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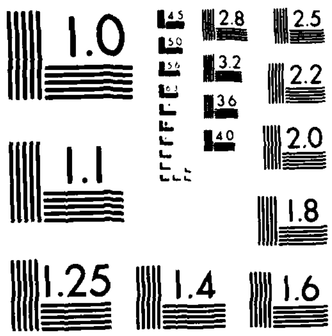

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THEORETICAL AND EXPERIMENTAL STUDIES OF THE MECHANICS OF VISCOELASTIC LIQUIDS

FINAL REPORT

BY

D. D. Joseph and G. S. Beavers

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

Theoretical and experimental studies of a wide range of problems in the rheometry of viscoelastic fluids are described. A device to measure the shear relaxation function has been invented (the wave speed meter). Investigations of the conditions under which the equations for the flow of (over)

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viscoelastic liquids can change type have been carried out. The results from experiments and analyses on a variety of problems associated with bicomponent flows of liquids with different viscosities are presented. These included liquid rollers, the flow of two fluids in a pipe, and Taylor instability with two immiscible liquids. Analysis and experiments on the motion of a viscoelastic liquid between eccentric rotating cylinders are described. Some analytical results for the extrudate swell problem are presented. The climbing constants for a wide range of viscoelastic liquids have been measured. Other projects include stability analyses for several flow geometries; experiments on viscoelastic liquid breakup; and experiments on the stability of a rotating and coning cylinder.

TABLE OF CONTENTS

1. INTRODUCTION

This report summarizes the major accomplishments of the research program supported under U.S. Army Research Office Grant Number DAAG 29-82-K-0051. The research has covered a broad range of problems associated with the rheological characterization and flow of non-Newtonian fluids from both analytical and experimental points of view.

Our research findings are given in Section 3 where we make reference to abstracts of manuscripts and journal articles which report this research. The abstracts are reproduced in Section 4 of this report. In Section 2 we describe a new device that we have invented and are developing for the measurement of the shear relaxation function of a viscoelastic fluid. This device is called a 'wave speed meter'. We think that the wave speed meter is a significant research accomplishment.

2. THE WAVE SPEED METER

The wave speed meter is a device for determining the propagation speed of a shear-induced wave in a viscoelastic fluid. The brief overview of the invention given below is extracted from the patent application.

A. Background of the Invention

Viscoelastic fluids such as polymer solutions and melts of polymers (molten plastic), can support the propagation of various kinds of waves of shear; in such fluids an event which takes place in one place in the fluid will take a certain time to reach another place. These wave speeds are a fundamental property of elastic fluids, but no instruments exist for directly measuring the wave speeds.

In an article entitled "Linearized Dynamics for Step Jumps of Velocity in Displacement of Shearing Flows of a Simple Fluid," Rheologica Acta, Vol. 21, pp. 228-250, 1982 (Ref. 1) A. Narian and D. D. Joseph, showed that the propagation speed of small amplitude waves of slip lines of velocity and displacement is given by

$$C = \sqrt{G(0)/\rho}$$

where ρ is the density of the fluid and G(0) is the instantaneous value of the shear relaxation modulus of the liquid that is being tested. There are some methods currently available to measure the relaxation function G(s) of some liquids, but these methods are not accurate for small values of the lapsed time s. By measuring the wave speed "C", the instantaneous value G(0) of the relaxation function G(s) can be determined.

B. Summary of the Invention

The present invention relates to an apparatus and method for determining the propagation speed of small amplitude waves in viscoelastic liquids.

In the apparatus, an amount of the test viscoelastic liquid is placed in a chamber defined by two relatively movable, facing surfaces that are independently supported with a known length gap between the surfaces. The surfaces are preferably left at rest. One of the surfaces is then displaced with a sudden velocity, and because the liquid carries shear, a shear wave is formed at the boundary layer of the liquid at the one surface by this sudden displacement of that surface.

The wave travels across the gap and when it reaches the independently supported second surface, the second surface is shifted or moved because of shear forces acting on the boundary layer of the second surface. The time when the movement of the second surface occurs is measured relative to the initial displacement of the first surface. The wave transit time thus can easily be determined. The wave speed can be calculated from the transit time and the distance travelled (the distance between the two surfaces).

In the present invention, concentric cylinders are utilized with facing spaced surfaces defining a chamber holding an annular band of viscoelastic liquid. A sudden acceleration is imparted to one of the cylinders for rotation about its central axis, and when the wave caused by shear at the first cylinder surface reaches the second cylinder, the second cylinder is caused to rotate. The time at which the rotation of the second cylinder occurs can be measured so that the elapsed time can be accurately determined.

The cylindrical form is advantageous in that cylinders provide large surface areas in a relatively small space, thereby insuring that the propagated wave will provide an impulse to the inner cylinder when it reaches the inner cylinder that will cause a reliable movement of the inner cylinder.

3. SUMMARY OF MAJOR RESEARCH RESULTS

3.1 Discontinuities in Viscoelastic Liquids

- Two analytical studies associated with the propagation of discontinuities in viscoelastic liquids have been developed. These are described in Ref. 1 and Ref. 2. Ref. 1 presents the linearized dynamics associated with step jumps in the velocity or displacement of the boundary of a fluid in a shearing motion. It is shown that the discontinuity will propagate into the interior of the liquid with a wave speed equal to $\sqrt{G(0)/\rho}$.

- A study has been conducted on the interpretation of popular experiments in rheometry, namely (i) stress-growth experiments involving a step shear rate, and (ii) stress-relaxation experiments involving a step shear rate. It has been shown by using a correct theory based on dynamics rather than kinematical assumptions that stress jumps and stress overshoot are possible with linear theories based on commonly used constitutive equations. Formulae for the limiting values of these jumps have been derived. This work is summarized in Ref. 3.

- The work of Ref. 3 has been extended to include the derivation of results for the amplitude of jumps and reflections for fluids sheared between concentric cylinders, and also the effects of curvature on the damping of cylindrical shock waves. This work is summarized in Ref. 4.

- An analysis for the propagation of discontinuities into incompressible linear viscoelastic solids subjected to a step shear displacement, similar to that for viscoelastic liquids discussed in Ref. 1, has been completed and is described in Ref. 5.

3.2 Hyperbolicity and Change of Type

- We are interested in the possibility that many mysteries of flow of viscoelastic liquids are associated with the appearance of real characteristics and a change of type of the equations, analogous to the sonic transition in gas dynamics. We have found cases where singular shear surfaces propagate along characteristics. In steady flow the vorticity is the variable which is affected by a change of type and may become discontinuous. This is discussed in Ref. 6 and Ref. 7.

- The investigations of the conditions under which the equations for the flow of viscoelastic liquids can change type have been extended to three flow configurations, namely sink flows (Ref. 8), flow past a flat plate (Ref. 9), and flow through channels (Ref. 10).

3.3 Bicomponent Flows and Encapsulation

- We are studying a variety of problems associated with bicomponent flows of liquids with different viscosities. The objective is to describe the motion and spatial arrangement of each component.

- In a series of experiments we have shown that the arrangements of components appear to be ones which extremalize in some mathematical sense. The extremalizing configurations are such as to minimize the shearing of high viscosity liquids by the spontaneous migration of low viscosity liquids into regions where the shearing is greatest. This can occur by sheet coating in which low viscosity liquid encapsulates high viscosity liquid, or through the formation of rigidly rotating masses of high viscosity liquid, or through the formation of emulsion of low viscosity liquid in a high viscosity foam. This work is described in Ref. 11 and Ref. 13.

- An analytical solution has been derived for the flow of two immiscible fluids of different viscosities and equal density through a circular pipe driven by a pressure gradient. This work is described in Ref. 12.

- We have studied the problem of the stability of liquid rollers in bicomponent flows of immiscible fluids. The most general qualitative result is that the roller is stable if the heavier fluid is outside. We have also carried out a numerical study of the flow of two immiscible liquids situated between concentric cylinders when the outer cylinder is fixed and the inner cylinder rotates. The results of these investigations are described in Ref. 14.

- The stability of plane Poiseuille flow of two immiscible fluids with different viscosities has been investigated using linearized stability analysis. It is found that plane Poiseuille flow can be unstable no matter how small the Reynolds number is. It is also found that there are two modes of disturbance dominated, respectively, by the two interfaces. The presence of another fluid may or may not be stabilizing, depending upon the values of depth and viscosity ratios. There are regions of stability when there are three layers with one of the liquids centrally located. This work is described in Ref. 15.

- The problem of Taylor instability with two immiscible liquids is being studied and we are attempting to develop formulae which govern the exchange of stabilities. An experiment to study the flow between horizontal concentric cylinders filled with two immiscible liquids, including torque measurements, is being developed. This project is still in progress.

- Experiments on the development and stability of rollers are continuing, including measurements of the relationship between torque and rotational speed during the development of the roller and its separation from the side-walls of the container.

- An experimental system to study the flow of two liquids down a vertical pipe has been constructed. The liquids enter the tube concentrically and are driven by independently adjustable head tanks. The flow rate of each liquid can be measured. The pipe is enclosed in a container of liquid which eliminates optical distortion through matching of refractive indices. This work is still in progress.

3.4 Fluid Motion Between Rotating Cylinders

- Analysis and experiments on the motion of a viscoelastic liquid between eccentric rotating cylinders are described in Ref. 17 and Ref. 18.

- The interfacial shape between two immiscible viscoelastic liquids in a vertical cylinder which oscillates about its axis has been measured experimentally and agrees well with theoretical predictions. This work is described in Ref. 19.

We consider the motion of two rings of liquids with different viscosities and densities lying between concentric cylinders that rotate with the same angular velocity Ω. Gravity is neglected and interfacial tension is included. We show that rigid motions are globally stable and that the shape of the interface which separates the two fluids is determined by a minimizing problem for a potential P defined as the negative of the sum of the kinetic energies of two rigid motions plus the surface energy of the interface. We show that the stable interface between fluids has a constant radius when heavy fluid is outside and J is larger than one, where $J = -d^3\{\{\rho\}\}\Omega^2/T$ where d is the mean radius, $\{\{\rho\}\} < 0$ the density difference and T the surface tension. When J is negative the heavy fluid is inside and the interface must be corrugated. The potential of flows with heavy fluid outside is smaller, thus relatively more stable, than when light fluid is outside, whenever J is large or for any J when the volume ratio m of heavy to light fluid is greater than one. These results give partial explanation of the stability and shape of rollers of viscous oils rotating in water and the corrugation of the free surface of films coating rotating cylinders.

Ref. 15

F. Rosso, P. T. Than, and D. D. Joseph, "Instability of Poiseuille flow of two immiscible liquids with different viscosities in a channel," to appear in Journal of Fluid Mechanics, 1985.

We study the stability of plane Poiseuille flow of two immiscible liquids of different viscosities and equal densities. The problem is like one considered by C. S. Yih who found that flow in two layers of equal thickness was always unstable. We find regions of stability when there are three layers with one of the fluids centrally located. We view our contribution as a study of selection of stable steady flow from a nonunique continuum of Poiseuille flows all of which satisfy the steady Navier Stokes and which differ from another in the number and thickness of layers of different viscosity. Experiments have shown that there is a tendency for the less viscous fluid to encapsulate the more viscous one. This arrangement of components, with the more viscous fluid in the center of the channel maximizes the mass flux for a fixed pressure gradient. A linear stability analysis of centrally located configuration to long waves is carried out by the analytic methods introduced by Yih (1). The stability results depend on the viscosity and volume prescribed interface shapes. The question therefore arises, which of these solutions are stable and thus observable. Experiments have shown a tendency for the thinner fluid to encapsulate the thicker one. This has been "explained" by the viscous dissipation principle, which postulates that the amount of viscous dissipation is minimized for a given flow rate. For a circular pipe, this predicts a concentric configuration with the more viscous fluid located at the core. A linear stability analysis, which is carried out numerically, shows that this configuration is stable when there is more of the thin fluid. Therefore the dissipation principle does not always hold, and the volume ratio is a crucial factor.

Ref. 13

D. D. Joseph, K. Nguyen and G. S. Beavers, "Flow of two fluids," Proceedings of the 1983 Scientific Conference on Chemical Defense Research, Aberdeen Proving Ground, 1984.

The arrangements of components which are realized in the flow of two liquids with very different viscosities seems to be such as to relieve high stresses which would otherwise develop in the more viscous component. This frequently requires fracture and tearing of the more viscous component. Sometimes this process of stress relief leads to sheet lubrication of moving solids with the less viscous component, or to the formation of rotating rigid bodies of viscous fluids which we call rollers, or to lubrication by low viscosity dynamically maintained emulsions.

Ref. 14

D. D. Joseph, Y. Renardy, M. Renardy, and K. T. Nguyen, "Stability of rigid motions and rollers in bicomponent flows of immiscible liquids," to appear in Journal of Fluid Mechanics, 1985.

The arrangement of components in steady flow of immiscible liquids is typically nonunique. The problem of selection of arrangements is defined here and is studied by variational methods under the hypothesis that the realized arrangements are the ones which maximize the speed on exterior boundaries for prescribed boundary tractions, or the ones which minimize the tractions for prescribed speeds. The arrangements which minimize tractions also minimize the dissipation by putting low viscosity liquid in regions of high shear. The variational problem is used as a guide to intuition in the design and interpretation of experiments when results of analysis of stability are unavailable. In fact we always observe some kind of shielding of high viscosity liquid. This can occur by sheet coating in which low viscosity liquid encapsulates high viscosity liquid, or through the formation of rigidly rotating masses of high viscosity liquid which we call rollers. In other cases we get emulsions of low viscosity liquid in a high viscosity foam. The emulsions arise from a fingering instability. The low viscosity liquid fingers into the high viscosity liquid and then low viscosity bubbles are pinched off the fingers. The emulsions seem to have a very low effective viscosity and they shield the high viscosity liquid from shearing. In the problem of Taylor instability with two fluids low viscosity Taylor cells are separated by stable high viscosity rollers.

Ref. 12

D. D. Joseph, M. Renardy and Y. Renardy, "Instability of the flow if immiscible liquids with different viscosities in a pipe," Journal of Fluid Mechanics, Vol. 141 , pp. 309-317, 1984.

We study the flow of two immiscible fluids of different viscosities and equal density through a pipe under a pressure gradient. This problem has a continuum of solutions corresponding to arbitrarily

Ref. 10

J. Y. Yoo and D. D. Joseph, "Hyperbolicity and change of type in the flow of viscoelastic fluids through channels," submitted to Journal of Non-Newtonian Fluid Mechanics, 1984.

> We consider steady flow of an upper convected Maxwell fluid through a channel with wavy walls. The vorticity of this flow will change type when the velocity in the center of the pipe is larger than a critical value defined by the propagation of shear waves. There is then a region around the pipe axis in which the vorticity equation is hyperbolic and a low speed region near the walls where the vorticity equation is elliptic. We linearize the problem for small amplitude waviness and the linearized problem is solved in detail. The characteristic nets depend on the viscoelastic "Mach" number which is the ratio ($M = U/c$) of the unperturbed maximum velocity U to the speed of shear waves c into the fluid at rest and the elasticity number E. There is a supercritical (hyperbolic) region around the center of the channel when $M > 1$. When $M \gg 1$, the diameter of this hyperbolic region is small when E is large, and large when E is small. Regions of positive and negative vorticity are swept out along forward facing characteristics in the hyperbolic region. There is rapid damping of vorticity in the hyperbolic region away from the boundary when $M \gg 1$ and the Weissenberg number $W = M\sqrt{E} \leq O(1)$. (The Weissenberg number is proportional to the relaxation time of the fluid.)
>
> The rate of damping of vorticity decreases as W is increased. Flows with high M appear to be more "elastic" when W is large in the sense that the damping is suppressed as the relaxation time of the fluid is increased.

4.3 Bicomponent Flows and Encapsulation

Ref. 11

D. D. Joseph, K. Nguyen and G. S. Beavers, "Non-uniqueness and stability of the configuration of flow of immiscible fluids with different viscosities," Journal of Fluid Mechanics, Vol. 141, pp. 319-345, 1984.

We consider the problem of steady fast flow of a family of Oldroyd fluids into a hole and show that the field of flow is partitioned into elliptic (subcritical) and hyperbolic (supercritical) regions. We analyze the characteristics and show that the vorticity changes type as in the experiments of Metzner, Uebler, and Fong (1969).

Ref. 9

D. D. Joseph, "Hyperbolicity and change of type in the flow of viscoelastic fluids past a flat plate," to appear in Archive for Rational Mechanics and Analysis, 1985.

We consider the problem of flow of viscoelastic liquids along a flat plate. The problem is the same one for which Newtonian fluids lead to the Blasius solution for the boundary layer on the plate. The vorticity of steady flows with instantaneous elasticity perturbing uniform flow will change type from elliptic to hyperbolic when the ratio of the velocity of the free stream to the velocity of propagation of shear waves into a fluid at rest exceeds unity. The perturbation problem may be solved subject to prescribed vorticity and zero normal velocity on the plate by transform methods. An explicit solution for a Maxwell model with prescribed constant vorticity on the plate is given. Formulae for the slope of the leading characteristic and the decay of the amplitude of the vorticity are expressed in terms of instantaneous values of the relaxation modulus and its derivatives in the case of a general fluid. The corresponding nonlinear problem is discussed and arguments are presented suggesting that the true flow is likely to have diffusive region near the plate, with a "transonic" transition away from the plate at points where the fluid velocity exceeds the wave velocity.

Ref. 7

D. D. Joseph, "Hyperbolic phenomena in the flow of viscoelastic fluids," Proceedings of the Symposium on Viscoelasticity and Rheology, Mathematics Research Center, University of Wisconsin, 1984.

This paper treats the problem of hyperbolicity, change of type and nonlinear wave propagation in the flow of viscoelastic fluids. Rate equations for fluids with and without instantaneous elasticity are derived and discussed. The equations of fluids with instantaneous elasticity are hyperbolic in unsteady flow and can change type in steady flow. The wave speeds depend on velocities and stresses. Some estimates of wave speeds into states of rest are given. For many of the popular models of fluids the vorticity is the field variable which changes type. The vorticity of all fluids with instantaneous elasticity can change type in motions which perturb rigid ones. Experiments and analysis exhibiting vorticity of changing type are exhibited. The linearized viscoelastic problem is governed by equations having the properties of a telegraph equation. The damping is small when the fluid is very elastic. Elastic fluids have a long memory, a large time (Weissenberg number) for relaxation. The damping is rapid when the relaxation time is small even when the flow is very supercritical. It is shown that steady flow around a body is of "transonic" type. The linearized problem for flow over a flat plate is reduced to an integral equation for the vorticity distribution on the plate. The problem of nonlinear wave propagation is discussed and the problems of nonlinear smoothing and shocking are considered. It is shown (by M. Slemrod) that the shocks of vorticity can arise from smooth data in some models and shocks of velocity in other models.

Ref. 8

J. Y. Yoo, M. Ahrens, and D. D. Joseph, "Hyperbolicity and change of type in sink flow," to appear in Journal of Fluid Mechanics, 1985.

Ref. 5

A. Narian and D. D. Joseph, "Classification of linear viscoelastic solids based on a failure criterion," Journal of Elasticity, Vol 14, pp. 19-26, 1984

An isotropic, incompressible linear viscoelastic solid subjected to a step shear displacement fails if the relaxation function G(s) is such that $0 < G(0) < \infty$ and $-\infty < G'(0) \leq 0$. In this case, the discontinuity in displacement propagates into the interior of the body. The discontinuity will not propagate however if $G(0) = \infty$ or $G'(0) = -\infty$. In the former case there is a diffusion like smoothing of discontinuous data characteristic of parabolic equations. The case $G(0) = \infty$ may be achieved by composing the kernel as a sum of a smooth kernel and a delta function at the origin times a viscosity coefficient. If the viscosity is small, the smoothing will take place in a propagating layer which scales with the small viscosity. The case of $G'(0) = -\infty$ is interesting in the sense that the solution is C^∞ smooth but the boundary of the support of the solution propagates at a constant wave speed. If $0 < G(0) < \infty$ and $-\infty < G'(0) < 0$, then the material accommodates stress waves under step traction leading to an elastic steady state.

4.2 Hyperbolicity and Change of Type

Ref. 6

D. D. Joseph, M. Renardy, and J-C. Saut, "Hyperbolicity and change of type in the flow of viscoelastic fluids," Archive for Rational Mechanics and Analysis, Vol. 87, No. 3, 1985.

The equations governing the flow of viscoelastic liquids are classified according to the symbol of their differential operators. Propagation of singularities is discussed and conditions for a change of type are investigated. The vorticity equation for steady flow can change type when a critical condition involving speed and stresses is satisfied. This leads to a partitioning of the field of flow into subcritical and supercritical regions, as in the problem of transonic flow.

This paper extends our earlier work on the propagation of jumps in velocity and displacement for shearing deformations imposed impulsively at the boundary of viscoelastic fluids and solids obeying constitutive equations in integral form with arbitrary kernels of fading memory type. The earlier work is briefly reviewed in §1, and we give new results. In §2 we relate old results to experiments. The limiting velocity distribution for start-up of Couette flow between parallel plates is a linear shear. It is common practice to assume that the real motion is close to linear shear long before the stress approaches its asymptotic steady state value. When the simplified kinematics are assumed, the evolution of the wall shear stress is determined by material functions, independent of deformation. These material functions are then determined by experimental measurements. We argue that in some cases only very special features of the material functions can be determined by this method because (in all cases) the early time behavior of the motion is incorrectly given by the kinematic assumption. The assumption that the early part of the stress response can be ignored is at best an approximation when the dynamics shows the presence of a delta function singularity in the wall shear stress at time t=0 and at subsequent discrete times of reflection off bounding walls. This delta function contribution cannot be ignored even if the steady state is achieved rapidly. In fact the early time behavior is crucial in the determination of the material functions and it can be obtained from experiments only by using a correct theory based on dynamics rather than kinematical assumptions. When this is done it is possible to interpret data showing stress overshoot with linear theories based on commonly used constitutive equations and to interpret early oscillations in the observed values of material functions in terms of repeated reflections off bounding walls. The foregoing critical remarks apply equally to the interpretation of stress relaxation experiments and other experiments involving impulsive changes in velocity and displacement. In §3 we derive formulae for the amplitude of jumps and reflections for fluids sheared between concentric cylinders. In §4 we develop integral methods of solution analogous to Duhamel integrals for inverting start-up problems with arbitrary data perturbing rest. In §5 we apply our analysis to start-up for viscoelastic solids and show how the creep depends on the kernel of the integral equation.

Ref. 3

A. Narian and D. D. Joseph, "Remarks about the interpretation of stress growth experiments in viscoelastic fluids," Rheologica Acta, Vol. 22, pp. 528-538, 1983.

The limiting velocity distribution for start-up of Couette flow between parallel places is a linear shear. It is common practice to assume that the real motion is close to linear shear long before the stress approaches its asymptotic steady state value. When the simplified kinematics are assumed, the evolution of the wall shear stress is determined by material functions, independent of deformation. These material functions are then determined by experimental measurements. We argue that in many cases only very special features of the material functions can be determined by this method because in all cases the early time behavior of the motion is incorrectly given by the kinematic assumption. The assumption that the early part of the stress response can be ignored is at best an approximation when the dynamics shows the presence of a delta function singularity in the wall shear stress at time t=0 and at subsequent discrete times of reflection off bounding walls. This delta function contribution cannot be ignored even if the steady state is achieved rapidly. In fact the early time behavior of the material functions can be obtained from experiments only by using a correct theory based on dynamics rather than kinematical assumptions. When this is done it is possible to interpret data showing stress jumps with linear theories based on commonly used constitutive equations and to interpret early oscillations in the observed values of material functions in terms of repeated reflections off bounding walls. The foregoing remarks apply equally to the interpretation of stress relaxation experiments and other experiments involving impulsive changes in velocity and displacement.

Ref. 4

A. Narian and D. D. Joseph, "Linearized dynamics of shearing deformations perturbing rest in viscoelastic materials," Transactions of the 28th Conference of Army Mathematicians, pp. 499-530, 1983.

4. PUBLICATIONS

The following publications have been produced during the grant period.

4.1 Discontinuities in Viscoelastic Materials

Ref. 1

A. Narian and D. D. Joseph, "Linearized dynamics for step jumps of velocity and displacement of shearing flows of a simple fluid," Rheologica Acta, Vol. 21, pp. 228-250, 1982.

> We consider linearized dynamics associated with step jumps in the velocity or displacement of the boundary of a fluid in a shearing motion. The discontinuity will propagate into the interior with a speed $C = \sqrt{G(0)/\rho}$ (ρ is the density) if the initial values $G(0)$ and $G'(0)$ of the fading memory kernels are bounded, $0 < G(0) < \infty$, $-\infty < G'(0) < 0$. If $G(0) \neq \infty$ but $G'(0) = -\infty$, then the boundary of the support of the solution still propagates with the speed C. However, the solutions on both sides of the boundary match together in a C^∞ fashion. If $G(0) \neq \infty$ but $G'(0) = 0$, the amplitude of the discontinuity will not damp as in a purely elastic fluid. If $G(0) = \infty$, the step change is felt immediately throughout the fluid, without shocks, as in Navier-Stokes fluids. This same type of parabolic behavior can be achieved by a small Newtonian contribution added to the integral form of the stress but if this contribution is small, a smooth transition layer around the shock will propagate with the speed C. In the case of step displacement, from rest to rest, singular surfaces of infinite velocity can propagate into the interior with speed of propagation C. The singular surfaces undergo multiple reflections off bounding walls, but the final steady state reached asymptotically is in universal form independent of material.

Ref. 2

M. Renardy, "Some remarks on the propagation and non-propagation of discontinuities in linearly viscoelastic liquids," Rheologica Acta, Vol. 21, pp. 251-254, 1982.

> The equations of linear viscoelasticity with a bounded memory kernel have been shown to propagate singularities in a similar way as hyperbolic equations. In this paper, we investigate a model problem for a certain class of unbounded memory kernels. It is shown that C^∞-solutions are obtained, although there is a discontinuity in the boundary conditions.

get a direct measurement of the breaking strength and at the same time observe the mechanics of the breaking process with high-speed video recording.

3.10 Stability of a Rotating and Coning Cylinder

- We are carrying out an experimental investigation of the stability of a cylinder which is both rotating and coning for the situation in which the cylinder contains two immiscible fluids. The flight instability in the spinning projectile is caused by the high viscosity tangential shear stress. This tangential shear stress can be reduced by using another immiscible and less viscous fluid. Following the concept of encapsulation, the less viscous fluid tends to move to the highest sheared region which is at the cylinder wall. The results show that the despin moment varies nonlinearly with the coning rate and depends on the range of canister spin rate. The despin moment versus viscosity curve for the two-fluid case shows the same effect as the single fluid case. The value of the despin moment converges to the values of the despin moment in the solid/liquid cases. This work is still in progress and has not yet been published.

3.11 Flows in Saturated Porous Media

- Two studies related to the quadratic drag for flow through saturated porous media are described in Ref. 28 and Ref. 29.

- A linear stability analysis for a two-layer Bénard problem is presented in Ref. 26.

- A stability analysis of a liquid film coating the inner surface of a rotating cylinder is being carried out. The coating flow inside a horizontal rotating cylinder is stable when the rate of rotation is high because of centrifugal effects. However, at low rotational speeds the steady coating flow is unstable. We are computing the critical rotation rate at which this instability occurs. We have derived the equations to determine the eigenvalue up to second order in gravity. We are now developing algorithms for the numerical solution of the equations, using Chebyshev polynomials.

3.9 Viscoelastic Liquid Breakup

- Several experimental techniques to study the breakup of viscoelastic liquids have been attempted with the objective of identifying suitable techniques for the development into laboratory procedures for the generation of quantitative information on breakup characteristics. Three techniques which have been tried and which show promise for further development are an impact system, a small scale aerodynamic stripping technique, and a shaker system. This work is described in Ref. 27.

- We are continuing to seek new experimental techniques to measure the breaking strength of liquids, including observations on the fracturing of rollers of viscous and viscoelastic liquids as they break away from rigid boundaries. We are designing an experimental device with which we shall attempt to

(iii) The flow rate can be controlled very accurately because the hydraulic system used to drive the test fluid through the capillary is itself driven by an MTS Universal Testing Machine with computer control.

3.6 Climbing Constants

- We have measured the values of the climbing constants for many viscoelastic liquids, including different concentrations of a polymer in a particular solvent, and the same polymer in different solvents. We have developed simple formulae for computing the first normal stress difference and the extensional viscosity at low rates of deformation using the measured climbing constant only. This work is presented in Ref. 22.

3.7 Fading Memory

- The problem of fading memory has been considered, and two solutions have been developed to the restricted problem of fading memory which deals with the forms of the stresses which arise for certain classes of allowed deformations. This work is presented in Ref. 23.

3.8 Stability Analyses

- Stability of plane Couette flow: an analysis of perturbations of plane Couette flow of viscoelastic liquids with fading memory has been carried out, and is described in Ref. 24.

- The stability of viscometric flow using a short memory assumption has been investigated and it has been found that the assumption cannot be justified for some popular rheological models (Ref. 25).

3.5 Jets and Extrudate Swell

- An exact asymptotic solution for the jet diameter of a liquid jet falling under gravity into an immiscible liquid has been developed, and compared with experimental data of J. Matta. This work is described in Ref. 20.

- The problem of extrudate swell of a viscoelastic fluid from a round pipe has been analysed by domain perturbation methods. This work is summarized in Ref. 21. Perturbation problems up to second-order in the flow rate, $\varepsilon = \frac{Q}{2\pi}$, are derived and solved by finite elements. We find the expression describing the surface shape of the round jet at slow flow, depends not only on the material constant α_1 but also on α_2.

- Although many experimental data on extrudate swell have been published, it appears that no systematic experimental program has been carried out. We have now constructed an experimental apparatus with which we shall carry out a systematic program of experiments. The apparatus is similar to a capillary extruder with the following special features:
  - (i) The fluid reservoir has a floating piston so that it can be adapted easily to our hydraulic system as well as to the gas-pressurized system used by Dr. Joseph Matta at the Chemical Systems Division, Aberdeen Proving Ground.
  - (ii) The capillary is built such that test liquid can be extruded into air or into another liquid at any angle from vertically upward to vertically downward.

ratio in a fairly complicated way. The flow with the high viscosity fluid centrally located is stable when the lubricating layer on the pipe wall is not too thick. Centrally located layers of less viscous fluid, called fingering flows, can also be stable if there is not too much fluid in the center. Stable fingering flows are less stable than stable lubricating flows when the volume and viscosity ratio of the two fluids is such that stable fingering and lubricating flows are both possible. For still other volume and viscosity ratios the lubricating and fingering flows are both unstable.

Ref. 16

M. Renardy and D. D. Joseph, "Hopf bifurcation in two-component flow," submitted to Journal of Differential Equations, 1984.

The stability of viscosity-stratified bicomponent flow has been studied by long wave asymptotics, by short wave asymptotics, and numerically. These studies have shown that interfacial instabilities arise from the viscosity difference between the two fluids. If the surface tension between the fluids is non-zero, then Hopf type bifurcation leading to traveling interfacial waves are expected. In this paper we prove a rigorous theorem establishing the existence of bifurcating solutions of this nature.

4.4 Fluid Motion Between Rotating Cylinders.

Ref. 17

A. Siginer, "Free surface on a simple fluid between rotating eccentric cylinders. Part I: Analytical solution," Journal of Non-Newtonian Fluid Mechanics, Vol. 15, pp. 93-108, 1984.

The steady motion of a simple fluid between vertical cylinders which rotate about non-concentric axes is solved by means of domain perturbations. The theory is developed as a perturbation of the rest state in powers of the angular frequency Ω of the inner cylinder, and the solution is carried out to $0(\Omega^2)$. The stress is expanded in a series of Rivlin-Ericksen tensors. At the second order only one material parameter, the climbing constant, enters the analysis. A procedure is developed for predicting the shape of the free surface on the fluid. Secondary motions generated by the eccentricity are shown to appear at the second order.

Ref. 18

A. Siginer and G. S. Beavers, "Free surface on a simple fluid between rotating eccentric cylinders. Part II: Experiments," Journal of Non-Newtonian Fluid Mechanics, Vol. 15, pp. 109-126, 1984.

This paper summarizes the results from an experimental investigation of the effects of eccentricity and rotational speed on the free surface shape on a viscoelastic liquid between eccentric cylinders. In the experimental geometry the inner cylinder rotates and the outer cylinder is stationary. The experiments show that there is a circumferential pressure gradient (the lubrication effect) which has a dominant influence on the free surface shape at all eccentricities and rotational speeds. For a liquid with small normal stress effects the normal-stress induced component of the deformation tends to be overwhelmed by the lubrication effect, whereas a liquid with large normal stress effects exhibits characteristic normal-stress induced deformations at small eccentricities and rotational speeds. There is good agreement between experiment and second order predictions for the large normal stress liquid under these conditions. The ranges of eccentricities and rotational speeds for which second order theory describes the low normal stress liquid appear to be much more limited and are difficult to reproduce experimentally.

Ref. 19

H. A. Tieu, D. D. Joseph and G. S. Beavers, "Interfacial shapes between two superimposed rotating simple fluids," Journal of Fluid Mechanics, Vol. 145, pp. 11-70, 1984.

The interfacial shape of two immiscible simple fluids in a vertical cylinder which oscillates about its axis is investigated using the theory of domain perturbations. The perturbation stresses are expressed by integrals over the history of the deformation. At first order the azimuthal velocity field satisfies the requirements of continuity in velocity and shear stresses across the interface. At second order, the solution consists of a mean part and a time periodic part varying at twice the frequency of the cylinder. The mean problem is inverted for the mean secondary flow, pressure and interfacial shape. Experimental data for two polymeric oils (TLA227 and STP) show qualitative agreement with theoretical predictions for the mean interfacial shapes.

4.5 Jets and Extrudate Swell

Ref. 20

D. D. Joseph, K. Nguyen and J. E. Matta, "Jets into liquid under gravity," Journal of Fluid Mechanics, Vol. 128, pp. 443-468, 1983.

We study the flow of a heavy, viscous, possibly non-Newtonian axisymmetric jet of liquid of density ρ falling under gravity g into a lighter liquid of density $\tilde{\rho}$. If the change in the momentum of the entrained lighter liquid is neglected the jet will ultimately reach a modified Torricelli limit with a speed given by

$$U(x) = \left[2 \frac{\delta\rho}{\rho} gx\right]^{\frac{1}{2}}$$

and an asymptotic radius

$$a(x) = \left[\frac{2Q^2}{\frac{\delta\rho}{\rho} gx}\right]^{\frac{1}{4}} \qquad (1)$$

where x is the downstream distance, $\delta\rho = \rho - \tilde{\rho} > 0$ and $2Q$ is the volume flow. An exact asymptotic solution perturbing the Torricelli limit with effects of surface tension, viscosity and elasticity is given in powers of $x^{-\frac{3}{4}}$. An extended unsteady problem including effects of entrainment is formulated in terms of nonlinear ordinary differential equations which also account for weak radial variations of the velocity across the cross-section of the jet. These equations are solved in a boundary layer approximation which gives

$$a(x) \cong 1.171 \left(\frac{\tilde{\rho}}{x}\right)^{1/20} \frac{Q^{3/10}\tilde{\mu}^{1/5}}{(\delta\rho g)^{1/4}} \qquad (2)$$

where $\tilde{\mu}$ is the viscosity of the ambient fluid. Eq. (1) is in agreement with experimental observations of jets of liquid into air. Eq. (2) is in agreement with experimental observations of jets of liquids into liquids.

Ref. 21

H. A. Tieu and D. D. Joseph, "Extrudate swell for a round jet with large surface tension," Journal of Non-Newtonian Fluid Mechanics, Vol. 13, pp. 203-222, 1983.

The problem of extrudate swell of a viscoelastic fluid from a round pipe is studied by the method of domain perturbations. The perturbation problems are solved by a finite element method through second-order in the flow rate parameter ε for small flow rates. The analysis extends the work of Sturges on swelling in two dimensional channels to round capillary tubes. In perturbation studies for small ε, the rheology of the fluid may be expressed by three parameters, the viscosity and the two constants α_1 and α_2 appearing at order two in the expansion of the extra stress around zero shear. Surface tension has an important influence on the shape of the jet at low speeds. The shape of the surface on a round jet depends on α_1 and α_2, in the plane jet only on α_1. The analysis predicts that no matter what the constitutive equation may be, the jet will first contract if the radius of the pipe is sufficiently small. The contraction takes place in a length less than 1/10 the diameter of the jet and is followed by a swell. The contraction is usually small and may be hard to observe. There are five different contributions to the jet shape at second order but only the viscoelastic ones persist as the pipe radius goes to zero.

4.6 Climbing Constants

Ref. 22

D. D. Joseph, G. S. Beavers, A. Cers, Carolyn Dewald, Anne Hoger, P. T. Than, "Climbing constants for various liquids," Journal of Rheology, Vol. 28(4), pp. 325-345, 1984.

In this paper we present tables of values of the climbing constant

$$\hat{\beta} = 3\alpha_1 + 2\alpha_2$$

where α_1 and α_2 are the parameters of the second order approximation to the stress in a slow, slowly varying flow of any simple non-Newtonian fluid. The parameter $\hat{\beta}$ arises in the analysis of rod climbing, and it is proportional to the height of climb in slow steady flow. A method is presented for computing the first normal stress difference and the extensional viscosity at low rates of deformation, using $\hat{\beta}$ alone.

4.7 Fading Memory

Ref. 23

J. C. Saut and D. D. Joseph, "Fading Memory," Archive for Rational Mechanics and Analysis, Vol. 81(1), pp. 53-95, 1983.

Fading memory expresses the intuitive idea that the recent rather than remote history of the deformation of a material body should have a greater effect on the present stress. The problem of fading memory is to give a useful mathematical formulation of this intuitive idea. The restricted problem of fading memory deals with the forms of stress which arise when the class of allowed deformations are small or slow in a sense specified differently in different theories. In this paper we give two new solutions to the restricted problem of fading memory, hereafter called the problem of fading memory. Our first solution is in a weighted Sobolev space. Our second solution is on a locally convex topological vector space. Both solutions are framed on subsets of the weighted Hilbert space used in the celebrated theory of Coleman and Noll (1961). In each of the new solutions we find as special cases, rate materials (including Newtonian fluids), integral materials of the Coleman & Noll type ("smooth" $\mathbb{L}^2_h$ kernels) and mixtures of these. The mixed constitutive equations are said to describe "materials of mixed type of order n and degree k." Dynamics associated with constitutive equations of mixed type are studied for shear flows in a linearized approximation. The dynamic equations have shock like solutions in the Coleman-Noll limit which are smoothed by rate terms (viscosity) in the materials of mixed type.

4.8 Stability Analyses

Ref. 24

J. Dunwoody and D. D. Joseph, "Systematic linearization for stability of flows of viscoelastic fluids." Archive for Rational Mechanics and Analysis, Vol. 86, pp. 65-84, 1984.

The phenomenon of melt fracture occurring in the process of polymer extrusion (see Tordella, 1963) has attracted the attention of research workers in the past two decades. In order to understand the mechanisms which might give rise to this phenomenon, the behavior of perturbations of plane Couette flow of visco-elastic fluids with fading memory has been studied by various authors. Among them are Coleman and Gurtin (1968) who proposed and studied the formation of shear shocks and Dunwoody (1970) who took a similar view, but incorporated heating effects. Slemrod (1978, 1979) also considered the physical conditions necessary to produce the existence of non-smooth solutions to the perturbation problem, and proposed a further mechanism for instability based on the proposition that the shear stress in the basic flow is not a convex function of the shear rate.

The most recent studies of this stability problem have been by Akbay, Becker, Krozer and Sponagel (1980) and Akbay and Sponagel (1982), both based on an approximate constitutive theory for slow flows of fluids with short memory proposed by Akbay and Becker (1979) and Becker (1980). In both cases it is assumed that stability can be studied by linear theory using standard spectral analysis of an eigenvalue problem and other approximations. Here we adopt the same approach, except that we have derived linear stability equations for infinitesimal perturbations of the history of a simple shear flow of a viscoelastic fluid with fading memory without further approximations. Simplifying assumptions with regard to material response are only introduced in order to draw conclusions from our exact analysis of stability in the final section (7). We show that periodic disturbances of long wave length in the flow direction may lead to instability when the modified Weissenberg number function (of κ)

$$\frac{\eta}{\left[\frac{d(\eta\kappa)}{d\kappa}\right]}\left[\frac{\kappa}{\gamma}\;\frac{d}{d\kappa}\left(\frac{N_1}{\kappa}\right)\right]^2$$

where κ is the shear rate, $\eta(\kappa)$ is the shear viscosity function and $N_1(\kappa)$ is the first normal stress difference, is sufficiently large.

Ref. 25

M. Ahrens, D. D. Joseph, M. Renardy, and Y. Renardy, "Remarks on the stability of viscometric flow," Rheologica Acta, Vol. 23, pp. 345-354, 1984.

We study the stability of viscometric flow using the type of short memory introduced by Akbay, Becker, Krozer and Sponagel (1-7). The instability found by these researchers is recognized as a change of type leading to non-evolutionary character of the governing equations.

We also address the question of justification for the short memory assumption and find that it cannot be justified for some of the more popular rheological models.

Ref. 26

Y. Renardy and D. D. Joseph, "Oscillatory instability in a two-fluid Bénard problem," submitted to Physics of Fluids, 1984.

A linear stability analysis for a two-layer Bénard problem is considered. The equations are not self-adjoint. The system can lose stability to time-periodic disturbances. For example, it is shown numerically that, when the viscosities and coefficients of cubical expansion of the fluids are different, a Hopf bifurcation can occur, resulting in a pair of travelling waves or a standing wave. This may have application in the modelling of convection in the Earth's mantle.

4.9 Viscoelastic Liquid Breakup

Ref. 27

G. S. Beavers, D. D. Joseph, and C. G. Dewald, "Laboratory techniques to study viscoelastic liquid breakup," Proceedings of the 1983 Scientific Conference on Chemical Defense Research, Aberdeen Proving Ground, 1984.

Small scale laboratory experiments are being developed which can be used to predict large scale liquid breakup behavior under conditions similar to munition functioning. Two techniques which show good potential are aerodynamic stripping of liquid drops and drop breakup by impact loading. Both techniques can be carried out with experimental equipment that can be made compact enough to allow experiments to be performed in a fume hood. The stripping experiment cannot be controlled as well as the impact experiment. The two experiments taken together seem to have a good potential for generating very useful comparative data on breakup.

4.10 Flows in Saturated Porous Media

Ref. 28

D. D. Joseph, D. A. Nield and G. Papanicolaou, "Nonlinear equation governing flow in a saturated porous medium," Water Resources Research Journal, Vol. 18, p. 1049, 1982.

It is argued that the appropriate generalization of Darcy's law when inertia effects are included takes the form

$$\nabla p = -\frac{\mu}{k}\underset{\sim}{V} - \frac{\rho c}{k^{\frac{1}{2}}}\left|\underset{\sim}{V}\right|\underset{\sim}{V}, \qquad \text{div } \underset{\sim}{V} = 0,$$

where k is the permeability of the medium, and the "form drag constant" c is a coefficient which is independent of the pressure p, the seepage velocity $\underset{\sim}{V}$, and the density ρ and viscosity μ of the fluid, but which is dependent on the geometry of the medium. We formulate a nonlinear extension of Brinkman's self consistent theory for the flow of a viscous fluid through a swarm of spherical particles. We equate the drag per unit volume given by the right hand side of the first of the above equations to the total drag ND on the N particles contained within that unit volume, in an infinite region Ω, where D is the drag on a a single particle placed in a velocity field $\underline{v}$ subject to

$$\rho(\underline{v}.\nabla)\underline{v} + \text{grad } p = \mu\nabla^2\underline{v} - \frac{\mu}{k}\underline{v} - \frac{c\rho}{k^{\frac{1}{2}}}|\underline{v}|\underline{v},$$

$$\text{div } \underline{v} = 0, \quad \underline{v}\Big|_{\partial\Omega} \text{ is a prescribed constant,}$$

where μ is the viscosity. Without solving these equations, we obtain an estimate for c from the known experimental drag law for a solid sphere placed in a uniform stream.

Ref. 29

D. A. Nield and D. D. Joseph, "Effects of quadratic drag on convection in a saturated porous medium," to appear in Physics of Fluids, 1985.

The effects of inertia, involving a drag which is quadratic in the velocity, on convection in a fluid-saturated porous medium, are considered. It is shown that the effect of quadratic drag is physically significant for natural convection, at realistic values of the Rayleigh number, in a thin layer of a medium whose overall Prandtl number is small. The qualitative effect of quadratic drag on the global stability of the conduction regime, and on bifurcation into the convection regime, is reported. Convection in an inclined slab of material is also discussed.

5. PARTICIPATING PERSONNEL

The following personnel have participated in this research at some time during the term of the grant:

Professor D. D. Joseph	Principal investigator
Professor G. S. Beavers	Principal investigator
Professor J. Y. Yoo	Visiting Professor
A. Narain	Research assistant
K. T. Nguyen	Research assistant
A. Tieu	Research assistant
P. Than	Research assistant
C. G. Dewald	Research assistant
O. Riccius	Research assistant
M. Ahrens	Research assistant

6. THESES COMPLETED DURING THE GRANT PERIOD

A. Narain; Ph.D., 1983 Thesis title: "Shearing deformation of simple materials."

A. H. Tieu; Ph.D., 1983. Thesis title: "Second-order effects in complex rheological flows."

K. T. Nguyen; Ph.D., 1984. Thesis title: "Bicomponent flow problems in rheological fluid mechanics."

C. G. Dewald; M.S., 1984. Thesis title: "Evaluation of rheological methods for predicting liquid breakup."

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