} = \int_{U_0}^{\infty} \frac{p_l}{\sigma^2} e^{-\frac{p_l^2 + A_l^2}{2\sigma^2}} I_0\left(\frac{A_l p_l}{\sigma^2}\right) dp_l. \quad (6.3.2)$$

For computing this integral we will use formula, obtained by U. R. Bennot [29]:

$$\begin{aligned} \int_0^U U \exp\left(-\frac{U^2 + a^2}{2}\right) I_0(aU) dU &= \\ &= \exp\left(-\frac{U^2 + a^2}{2}\right) \sum_{v=1}^{\infty} I_v(aU). \end{aligned}$$

Integral of such type is expressed as Q-functions, described by Markum [43], for which there are graphs/curves.

Applying it to (6.3.2) and taking into account that

$$\int_{U_0}^{\infty} W_{cm}(p_l) dp_l = 1 - \int_0^{U_0} W_{cm}(p_l) dp_l,$$

we will obtain that

$$p_{icm} = 1 - e^{-\frac{A_l^2 + U_0^2}{2\sigma^2}} \sum_{v=1}^{\infty} \left(\frac{U_0}{A_l}\right)^v I_v\left(\frac{A_l U_0}{\sigma^2}\right). \quad (6.3.3)$$

Graph/curve of this function is represented in Fig. 6.6 for different ratios of signal to interference.

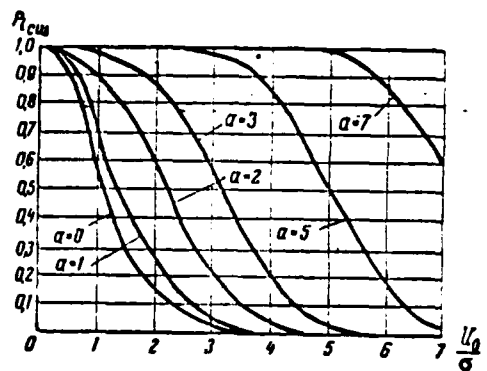


Fig. 6.6. Graphs/curves of the probability of threshold crossing U_0/σ by signal with an amplitude of A_1 ($a = \frac{A_1}{\sigma}$)

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From the graphs/curves it is evident that the probability of state 1 on the assigned threshold level U_0/σ increases with an increase in relation $\frac{\bar{A}_1}{\sigma} = a$.

Probability of nonexcess of envelope of mixture signal + noise of threshold level U_0 , obviously, is equal to

$$q_{1cw} = 1 - p_{1cw} = e^{-\frac{A_1^2 + U_0^2}{2\sigma^2}} \sum_{v=1}^{\infty} \left(\frac{U_0}{A_1}\right)^v I_v\left(\frac{A_1 U_0}{\sigma^2}\right). \quad (6.3.4)$$

Probability of excess of envelope of fluctuating noise of threshold value

$$p_{w1} = \int_{U_0}^{\infty} W_w(\rho_1) d\rho_1$$

where $W_m(\rho_i)$ - function of density distribution of envelope of noise, determined by expression (5.1.10), when $A_i=0$

$$p_{w_i} = \int_{U_i}^{\infty} \frac{\rho}{\sigma^2} e^{-\frac{\rho^2}{2\sigma^2}} d\rho = e^{-\frac{U_i^2}{2\sigma^2}}. \quad (6.3.5)$$

Probability of the nonexcess of the envelope of the fluctuating noise of the threshold value

$$q_{w_i} = 1 - p_{w_i}.$$

Let us determine structure of binary feeler in accordance with threshold value of coefficient of plausibility.

A. Structure of optimum feeler.

Unknown structure of detector we will determine, on the basis of sufficiently general/common/total criteria, which characterize its work.

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As is known, the criterion of the minimum of the average/mean risk, which in our case is reduced to the comparison of likelihood ratio with the threshold λ , (section 6.1), is such criterion.

However, for computing coefficient of plausibility (6.1.21) it is necessary to know functions of distribution of signals, which exceed by n tests k of times threshold level U_0 , for two cases: analyzed random process corresponds to mixture signal + noise or only to pure/clean noise. Since in each of these cases during the separate testing there are the always two incompatible outcomes with probabilities p_{icm} ; q_{icm} and p_{wi} ; q_{wi} respectively, are valid the relationships/ratios:

$$p_{icm} + q_{icm} = 1; \quad p_{wi} + q_{wi} = 1.$$

Subsequently we will assume that probabilities in question do not depend on index i , i.e., signal amplitudes in separate impulses/momenta/pulses of packet in question are identical

$A_1 = A_2 = \dots = A_i = \dots = A_n$, which is observed with motionless antenna or with antenna, which rotates within limits of small sector, where amplification factor can be considered constant. In this case for determining the unknown functions of plausibility it is possible to use Bernoulli's diagram. Let us recall it in application to the case in question.

Let us conditionally name/call onset of one of outcomes, when output signal accepts state 1 success. Probability of this event p_{icm} . We will call the opposite case failure. Thus, in separate experiment (appearance of a signal in the elementary section of range) can occur

only the two incompatible outcomes with probabilities p_{cm} and q_{cm} .

As a result of experiment success can begin or not begin.

Let us assume are conducted n experiments, where n - quantity of impulses/momenta/pulses in packet. It is necessary to determine probability $P(n, k)$ of the fact that in the n tests the success will begin accurately k of times. In this case tests we assume/set by independent variables, and value $p_{cm_1} = p_{cm_2} = \dots = p_{cm_i} = \dots = p_{cm_n}$ the probabilities of outcomes do not depend on the number of testing.

For determining unknown probability $P(n, k)$ let us find probability of onset of success in any fixed/recorded by k numbers of tests at first.

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This probability is equal to product of probabilities of success in k numbers and failure in rest, i.e., in $(n-k)$, numbers:

$$p_{cu}^k q_{cu}^{(n-k)} = p_{cu}^k (1 - p_{cu})^{n-k}.$$

Since event examined is only one special case of combination of successes and failures into any k numbers of tests, it is necessary to still consider other cases, where there will be k of successes into any k numbers of tests. A number of such cases is equal to a number of combinations of n on k . Thus, finally we have:

$$P_{cu}(n, k) = C_n^k p_{cu}^k (1 - p_{cu})^{n-k}, \quad (6.3.6)$$

where binomial coefficients C_n^k are a number of combinations

$$C_n^k = \frac{n!}{k!(n-k)!}.$$

Consequently, distribution function for signals, which correspond to excess by pure/clean noise of threshold U , k of times from n tests, on the basis of analogous reasonings will be equal to

$$P_u(n, k) = C_n^k p_u^k (1 - p_u)^{n-k}. \quad (6.3.7)$$

Coefficient of plausibility on the basis (6.1.21)

$$\frac{p_{cu}^k (1 - p_{cu})^{n-k}}{p_{u}^k (1 - p_u)^{n-k}} = \frac{P_{cu}(n, k)}{P_u(n, k)} > l_0 \quad (6.3.8)$$

Passing by logarithmic operation to another, more convenient criterion, we will obtain

$$k \ln \frac{p_{cu}}{p_u} + (n - k) \ln \frac{q_{cu}}{q_u} > \ln l_0$$

Solving this inequality relative to k , we have

$$k \left(\ln \frac{p_{cu}}{p_u} - \ln \frac{q_{cu}}{q_u} \right) > \ln l_0 - n \ln \frac{q_{cu}}{q_u},$$

whence finally we obtain

$$k > k_0 \quad (6.3.9)$$

where

$$k_0 = \frac{\ln l_0 - n \ln \frac{1 - p_{cu}}{1 - p_u}}{\ln \frac{p_{cu}}{p_u} - \ln \frac{1 - p_{cu}}{1 - p_u}} \quad (6.3.10)$$

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Since k is quantity of cases of threshold crossing of quantization U , diagram of detection in elementary section of range is reduced in this case simply to counter of standard impulses/momenta/pulses with modulus/module of translation k . Excess k , indicates the presence of target in this elementary section. This fact differs in terms of appearance on the counter of the output

signal, which appears each time, when a quantity of entered impulses/momenta/pulses exceeds the modulus/module of its translation.

We assume/set values l , and n , entering formula (6.3.10), known, and probabilities p_{ciu} and p_{in} are determined by functions of distribution of mixture signal + noise also of pure/clean noise, i.e., by statistical characteristics of analyzed process. For the incoherent signal with the constant amplitude corresponding probabilities p_{ciu} and p_{in} are determined by formulas (6.3.3) and (6.3.5).

Value of threshold U , is established/installed in threshold cascade/stage, arranged/located on output of receiving channel of radar. From this cascade/stage into the diagrams of the detection of signal only the impulses/momenta/pulses of standard form come, if occurred excess by the signal of threshold U . Thus, detector in each elementary section of range is usual counter with the modulus/module of translation k . In this counter the signals are accumulated after n of the pulse repetition periods of radar, after which the accumulated sum is dumped in 0, and accumulation after n of the following repetition periods begins. For the realization of the current accumulation consecutively/serially for h adjacent positions (scannings/sweeps) it is necessary to have h of the switched counters

with modulus/module k , each.

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B. Evaluation/estimate of the figures of merit of detection.

Figures of merit of different methods of detection of signals against the background of interferences are determined by probability of false alarm F and by correctness of detection D , and also by their dependence on signal-to-noise ratio and on parameters of device/equipment. In the case of the detection of radar signals according to the method of binary integration by parameters of the device/equipment of detection we will understand a quantity of impulses/momenta/pulses in packet n , and also a threshold value of accumulation k , and quantization U .

For determining probabilities of false alarm F and correct detection D it is necessary to know resulting functions of array of signals, which exceeded k of times from n level of quantization U , and entered storage device/equipment - counter with threshold k . In this case we should know two forms of such of the distribution function, that correspond to the excess of the level of quantization by pure/clean noise and to the excess of mixture signal + noise.

These distribution functions were examined above, in p. A §6.3 were determined by formulas (6.3.6) and (6.3.7). For the mixture signal + the noise

$$P_{cw}(n, k) = C_n^k p_{cw}^k (1 - p_{cw})^{n-k},$$

for the pure/clean noise

$$P_w(n, k) = C_n^k p_w^k (1 - p_w)^{n-k}.$$

Value of false alarm F and correct detection D is graphically determined by areas, limited respectively by functions (6.3.7) and (6.3.6) and by axis intercept of abscissas, situated more to the right of value $k=k_0$, called threshold of calculation. Since the variable k changes integral, determination of area is reduced to the addition of the ordinates of binomial distribution (6.3.6) and (6.3.7). Summarizing the "remainders/residues" of ordinates after $k=k_0$, we will obtain:

$$D = \sum_{k=k_0}^n C_n^k p_{cw}^k (1 - p_{cw})^{n-k}, \quad (6.3.11)$$

$$F = \sum_{k=k_0}^n C_n^k p_w^k (1 - p_w)^{n-k}. \quad (6.3.12)$$

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These probabilities for different values of n and $k, k < n$ can be expressed through incomplete beta function, for which there are tables [32]:

$$D = I_{p_{cw}}(k_0; n - k_0 + 1);$$

$$F = I_{p_w}(k_0; n - k_0 + 1),$$

where

$$I_p(a, b) = \int_0^p \frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)} x^{a-1} (1-x)^{b-1} dx$$

there is incomplete beta function.

With relatively greater n for calculating probabilities D and F it is possible to use representation of binomial distribution as expansion of Edgeworth in terms of Hermite's functions. Use/application of these series/rows conveniently also for conducting of investigations for the purpose of the optimization of different parameters of detector.

1. Representation of binomial distribution by the resolution of Edgeworth.

For facilitating calculations, connected with determination of criteria of quality of detection (D and F), it is expedient to use approximate representation of binomial distribution with series of Edgeworth, who gives expansion of distribution function in terms of orthogonal polynomials of Hermite with coefficients, which are totality of moments/torques of analyzed distribution.

Sense of introduction of Edgeworth series consists to replacement of discontinuous distribution by continuous, what is

advisable for simplification in computational and mathematical operations/processes, which appear with different kind investigations. This replacement is based on the equality of moments/torques α_{ip} ($i=1, 2, 3 \dots$) discontinuous distribution and moments/torques α_i ($i=1, 2, 3 \dots$) continuous distribution. The moments/torques of discontinuous distribution can be easily calculated with known probabilities p_{cm} and p_m (see below), therefore, can be determined the coefficients of the Edgeworth series.

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Essential specific feature of this resolution is the decrease of the value of the terms of series/row with an increase in the size of sample n , the terms of series/row decrease inversely proportionally $n^{-\nu}$, where ν - number of the term of series/row. With infinite n all terms of the series/row vanish, except the first, which corresponds to the density function of normal distribution. Thus, these expansions give the distribution, which asymptotically approaches normal.

In our case discontinuous distributions are $P_{cm}(n, k)$, $P_m(n, k)$ [(6.3.6) and (6.3.7)], which for simplification we will designate in further simply $P(n, k)$. These distributions must be represented in

the form of the continuous distribution $P(X)$, which searches for in the form of the resolution of Edgeworth, which has the following form:

$$P(x) = \sum_{\nu=0}^{\infty} \frac{B_{\nu 0}}{\nu!} \varphi_{\nu}(y), \quad (6.3.13)$$

where x - variable, which corresponds to the scale by the variable k , which is only changed not discretely, but continuously $k=x$;

y - standardized/normalized variable, determined by the relationship/ratio

$$y = \frac{x - a_{1p}}{\sigma_p}, \quad (6.3.14)$$

where a_{1p} and σ_p - respectively the average/mean and root-mean-square divergences of the random variables, distributed according to the law of Bernoulli.

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Let us note that $\sigma_p = a_{2p} - a_{1p}^2$, in this case a_{1p} and a_{2p} is moments/torques of distribution $P(n, k)$, but not $P(X)$ [which there is continuous analog of discontinuous distribution $P(n, k)$]; B_{ν} - coefficient of expansion, and $\varphi_{\nu}(y)$ - Hermite's function of ν order [30]:

$$\varphi_{\nu}(y) = e^{-\frac{y^2}{2}} H_{\nu}(y), \quad (6.3.15)$$

where $H_n(y)$ there is n Hermite's polynomial:

$$H_n(y) = (-1)^n e^{\frac{y^2}{2}} \frac{d^n}{dy^n} e^{-\frac{y^2}{2}}; \quad (6.3.16)$$

$$H_0 = 1; H_1 = y; H_2 = y^2 - 1; H_3 = y^3 - 3y; \\ H_4 = y^4 - 6y^2 + 3 \text{ и т. д.} \quad (6.3.17)$$

Key: (1). etc.

Approximate representation of binomial distribution by expansion of Edgeworth leads to replacement of broken line $P(n, k)$, which determines distribution of Bernoulli's law, with smooth curve $P(X)$. In this case if moments/torques of both distribution functions coincide, then smooth curve $P(X)$ will little deviate from broken $P(n, k)$. However, having at its disposal continuous function $P(X)$, it is possible by simple integration to find the interesting us criteria of the quality of detection and other parameters. Expressions for the coefficients of series/row (6.3.13) are determined from the conditions of orthogonality. For determining B , let us multiply both parts of equality (6.3.13) on $H_k(y)$ and will integrate in the two-way infinite gap/interval. Taking into account (6.3.15) we will obtain

$$\int_{-\infty}^{\infty} H_k(y) P(x) dy = \sum_{n=0}^{\infty} \frac{1}{\sqrt{n!}} B_n \int_{-\infty}^{\infty} H_n(y) H_k(y) e^{-\frac{y^2}{2}} dy. \quad (6.3.18)$$

As is known [30], Hermite's polynomials are orthogonal with weight of $e^{-\frac{y^2}{2}}$ in interval $[-\infty, \infty]$, i.e.

$$\int_{-\infty}^{\infty} e^{-\frac{y^2}{2}} H_\nu(y) H_k(y) dy = \begin{cases} 0 & \text{при } \nu \neq k, \\ \sqrt{2\pi} & \text{при } \nu = k. \end{cases} \quad (6.3.19)$$

Key: (1). with.

On the basis of this relationship/ratio it follows that in right side of equality (6.3.18) all terms will be equal to zero, except one, for which $\nu=k$.

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Then taking into account (6.3.19) it follows that

$$B_\nu = \frac{B_\nu}{\sqrt{2\pi} \sigma_p}, \quad (6.3.20)$$

where

$$B_\nu = \int_{-\infty}^{\infty} P_\nu(x) H_\nu(y) dx, \quad (6.3.21)$$

in this case

$$dy = \frac{dx}{\sigma_p}.$$

Substituting in (6.3.21) Hermite's polynomials (6.3.17) and integrating in limits indicated, we will obtain

$$B_0 = \int_{-\infty}^{\infty} P(x) dx = 1,$$

since area, limited by density function of probability distribution, is always equal to one. Let us switch over to computation B_1 .

According to (6.3.21)

$$B_1 = \int_{-\infty}^{\infty} y P(x) dx.$$

Substituting here y from (6.3.14) and taking into account that on approximation/approach conditions moments/torques of distribution $P(k, n)$ and $P(X)$ are equal to one to another, we will obtain

$$B_1 = \frac{1}{\sigma_p} \left[\int_{-\infty}^{\infty} x P(x) dx - a_{1p} \int_{-\infty}^{\infty} P(x) dx \right] = 0,$$

since first moments/torques of function $P(X)$ and $P(n, k)$ are equal to one to another, i.e.,

$$a_1 = \int_{-\infty}^{\infty} x P(x) dx = a_{1p} \quad (6.3.22)$$

Thus, second term is always absent from expansion of Edgeworth, if variable y is calibrated according to relationship/ratio (6.3.14).

Let us switch over to computation of following coefficient. According to (6.3.21)

$$B_2 = \int_{-\infty}^{\infty} (y^2 - 1) P(x) dx.$$

Substituting here y from (6.3.14), we will obtain

$$B_2 = \int_{-\infty}^{\infty} \frac{1}{\sigma_p^2} [x^2 - 2xz_{1p} + z_{1p}^2] P(x) dx - \int_{-\infty}^{\infty} P(x) dx.$$

Taking into account that

$$\alpha_\nu = \int_{-\infty}^{\infty} x^\nu P(x) dx \quad (6.3.23)$$

there is moment/torque of ν order of function $P(X)$, and

$$M_\nu = \int_{-\infty}^{\infty} (x - z_1)^\nu P(x) dx \quad (6.3.24)$$

there is central moment of ν order of the same function, then taking into account (6.3.22) we will obtain

$$M_2 = \int_{-\infty}^{\infty} [x^2 - 2xz_{1p} + z_{1p}^2] P(x) dx = \alpha_2 - \alpha_1^2$$

but very value of coefficient

$$B_2 = \frac{M_2}{\sigma_p^2} - 1 = 0,$$

since second central moments of distributions $P(n, k)$ and $P(X)$ on condition are equal to one to another, i.e., $M_2 = \sigma_p^2 = M_{2p}$. Thus, the third member will also be absent from the expansion of Edgeworth, if variable in is calibrated according to (6.3.14).

From expression (6.3.24) relationships/ratios between simple and central moments of any order can be obtained.

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Actually/really, opening in (6.3.24) the brackets according to the formula of Newton's binomial expression, integrating polynomials piecemeal and producing the necessary conversions, we will obtain

$$\begin{aligned}
 M_2 &= x_2 - x_1^2; \\
 M_3 &= x_3 - 3x_1x_2 + 2x_1^3; \\
 M_4 &= x_4 - 4x_1x_2 + 6x_1^2x_2 - 3x_1^4; \\
 M_5 &= x_5 - 5x_1x_2 + 10x_1^2x_2 - 10x_1^3x_2 + 4x_1^5; \\
 M_6 &= x_6 - 6x_1x_2 + 15x_1^2x_2 - 2x_1^3x_2 + 15x_1^4x_2 - 5x_1^6; \\
 &\dots
 \end{aligned}
 \tag{6.3.24}$$

Let us now move on to computation of coefficient B_3 . According to (6.3.21)

$$B_3 = \int_{-\infty}^{\infty} (y^3 - 3y) P(x) dx.$$

Taking into account (6.3.14) this expression can be rewritten in the following form:

$$B_3 = \frac{1}{\sigma_p^3} \int_{-\infty}^{\infty} (x - a_1)^3 P(x) dx - 3 \frac{1}{\sigma_p} \int_{-\infty}^{\infty} (x - a_1) P(x) dx.$$

Since second term in this formula is equal to zero, finally

$$B_3 = \frac{M_3}{\sigma_p^3}. \tag{6.3.25}$$

Let us switch over to computation of following coefficient.
According to (6.3.21) the coefficient

$$B_4 = \int_{-\infty}^{\infty} (y^4 - 6y^2 + 3) P(x) dx.$$

After substituting y from (6.3.14) and after fulfilling conversions, we will obtain

$$B_4 = \frac{M_4}{\sigma_p^4}. \quad (6.3.26)$$

By analogous computations can be obtained expressions for other leading coefficients:

$$\begin{aligned} B_5 &= \frac{M_5}{\sigma_p^5} + 10 \frac{M_3}{\sigma_p^3}; \\ B_6 &= \frac{M_6}{\sigma_p^6} - 15 \frac{M_4}{\sigma_p^4} + 30, \end{aligned} \quad (6.3.27)$$

.....

where $M_1, M_2, \dots, M_k$ - central moments of continuous distribution $P(X)$, which always can be expressed through simple moments/torques in accordance with formula (6.3.24).

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As noted, these simple moments/torques must correspond to moments/torques α_{np} and M_{np} of discontinuous distribution $P(n, k)$. Therefore for computing the coefficients B_k it is necessary to know

moments/torques $P(n, k)$.

Moment/torque of first order of binomial distribution is determined by following formula:

$$\alpha_{1p} = \sum_{k=0}^n k P(k, n).$$

After substituting here expression $P(k, n)$ from formula (6.3.6) or (6.3.7), we will obtain

$$\alpha_{1p} = \sum_{k=0}^n k C_n^k p^k q^{n-k} \quad (6.3.28)$$

(for generality of reasonings indices in probabilities p_{cm} and q_{cm} are omitted, since obtained subsequently results are applicable for general view of binomial (discrete/digital) distribution, used in different cases).

During computation of formula (6.3.28) we will proceed from Newton's binomial expression

$$(p + q)^n = \sum_{k=0}^n C_n^k p^k q^{n-k}.$$

Differentiating both parts of equality on p , we will obtain

$$n(p + q)^{n-1} = \sum_{k=0}^n k C_n^k p^{k-1} q^{n-k}.$$

After multiplying now equality on p , we will obtain

$$np(p+q)^{n-1} = \sum_{k=0}^n k C_n^k p^k q^{n-k}. \quad (6.3.29)$$

Right side of this equality coincides with expression (6.3.28), which characterizes moment/torque α_{1p} regarding. Consequently,

$$\alpha_{1p} = np(p+q)^{n-1}.$$

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However, since in our case of p and q there are probabilities of conflicting events, then always $p+q=1$, and then finally

$$\alpha_{1p} = np. \quad (6.3.30)$$

Let us switch over to computation of second moment/torque.

Regarding

$$\alpha_{2p} = \sum_{k=0}^n k^2 C_n^k p^k q^{n-k}. \quad (6.3.31)$$

For computing this moment/torque let us differentiate on p of formula (6.3.29):

$$n(p+q)^{n-1} + np(n-1)(p+q)^{n-2} = \sum_{k=0}^n k^2 C_n^k p^{k-1} q^{n-k}.$$

After multiplying both parts of equality on p , we will obtain

$$\begin{aligned} np(p+q)^{n-1} + np^2(p+q)^{n-1}(n-1) = \\ = \sum_{k=0}^n k^2 C_n^k p^k q^{n-k}. \end{aligned} \quad (6.3.32)$$

Since right side of this equality coincides with expression of moment/torque α_{2p} from (6.3.31), left frequently also must be equal to unknown moment/torque; taking into account that $p+q=1$, we have

$$\alpha_{2p} = np + n(n-1)p^2. \quad (6.3.33)$$

Let us calculate third moment/torque α_{3p} . Regarding

$$\alpha_{3p} = \sum_{k=0}^n k^3 C_n^k p^k q^{n-k}. \quad (6.3.34)$$

For computation of α_3 we differentiate (6.3.32) on p and then let us multiply on p :

$$\begin{aligned} np(p+q)^{n-1} + np^2(n-1)(p+q)^{n-2} + \\ + n(n-1)p[2p(p+q)^{n-2} + p^2(n-2)(p+q)^{n-3}] = \\ = \sum_{k=0}^n k^3 C_n^k p^k q^{n-k}. \end{aligned} \quad (6.3.35)$$

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Since right sides (6.3.35) and (6.3.34) coincide, then, equalizing left sides of equalities taking into account, that $p+q=1$,

we will obtain

$$\alpha_3 = np + 3n(n-1)p^2 + n(n-1)(n-2)p^3 \quad (6.3.36)$$

It is possible to obtain expressions for moments/torques of higher order by analogous computations. In particular, differentiating expression (6.3.35) on p , it is possible to obtain formula for α_4 and so forth.

Let us calculate now central moments of distribution. In accordance with (6.3.24a) we have:

$$\begin{aligned} M_{2p} &= \sigma_p^2 = \alpha_{2p} - \alpha_{1p}^2 = npq, \\ M_{3p} &= \alpha_{3p} - 3\alpha_{1p}\alpha_{2p} + 2\alpha_{1p}^3. \end{aligned} \quad (6.3.37)$$

After substituting here expressions for moments/torques from (6.3.36) (6.3.33) and (6.3.30) and after producing necessary conversions, we will obtain

$$M_{2p} = np + np^2(2n-3) + p^3n(2-n^2). \quad (6.3.38)$$

Knowing moments/torques, it is possible to calculate coefficients of Edgeworth series. Subsequently we will be restricted to the section/segment of series/row to the fourth term inclusively ($\nu=3$). Since $B_0=1$, and $B_1=B_2=0$, then the section/segment of series/row will take the form:

$$P_n(x) = \frac{1}{\sqrt{2\pi} \sigma_p} \left\{ \varphi_0(y) - \frac{B_3}{3!} \varphi_3(y) + \dots \right\}. \quad (6.3.39)$$

Value B_3 is determined from (6.3.35) as a result of substitution there (6.3.38) and (6.3.37)

$$B_3 = \frac{1 + p(2n-3) + (2-n^2)p^2}{(1-p) \{ np(1-p) \}}. \quad (6.3.40)$$

Criteria of quality of detection can be found as a result of termwise integration of series/row (6.3.39).

2. Computation of the figures of merit of detection D and F.

For computing probability of correct detection D and false alarm F it is necessary to have two forms of the function of distribution (and respectively two types of expansion of Edgeworth). One - for the functions of distribution $P_c(x)$, which correspond to the presence of signal and noise, and another $P_m(x)$ - to the presence only of noise.

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The form of the types of expansion is identical, they are distinguished by the fact that one depends on probability p_{cm} , and another - from p_m . Let us examine this in more detail based on the example of the computation of the probability of correct detection.

As is known,

$$D = \int_{x_0}^{\infty} P_c(x) dx, \quad (6.3.41)$$

in this case

$$P_c(x) = \frac{1}{\sqrt{2\pi}\sigma_c} \sum_{v=0}^{\infty} \frac{B_{vc}}{v!} \varphi_v(y_c), \quad (6.3.42)$$

where

$$y_c = \frac{x - a_{1c}}{\sigma_c}, \quad (6.3.43)$$

and

$$\begin{aligned} \sigma_c^2 &= np_{cw}(1 - p_{cw}), \\ a_{1c} &= np_{cw}, \\ a_{2c} &= np_{cw} + n(n-1)p_{cw}, \\ a_{3c} &= np_{cw} + 3n(n-1)p_{cw}^2 + n(n-1)(n-2)p_{cw}^3, \\ &\dots \end{aligned} \quad (6.3.44)$$

Here probability p_c is determined by formula (6.3.1).

Substituting these expressions into the formulas for the central moments and the coefficients of the Edgeworth series, we will obtain expression for B_{vc} . In particular, according to (6.3.40)

$$B_{3c} = \frac{1 - p_{cw}(2n-3) + p_{cw}^2(2-n^2)}{(1 - p_{cw})\sqrt{np_{cw}(1 - p_{cw})}}.$$

After substituting (6.3.42) in (6.3.41) and after considering that $\frac{dx}{\sigma_c} = dy$, we will obtain

$$D = \frac{1}{\sqrt{2\pi}} \sum_{v=0}^{\infty} \frac{B_{vc}}{v!} \int_{y_{0c}}^{\infty} \varphi_v(y_c) dy_c, \quad (6.3.45)$$

where changed according to (6.3.43) integration limit

$$y_{0c} = \frac{k_0 - a_{1c}}{a_c} \quad (6.3.45')$$

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For Hermite functions of any order, except $\nu=0$, it is not difficult to obtain following relationship/ratio [see (6.3.15) (6.3.16)]:

$$\int_{y_{0c}}^{\infty} \varphi_{\nu}(y) dy = -[\varphi_{\nu-1}(\infty) - \varphi_{\nu-1}(y_{0c})].$$

Since with upper limit $\varphi_{\nu-1}(\infty)=0$ with any ν ,

$$\int_{y_{0c}}^{\infty} \varphi_{\nu}(y) dy = \varphi_{\nu-1}(y_{0c}).$$

However, integral of function of zero-order Hermite is reduced, as is known, to Kramp function [19; 32]:

$$\Phi_0(y_{0c}) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{y_{0c}} e^{-\frac{y^2}{2}} dy,$$

i.e.

$$\frac{1}{\sqrt{2\pi}} \int_{y_{0c}}^{\infty} e^{-\frac{y^2}{2}} dy = 1 - \Phi_0(y_{0c}).$$

After substituting these expressions in (6.3.45), we will obtain

$$D = 1 - \Phi_0(y'_{0e}) + \sum_{v=1}^{\infty} \frac{B_{ve}}{v!} \frac{\varphi_{v-1}(y'_{0e})}{\sqrt{2\pi}}, \quad (6.3.46)$$

where taking into account (6.3.45') and (6.3.44)

$$y'_{0e} = \frac{k_0 - np_{em}}{\sqrt{np_{em}(1-p_{em})}}.$$

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It is possible to calculate probability of false alarm

$$F = 1 - \Phi_0(y'_{0m}) + \sum_{v=1}^{\infty} \frac{B_{vm}}{v!} \frac{\varphi_{v-1}(y'_{0m})}{\sqrt{2\pi}}, \quad (6.3.47)$$

via analogous reasonings where

$$y'_{0m} = \frac{k_0 - np_m}{\sqrt{np_m(1-p_m)}},$$

and values of coefficients B_{vm} are calculated in accordance with formulas given above, if we in them substitute moments/torques, which characterize noise distribution:

$$\begin{aligned} \sigma_m^2 &= np_m(1-p_m); \\ \alpha_{1m} &= np_m; \\ \alpha_{2m} &= np_m + n(n-1)p_m; \\ \alpha_{3m} &= np_m + 3n(n-1)p_m^2 + n(n-1)(n-2)p_m^3; \\ &\dots \end{aligned}$$

Here probability p_m is determined by formula (6.3.5).

Tables of functions of Kramp and Hermite are given in the

literature [19].

Taking into account that in expansion of Edgeworth second and third members are equal to zero ($B_1=B_2=0$), it is expedient as approximate result to utilize components/terms/addends of formula (6.3.46) or (6.3.47). As a result we will obtain the following relationships/ratios:

$$D \approx 1 - \Phi_0 \left(\frac{k_0 - np_{cm}}{\sqrt{np_{cm}(1-p_{cm})}} \right), \quad (6.3.48)$$

$$F \approx 1 - \Phi_0 \left(\frac{k_0 - np_{m}}{\sqrt{np_{m}(1-p_{m})}} \right). \quad (6.3.49)$$

These formulas can be utilized for determining different parameters of circuit of detector, in particular threshold number k , at assigned probabilities F and D and so forth.

C. Determination of the most advantageous threshold of quantization.

We will consider that optimum threshold value of quantization U , will be such, which with smallest quantity of impulses/momenta/pulses in packet will ensure given value of probability of correct detection D and minimum value of false alarm F . This value U , is optimum in the sense of criterion Neumann-Pearson, providing in this case minimum n .

Let us examine computation of optimum U , in accordance with procedure, proposed in [33], based on example of particular value $D=0.5$. Then from (6.3.48) it follows that

$$\Phi_0\left(\frac{k_0 - np_{cm}}{\sqrt{np_{cm}(1-p_{cm})}}\right) = 0.5.$$

Consequently,

$$\frac{k_0 - np_{cm}}{\sqrt{np_{cm}(1-p_{cm})}} = 0,$$

whence

$$k_0 = np_{cm}^*. \quad (6.3.50)$$

After substituting this relationship/ratio in (6.3.49), we will obtain

$$F = 1 - \Phi_0\left(\sqrt{n} \frac{p_{cm} - p_m}{\sqrt{p_m(1-p_m)}}\right). \quad (6.3.51)$$

So that probability of false alarm would be minimum, it is necessary to obtain maximum value of function $\Phi_0\left(\sqrt{n} \frac{p_{cm} - p_m}{\sqrt{p_m(1-p_m)}}\right)$. For this its argument must have maximum value. Since the sample size is limited, the greatest value of argument can be obtained due to an increase in the second factor, i.e., maximum must be the relationship/ratio

$$\frac{p_{cm} - p_m}{\sqrt{p_m(1-p_m)}} = \max. \quad (6.3.52)$$

Expression (6.3.52) depends exclusively on statistic structure of mixture signal + noise (its distribution functions), and also value of threshold U_0 . Establishing/installing dependence explicitly between p_{cm} and p_m , they calculate the extremum of expression (6.3.52), whence is determined optimum value p_{m0} . For the establishment of conformity between p_{cm} and p_m is utilized their overall dependence on threshold U_0 . Let us assume that this dependence exists and can be designated in the form

$$p_{cm} = \psi(p_m). \quad (6.3.53)$$

FOOTNOTE 1. This procedure can be propagated also to other values of $D=\beta$, if we in (6.3.50) place dependence $k_0 = \mu(\beta) p_{cm}$, where $\mu(\beta)$ - the constant, which depends on β . ENDFOOTNOTE.

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After substituting this expression in (6.3.52), we will obtain function only of one variable p_m

$$\frac{\psi(p_m) - p_m}{\sqrt{p_m(1-p_m)}} = \varphi(p_m). \quad (6.3.54)$$

From the solution of extreme problem for this expression optimum value $p_m = p_{m0}$, which rotates this function, is found into the maximum.

Character of computations and structure of final results depend on concrete/specific/actual form of the function of distribution $W_{cm}(\rho_i)$ and $W_{ur}(\rho_i)$. For example, for the distribution functions, which correspond to generalized Rayleigh distribution (5.1.10), as noted above, they occur the following relationships/ratios:

$$p_{cw} = \int_{U_0}^{\infty} \frac{\rho}{\sigma^2} \exp\left(-\frac{\rho^2 + A^2}{2\sigma^2}\right) I_0\left(\frac{A\rho}{\sigma^2}\right) d\rho,$$

$$p_w = \int_{U_0}^{\infty} \frac{\rho}{\sigma^2} \exp\left(-\frac{\rho^2}{2\sigma^2}\right) d\rho = \exp\left(-\frac{U_0^2}{2\sigma^2}\right).$$

From second formula relationship/ratio between probability and threshold of quantization U_0 , is obtained in the form

$$U_0 = \sigma \sqrt{2 \ln \frac{1}{p_{w0}}}.$$

Let us switch over to the determination of the dependence between p_{ur} and p_{cw} . After producing obvious conversions in the first of two formulas, we will obtain

$$p_{cw} = 1 - \int_0^{U_0} \frac{\rho}{\sigma^2} \exp\left(-\frac{\rho^2 + A^2}{2\sigma^2}\right) I_0\left(\frac{A\rho}{\sigma^2}\right) d\rho.$$

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Let us conduct the replacement of the variable

$$t = \exp\left(-\frac{\rho^2}{2\sigma^2}\right); \quad dt = -\frac{\rho d\rho}{\sigma^2} \exp\left(-\frac{\rho^2}{2\sigma^2}\right);$$

$$d\rho = \frac{-\sigma^2 dt}{\rho \exp\left(-\frac{\rho^2}{2\sigma^2}\right)};$$

$$\ln t = -\frac{\rho^2}{2\sigma^2}; \quad \ln \frac{1}{t} = \frac{\rho^2}{2\sigma^2}; \quad \rho = \sigma \sqrt{2 \ln \frac{1}{t}};$$

with $\rho=0$; $t=1$ taking into account (6.3.5) with $\rho=U$, we obtain

$$t = t_0 = e^{-\frac{U^2}{2\sigma^2}} = p_{uw}.$$

Then form of the function (6.3.53) for case (generalized functions of Rayleigh distribution) in question will take form

$$p_{cw} = 1 - \int_{p_{uw}}^1 \exp\left(-\frac{a^2}{2}\right) I_0\left(a \sqrt{2 \ln \frac{1}{t}}\right) dt, \quad (6.3.55)$$

where $a=A/\sigma$ there is signal-to-noise ratio. The graph/diagram of dependence p_{cw} on p_{uw} , determined by formula (6.3.55), is given in Fig. 6.7 for the different value of signal-to-noise ratio.

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Substituting (6.3.55) in (6.3.54), it is possible to show [33] that with low signals when $a \ll 1$, maximum value $\varphi(p_{uw})$ begins with

$$p_{uw0} = 0.2, \quad (6.3.56)$$

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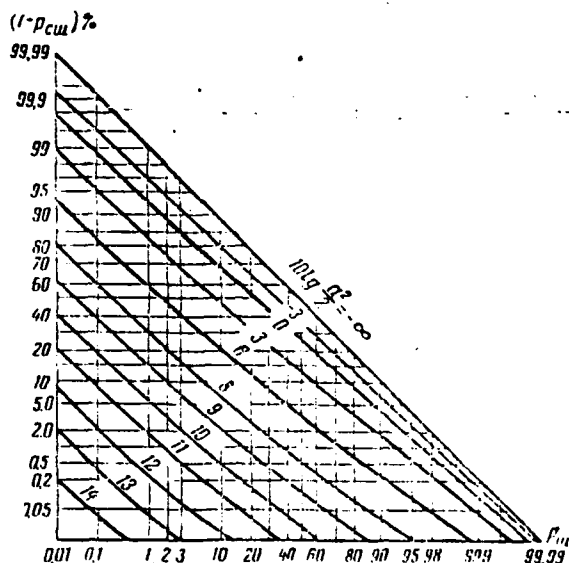


Fig. 6.7. Graph/diagram of dependence (6.3.55).

In the case of low signals the same value p_{min} can be obtained by simpler, more approximate method, based on approximate representation of generalized Rayleigh distribution (5.1.10). When $A/\sigma \ll 1$, relationship/ratio (5.1.10) can be represented approximately in the form [28]

$$W_{cw}(p) = \frac{p}{\sigma^2} \left(1 - \frac{A^2}{2\sigma^2}\right) \exp\left[-\frac{p^2}{2\sigma^2} \left(1 - \frac{A^2}{2\sigma^2}\right)\right]. \quad (6.3.57)$$

Then

$$p_{cw} = \int_{U_0}^{\infty} \frac{p}{\sigma^2} \left(1 - \frac{A^2}{2\sigma^2}\right) \exp\left[-\frac{p^2}{2\sigma^2} \left(1 - \frac{A^2}{2\sigma^2}\right)\right] dp.$$

After fulfilling integration, we will have

$$p_{cu} = \exp \left[-\frac{U_0^2}{2\sigma^2} \left(1 - \frac{A^2}{2\sigma^2} \right) \right].$$

After comparing this relationship/ratio with formula (6.3.5), we will obtain simple correlation

$$p_{cu} = p_u \cdot \frac{1 - \frac{A^2}{2\sigma^2}}{\frac{p_u}{a^2}} = \frac{p_u}{\frac{a^2}{2}}. \quad (6.3.58)$$

After substituting this dependence in (6.3.54), we will obtain

$$\varphi(p_u) \approx \frac{a^2}{2} \sqrt{\frac{p_u}{1-p_u}} (-\ln p_u). \quad (6.3.59)$$

Optimum value p_u is determined from condition

$$\frac{d}{dp_u} \varphi(p_u) = 0.$$

This equality after substitution here (6.3.59) is reduced to transcendental equation

$$p_{u0} = e^{2(p_{u0}-1)},$$

solution of which is value $p_{u0} \approx 0.2$, indicated above. Hence most advantageous threshold value according to (6.3.5)

$$U_{opt} = \sigma \sqrt{2 \ln 5}. \quad (6.3.60)$$

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D. Determination of the parameters of detector.

Calculation of parameters of detector in this case is reduced to determination of threshold of accumulation k , with permissible number of impulses/momenta/pulses in packet n from preset parameters of quality of detection (D and F) depending on relationship/ratio of signal to noise $A/\sigma=a$. The probabilities of the correct detection D and the false alarm F are assigned on the basis of the conditions for the tactical-technical use of a device/equipment of detection. During their assignment full weight of the considerations, which determine the undesirability of false alarm or permissible probability of the passage of target $\bar{D}=1-D$, must be considered.

Value of signal-to-noise ratio is determined from energy characteristics of radar and can be calculated with the aid of equation of range. Virtually most interesting the cases corresponds to a small level of the signal, when the target detected is located at maximum range. For this case we have determined the most advantageous threshold value of quantization $U_{0 \text{ opt}}$ and probability

$p_{uo}=0.2$. corresponding to it. Furthermore, in work [33] it shows that the same value p_{uo} can be utilized, also, on the high signal level. Therefore subsequently we will use this value p_{uo} as the optimum.

Necessary values k_0 for large n ($n>5$) are designed from approximation formulas (6.3.48) and (6.3.49); for any n are utilized formulas of binomial distribution (6.3.11) and (6.3.12), suitable in all cases.

During use of formulas (6.3.48) and (6.3.49) calculation for signals, distributed according to formula (5.1.10), is conducted in this sequence:

1. In terms of the given value of $a=A/\sigma$, taking into account that $p_{uo}=0.2$, is determined probability p_{em} which characterizes excess by mixture signal + the noise of the threshold of quantization U_0 . Determination p_{em} is conducted with the aid of the graph/curve (Fig. 6.7), and for small signal-to-noise ratios - with the aid of simple correlation (6.3.58).

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2. In terms of given values of D and F boundary extreme values of arguments of probability integral, which ensure obtaining assigned

magnitudes D and F, are determined. In accordance with (6.3.58) and (6.3.59)-the arguments of probability integrals must satisfy the following relationships/ratios:

$$y_{0c} \geq \Phi_0^{-1}(1-D),$$

$$y_{0w} \leq \Phi_0^{-1}(1-F),$$

where $\Phi_0^{-1}(y)$ - the function, reverse/inverse to the integral of Kramp, and arguments are respectively equal to:

$$y_{0c} = \frac{k_0 - np_{cw}}{\sqrt{np_{cw}(1-p_{cw})}}, \quad (6.3.61)$$

$$y_{0w} = \frac{k_0 - 0,2n}{\sqrt{n0,16}}, \quad (6.3.62)$$

in this case in the formula for y_{0c} value p_{cw} was determined in p. 1.

3. From formulas (6.3.61) and (6.3.62) determine smallest values of n and $k_0 < n$, that satisfy these relationships/ratios.

Parameters of feeler for any n should be selected with the aid of formulas of binomial distribution (6.3.11) and (6.3.12), since they are most general/most common/most total. To utilize them is expedient with a small number of impulses/momenta/pulses in the packet, when the preceding/previous simplified formulas prove to be insufficiently precise. For convenience in the use of the formulas of binomial distributions in work [34] the series of the graphs/curves (Fig. 6.8-6.11), which mutually connect values a, n, F and D, is calculated. The calculation of the given graphs/curves was conducted

in the following order. Were fixed/recorded probabilities D and F, and also numbers of impulses/momenta/pulses n. Then according to the formula [of type (6.3.11) or (6.3.12)]

$$P(n, p, k_0) = \sum_{k=k_0}^n C_n^k p^k (1-p)^{n-k}$$

was conducted the calculation of the auxiliary tables of this sum for different combinations k_0 and p .

From these tables of the values of the probabilities p are selected for each value k_0 , such values $p=p_1$ and $p=p_2$ ($p_2 > p_1$), with which

$$P(n, p_1, k_0) = F$$

and

$$P(n, p_2, k_0) = D.$$

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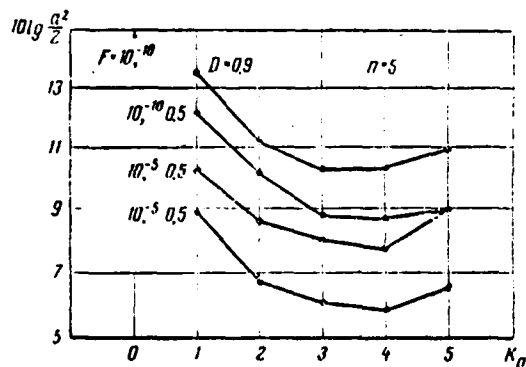


Fig. 6.8.

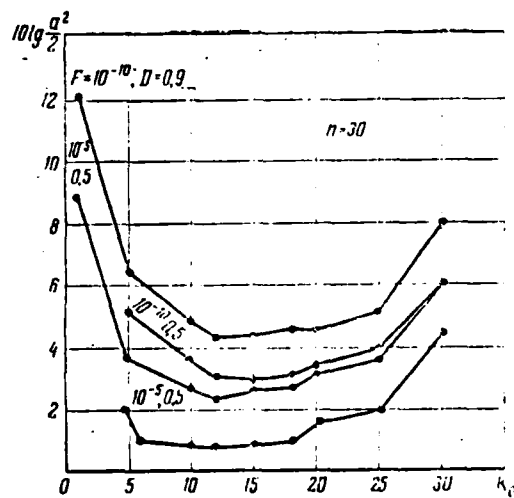


Fig. 6.9.

Fig. 6.8. Graphs/curves of interdependence a , n , F and D .Fig. 6.9. Graphs/curves of interdependence a , n , F and D .

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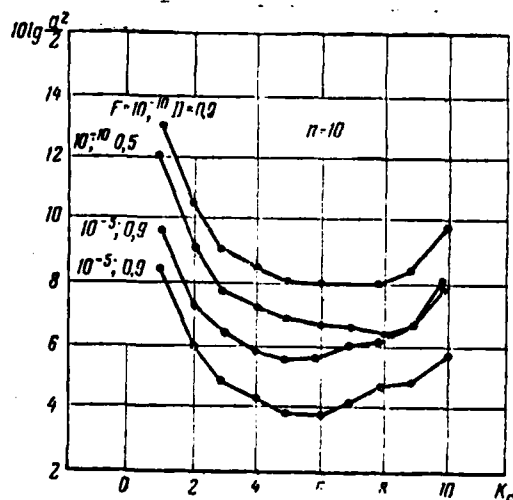


Fig. 6.10.

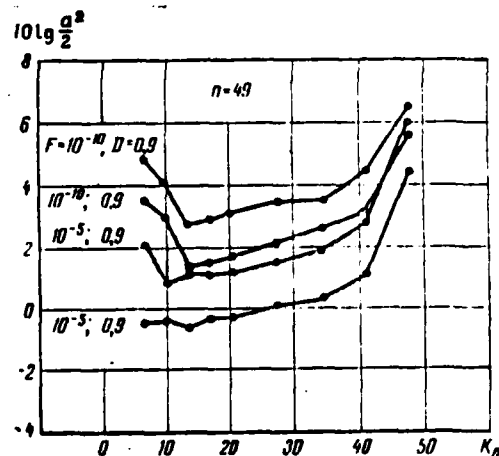


Fig. 6.11.

Fig. 6.10. Graphs/curves of interdependence a , n , F and D .Fig. 6.11. Graphs/curves of interdependence a , n , F and D .

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Here occurs obvious conformity: $p_1 = p_{10}$ and $p_2 = p_0$. However, in the known values of probabilities p_0 and p_{10} there can be specifically necessary signal-to-noise ratio a , that corresponds to the fixed value of k_0 . The parameter a is determined from formula (6.3.58) or with the aid of the graph/curve, given in Fig. 6.7. Furthermore,

according to formula (6.3.62) can be determined the value of the threshold of quantization U_0 , which corresponds to the data k_0 .

From given graphs/curves it follows that for each n there is interval of optimum ones threshold value of accumulation k_0 , with which a (signal-to-noise ratio) is minimal. The optimum value k_0 can be determined by empirical dependence [34]

$$k_{0, \text{opt}} \approx 1,5 \sqrt{n}.$$

Thus, for fixed/recorded D , F and n it is possible to speak about optimum totality of thresholds of accumulation k_0 and quantization $U_0 = \sigma \sqrt{2 \ln \frac{1}{p_m}}$, that correspond to minimum signal-to-noise ratio.

Especially simply are selected parameters for some quotients threshold value of quantization k_0 . Thus, for instance, with $k_0=1$ relationship/ratio (6.3.11) and (6.3.12) they take the following form:

$$\begin{aligned} D &= 1 - (1 - p_c)^n; \\ F &= 1 - (1 - p_m)^n. \end{aligned}$$

This threshold of accumulation is accepted during detection of packets with small number of impulses/momenta/pulses and corresponds to criterion "at least one of n ". If D and F are assigned, then the probability

$$p_m = 1 - (1 - F)^{\frac{1}{n}};$$

$$p_c = 1 - (1 - D)^{\frac{1}{n}}.$$

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Knowing these probabilities, it is possible to determine threshold of quantization U , and signal-to-noise ratio a , with which can be achieved/reached preset parameters of quality D and F with selected n . The parameter a and threshold U , are determined from the graph/curve Fig. 6.7.

Another special case of threshold of quantization $k, = n$ corresponds to very rigid criterion of detection of type "n of impulses/momenta/pulses from n". In this case

$$D = p_c^n; \quad F = p_m^n;$$

These relationships/ratios, just as in the preceding case, make it possible to determine values U , and and by analogous methods.

However, it must be noted that in usual formulation of problem of determining parameters of detector value of threshold k , is not initial, since it does not determine detection time or other tactical-technical parameters of feeler (for example, D and F). The

detection time, as is known, is determined by a quantity of impulses/momenta/pulses n in the packet (with sample size), whose permissible value is assigned as the initial parameter. In accordance with assigned n , F and D it is necessary to establish/install and U , ensuring reaching/achievement given ones by F and D in the minimum signal-to-noise ratio.

Presence of optimum k , can be explained as follows. If the threshold of accumulation k , is selected small, then this means that must be reduced the number of threshold crossings of quantization U , both from the signal ones and from the noise overshoots. This can be achieved/reached as a result of an increase in the threshold of quantization (limitation). But in this case signal can be discovered with assigned magnitude D only in such a case, when signal-to-noise ratio is great (left segment of a curve of Fig. 6.8-6.11).

With high value of k , level of quantization must be small, which makes it possible to expect increase in quantity of noise overshoots, which exceeded threshold of limitation.

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However, so that the probability of the detection of useful signal would reach desired value of D , was necessary, just as is earlier,

the high value of signal-to-noise ratio (in this case the right region of the curves Fig. 6.8-6.11). In such a manner both in the first and in the second case the level of signal-to-noise ratio must be high. Then should be expected optimum threshold value k_{opt} , which corresponds to minimum signal-to-noise ratio and which is located in the gap/interval between one and n .

§6.4. Optimum detection of incoherent impulses/momenta/pulses with the fluctuating (flickering) amplitude.

Echo pulse with flickering amplitude is observed when diffusing surface of target in pause between impulses/momenta/pulses sufficiently rapidly changes its value. This will occur when the duration of the pulse repetition period of radar T considerably exceeds the time of the correlation of these random changes, i.e., when the correlation between the impulses/momenta/pulses within the packet is absent. In this case the functions of the distribution of mixture signal + noise for the separate impulse/momentum/pulse are determined by the relationships/ratios, obtained in §5.2, namely:

$$W_{\text{cw}}(\rho_i) = \frac{\rho_i}{1 + B_i^2} e^{-\frac{\rho_i}{2(1+B_i^2)}},$$

where $B_i = \frac{\sigma_{ci}}{\sigma}$, and σ_{ci} and σ - dispersions a change in the signal and interference (noise); the function of noise distribution with ($B=0$)

$$W_m(\rho_i) = \rho_i \exp\left(-\frac{\rho_i^2}{2}\right).$$

This form of the function of distribution with certain approximation/approach they can characterize statistical properties of sum of fluctuating interferences and signal, reflected from aircraft, some types of rockets and from other targets, which rapidly change their diffusing surface.

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Structure of optimum receiver, as before will be determined from threshold criterion for likelihood ratio:

$$\prod_{i=1}^n \frac{W_{\text{sig}}(\rho_i)}{W_m(\rho_i)} > l_0.$$

Assuming that root-mean-square changes in signal of separate impulses/momenta/pulses are identical, i.e., $B_1=B_2=\dots=B_n=B$, since $\sigma_{1c}=\sigma_{2c}=\dots=\sigma_{nc}=\sigma_c$, it is possible to write criterion of likelihood ratio for packet of n impulses/momenta/pulses in the form

$$\frac{1}{(1+B^2)^n} \prod_{i=1}^n \exp \frac{\rho_i^2 B^2}{2(1+B^2)} > l_0 \quad (6.4.1)$$

After passing with the aid of logarithmic operation to more convenient criterion, we will obtain

$$n \ln \frac{1}{1+B^2} + \sum_{i=1}^n \frac{\rho_i^2 B^2}{2(1+B^2)} > \ln l_0.$$

or after conversion

$$\sum_{i=1}^n \frac{\rho_i^2 B^2}{1+B^2} > \ln l_0 (1+B^2)^n.$$

Leaving on the left side of equality only selective values of received signal, finally we have

$$\sum_{i=1}^n \rho_i^2 \geq C(n), \quad (6.4.2)$$

where

$$C(n) = \frac{2}{B^2} (1+B^2) \ln l_0 (1+B^2)^n. \quad (6.4.3)$$

Threshold value $\underset{C}{s}(n)$ can be calculated through known values B , n and l_0 . Depending on the value of the echo signal, i.e., from value B , can be obtained simpler expressions for the threshold coefficient $\underset{C}{s}(n)$.

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With low signal, when $\sigma_r \ll \sigma_{\text{int}}$ and $B \ll 1$, from (6.4.3) follows, that

$$C(n) = -\frac{2}{B^2} \ln l_0. \quad (6.4.4)$$

from which it is clear that threshold $\underset{C}{s}(n)$ does not depend on number of selective values n .

With large signal, when $\sigma_c \gg \sigma_{iw}$ and $B \gg 1$, from (6.4.3) follows that

$$C(n) = 2 \ln l_0 B^{2^n}. \quad (6.4.5)$$

As can be seen from (6.4.2), structure of optimum feeler is reduced to addition of squares of selective values (p_i^2) and comparison of sum with threshold. This working/treatment is necessary both for the weak and for the strong signal. The same processing of signal was performed during the detection of incoherent impulses/momenta/pulses with the constant amplitude on a small level of signal (6.2.7). Difference consists only of the value of the threshold of accumulation $\zeta(n)$, determined in each case by its relationships/ratios. However, the structure of the diagram of optimum feeler will be the same as in Fig. 6.3, i.e., envelopes are squared, then they are supplied on the converter of the type "voltage - the code" and then they are summarized.

Computation of figures of merit in this case can be carried out by the same methods as in §6.2, p. B, since processing signal here also is reduced to addition of squares of selective values.

§6.5. Detection of incoherent pulse signals with the quantization on ν levels.

Detection according to method of binary integration is convenient from point of view of technical realization; however, amplitude quantization used here to two levels leads to known loss of information relative to content of signal in noise.

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True, this loss of information can be compensated by increase of quantity of impulses/momenta/pulses in packet; however, this leads to increase in duration of detection, which is not always acceptably for tactical reasons.

Ideal integration, examined in §6.2, is more effective method of detection, but it requires use/application of complicated devices/equipment of analog memorization (charge-storage tubes, magnetic drums, etc.) or conversions of analog quantity into discrete/digital (PND), whose use/application in number of cases is undesirable.

Use/application in schematic of detector of storage devices/equipment, which use continuous integrators, in number of cases is undesirable in connection with possibility of their "saturation" and partial loss of initial information.

In connection with this method of detection of pulse signals with quantization by several levels is of interest.

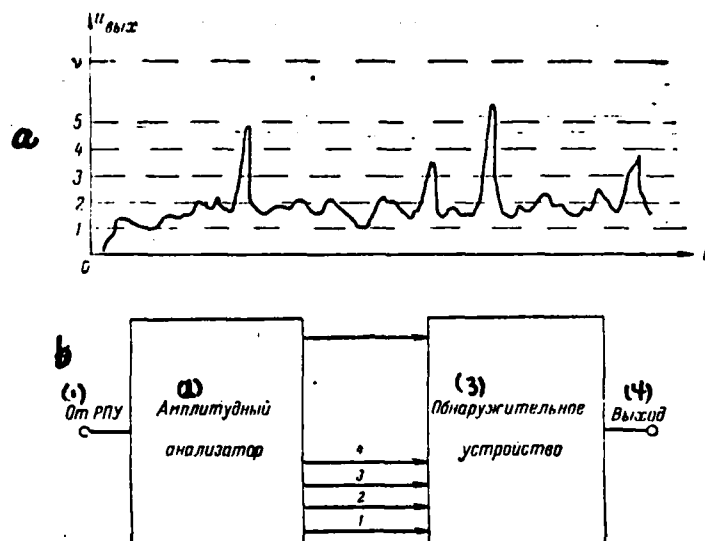


Fig. 6.12. Detection with the quantization on ν levels.

Key: (1). From RPU. (2). Pulse-height analyzer. (3). Detector. (4). Output.

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With this method samples/specimens of envelopes being investigated in each elementary section preliminarily are quantized according to amplitude sign/criterion on ν levels (Fig. 6.12a) with the aid of N-channel pulse-height analyzer. Signals from the output of analyzer enter the detector (Fig. 6.12b), which must make a decision about presence or absence of signal on the base of the analysis of the totality of the signals, which come from analyzer.

Appearance at given moment of signal at output of separate channel of pulse-height analyzer is determined by current level of signal amplitude at input. If input signal level is found in the range of the i interval of quantization, then at the output of the i channel of the pulse-height analyzer the impulse/momentum/pulse of standard form will appear. A quantity of signals, which entered into different channels, depends on the statistical properties of input signal. If to the input of analyzer only noise signal is supplied, then, apparently, the greatest quantity of signals should be expected in the channels of analyzer, which correspond to small levels of amplitude. Under the influence of mixtures signal + noise the greatest quantity should be expected in the channels of larger level. If we fix n of the samples/specimens of input signal, which are subject to investigation in each elementary range channel, then on the base of analysis and calculation of impulses/momenta/pulses in the separate channels of the pulse-height analyzer it is possible to make conclusion about that, presents the totality being investigated the samples/specimens of noise or the samples/specimens of mixture signal + noise.

Determination of block diagram of detector, which makes decision about presence or absence of signal according to data of signal on

output of analyzer, is our most immediate task.

A. Structure of optimum feeler.

Block diagram of optimum detector, as usual, is determined from most general consideration - criterion of minimum of average/mean risk.

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This criterion, as is known, is reduced to the comparison of likelihood ratio with the threshold l_0 . (S6.1):

$$\frac{W_{cnn}}{W_{nn}} > l_0$$

where W_{cnn} and W_{nn} - function of the plausibility of the n -dimensional selections of mixture signal + the noise and of pure/clean noise, supplied to the device/equipment of detection.

Entering this inequality functions of plausibility in the case of experiment and detection of signal at output ν -channel pulse-height analyzer are probability $P(k_1, k_2, \dots, k_n)$ of outcome, with which of n of samples/specimens (signals) being investigated exactly k_1 samples/specimens (signals) it will prove to be in first channel, is exact k_2 - in second channel ... and is exact k_n samples/specimens into ν channel. This probability is determined

by the multinomial law of distribution of probabilities, which takes form [19, 28]

$$P(k_1, k_2, \dots, k_v; n) = \frac{n!}{k_1! k_2! \dots k_v!} p_1^{k_1} p_2^{k_2} \dots p_v^{k_v}, \quad (6.5.1)$$

where p_i - probability of the entry of the sample/specimen being investigated into the i channel of analyzer.

In the case of detection of radar signals are two types of samples/specimens of signal: samples/specimens of mixture signals + noise also of noise. Will respectively occur two multinomial distributions: signal + the noise

$$P_{cw}(k_1, k_2, \dots, k_v; n) = W_{cw}$$

and the noise

$$P_w(k_1, k_2, \dots, k_v; n) = W_w.$$

Let p_{wi} and p_{cwi} be probabilities of fact that samples/specimens of signals, which respectively relate to noise and totality signal + noise, are within the limits of i interval.

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Let us designate the functions of distribution of both signal aspects at the input of the pulse-height analyzer respectively through W_w and W_{cw} , then the probabilities indicated will be determined by the obvious relationships/ratios:

$$p_{cm_i} = \int_{\Delta p_i} W_{cm}(\rho) d\rho; \quad p_{m_i} = \int_{\Delta p_i} W_m(\rho) d\rho, \quad (6.5.2)$$

where $\Delta p_i = p_{i+1} - p_i$ - range (width) of the i interval of quantization. The functions of distribution W_{cm} and W_m are determined by the statistic structure of signal and interference; some forms of these functions are given in Chapter 5.

Multinomial distribution, which corresponds to noise samples/specimens at input of analyzer, is equal

$$W_m = P_m(k_1, k_2, \dots, k_v; n) = \frac{n!}{k_1! k_2! \dots k_v!} \prod_{i=1}^v p_{m_i}^{k_i}, \quad (6.5.3)$$

and distribution for mixture signal plus noise

$$W_{cm} = \frac{n!}{k_1! k_2! \dots k_v!} \prod_{i=1}^v p_{cm_i}^{k_i}. \quad (6.5.4)$$

Then the coefficient of plausibility in accordance with (6.1.18)

$$\prod_{i=1}^v \left(\frac{p_{cm_i}}{p_{m_i}} \right)^{k_i} > l_0. \quad (6.5.5)$$

Passing to more convenient logarithmic criterion, we obtain formula, which determines structure of optimum receiver:

$$z = \sum_{i=1}^v k_i \delta_i > \ln l_0, \quad (6.5.6)$$

where

$$\delta_i = \ln \frac{p_{cm_i}}{p_{m_i}}. \quad (6.5.7)$$

From obtained relationship/ratio it is evident that structure of optimum receiver, as in preceding/previous cases, it was obtained threshold, i.e., diagram must accumulate specific sum and then compare it with threshold I_0 . Working/treatment itself was reduced to accumulation k_i of the impulses/momenta/pulses, obtained in the separate i channels with the subsequent addition of the results of accumulation with weight coefficients δ_i .

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Since probabilities p_{ei} and p_{mi} characterize an average number of expected appearances of a signal in the i interval in the presence and the absence of signal, the value of weight factor proves to be equal to the logarithm of the ratios of expected values k_i in the presence of signal to expected value k_i in its absence. Thus, the sense of weight coefficient consists of the emphasis of the weight of the large amplitudes of the accumulated signal in comparison with small ones. This will become obvious, if one considers that on the high levels of signals $p_{ei} > p_{mi}$ and with small ones, on the contrary, $p_{mi} > p_{ei}$. However, the solution about the presence of signal in this elementary section is accepted when the resulting sum z will exceed threshold $\ln I_0$.

Exemplary/approximate diagram, which realizes criterion (6.5.6), is represented in simplified view of Fig. 6.13. Diagram consists of the pulse-height analyzer with ν outlets. In each channel is located the generator of the series of standard impulses/momenta/pulses (GSSI), which puts out pulse train under the influence on it of signal from the output of the i channel of analyzer, and also counter C_i , which come the impulses/momenta/pulses from GSSI.

Quantity of impulses/momenta/pulses in series on specific scale corresponds to value of weight coefficient α_i . Since the signals from the output of analyzer enter comparatively slowly with the pulse repetition frequency of RLS, quantity of impulses/momenta/pulses in the series, which characterizes α_i can be selected if necessary sufficient to large ones. The account of weight factor α_i with other means, in particular with the aid of the multiplying circuits, is connected with the great circuit difficulties.

Resulting accumulation of impulses/momenta/pulses usually is conducted in schematic of trigger counter C_i with modulus/module of translation, equal to threshold of accumulation l_i .

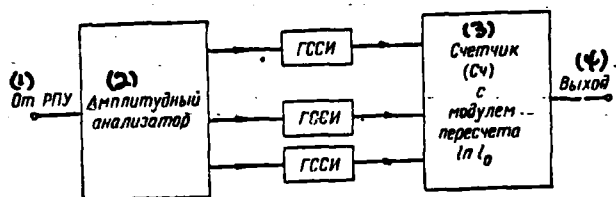


Fig. 6.13. Exemplary/approximate diagram of feeler.

Key: (1). From RPU. (2). Pulse-height analyzer. (3). Counter (mf/SCh) with modulus/module of translation $\ln l_0$. (4). Output.

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If a quantity of impulses/momenta/pulses exceeds threshold, then counter puts out the standard impulse/momentum/pulse, which testifies about the presence of useful signal in this elementary section of range. If it does not exceed, then standard impulse/momentum/pulse is absent.

Values of weight factors must be known previously, since they are determined by intervals of quantization and by statistical properties of signals, which we assume/set by given ones.

Relationships/ratios (6.5.6) and (6.5.7) determine structure of processing different form of uncorrelated signals during detection in

the case of multi-threshold quantization. The computation of weight coefficients for concrete/specific/actual signal aspect makes it possible to usually simplify expression for δ_i . In this case are simplified sometimes the formulas, which determine the structure of working/treatment. Let us give an example of the computation of weight factors during the detection of the nonfluctuating impulses/momenta/pulses, the function of distribution of which is determined by formula (5.1.10).

Let us preliminarily note that probabilities p_{cwi} and p_{wi} areas being under functions of distribution $W_{cw}(\rho)$ and $W_w(\rho)$ in interval of axis of abscissas $\Delta\rho_i$ can be represented in the form:

$$p_{cwi} = W_{cw}(\rho_{i1}) \Delta\rho_i \quad (6.5.8)$$

$$p_{wi} = W_w(\rho_{i2}) \Delta\rho_i$$

where ρ_{i1} and ρ_{i2} - internal points of this interval. With the small intervals of quantization, and also if the distribution functions can be considered changing linearly in these sections, can occur the equality

$$\rho_{i1} = \rho_{i2} = \rho_{i0}$$

where ρ_{i0} - the midpoint of interval $\Delta\rho_i$.

FOOTNOTE ¹. The need for the introduction of second index (1 or 2) in ρ_{i1} and ρ_{i2} is caused by the nonlinearity of density function $W_{cw}()$ and $W_w()$, since with identical $\Delta\rho_i$ of point ρ_{i1} and ρ_{i2} that satisfy

(6.5.8), cannot coincide one with another; therefore will have to distinguish them special index. ENDFOOTNOTE.

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In this case relationships/ratios (6.5.8) can be represented in the form

$$p_{cm_i} \approx W_{cm}(\rho_{i0}) \Delta \rho_i, \quad (6.5.9)$$

$$p_{un_i} \approx W_{un}(\rho_{i0}) \Delta \rho_i.$$

Substituting (6.5.9) in (6.5.7), we have

$$\delta_i = \ln \frac{W_{cm}(\rho_{i0})}{W_{un}(\rho_{i0})}. \quad (6.5.10)$$

Utilizing now these or other values of functions of distribution $W_{cm}(\rho_{i0})$ and $W_{un}(\rho_{i0})$ for specific signal aspect, we will obtain concrete/specific/actual expressions of weight coefficients, in particular, for signals with constant amplitude, substituting in (6.5.10) function of distribution (5.1.9), we obtain

$$\delta_i = -\frac{a}{2} + \ln I_0(a\rho_{i0}), \quad (6.5.11)$$

where $\rho_{i0} = \frac{P_i}{\sigma}$; - $a = A/\sigma$ - signal-to-noise ratio.

Substituting (6.5.11) in (6.5.6), it is possible to obtain further simplifications in formula for δ_i . Taking into account,

$\sum_{i=1}^n k_i = n$, we will obtain

$$\sum_{i=1}^n k_i \ln I_0(a p_{i0}) > \ln I_0 + \frac{na^2}{2}. \quad (6.5.12)$$

Utilizing asymptotic representations for logarithm of Bessel functions, given in formula (6.2.6), it is possible to obtain simpler expressions δ_i , which correspond to channels of analyzer with small and large amplitudes, and, substituting them in (6.5.12), respectively structure of working/treatment in the case of low and large signals.

With low signals, when $a \ll 1$, formula (6.5.12) passes into inequality

$$\sum_{i=1}^n k_i p_{i0}^2 > C, \quad (6.5.13)$$

where

$$C = 2 \frac{\ln I_0}{a^2} + n.$$

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But in the case of high levels of signals, when $a \gg 1$, formula (6.5.12) can be rewritten in the form

$$\sum_{i=1}^n k_i p_{i0} > C_1, \quad (6.5.14)$$

where

$$\bar{C}_t = \frac{\ln I_0}{a} + \frac{\pi}{2}$$

From formulas (6.5.13) and (6.5.14) it follows that weight coefficient in the case of low and large signals is respectively equal to ρ_{10}^2 and ρ_{10}^{-1} .

B. Figures of merit of detection.

Quantitative characteristics of detector are determined by figures of merit of detection, i.e., by probabilities of correct detection D and false alarm F, by their dependence on circuit parameters. By circuit parameters here are understood a quantity of impulses/momenta/pulses in packet n, a signal-to-noise ratio a and number of levels of quantization ν .

Fig. 6.14. gives graphs/curves, which characterize dependence of figures of merit on parameters of device/equipment of detection a, n and ν for particular value of figures of merit $D=0.5$ and $F=10^{-4}$ [28]. Are here for the comparison cited the data of the analogous figures of merit for the detector, which uses nonquantized data, i.e., using the ideal integrator (see §6.2).

FOOTNOTE ¹. The values of these values can be easily established/installed from the diagram Fig. 6.12b, which characterizes the levels of quantization. ENDFOOTNOTE.

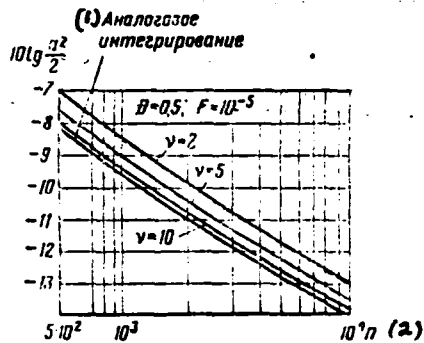


Fig. 6.14. Graphs/curves of the figures of merit of detection with the quantization on ν levels.

Key: (1). Analog integration. (2). p.

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From given graphs/curves it is distinctly evident that already with small number of discrete/digital channels of quantization ($\nu=5-10$) detectors of such type in their quantitative characteristics approach figures of merit of ideal integrator. This means that the virtually multichannel detector with the pulse-height analyzer with a small number of channels to a considerable extent realizes the potential possibilities of the optimum processing of the nonquantized pulse signals.

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Chapter 7.

MEASUREMENT OF ANGLE IN THE PRESENCE OF INTERFERENCES.

As is known, azimuth of target can be measured because of directional effect of antenna system. However, it cannot be considered that the accuracy of the determination of angular coordinates of radar is determined only by the parameters of the radiation pattern of its antenna system. This position would be possible if on radar system the signals, which have random character, did not act. Actually/really, in the absence of random disturbances the envelope of the impulses/momenta/pulses, reflected, for example, from the pinpoint target, with a sufficient degree of accuracy corresponds to the antenna radiation pattern and in the known form of ray/beam it is possible to error-free determine the value of the azimuth of target on the "angular" position of the packet of the echo pulses.

However, on radar usually operate two random processes, fluctuations of which do not have causal connection/communication with determined angular coordinate. Such random processes include the noises in input circuits of receiver and the fluctuation of the echo from the target signal. In some cases it is possible to consider that the echo signal has stable, constant value and then as the random signal only some noises will be present. Subsequently let us examine both cases, although let us begin naturally from the simpler, with which the amplitude of the echo pulses is constant.

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In both cases at output of receiving channel of radar is obtained totality of impulses/momenta/pulses - burst of pulses, whose amplitude changes, on one hand, with known regular shape - as a result of changing position of antenna system, and on the other hand, randomly - as a result of effect of fluctuating signals. Consequently, at the output of receiving channel is obtained the resulting random process, whose intensity nevertheless depends to a certain degree on direction to the oriented object θ .

According to results of observation and treating this random process they compose judgment about value of angle θ . This angle is actually the parameter of the random process in question; therefore

the determination of angular coordinate is conducted in accordance with the methods of the evaluation/estimate of the parameters of random process, used in the mathematical statistics.

Furthermore, us interests measuring error, caused on presence of interferences, and also form of optimum working/treatment, at which is reached maximum theoretical accuracy of measurement. It is obvious, this there will be the working/treatment, which utilizes a maximum of the information, which is contained in the echo pulses.

Fundamental task, which one must solve subsequently, is reduced to following: let us assume that in assigned interval of range - range gate - is signal from individual target. It does request itself: what it is necessary to produce the operations/processes above the echo signal in order to fulfill the measurement of angular coordinate?

Mode of vibration, available at output of radio-receiving channel of the RLS, is known. Known also that this oscillation consists of the additive mixture of interference $\sigma_m(t)$, which does not depend on the azimuth, and signal $U_c(t, \theta)$, which depends on the azimuth θ . It is obvious, total signal depends on the parameter θ . Signal aspect, and also their distribution function we assume known ones. It is necessary to determine the diagram of data processing and

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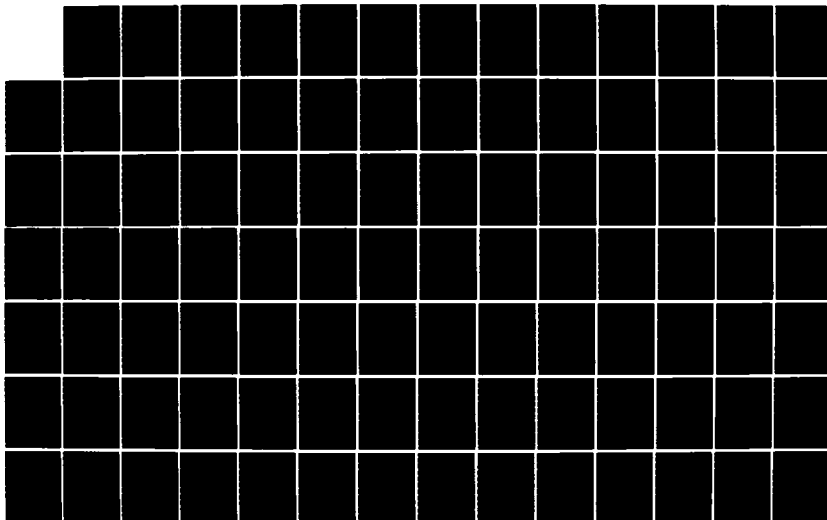
DIGITAL METHODS OF THE OPTIMUM PROCESSING OF RADAR
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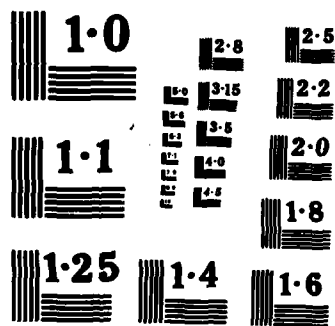
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the rule of making decisions, which guarantees obtaining the evaluation/estimate of angular coordinate θ best in this case, and to also calculate its dispersion.

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In view of the generality of the problem of the azimuth determination of target and statistical evaluation of the parameters of the signal of random structure let us give the presentation of some fundamental elementary questions of mathematical statistics, which concern evaluation/estimate, the parameters they will be required us subsequently.

It is necessary to also note that series/row of monographs and separate works in periodicals is dedicated to theoretical questions of measurement of angular coordinates in presence of interferences (and also to evaluation of other parameters of signal). From the Soviet literature first of all should be noted the monographs [48, volume II] and [50], and also various articles [35, 64, 67, 69, etc.]. In the foreign press these questions were examined in articles [21, 56, 58, 60, etc.]. Some results from these works are given subsequently presentation.

§ 7.1. Evaluation of the parameters of the signal of random

structure.

In mathematical statistics tasks of determining different characteristics of random variables according to experimental data are examined. Thus, for instance, as a result of processing of experimental data are determined the distribution laws or other most important numerical characteristics of the random variable: mathematical expectation, dispersion, etc. The only source of information for these calculations is experiment - observation.

However, frequently the number of observations, i.e., space of statistical data, is limited. This is connected with the short duration of observations, caused by the fact that the random value can be observed only during the small time interval: so, the rapidly flying target is located in the opening of the diagram of the antenna of radar the very limited time interval. In certain cases obtaining the large space of statistical data is connected with high costs and complexity of the experimental setup. Therefore at performance calculation of the parameters of the random variables of especially important is the task of the best processing of obtained experimental data, i.e., on the basis of the limited space experimental data, that carry random character, it is necessary to give the best estimator of the parameters of random variable being investigated.

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Subsequently we will assume that form of law of random number distribution is known previously, it is necessary to find only some parameters, on which depends this distribution law. In this case we will be interested mainly in the procedure of processing the observed data for obtaining the best estimator of the parameter being investigated at the limited space of experimental data (limited sample size).

Before passing to presentation of this material, let us refine some questions of terminology.

Let there be certain depending on parameter θ random variable ρ or random function $\rho(t)$, law of distribution of which also depends on this unknown of parameter θ . As a result of experiment we will have certain totality of the observed values of random variables, and for the function - instantaneous values of its realization:

$$p(\rho_1, \rho_2, \dots, \rho_n).$$

In terms of these values judgment about value of true value of parameter θ or another any characteristic of random variable (for example, average/mean value of random variable, its dispersion, etc.) is comprised. For this is necessary above the random variables $\rho_1,$

$p_1, \dots, p_n$ to produce some computational operation/process, as a result of which will be obtained approximate value $\hat{\theta}$ of the unknown parameter θ . This approximate value, calculated on the base of the limited number of experimental data, will always contain the element of chance. We will call this approximate value $\hat{\theta}$ subsequently the evaluation/estimate of the parameter θ . It is obvious that the estimate is the function of the observed values

$$\hat{\theta} = f(p_1, p_2, \dots, p_n). \quad (7.1.1)$$

In this case it is important to emphasize that evaluation/estimate $\hat{\theta}$ is also random variable, since it is function of observed values of $p_1, p_2, \dots, p_n$ that have random character. In one test series of value ($\hat{\theta}$) they can have one value, while in another - another value.

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Respectively the values of evaluations/estimates in each case will be different. Therefore it is not possible to identify the concept of evaluation/estimate with the concept of the approximate value of the true value of the parameter θ .

Evaluation/estimate itself is random variable and in certain cases can be close to true value of unknown parameter, and sometimes to considerably from it differ ¹. The function of the distribution of evaluation/estimate depends on the law of the random number distribution ρ , number of experiments, i.e., from the size of sample of the random variables n , and also from the form of function itself $f(\rho_1, \rho_2, \dots, \rho_n)$, and hence also from the most unknown parameter θ , since the selection of the form of the function $f()$, called the sometimes decision function, depends on that, what precisely parameter it is rated/estimated. For the different parameters of signal the form of the function (7.1.2) can be different.

In theory of estimations important is selection of form of the function $f()$, in accordance with which is calculated evaluation/estimate of parameter. Depending on the form of the function the evaluation/estimate possesses different properties, it can be "good" or vice versa. From the obvious considerations it is possible to say that the best estimator will be such that with a small quantity of experimental data as little as convenient differs from the true value of the parameter. However, this evaluation/estimate to obtain difficultly. Therefore it remains to only require so that the scatter of the values of evaluation/estimate would be small relative to the average/mean level (mathematical expectation), which corresponds to the true value of

evaluation/estimate. In other words, desirable so that on the average evaluation/estimate $\hat{\theta}$ would be equal to the true value of the parameter θ , and also that the scatter of the values of random evaluation/estimate as far as possible would be minimum, i.e., the dispersion of evaluation/estimate must be limited.

Considerations indicated find their reflection in following requirements, presented to evaluations/estimates.

Consistency of estimator.

FOOTNOTE ¹. Since the evaluation/estimate is also random variable, then it is inappropriate to raise the question about magnitude of error in the evaluation/estimate. It is possible to speak only about the probability of the fact that the value of evaluation/estimate will not exceed the limits of certain, so-called confidence interval.
ENDFOOTNOTE.

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Evaluation/estimate is called justified, if with an increase in the number of experiments n it approaches (it converges on the probability) ¹ to the true value of the parameter:

$\lim_{n \rightarrow \infty} \hat{\theta} \rightarrow$ (on the probability) θ .

Nondisplacement of evaluation/estimate. Evaluation/estimate is called unbiased, if its mathematical expectation corresponds to the true value θ of the evaluated parameter:

$$M[\hat{\theta}] = \theta.$$

In this case, using value $\hat{\theta}$ instead of θ , we will not make systematic error to side of overestimate or understating.

Effectiveness of evaluation/estimate. Evaluation/estimate is called effective, if its dispersion is less in comparison with the dispersion of other evaluations/estimates (with the same number of observed values)

$$D[\hat{\theta}] = \min.$$

In a number of cases it is impossible to simultaneously satisfy all these requirements. Therefore in each case it is necessary to carry out preliminary analysis and critical examination of evaluation/estimate from all points of view. Sometimes, for example, it can seem that efficient estimator exists, but it is calculated by complicated shape, and then they are satisfied by another evaluation/estimate, whose dispersion somewhat exceeds the dispersion of the first evaluation/estimate. Sometimes in the interests of

simplicity of calculations can be used even somewhat biased estimates.

A. On the procedure of the determination of evaluations/estimates.

Let us examine also the question about determination of optimum evaluations/estimates from position of theory of statistical solutions. In this case the evaluation/estimate adopted must be optimum from the point of view of the minimum of the losses of information about the azimuth of the target, which is contained in the echo radar signals.

FOOTNOTE 1. Value $\hat{\theta}$, they speak, converges on the probability to value θ with $n \rightarrow \infty$, if with as small as desired ϵ the probability of inequality $|\hat{\theta} - \theta| < \epsilon$ with increase in n unlimitedly approaches 1, i.e., $P_n(|\hat{\theta} - \theta| < \epsilon) \xrightarrow{n \rightarrow \infty} 1$. ENDFOOTNOTE.

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As is known, the theory of statistical solutions makes it possible to minimize the average/mean risk, connected with obtaining of certain "loss" in the presence of erroneous solutions; assuming that this criterion is sufficiently to general/common/total ones, let us accept it subsequently as the initial during the determination of

evaluation/estimate.

Subsequently we will be restricted to examination of evaluation/estimate only of one parameter $\hat{\theta}$, by assuming that informative parameter of signal is only angle of bearing θ . Let $\rho(t)$ be the observed oscillation, which consists of the mixture of the signal of known form and noise. Is conducted the observation of this signal in p the discrete/digital points, which correspond to moments/torques $t_1, t_2, \dots, t_n$ they are obtained by p of the values of value $\rho(t) = \rho(\rho_1, \rho_2, \rho_3, \dots, \rho_n)$, which can be considered as the realization of pi-measured random variable. On the basis of the observed values r it is necessary to make a decision about the value of the parameter θ , i.e., to find its evaluation/estimate.

For obtaining evaluation/estimate it is necessary to form function from selective values of received signal

$$f(\rho_1, \rho_2, \dots, \rho_n) = \hat{\theta}. \quad (7.1.2)$$

Form of this function must be selected optimally. In the future $f()$ we will call decision function.

It should be pointed out that, apparently, any selection of function $f()$ does not guarantee against making of erroneous decisions, consisting in the fact that evaluation/estimate $\hat{\theta}$ will

differ from true value of parameter θ . In the theory of statistical solutions, as it is known [15, 16, 17], they assume that the inaccuracy of solution is connected with obtaining certain of the "loss", which depends on the value of the error of solution, i.e., from the difference $\theta - \hat{\theta}$.

To account for importance of that go another error is introduced into examination as in § 6.1, the "value" of board for error $r(\theta, \hat{\theta})$, which depends on values of parameter θ and its evaluation/estimate $\hat{\theta}$ (more precise, from their difference).

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We accept that function $r(\theta, \hat{\theta})$ increases with an increase in the difference between θ and $\hat{\theta}$ or it remains constant, but it does not decrease. Let us assume that we selected the decision function of form $f(\rho) = \hat{\theta}$, then the value of error can be rewritten in the form

$$r(\theta, \hat{\theta}) = r[\theta, f(\rho)]. \quad (7.1.3)$$

Let us designate the a posteriori density of unconditional probability of appearance of observed value r with true value of parameter θ through $W(\rho, \theta)$. It is obvious, in this case

$$\int \int W(\rho, \theta) d\rho d\theta = 1^*. \quad (7.1.4)$$

For observer parameter θ is random; therefore probability that in observed signal parameter θ will be located in interval $\Delta\theta$, and multidimensional value ρ - in interval $\Delta\rho$, it is equal to

$$W(\rho, \theta) \Delta\rho \Delta\theta.$$

In this case loss from acceptance by us of decision function with given values of $\Delta\rho$ and $\Delta\theta$ is equal to

$$r[\theta, f(\rho)] W(\rho, \theta) \Delta\rho \Delta\theta. \quad (7.1.5)$$

Hence the average loss or so-called average/mean risk from selected function $f(\rho)$ with different values θ and ρ is obtained by averaging (addition) of losses during all possible situations, i.e., with all possible values of values θ and ρ .

Assuming/setting distribution $W(\rho, \theta)$ by continuous and passing from finite increments in limit to differentials $d\rho$, $d\theta$, it is possible instead of sum of expressions (7.1.5) to switch over to integral. Then average/mean risk from the acceptance of the decision function of form $f(\rho)=\hat{\theta}$ is equal to

$$R = \int \int r[f(\rho), \theta] W(\rho, \theta) d\rho d\theta. \quad (7.1.6)$$

Our further task consists in selection of optimum evaluation/estimate and with respect to optimum decision function $f(\rho)$. During their selection we will consider optimum such

evaluation/estimate, during which average/mean risk will be minimum.

FOOTNOTE ¹. Since here ρ - value multidimensional, integral (7.1.4) will be also multidimensional. ENDFOOTNOTE.

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As it follows from (7.1.6), form of decision function depends substantially on how selected form of the function of value of risk $r(\theta, \hat{\theta})$, i.e., how loss from incorrect solution is rated/estimated. Since the assignment to function of value can be produced by diverse methods, hence there ensue many optimum decision functions (7.1.2). The selection of one or the other method of the assignment to function of value is determined by the tactical considerations, presented to the radar system as a whole.

Designation/purpose of value of error is critical stage of investigation, with which is considered entire/all totality of considerations, which determine undesirability of error of one or the other form. However, not always can these substantiations, which ensue from the analysis of the combat use of radar equipment, be given in all cases full-valued. Therefore is given below the selection of the optimum decision function for the particular method of the assignment to the function of the value of the error, when

value is constant and does not depend on difference $\theta - \hat{\theta}$.

In this case function of value of error is determined by following relationship/ratio:

$$r(\theta - \hat{\theta}) = 1 - \delta(\theta - \hat{\theta}),$$

where

$$\delta(\theta - \hat{\theta}) = \begin{cases} 1 & \text{при } \theta = \hat{\theta}, \\ 0 & \text{при } \theta \neq \hat{\theta}. \end{cases} \quad (7.1.7)$$

Key: (1). with.

Hence it follows that for the correct solutions, when $\theta = \hat{\theta}$, the value of error $r(\theta, \hat{\theta}) = 0$; for all incorrect solutions the value of error is identical independent of the value of difference $|\theta - \hat{\theta}|$. After substituting (7.1.7) into (7.1.6), we will obtain

$$R = \int_0^1 \int_0^1 [1 - \delta(\theta - \hat{\theta})] W(\rho, \theta) d\rho d\theta. \quad (7.1.8)$$

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Taking into account relationship/ratio (7.1.4), we obtain, that

$$R = 1 - \int_0^1 d\rho \int_0^1 W(\rho, \theta) \delta(\theta - \hat{\theta}) d\theta.$$

After taking into consideration, that

$$\int_0^1 W(\rho, \theta) \delta(\theta - \hat{\theta}) d\theta = W(\rho, \hat{\theta}),$$

finally we will obtain

$$R = 1 - \int_0^1 W(\rho, \hat{\theta}) d\rho. \quad (7.1.9)$$

Hence it follows that minimum of average risk is reached when $W(\rho, \hat{\theta})$ reaches maximum. As is known, the unconditional density of the distribution of the probabilities

$$W(\rho, \theta) = W_0(\rho) W(\theta), \quad (7.1.10)$$

where $W(\theta)$ - the a priori density of the distribution of probability for the parameter θ , and probability $W_0(\rho)$ is the density of the conditional probability of the appearance of the observed value ρ when the parameter of this oscillation has true value θ . This probability is called the function of plausibility $L(\rho, \theta)$

$$W_0(\rho) = L(\rho, \theta).$$

Natural to assume as best estimator $\hat{\theta}$ such, for which a posteriori probability reaches greatest value, since this minimizes the average risk (7.1.9). Then for the evaluation/estimate $\hat{\theta}$ should be accepted the root of the equation

$$\frac{dW(\rho, \theta)}{d\theta} = 0, \theta = \hat{\theta}. \quad (7.1.11)$$

Method of determining evaluation/estimate thus is called method of maximum of a posteriori it is probabilistic. However, the practical use/application of this method requires the a priori knowledge of function $W(\theta)$, i.e., the a priori probability of the

parameter θ .

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However, in the majority of the cases a priori information about the parameter θ is absent, in connection with which the use/application of this method causes definite difficulties. Therefore frequently is utilized another method, the so-called method of the maximum of plausibility.

According to this method a priori distribution $W(\theta)$ they accept uniform, i.e., assume that

$$W(\theta) = \text{const.}$$

Then taking into account (7.1.10) relationship/ratio (7.1.1 1) passes into equality

$$\frac{\partial L(\rho, \theta)}{\partial \theta} = 0_{\theta = \hat{\theta}} \quad (7.1.12)$$

which determines optimum evaluation/estimate. Thus, in the method of the maximum of plausibility for the optimum evaluation/estimate is accepted the value θ , at which is reached the maximum of the function of plausibility $L(\rho, \theta)$, i.e., the root of equation (7.1.12).

Since the function of plausibility has exponential character usually and contains product of several factors, for convenience in

computations it is expedient to search for maximum not for the very function of plausibility, but its logarithm. Then (7.1.12) it passes in

$$\frac{\partial}{\partial \theta} \ln L(\rho, \theta) = 0_{\theta = \hat{\theta}} \quad (7.1.13)$$

This relationship/ratio is initial for evaluation/estimate angular-singing of coordinate when

$$\frac{\partial^2}{\partial \theta^2} \ln L(\rho, \theta) < 0.$$

Maximum likelihood estimates possess a number of remarkable properties, highly useful during azimuth determination. In mathematical statistics [18, 19] it is proven, that under the general conditions for maximum likelihood the estimates possess the following properties:

1. Evaluations/estimates are asymptotically effective, i.e., with an increase in the selected data ($n \rightarrow \infty$) the dispersion of evaluation/estimate approaches the minimum.

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2. Evaluation/estimate is justified with $n \rightarrow \infty$.

3. For evaluation/estimate of parameter with true value θ

correctly following relationship/ratio at $n \rightarrow \infty$.

$$P(|\hat{\theta} - \theta| > \epsilon) \rightarrow 0.$$

4. Evaluation/estimate $\hat{\theta}$ with $n \rightarrow \infty$ has asymptotically normal distribution.

Thus, evaluations/estimates, obtained according to method of maximum plausibility, possess properties of asymptotic nondisplacement, effectiveness, justifiability and asymptotic normality. Because of these important positive properties the method in question is isolated among others as one of the most widely used during the solution practical engineering tasks.

It should be noted that method of maximum of plausibility, obtained for specific value of error, determined by relationship/ratio (7.1.7), can be also obtained from solution of extreme problem for a posteriori probability, determined by bayes' formula.

Sense of method of maximum plausibility lies in the fact that is selected this evaluation/estimate of parameter θ , with which value of function of plausibility $L(\rho_1, \rho_2, \dots, \rho_n; \hat{\theta})$ is maximal. In this case the maximum of the probability density of selection is reached, but not the "most probable value" of the evaluated parameter.

B. On the accuracy of the evaluation/estimate of the parameters at the given number of observations.

With small number of observed values accuracy of evaluation/estimate decreases. As the accuracy let us accept the dispersion of evaluation/estimate $D(\hat{\theta})$. This is expedient because for the asymptotically normal and unbiased estimates it is natural to count the dispersion as the modulus of precision.

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In courses of mathematical statistics [18] it is proven, that for characteristic of accuracy of unbiased and asymptotically unbiased estimates Rao-Cramer inequality, which gives lower boundary of dispersion of evaluation/estimate

$$D(\hat{\theta}) = E(\theta - \hat{\theta})^2 \geq \frac{1}{E\left(-\frac{\partial^2 \ln L}{\partial \theta^2}\right)}. \quad (7.1.4)$$

can be used.

Value $E\left(-\frac{\partial^2 \ln L}{\partial \theta^2}\right)$ is mathematical expectation of second particular derivative of function of plausibility from parameter

$$E\left(-\frac{\partial^2 \ln L}{\partial \theta^2}\right) = J = - \int \dots \int \frac{\partial^2 \ln L}{\partial \theta^2} d\rho_1, d\rho_2, \dots, d\rho_n.$$

It is called informational quantity of Fisher (on name of

scientist, for the first time its examined).

Since function of plausibility depends on space of selected data, dispersion of evaluation/estimate $D(\hat{\theta})$ will also be function of number of observations of random variable.

Evaluation/estimate, for which in equality (7.1.14) occurs equal sign, is called effective. In literature [53] the necessary and sufficient conditions of obtaining efficient estimators are formulated. However, in the measurement of angular coordinates there are the foundations for assuming on the basis of the results of mathematical experiments, that the obtained estimates considerably do not differ from effective ones.

C. On the structure of processing signal and the direction-finding methods.

Determination of angular coordinate is conducted as a result of one or the other processing of signals at the output of receiving channel of radar (direction-finding device/equipment). The character of the working/treatment, produced, in particular, with the aid of the digital computers, is determined by the structure of the algorithm, utilized for determining the coordinate. This algorithm subsequently we will determine on the base of the method of the

maximum of the plausibility (see § 7.1, p. A), which gives the optimum evaluation/estimate of the angular coordinate of target.

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Diagram, which realizes automatic computation of evaluation/estimate of angle according to method of maximum plausibility, and is actually schematic of working/treatment and measurement of angular coordinate. According to (7.1.13) this evaluation/estimate is calculated from the known selective values of the signals of packet $\rho_1, \rho_2, \dots, \rho_n$ with the aid of the relationship/ratio, called sometimes the equation of the maximum of plausibility

$$\frac{\partial \ln L(\rho_1, \rho_2, \dots, \rho_n; \theta)}{\partial \theta} = 0.$$

The specific form of equation of plausibility is determined by structure of expression of function of plausibility $L(\rho_1, \rho_2, \dots, \rho_n; \theta)$. The latter depends on the character of signals and interference, form of the radiation pattern of antenna system, and also on the method of scanning the antenna of radar. Hence it follows that the different methods of direction finding relate between themselves as special cases of the method of the maximum of plausibility, determining in the equation of plausibility only the form of the function $L(\rho_1, \rho_2, \dots, \rho_n; \theta)$. But The method based on

the computation of the equation of plausibility is most general/most common/most total and makes it possible to consecutively/serially conduct a comparative characteristic of the diverse methods of measurement of angular coordinate.

In radars the methods of developing/scanning ray/beam also determine form of the function of plausibility. Actually these methods determine a quantity of selective values (impulses/momenta/pulses), which fall in one or another the section of radiation pattern. It must be noted that here the greatest possible variety of the methods of the "distribution of the impulses/momenta/pulses" on the width of radiation pattern, the advisability of using each of which should be solved especially taking into account of their structural/design and operating characteristics, and also parameters of signal and interference.

Existing methods of measurement are frequently connected with working/treatment of packet, obtained as a result of uniform rotation of antenna (method of linear scanning). In this packet not all pulses bear identical information about angular target position. The signals, arranged/located in the sections, where the slope/transconductance of direction-finding characteristic is small, i.e., in the sections in the direction of the maximum and minimum value of radiation pattern, bear the smallest information.

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Therefore it is desirable to apply other methods, i.e., the packet of another form, in which more fully would be utilized the information of the echo signals. In connection with this the method of the so-called discrete/digital scanning, examined below, is of interest. To these two methods, apparently, can be brought, in a certain sense, other direction-finding methods.

Computation of evaluations/estimates of measured angle according to method of maximum plausibility for radar systems is most expedient to carry out with the aid of electronic digital computers. In this case different modifications of this method can be realized by relatively simple means.

Analysis of different diagrams and methods of measuring angular coordinates should be examined in light of effect of factors, which determine form of the function of plausibility. In connection with this subsequently the methods in question are divided into two groups in accordance with the methods of the survey/coverage of space, namely: direction finding during the linear (circular) scanning of antenna and direction finding during the discrete/digital scanning;

as it shows below, the latter/last case includes also the direction finding in the multichannel system.

In first case burst of pulses is formed as a result reflection of signal from target during uniform (circular) rotation of antenna. In the second case the burst of pulses is formed as a result of the reflections of signals from the target in several discrete/digital positions, and in particular in two antenna positions. Of this case the packet consists of the totality of two groups of pulses, which have, as a rule, a different level of amplitude. Only in one case, when direction to the target in these two positions coincides with the equisignal direction, the signal amplitude of entire packet will have identical level. Knowing the equation of the antenna radiation pattern and the relationship/ratio between the values of the impulses/momenta/pulses, obtained in the different antenna positions, it is possible to measure the angular coordinate. This direction-finding method subsequently we will call the "method of discrete/digital scanning", or the "method of two antenna positions".

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It must be noted that the measurement of angular coordinates begins always after is produced target detection, i.e., after in device/equipment of detection fact of presence of echo signals on

specific distance was recorded. In this case target detection in the radar systems because of the directional effect of antenna system is always connected at least with the rough determination of the angular coordinate of target. By other words: after revealing/detecting target, always it is possible (at least approximately, with an accuracy to the width of the opening of radiation pattern) to indicate angular coordinate - azimuthal target position. Difficulty in the fulfillment of this can be caused only by a essential difference in the technical methods of determining the coordinates with target detection. Let us assume that during the detection it is possible to determine angular target position with an accuracy to the angle, equal to the width of the opening of radiation pattern. Further task of the measuring circuit of angle consists only of the refinement of azimuth to the values, determined by the necessary accuracy, i.e., in the decrease of error to the value, determined by the portions of the width of the antenna radiation pattern.

From aforesaid it follows that procedure of measurement of coordinates comes into force only after took place detection of reflecting target and in direction θ_n in question is stated/established presence of echo pulses. We assume the direction θ_n to be known with an accuracy to the width of the antenna radiation pattern. The task of the measuring circuit of angular coordinate (SIUK) consists of further refinement of angular target position. In

this case the procedure of working/treatment and the maximally obtainable accuracy, which can be obtained during the use of one or the other method for the signals of different structure, interest us.

Questions of detection of signals against the background of noises are presented in Chapter 6. Therefore let us assume that the packet of the echo pulses was by that or another method discovered and put out in the gate/strobe, which on the angle and the range corresponds to the defined channel of the elementary section of range, and the values of gates both on the angle and on the range have such sizes/dimensions, that they knowingly encompass the analyzed totality of the pulses reflected from the individual target. The character of the reflecting objects and the structure of the signals echo from them are examined in Chapter 5.

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Passing to the determination of algorithms and measuring circuits of angular coordinates for different cases of direction finding of targets, it is necessary to have in mind that methods of solving equation of maximum of plausibility (7.1.13) and respectively character of algorithms depend on completeness of a priori information about signal and interference. Subsequently, we will consider that the characteristics of signal and interference, their

distribution function are completely known to us, and also the equation of the antenna radiation pattern. Only the angular coordinate θ , which is subject to measurement, is unknown. But if some parameters of signal or interference prove to be unknowns, then the character of optimum algorithm will be different, in this case the accuracy of measurement in this case will be, obviously, lower than with the complete a priori information.

D. On the conformity of the stages of pulse of packet and their envelope.

With direction finding by linear scanning important is question about conformity between angular position of impulses/momenta/pulses of packet with respect to position of envelope of packet, determined by characteristic of radiation pattern. The discussion deals with phase relationship between the impulses/momenta/pulses of packet with respect to the envelope phase of packet. Therefore before passing to the presentation of algorithms and measuring circuits of coordinates, is necessary to in detail dwell on this question and to examine the character of the angular distribution of pulse signals with respect to their envelope - antenna radiation pattern.

Let us assume that θ is true angular target position, calculated off arbitrary initial direction, and $\Delta\theta$ - the angle at which is moved

pattern of antenna of radar in interval between two adjacent impulses/momenta/pulses.

If angular velocity Ω of rotation of antenna is constant and pulse repetition period T remains constant, then

$$\Delta\theta = T\Omega.$$

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In this case assigned sector of survey/coverage F_0 can be conditionally divided into some bearing intervals by width each of $\Delta\theta$ (Fig. 7.1). Let the number of such intervals be

$$k = \frac{F_0}{\Delta\theta},$$

moreover k it can be whole or fractional, if F_0 is not multiple to $\Delta\theta$; subsequently in this case we will consider the nearest smaller whole part of k .

Obtained as a result of this laying out reference points can be considered as initial scale of reading of angular coordinate.

Besides supporting/reference ones, are other marks, connected with sounding and echo from target pulses.

During analysis of procedure of measurement it is expedient to

distinguish two cases, characterizing position sounding pulses with respect to selected angular intervals of raster (Fig. 7.1).

1. Determined position of echo pulses.
2. Random position of echo pulses.

In first case position of echo pulses can be agreed to with respect to selected reference points, in second case it is random.

This separation is defined by both the design features of radars and by some aspects of theoretical analysis, which appear in examination of one or the other case.

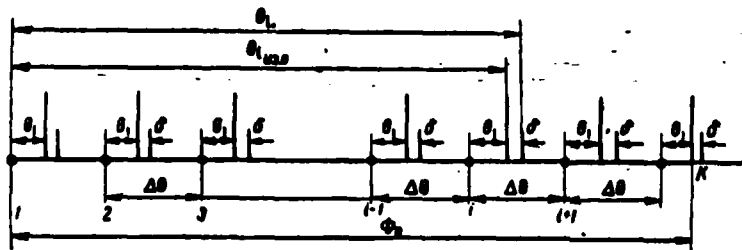


Fig. 7.1. Pulses with the determined packet.

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However, the fact is characteristic for both cases that within each interval $\Delta\theta$ during one period of irradiation only the one echo pulse will be located. It appears at that moment/torque, when diagram will be located in this interval. The position of diagram is determined by the angular position of the direction of maximum radiation/emission. Let us designate the position of the emitted impulse/momentum/pulse within the i interval through $\theta_{i_{NSA}}$, then

$$\theta_{i_{NSA}} = \theta_1 + \Delta\theta(i-1),$$

where θ_1 - angular interval between the sounding pulse and $(i-1)$ th mark.

In first case in determined position emitted pulse in each interval $\Delta\theta$ will occupy one and the same specific position, being located on identical range θ_1 from left nearest mark (Fig. 7.1). In

the random position of pulse θ_1 it is a random variable in each interval.

In first case it is possible to consider that marks are rigidly synchronized with sounding pulses and is always accurately known angular position of sounding pulse and respectively position of maximum of antenna radiation pattern at moment of radiating/emitting sounding pulse. Structurally this can be achieved/reached, in particular, by constancy and by harmonic order of pulse repetition and rotation of antenna ¹.

In the second case position of sounding pulse within interval $\Delta\theta$ is accurately unknown.

Echo pulse is located from sounding pulse in interval

$$\delta = \Omega \tau_0$$

where $\tau_0 = \frac{2R}{c}$, and Ω - rate of change of angular antenna position.

Then angular position of the echo pulse

$$\theta_l = \Delta\theta(l-1) + \theta_1 + \delta. \quad (7.1.15)$$

With time constant of time lag τ , within limits of one packet always it is possible to know angular position of echo pulse; value δ plays role of constant shift/shear, which can be taken into consideration in obtaining of final result here. For simplification

in further reasonings it is possible to place by the equal to zero and to consider the emitted impulses/momenta/pulses (Fig. 7.1) as the impulses/momenta/pulses, which correspond to the angular position of the echo signals.

FOOTNOTE ¹. In this case rigid synchronization between the rotation of antenna and the frequency of pulsing is assumed. ENDFOOTNOTE.

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This is appropriate to make also for such radar systems, whose pulse repetition period is so great that angular position between impulses/momenta/pulses $\Delta\theta$ is considerably more than interval of time lag δ , i.e., when $\Delta\theta \gg \delta$.

Finally, it should be noted that value θ_1 in the determined position of impulses/momenta/pulses plays role of constant shift/shear, which can be taken into consideration in obtaining final of result. Simplifications for in this case it is possible to consider that the impulses/momenta/pulses coincide with the supporting/reference marks, i.e., $\theta_1 = 0$, and the selected marks of the scale of angles will correspond to the angular position of received pulses. Moreover, it is appropriate to place for the simplification which $\delta + \theta_1 = 0$, then for the determined case relationship/ratio

(7.1.15) passes into the formula

$$\theta_i = \Delta\theta(i-1), \quad (7.1.15')$$

where i - the instantaneous value of the number of impulse/momentum/pulse in the packet. Reference point i can be assumed/set usually by that coinciding with the extreme impulse/momentum/pulse of the packet, advanced in the gate/strobe.

Let the form of pattern of antenna, which determines intensity of received signal, be

$$Q(\theta_i - \theta).$$

This function coincides with diagram of power gain with bilateral landing run of signal. Let us assume that this function is calibrated by condition $Q(0)=1$ and is even relative to zero argument. For the simplification and a comparative evaluation of different direction-finding methods (linear scanning, the method of two positions, etc.) we will consider that a quantity of impulses/momenta/pulses in the packet is always even and a number of impulses/momenta/pulses to the right and to the left relative to the center of packet is equal. Let us note that for the analysis strictly of the method, the linear scanning of such conditions of the symmetry of packet is not necessary, and it is possible to measure the angle with the asymmetric packet; however, these conditions are useful during the analysis and a comparative evaluation of the method of two positions (discrete/digital scanning).

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Now let us pause at special features/peculiarities of another version, which corresponds to random position of echo pulses, in which phase of pulse signal of packet is by chance with respect to enveloping packet itself.

In second case, when position of impulses/momenta/pulses of packet is not determined, value θ_1 in formula (7.1.15) is by chance. Rigid synchronization between the rotation of antenna and the pulse repetition frequency here is absent. Then the position of the echo pulse is random with respect to the supporting/reference marks (Fig. 7.2). Let the vertical lines in this figure characterize the position of the sounding and echo pulses; the latter are located from those emitted in the interval δ . If we disregard/neglect the time lag of the echo signal, then value θ_1 will characterize the position of received pulse. Within the interval between the marks value θ_1 can take any random values, in this case it is possible to assume that it will be distributed in the section $\Delta\theta$ evenly. Then it is obvious that also value θ_1 will be also random and, as it follows from (7.1.15), it will be distributed within $\Delta\theta$ with the constant density.

It is natural, apparently, that accuracy of measurement in first case, in determined position of impulse/momentum/pulse, can be higher than the secondly, when position of impulses/momenta/pulses is by chance. In this case the density function of the distribution of bursts of pulses in each the cases will be different, since for the signals with the random position of impulse/momentum/pulse the distribution function depends on the results of averaging from random parameter θ ; in the limits of the width of interval $\Delta\theta$.

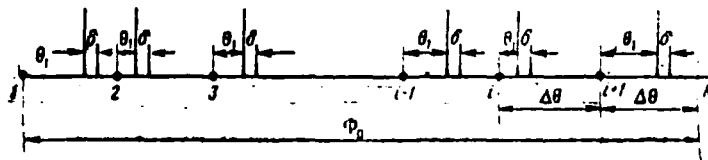


Fig. 7.2. Reciprocal location of impulses/momenta/pulses with the indeterminate packet.

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From an analytical point of view, this fact as a result leads to the need for utilizing in the equation of plausibility the averaged distribution functions and, therefore, to the complication of the apparatus for experiment. With the low values of $\Delta\theta$ this deterioration in the accuracy will be insignificant, while with large $\Delta\theta$ and a small quantity of impulses/momenta/pulses, which fall to the width of diagram, measuring error can be essential.

In summary in first method, in determined position of impulses/momenta/pulses, only random changes in signal amplitude at output of radar channel are source of error in measurement. However, in the second method, in the random position of impulses/momenta/pulses, besides the source of errors indicated, appears the even supplementary reason for the onset of the errors, connected with the random position of impulse/momentum/pulse with

respect to the enveloping packet and, therefore, with random value θ_i within the interval $\Delta\theta$. This method is less precise, and its analysis - more complicated. Therefore, if it is necessary to have the high accuracy of the measurement of angle, it is necessary to utilize radar systems with the determined position of impulses/momenta/pulses.

Subsequently let us pause at measurement of angle with determined packet, since this method makes it possible to realize potential accuracy of measurement of angular coordinate and thereby to determine special features/peculiarities of measurement with small quantity of impulses/momenta/pulses in packet. Measurement with the indeterminate packet for the signal, reflected from the target with the constant diffusing surface, is examined in work [20].

§7.2. Algorithms of the measurement of angular coordinates during the linear scanning of antenna.

A. Target with constant diffusing surface.

We assume that in process of measurement of angular coordinate from output of radar channel to measuring circuit enters totality of fluctuating noise and pulse signals, reflected from target.

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We assume/set target reflecting properties constants and time-independent. Then the amplitude of the echo signal will be constant, changing only as a result of rotating the antenna.

Determination of optimum algorithm of working/treatment of packet from n impulses/momenta/pulses is our problem, obtained as a result of one survey/coverage of antenna. The algorithms of working/treatment of the packets, obtained with other periods of survey/coverage, will be analogous. During the solution of this problem we assume that the parameters of signal, interference and radiation pattern of antenna system are the given ones. Only the angular coordinate of target θ , which is determined as a result of processing the signals, reflected from the target, is unknown.

Echo pulses store/add up to fluctuating noises of input circuits of receiver, are amplified and enter measuring circuit of angular coordinate (SIUK). In this case it is assumed that into the channel SIUK from the output of the elementary section of range only the impulses/momenta/pulses of one workable packet are supplied. This position is achieved by the diagram of special strobing/gating along the azimuth, in this case the position of angular gate/strobe and respectively oriented antenna can be established/installed according

to the data of the device/equipment of the channel of detection, which usually makes it possible already with certain approximation/approach to judge angular target position.

Concrete/specific/actual schematics of strobing/gating and coupling channel of measurement of angular coordinates with other elements/cells of radar here are not examined. Below is set forth the operating principle of the devices/equipment of the measurement of the angular coordinates, which use working/treatment of packet on the base of the algorithms, obtained by the method of the maximum of plausibility. In this case it is assumed that the circuit realization of this working/treatment is realized with the aid of elements/cells and nodes of electronic digital engineering.

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Oscillations/vibrations diagram of working/treatment enter in the form of mixture signal + noise, function of distribution of which according to (5.1.10) is subordinated to generalized Rayleigh distribution:

$$W(p_i) = \frac{p_i}{\sigma^2} \exp\left(-\frac{p_i^2 + A_i^2}{2\sigma^2}\right) I_0\left(\frac{A_i p_i}{\sigma^2}\right),$$

where A_i - nonfluctuating amplitude of i signal of packet, which does not depend on antenna position with respect to direction to reflecting object. Taking into account radiation pattern

$$A_i = A_0 Q(\theta_i - \theta), \quad (7.2.1)$$

where A_0 - value of signal in the direction of maximum, and θ_i - known antenna position at the moment of the reception/procedure of the i impulse/momentum/pulse, determined by formula (7.1.15).

In process of measurement we will arrange/locate n with selective values of signals $\rho_1, \rho_2, \dots, \rho_i, \dots, \rho_n$ obtained in different antenna positions. These signals compose the packet of the echo pulses. We assume that the pulse repetition period is much more than the interval of the correlation of the noises of receiver. In this case the function of the plausibility of packet under the condition of the statistical independence of signals in the selection according to (5.1.10) will take the form

$$L(\rho_1, \rho_2, \dots, \rho_n; \theta) = \prod_{i=1}^n \frac{\rho_i}{\sigma^2} \exp\left(-\frac{\rho_i^2 + A_i^2}{2\sigma^2}\right) I_0\left(\frac{A_i \rho_i}{\sigma^2}\right). \quad (7.2.2)$$

Unknown value of angular coordinate, is more precise, its evaluation/estimate $\hat{\theta}$, obtained in accordance with method of maximum plausibility §7.1, is found from equation (7.1-13)

$$\frac{\partial \ln L(\rho_1, \rho_2, \dots, \rho_n; \theta)}{\partial \theta} = 0_{\theta=\hat{\theta}}$$

Logarithm of function of plausibility

$$\ln L(\rho_1, \rho_2, \dots, \rho_n; \theta) = \sum_{i=1}^n \left[\ln \frac{\rho_i^2}{\sigma^2} - \frac{\rho_i^2}{2\sigma^2} - \frac{A_i^2}{2\sigma^2} + \ln I_0\left(\frac{A_i \rho_i}{\sigma^2}\right) \right]. \quad (7.2.3)$$

Derivative of logarithm of modified Bessel function first zero-order type is determined by relationship/ratio of form

$$\frac{\partial \ln I_0[u(0)]}{\partial \theta} = \frac{\partial \ln I_0(u)}{\partial u} \frac{\partial u}{\partial \theta} = \frac{I_1(u)}{I_0(u)} \frac{\rho_l}{\sigma^2} A'_{l_0}(0)^*. \quad (7.2.4)$$

FOOTNOTE 1. During conclusion/output (7.2.4) was considered that

$$\frac{d I_n(x)}{dx} = -\frac{n}{x} I_n + I_{n-1}, \text{ but } I_{n-1} = \frac{2n I_n}{x} + I_{n+1}, \quad I'_n = \frac{n}{x} I_n + I_{n+1} \quad \text{For } n=0 \quad I'_0 = I_1.$$

ENDFOOTNOTE.

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After substituting (7.2.3) and (7.2.4) in relationship/ratio (7.1.13), we will obtain

$$\frac{\partial \ln L}{\partial \theta} = \sum_{i=1}^n \left[-\frac{A_i}{\sigma^2} \frac{d A_i}{d \theta} + \frac{I_1 \left(\frac{\rho_l A_i}{\sigma^2} \right)}{I_0 \left(\frac{\rho_l A_i}{\sigma^2} \right)} \frac{\rho_l}{\sigma^2} \frac{d A_i}{d \theta} \right] = 0, \quad (7.2.5)$$

Hence

or

$$\sum_{i=1}^n [C_i \rho_l - A_i A'_i] = 0, \quad (7.2.6)$$

where

$$\sum_{i=1}^n C_i \rho_l = \sum_{i=1}^n A_i A'_i, \quad (7.2.7)$$

but

$$C_i = \frac{I_1(u_i)}{I_0(u_i)} A'_i, \quad (7.2.8)$$

$$u_i = \frac{\rho_l A_i}{\sigma^2}$$

is weight coefficient, by which are multiplied selections p_i obtained from output of receiving channel of radar. Formulas (7.2.6) and (7.2.7) are decision functions, whence the unknown evaluation/estimate of angular coordinate is determined. Thus, the value of angle θ , which satisfies expression (7.2.7), serves as evaluation/estimate according to the maximum of plausibility.

For solving this equation it is necessary selective values of signal p_i to multiply by weight coefficient C_i and obtained products to accumulate. Then the obtained result must be compared with the sum, which stands in the right side of equality (7.2.7). This sum does not depend on received signal and for the antenna radiation pattern of unknown form can be calculated previously. However, its value depends on the unknown coordinate θ ; furthermore, weight coefficients C_i by which are multiplied the selective values of signal, also depend on the angle θ and on selective value p_i .

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Therefore the direct solution of this equation is hindered/hampered and it is expedient to solve by the method of "tests/samples and search" for some assumed values θ with the subsequent account of the

obtained errors in solution.

Thus, in resolving this equation we consider known: equation of radiation pattern and respectively value A_i ($i=1, 2, \dots, n$), and also p_i and σ . Unknown is here θ , that satisfies equality (7.2.7).

One of possible methods of solving this equation method of "tests/samples and search" is of assignment of series/row of assumed values $\theta=\theta_1; \theta=\theta_2; \theta=\theta_3$, and so forth, for which are calculated sums of right and left parts of equality (7.2.7). Then the obtained results are compared for the appropriate values θ and the part of the equality, that give the best coincidence between themselves, is determined the unknown value of the evaluation/estimate of angle $\hat{\theta}$. It is most expedient to produce such computations in view of their large space in the digital computers, in this case selective values p_i are converted with the aid of the devices/equipment PND into the digital code and are written/recorded in the memory unit. For accelerating the calculations it is possible also previously to calculate and to carry into the memory unit of value $\bar{A}_i$ and A_{i0} calculated for appropriate assumed values θ ($\theta=\theta_1; \theta=\theta_2, \dots$). Below, for the systematic targets, as an example of synthesis one of the possible structures of algorithm for the simplified equation of type (7.2.7) is given. This structure is given on the assumption that the parameters of the ray/beam of antenna and signal level or

signal-to-noise ratio are known.

It must be noted that value of coefficient C_i depends on signal level. At small intensities, when the argument of the Bessel functions is small

$$\frac{C_i}{A_i} = \frac{I_1\left(\frac{1}{\sigma^2} \rho_i A_i\right)}{I_0\left(\frac{1}{\sigma^2} \rho_i A_i\right)} \quad (7.2.9)$$

and it vanishes, while on the high levels the value of this coefficient monotonically approaches one.

FOOTNOTE ¹. In this case it is assumed that (7.2.7) they have unique solution for $\theta=\theta_1$; $\theta=\theta_2$; $\theta=\theta_3$, ... ENDFOOTNOTE.

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Thus, weight factor C_i emphasizes the effect of large signals on the result of measurement and weakens/attenuates the effect of small ones.

Generally coefficient C_i depends on selective values ρ_i and must be calculated each time after obtaining of corresponding selective value of signal. This, naturally, it complicates working/treatment; therefore in certain cases [21] they use the precomputed values of this coefficient, calculated for expected values $\left(\frac{\rho_i A_i}{\sigma^2}\right)$.

Components/terms/addends of right side of equality (7.2.7) for widespread diagrams of antennas change in the manner that it is shown in Fig. 7.3. From the graph/curve it is evident that the point, at which the diagram has a maximum, corresponds to transition/junction through zero and gives sign change of components/terms/addends. Since the selections enter in the time gradually in proportion to the rotation of antenna, with the uniform rotation they will enter through the identical intervals. Therefore graph/curve (Fig. 7.3) can be considered as the function of time. In this case the sum on the left side of equality (7.2.8) there will also be the function of time. For the symmetrical radiation patterns of area, limited by the positive and negative branch of graph/curve (Fig. 7.3), must be identical and the sum of the ordinates, arranged/located to the right and to the left of the point of intersection, it is equal to zero.

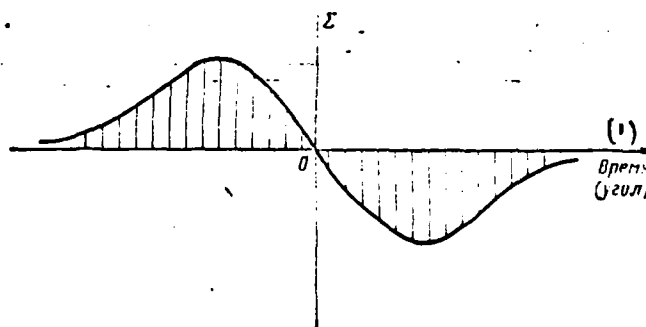


Fig. 7.3. Values of the sum of terms in the equation of plausibility.

Key: (1). Time (angle).

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Let us examine specific example. Let us assume that the diagram has bell-shaped form and is determined by the function of zero-order Hermite

$$Q(\theta - \theta_i) = e^{-p(\theta - \theta_i)^2}, \quad (7.2.10)$$

where r - parameter of the approximation:

$$p = \frac{1}{\varphi_d} \sqrt{\ln \frac{1}{Q_d}}, \quad \varphi_d = \theta - \theta_{id}, \quad (7.2.11)$$

Then

$$A_i = A_0 e^{-p^2(\theta - \theta_i)^2}, \quad A'_i = 2p^2(\theta_i - \theta) A_p, \quad (7.2.12)$$

and equation (7.2.7) passes into following equality:

$$\sum_{i=1}^n C_i \rho_i = \sum_{i=1}^n A_i^2 2p^2 (\theta_i - \theta), \quad (7.2.13)$$

where weight factor

$$C_i = A_i^2 2p^2 (\theta_i - \theta) \frac{f_1(u_i)}{f_0(u_i)}. \quad (7.2.14)$$

Assuming that coefficients C_i are calculated previously according to expected selective values and are known constant numbers, then solution of problem is simplified.

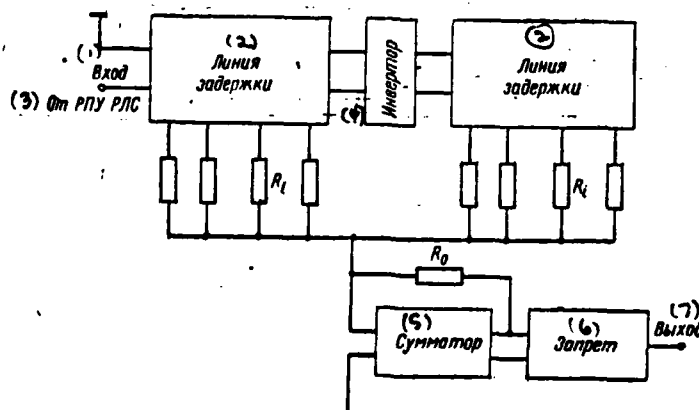


Fig. 7.4. Block diagram of the simplified device/equipment for determining the angle from the maximum of plausibility.

Key: (1). Input. (2). Delay line. (3). From RPU of RLS. (4). Inverter. (5). Summator. (6). Exclusion. (7). Output.

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In particular, in work [21] for this case the simulating circuit of the azimuth determination of target, depicted in Fig. 7.4 [21], is proposed. Diagram is suitable for radars with the constant repetition period. Two delay lines with the removals/outlets, been connected in series through the inverter are the fundamental network elements. Inverter serves for forming the negative weighting function. A quantity of removals/outlets is equal to a quantity of selective values of signal in the packet. From output terminals of the

removals/outlets of line the signal enters through weight resistors/resistances $R_1, R_2, \dots, R_n$, summing amplifier (or resistor/resistance), as which can be used usual scale amplifier. Weight resistors/resistances are selected depending on the value of weight coefficient C_i by such shape, in order to

$$\frac{R_0}{R_i} = C_i.$$

At output of adder instantaneous value of sum of weighed impulses/momenta/pulses, which appear at output of receiving channel, is formed.

Change in instantaneous value of sum has following character. At first in proportion to the entry of target into the zone of the diagram of antenna sum gradually increases. Let us note that the weight factor depends on derived radiation pattern; therefore the greatest value of components/terms/addends will be in those sections, where the slope/transconductance of radiation pattern is greatest. With the approach of target to the middle, the center of radiation pattern the sum increases more slowly, since the derivative at this point is close to zero. After the point of maximum the sum already decreases, since from delay line, which is located beyond the inverter, the signals, which have opposite polarity, come. Subsequently the sum will be equal to zero. This means that the selective values of signal will be multiplied by the weight

coefficients, which will reduce sum to zero, i.e., packet "will become in its place" in accordance with decision function (7.2.7). The value of the angle, at which sum (7.2.6) became zero, corresponds to the unknown evaluation/estimate $\hat{\theta}$, which determines angular target position.

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Then resulting sum will have opposite sign and, after traversing maximum, it will become equal to zero.

Moment/torque of passage through zero it is possible to record automatically with the aid of threshold valve/gate, cut off by output signal of adder. When the output signal of this device/equipment is equal to zero, then valve/gate will be opened and will pass the signal of mark.

Large number of delay lines in this diagram and requirement of constancy of pulse repetition frequency decrease practical value of diagram.

Some simplifications in structure of processing signal can be obtained on high level of signal, when $A_{ip1} \gg \sigma^2$. In this case

$$\ln I_0 \left(\frac{A_i \rho_i}{\sigma^2} \right) \approx \frac{\rho_i A_i}{\sigma^2}$$

After substituting this relationship/ratio in (7.2.3) and differentiate of function of plausibility, we will obtain

$$\frac{\partial \ln L}{\partial \theta} = \frac{1}{\sigma^2} \sum_{i=1}^n [\rho_i A_i - A_i A'_i].$$

Hence equation of plausibility will be

$$\sum_{i=1}^n A'_i (\rho_i - A_i) = 0, \quad (7.2.15)$$

or

$$\sum_{i=1}^n A'_i \rho_i = \sum_{i=1}^n A'_i A_i. \quad (7.2.15a)$$

This equation is solved, just as is earlier, by method of "tests/samples and search", but its structure is simpler than equation (7.2.8). In the latter/last equality the right side does not depend on selective values ρ_i and it can be calculated previously for several values of predicted angles θ_m ($m=1, 2, 3, \dots$). Factor A'_i with selective values ρ_i also does not depend on them and therefore it can be calculated for the same values θ_m .

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Sense of solution of equation (7.2.15) lies in the fact that it is necessary to find this location of radiation pattern relative to selective values ρ_i (i.e. to find similar θ), with which would be

fulfilled relationship/ratio (7.2.15). In this case θ , it is considered known. One of the versions of the structure of the algorithm of equation (7.2.15a) is given in Fig. 7.5.

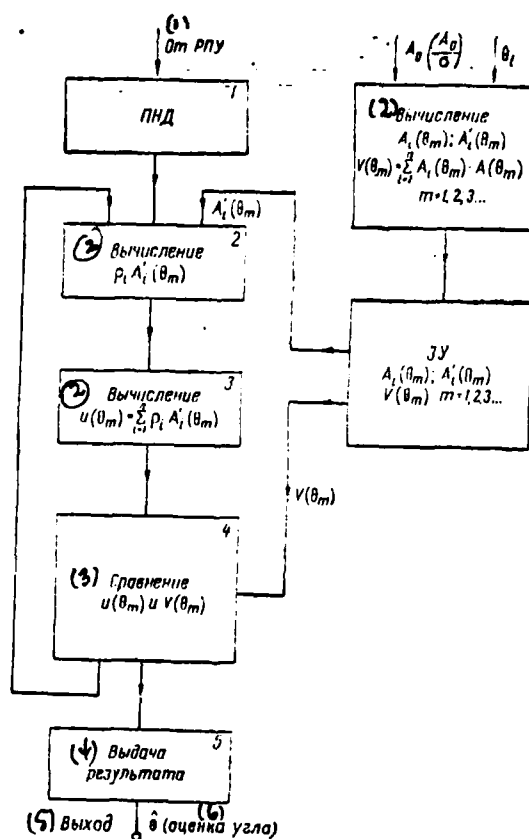


Fig. 7.5. Structure of algorithm in the measurement of angle.

Key: (1). From RPU. (2). Computation. (3). Comparison and (4). Readout. (5). Output. (6). (evaluation/estimate of angle).

B. Target with the flickering reflecting surface.

This case of target with known approximation/approach occurs, for example, with reflection of waves of centimeter band from aircraft, whose sizes/dimensions are great in comparison with scattered wave. Actually/really, in Chapter 5 it was indicated that for the waves of centimeter band the aircraft can be represented in the form of the body, which consists of many separate "reflectors", which create "ghost images".

As a result of vibration of housing and aircraft yawing in course "ghost images" are changed their position and reflections from many separate "reflectors" they come to antenna of radar in the form of signals (sinusoids) with random phases, which during addition give resulting signal with fluctuating amplitude.

Analogous character of reflection can be expected, also, for other irregularly moving objects, which have sizes/dimensions much more than scattered wavelength, and as a result of vibration, hunting and other changes in position of housing is created to fluctuation of reflecting surface of object.

Subsequently we assume that time of correlation of random changes in diffusing surface is less than duration of interval

between impulses/momenta/pulses of radars. In this case the impulses/momenta/pulses, reflected from this target, can be considered as the independent random signals. This impulse/momentum/pulse, entering early-warning radar receiver, stores/adds up to the noises at the input of receiver and is formed the resulting serrated signal, which has random structure.

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According to §5.2 density function of distribution of this enveloping totality of signal and interference are determined by law "chi-square" with two degrees of freedom (5.2.10)

$$W(\rho_i) = \frac{\rho_i}{\sigma_{c_i}^2 + \sigma_w^2} e^{-\frac{\rho_i^2}{\sigma_{c_i}^2 + \sigma_w^2}},$$

and phase

$$\eta(\psi) = \begin{cases} \frac{1}{2\pi}, & \text{если } 0 \leq \psi < 2\pi \\ 0, & \text{если } \psi \text{ вне интервала } (0-2\pi). \end{cases}$$

Key: (1). if. (2). 0, if ψ out of interval (0-2 π).

here σ_{c_i} - rms value of the i signal, reflected from the target with the changing diffusing surface;

σ - rms value of noise;

p_i - value of the i signal of packet.

Value of rms value of fluctuations of echo signal σ_{ei} depends on angular antenna position $\theta - \theta_i$ at moment of reception/procedure of i impulse/momentum/pulse. If the equation of the antenna radiation pattern, calibrated relative to its maximum, will be

$$Q(\theta - \theta_i),$$

the rms value of the i signal

$$\sigma_{ei} = \sigma_0 Q(\theta - \theta_i), \quad (7.2.16)$$

where σ_0 - maximum value of a root-mean-square change in the echo signal. When the axis of the diagram of antenna is directed toward the target, then $\sigma_{ei} = \sigma_0$. We assume/set value σ_0 in further reasonings known.

Taking into account (7.2.16) and (5.2.10) resulting density function of distribution for i signal of packet can be represented in the form

$$W(p_i) = \frac{p_i}{1 + \frac{\sigma_0^2}{\sigma_i^2} Q^2(\theta - \theta_i)} \exp \left[-\frac{\frac{p_i^2}{2}}{1 + \frac{\sigma_0^2}{\sigma_i^2} Q^2(\theta - \theta_i)} \right], \quad (7.2.17)$$

where θ_i - known angular antenna position at moment of reception/procedure of i impulse/momentum/pulse of packet.

In measurement of angular coordinate packet from n echo pulses

is processed.

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The function of the plausibility of this packet with the statistical independence of impulses/momenta/pulses is determined by the following expression:

$$L(\rho_1, \rho_2, \dots, \rho_n; \theta) = \prod_{i=1}^n \frac{\rho_i}{1 + \frac{\sigma_0^2}{\sigma^2} Q^2(\theta - \theta_i)} \exp \left\{ \frac{-\frac{\rho_i^2}{2}}{1 + \frac{\sigma_0^2}{\sigma^2} Q^2(\theta - \theta_i)} \right\}, \quad (7.2.18)$$

where θ - unknown value, which plays the role of the parameter of the function of plausibility.

According to (7.1.13) unknown evaluation/estimate of parameter $\hat{\theta}$ in accordance with method of maximum plausibility is root of equation

$$\frac{\partial \ln L(\rho_1, \rho_2, \dots, \rho_n; \theta)}{\partial \theta} = 0 \Big|_{\theta = \hat{\theta}}.$$

Logarithm of function of plausibility according to (7.2.18)

$$\ln L = \sum_{i=1}^n \left[\ln \rho_i - \ln(1 + k_i^2) - \frac{\rho_i^2}{2(1 + k_i^2)} \right],$$

but derivative of logarithm of function of plausibility

$$\frac{\partial \ln L}{\partial \theta} = \sum_{i=1}^n \left[\frac{2k_i k_i'}{1 + k_i^2} - \frac{\rho_i^2 k_i k_i'}{(1 + k_i^2)^2} \right], \quad (7.2.19)$$

where $k_i = k_m Q(\theta - \theta_i)$, and $k_m = \frac{\sigma_0}{\sigma}$.

First term does not depend on selective values of signal, and with symmetrical diagram this sum is equal to zero. In certain cases [22] they assume that it is small or vanishes with a large quantity of impulses/momenta/pulses in the packet. However, if the direction of the middle of packet does not coincide with the true azimuth of target, then this component/term/addend will not be small. It is possible to only say that the average/mean value of this sum with a large quantity of surveys/coverage will be equal to zero.

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If we accept the assumption that this sum is equal to zero (assuming a quantity of impulses/momenta/pulses in the packet large), then the initial algorithm of processing signal for determining the evaluation/estimate of the azimuth of target we will obtain in the following form:

$$\sum_{i=1}^n p_i^2 \frac{k_i k'_i}{1 + k_i^2} = 0. \quad (7.2.20a)$$

However, in the case of small quantity of impulses/momenta/pulses in packet adopted assumption proves to be invalid and initial algorithm of working/treatment is determined by relationship/ratio

$$\sum_{i=1}^n \frac{k_i k'_i}{1 + k_i^2} \left(2 - \frac{p_i^2}{1 + k_i^2} \right) = 0. \quad (7.2.20)$$

These equations, just as equation (7.2.7), are solved by method of "tests/samples and search" with subsequent account of obtained errors. For solving this equation with the aid of the electronic digital computers it is necessary to convert with the aid of PND selective values $p_1, p_2, \dots, p_n$ into the binary code and to memorize them. Then, being assigned by different values $\theta = \theta_1; \theta = \theta_2; \dots$ they calculate the left side of equation (7.2.20). The value θ , which in the best way satisfies these equalities, can be accepted as the unknown value of evaluation/estimate. The fulfillment of this procedure in this case can be simplified, since the sum

$$2 \sum_{i=1}^n \frac{k_i (0 - \theta_i) k'_i (0 - \theta_i)}{1 + k_i^2 (0 - \theta_i)} = V(n, \theta) \quad (7.2.21)$$

does not depend on selective values, it can be calculated previously for the series/row of values $\theta = \theta_1; \theta = \theta_2; \dots$ and it is recorded in the memory unit. As a result are obtained several values of sums of $V_1(n; \theta_1); V_2(n; \theta_2) \dots$, which are memorized. From the selective values $p_1, p_2, \dots, p_n$ is calculated immediately after reception/procedure the sum

$$\sum_{i=1}^n \frac{k_i k'_i}{(1 + k_i^2)^2} \rho_i^2 = U(n; p_1, p_2, \dots, p_n; \theta). \quad (7.2.22)$$

This sum also is calculated for several values θ .

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Is obtained series/row of values $U_1, U_2, U_3, \dots$

Since equation (7.2.20) is reduced taking into account (7.2.21) and (7.2.22) to equality

$$V(n; \theta) = U(n; \rho_1, \rho_2, \dots, \rho_n; \theta), \quad (7.2.23)$$

then determination of evaluation/estimate can be brought to consecutive comparison of values $V_m(n; \theta_m)$ and $U_m(n; \rho_1, \dots, \rho_n; \theta_m)$; $m=1, 2, \dots$. The closest values of these values determine the value of angle $\hat{\theta}$, which can be accepted as the unknown value of the evaluation/estimate of azimuth.

Expressions for sums (7.2.21) and (7.2.22) can be simplified for case of strong signal, when $\sigma_{ci} \gg \sigma$ and $k_i \gg 1$ for all $i=1-n$. In this case

$$V(n; \theta) = 2 \sum_{i=1}^n \frac{Q'(\theta - \theta_i)}{Q(\theta - \theta_i)} \quad (7.2.24)$$

and

$$U(n; \rho_1, \rho_2, \dots, \rho_n; \theta) = \sum_{i=1}^n \frac{Q'(\theta - \theta_i)}{Q(\theta - \theta_i)} \frac{\rho_i^2}{k_i(\theta - \theta_i)}. \quad (7.2.25)$$

Methods of measurement examined here of angle utilize information, which is contained in amplitudes of selective values of workable signal. This necessitates the applying of the fairly complicated transformation circuits of analog quantities into the discrete/digital (PND); therefore it is desirable to utilize other, simpler in circuit sense methods of working/treatment, which do not

require complicated nodes in its composition. As this method can be used the method of measurement for quantized data, which is examined in §7.7.

§7.3. Accuracies of the measurement of angular coordinates during the linear scanning.

Let us conduct research of this question in the same sequence, in which algorithms of measurement of angles for different signal aspect were examined, i.e., let us at first examine evaluation/estimate of accuracy of direction finding of targets with constant, and then with which fluctuates by reflecting surfaces.

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As modulus of precision of measurement of angular coordinate dispersion $D(\hat{\theta})$ of evaluation/estimate of measured angle θ is accepted. According to Rao-Cramer theorem [18] the dispersion of evaluation/estimate is inversely proportional to an informational quantity of Fisher j , i.e.,

$$D(\hat{\theta}) \geq \frac{1}{j},$$

and

$$j = E \left(- \frac{\partial^2 \ln L}{\partial \theta^2} \right).$$

Here $E()$ indicates the mathematical expectation of the value, included in the brackets.

A. Accuracy of the measurement of angle with the constant diffusing surface of circuit.

In view of unwieldiness of computation of accuracy of direction finding in general form let us examine this question for two special cases: for small and large amplitude of received signal.

Let us at first calculate evaluation/estimate for case, when signal has small intensity, i.e., $A_i \ll \sigma$, and proves to be valid following approximate relationship/ratio:

$$\ln I_0 \left(\frac{p_i A_i}{\sigma^2} \right) \approx \frac{p_i^2 A_i^2}{4\sigma^4}.$$

In this case formula (7.2.4) can be rewritten in the following form:

$$\ln L = \sum_{i=1}^n \left[\ln \frac{p_i^2}{\sigma^2} - \frac{p_i^2}{2\sigma^2} - \frac{A_i^2}{2\sigma^2} + \frac{p_i^2 A_i^2}{4\sigma^4} \right]. \quad (7.3.1)$$

But derivative of logarithm of function of plausibility

$$\frac{\partial \ln L}{\partial \theta} = \sum_{i=1}^n \left[\frac{2A_i A_i' p_i^2}{\sigma^4} - \frac{A_i A_i'}{\sigma^2} \right]. \quad (7.3.2)$$

Mathematical expectation for first case

$$E\left(-\frac{\partial^2 \ln L}{\partial \theta^2}\right) = \frac{1}{\sigma^2} \sum_{i=1}^n \{A_i^2 + A_i A_i^*\} \left[1 - \frac{1}{2\sigma^2} E(p_i^2)\right]. \quad (7.3.3)$$

In this expression is contained mathematical expectation from square of random variable $E(p_i^2)$; in this case random variable itself p_i it is distributed according to generalized Rayleigh's law. For computing this mathematical expectation we will use the formulas of the functional conversions of random variables, after considering in this case, that the mathematical expectation from the random variable to first degree $E(p_i)$ is known, since it corresponds to the first moment/torque of the distribution function. In general form the expression of the k moment/torque of the function of random number distribution, distributed according to generalized Rayleigh's law, is determined by the expression

$$m_k = \frac{1}{\sigma^2} \int_0^\infty e^{-\frac{A^2}{2\sigma^2}} p^{k+1} e^{-\frac{p^2}{2\sigma^2}} I_0\left(\frac{Ap}{\sigma^2}\right) dp.$$

This formula [23] takes also form

$$m_k = (2\sigma^2)^{\frac{k}{2}} \Gamma\left(1 + \frac{k}{2}\right) {}_1F_1\left(-\frac{k}{2}; 1; -\frac{A^2}{2\sigma^2}\right), \quad (7.3.4)$$

where ${}_1F_1$ - degenerate hypo-geometric function [24]. In particular, the second moment/torque with $k=2$

$$m_2 = 2\sigma^2 \left(1 + \frac{A^2}{2\sigma^2}\right), \quad (7.3.4a)$$

and the first moment/torque with $k=1$

$$m_1 = \sigma \sqrt{\frac{\pi}{2}} \left[\left(1 + \frac{A^2}{2\sigma^2}\right) I_0\left(\frac{A^2}{4\sigma^2}\right) + \frac{A^2}{2\sigma^2} I_1\left(\frac{A^2}{4\sigma^2}\right) \right] e^{-\frac{A^2}{4\sigma^2}}.$$

Utilizing asymptotic expansion of Bessel function, we will obtain expression for first moment/torque under condition $A \gg \sigma$ in the form

$$m_1 = A \left(1 + \frac{\sigma^2}{2A^2} \right). \quad (7.3.46)$$

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Function of distribution of square of random variable in accordance with formulas of conversion of function of random number distribution will be equal (with $k=2$)

$$E(\rho_i^2) = 2\sigma^2 + A_i^2. \quad (7.3.48)$$

After substituting this relationship/ratio into formula (7.3.3), and then in (7.1.14), we will obtain formula of dispersion of evaluation/estimate of angle for case of weak signal with direction finding by method of linear scanning (reflecting diffusing surface of target it has constant value)

$$D(\hat{\theta}) \geq \frac{-2\sigma^4}{\sum_{i=1}^n [A_i^4 A_i^2 + A_i^2 A_i^4]}. \quad (7.3.5)$$

Let us switch over to numerical examples. All further reasonings let us conduct based on the example of the radiation pattern of bell-shaped form, which is determined

by the function of zero-order Hermite

$$A_i = A_0 e^{-\varphi_i^2}, \quad (7.3.6)$$

where

$$\varphi_i = p [\Delta\theta (i-1) - \theta], \quad (7.3.6a)$$

and r - parameter of approximation, determined half of the width of diagram φ_d calculated at the level d :

$$p = \frac{1}{\varphi_d} \sqrt{\ln \frac{1}{d}}.$$

Taking into account that

$$A_i'(\theta) = A_i 2p\varphi_i, \quad A_i''(\theta) = 2p^2 A_i(\theta) (2\varphi_i^2 - 1); \quad (7.3.7)$$

$$A_i' A_i' + A_i^3 A_i = 2A_i^4 p^2 (4\varphi_i^2 - 1),$$

and by substituting these expressions, and also formula (7.3.6) in (7.3.5), we will obtain the dispersion of the evaluation/estimate of angle with the direction finding of the weak signal, measured by radar system from the antenna, having diagram bell-shaped form:

$$D(\hat{\theta}) \geq \frac{\sigma^2}{A_0^4 p^2 \sum_{i=1}^n (1 - 4\varphi_i^2) e^{-4\varphi_i^2}}. \quad (7.3.8)$$

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For simplification in further reasonings let us assume that reference point of angle θ coincides with position of first extreme impulse/momentum/pulse of packet. In this case we consider that angular target position coincides with the middle of radiation pattern and burst of pulses is symmetrical relative to center, i.e.,

$$\theta = \frac{\Delta\theta}{2} (n-1). \quad (7.3.9)$$

After substituting this expression in (7.3.6a), we will obtain formula for the discrete/digital variable, which characterizes a change in the angle:

$$\varphi_i = p\Delta\theta \left(i - \frac{n+1}{2} \right), \quad (7.3.10)$$

where initial discrete/digital the variable i changes through one within the limits 1-n. After rewriting (7.3.8) taking into account (7.3.10), we will obtain

$$D(\hat{\theta}) \geq \frac{\sigma^4}{A_0^4 p^4 \sum_{i=1}^n \left[1 - 4b^2 \left(i - \frac{n+1}{2} \right)^2 \right] e^{-4b^2 \left(i - \frac{n+1}{2} \right)^2}}, \quad (7.3.11)$$

where $b = p\Delta\theta$.

Since in this formula factor, which contains discrete/digital

variable, is even, expression can be simplified, and quantity of components/terms/addends in sum of denominator is reduced.

From structure of expressions (7.3.11) it follows that with $b=0$ dispersion of evaluation/estimate will be minimum, and with $b>0$ denominator decreases and dispersion of evaluation/estimate increases. However, case $b=0$ corresponds so that the raster angle $\Delta\theta$ is equal to zero and a quantity of impulses/momenta/pulses in the packet must be infinitely great, which is virtually unrealizable. Therefore case $b=0$ should be examined only as the idealization of mode/conditions with a large quantity of impulses/momenta/pulses in the packet. But since questions of the accuracy of measurement have the greatest interest with the limited number of impulses/momenta/pulses in the packet, case $b=0$ is not of great interest, since usually they try to achieve the required accuracy with the smallest number of impulses/momenta/pulses.

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The dependence of the accuracy of measurement from a quantity of impulses/momenta/pulses in the packet, and also from the parameters of diagram and signal-to-noise ratio A_s/σ is determined by formula (7.3.11) (for the symmetrical packet). If direction to the oriented object does not coincide with the middle of packet, then the

corresponding relationship/ratio for the evaluation/estimate can be obtained from formula (7.3.5):

Let us switch over to computation of dispersion of evaluation/estimate of angle for case of strong signal, when it is possible to consider that

$$\ln I_0 \left(\frac{A_i \rho_i}{\sigma^2} \right) \approx \frac{A_i \rho_i}{\sigma^2}.$$

In this case logarithm of function of plausibility

$$\ln L = \sum_{i=1}^n \ln \frac{\rho_i}{\sigma^2} - \frac{\rho_i^2}{2\sigma^2} - \frac{A_i^2}{2\sigma^2} + \frac{\rho_i A_i}{\sigma^2}, \quad (7.3.12)$$

and derivative of logarithm of function of plausibility

$$\frac{\partial \ln L}{\partial \theta} = \sum_{i=1}^n \left(\frac{A_i' \rho_i}{\sigma^2} - \frac{A_i' A_i}{\sigma^2} \right). \quad (7.3.13)$$

Second derivative of logarithm of function of plausibility

$$\frac{\partial^2 \ln L}{\partial \theta^2} = \sum_{i=1}^n \left(\frac{\rho_i A_i'}{\sigma^2} - \frac{A_i A_i' + A_i'^2}{\sigma^2} \right).$$

Mathematical expectation from second partial derivative of logarithm of function of plausibility

$$E \left(- \frac{\partial^2 \ln L}{\partial \theta^2} \right) = \frac{1}{\sigma^2} \sum_{i=1}^n \left[A_i'^2 - A_i' [E(\rho_i) - A_i] \right]. \quad (7.3.14)$$

Mathematical expectation from random variable $E(\rho_i)$, entering in (7.3.14), for signals of large intensity $A_i \gg \sigma$ is determined by relationship/ratio (7.3.4b)

$$E(\rho_i) \approx A_i \left(1 + \frac{\sigma^2}{2A_i^2} \right).$$

After substituting this relationship/ratio in (7.3.14), and then in (7.1.14), we will obtain expression for dispersion of evaluation/estimate of measured angle for strong signal with direction finding by method of linear scanning:

$$D(\hat{\theta}) \geq \frac{1}{\frac{1}{\sigma^2} \sum_{i=1}^n \left(A_i^2 - \frac{\sigma^2 A_i^2}{2A_i} \right)} \quad (7.3.15)$$

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From this formula calculated relationships/ratios for different form of radiation patterns of antenna systems, utilized for direction finding of echo signal, can be obtained. In particular, for the bell-shaped radiation pattern, determined by relationship/ratio (7.3.6), and packet with symmetrical envelope (packet it is symmetrical relative to the center, which coincides with the direction to the oriented signal), it is possible to calculate the dispersion of evaluation/estimate, if we substitute relationships/ratios (7.3.10) and (7.3.6) in (7.3.7) and then in (7.3.15). After producing the necessary computations, we will obtain

$$D(\hat{\theta}) \geq \frac{1}{p^2 \left[n + \sum_{i=1}^n 2b^2 \left(i - \frac{1+n}{2} \right)^2 \left[2 \left(\frac{A_0}{\sigma} \right)^2 e^{-2b^2 \left(i - \frac{1+n}{2} \right)^2} - 1 \right] \right]} \quad (7.3.16)$$

The terms in the denominator of expression (7.3.11) can have both positive and negative values. The presence of negative components/terms/addends, naturally, decreases the denominator and increases the measuring error of angle by the method of linear scanning. Negative components/terms/addends appear when

$$1 - 4b^2 \left(i - \frac{n+1}{2} \right)^2 < 0. \quad (7.3.17)$$

Fulfilling of this inequality depends on values of b and n .

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For increasing the accuracy of direction finding according to the method of linear scanning the parameters of the orienting system must be selected in such a way that for the impulses/momenta/pulses of packet with a small amplitude, i.e., for the extreme impulses/momenta/pulses of packet, determined by index $i=1$ or $i=n-1$, expression in brackets (7.3-11) would be positive, i.e.,

$$1 - b^2(n-1)^2 > 0. \quad (7.3.18)$$

In this case for mean momentum of packet expression (7.3.17) will be more positive and negative terms in denominator will not be. The presence of there negative terms speaks about the irrational use of impulses/momenta/pulses in the packet. Actually/really, the

impulses/momenta/pulses, which have a small amplitude, increase the sample size, without decreasing in this case the dispersion of evaluation/estimate. The level of diagram, lower than which to utilize signals for the direction finding is inexpedient (in a small signal-to-noise ratio), and the pulse amplitudes corresponding to it can be found from the limiting value of b , which satisfies the condition of the absence of negative components/terms/addends in denominator (7.3.16), i.e., from the condition

$$b < \frac{1}{n-1}. \quad (7.3.18a)$$

Then "critical" value of level of diagram d_0 can be determined from value $b_0 = p\Delta\theta_0$, where

$$p = \frac{1}{2\varphi_{d_0}} \sqrt{\ln \frac{1}{d_0}}, \quad (7.3.19)$$

and $2\varphi_{d_0}$ - width of diagram, calculated at the level $d_0 < 1$.

For determining critical level of diagram d_0 we assume that impulses/momenta/pulses of packet are located accurately in solution/opening of diagram, then

$$\Delta\theta = \frac{2\varphi_{d_0}}{n-1}. \quad (7.3.20)$$

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From relation (7.3.19) and formula for p it follows that

$$b_0 = \frac{2}{n-1} \sqrt{\ln \frac{1}{d_0}}, \quad (7.3.21)$$

From this equality taking into account (7.3.18a) we will obtain

$$\sqrt{\ln \frac{1}{d_0}} < 0.5, \quad (7.3.22)$$

whence $d_0 > e^{-0.25} = 0.78$. Thus, at small intensities of signal the impulses/momenta/pulses of packet it is expedient to arrange/locate within the sector of diagram, limited by the ordinates:

$$d \geq d_0 = 0.78. \quad (7.3.23)$$

The use of a wider section of diagram does not improve the accuracy of direction finding.

Analogous type limitations on small signal level can be obtained also for antenna radiation patterns of another form, which differ from bell-shaped.

Let us examine now computation of dispersion of angle with direction finding of fluctuating target.

B. Evaluation/estimate of the accuracy of the measurement of the angle of the fluctuating target.

For evaluation/estimate of accuracy of measurement of angle we should define informational quantity of Fisher (7.1.14), as is known.

For this it is necessary to at first find the second derivative of the function of plausibility. On the basis of formula (7.2.19)

$$\frac{\partial^2 \ln L}{\partial \theta^2} = \sum_{i=1}^n \left\{ \frac{\partial^2}{\partial t^2} \left[k_i' \left(1 - \frac{4k_i}{d_i} \right) + k_i k_i' \right] - \right. \\ \left. - \frac{2}{d_i} \left[k_i' \left(1 - \frac{2k_i}{d_i} + k_i k_i' \right) \right] \right\}. \quad (7.3.24)$$

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If one considers that average/mean value from square of random variable, distributed according to formula (5.2.10), is equal

$$E(p_i^2) = 2d_i, \quad (7.3.25)$$

then, after substituting (7.3.25) and (7.3.24) in (7.1.14), we will obtain expression for informational quantity of Fisher in the form

$$j = \sum_{i=1}^n \frac{4k_i k_i'}{d_i^2}. \quad (7.3.26)$$

And finally dispersion of evaluation/estimate

$$D(\hat{\theta}) \geq \frac{0.25}{\sum_{i=1}^n \frac{k_i^2 k_i'}{d_i^2}}. \quad (7.3.27)$$

Let us give specific example of calculation for antennas with radiation pattern of bell-shaped form, determined by expression (7.3.6):

$$Q_i = e^{-\varphi_i^2}, \quad (7.3.28)$$

where φ_i is determined by formula (7.3.6a)

$$\varphi_i = p [\Delta\theta (i-1) - \theta].$$

As is known from (7.2.19),

$$k_i = k_m Q_i, \text{ and } k'_i = k_m Q'_i.$$

Taking into account that

$$Q_i = 2p\varphi_i Q_i \text{ and } Q'_i = 2p^2(2\varphi_i^2 - 1) Q_i, \quad (7.3.29)$$

we will obtain for dispersion of evaluation/estimate expression

$$D(\hat{\theta}) \geq \frac{0,25}{4p^2 k_m^4 \sum_{i=1}^n \frac{\varphi_i^2 R_i^4}{(1 + k_m^2 Q_i^2)^3}}. \quad (7.3.30)$$

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Let us calculate this formula for angular coordinate of target, which coincides with axis of symmetry of packet. In this case

$$\theta = \frac{\Delta\theta}{2} (n-1) \quad (7.3.31)$$

and

$$\varphi_i = p\Delta\theta \left(i - \frac{1+n}{2} \right),$$

then

$$D(\hat{\theta}) \geq \frac{0,25}{4p^2 k_m^4 \sum_{i=1}^n \frac{b^2 \left(i - \frac{1+n}{2} \right)^2 e^{-4b^2 \left(i - \frac{1+n}{2} \right)^2}}{\left[1 + k_m^2 e^{-2b^2 \left(i - \frac{1+n}{2} \right)^2} \right]^3}}, \quad (7.3.32)$$

where $b = p\Delta\theta$.

This relationship/ratio can be simplified for case of weak and strong signal. In first case $k_m \ll 1$, and then expression (7.3.32) takes the form

$$D(\hat{\theta}) \geq \frac{1}{4p^2 k_m^4 \sum_{i=1}^n b^2 \left(i - \frac{1+n}{2}\right)^2 e^{-4b^2 \left(i - \frac{1+n}{2}\right)^2}}. \quad (7.3.33)$$

However, for strong signal when occurs inequality $k_m Q_i \gg 1$, valid for all i ,

$$D(\hat{\theta}) \geq \frac{0,25}{4p^2 b^2 \sum_{i=1}^n \left(i - \frac{1+n}{2}\right)^2} = \frac{1}{J_{a.c.}(\hat{\theta})}. \quad (7.3.34)$$

Whence ratio of dispersion to square of half of width of diagram

$$\frac{D(\hat{\theta})}{\sigma_d^2} \geq \frac{0,25}{4b^2 \ln \frac{1}{d} \sum_{i=1}^n \left(i - \frac{1+n}{2}\right)^2}.$$

Since is correct relationship/ratio

$$\sum_{i=1}^n \left(i - \frac{1+n}{2}\right)^2 = \frac{n}{12} (n^2 - 1),$$

then

$$\frac{D(\hat{\theta})}{\sigma_d^2} \geq \frac{0,75}{b^2 n (n^2 - 1) \ln \frac{1}{d}}.$$

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Taking into account that for $b = \rho \Delta \theta$ finally we correctly obtain relationship/ratio

$$b = \frac{\Delta \theta}{\sigma_d} \sqrt{\ln \frac{1}{d}} \approx \frac{2}{n} \sqrt{\ln \frac{1}{d}},$$

$$\frac{D(\hat{\theta})}{\sigma_d^2} \geq \frac{0.75n}{4(n^2-1)n^2 \frac{1}{n^2}}$$

Hence, in particular, it follows that ratio of variance of error to square of half of width of diagram decreases, approximately/exemplarily, as $1/n$.

57.4. Measurement of angle during the discrete/digital scanning.

Together with linear scanning of antenna discrete/digital scanning for measuring angular coordinate can be used. During the discrete/digital scanning the antenna of radar accepts the series/row of those fixed/recorded position relative to direction to the target. In particular, let us suppose that it occupies only two fixed/recorded positions (Fig. 7.6a), in each of which the direction of maximum radiation/emission is directed at angle α_1 and α_2 ($\alpha_2 > \alpha_1$). Angles are counted off from the arbitrary initial direction, and the difference

$$\alpha_2 - \alpha_1 = \Delta\alpha$$

we will call subsequently the angle of discrete/digital scanning.

Known method of measurement of angle with the aid of equisignal sector is special case of method of discrete/digital scanning. This method although provides in a number of cases the necessary accuracy

of measurement however it requires the expenditure of relatively long time for the fulfillment of separate measurement.

Actually/really, with this method it is necessary so to arrange antenna system of direction finder so that equisignal direction would coincide with direction to target.

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The "groping" of equisignal sector and the output of antenna to the equisignal direction require the series/row of the commutations of the position of radiation pattern so that in each of the discrete/digital antenna positions could be fixed during the specific time interval. As a whole on entire process of measurement sufficiently considerable time here is spent.

But if duration of observation of target will be limited for any reasons, then use/application of method of equisignal sector will prove to be hardly appropriate and advisable. In this case it is desirable to apply such methods, which would give high accuracy with the limited time of measurement.

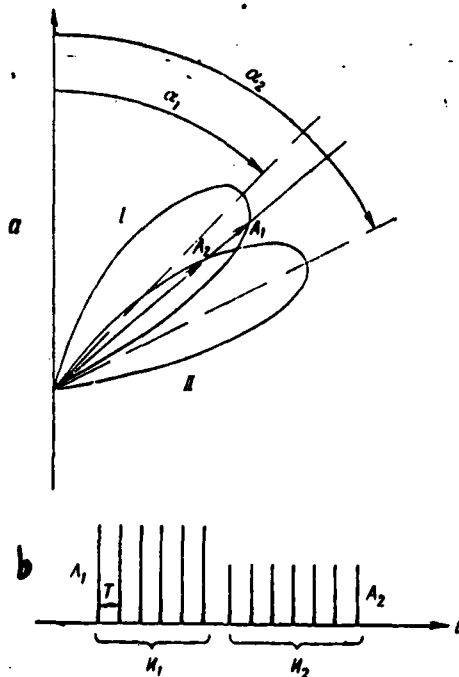


Fig. 7.6. To the method of two positions.

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From this point of view deserves attention the method of discrete/digital scanning, with which the measurement of angle is made as a result of processing the signals, obtained with several ones, in particular in two, antenna positions. In this case the equation of radiation pattern must be known, and direction to the target must be within the limits of angle $\Delta\alpha$.

Direction-finding method in question with small quantity of impulses/momenta/pulses in packet can ensure in certain cases high accuracy of direction finding, than method of linear scanning, that also determines its known practical value. In this case the use of diagrams of electronic beam control of antenna facilitates the structural/design realization of this system.

Furthermore, method of discrete/digital scanning in theoretical plan/layout is in a sense initial for analysis of multichannel (monopulse) system of direction finding, where antenna position fixed and processing signal is performed in channel of each antenna. But in two positions this method is to a certain extent the generalization of the method of equisignal sector, which is obtained hence as a special case.

Let us pass to presentation of method of discrete/digital scanning in two antenna positions.

Let antenna in each of two positions be located in period T_1 and T_2 . In this case at the receiving channel enter respectively n_1 and n_2 of the impulses/momenta/pulses, reflected from the target. We will consider that the antenna passes of one state to another abruptly, then the character of burst of pulses will take the form, indicated in Fig. 7.6b. Knowing the equation of the antenna radiation pattern

and the relationship/ratio between the values of the impulses/momenta/pulses, obtained in the different antenna positions, it is possible to determine the angular coordinate of target. However, the presence of interferences in the receiving channel distorts received pulses and is introduced error into the measurement of angular coordinate.

Process of direction finding is reduced to reception/procedure n_1 impulses/momenta/pulses in one antenna position and n_2 with other with their subsequent working/treatment. In the general case of $n_1 \neq n_2$, but

$$n_1 + n_2 = n, \quad (7.4.1)$$

where n - number of impulses/momenta/pulses in the packet.

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As a result of reception of signals at output of radar channel we will have N of selective values of signal, which consist of two totalities. One of them

$$p_1^{(1)}, p_2^{(1)}, \dots, p_{n_1}^{(1)} \quad (7.4.2)$$

corresponds to the first antenna position - in the direction α_1 , and another totality

$$p_1^{(2)}, p_2^{(2)}, \dots, p_{n_2}^{(2)} \quad (7.4.2a)$$

corresponds to antenna position in the direction α_2 . In this case the

level of received signals in each the cases will be proportional to the specific values of the radiation pattern, which are respectively equal to

$$Q(\theta - \alpha_1); Q(\alpha_2 - \theta). \quad (7.4.3)$$

Our most immediate task is determination of structure of optimum processing of selective values of signal (7.4.2) and (7.4.2a), necessary for measurement angular coordinate of target.

Structure of optimum working/treatment as in the preceding case (§ 7.1 and 7.2), will be determined from maximum likelihood equation (7.1.13)

$$\frac{\partial \ln L(p_1^{(1)}, p_2^{(1)}, \dots, p_{n_1}^{(1)}, p_1^{(2)}, p_2^{(2)}, \dots, p_{n_2}^{(2)}; \theta)}{\partial \theta} = 0.$$

Concrete/specific/actual algorithm, and also diagram of processing signal for measuring angle depend on character of echo signals and interferences, since in each of these cases function of plausibility is different. Subsequently we assume that for the time of the formation of packet the angular coordinate of target virtually will not change.

A. Measurement of the angular coordinate of target with the constant diffusing surface.

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Function of plausibility, i.e., joint probability density of values $\rho_1^{(1)}, \rho_2^{(1)}, \dots, \rho_{n_1}^{(1)}$ and $\rho_1^{(2)}, \rho_2^{(2)}, \dots, \rho_{n_2}^{(2)}$ with value of parameter θ and at condition of statistical independence of signals in selection, in accordance with formulas (5.1.10) and (7.4.3) will take form

$$\begin{aligned} L(\rho_1^{(1)}, \rho_2^{(1)}, \dots, \rho_{n_1}^{(1)}, \rho_1^{(2)}, \rho_2^{(2)}, \dots, \rho_{n_2}^{(2)}; \theta) = \\ = \prod_{i=1}^{n_1} \frac{\rho_i^{(1)}}{\sigma^2} \exp\left(-\frac{\rho_i^{(1)^2} + A_1^2(\theta)}{2\sigma^2}\right) I_0\left(\frac{\rho_i^{(1)} A_1(\theta)}{\sigma^2}\right) \times \\ \times \prod_{i=1}^{n_2} \frac{\rho_i^{(2)}}{\sigma^2} \exp\left(-\frac{\rho_i^{(2)^2} + A_2^2(\theta)}{2\sigma^2}\right) I_0\left(\frac{\rho_i^{(2)} A_2(\theta)}{\sigma^2}\right). \quad (7.4.4) \end{aligned}$$

where

$$A_1(\theta) = A_0 Q(\theta - \alpha_1) = A_1; \quad A_2(\theta) = A_0 Q(\alpha_2 - \theta) = A_2. \quad (7.4.5)$$

Value A , corresponds to signal amplitude in direction of maximum of diagram.

Logarithm of function of plausibility

$$\begin{aligned} \ln L = \sum_{i=1}^{n_1} \left[\ln \frac{\rho_i^{(1)}}{\sigma^2} - \frac{\rho_i^{(1)^2} + A_1^2}{2\sigma^2} + \ln I_0\left(\frac{\rho_i^{(1)} A_1}{\sigma^2}\right) \right] + \\ + \sum_{i=1}^{n_2} \left[\ln \frac{\rho_i^{(2)}}{\sigma^2} - \frac{\rho_i^{(2)^2} + A_2^2}{2\sigma^2} + \ln I_0\left(\frac{\rho_i^{(2)} A_2}{\sigma^2}\right) \right]. \quad (7.4.6) \end{aligned}$$

Let us examine case of strong signal, when $\frac{\rho_i A_i}{\sigma^2} \gg 1$. In this case according to (6.2.6)

$$\ln I_0\left(\frac{\rho_i A_i}{\sigma^2}\right) \approx \frac{\rho_i A_i}{\sigma^2}.$$

After substituting this relationship/ratio in (7.4.6) and then in (7.1.13), we will obtain

$$\begin{aligned} \frac{\partial \ln L}{\partial \theta} = & \sum_{i=1}^{n_1} \left[\frac{p_i A_1'}{\sigma^2} - \frac{A_1 A_1'}{\sigma^2} \right] + \\ & + \sum_{i=1}^{n_2} \left[\frac{p_i^2 A_2'}{\sigma^2} - \frac{A_2 A_2'}{\sigma^2} \right] = 0, \end{aligned} \quad (7.4.7)$$

where A_1' and A_2' is derivatives on θ .

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Producing obvious conversions and reducing by σ^2 , the equation of the maximum of plausibility (7.4.7) can be rewritten in the following form:

$$\sum_{i=1}^{n_1} p_i^{(1)} + \mu(\theta) \sum_{i=1}^{n_2} p_i^{(2)} = \psi(\theta), \quad (7.4.8)$$

where

$$\mu(\theta) = \frac{Q'(\alpha_2 - \theta)}{Q'(\theta - \alpha_1)}; \quad \psi(\theta) = A_1 n_1 + \mu(\theta) A_2 n_2. \quad (7.4.9)$$

Since parameters of signal, interference and equation of antenna directivity are considered given ones, and number of impulses/momenta/pulses in each antenna position known, form of the function $\mu(\theta)$ and $\psi(\theta)$ should be considered known. Unknown is here only the value of angle θ , i.e., the argument of these functions.

Equality (7.4.8) determines structure of processing signal in

measurement of angular coordinate according to method of maximum plausibility in two antenna positions. Working/treatment is reduced to the addition of the selective values of signal in both antenna positions, i.e., to the formation/education of values $\sum_{i=1}^{n_1} \rho_i^{(1)}$ and $\sum_{i=1}^{n_2} \rho_i^{(2)}$, with the subsequent solution of equation (7.4.8) relatively θ . This solution is also expedient to realize by the method of "tests/samples and search". However, it must be noted that the computations obtained here are simpler than in the preceding case, i.e., in the case of linear scanning, since there is no need for producing the method of "tests/samples and search" in this space, as this occurred in §7.2 (7.2.7 or 7.2.15). Actually/really, here the sums of selective values are the coefficients of equation (7.4.8). These sums can be calculated as a result of processing received signals in the diagrams PND and the summator. Thus, and the coefficients of equation (7.4.8) will be known, and the form of functions themselves. Their numerical values can be calculated previously for the different ones θ . Solution in this case is reduced to the determination of the values of functions $\psi(\hat{\theta})$ and $\mu(\hat{\theta})$ and, consequently, also angle $\hat{\theta}$, in the best way satisfying equation (7.4.8).

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Solution of these problems in known parameters of distribution

(5.1.10) more conveniently fulfilling with the aid of electronic computers, for which relationships/ratios (7.4.8) and (7.4.9) are initial for composition of algorithm and program of work of machine.

Let us examine algorithm of processing signal with direction finding according to method of equisignal sector, which resultant as special case. Actually/really, when the source of the oriented oscillations/vibrations is located on the axis of equisignal sector and $\theta = \theta_p$, corresponds to equisignal direction, occur the following relationships/ratios:

$$\theta_p = \alpha_1 + \frac{\Delta\alpha}{2} = \alpha_2 - \frac{\Delta\alpha}{2},$$

$$A_1(\theta_p) = A_2(\theta_p); A_1'(\theta_p) = -A_2'(\theta_p);$$

$$\mu(\theta_p) = -1; \psi(\theta_p) = A_1(\theta_p)(n_1 - n_2).$$

Then relationship/ratio (7.4.8) passes into following equality:

$$\sum_{i=1}^{n_1} \rho_i^{(1)} - \sum_{i=1}^{n_2} \rho_i^{(2)} = (n_1 - n_2) A_1(\theta_p). \quad (7.4.10)$$

If quantity of impulses/momenta/pulses in each antenna position is equal to, i.e. $n_1 = n_2 = n/2$, then algorithm of working/treatment is reduced to establishment of equality

$$\sum_{i=1}^{\frac{n}{2}} \rho_i^{(1)} = \sum_{i=1}^{\frac{n}{2}} \rho_i^{(2)}, \quad (7.4.11)$$

fulfilling of which means that unknown value of angle

$$\theta_p = \alpha_1 + \frac{\Delta\alpha}{2}.$$

It must be noted that in the case of direction finding by method of equisignal sector with $n_1 = n_2$, for solving equation of plausibility there is no need for a priori knowledge of parameters of signal (signal-to-noise ratio) and radiation pattern of antenna system. In this case the obtained solutions relate to the special type solution, that correspond to the formulation of the problem with not completely known data.

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Such type of solution have values when at our disposal sufficient completeness of a priori information about signal or interference is absent. Frequently proves to be unknown the level of signal or interference or the two together, etc. The problem of obtaining the optimum algorithm of processing signal with not completely known data in accordance with the terminology, accepted in the literature [73], relates to the class of nonparametric type problems.

Solution of nonparametric problem, given by method of equisignal sector, is correct for unique point and for operating reasons not in all cases is applicable, since "groping" of this point, as noted earlier, it is connected with considerable expenditure of time. This increase in the time it is possible to examine, in the known case as

"board" for ignorance of the parameters of the problem.

Let us examine procedure of working/treatment at small intensities of signal, when $\frac{\rho_i A}{\sigma^2} \ll 1$. In this case occurs following approximate relationship/ratio (6.2.6):

$$\ln I_0 \left(\frac{\rho_i A}{\sigma^2} \right) \approx \frac{\rho_i^2 A^2}{4\sigma^4}.$$

Then from (7.4.6) it follows that

$$\begin{aligned} \ln L = & \sum_{i=1}^{n_1} \left[\ln \frac{\rho_i^{(1)}}{\sigma^2} - \frac{\rho_i^{(1)2}}{2\sigma^2} - \frac{A_1^2}{2\sigma^2} + \frac{(\rho_i^{(1)} A_1)^2}{4\sigma^4} \right] + \\ & + \sum_{i=1}^{n_2} \left[\ln \frac{\rho_i^{(2)}}{\sigma^2} - \frac{\rho_i^{(2)2}}{2\sigma^2} - \frac{A_2^2}{2\sigma^2} + \frac{(\rho_i^{(2)} A_2)^2}{4\sigma^4} \right]. \end{aligned}$$

Substituting this equality in (7.1.13), we have

$$\begin{aligned} \frac{\partial \ln L}{\partial \theta} = & \sum_{i=1}^{n_1} \left[\frac{A_1 A_1'}{\sigma^2} \left(\frac{\rho_i^{(1)}}{2} - 1 \right) \right] + \\ & + \sum_{i=1}^{n_2} \left[\frac{A_2 A_2'}{\sigma^2} (\rho_i^{(2)} - 1) \right] = 0. \end{aligned} \quad (7.4.12)$$

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After fulfilling obvious conversions and after shortening on σ , and then divide by $A_1 A_1'$, we will obtain relationship/ratio, analogous (7.4.8):

$$\sum_{i=1}^{n_1} \rho_i^{(1)} + \gamma_1(\theta) \sum_{i=1}^{n_2} \rho_i^{(2)} = F(\theta), \quad (7.4.13)$$

where

$$\eta(\theta) = \frac{A_2 A_2'}{A_1 A_1'}; F(\theta) = 2\sigma^2 [n_1 + \eta(\theta) n_2]. \quad (7.4.13')$$

Relationship/ratio (7.4.12) determines algorithm (structure) of processing signal with direction finding on two antenna positions feeble in the case of signal. The value $\hat{\theta}$, which satisfies this relationship/ratio, is the best estimator of the measured angle θ , since (7.4.12) it is obtained from the criterion of the maximum of the function of plausibility. The character of working/treatment here is the same, as for the strong signal; however, the form of the function (7.4-13) differs from analogous functions for the case of strong signal.

Let us examine now algorithm of working/treatment with direction finding according to method of equisignal sector with weak signal. If the oriented source is located on the axis of equisignal sector and $\theta = \theta_p$ corresponds to equisignal direction, then

$$A_1(\theta_p) = A_2(\theta_p); \quad A_1'(\theta_p) = -A_2'(\theta_p); \\ \eta(\theta_p) = -1; \quad a F(\theta_p) = (n_1 - n_2) 2\sigma^2$$

and relationship/ratio (7.5.12) passes into the simpler equality

$$\sum_{i=1}^{n_1} \rho_i^{(1)*} - \sum_{i=1}^{n_2} \rho_i^{(2)*} = 2\sigma^2 (n_1 - n_2). \quad (7.4.14)$$

With equality $n_1 = n_2$, formula (7.4.14) passes into equality (7.4.11) and algorithms of working/treatment for method of equisignal

sector with weak and strong signal they coincide.

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B. Measurement of the angular coordinate of the fluctuating (flickering) target.

Function of plausibility of selection (7.6.2a) with value of parameter θ and at condition of statistical independence of signals in selection will be defined, as is known, by product of functions of density distribution of separate selective random variables in selection. Since each of the random variables in the example in question is distributed according to the law "chi-square" (5.2.10),

$$\begin{aligned}
 L(\rho_1^{(1)}, \rho_2^{(1)}, \dots, \rho_{n_1}^{(1)}, \rho_1^{(2)}, \rho_2^{(2)}, \dots, \rho_{n_2}^{(2)}; \theta) = \\
 = \prod_{i=1}^{n_1} \frac{\rho_i^{(1)}}{1+k_1^2} \exp \left\{ -\frac{\rho_i^{(1)^2}}{2(1+k_1^2)} \right\} \times \\
 \times \prod_{i=1}^{n_2} \frac{\rho_i^{(2)}}{1+k_2^2} \exp \left\{ -\frac{\rho_i^{(2)^2}}{2(1+k_2^2)} \right\}, \quad (7.4.15)
 \end{aligned}$$

where

$$k_1 = k_m Q(\theta - \alpha_1); \quad k_2 = k_m Q(\theta - \alpha_2).$$

Logarithm of function of plausibility

$$\begin{aligned}
 \ln L = \sum_{i=1}^{n_1} \left[\ln \rho_i^{(1)} - \ln d_1 - \frac{\rho_i^{(1)^2}}{2d_1} \right] + \\
 + \sum_{i=1}^{n_2} \left[\ln \rho_i^{(2)} - \ln d_2 - \frac{\rho_i^{(2)^2}}{2d_2} \right],
 \end{aligned}$$

where

$$d_1 = 1 + k_1^2, \quad d_2 = 1 + k_2^2.$$

After substituting this expression in equation of plausibility (7.1.13), i.e., differentiate of logarithm of function of plausibility and after equating it with zero, we will obtain

$$\begin{aligned} \frac{\partial \ln L}{\partial \theta} &= \sum_{i=1}^{n_1} \left[\frac{k_1 k_1'}{d_1} \left(\frac{\rho_i^{(1)\theta}}{d_1} - 2 \right) \right] + \\ &+ \sum_{i=1}^{n_2} \left[\frac{k_2 k_2'}{d_2} \left(\frac{\rho_i^{(2)\theta}}{d_2} - 2 \right) \right] = 0. \end{aligned} \quad (7.4.16)$$

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After producing obvious conversions, this relationship/ratio can be rewritten in the form

$$\frac{k_1 k_1'}{d_1^2} \sum_{i=1}^{n_1} \rho_i^{(1)\theta} + \frac{k_2 k_2'}{d_2^2} \sum_{i=1}^{n_2} \rho_i^{(2)\theta} = 2 \left(\frac{k_2 k_2'}{d_2} n_2 + \frac{k_1 k_1'}{d_1} n_1 \right), \quad (7.4.17)$$

which can be brought to equation of form (7.4.13)

$$\sum_{i=1}^{n_1} \rho_i^{(1)\theta} + \eta(\theta) \sum_{i=1}^{n_2} \rho_i^{(2)\theta} = F(\theta)$$

only with other functions $\eta(\theta)$ and $F(\theta)$, namely:

$$\eta(\theta) = \frac{k_2 k_2'}{k_1 k_1'} \left(\frac{d_1}{d_2} \right)^2$$

and

$$F(\theta) = 2 [d_1 n_1 + d_2 n_2 \eta(\theta)]. \quad (7.4.18)$$

Form of these functions is known. The parameters of the antenna of received signal (signal-to-noise ratio) and a quantity of impulses/momenta/pulses in the packet we assume given, then only the value of angle θ is here unknown. Solution (7.4.13) is realized by the method of "tests/samples and search". However, as noted earlier (57.4, p. A), the solution of equation of this type is simpler than equation (7.2.7) and (7.2.20), that are obtained in the measurement of angle by the method of linear scanning, since in equation (7.4.13) coefficients are the sums of selective values $\sum_{i=1}^{n_1} \rho_i^{(1)\theta}$ and $\sum_{i=1}^{n_2} \rho_i^{(2)\theta}$, which comparatively simply can be calculated with the aid of the converters of the type PND and the digital summator.

In the case of strong signal, when level of received signal considerably exceeds noise level and is correct inequality $k_i \gg 1$, then $d_1 = k_1$; $d_2 = k_2$, relationships/ratios (7.4.18) pass into following equalities:

$$\eta(\theta) = \left[\frac{Q'(\theta - \alpha_1)}{Q(\theta - \alpha_2)} \right] \frac{Q'(\theta - \alpha_2)}{Q'(\theta - \alpha_1)};$$

$$F(\theta) = 2 [K_1 n_1 + K_2 n_2 \eta(\theta)]. \quad (7.4.19)$$

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In the case of low signal, when $k_m \ll 1$ and $d_1 = 1 = d_2$, relationship/ratio (7.4.18) takes following form:

$$\eta(\theta) = \frac{k_2 k_2'}{k_1 k_1'}; \quad F(\theta) = 2 [n_1 + n_2 \eta(\theta)]. \quad (7.4.20)$$

Formulas (7.4.17), (7.4.18) are initial for determining program of processing of signal by specialized computer(s) in measurement of angular coordinate.

It is of interest to also examine algorithm of processing signal with direction finding according to so-called method of equisignal sector, which resultant as special case. If the source of the oriented oscillations/vibrations is located on the axis of equisignal sector and $\theta = \theta_p$ corresponds to equisignal direction, then occur the following relationships/ratios (Fig. 7.6):

$$\theta_p = \alpha_1 + \frac{\Delta\alpha}{2} = \alpha_2 - \frac{\Delta\alpha}{2}; k_1'(\theta_p) = -k_2'(\theta_p);$$

$$k_1(\theta_p) = k_2(\theta_p); d_1(\theta_p) = d_2(\theta_p).$$

Then relationship/ratio (7.4.17) passes into equality

$$\sum_{l=1}^{n_1} \rho_l^{(1)*} - \sum_{l=1}^{n_2} \rho_l^{(2)*} = 2d_2(n_2 - n_1). \quad (7.4.21)$$

If quantity of impulses/momenta/pulses in each position of impulses/momenta/pulses of antenna is equal, i.e., $n_1 = n_2 = n/2$, then algorithm of working/treatment and in this case is reduced to establishment of equality, analogous (7.4.11):

$$\sum_{l=1}^{\frac{n}{2}} \rho_l^{(1)*} = \sum_{l=1}^{\frac{n}{2}} \rho_l^{(2)*}. \quad (7.4.22)$$

One of versions of block diagram of processing signal in measurement of angle in accordance with equation (7.4.13) is represented in the diagram Fig. 7.7.

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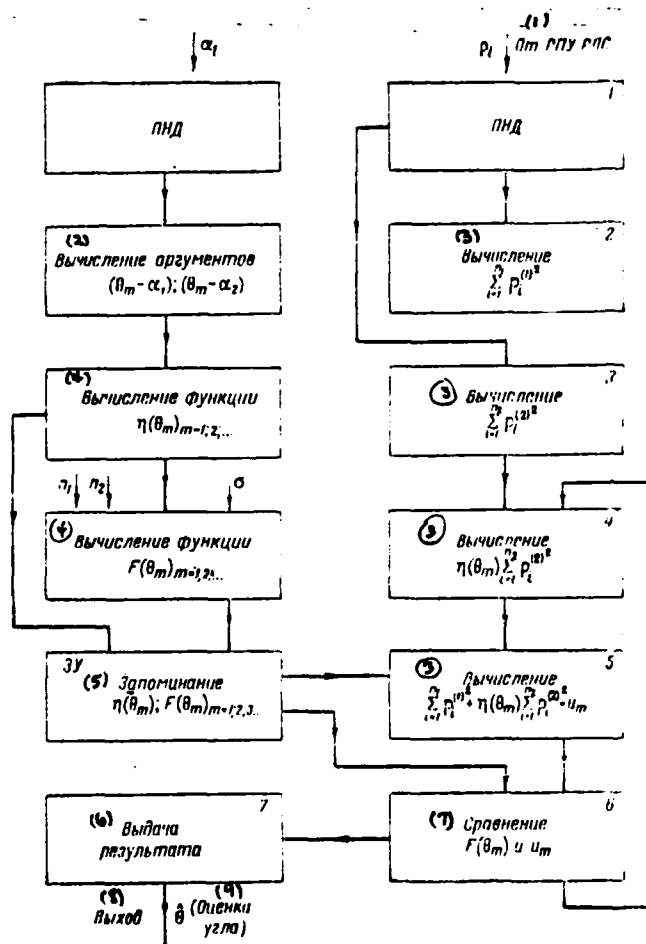


Fig. 7.7. Structure of algorithm during discrete/digital scanning.

Key: (1). From RPU RLS. (2). Computation of arguments. (3). Computation. (4). Computation of function. (5). Memorization. (6). Readout. (7). Comparison ... and ... (8). Output. (9). (Evaluation/estimate of angle).

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During the solution of the equation indicated we assume/set by the given ones: the parameters of the signal and the interferences (their distribution function), the equation of the antenna radiation pattern, value α_1 and α_2 , a quantity of impulses/momenta/pulses in each antenna position n_1 and n_2 . This equation is solved by the method of "tests/samples and search" for assumed values $\theta = \theta_m$. Tentatively $m = \Delta\alpha / \delta\theta$, where $\delta\theta$ - required accuracy of the measurement of angle, since the usually oriented target is located in the interval of the values of angles $\alpha_2 - \alpha_1 = \Delta\alpha$.

It should be noted that in assigned parameters antennas and selective values of signals of function $\eta(\theta)$ and $F(\theta)$ can be calculated previously and carried into memory unit for different values $\theta = \theta_m$; $m = 1, 2, 3, \dots$. If during comparison $F(\theta_m)$ and U_m these values proved to be not equal to one another, then computations are repeated again, but already for another value m and respectively for others θ_m and functions η and F .

Structure of processing signal allows subsequently on the basis of given algorithm to determine structure of specialized electronic

digital computer, used for solution of problem in question.

§7.5. Accuracy of the measurement of angle during the discrete/digital scanning.

As modulus of precision angular-singing of coordinate, just as in §7.3, is accepted dispersion of evaluation/estimate of measured angle of $D(\hat{\theta})$ (7.1.14). Research of this question let us conduct for two forms of the echo signal: with the constant and fluctuating amplitude.

A. Accuracy of measurement with the constant diffusing surface of target.

For determining dispersion of evaluation/estimate it is necessary in accordance with (7.1.13) to calculate informational quantity of Fisher

$$J = E \left(- \frac{\partial^2 \ln L}{\partial \theta^2} \right), \quad (7.5.1)$$

where symbol $E ()$ indicates average/mean value of random variable, included in brackets.

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Second derivative of logarithm of function of plausibility

according to (7.4.7) will take for case of strong signal following form:

$$\begin{aligned} \frac{\partial^2 \ln L}{\partial \theta^2} &= \frac{1}{\sigma^2} \sum_{i=1}^{n_1} \{A_1^* (p_i^{(1)} - A_1) - A_1^* \} + \\ &+ \frac{1}{\sigma^2} \sum_{i=1}^{n_2} \{A_2^* (p_i^{(2)} - A_2) - A_2^* \}. \end{aligned}$$

Average/mean value of second partial derivative

$$\begin{aligned} E \left(-\frac{\partial^2 \ln L}{\partial \theta^2} \right) &= \frac{1}{\sigma^2} \sum_{i=1}^{n_1} \{A_1^* - [E(p_i^{(1)}) - A_1] A_1^* \} + \\ &+ \frac{1}{\sigma^2} \sum_{i=1}^{n_2} \{A_2^* - [E(p_i^{(2)}) - A_2] A_2^* \}. \quad (7.5.2) \end{aligned}$$

Average/mean values of random variables $p_i^{(1)}$ and $p_i^{(2)}$ distributed according to generalized Rayleigh's law (5.1.10), with $A_1 > \sigma$ and $A_2 > \sigma$ can be represented in the form [23]:

$$E(p_i^{(1)}) \approx A_1 \left(1 + \frac{\sigma^2}{2A_1^2} \right); \quad E(p_i^{(2)}) \approx A_2 \left(1 + \frac{\sigma^2}{2A_2^2} \right). \quad (7.5.3)$$

After substituting formulas (7.5.3), (7.5.2) into expressions for informational quantity of Fisher and after producing obvious conversions, we will obtain

$$J = \frac{n_1}{\sigma^2} \left(A_1^* - \frac{\sigma^2 A_1^*}{2A_1^2} \right) + \frac{n_2}{\sigma^2} \left(A_2^* - \frac{\sigma^2 A_2^*}{2A_2^2} \right),$$

but dispersion of evaluation/estimate

$$D(\hat{\theta}) \geq \frac{\sigma^2}{k_1 \left(A_1^* - \frac{\sigma^2 A_1^*}{2A_1^2} \right) + k_2 \left(A_2^* - \frac{\sigma^2 A_2^*}{2A_2^2} \right)}. \quad (7.5.4)$$

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DIGITAL METHODS OF THE OPTIMUM PROCESSING OF RADAR
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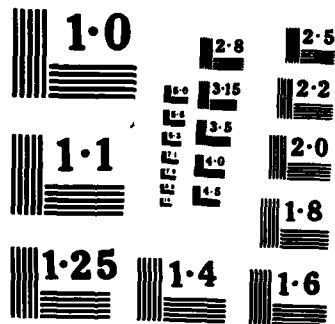
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Let us produce now computations for case of weak signal. In accordance with formula (7.4.12) the second partial derivative

$$\frac{\partial^2 \ln L}{\partial \theta^2} = \frac{1}{\sigma^2} \sum_{i=1}^{n_1} (A_1'' + A_1 A_1') \left(\frac{\rho_i^{(1)2}}{2\sigma^2} - 1 \right) + \\ + \frac{1}{\sigma^2} \sum_{i=1}^{n_2} (A_2'' + A_2 A_2') \left(\frac{\rho_i^{(2)2}}{2\sigma^2} - 1 \right).$$

Average/mean value of second partial derivative

$$E \left(- \frac{\partial^2 \ln L}{\partial \theta^2} \right) = \frac{1}{\sigma^2} \sum_{i=1}^{n_1} (A_1'' + A_1 A_1') \left[1 - \frac{1}{2\sigma^2} E(\rho_i^{(1)2}) \right] + \\ + \frac{1}{\sigma^2} \sum_{i=1}^{n_2} (A_2'' + A_2 A_2') \left[1 - \frac{1}{2\sigma^2} E(\rho_i^{(2)2}) \right]. \quad (7.5.5)$$

Value $E(\rho_i^2)$ can be determined in accordance with known formulas of conversion of random variables. If is known the density of distribution $W(\rho_i)$ of random variable ρ_i , then the density function of the distribution of the square of random variable $y = \rho_i^2$ it is

$$V(y) = W(\sqrt{y}) \frac{d\rho_i}{dy}.$$

Then average of converted random variable

$$E(\rho_i^2) = \int_0^\infty y V(y) dy = \int_0^\infty \rho^2 W(\rho) d\rho,$$

which corresponds to second moment/torque of distribution $W(\rho)$. For the random variable, distributed according to generalized Rayleigh's law, the moment/torque of second order (7.3.4c)

$$E(\rho_i^2) = 2\sigma^2 + A_i.$$

where A_i - signal amplitude in this or another antenna position.

Then

$$E(\rho_i^{(1)}) = 2\sigma^2 + A_{1i}^2, \quad E(\rho_i^{(2)}) = 2\sigma^2 + A_{2i}^2.$$

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After substituting these values into expression for informational quantity of Fisher (7.5.5) and after producing necessary simplifications, we will obtain expression for dispersion of evaluation/estimate in the form

$$D(\hat{\theta}) \geq \frac{-2\sigma^4}{n_1 [(A_1^1 A_1)^2 + A_1^3 A_1^3] + n_2 [(A_2^1 A_2)^2 + A_2^3 A_2^3]}. \quad (7.5.6)$$

Formulas (7.5.4) and (7.5.6) are calculated during error analysis of determination of angular coordinates. For computing the errors it is necessary to know the concrete/specific/actual expressions of directional characteristic of antenna systems. Let us give examples of calculation.

Examples of calculation of evaluation/estimate of accuracy of measurement of angular coordinate.

Let us assume that equation of radiation pattern Q is determined by function of zero-order Hermite (bell-shaped type diagram):

$$\begin{aligned} Q_1 &= e^{-\varphi_1}; \quad Q_2 = e^{-\varphi_2}; \\ \varphi_1 &= p(\theta - \alpha_1); \quad \varphi_2 = p(\alpha_2 - \theta), \end{aligned} \quad (7.5.7a)$$

where p - parameter of approximation ($p = \frac{1}{\varphi_d} \sqrt{\ln \frac{1}{d}}$);

$2\varphi_d$ - solution/opening of diagram at the level d .

For equation of diagram accepted can be obtained following relationships/ratios:

$$\begin{aligned} A_1 &= A_0 e^{-\varphi_1}; \quad A_2 = A_0 e^{-\varphi_2}; \\ A_1' &= -2\varphi_1 p A_1; \quad A_2' = 2\varphi_2 p A_2; \\ A_1' &= (2\varphi_1^2 - 1) 2p^2 A_1; \quad A_2' = (2\varphi_2^2 - 1) 2p^2 A_2. \end{aligned} \quad (7.5.7b)$$

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After substituting (7.5.7b) in (7.5.4), we will obtain for case of strong signal

$$\begin{aligned} A_1' - \sigma^2 \frac{A_1'}{2A_1} &= p^2 [2\varphi_1^2 (2A_1^2 - \sigma^2) - \sigma^2]; \\ A_2' - \sigma^2 \frac{A_2'}{2A_2} &= p^2 [2\varphi_2^2 (2A_2^2 - \sigma^2) - \sigma^2], \end{aligned}$$

and dispersion

$$D(\hat{\theta}) \geq \frac{\sigma^2}{p^2 \{n_1 [2\varphi_1^2 (2A_1^2 - \sigma^2) - \sigma^2] + n_2 [2\varphi_2^2 (2A_2^2 - \sigma^2) - \sigma^2]\}}. \quad (7.5.8)$$

With the direction finding by the method of the equisignal sector

$$\theta = \theta_p = \alpha_1 + \frac{\Delta \alpha}{2}; \quad \varphi_1 = -\varphi_2 = 0.5c; \quad c = p\Delta \alpha; \quad A_1 = A_2.$$

Then, after substituting this in (7.5.8), we will obtain

$$D(\hat{\theta}) \geq \frac{1}{p^2 \left[\frac{A_0^2}{\sigma^2} c^2 e^{-\frac{c^2}{2}} - \left(\frac{c^2}{2} + 1 \right) \right] (n_1 + n_2)}$$

On high level of signal, when $A_0 > \sigma$, it is possible to disregard second term in denominator in comparison with the first, and then approximately inequality for dispersion will be

$$D(\hat{\theta}) \geq \frac{\sigma^2}{p^2 A_0^2 n} F(c), \quad \text{where} \quad F(c) = \frac{e^{-\frac{c^2}{2}}}{c^2}. \quad (7.5.9)$$

Hence it is possible to find optimum value of bias/displacement of diagram $\Delta\alpha$ (angle of oscillation of diagram), with which right side of inequality and, consequently, also error of direction finding according to method of equisignal sector will be minimum. This optimum value will be located from the condition

$$\frac{dF(c)}{dc} = \frac{1}{c^3} e^{-\frac{c^2}{2}} (c^2 - 2) = 0,$$

whence

$$c_{opt} = \sqrt{2} \quad \text{or} \quad \Delta\alpha = \frac{\sqrt{2} \sigma}{p}. \quad (7.5.10)$$

Second derivative $F(c)$ is not investigated, since from structure $F(c)$ it is evident that $F(c)$ will have two maximum values with $c=0$ and $c=\infty$ and one minimum, whose coordinate will be located from equality $(c^2-2)=0$ its solution is given in latter/last relationship/ratio.

FOOTNOTE⁻¹. This coincides with the coordinate of the maximum of the slope/transconductance of the diagram of antenna, since

$A_1' = 2p^2 A_1 (2\varphi_1^2 - 1) = 0$, $(2\varphi_1^2 - 1) = 0$, and since $\varphi_1^2 = \frac{c^2}{4}$, $c^2/2=1$, whence it follows

(7.5.10). ENDFOOTNOTE.

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B. Accuracy of the measurement of angle with fluctuating surface of target.

According to formula (7.4.16) second derivative of logarithm of function of plausibility

$$\begin{aligned} \frac{\partial^2 \ln L}{\partial \theta^2} = & -2n_1 \left[\frac{K_1^2}{d_1} \left(1 - \frac{2k_1^2}{d_1} \right) + \frac{k_1 k_1^*}{d_1} \right] + \\ & + \frac{1}{d_1^2} \left[k_1^* \left(1 - \frac{4k_1^2}{d_1} \right) + k_1 k_1^* \right] \sum_{i=1}^{n_1} \rho_i^{(1)*} - \\ & - \frac{2n_2}{d_2} \left[K_2^2 \left(1 - 2 \frac{k_2^2}{d_2} \right) + k_2 k_2^* \right] + \\ & + \frac{1}{d_2^2} \left[k_2^* \left(1 - \frac{4k_2^2}{d_2} \right) + k_2 k_2^* \right] \sum_{i=1}^{n_2} \rho_i^{(2)*}, \quad (7.5.11) \end{aligned}$$

where, as is known,

$$d_1 = 1 + k_1^2; \quad d_2 = 1 + k_2^2.$$

Since average/mean value from square of random variable, distributed according to the law "chi-square",

$$E(\rho_i^{(1)*}) = 2d_1; \quad E(\rho_i^{(2)*}) = 2d_2,$$

the expression for dispersion of evaluation/estimate will be obtained in the following form:

$$D(\hat{\theta}) \geq \frac{0,25}{\frac{n_1 k_1^2 k_1'^2}{d_1^2} + \frac{n_2 k_2^2 k_2'^2}{d_2^2}} \quad (7.5.12)$$

This relationship/ratio is correct for different form of radiation patterns of antenna system. However, concrete/specific/actual values can be obtained only for the specific form of the characteristics of antennas.

Let us give example of calculation of accuracy of evaluation/estimate of measurement angular-singing of coordinate for radiation pattern of bell-shaped type (7.5.7a).

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For this form of the equation of diagram can be obtained the following relationships/ratios:

$$\begin{aligned} k_1' &= -k_1 2p\varphi_1; \quad k_2' = k_2 2p\varphi_2; \\ (k_1 k_1')^2 &= 4p^2 \varphi_1^2 k_1^4; \quad (k_2 k_2')^2 = 4p^2 \varphi_2^2 k_2^4. \end{aligned} \quad (7.5.13)$$

If we substitute (7.5.13) in (7.5.12), then dispersion of evaluation/estimate can be recorded in the following form:

$$D(\hat{\theta}) \geq \frac{0,25}{k_m^4 4p^2 \left[\frac{n_1 e^{-4\varphi_1^2}}{d_1^2} + \frac{n_2 e^{-4\varphi_2^2}}{d_2^2} \right]} = I_1. \quad (7.5.14)$$

This relationship/ratio is correct for evaluation/estimate of different values of angular coordinate. The evaluation/estimate of the error of the angle, which corresponds to equisignal sector, is of interest. In this case occur the following relationships/ratios:

$$\theta = \theta_p = \alpha_1 + \frac{\Delta\alpha}{2}; \varphi_1 = -\varphi_2 = 0,5c; c = p\Delta\alpha;$$

$$\varphi_1^2 = \varphi_2^2, d_1 = d_2 = 1 + k_m^2 e^{-\frac{c^2}{2}}.$$

After substituting these relationships/ratios in (7.5.14), we will obtain expression for dispersion of evaluation/estimate of angle, which corresponds to angular position of equisignal sector:

$$I_1 = \frac{0,25 \left(1 + k_m^2 e^{-\frac{c^2}{2}} \right)^2}{k_m^4 p^2 (n_1 + n_2) c^2 e^{-c^2}}. \quad (7.5.15)$$

On small level of signal, when $k_m < 1$, relationship/ratio (7.5.15) will take following form:

$$I_1 = \frac{0,25 e^{c^2}}{k_m^4 p^2 n c^2}. \quad (7.5.16)$$

From this formula it is possible to determine optimum bias/displacement of diagram, with which right side of inequality and, consequently, also error of direction finding according to method of equisignal sector will be smallest.

$$\frac{d}{dc} \left(\frac{1}{c^2} e^{c^2} \right) = 0,$$

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This value will be located from the condition

whence is analogous (with 7.5.10) $c_{opt} = \sqrt{2}$, and the optimum angle of shift of the antenna

$$\Delta \alpha_{opt} = \frac{\sqrt{2}}{p}.$$

Obtained value coincides with coordinate of maximum slope/transconductance of antenna radiation pattern (7.5.10).

For strong signal, when $k_m^{-2} \gg 1$, for all i

$$D(\hat{\theta}) \geq \frac{0.25}{c^2 p^2 (n_1 + n_2)}. \quad (7.5.17)$$

§7.6. Comparative data of the accuracy of the measurement of angles.

Above were examined two methods of measuring angular coordinates, namely: measurement with linear and during discrete/digital scanning. In each of these methods occurs different character of the "distribution" of impulses/momenta/pulses according to the diagram of antenna and the different form of the packet of the pulse signals, supplied to the diagram of processing, respectively is obtained. For both methods the block diagrams of the algorithms of processing signal were examined and an evaluation of the accuracy of

measurement was made.

It is interesting to compare accuracies of measurement, obtained with different direction-finding methods, in connection with the fact that each of these methods has different structural/design and tactical characteristics. Therefore during the selection of one or the other method of scanning for any specific radar system it is desirable to know, what accuracy can be achieved/reached in each of these specific cases and which of these methods it is best.

As modulus of precision in each case is accepted dispersion of evaluation/estimate of measured angle of $D(\hat{\theta})$, which, as noted above, is determined through informational quantity of Fisher $j(\hat{\theta})$

$$D(\hat{\theta}) \geq \frac{1}{j(\hat{\theta})}.$$

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In this case, the greater the informational quantity, the less dispersion of evaluation/estimate can be obtained and the greater the accuracy of measurement.

As comparative characteristic of methods of measurement is accepted relation of informational quantities of Fisher:

$$m = \frac{J_{\text{a.c.}}(\hat{\theta})}{J_{\text{a.c.}}(\hat{\theta}_0)}, \quad (7.6.1)$$

where $J_{\text{a.c.}}(\hat{\theta})$ - informational quantity for evaluation/estimate of angle, measured by method of linear scanning;

$J_{\text{a.c.}}(\hat{\theta})$ - informational quantity for evaluation/estimate of angle, obtained during discrete/digital scanning (by method of equisignal sector). Obviously, if value m is more than one, then the method of discrete/digital scanning gives greater accuracy than the method of linear scanning, and, on the contrary, if it is less than one, then the second method is more precise than the first. The greater the value m , the more precise the measurements during the discrete/digital scanning in comparison with the method of linear scanning are obtained.

Subsequently is determined dependence of m on parameters of direction-finding system, and in particular from quantity l of impulses/momenta/pulses in packet, utilized for determining angular coordinate. In the radar systems the time, spent on the measurement of angle with the appropriate accuracy, is determined by this number of impulses/momenta/pulses. This value is important, since the accuracy, attained for the specific time interval, is one of the fundamental indices of any method of measurement. The less this time interval, the more effective the measuring system of angular

coordinate, and vice versa. With the infinite time, spent on the observations of target and measurement of its coordinate, obviously, can be achieved/reached how conveniently high accuracy with any method of measurement and the preferability of one or the other method will be rated/estimated, apparently, only by its design and energy parameters.

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However, if time of measurement according to one or the other reasons is limited, then, naturally, becomes important for radar systems this characteristic, as quantity of impulses/momenta/pulses in packet, necessary for achievement of assigned accuracy of measurement of angular coordinate.

Comparative evaluations, carried out for both methods, show that in majority of cases during discrete/digital scanning of antenna can be obtained higher accuracy of measurement, than with linear. Let us show this based on the example of the evaluation/estimate of the accuracy of the direction finding of the fluctuating flickering target, with which are obtained simpler linings/calculations.

Comparative characteristics of the accuracy of measurement for the fluctuating targets.

Let us examine case of direction finding of strong signal. An informational quantity of Fisher for this signal during the linear scanning is determined by formula (7.3.34)

$$j_{\text{a.c.}} = 16p^2 \sum_{i=1}^n b^2 \left(i - \frac{1+n}{2} \right)^2.$$

Here $b = p\Delta\theta$ where r - parameter of approximation, and $\Delta\theta$ - the raster angle:

$$\Delta\theta = \frac{2\varphi_d}{n-1}, \quad (7.6.2)$$

where $2\varphi_d$ - effective width of diagram at the level d , and n - quantity of impulses/momenta/pulses in the packet.

Informational quantity for case of strong signal during discrete/digital scanning (for equisignal direction) is determined by formula (7.5.17)

$$j_{\text{a.c.}} = 4c^2 p^2 n,$$

where $c = p\Delta\alpha$.

After substituting informational quantities (7.3.34) and (7.5.17) into formula (7.6.1), we will obtain value of comparative

evaluation

$$m = \frac{c^2}{4b^2} \frac{n}{\sum_{i=1}^n \left(i - \frac{1+n}{2} \right)^2} \quad (7.6.3)$$

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It is necessary to keep in mind that $\frac{c}{b} = \frac{\Delta \alpha}{\Delta \theta}$, and taking into account (7.6.2) $c/b = \xi(n-1)$, where $\xi = \frac{\Delta \alpha}{2\varphi_d}$ is ratio of the angle of discrete/digital scanning to the width of radiation pattern.

Furthermore, is a following equality:

$$\sum_{i=1}^n \left(i - \frac{1+n}{2} \right)^2 = \frac{n}{12} (n^2 - 1).$$

After substituting these formulas in (7.6.3), we will obtain

$$m = 3\xi \frac{n-1}{n+1}.$$

If

$$3\xi > \frac{5}{3},$$

then for $n \geq 4$ there will be $m > 1$. Since this condition easily is satisfied, with any number of impulses/momenta/pulses of packet $n > 4$ the accuracy of direction finding during the linear scanning is obtained worse than with the equisignal sector. However, with large n dependence on a number of impulses/momenta/pulses decreases. This position was proved here based on the example of the direction

finding of the rapidly fluctuating signal with the large intensity (signal-to-noise ratio of more than one). However, apparently, this position will be correct and for other cases, which is confirmed by the results of the investigation of other particular examples [35].

57.7. Measurement of the angular coordinate of the fluctuating target with quantized data.

Methods of measurement examined above of angle are connected with working/treatment of nonquantized impulses/momenta/pulses, obtained directly from output of receiving channel of RLS. This working/treatment causes the use/application of compound circuits of the conversion of analog quantity into the discrete/digital, which in a number of cases complicates measurements.

Tendency toward simplification in device/equipment of processing signal leads to methods of measuring angular coordinate with quantized data [22, 57].

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In this case the output signal of radar channel is quantized according to the amplitude sign/criterion to two levels, that correspond to the threshold of quantization $U_{\cdot}$.

Quantization of signal is realized in special threshold cascade/stage - amplifier-limiter (§4.5, Fig. 4.25), at output of which standard impulses/momenta/pulses are obtained, if input signal exceeded level of quantization. Thus, at the output of threshold cascade/stage is obtained the so-called quantized packet, which consists of the passages or the standard impulses/momenta/pulses, i.e., of zero or ones.

It is natural that measurement of angular coordinate with quantized packet, apparently will give smaller accuracy than with that not quantized, since information, placed in amplitudes of received signal, here is eliminated from examination. However, the use/application of quantization considerably simplifies further processing of signal in connection with the possibility to utilize here diagrams of simpler form. For obtaining the same accuracy of the measurement of angle, as with the nonquantized packet, a quantity of impulses/momenta/pulses in the quantized packet probably it is obvious, more in the comparison with the first case, i.e., the duration of the measurement of angle during processing of quantized data increases.

Let us switch over to determination of algorithm of processing

signal with numbering from method of maximum of function of plausibility. In the subsequent reasonings we assume that the measurement of angle is made with the linear scanning of antenna and the determined packet.

Let p_i be probability of excess by i signal of packet of threshold of quantization U_i . The same value characterizes the probability of the appearance of a standard impulse/momentum/pulse - one, on the i position of packet, q_i is probability of nonexcess by the i signal of the packet of threshold U_i . It is obvious,

$$p_i = 1 - q_i.$$

These probabilities can be easily calculated, if are known to density function $W(p_i)$ distributions of signal during i observation, i.e., on i position of packet, in accordance with formula

$$p_i = \int_{U_i}^{\infty} W(p_i) dp_i. \quad (7.7.1)$$

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Subsequently during determination of structure of working/treatment we assume/set p_i and q_i by known values, since density function of distribution $W(p_i)$ are given ones and for different characteristics of signal and interference are given in preceding/previous sections.

Function of plausibility of quantized packet, as before will be determined under the assumption of statistical independence of selective signals in adjacent observations. In this case it is determined by the product of the probability of the appearance of signals, which form part of packet. If we designate through H many standard impulses/momenta/pulses, i.e., ones, and through M many zeros - passages, then the function of the plausibility

$$L = \prod_{i=1}^H p_i \prod_{i=1}^M q_i \quad (7.7.2)$$

With this

$$H + M = n,$$

where n - quantity of impulses/momenta/pulses in packet.

If we use value

$$Q = \prod_{i=1}^n q_i \quad (7.7.3)$$

which determines probability of obtaining zeros - passages with all positions of impulses/momenta/pulses of packet, then formula (7.7.2) can be rewritten in the form

$$L = Q \prod_{i=1}^H \frac{p_i}{q_i}.$$

Passing with the aid of selective values x_i to entire selection

n, logarithm of function of plausibility can be recorded in the following form:

$$\log L = \ln Q + \sum_{i=1}^n x_i (\ln p_i - \ln q_i),$$

where selective value of signal at output of quantizer

$$x_i = \begin{cases} 1 & \text{при наличии стандартного импульса;} \\ 0 & \text{при пропуске.} \end{cases}$$

Key: (1). in the presence of standard impulse/momentum/pulse. (2). with passage.

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Derivative of logarithm of function of plausibility

$$\frac{\partial \ln L}{\partial \theta} = \sum_{i=1}^n x_i \left(\frac{1}{p_i} \frac{\partial p_i}{\partial \theta} - \frac{1}{q_i} \frac{\partial q_i}{\partial \theta} \right) + \frac{\partial \ln Q}{\partial \theta}. \quad (7.7.4)$$

Since

$$\frac{\partial p_i}{\partial \theta} = - \frac{\partial q_i}{\partial \theta} = p_i'$$

and

$$\frac{\partial \ln Q}{\partial \theta} = - \sum_{i=1}^n \frac{p_i'}{q_i},$$

that

$$\sum_{i=1}^n x_i p_i' \left(\frac{1}{p_i} + \frac{1}{q_i} \right) = \sum_{i=1}^n \frac{x_i p_i'}{p_i q_i}.$$

Equation of plausibility (7.7.4) in this case will take form

$$\sum_{i=1}^n \frac{p_i}{q_i} \left(\frac{x_i}{p_i} - 1 \right) = 0, \quad (7.7.5)$$

or

$$\sum_{i=1}^n x_i \psi_i = V(\theta, n, U_0),$$

where sum

$$\sum_{i=1}^n \frac{p_i}{q_i} = V(\theta, n, U_0) \quad (7.7.6)$$

does not depend on selective values x_i and ψ_i - weight factor

$$\psi_i = \frac{p_i}{q_i p_i}. \quad (7.7.7)$$

Equations (7.7.5) can be solved by method of "tests/samples and search". For facilitating the calculations of value (7.7.6) and (7.7.7) it is possible to calculate previously for several assumed values $\theta = \theta_1, \theta = \theta_2, \dots, \theta = \theta_m$ and to carry into the memory.

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In accordance with those obtained in the measurement of angle by selections $x_1, x_2, \dots, x_n$ can be calculated the sum

$$U(x_1, x_2, \dots, x_n, n, \theta, U_0) = \sum_{i=1}^n x_i \psi_i \quad (7.7.8)$$

for several assumed values $\theta = \theta_m$. The unknown evaluation/estimate of angle $\theta_m = \hat{\theta}$ with this method is determined from the best conformity of sums (7.7.6) and (7.7.8).

In certain cases, for example with large number of impulses/momenta/pulses in packet, they propose [22] that

$$V(\theta, n, U_0) = 0.$$

then somewhat is simplified solution (7.7.5), i.e., in this case only those values of sums (7.7.8), which correspond to zero, are located:

$$\sum_{i=1}^n x_i \psi_i = 0^*.$$

For explaining character of computations obtained here it is systematic expedient to examine diagram of one of versions of algorithm of processing signal in accordance with equation (7.7.5). The latter is represented in Fig. 7.8. As is evident, the structure of algorithm is identical for different signal aspects and interference. Difference consists only of the fact that the expressions for probabilities p_i and q_i for different functions of the distribution of signal and interference will be different. The solution of equation (7.7.5) with the aid of the given algorithm is conducted by the method of "tests/samples and search" for assumed values $\theta = \theta_m$; $m=1, 2, \dots$. Tentatively $m = \Delta / \delta\theta$, where $\delta\theta$ - required accuracy of the measurement of angle. During the solution of the equation indicated are considered known, i.e., assigned, the functions of the distribution of signal and interference, the equation of the diagram of antenna, a quantity of impulses/momenta/pulses in packet n , a value of angles α_i , where $i=1-n$. During the solution of equation (7.7.5) of the value of

$V(\theta_m; U_r)$.

expressions $p_i; q_i; R_i; \psi_i$ as not depending on selective values it is expedient to calculate previously for different values $\theta = \theta_m$ and to record into the memory unit.

FOOTNOTE ¹. The evaluation/estimate of the accuracy of the measurement of angle for particular signal aspect with the binary quantization of amplitudes is examined in [57]. ENDFOOTNOTE.

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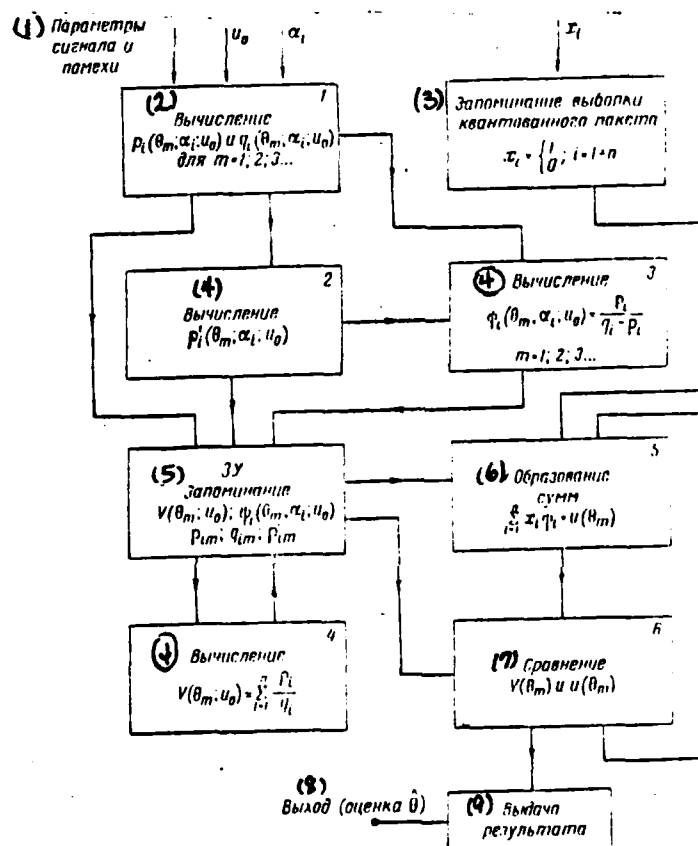


Fig. 7.8. Diagram of algorithm with quantized packet.

Key: (1). Parameters of signal and interference. (2). Computation and for $m=1; 2; 3 \dots$. (3). Memorization of selection of quantized packet. (4). computation. (5). Memorization. (6). Formation/education of sums. (7). Comparison and (8). Output (evaluation/estimate θ). (9). Readout.

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After the formation/education of sum $\sum_{i=1}^n x_i \phi_i = U(\theta_m)$ taking into account the obtained selection of signals in the quantized packet is conducted comparison $U(\theta_m)$ and $V(\theta_m; U_0)$. If these conducted the ranks they proved to be not equal to one another, then computations are repeated again, but already for another value θ_m . Since with the quantized packet the value of the signals in the selection takes value of 0 or 1, formation/education of sum of p. 5 is reduced to the addition of values ϕ_i for those values of i for which $x_i = 1$.

Expressions for probabilities p_i and q_i are determined by concrete/specific/actual form of the function of distribution of signal and interference. Thus, for instance, for the fluctuating target the function of the distribution of the i impulse/momentum/pulse of packet according to (7.2.17)

$$W(p_i) = \frac{p_i}{1 + k_i^2} \exp \left[-\frac{p_i^2}{2(1 + k_i^2)} \right],$$

where

$$k_i = k_m Q(\theta - \theta_i), \text{ and } k_m = \frac{\sigma_0}{\sigma}.$$

After substituting this expression in (7.7.1) and after producing integration, we will obtain

$$p_l = \int_{U_0}^{\infty} \frac{p_l}{1+k_l^2} \exp \left[-\frac{p_l^2}{2(1+k_l^2)} \right] dp_l = \\ = \exp \left[-\frac{U_0^2}{2(1+k_l^2)} \right], \quad (7.7.9)$$

and derivative

$$p_l' = -\frac{U_0^2 k_l k_l'}{(1+k_l^2)^2} p_l. \quad (7.7.10)$$

For target with constant diffusing surface on high signal level in accordance with (5.1.10) density function of distribution of i impulse/momentum/pulse of packet (5.1.10)

$$W(p_l) = \frac{p_l}{\sigma^2} \exp \left[-\frac{(p_l - A_l)^2}{2\sigma^2} \right] I_0 \left(\frac{p_l A_l}{\sigma^2} \right).$$

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Substituting this expression into 7.7.1 and integrating, we obtain

$$p_l = e^{-x_l^2} + \frac{A_l}{\sigma \sqrt{\pi}} V \pi [1 - \Phi(x_l)],$$

where

$$\Phi(x_l) = \frac{2}{V\pi} \int_0^{x_l} e^{-t^2} dt \quad \text{and} \quad x_l = \frac{U_0 - A_l}{\sigma \sqrt{2}},$$

in this case derivative

$$p_i = \frac{A_0}{\sigma \sqrt{2}} Q_i(0 - \theta_i) \left\{ V \pi [1 - \Phi(x_i)] + e^{-x_i^2} \left[2x_i + \frac{2A_0}{\sigma \sqrt{2}} Q(0 - \theta_i) \right] \right\}.$$

For other functions distributions of expression for probabilities p_i can be obtained by analogous computations. These expressions determine in the diagram of algorithm (Fig. 7.8) the structure of concrete/specific/actual computations during processing of signal for measuring the angular coordinate of target.

On the basis of given algorithm in known parameters of distributions it is possible to determine structure and program of specialized electronic computer, used for processing of signal in measurement of angular coordinate.

Let us examine now measurement of angle during discrete/digital scanning of antenna, when ray/beam occupies two fixed/recorded positions. In each of these cases the direction of maximum radiation/emission takes value α_1 and α_2 , in this case $\alpha_2 > \alpha_1$, and $\alpha_2 - \alpha_1 = \Delta\alpha$ there is an angle of discrete/digital scanning. Angles α_1 and α_2 are counted off from the arbitrary initial direction. Transition/junction from one position to another is realized by a jump. It is assumed that the oriented object is within the limits of angle $\Delta\alpha$. In each antenna position into the receiving channel of

radar it enters with respect to n_1 and n_2 of the echo pulses, on which are superimposed the interferences of random form. The impulses/momenta/pulses accepted are quantized to two levels according to the amplitude sign/criterion relative to threshold U , - level of quantization. The determination of the form of the operations/processes, which it is necessary to produce above the quantized echo pulses for determining the angular coordinate of the oriented object (i.e. for determining the structure of working/treatment), and the computation of limits of accuracy of measurement, is our problem.

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Subsequently we assume that the functions of array of signal and interference are completely known. For the discrete/digital scanning the function of the plausibility

$$L = \left(\prod_{l=1}^{H_1} p_l^{(1)} \prod_{l=1}^{M_1} q_l^{(1)} \right) \left(\prod_{l=1}^{H_2} p_l^{(2)} \prod_{l=1}^{M_2} q_l^{(2)} \right), \quad (7.7.11)$$

where $H_1(H_2)$ - many signals accepted in the antenna position $\alpha_1(\alpha_2)$;

$M_1(M_2)$ - many passages in the antenna position $\alpha_1(\alpha_2)$;

$n_1(n_2)$ - a number of impulses/momenta/pulses, accepted in the antenna position $\alpha_1(\alpha_2)$;

$n_1 + n_2 = n$ - number of impulses/momenta/pulses in the packet;

$$p_i^{(j)} = \int_{U_i}^{\infty} f_i(\rho_i^{(j)}, \eta) d\rho_i^{(j)} \quad (7.7.12)$$

there is probability of excess by the signal of threshold U_i in the j antenna position ($j=1, 2$);

$$q_i^{(j)} = 1 - p_i^{(j)}, \quad (7.7.13)$$

$f_i(\rho_i^{(j)}, \eta)$ - density function of array of signal and interference during the i observation.

Unknown evaluation/estimate $\hat{\theta}$ of angular coordinate in accordance with method of maximum plausibility will be located from relationship/ratio:

$$\frac{\partial \ln L}{\partial \theta} = 0 \Big|_{\theta = \hat{\theta}} \quad (7.7.14)$$

After substituting here (7.7.11), we will obtain by analogy with reasonings of preceding/previous case

where
$$\sum_{i=1}^{n_1} x_i^{(1)} + \psi \sum_{i=1}^{n_2} x_i^{(2)} = \mu, \quad (7.7.15)$$

$$\psi = \frac{p_1 q_1 p_2'}{q_2 p_2 p_1'}; \mu = \frac{(n_1 p_1' q_2 + n_2 p_2' q_1) p_1}{p_1 q_2};$$

$$p_1^{(1)'} = p_1'; \quad p_1 = p_1^{(1)}; \quad q_1 = q_1^{(1)};$$

$$p_2' = p_2^{(2)'}; \quad p_2 = p_2^{(2)}; \quad q_2 = q_2^{(2)}.$$

Here in $x_j^{(j)}$ ($j=1; 2$) - random variable, which takes only two values:

$$x_j^{(j)} = \begin{cases} 1 & \text{при наличии сигнала;} \\ 0 & \text{при пропуске сигнала.} \end{cases}$$

Key: (1). in the presence of signal. (2). with passage of signal.

Relationship/ratio (7.7.15) determines structure of processing signal in measurement of angular coordinate. It is initial for determining the program of processing signal with the aid of the specialized computer. equation (7.7.15) relatively θ is expedient to solve by the method of "tests/samples and search". For facilitating the calculations of value ϕ and μ it is possible to calculate previously for several values $\theta=\theta_1; \theta_2; \theta_3, \dots$ and to carry into the computer memory.

However, it must be noted that computations obtained here prove to be simpler than with nonquantized data, since sums $\sum_{i=1}^{n_1} x_i^{(1)}$ and $\sum_{i=1}^{n_2} x_i^{(2)}$ are fixed/recorded not with summator, but by usual binary counter. The concrete/specific/actual form of the function ϕ and μ depends on the form of the function of the density of array of signal and interference and on the type of the antenna radiation pattern.

As criterion of accuracy of measurement of angular coordinate let us accept dispersion of evaluation/estimate of measured angle. According to Rao-Cramer theorem [18] for the dispersion of evaluation/estimate occurs following inequality (7.1.14):

$$D(\hat{\theta}) \geq \frac{1}{-E\left(\frac{\partial^2 \ln L}{\partial \theta^2}\right)}.$$

Substituting here (7.7.11), we have:

$$D(\hat{\theta}) \geq \frac{1}{\frac{n_1 p_1}{q_1 p_1} + \frac{n_2 p_2}{q_2 p_2}}. \quad (7.7.16)$$

Let us examine example of computation of evaluation/estimate of accuracy of measurement of angular coordinate with direction finding of fluctuating target. The density function of the probability of distributing the signals, reflected from such targets, together with the noise takes form (7.2.17).

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By analogy (7.7.9)

$$p_j = \int_{U_0}^{\infty} f_i(\rho_i^{(j)\theta}) d\rho_i^{(j)} = \exp \left\{ -\frac{U_0^2}{2(1+k_j^2)} \right\}, \quad (7.7.17)$$

$$p_j' = \frac{dp_j}{d\theta} = \frac{U_0^2 k_j k_j'}{(1+k_j^2)^2} p_j, \quad (7.7.17a)$$

where

$$j = 1, 2; \quad k_1' = k_m Q_{1\theta}; \quad k_2' = k_m Q_{2\theta}.$$

After substituting obtained relationships/ratios in (7.7.16), we will obtain

$$D(\hat{\theta}) \geq \frac{1}{U_0^4 \left[\frac{n_1 k_1^2 k_1'}{(1+k_1^2)^4} \frac{p_1}{q_1} + \frac{n_2 k_2^2 k_2'}{(1+k_2^2)^4} \frac{p_2}{q_2} \right]}. \quad (7.7.18)$$

Example of calculation of accuracy let us give for radiation pattern of bell-shaped type, for which

$$\begin{aligned} Q_1 &= e^{-\varphi_1^2}; \quad Q_2 = e^{-\varphi_2^2}; \\ \varphi_1 &= p(\theta - \alpha_1); \quad \varphi_2 = p(\alpha_2 - \theta), \end{aligned} \quad (7.7.19)$$

where r - parameter of approximation

$$\left(p = \frac{1}{\varphi_d} \sqrt{\ln \frac{1}{d}} \right);$$

$2\varphi_d$ - solution/opening of radiation pattern at the level d.

Let us examine evaluation/estimate of accuracy in measurement angular-singing of coordinate by method of equisignal sector. In this case

$$\begin{aligned}\theta &= \alpha_1 + \frac{\Delta\alpha}{2}; \quad \varphi_1 = \varphi_2; \quad k_1 = k_2 = k_m e^{-\epsilon^2} = k; \\ \varphi_1^2 &= \varphi_2^2 = p^2 \frac{\Delta\alpha^2}{4} = c^2; \\ p_1 &= p_2; \quad q_1 = q_2; \quad Q_1 = Q_2 = e^{-\epsilon^2}; \quad (7.7.20) \\ Q_1' &= -2pcQ_1, \quad Q_2' = 2pcQ_2.\end{aligned}$$

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After substituting these relationships/ratios, and also formulas (7.7.17) in (7.7.18), we will obtain

$$D(\hat{\theta}) \geq \frac{(1+k^2)^4}{nk^2k^4} H(U_0, k), \quad (7.7.21)$$

where

$$H(U_0, k) = \frac{1 - \exp\left\{-\frac{U_0^2}{2(1+k^2)}\right\}}{U_0 \exp\left\{-\frac{U_0^2}{2(1+k^2)}\right\}}.$$

In this expression factor $H(U_0, k)$ depends on threshold of quantization U_0 , and from relation of dispersions of signal and noise $k_m = \frac{\sigma_c}{\sigma_m}$. It is possible to decrease the variance of error by the reasonable selection of the threshold of quantization. Selection U_0 should be produced from the expression of the outer limit of factor H , which will be located from the equation

$$\frac{\partial H(U_0, k)}{\partial U_0} = 0. \quad (7.7.22)$$

This extremum corresponds to minimum value of factor $H(U_0, k) = H_{\min}$, since it is not difficult to show that second derivative

$$\frac{\partial^2 H(U_0, k)}{\partial U_0^2} > 0.$$

After producing necessary computations, we will obtain that condition of minimum H is reduced to equality

$$\frac{U_0^2 \exp \left\{ -\frac{U_0^2}{2(1+k^2)} \right\} \left[\exp \left\{ -\frac{U_0^2}{2(1+k^2)} \right\} + \frac{U_0^2}{1+k^2} - 1 \right]}{U_0^2 \exp \left\{ -\frac{U_0^2}{2(1+k^2)} \right\}} = 0,$$

trivial solution of which are $U_0 = 0$ and $U_0 = \infty$. However, they are not of practical interest, since they correspond to the absence of threshold or to the completely closed channel.

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The value interesting us will be located from the condition of equality to zero values in the brackets, i.e., from the equation

$$2e^{-aU_0^2} = 2 - aU_0^2,$$

where

$$a = \frac{1}{2(1+k^2)}.$$

Graphical solution of this equation gives

$$U_{0 \text{ opt}} = \sqrt{3,2(1+k_m^2 Q^2)}. \quad (7.7.23)$$

As is evident, optimum threshold value depends on signal-to-noise ratio (k_m) , and also on equation of radiation pattern. After substituting (7.7.23) in (7.7.21), we will obtain

$$H_{\min} = \frac{4}{U_{0\text{opt}}^4}. \quad (7.7.24)$$

In turn, substituting $H_{\min}$ in (7.7.21) and taking into account formulas (7.7.22) and (7.7.20), we obtain

$$D(\hat{\theta}) \geq \frac{(1 + k^2)^2}{(3.2)^2 n c^2 p^2 k^4}, \quad (7.7.25)$$

in this case

$$k = k_m Q(c).$$

From this formula it is possible to determine optimum "disintegration" between antennas - space of discrete/digital scanning Δa_{opt} , which provides minimum of variance of error. This value, obviously, must correspond to the minimum value of the factor

$$\frac{(1 + k^2)^2}{c^2 k^4},$$

which is found from the condition

$$\frac{\partial}{\partial c} \left[\frac{(1 + k^2)^2}{c^2 k^4} \right] = 0.$$

It is not difficult to show that this equation is equivalent to equality

$$4c^2 - k^2 - 1 = 0,$$

whence

$$c_{opt}^2 = \frac{1 + k_{opt}^2}{4},$$

where

$$k_{opt} = k_m e^{-c_{opt}^2}.$$

Hence it is apparent that optimum value c_{opt} depends on signal-to-noise ratio (k_m) and equation of radiation pattern. For the specific case, when antenna, for example, has bell-shaped type diagram, equation (7.7.26) can be solved graphically. Result will be dependence c_{opt} as function k_m .

In the case of strong signal, which is for measurements of angle of greatest interest, it is possible to consider that $k \gg 1$, and from (7.7.26) follows that $c_{opt}^2 = \frac{1}{4}$. Then from (7.7.25) taking into account (7.7.19) it is possible to represent relative value of error in the form of the simple correlation

$$\frac{D(\hat{\theta})}{\vartheta_d^2} \geq \frac{1}{2.5n \ln \frac{1}{d}},$$

which characterizes the effect of fluctuating interferences on the accuracy of measurement the angular-singing of coordinate. Hence it is apparent that the relative error does not depend on k , and

decreases with increase in n .

§7.8. Measurement of angle in the multichannel systems.

Tendency toward decrease of time, spent on measurement angular-singing of target coordinate by radars, leads to use/application of multichannel systems [21]. Such systems have several antennas, which work to the reception/procedure and the transmission, or one transmitting several receiving antennas, called sometimes partial [45, 46]. Systems in application to the first case we will examine. The receiving channel of each antenna is self-contained, and the transmitting can be general/common/total for all antennas.

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With the multichannel direction finding the reception of the echo signal is realized by several antennas simultaneously. The antennas of multichannel system it is possible to arrange/locate one relative to another differently. They can be displaced to the identical or to different angle. They can be also broken into two groups on m_1 and m_2 in each, and separate groups are displaced on the angle to value $\Delta\alpha$ or arranged/located in any other manner.

Let us assume that there are by m of motionless antennas with identical radiation patterns $Q(\theta)$, arranged/located at angle β_i , $i=1, 2, \dots, m$ relative to selected initial direction, and target is moved with respect to this system. Each of the antennas will accept the echo signal, whose value A_i will be determined by antenna location with respect to the direction to the oriented object:

$$A_i = A_0 Q(\theta - \beta_i).$$

Thus, from each sounding pulse they obtain in receiving system m echo pulses with different amplitudes. On the amplitude ratio of these signals it is possible to determine angular target position. The totality of impulses/momenta/pulses at the output of receiving system composes the packet of the echo pulses, which is analogous to the packet, examined earlier during the discrete/digital or linear scanning, with the only difference that the the there echo pulses were obtained nonsimultaneously in proportion to the development/scanning the ray/beam of antenna, and here - simultaneously.

Impulse/momentum/pulse, accepted by each antenna, is amplified in special channel and then it enters diagram of working/treatment (Fig. 7.9). In each channel the fluctuating noises, which we assume/set by statistically independent variables, occur. These noises store/add up to the received signal and impede the

determination of the unknown angular coordinate. Determination of the structure of working/treatment of the echo pulses for measuring the angle in the presence of interferences for one or the other location of the diagrams of antenna and issuing recommendations about the best antenna location from the point of view of the accuracy of direction finding are our task.

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The analogy between the bursts of pulses, obtained during the scanning in the single-channel system and with the motionless antennas in the multichannel system, is starting point for this investigation. This makes it possible to transfer the results of investigations, obtained in the first case, for the multichannel method and to establish/install the structure of processing and the accuracy of measurement during different location of antenna systems (partial diagrams).

Let us examine group antenna location, which is analog of discrete/digital scanning of single-channel system, and uniform antenna location, which corresponds to linear scanning with determined packet.

First case. Let there be in the multichannel system with the group antenna location m , antennas, arranged/located at angle β_1 , and m , at angle β_1 .

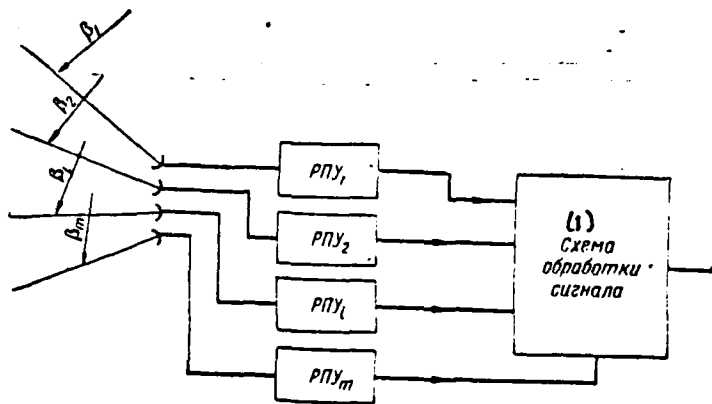


Fig. 7.9. Diagram of multichannel direction finding.

Key: (1). Diagram of processing signal.

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For the signals, reflected from the rapidly fluctuating target, the function of array of the echo signal with the fluctuating noises of the i channel, as is known, will take form (7.2.17)

$$f(\rho_i) = \frac{\rho_i}{\sigma_{c_i}^2 + \sigma^2} e^{-\frac{\rho_i^2}{2(\sigma_{c_i}^2 + \sigma^2)}},$$

where σ_{c_i} - signal level in the channel of the i antenna, depending on its position β_i and direction to the target θ , and according to (7.2.16) $\sigma_{c_i} = \sigma_0 Q(\theta - \beta_i)$. Thus, at the output of each i channel of antenna is obtained signal ρ_i , which is the observed value of one signal in the selection, while the packet of the echo pulses at the output of

all antennas is nothing else but the selection of the observed values with space m . The function of the plausibility of this selection with the group antenna location coincides with the function of the plausibility of packet, obtained during the discrete/digital scanning of antenna, i.e., with expression (7.4.15), if we assume in it

$$n_1=m_1; n_2=m_2; \alpha_1=\beta_1 \text{ and } \alpha_2=\beta_2.$$

Consequently, is analogous structure of working/treatment, determined by equation of maximum of plausibility (7.4.13). Taking into account the replacement indicated the structure of working/treatment is determined by the following expression:

$$\sum_{l=1}^{m_1} \rho_l^{(1)*} + \eta(\theta) \sum_{l=1}^{m_2} \rho_l^{(2)*} = F(\theta), \quad (7.8.1)$$

where $\eta(\theta)$ and $F(\theta)$ are determined by formulas (7.4.17) and (7.4.18), in which

$$k_1 = k_m Q(\theta - \beta_1) \text{ and } k_2 = k_m Q(\beta_2 - \theta),$$

and the dispersion of evaluation/estimate according to (7.5.12)

$$D(\hat{\theta}) \geq \frac{0.25}{\frac{m_1 k_1^2 k_1'^2}{d_1^2} + \frac{m_2 k_2^2 k_2'^2}{d_2^2}} \quad (7.8.2)$$

For radiation pattern of bell-shaped form (7.2.10) dispersion of evaluation/estimate takes form (7.5.14), and in the case of strong

signal, when $k_1 \gg 1$ and $k_2 \gg 1$, it takes very simple form

$$D(\hat{\theta}) \geq \frac{0,25}{4\rho^2 (\varphi_1^2 m_1 + \varphi_2^2 m_2)} \quad (7.8.3)$$

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For equisignal direction $\varphi_1 = -\varphi_2 = c_1/2$, where $c_1 = \rho\Delta\beta$ and $m_1 = m_2$

$$D(\hat{\theta}) \geq \frac{0,25}{\rho^2 c_1^2 m}, \quad (7.8.3a)$$

or

$$\frac{D(\hat{\theta})}{\varphi_d^2} \geq \frac{0,25\epsilon^2}{\ln^2 \frac{1}{d} m},$$

where

$$\epsilon = \frac{\varphi_d}{\Delta\beta}.$$

Second case - multichannel system with uniform antenna location.

All relationships/ratios, which characterize the procedure of working/treatment and the accuracy of measurement, are determined by the analogous relationships/ratios, which correspond to the linear scanning of antenna with the determined packet. Only in these formulas instead of the angular position of impulse/momentum/pulse θ , it is necessary to substitute angular antenna position β , and for a quantity of impulses/momenta/pulses in packet n - a quantity of antennas m . The value of raster angle in the multichannel system corresponds to the angular interval between the antennas. Then the likelihood function of totality m of the taken impulses/momenta/pulses at the output of receiving antennas is determined by formula (7.2.18), and the equation of the maximum

likelihood, which characterizes the structure of working/treatment taking into account the replacement indicated above:

$$\sum_{i=1}^m \frac{k_i k_i'}{1 + k_i^2} \left(2 - \frac{\rho_i^2}{1 + k_i^2} \right) = 0. \quad (7.8.4)$$

It must be noted that solution of equations (7.8.1) and (7.8.4) can be conducted just as analogous equations for single-channel systems: by iterative method or by method of "tests/samples and search". However, with the uniform antenna location in the solution is certain difference from the analogous case during the linear scanning.

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This difference lies in the fact that in the first case the packet can be substantially asymmetric relative to the impulses/momenta/pulses, obtained from the average/mean antenna, while during the linear scanning it in essence is symmetrical relative to its center.

Actually/really, with direction finding let us assume of pinpoint target with the aid of single-channel system during linear scanning of ray/beam maximum of envelope of packet, in any case in the absence of interferences, will be directed toward target, i.e., will coincide with bearing direction. It is necessary to keep in mind

that by envelope is understood the imaginary line, passing through the apexes/vertexes of the echo pulses. However, in the multichannel systems under the same conditions the apex/vertex of envelope can be located in view of the immobility of antennas both on the edge and on the middle of packet in the dependence on the location of target with respect to by antenna to system. This fact must be considered during the calculation of the errors of system. Actually/really, if in the single-channel system the calculation of errors was conducted for the values θ , arranged/located only near the center of packet, then for the multichannel systems the calculation of error must be conducted in the limits of the considerably larger range of angles - in the limits of the width of packet. However, the value of the dispersion of evaluation/estimate with the uniform antenna location by analogy with expression (7.3.27) is determined by the relationship/ratio

$$D(\hat{\theta}) \geq \frac{0,25}{\sum_{i=1}^m \frac{k_i^2 k_i^4}{d_i^2}} \quad (7.8.5)$$

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For bell-shaped radiation pattern it passes by analogy (7.3.30) into formula

$$D(\hat{\theta}) \geq \frac{1}{16 p^2 k_m^4 \sum_{i=1}^m \frac{\varphi_i^2 k_i^4}{d_i^2}} \quad (7.8.6)$$

and further for high signal level, which exceeds noises, into

expression

$$D(\hat{\theta}) \geq \frac{1}{16p^2 \sum_{i=1}^m \varphi_i^2}, \quad (7.8.7)$$

where

$$\varphi_i = p(\beta_i - \theta),$$

i.e. φ_i depends on location of target with respect to antennas of multichannel system. For the specific values of angular coordinate the concrete/specific/actual expressions of the dispersion of evaluation/estimate can be obtained. In particular, for the middle of packet $\theta = \Delta_1/2(m-1)$ (θ it is counted off from the direction of the maximum of the diagram of outer antenna) formula it takes the form, analogous (7.3.34):

$$D(\hat{\theta}) \geq \frac{0.75}{p^2 b_1^2 m (m^2 - 1)}, \quad (7.8.8)$$

where $b_1 = p\Delta_1$, $\Delta_1 = \frac{\beta_m - \beta_1}{m-1}$; in this case was accepted $b_i = \Delta_1(i-1)$. For the "extreme" location of the target, when bearing direction coincides with the maximum of outer antenna $\theta=0$ [or $\theta=\Delta_1(m-1)$],

$$D(\hat{\theta}) \geq \frac{3}{8p^2 b_1^2 m (2m^2 - 3m + 1)}. \quad (7.8.9)$$

In general case of arbitrary location of target in limits of width of packet of diagrams it is possible to assume

$$\theta = \mu \Delta_1 (m-1),$$

where $\mu=0-1$ - parameter, which determines target position.

Then

$$\varphi_i = p\Delta_1 [i-1-\mu(m-1)]$$

and informational quantity

$$j(\mu, m) = 16p^2 b_1^2 \sum_{i=1}^m [i - 1 - \mu(m-1)]^2. \quad (7.8.10)$$

After fulfilling necessary conversions, we will obtain

$$j(\mu, m) = 16p^2 b_1^2 \left[\frac{m(m+1)(2m+1) - 6m^2}{6} + m(m-1)^2 \mu(\mu-1) \right], \quad (7.8.11)$$

in this case

$$D(\hat{\theta}) \geq \frac{1}{j(\mu, m)}. \quad (7.8.12)$$

Hence as special case ensues/escapes/flows out formula (7.8.8) with $\mu=1/2$, and (7.8.9) with $\mu=0$ and $\mu=1$.

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From structure of expression (7.8.11) it follows that accuracy of measurement depends on target position, i.e., from parameter μ . The character of dependence is determined by the behavior of factor $\mu(\mu-1)$. In the assigned range of change μ this factor is negative; therefore its greatest value corresponds to the smallest informational quantity and the greatest error, and a small quantity, on the contrary, gives the greatest accuracy. The value of it monotonically and symmetrically changes relative to the point of maximum, which corresponds $\mu=1/2$, and at which factor value is equal

to $1/2$. On the edges of packet, i.e., with $\mu=1$ and $\mu=0$, factor value is equal to zero, and an informational quantity grows/rises.

Comparison of multichannel system with uniform and group antenna locations is reduced to comparison of dispersion of evaluations/estimates. From formulas (7.8.3a) and (7.8.8) it follows that group antenna location gives the high accuracy of direction finding, than uniform one.

Accuracy of direction finding indicated was reached in multichannel systems with the aid of altogether only one sounding pulse. With an increase in the quantity of sounding pulses the accuracy, naturally, can be increased. Taking into account the asymptotic normality of maximum likelihood estimates and assuming/setting the measurements by equally accurate ones, we will obtain that with the n impulses/momenta/pulses the dispersion of evaluation/estimate decreases in n of times. Thus, the dispersion of evaluation/estimate for the multichannel system in this case can be by the specific relationship

$$D_n(\hat{\theta}) = \frac{D_m(\hat{\theta})}{n},$$

where $D_m(\hat{\theta})$ - dispersion, determined by one of given above relationships/ratios (7.8.3) (7.8.8) or (7.8.12) depending on the character of the location of diagrams. Latter/last formula together with relationships/ratios indicated above for the dispersion of

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evaluation/estimate in different cases makes it possible to determine a number of channels and a quantity of impulses/momenta/pulses in the packet, i.e., the time of direction finding, necessary for obtaining of the required accuracy of the measurement of angular coordinate.

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