AD-A153 157 A TRIVARIATE VERSION OF 'LEVY'S EQUIVALENCE(U) NORTH CAROLINA UNIV AT CHAPEL HILL CENTER FOR STOCHASTIC PROCESSES G SIMONS FEB 85 TR-90 AFOSR-TR-85-0295 1/1

UNCLASSIFIED F49620-82-C-0009 F/G 12/1 NL

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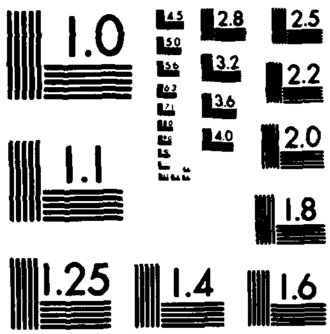
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# CENTER FOR STOCHASTIC PROCESSES

Department of Statistics
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A Trivariate Version of 'Lévy's Equivalence'

by

Gordon Simons

Technical Report No. 90

February 1985

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE

# REPORT DOCUMENTATION PAGE

| | |
|---|---|
| 1a. REPORT SECURITY CLASSIFICATION<br>UNCLASSIFIED | 1b. RESTRICTIVE MARKINGS |
| 2a. SECURITY CLASSIFICATION AUTHORITY | 3. DISTRIBUTION/AVAILABILITY OF REPORT<br>Approved for public release; distribution unlimited. |
| 2b. DECLASSIFICATION/DOWNGRADING SCHEDULE | |
| 4. PERFORMING ORGANIZATION REPORT NUMBER(S)<br>TR No. 90 | 5. MONITORING ORGANIZATION REPORT NUMBER(S)<br>AFOSR-TR- 85-0295 |
| 6a. NAME OF PERFORMING ORGANIZATION<br>University of North Carolina — 6b. OFFICE SYMBOL (If applicable) | 7a. NAME OF MONITORING ORGANIZATION<br>Air Force Office of Scientific Research |
| 6c. ADDRESS (City, State and ZIP Code)<br>Center for Stochastic Processes, Department of Statistics, Chapel Hill NC 27514 | 7b. ADDRESS (City, State and ZIP Code)<br>Directorate of Mathematical & Information Sciences, Bolling AFB DC 20332-6448 |
| 8a. NAME OF FUNDING/SPONSORING ORGANIZATION<br>AFOSR — 8b. OFFICE SYMBOL (If applicable) NM | 9. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER<br>F49620-82-C-0009 |
| 8c. ADDRESS (City, State and ZIP Code)<br>Bolling AFB DC 20332-6448 | 10. SOURCE OF FUNDING NOS.<br>PROGRAM ELEMENT NO. 61102F; PROJECT NO. 2304; TASK NO. A5; WORK UNIT NO. |

11. TITLE (Include Security Classification)
A TRIVARIATE VERSION OF 'LEVY'S EQUIVALENCE'

12. PERSONAL AUTHOR(S)
Gordon Simons

| 13a. TYPE OF REPORT | 13b. TIME COVERED | 14. DATE OF REPORT (Yr., Mo., Day) | 15. PAGE COUNT |
|---|---|---|---|
| Technical | FROM ______ TO ______ | FEB 85 | 2 |

16. SUPPLEMENTARY NOTATION

| 17. COSATI CODES | | | 18. SUBJECT TERMS (Continue on reverse if necessary and identify by block number) |
|---|---|---|---|
| FIELD | GROUP | SUB. GR. | Wiener process; Levy's equivalence; Tanaka's formula. |

19. ABSTRACT (Continue on reverse if necessary and identify by block number)


It is shown that the trivariate stochastic processes $\{(M_t, W_t, M_t, \Theta_t, t \geq 0\}$ and $\{(|W_t|, L_t, T_t), t \geq 0\}$ have the same distributions when $W = \{W_t, t\} \geq 0$ is a Wiener process, $M_t$ is the maximum value attained by W over the time interval $[0,t]$, $\Theta_t$ is the time the maximum value is attained, $L_t$ is the local time of W at level zero and time t, and $T_t$ is the last time W is zero in the time interval $[0,t]$. A straightforward proof, based on "Tanaka's formula", establishes this result by deriving an almost sure version of the equivalence.


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| 20. DISTRIBUTION/AVAILABILITY OF ABSTRACT<br>UNCLASSIFIED/UNLIMITED ☒ SAME AS RPT. ☐ DTIC USERS ☐ | 21. ABSTRACT SECURITY CLASSIFICATION<br>UNCLASSIFIED |
| 22a. NAME OF RESPONSIBLE INDIVIDUAL<br>MAJ Brian W. Woodruff | 22b. TELEPHONE NUMBER (Include Area Code) (202) 767- 5027 — 22c. OFFICE SYMBOL NM |

DD FORM 1473, 83 APR — EDITION OF 1 JAN 73 IS OBSOLETE.

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A Trivariate Version of 'Lévy's Equivalence'



by Gordon Simons*
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Summary

It is shown that the trivariate stochastic processes $\{(M_t - W_t, M_t, \Theta_t), t \geq 0\}$ and $\{(|W_t|, L_t, T_t), t \geq 0\}$ have the same distributions when: $W = \{W_t, t \geq 0\}$ is a Wiener process, $M_t$ is the maximum value attained by W over the time interval $[0,t]$, $\Theta_t$ is the time the maximum value is attained, $L_t$ is the local time of W at level zero and time t, and $T_t$ is the last time W is zero in the time interval $[0,t]$. A straightforward proof, based on "Tanaka's formula", establishes this result by deriving an almost sure version of the equivalence.

AMS 1980 *subject classification.* Primary 60G17 Secondary 60H05

*Keywords and phrases.* Wiener process, Lévy's equivalence, Tanaka's formula

*This research has been supported by the National Science Foundation Grant No. DMS-8400602.

As described by Knight (1981), "Lévy's equivalence" refers to the equality in distribution of the bivariate stochastic processes $\{(M_t - W_t, M_t), t \geq 0\}$ and $\{(|W_t|, L_t)\ t \geq 0\}$. Here, $W = \{W_t, t \geq 0\}$ is a (standard) Wiener process, $M_t = \max_{0 \leq s \leq t} W_s$, and $L_t$ is the local time of W at level zero and time t. In a recent paper [3], the author presents an elementary derivation of a discrete analogue of this result, for a symmetric simple random walk, which he then uses to derive Lévy's equivalence. The objective here is to point out that there is a trivariate version of Lévy's equivalence, which states that the processes $\{(M_t - W_t, M_t, \Theta_t), t \geq 0\}$ and $\{(|W_t|, L_t, T_t), t \geq 0\}$ have the same distributions, where $\Theta_t \in [0,t]$ is the time at which the maximum $M_t$ is attained, and $T_t$ is the last zero of W in the time interval [0,t]. Additional keywords: Stochastic processes; distribution functions; Wiener process.

The proof depends on Tanaka's formula (cf. McKean (1969), page 68), which says

$$L_t = |W_t| + \tilde{W}_t \qquad , \ t \geq 0,$$

where $\tilde{W} = \{\tilde{W}_t, t \geq 0\}$ is a new Wiener process defined by

$$\tilde{W}_t = \int_0^t h(W_s) dW_s \qquad , \ t \geq 0,$$

with $h(\cdot) = -\mathrm{sign}(\cdot)$ (cf. McKean (1969), page 29). Observe that

$$\tilde{W}_s - \tilde{W}_t = |W_t| - |W_s| - (L_t - L_s) \leq |W_t|, \ s \in [0,t].$$

The inequality is an equality if and only if $s = T_t$. Thus

The inequality is an equality if and only if $s = T_t$. Thus

$$(1) \qquad (\tilde{M}_t - \tilde{W}_t,\ \tilde{M}_t,\ \tilde{\Theta}_t) = (|W_t|,\ L_t,\ T_t) \qquad ,\ t \geq 0,$$

where $\tilde{M}_t = \max_{0 \leq s \leq t} \tilde{W}_s$, and $\tilde{\Theta}_t \in [0,t]$ is the time of the maximum. It should be emphasized that (1) is an almost sure identity in t for two trivariate stochastic processes. Consequently, $\{(M_t - W_t,\ M_t,\ \Theta_t),\ t \geq 0\}$ and $\{(|W_t|,\ L_t,\ T_t),\ t \geq 0\}$ have the same distributions as asserted.

## REFERENCES


[1] Knight, F. (1981), Essentials of Browian Motion and Diffusion, Mathematical Surveys No. 18 (American Math. Soc., Providence).

[2] McKean, H.P. (1969), Stochastic Integrals, (Academic Press, New York).

[3] Simons, G. (1983), A discrete analogue and elementary derivation of 'Lévy's equivalence' for Browian motion, Statistics and Probability Letters 1, 203-206.

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