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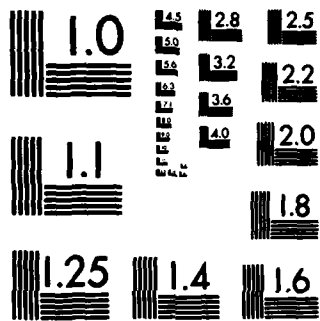
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Francis Labrune

February 1984

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# COMPARISON OF VALUES OF A SPOT MARKET FOR ELECTRICITY

Francis Labrune

February 1984

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The author, Francis Labrune, is a graduate fellow in The Rand Graduate Institute. This paper was prepared for an RGI course, Game Theory and Applications, taught by Dr. Lloyd Shapley in the Spring Quarter of 1983.



## I. INTRODUCTION

This paper is concerned with a market for electricity that has been successfully operating in Florida since 1978 -- the Florida Energy Broker. [1] The Broker provides an hourly spot market for energy traded at the wholesale level between utilities with bilateral contracts.

The Florida Energy Broker is a central computer matching bids and offers made by participating utilities. Participating utilities as a whole realize important savings. In 1980 savings to utilities exceeded \$40 million, roughly two percent of the total fuel bill. These savings are distributed to participating utilities according to a predefined policy.

This paper compares the distribution of savings resulting from the Broker policy and two alternative policies: one is defined as the Shapley value applied to the Broker market considered as a cooperative game with possibilities of coalitions, another results from the application of the market clearing price which is the core of the previous game.

Section II is a general description of the Florida Energy Broker. Section III is a computation of the Broker value for an example generated from available data for a given hourly spot market. Section IV applies the Shapley value to the Florida Energy Broker market and describes the Monte Carlo simulation method used to estimate this value. The corresponding PL/I routine is listed in the appendix.

Section V is a comparison between the different values.

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[1] See Rand Note N-1817-DOE, *A spot market for electricity: preliminary analysis of the Florida Energy Broker*, Linda Cohen - February 1982.

## II. THE FLORIDA ENERGY BROKER

### DESCRIPTION

The role of the Florida Energy Broker is to coordinate and enhance sales and purchases of economy energy by FCG (Florida Electric Power Coordinating Group, Inc.) utilities. Its mechanisms for coordination incorporate two basic policies. First the Broker attempts to secure hourly operating efficiency for the entire system, by ensuring that no energy remains untraded whose incremental cost to a potential selling utility is less than the avoided cost to a potential purchaser. Second, purchasing utilities agree to an explicit policy for distributing the net savings.

The FCG broker arranges hour by hour energy contracts .At fifteen minutes before each hour utilities enter buy and sell quotes with the Broker. The quotes are intended to represent marginal incremental and decremental cost, including incremental fuel cost and incremental transmission loss adjustments.

Because marginal costs vary with the level of production, each utility can send in several quotes (up to three bids and three sell offers per utility), each specifying a quantity of energy and a bid or offer price.

The Broker sorts sell offers and arranges them from lowest cost to highest cost. Buy quotes are ranked from highest bid to lowest bid. Bids and offers are then matched on a high-low basis: the highest buy bid with the lowest sell offer, the next highest buy bid with the next lowest sell offer and so on. Matches are arranged so that the first match maximizes net savings among all possible matches, and so on until all savings are exhausted.

**SAVINGS DISTRIBUTION POLICY**

For each matched bidding and sell-offering utility, the price is determined as follows:

$$\text{Price} = 1/2 (\text{buyer's bid} + \text{seller's offer})$$

so that each participating utility in a particular trade gets half of the savings resulting from the match.

### III. BROKER VALUE

In order to simplify our own calculations and due in part to the lack of adequate data, we do not consider the following constraints taken into account for the actual Broker calculations:

- o We do not include third-party cost for wheeling and transmission losses.
- o We do not require the existence of a minimum spread between match and sell quotes to counteract underestimations in some quotes.
- o We do not limit trades to utilities having pre-existing bilateral contracts with each other.

The example we consider is directly generated from the example pictured in Figure 2 of N-1817-DOE describing the Broker during one hour on February 12, 1980.

We list the bids and sell offers in Table 1 and plot them in Figure 1.

The highest bid is entered by utility 1, 10 megawatts at \$55/MWH; the lowest offer was entered by utility 10, 200 megawatts at \$18/MWH. The Broker matches utility 1 with utility 10 for a trade of 10 megawatts at a price of  $(55+18)/2=\$36.5/\text{MWH}$ , resulting in total savings of :

$$(55-18) * 10 = \$370$$

i.e., individual savings of \$185. for each one. After this trade, demand for utility 1 is satisfied. Utility 10 however is still offering 190 megawatts at a price of \$18/MWH. Utility 10 is then matched with the next highest bid, to wit utility bid, 30 megawatts at \$50/MWH. This match results in a trade of 30 megawatts at  $(50+18)/2=\$34/\text{MWH}$ , and in savings of \$480 for both utilities.

Table 1

BIDS AND SELL OFFERS FOR A ONE HOUR SPOT MARKET

UTILITY	BIDS		SELL OFFERS	
	PRICE	QUANTITY	PRICE	QUANTITY
1	55	10		
1	38	10		
2	50	30		
3	47	3		
4	42	10		
5	40	93		
6	37	50		
7	36	400	31	210
8	31	31	39	30
8			41	35
9	27	10	30	6
10	15	38	18	200
10			32	100
10			36	50
11			33	50
11			34	50
11			35	25

The broker keeps matching utilities following the same algorithm until the lowest remaining offer is higher than the highest remaining bid. In our example the last trade occurs between utility 7 bidding 400 megawatts at \$36/MWH and utility 11 offering 50 megawatts at \$34/MWH. At this stage, utility 7 has already bought in other trades and is still bidding for 40 megawatts. Utility 11 is still offering its 50 megawatts at \$34/MWH. This results in a trade of 40 megawatts at \$35/MWH, utility 7 and utility 11 realizing savings of \$40 each.

All the savings are listed by utility in Table 2.

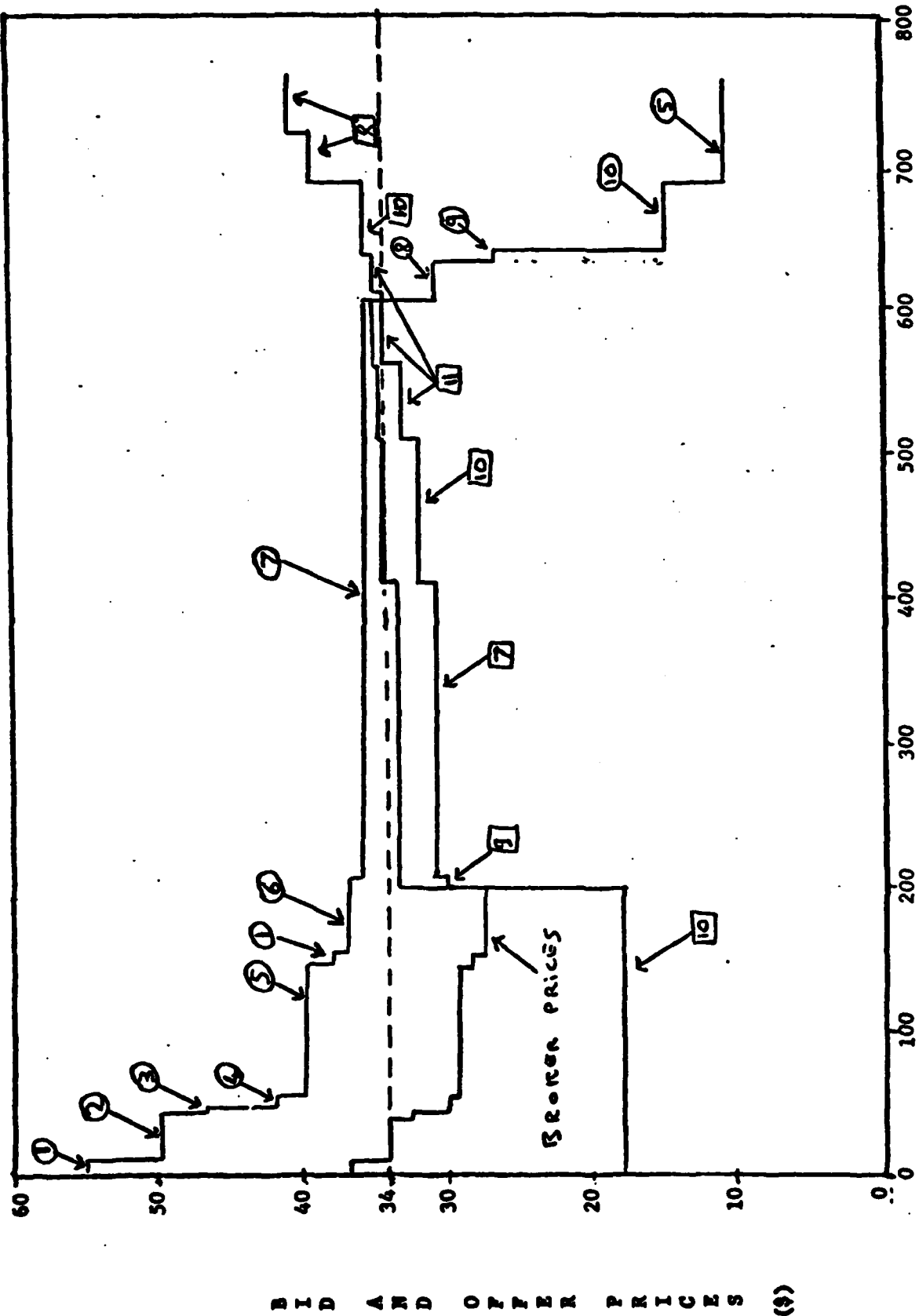


Figure 1: Set of bids and offers for a one hour spot market - Broker prices.  
 (from Fig. 2 in Rand Note N-1817-DOE)  
 ① = bid by utility i - - ① = sell offer by utility j

B I D A N D O F F E R P R I C E S (\$)

#### IV. THE FLORIDA ENERGY BROKER AS A COOPERATIVE GAME

We now consider the same Florida Energy Broker market as a cooperative game with 15 players: 10 bidders and 5 offerers. A utility making at least one bid and one offer is represented in the game as two different players. We assume that the energy produced by the different players is perfectly interchangeable. As stated earlier we assume that any utility can sell to any other i.e., we assume that each bidding utility has a bilateral contract with each offering utility and vice-versa. We further assume that each utility has a linear utility for money.

Our problem is to define how the general profit in the game inherent from the different marginal costs is to be distributed among the players, given the theoretical possibility of coalitions, i.e., subsets of players setting up their own markets. We are interested in finding the Shapley Value for this energy market game.

We first define the characteristic function of the game. The value for a pair bidder-offerer  $(B_i, O_j)$  is defined as the joint profit

to  $B_i$  and  $O_j$  which we write as:

$$V((B_i, O_j)) = \max ( 0, x_{ij} * (b_i - o_j) )$$

where  $i=1, \dots, 10$  and  $j=7, \dots, 11$ ,  $x_{ij}$  is the number of megawatts traded

between  $B_i$  and  $O_j$ , and  $b_i$  and  $o_j$  are respectively the bid and offer prices.

We similarly define  $V(P)$  for any set  $P$  of players as the maximum total profit that they can achieve trading among themselves.  $V(P)$  is equal to the area between the bid-demand and offer-supply curves obtained by plotting the bids of utilities in  $P$  in decreasing order and the offers in increasing order as shown in Figure 2.

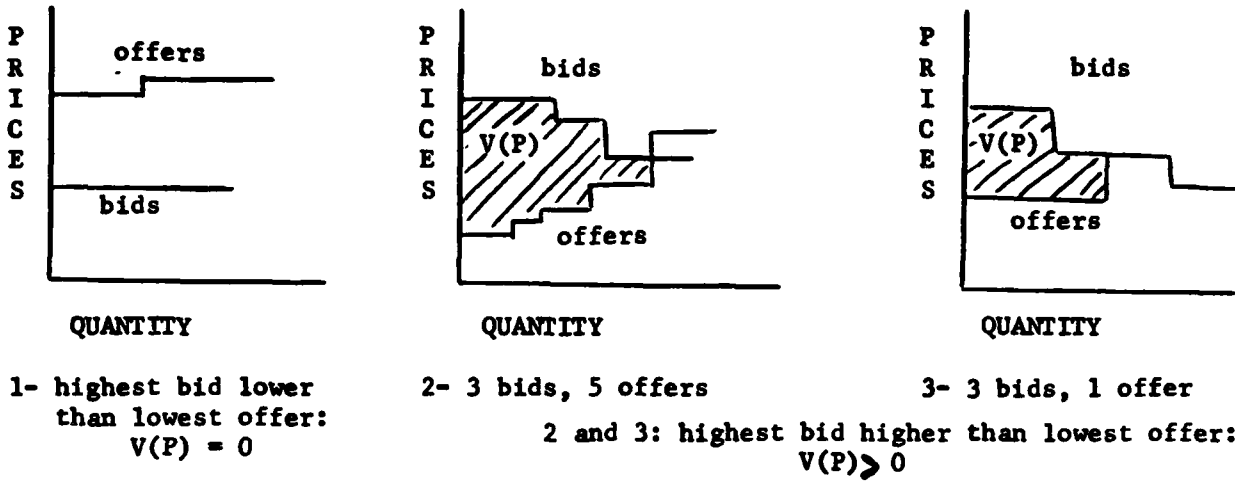


Fig. 2 -- Example of game values for different sets of players

The value of the coalition of all the players in the game is equal to the total savings resulting from the Broker, as the total Broker savings as well are equal to the area between between the total demand (=bid) and supply (=offer) curves.

**Value Computation**

The value  $S(i)$  to a particular utility is defined as its expected or averaged contributons to the worth of a typical coalition  $C$ , which can be written as:

$$S(i) = E \{V(C) - V(C - \{i\})\} \quad i \in C$$

The exact formula defining the value to a particular player i is:

$$S(i) = \frac{1}{n} * \sum_{s=1}^n \frac{1}{c(s)} \sum_{\substack{C \ni i \\ |C|=s}} [V(C) - V(C - \{i\})]$$

$$\text{where } c(s) = \binom{n-1}{s-1} = \frac{(n-1)!}{(n-s)!(s-1)!} = \text{number of coalitions of size } s.$$

Given the large number of possible coalitions, an exact calculation of these expectations is practically impossible. But we can generate these weights in the form of probabilities: we shuffle the players, line them up and take C-{i} to be the set of players in front of player i. In the case of our energy market, this process assumes that the trading utilities arrive at the market place one by one in a random order and that each utility gets credit for the full increase in total profit (if any) that its arrival makes possible. Under this scheme, a player's expected gain is exactly its value, assuming all order of arrival are equally likely.

In order to estimate the expected values, we have written a Monte Carlo routine to simulate the Florida Energy Broker market. Using a random number generator, we generate numbers representing utilities as they enter the market one by one. After each new entry, we compute the area between the demand and supply curves obtained from the bids and offers of utilities already in the market. The difference between the area computed before and after a new utility entry represents the full increase in total profit to be credited to the last entered utility. We run 5000 market simulations. We list the expected game value to each player in table 2.

When a player is both a bidder and an offerer we enter its bids separately from its offers to be able to look at the benefits it gets as a buyer separately from the benefits it gets as a seller. But we enter all the bids or all the offers of a given player at once, considering the set of bids or offers as respectively the player demand or supply curve. We analyse the results in Section VI.

## V. MARKET-CLEARING-PRICE VALUE

The clearing price value for this competitive market is given by the intersection of the demand and supply curves drawn on figure 2, i.e., the intersection of the bids and sell offers curves: \$34/MWH. This value is also the core of the game, i.e., the outcome (unique in this case) which no coalition of players can improve upon by setting up their own market. We compute the benefits that would occur to each utility, should this market clearing price be in effect and we list the results in Table 2.

Table 2

VALUES AND EFFECTIVE PRICES FOR A ONE HOUR SPOT MARKET

---

UTILITY(*)	GAME VALUE	STANDARD ERROR	BROKER VALUE	MARKET CLEARING PRICE VALUE	EFFECTIVE PRICE FOR GAME VALUE	EFFECTIVE PRICE FOR BROKER VALUE
1(b)	244.09	2.41	285	250	34.30	32.50
2(b)	443.76	3.96	480	480	35.20	34.00
3(b)	42.00	0.92	43.5	39	33.00	32.50
4(b)	85.09	1.26	120	80	33.49	30.00
5(b)	642.53	9.05	1023	558	33.09	29.00
6(b)	233.16	4.40	439	150	32.34	28.22
7(b)	1203.90	16.70	840	800	32.99	33.90
7(o)	631.55	7.28	525	630	33.46	33.50
8(b)	53.57	1.89	0	-	-	-
8(o)	24.52	1.17	0	-	-	-
9(b)	11.60	0.42	0	-	-	-
9(o)	28.23	0.46	21	24	34.70	33.50
10(b)	0.00	0.00	0	-	-	-
10(o)	2612.80	22.26	2569.5	3400	31.38	31.23
11(o)	204.21	3.36	115	50	36.57	35.08

---

(\*) b=bidding utility, o=offering utility

## VI. VALUE COMPARISONS

In order to use a unique scale of comparison for the different utilities, we calculate the effective price per MWH faced by each purchaser and each seller both under the broker policy and the game value policy. The effective price is obtained as follows: for a given purchaser, we take the total bid value of the energy actually purchased, i.e., the total of all products (bid price)x(bid quantities); we respectively subtract either the expected player game value from the total bid value or the broker value from the total bid value; and we divide the result by the number of megawatts purchased. The result is the effective price under either policy.

Let us take utility 1 for example. Utility 1 enters two bids: 10 megawatts at \$55. and 10 megawatts at \$38. Figure 2 shows that utility 1 actually purchases these 20 megawatts. So, the total value of its actually purchased energy is \$930. The expected game value is \$244.09. Utility 1 faces an expected effective price of:

$$(930-244.09) / 20 = \$34.30$$

Similarly, the broker value is \$285, i.e., utility 1 faces an effective broker price of:

$$(930-285) / 20 = \$32.50$$

For a given seller, we do the symmetric computation, considering only the energy actually sold on the market, adding to the total offer value respectively the expected game value or the broker value before dividing by the number of megawatts actually sold. We list effective prices for all active market utilities in Table 2. The calculation does not apply to utility 8, nor to the bid component of units 9 and 10.

A total of 6461 megawatts are traded on the market.

The comparison of effective prices shows that for each purchaser or seller but utility 7, the effective price under the game value policy is higher than the effective price under the broker policy. It means as we can read it from Table 2, that purchasers expect less under the game value policy than under the broker policy. And sellers expect more under the game value policy than under the broker policy. Given that the whole model is symmetric between buyers and sellers, these differences between buyers's and sellers's expectations might be explained by the differences in their marginal cost curves. Many bidders need to get small amounts of energy to satisfy a peak demand.

Already under the broker policy the credit received by selling utilities gives all utilities an incentive to produce cheaper energy in order to enter the active market and get some credit. As under the game value policy selling utilities get even more credit, the incentive to reduce energy production cost is even bigger.

Effective prices under the broker policy are generally lower than the market clearing price. As a result, under the market clearing policy, the purchasers generally get more credit than under the broker policy, and the sellers generally get less credit. No general pattern emerges from the comparison of the game value policy with the market clearing price policy. Utility 10 does not get any credit as a purchaser under any policy. This is because utility 10's only bid price, \$15 for 38 megawatts is uniformly lower than all sell offers. Utility 8 both as a purchaser and a seller and utility 9 as a purchaser do not get any credit under the broker policy, but their expected game value is different from 0. Even if they do not participate actively in the market, they can influence the market if they associate in some coalitions: they have some bargaining power. The broker policy does not credit this power. The game value policy suggests that offering utilities with an incremental cost too high to enable them to actively participate in the market but such that their incremental cost is lower than at least one bidder's decremental cost should be credited in some way. Similarly, the game value policy suggests that utilities with a decremental cost too low to enable them to actively participate in the market but such that their decremental cost is higher than at least one

offering utility's incremental cost should be credited as well. This could be achieved by having some utilities give them side payments. We note that these payments to "non-participants" represent only about 1.5% of total savings. This would encourage them to keep participating in the broker's market with bids or sell offers. They might directly contribute to savings in another one hour market. Such a contribution could be considered as a reimbursement for these side-payments, given that they would generate savings for at least another utility.

## VII. CONCLUSION

The application of game theory methods to the Florida Energy Broker market suggests a different allocation of savings to utilities, giving less credit to purchasers, more credit to sellers and some credit to some utilities at the border of the active market which do not get any credit under the broker policy.

The distribution of savings under the market clearing price policy as well would generally give less credit to purchasers and more to sellers than the broker policy. The market clearing price value although different from the expected game value, does not show a systematic pattern of either similarity to or difference from the game value. These conclusions are drawn from the study of a single one hour spot market. In order to test the sensitivity of the above conclusions, it would be necessary to do the same comparison for many other such one hour markets, at different times of the day, and different periods of the year when utilities demand and supply curves are different, leading to different patterns of marginal costs. A problem arising when attempting such a study would be the difficulty to get data. The sets of bids and offers are known only to the broker, and are not published, to prevent any utility to have the knowledge of other utilities' bids and/or sell offers.

VIII. APPENDIX

PL/I ROUTINE FOR THE MONTE CARLO SIMULATION OF THE BROKER MARKET

```
// EXEC PLIOCLG,LIBL='SYS1.CSDFNLIB'  
//PLI.SYSIN DD *  
GAME:PROCEDURE OPTIONS(MAIN);  
DECLARE TOTAL_VALUE(1:15) FLOAT BIN(21) STATIC  
      INIT(0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.);  
DECLARE GAME_VALUE(1:15) FLOAT BIN(21) STATIC;  
DECLARE (I,NN,NP,NP_IN_GAME) BINARY FIXED(15);  
DECLARE (AREA,PREVIOUS) FLOAT BIN(21);  
DECLARE (IBID,IBID_MAX) BINARY FIXED(15);  
DECLARE (IOFFER,IOFFER_MAX) BINARY FIXED(15);  
DECLARE (CURBID,CUROFFER) BINARY FIXED(15);  
DECLARE BID_PRICE(1:15) BINARY FIXED STATIC  
      INIT(55,50,47,42,40,38,37,36,31,27,15,11,0,0,0);  
DECLARE BID_QUANT(1:15) BINARY FIXED STATIC  
      INIT(10,30,3,10,93,10,50,400,31,10,38,68,0,0,0);  
DECLARE BID_IND(1:16) BINARY FIXED STATIC;  
DECLARE OFFER_PRICE(1:15) BINARY FIXED STATIC  
      INIT(18,30,31,32,33,34,35,36,39,41,0,0,0,0,0);  
DECLARE OFFER_QUANT(1:15) BINARY FIXED STATIC  
      INIT(200,6,210,100,50,50,25,50,30,35,0,0,0,0,0);  
DECLARE OFFER_IND(1:16) BINARY FIXED STATIC;  
DECLARE PLAYER_IND(1:15) BINARY FIXED STATIC;  
DECLARE FIRST_OFFER_ADDRESS(1:15) BINARY FIXED  
      INIT(0,0,0,0,0,0,0,0,0,0,9,7,13,5,1);  
DECLARE FIRST_BID_ADDRESS(1:15) BINARY FIXED  
      INIT(1,4,6,8,10,13,15,17,19,21,0,0,0,0,0);  
DECLARE KEY_OFFER(1:25) BINARY FIXED  
      INIT(1,4,8,0,2,0,3,0,5,6,7,0,9,10,0,0,0,0,0,0,0,0,0,0,0);  
DECLARE KEY_BID(1:25) BINARY FIXED  
      INIT(1,6,0,2,0,3,0,4,0,5,12,0,7,0,8,0,9,0,10,0,11,0,0,0,0);  
DECLARE NOT_NEW BIT(1);  
DECLARE R FLOAT BIN(21);  
DECLARE RANDOM ENTRY EXTERNAL;  
DECLARE RANK BIN FIXED(15);  
DECLARE MONTE_CARLO FIXED BIN;  
DECLARE SIGMA_XSQUARE(1:15) FLOAT BIN(21) STATIC  
      INIT(0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.);  
DECLARE ERROR(1:15) FLOAT BIN(21);  
IOFFER_MAX=10;  
IBID_MAX=12;  
NP=15;  
NN=2000;  
DO MONTE_CARLO=1 TO NN;  
  PREVIOUS=0.;
```

```
NP_IN_GAME=0;
DO I = 1 TO NP;
  PLAYER_IND(I)=0;
END;
DO I=1 TO 16;
  BID_IND(I)=0;
  OFFER_IND(I)=0;
END;
DO WHILE(NP_IN_GAME<NP);
  NOT_NEW='1'B;
  DO WHILE(NOT_NEW);
    CALL RANDOM(R);
    RANK=MOD(TRUNC(R*1000),NP)+1;
    IF (PLAYER_IND(RANK)=0) THEN
      DO;
        NOT_NEW='0'B;
        PLAYER_IND(RANK)=1;
        NP_IN_GAME=NP_IN_GAME+1;
        CALL SETIND;
      END;
    END;
  END;
  CALL VALUE;
  TOTAL_VALUE(RANK)=AREA-PREVIOUS+TOTAL_VALUE(RANK);
  SIGMA_XSQUARE(RANK)=SIGMA_XSQUARE(RANK)+(AREA-PREVIOUS)**2;
  PREVIOUS=AREA;
END;
END;
DO I = 1 TO NP;
  GAME_VALUE(I)=TOTAL_VALUE(I)/NN;
  ERROR(I)=SQRT((SIGMA_XSQUARE(I)-(TOTAL_VALUE(I)**2)/NN)/(NN-1));
  ERROR(I)=ERROR(I)/SQRT(NN-1);
END;
PUT FILE(SYSPRINT) EDIT(' PLAYER* TOTAL GAIN * AVERAGE GAIN ',
  '* ERROR')(SKIP,A,A);
DO I=1 TO NP;
  PUT EDIT(I,TOTAL_VALUE(I),GAME_VALUE(I),ERROR(I))
    (SKIP,X(3),P'99',X(4),F(10,0),X(4),F(10,2),X(6),
    F(10,2));
END;
VALUE:PROCEDURE;
IOFFER=0;
IBID=0;
CUROFFER=0;
CURBID=0;
AREA=0.;
DO WHILE(IOFFER<=IOFFER_MAX & IBID <=IBID_MAX);
  IF CUROFFER=0 THEN
    DO;
      IOFFER=IOFFER+1;
      DO WHILE(IOFFER<=IOFFER_MAX & OFFER_IND(IOFFER)=0);
        IOFFER=IOFFER+1;
      END;
    END;
```

```
      IF IOFFER<=IOFFER_MAX THEN
        CUROFFER=OFFER_QUANT(IOFFER);
      END;
    IF CURBID=0 THEN
      DO;
        IBID=IBID+1;
        DO WHILE(IBID<=IBID_MAX & BID_IND(IBID)=0);
          IBID=IBID+1;
        END;
        IF IBID<=IBID_MAX THEN
          CURBID=BID_QUANT(IBID);
        END;
      IF (IOFFER<=IOFFER_MAX & IBID<=IBID_MAX
        & BID_PRICE(IBID) > OFFER_PRICE(IOFFER)) THEN
        DO;
          IF CURBID>=CUROFFER THEN
            DO;
              AREA=AREA+(CUROFFER*(BID_PRICE(IBID)-OFFER_PRICE(IOFFER)));
              CURBID=CURBID-CUROFFER;
              IF CUROFFER=CURBID THEN
                CURBID=0;
                CUROFFER=0;
              END;
            ELSE
              DO;
                AREA=AREA+(CURBID*(BID_PRICE(IBID)-OFFER_PRICE(IOFFER)));
                CUROFFER=CUROFFER-CURBID;
                CURBID=0;
              END;
            END;
          END;
        ELSE
          DO;
            IBID=IBID_MAX+1;
            IOFFER=IOFFER_MAX+1;
          END;
        END;
      END VALUE;
      SETIND:PROCEDURE;
      DECLARE RRR FIXED BIN(15);
      RRR=FIRST_BID_ADDRESS(RANK);
      IF (RRR!=0) THEN
        DO WHILE(KEY_BID(RRR)!=0);
          BID_IND(KEY_BID(RRR))=1;
          RRR=RRR+1;
        END;
        RRR=FIRST_OFFER_ADDRESS(RANK);
        IF (RRR!=0) THEN
          DO WHILE(KEY_OFFER(RRR)!=0);
            OFFER_IND(KEY_OFFER(RRR))=1;
            RRR=RRR+1;
          END;
        END SETIND;
      END GAME;
    /*
```

**END**

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