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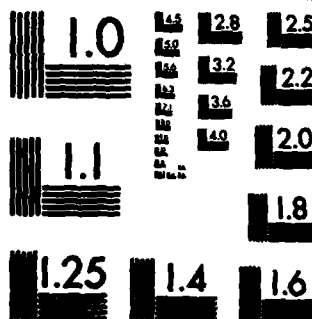
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ELECTROMAGNETIC TRANSMISSION THROUGH A
SLOT IN A PERFECTLY CONDUCTING PLANE

Interim Technical Report

by

Joseph R. Mautz
Roger F. Harrington

December 1984

Department of
Electrical and Computer Engineering
Syracuse University
Syracuse, New York 13210

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number) → When a plane wave is incident on a slot in a perfectly conducting ground plane of infinite extent, three quantities of interest are the tangential electric field in the aperture, the transmission coefficient, and the scatter- ing cross section. It is assumed that the tangential electric field in the aperture is transverse to the slot axis and depends only on the coordinate along the slot axis. A computer program is presented to calculate the tangen- tial electric field in the aperture, the transmission coefficient, and the		

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scattering cross section. This program is described and listed. Sample input and output data are included for the convenience of the user.

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ELECTROMAGNETIC TRANSMISSION THROUGH A SLOT IN A PERFECTLY CONDUCTING PLANE

I. INTRODUCTION

Although the computer program in [1] can treat the case where the rectangular aperture is one subsection wide, i.e., $L_y = 1$ in Fig. 1 of [1], this computer program is long and complicated. A computer program was written specifically for the $L_y = 1$ case. Relatively short, this program is described and listed here. It consists of a main program and the subroutines YMAT, PLANE, DECOMP, and SOLVE.

The formulas that are programmed are presented in Section II. The main program is described and listed in Section III, the subroutine YMAT in Section IV, the subroutine PLANE in Section V, and the subroutines DECOMP and SOLVE in Section VI.

II. FORMULATION

The magnetic current $\underline{M}$ on the $z < 0$ side of the aperture (see [2, Fig. 2]) is expressed as

$$\underline{M} = \sum_{j=1}^{L_x-1} v_j \underline{M}_j^x \quad (1)$$

where $\{v_j\}$ are unknown coefficients and $\{\underline{M}_j^x\}$ are expansion functions defined by [1, Eq. (10)]

$$\underline{M}_j^x = \underline{u}_x T_j(x) P(y), \quad j=1,2,\dots,L_x-1 \quad (2)$$

Here, $\underline{u}_x$ is the unit vector in the x-direction, $T_j(x)$ is the triangle function defined by

$$T_j(x) = \begin{cases} \frac{x - (j-1)\Delta x}{\Delta x} & (j-1)\Delta x \leq x \leq j\Delta x \\ \frac{(j+1)\Delta x - x}{\Delta x} & j\Delta x \leq x \leq (j+1)\Delta x \\ 0 & |x - j\Delta x| \geq \Delta x \end{cases} \quad (3)$$

and $P(y)$ is the pulse function defined by

$$P(y) = \begin{cases} 1 & 0 \leq y < \Delta y \\ 0 & \text{all other } y \end{cases} \quad (4)$$

Here, Δx and Δy are, respectively, the aperture subsection lengths in the x and y directions.

The magnetic field incident on the aperture-perforated conducting plane of [1, Fig. 1] is either $\underline{H}_{\theta y}$ or $\underline{H}_{xx}$ where

$$\underline{H}_{\theta y} = \underline{u}_{\theta}(\theta^{inc}) e^{jk(x \cos \theta^{inc} + z \sin \theta^{inc})}, \quad \pi \leq \theta^{inc} < 2\pi \quad (5)$$

$$\underline{H}_{xx} = \underline{u}_x e^{jk(y \cos \phi^{inc} + z \sin \phi^{inc})}, \quad \pi \leq \phi^{inc} < 2\pi \quad (6)$$

Here, k is the wave number. In (5), the incident wave comes from the direction for which $y=0$ and $\theta=\theta^{inc}$. In (6), the incident wave comes from the direction for which $x=0$ and $\phi=\phi^{inc}$. In (5), $\underline{u}_{\theta}(\theta^{inc})$ is the unit vector in the θ direction evaluated at $\theta=\theta^{inc}$. In (5) and (6), the first subscript on $\underline{H}$ denotes the polarization of the magnetic field, and the second subscript denotes the plane of incidence. In (5), the plane of incidence is the $y=0$ plane. In (6), the plane of incidence is the $x=0$ plane. Called the incident magnetic field, the magnetic field (5) or (6) is the field that would exist in free-space, i.e., in the absence of the aperture-perforated conducting plane.

When the incident magnetic field is given by (5), the coefficients $\{V_j\}$ in (1) are the elements of the column vector $\vec{V}$ that satisfies

$$Y\vec{V} = -(\vec{P}^{inc})_{\theta_y} \quad (7)$$

where the i th element of the column vector $(\vec{P}^{inc})_{\theta_y}$ is called $(P_i^{inc})_{\theta_y}$ and is given by [1, Eq. (59)]

$$(P_i^{inc})_{\theta_y} = 2\Delta x \Delta y \sin \theta^{inc} \left(\frac{\sin \frac{k\Delta x \cos \theta^{inc}}{2}}{\frac{k\Delta x \cos \theta^{inc}}{2}} \right) e^{jk i \Delta x \cos \theta^{inc}}, \quad i=1,2,\dots,L_x-1 \quad (8)$$

In (7), Y is the square matrix whose ij element is called Y_{ij} and is given by [1, Eq. (23)]

$$\begin{aligned} Y_{ij} = & \frac{j\Delta x \Delta y}{\pi\eta} \left[\frac{1}{2} I_c(j-1) - \frac{1}{2} I_x(j-1+1) + \frac{(j-1+3/2)}{2} I_c(j-1+1) \right. \\ & + \frac{1}{2} I_x(j-1-1) - \frac{(j-1-3/2)}{2} I_c(j-1-1) + \frac{1}{(k\Delta x)^2} (I_c(j-1+1) \\ & \left. - 2I_c(j-1) + I_c(j-1-1)) \right] \quad (9) \end{aligned}$$

The j that appears in the factor $j\Delta x \Delta y / (\pi\eta)$ on the right-hand side of (9) is $\sqrt{-1}$. Each of the rest of the j 's on the right-hand side of (9) is the subscript j on Y_{ij} . In (9), η is the impedance of free space. Moreover, I_c and I_x are given by [1, Eqs. (27) and (28)]

$$I_c(1) = 2 \int_0^{y_u} dy \int_{x_l}^{x_u} dx \frac{e^{-j\sqrt{x^2+y^2}}}{\sqrt{x^2+y^2}} \quad (10)$$

$$I_x(i) = \frac{2}{k\Delta x} \int_0^{y_u} dy \int_{x_l}^{x_u} dx \frac{x e^{-j\sqrt{x^2+y^2}}}{\sqrt{x^2+y^2}} \quad (11)$$

where

$$y_u = k\Delta y/2 \quad (12)$$

$$x_u = (i + 1/2)k\Delta x \quad (13)$$

$$x_l = (i - 1/2)k\Delta x \quad (14)$$

It is evident that $I_c(i)$ is even in i and that $I_x(i)$ is odd in i .

When the incident magnetic field is given by (6), the coefficients $\{V_j\}$ in (1) are the elements of the column vector $\vec{V}$ that satisfies

$$Y\vec{V} = -(\vec{P}^{inc})_{xx} \quad (15)$$

where the i th element of the column vector $(\vec{P}^{inc})_{xx}$ is called $(P_i^{inc})_{xx}$ and is given by [1, Eq. (65)]

$$(P_i^{inc})_{xx} = -2\Delta x \Delta y \left(\frac{\sin \frac{k\Delta y \cos \phi^{inc}}{2}}{\frac{k\Delta y \cos \phi^{inc}}{2}} \right) e^{j(k\Delta y/2) \cos \phi^{inc}}, \quad i=1,2,\dots,L_x-1 \quad (16)$$

In (9),

$$2-L_x \leq j-1 \leq L_x-2 \quad (17)$$

but, because (9) is even in $j-1$,

$$0 \leq j-1 \leq L_x-2 \quad (18)$$

is sufficient. Therefore,

$$-1 \leq i \leq L_x-1 \quad (19)$$

is sufficient in (10) and (11).

The integrals (10) and (11) are evaluated by using the following four term approximation [1, Eq. (39)]

$$e^{-jr} \approx e^{-jr_1} [1 - j(r-r_1) - \frac{1}{2}(r-r_1)^2 + \frac{j}{6}(r-r_1)^3] \quad (20)$$

where

$$r = \sqrt{x^2 + y^2} \quad (21)$$

$$r_1 = |ik\Delta x| \quad (22)$$

Substitution of (20) into (10) gives [1, Eq. (42)]

$$I_c(i) = (C_1 U_1 + C_2 U_2 + C_3 U_3 + C_4 U_4) 2e^{-jr_1} \quad (23)$$

where

$$U_1 = 1 - \frac{r_1^2}{2} + jr_1 \left(1 - \frac{r_1^2}{6}\right) \quad (24)$$

$$U_2 = r_1 - j \left(1 - \frac{r_1^2}{2}\right) \quad (25)$$

$$U_3 = -\frac{1}{2} (1 + jr_1) \quad (26)$$

$$U_4 = \frac{j}{6} \quad (27)$$

$$C_n = \int_0^{y_u} dy \int_{x_l}^{x_u} dx r^{n-2}, \quad n = 1, 2, 3, 4 \quad (28)$$

Substitution of (20) into (11) gives

$$I_x(i) = (X_1 U_1 + X_2 U_2 + X_3 U_3 + X_4 U_4) \frac{2e^{-jr_1}}{k\Delta x} \quad (29)$$

where U_1 , U_2 , U_3 , and U_4 are given by (24)-(27) and

$$X_n = \int_0^{y_u} dy \int_{x_\ell}^{x_u} dx \, x r^{n-2}, \quad n=1,2,3,4 \quad (30)$$

Using the indefinite integrals [1, Eqs. (46)-(54)], we obtain

$$C_1 = A_{xu} - A_{x\ell} + A_{yu} \quad (31)$$

$$C_2 = y_u k \Delta x \quad (32)$$

$$C_3 = \frac{y_u}{3} (x_u r_4 - x_\ell r_3) + \frac{1}{6} (x_u^2 A_{xu} - x_\ell^2 A_{x\ell} + y_u^2 A_{yu}) \quad (33)$$

$$C_4 = \frac{1}{3} y_u (x_u r_4^2 - x_\ell r_3^2) \quad (34)$$

$$X_1 = \frac{1}{2} (y_u (r_4 - r_3) + x_u A_{xu} - x_\ell A_{x\ell}) \quad (35)$$

$$X_2 = r_1 C_2 \text{ sign } (i) \quad (36)$$

$$X_3 = y_u \left(\frac{r_4^3 - r_3^3}{12} + \frac{x_u^2 r_4 - x_\ell^2 r_3}{8} \right) + \frac{1}{8} (x_u^3 A_{xu} - x_\ell^3 A_{x\ell}) \quad (37)$$

$$X_4 = r_1 y_u k \Delta x \left(r_1^2 + \frac{(k \Delta x)^2}{4} + \frac{y_u^2}{3} \right) \text{ sign } (i) \quad (38)$$

where $\text{sign } (i)$ denotes the algebraic sign of i . If $i=0$, then $\text{sign } (i)$ is inconsequential because the term that multiplies it is zero. In (31)-(38),

$$r_3 = \sqrt{x_\ell^2 + y_u^2} \quad (39)$$

$$r_4 = \sqrt{x_u^2 + y_u^2} \quad (40)$$

$$A_{xu} = x_u \log \left(\frac{y_u + r_4}{|x_u|} \right) \quad (41)$$

$$A_{x\ell} = x_\ell \log \left(\frac{y_u + r_3}{|x_\ell|} \right) \quad (42)$$

$$A_{yu} = y_u \log \left(\frac{x_u + r_4}{|x_\ell + r_3|} \right) \quad (43)$$

Here, $\log$ denotes the natural logarithm.

Substituting $Y/2$ for Y^b in [2, Eq. (27)], we obtain for the complex power P_t transmitted through the aperture

$$P_t = \frac{1}{2} \tilde{V}[Y\vec{V}]^* \quad (44)$$

where $\tilde{V}$ is the transpose of $\vec{V}$ and $*$ denotes the complex conjugate.

Since the incident magnetic field is given by either (5) or (6), $Y\vec{V}$ is given by either (7) or (15) so that (44) reduces to

$$P_t = -\frac{1}{2} \vec{V}\vec{P}^* \quad (45)$$

where $\vec{P}$ is $(\vec{P}^{inc})_{\theta y}$ if the incident magnetic field is given by (5) and $\vec{P}$ is $(\vec{P}^{inc})_{xx}$ if the incident magnetic field is given by (6).

The transmission coefficient T is the ratio of the real power transmitted through the aperture to the real power P_{inc} incident on the aperture. Since the incident magnetic field is given by either (5) or (6), we obtain

$$P_{inc} = nL_x \Delta x \Delta y \cos \alpha \quad (46)$$

where α is the angle between the propagation vector of the incident wave and $\underline{u}_z$. Here, $\underline{u}_z$ is the unit vector in the z direction. Since the incident magnetic field is given by (5) or (6), inspection of [1, Fig. 1] reveals that

$$\cos \alpha = -\sin \beta \quad (47)$$

where β is either the angle θ^{inc} in (5) or the angle ϕ^{inc} in (6). From (45)-(47), we obtain

$$T = \frac{\text{Real}(\tilde{V}\vec{P}^*)}{2\eta L \Delta x \Delta y \sin \beta} \quad (48)$$

The scattering cross section $\tau(\theta, \phi)$ is the area that the incident power per unit area must be multiplied by in order to obtain the power which, when radiated omnidirectionally into the $z > 0$ half space, would produce the actual power per unit area at (θ, ϕ) . The above definition of τ leads to [1, Eq. (72)]

$$\tau/\lambda^2 = \frac{k^4}{32\pi^3 \eta^2} |\tilde{P}^m \vec{V}|^2 \quad (49)$$

where λ is the wavelength and $\tilde{P}^m$ is the transpose of a measurement vector $\vec{P}^m$. For the θ polarized pattern in the $y=0$ plane, τ is called $(\tau)_{\theta y}$ and $\tilde{P}^m$ is $(\vec{P}^m)_{\theta y}$ where $(\vec{P}^m)_{\theta y}$ is the column vector whose i th element is given by (8) with the angle of incidence θ^{inc} replaced by the observation or "measurement" angle θ^m .

$$(\tau)_{\theta y}/\lambda^2 = \frac{k^4}{32\pi^3 \eta^2} |(\tilde{P}^m)_{\theta y} \vec{V}|^2 \quad (50)$$

For the x polarized pattern in the $x=0$ plane, τ is called $(\tau)_{xx}$ and $\tilde{P}^m$ is $(\vec{P}^m)_{xx}$ where $(\vec{P}^m)_{xx}$ is the column vector whose i th element is given by (16) with the angle of incidence ϕ^{inc} replaced by the measurement angle ϕ^m .

$$(\tau)_{xx}/\lambda^2 = \frac{k^4}{32\pi^3 \eta^2} |(\tilde{P}^m)_{xx} \vec{V}|^2 \quad (51)$$

III. THE MAIN PROGRAM

The main program uses the subroutines YMAT, PLANE, DECOMP, and SOLVE to calculate the coefficients $\{V_j\}$ appearing in expression (1) for the magnetic current, the transmission coefficient (48), and the scattering cross sections per square wavelength (50) and (51). The main program is described and listed in this section. Sample input and output data are provided so that the user can verify that the program is running properly.

In the listing of the main program, line 4 defines the input data file and line 5 defines the output data file. The input data are read according to lines 10 and 11 which are

```

      READ(20,11) LX, LI, NTH, DX, DY, TH
11  FORMAT(3I3, 3E14.7)

```

Here, LX is L_x , DX is $\Delta x/\lambda$, and DY is $\Delta y/\lambda$ where λ is the wavelength, and, as in Section II, $L_x \Delta x$ is the length of the aperture in the x direction and Δy is the width of the aperture in the y direction. LX is a positive integer greater than or equal to 2. LI is either 1 or 2. If LI is 1, then the incident magnetic field is given by (5) and TH is θ^{inc} of (5). If LI is 2, then the incident magnetic field is given by (6) and TH is ϕ^{inc} of (6). The input variable TH is in degrees. The normalized cross section (50) is calculated at

$$\theta^m = (J-1)\pi/(NTH-1), \quad J = 1, 2, \dots, NTH \quad (52)$$

and is written on the output data file under the heading TAU1. The normalized cross section (51) is calculated at

$$\phi^m = (J-1)\pi/(NTH-1), \quad J = 1, 2, \dots, NTH \quad (53)$$

and is written on the output data file under the heading TAU2. The right-hand sides of both (52) and (53) are in radians.

Minimum allocations are given by

COMPLEX Y(N*N), P(2*N), B(N), V(N)

DIMENSION IPS(N)

where

$$N = LX - 1 \quad (54)$$

Line 20 stores by columns in Y the elements of the matrix $\frac{\pi\eta}{j\Delta x\Delta y} Y$ where Y appears in (7). Line 24 stores in P(1) to P(N) the elements of $\frac{1}{2\Delta x\Delta y} (\vec{P}^{inc})_{\theta y}$ where $(\vec{P}^{inc})_{\theta y}$ appears in (7). Here, N is given by (54). Line 24 also stores in P(N+1) to P(2*N) the elements of $\frac{1}{2\Delta x\Delta y} (\vec{P}^{inc})_{xx}$ where $(\vec{P}^{inc})_{xx}$ appears in (15). Equation (7) is recast as

$$[\frac{\pi\eta}{j\Delta x\Delta y} Y] \vec{V} = j2\pi\eta [\frac{1}{2\Delta x\Delta y} (\vec{P}^{inc})_{\theta y}] \quad (55)$$

Equation (15) is recast as

$$[\frac{\pi\eta}{j\Delta x\Delta y} Y] \vec{V} = j2\pi\eta [\frac{1}{2\pi\Delta x\Delta y} (\vec{P}^{inc})_{xx}] \quad (56)$$

The square matrix $[\frac{\pi\eta}{j\Delta x\Delta y} Y]$ is common to the left-hand sides of both (55) and (56). This is the matrix that resides in the computer program variable Y. The bracketed quantity on the right-hand side of (55) is the column vector that resides in the computer program variables P(1) to P(N) where N is given by (54). The bracketed quantity on the right-hand side of (56) is the column vector that resides in the computer program variables P(N+1) to P(2*N). If LI is 1, DO loop 16 performs

the multiplication by the factor $UV = 2j\pi\eta$ in (55) in order to store the elements of the right-hand side of (55) in $B(1)$ to $B(N)$. If LI is 2, DO loop 16 stores the elements of the right-hand side of (56) in $B(1)$ to $B(N)$. If LI is 1, lines 33 and 34 put in $V(1)$ to $V(N)$ the elements of the column vector $\vec{V}$ that satisfies (55). If LI is 2, lines 33 and 34 put in $V(1)$ to $V(N)$ the elements of the column vector $\vec{V}$ that satisfies (56). Since the elements of $\vec{V}$ are the $\{V_j\}$ of (1), it is evident that the coefficients $\{V_j\}$ in the expansion (1) for the magnetic current $\underline{M}$ will reside in $V(1)$ to $V(N)$.

Equation (48) is recast as

$$T = \frac{\text{Real} \left(\tilde{V} \frac{1}{2\Delta x \Delta y} \vec{P}^* \right)}{\eta L_x \sin \beta} \quad (57)$$

In (57), β is TH and $\frac{1}{2\Delta x \Delta y} \vec{P}$ is the column vector that stored in either $P(1)$ to $P(N)$ or $P(N+1)$ to $P(2*N)$ according as LI is either 1 or 2. With regard to (57), DO loop 17 accumulates $\tilde{V} \frac{1}{2\Delta x \Delta y} \vec{P}^*$ in $U1$. Line 44 puts T of (57) in the computer program variable T .

Equation (50) is recast as

$$(\tau)_{\theta y} / \lambda^2 = \frac{(k^2 \Delta x \Delta y)^2}{8\pi^3 \eta^2} \left| \frac{1}{2\Delta x \Delta y} (\tilde{P}^m)_{\theta y} \vec{V} \right|^2 \quad (58)$$

Equation (51) is recast as

$$(\tau)_{xx} / \lambda^2 = \frac{(k^2 \Delta x \Delta y)^2}{8\pi^3 \eta^2} \left| \frac{1}{2\Delta x \Delta y} (\tilde{P}^m)_{xx} \vec{V} \right|^2 \quad (59)$$

The index J of DO loop 19 obtains

$$\theta^m = (J-1)\pi / (NTH-1) \quad (60)$$

in (58) and

$$\phi^m = (J-1)\pi / (NTH-1) \quad (61)$$

in (59). The value of the right-hand side of (61) is the same as that of (60). Inside DO loop 19, line 53 puts this common value in TH. With regard to (58) and (59), line 54 puts the elements of $\frac{1}{2\Delta x \Delta y} (\tilde{P}^m)_{\theta y}$ in P(1) to P(N) and the elements of $\frac{1}{2\Delta x \Delta y} (\tilde{P}^m)_{xx}$ in P(N+1) to P(2*N). DO loop 21 lies inside DO loop 20 whose index is K. If K = 1, DO loop 21 accumulates $\frac{1}{2\Delta x \Delta y} (\tilde{P}^m)_{\theta y} \vec{V}$ in U1. If K = 2, DO loop 21 accumulates $\frac{1}{2\Delta x \Delta y} (\tilde{P}^m)_{xx} \vec{V}$ in U1. When K = 1, line 64 puts $(\tau)_{\theta y} / \lambda^2$ of (58) with θ^m given by (60) in TAU(1). When K = 2, line 64 puts $(\tau)_{xx} / \lambda^2$ of (59) with ϕ^m given by (61) in TAU(2).

```

001 C      LISTING OF THE MAIN PROGRAM
002      COMPLEX U,UV,Y(1600),P(100),B(40),V(40),U1,CONJG
003      DIMENSION IPS(40),TAU(2)
004      OPEN(UNIT=20,FILE='HAUTZ3.DAT')
005      OPEN(UNIT=21,FILE='HAUTZ4.DAT')
006      PI=3.141593
007      ETA=376.730
008      U=(0.,1.)
009      UV=2.*PI*ETA*U
010      READ(20,11) LX,LI,NTH,DX,DY,TH
011 11      FORMAT(3I3,3E14.7)
012      WRITE(21,12) LX,LI,NTH,DX,DY,TH
013 12      FORMAT(' LX LI NTH',5X,'DX',12X,'DY',12X,'TH'/1X,
014          1 3I3,3E14.7)
015      EK=2.*PI
016      DX=DX*EK
017      DY=DY*EK
018      P8=180./PI
019      TH=TH/P8
020      CALL YHAT(LX,DX,DY,Y)
021      WRITE(21,13) (Y(I),I=1,3)
022 13      FORMAT(' Y'/(1X,6E11.4))
023      N=LX-1
024      CALL PLANE(TH,LX,DX,DY,P)
025      WRITE(21,14) (P(I),I=1,3)
026 14      FORMAT(' P'/(1X,6E11.4))
027      IA=(LI-1)*N
028      IB=IA
029      DO 16 J=1,N
030      IB=IB+1
031      P(J)=UV*P(IB)
032 16      CONTINUE
033      CALL DECOMP(N,IPS,Y)
034      CALL SOLVE(N,IPS,Y,B,V)
035      WRITE(21,24) (V(I),I=1,N)
036 24      FORMAT(' COEFFICIENTS V OF MAGNETIC CURRENT ',
037          1  'EXPANSION FUNCTIONS'/(1X,6E11.4))
038      U1=0.
039      IB=IA
040      DO 17 J=1,N
041      IB=IB+1
042      U1=U1+V(J)*CONJG(P(IB))
043 17      CONTINUE
044      T=REAL(U1)/(LX*ETA*SIN(TH))
045      WRITE(21,18) T
046 18      FORMAT(' TRANSMISSION COEFFICIENT T=',E14.7)
047      CT=DX*DY/(PI*ETA)
048      CT=CT*CT/(8.*PI)
049      LTH=PI/(NTH-1)
050      WRITE(21,23)

```

```
051 23      FORMAT(' ANGLE',4X,'TAU1',7X,'TAU2')
052          DO 19 J=1,NTH
053          TH=(J-1)*DTH
054          CALL PLANE(TH,LX,DX,DY,P)
055          TH=TH*P8
056          J1=0
057          DO 20 K=1,2
058          U1=0.
059          DO 21 I=1,N
060          J1=J1+1
061          U1=U1+P(J1)*V(I)
062 21      CONTINUE
063          H=U1*CONJG(U1)
064          TAU(K)=CT*H
065 20      CONTINUE
066          WRITE(21,22) TH,(TAU(I),I=1,2)
067 22      FORMAT(1X,F7.2,2E11.4)
068 19      CONTINUE
069          STOP
070          END
```

INPUT DATA IN THE FILE NAUTZ3.DAT

```
5  1 19 0.5000000E-01 0.5000000E-01 0.2700000E+03
```

OUTPUT DATA IN THE FILE MAUTZ4.DAT

LX LI NTH DX DY TH
 5 1 19 0.5000000E-01 0.5000000E-01 0.2700000E+03
 Y
 -0.1531E+02-0.6526E-01 0.6646E+01-0.6463E-01 0.1312E+01-0.6274E-01
 P
 -0.1000E+01-0.1471E-06-0.1000E+01-0.2941E-06-0.1000E+01-0.4412E-06
 COEFFICIENTS V OF MAGNETIC CURRENT EXPANSION FUNCTIONS
 0.4511E+02 0.5916E+03 0.6238E+02 0.8153E+03 0.6238E+02 0.8153E+03
 0.4511E+02 0.5916E+03
 TRANSMISSION COEFFICIENT T= 0.1141263E+00
 ANGLE TAU1 TAU2
 0.00 0.0000E+00 0.2186E-02
 10.00 0.5885E-04 0.2186E-02
 20.00 0.2308E-03 0.2188E-02
 30.00 0.5016E-03 0.2190E-02
 40.00 0.8462E-03 0.2193E-02
 50.00 0.1228E-02 0.2196E-02
 60.00 0.1602E-02 0.2199E-02
 70.00 0.1918E-02 0.2202E-02
 80.00 0.2129E-02 0.2203E-02
 90.00 0.2204E-02 0.2204E-02
 100.00 0.2129E-02 0.2203E-02
 110.00 0.1918E-02 0.2202E-02
 120.00 0.1602E-02 0.2199E-02
 130.00 0.1228E-02 0.2196E-02
 140.00 0.8462E-03 0.2193E-02
 150.00 0.5016E-03 0.2190E-02
 160.00 0.2308E-03 0.2188E-02
 170.00 0.5885E-04 0.2186E-02
 180.00 0.2088E-15 0.2186E-02

IV. THE SUBROUTINE YMAT

The subroutine YMAT(LX, DX, DY, Y) stores by columns in Y the matrix $\frac{\pi\eta}{j\Delta x\Delta y}$ Y that appears in (55) and (56). The first three arguments of YMAT are input arguments. The aperture is LX subsections long in the x direction and one subsection wide in the y-direction. DX is $k\Delta x$ and DY is $k\Delta y$ where k is the wave number, Δx is the subsection length in the x direction, and Δy is the subsection width in the y direction.

Minimum allocations are given by

COMPLEX TC(LX+1), TX(LX+1), YXX(N), Y(N*N)

where N is given by (54).

Inside DO loop 16, $I_c(I-1)$ of (23) is put in the computer program variable TC(I+1). Here, I is the index of DO loop 16. Also inside DO loop 16, $I_x(I-1)$ of (29) is put in the computer program variable TX(I+1). The logic inside DO loop 16 is best understood by building up a table of variables in YMAT versus expressions in terms of variables in Section II.

Variables in YMAT	Expressions in Section II
I	i+1 where i appears in (23) and (29)
YU	y_u of (12)
XU	x_u of (13)
XL	x_l of (14)
R1	r_1 of (22)
U1	U_1 of (24)
U2	$U_2 C_2$ of (25) and (32)
U3	U_3 of (26)
U4	$\frac{1}{6}$ of (27)

EX	$2e^{-jr_1}$ of (23) and (29)
R3	r_3 of (39)
R4	r_4 of (40)
AXU	A_{xu} of (41)
AXL	A_{xl} of (42)
AYU	A_{yu} of (43)
C1	C_1 of (31)
C3	C_3 of (33)
C4	C_4 of (34)
TC(I+1)	$I_c(i)$ of (23) with $i = I-1$
X1	X_1 of (35)
X3	X_3 of (37)
X4	X_4 of (38)
TX(I+1)	$I_x(i)$ of (29) with $i = I-1$

Taking advantage of the fact that $I_c(i)$ is even in i , line 46 stores $I_c(-1)$ in TC(1). Taking advantage of the fact that $I_x(i)$ is odd in i , line 47 stores $I_x(-1)$ in TX(1).

Inside DO loop 20, lines 51 and 52 put in YXX(J-1) the square bracketed term on the right-hand side of (9) with $j-1 = J-2$. Now, we have

$$\frac{\pi\eta}{j\Delta x\Delta y} Y_{1j} = YXX(j-1+1), \quad j-1 = 0, 1, 2, \dots, L_x-2 \quad (62)$$

Because Y_{1j} is even in $(j-1)$, (62) becomes

$$\frac{\pi\eta}{j\Delta x\Delta y} Y_{1j} = YXX(|j-1|+1), \quad |j-1| = 0, 1, 2, \dots, L_x-2 \quad (63)$$

Inside nested do loops 23 and 21, line 59 puts $\frac{\pi\eta}{j\Delta x\Delta y} Y_{1j}$ of (63) in the computer program variable $Y(I + (J-1)*N)$ where N is given by (54).

```

001 C      LISTING OF THE SUBROUTINE YHAT
002      SUBROUTINE YHAT(LX,DX,DY,Y)
003      COMPLEX U,U1,U2,U3,U4,EX,TC(100),TX(100)
004      COMPLEX YXX(100),Y(1600)
005      DX2=DX*DX
006      N=LX-1
007      U=(0.,1.)
008      U4=.1666667*U
009      YU=.5*DY
010      YUD=YU*DX
011      YU2=YU*YU
012      YU3=.3333333*YU
013      YU4=.25*DX2+YU3*YU
014      DO 16 I=1,LX
015      IP=I+1
016      XU=(I-.5)*DX
017      XU2=XU*XU
018      XL=XU-DX
019      XL2=XL*XL
020      R1=(I-1)*DX
021      R2=R1*R1
022      RU1=1.-.5*R2
023      U1=RU1+R1*(1.-.1666667*R2)*U
024      U2=(R1-RU1*U)*YUD
025      U3=-.5-.5*R1*U
026      EX=2.*(COS(R1)-U*SIN(R1))
027      R7=XL2+YU2
028      R8=XU2+YU2
029      R3=SQRT(R7)
030      R4=SQRT(R8)
031      AXU=XU*ALOG((YU+R4)/XU)
032      AXL=XL*ALOG((YU+R3)/ABS(XL))
033      AYU=YU*ALOG((XU+R4)/(XL+R3))
034      C1=AXU-AXL+AYU
035      C3=YU3*(XU*R4-XL*R3)+.1666667*(XU2*AXU-XL2*AXL+YU2*AYU)
036      C4=YU3*(XU*R8-XL*R7)
037      TC(IP)=(C1*U1+U2+C3*U3+C4*U4)*EX
038      AXU=XU*AXU
039      AXL=XL*AXL
040      X1=.5*(YU*(R4-R3)+AXU-AXL)
041      X3=YU*(-.8333333E-1*(R8*R4-R7*R3)+.125*(XU2*R4-XL2*R3))
042      1 +.125*(XU2*AXU-XL2*AXL)
043      X4=R1*YUD*(R2+YU4)
044      TX(IP)=(X1*U1+R1*U2+X3*U3+X4*U4)*EX/CX
045 16      CONTINUE
046      TC(1)=TC(3)
047      TX(1)=-TX(3)
048      DO 20 J=2,LX
049      JM=J-1
050      JP=J+1
051      YXX(JM)=.5*(TC(J)-TX(JP)+(J-.5)*TC(JP)+TX(JM)-
052      1 (J-3.5)*TC(JM))+(TC(JP)-2.*TC(J)+TC(JM))/DX2
053 20      CONTINUE

```

```

054      JY=0
055      DO 23 J=1,N
056      DO 21 I=1,N
057      JY=JY+1
058      K=IABS(J-I)+1
059      Y(JY)=YXX(K)
060 21      CONTINUE
061 23      CONTINUE
062      RETURN
063      END

```

V. THE SUBROUTINE PLANE

The subroutine PLANE(TH, LX, DX, DY, P) stores in P(1) to P(N) the quantities

$$\frac{1}{2\Delta x \Delta y} (P_i^{inc})_{\theta_y} = \sin \theta^{inc} \left(\frac{\sin \left(\frac{k\Delta x \cos \theta^{inc}}{2} \right)}{\frac{k\Delta x \cos \theta^{inc}}{2}} \right)^2 e^{jki\Delta x \cos \theta^{inc}}, \quad i=1,2,\dots,N \quad (64)$$

where (64) comes from (8) and N is $(L_x - 1)$. The aperture is L_x subsections long in the x direction and one subsection wide in the y direction. In (64), k is the wave number, Δx is the subsection length in the x direction, and Δy is the width of the aperture in the y direction. Moreover, the subroutine PLANE stores in P(N+1) to P(2N) the quantities

$$\frac{1}{2\Delta x \Delta y} (P_i^{inc})_{xx} = - \left(\frac{\sin \frac{k\Delta y \cos \phi^{inc}}{2}}{\frac{k\Delta y \cos \phi^{inc}}{2}} \right) e^{j(k\Delta y/2) \cos \phi^{inc}}, \quad i=1,2,\dots,N \quad (65)$$

where (65) comes from (16). The angle ϕ^{inc} in (65) is assumed to be the same as θ^{inc} in (64). The first four arguments of PLANE are input arguments. In radians, TH is the common value of θ^{inc} and ϕ^{inc} in (64) and (65). Furthermore, LX is L_x , DX is $k\Delta x$, and DY is $k\Delta y$. LX is a positive integer greater than or equal to 2.

The minimum allocation for P is given by

COMPLEX P(2*N)

where N is given by (54).

Lines 5 and 6 set

CX = $k\Delta x \cos (TH)$

CY = $\frac{1}{2} k\Delta y \cos (TH)$

If $\cos (TH) \neq 0$, then lines 11 to 14 set

$$SX = \sin (TH) \left(\frac{\sin \left(\frac{k\Delta x \cos (TH)}{2} \right)}{\frac{k\Delta x \cos (TH)}{2}} \right)^2$$

$$SY = - \frac{\sin \left(\frac{k\Delta y \cos (TH)}{2} \right)}{\frac{k\Delta y \cos (TH)}{2}}$$

If $\cos (TH) = 0$, then lines 8 and 9 set SX and SY equal to the limits of the above two expressions as $\cos (TH)$ goes to zero, namely

$$SX = \sin (TH)$$

$$SY = - 1$$

Inside DO loop 13, line 19 puts in P(I) the right-hand side of (64) with 1 replaced by I where I is the index of DO loop 13. Note that the right-hand side of (65) does not depend on i. Line 21 puts in U1 the right-hand side of (65). DO loop 14 puts the right-hand side of (65) in P(N+1) to P(2*N) where N is given by (54).

```
001C      LISTING OF THE SUBROUTINE PLANE
002      SUBROUTINE PLANE (TH,LX,DX,DY,P)
003      COMPLEX U,U1,P(100)
004      CS=CCS (TH)
005      CX=LX*CS
006      CY=.5*DY*CS
007      IF (CS) 11,10,11
00810      SX=SIN (TH)
009      SY=-1.
010      GO TO 12
01111      SX=.5*CX
012      SX=SIN (SX) /SX
013      SX=SIN (TH) *SX*SX
014      SY=-SIN (CY) /CY
01512      U=(0.,1.)
016      N=LX-1
017      DO 13 I=1,N
018      S=I*CX
019      P(I)=SX*(COS (S) +U*SIN (S) )
02013      CONTINUE
021      U1=SY*(COS (CY) +U*SIN (CY) )
022      DO 14 J=1,N
023      I=J+N
024      P(I)=U1
02514      CONTINUE
026      RETURN
027      END
```

VI. THE SUBROUTINES DECOMP AND SOLVE

The subroutines DECOMP(N, IPS, UL) and SOLVE(N, IPS, UL, B, X) solve a system of N linear equations in N unknowns. The input to DECOMP consists of N and the N by N matrix of coefficients on the left-hand side of the matrix equation stored by columns in UL. The output from DECOMP is IPS and UL. This output is fed into SOLVE. The rest of the input to SOLVE consists of N and the column of coefficients on the right-hand side of the matrix equation stored in B. SOLVE puts the solution to the matrix equation in X.

Minimum allocations are given by

COMPLEX UL(N*N)

DIMENSION SCL(N), IPS(N)

in DECOMP and by

COMPLEX UL(N*N), B(N), X(N)

DIMENSION IPS(N)

in SOLVE.

More detail concerning DECOMP and SOLVE is on pages 46-49 of [3].

```

001 C      LISTING OF THE SUBROUTINE DECOMP
002      SUBROUTINE DECOMP(N,IPS,UL)
003      COMPLEX UL(1600),PIVOT,EM
004      DIMENSION SCL(40),IPS(40)
005      DO 5 I=1,N
006      IPS(I)=I
007      RN=0.
008      J1=I
009      DO 2 J=1,N
010      ULM=ABS(REAL(UL(J1)))+ABS(AIMAG(UL(J1)))
011      J1=J1+N
012      IF(RN-ULM) 1,2,2
013 1      RN=ULM
014 2      CONTINUE
015      SCL(I)=1./RN
016 5      CONTINUE
017      NM1=N-1
018      K2=0
019      DO 17 K=1,NM1
020      EIG=0.
021      DO 11 I=K,N
022      IP=IPS(I)
023      IPK=IP+K2
024      SIZE=(ABS(REAL(UL(IPK)))+ABS(AIMAG(UL(IPK))))*SCL(IP)
025      IF(SIZE-BIG) 11,11,10
026 10      EIG=SIZE
027      IPV=I
028 11      CONTINUE
029      IF(IPV-K) 14,15,14
030 14      J=IPS(K)
031      IPS(K)=IPS(IPV)
032      IPS(IPV)=J
033 15      KPP=IPS(K)+K2
034      PIVOT=UL(KPP)
035      KP1=K+1
036      DO 16 I=KP1,N
037      KP=KPP
038      IP=IPS(I)+K2
039      EM=-UL(IP)/PIVOT
040 18      UL(IP)=-EM
041      DO 16 J=KP1,N
042      IP=IP+N
043      KP=KP+N
044      UL(IP)=UL(IP)+EM*UL(KP)
045 16      CONTINUE
046      K2=K2+N
047 17      CONTINUE
048      RETURN
049      END

```

```

050 C      LISTING OF THE SUBROUTINE SOLVE
051      SUBROUTINE SOLVE(N,IPS,UL,B,X)
052      COMPLEX UL(1600),B(40),X(40),SUM
053      DIMENSION IPS(40)
054      NP1=N+1
055      IP=IPS(1)
056      X(1)=B(IP)
057      DO 2 I=2,N
058      IP=IPS(I)
059      IPB=IP
060      IM1=I-1
061      SUM=0.
062      DO 1 J=1,IM1
063      SUM=SUM+UL(IP)*X(J)
064 1      IP=IP+N
065 2      X(I)=B(IPB)-SUM
066      K2=N*(N-1)
067      IP=IPS(N)+K2
068      X(N)=X(N)/UL(IP)
069      DO 4 IBACK=2,N
070      I=NP1-IBACK
071      K2=K2-N
072      IPI=IPS(I)+K2
073      IP1=I+1
074      SUM=0.
075      IP=IPI
076      DO 3 J=IP1,N
077      IP=IP+N
078 3      SUM=SUM+UL(IP)*X(J)
079 4      X(I)=(X(I)-SUM)/UL(IPI)
080      RETURN
081      END

```

REFERENCES

- [1] J. R. Mautz and R. F. Harrington, "Electromagnetic Transmission through a Rectangular Aperture in a Perfectly Conducting Plane," Report TR-76-1, Department of Electrical and Computer Engineering, Syracuse University, Syracuse, NY 13210, February 1976.
- [2] R. F. Harrington and J. R. Mautz, "A Generalized Network Formulation for Aperture Problems," Report TR-75-13, Department of Electrical and Computer Engineering, Syracuse University, Syracuse, NY 13210, November 1975.
- [3] J. R. Mautz and R. F. Harrington, "Transmission from a Rectangular Waveguide into Half Space through a Rectangular Aperture," Report TR-76-5, Department of Electrical and Computer Engineering, Syracuse University, Syracuse, NY 13210, May 1976.

END

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