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ELASTIC RESPONSE OF A HALF-PLANE TO A BONDED
INTERFERENCE-FIT DISC OF THE SAME MATERIAL(U)
AERONAUTICAL RESEARCH LABS MELBOURNE (AUSTRALIA)
G S JOST ET AL. JUL 84 ARL/STRUC-R-406

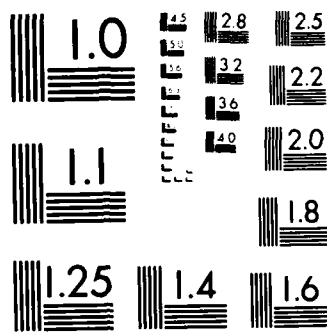
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**DEPARTMENT OF DEFENCE
DEFENCE SCIENCE AND TECHNOLOGY ORGANISATION
AERONAUTICAL RESEARCH LABORATORIES
MELBOURNE, VICTORIA**

STRUCTURES REPORT 408

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TO A BONDED INTERFERENCE-FIT DISC
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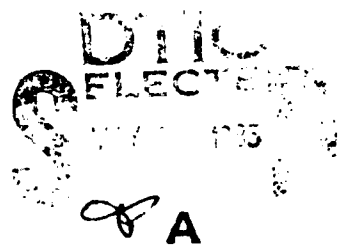
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by

G. S. JOST and R. P. CAREY

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1 Inch thick austenitic hand welded plate Cadillac 140.
Max. size electrode: 1/4 in. dia.



1 Inch thick austenitic hand welded plate Cadillac 167.
Max. size electrode: 5/16 in. dia.



1 Inch thick austenitic Unionmelt welded plate Gen. Motors Truck 34.



3/4 Inch thick austenitic Unionmelt welded plate Fisher U 37.



3/4 Inch thick austenitic Unionmelt welded plate Fisher U 39.



1/2 Inch thick austenitic hand welded plate Cadillac 178.
Max. size electrode: 5/32 in. dia.



1/2 Inch thick austenitic hand welded plate Gen. Motors Truck.
Max. size electrode: 3/16 in. dia.



1/2 Inch thick ferritic hand welded plate Fisher H-114.
Max. size electrode: 1/4 in. dia.



1/2 Inch thick austenitic Unionmelt welded plate Gen. Motors Truck 3.



1/2 Inch thick ferritic Unionmelt welded plate Gen. Motors Truck Y.

WTN.639-6677

Figure 9. Typical Weld Joint Fractures.

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**ELASTIC RESPONSE OF A HALF-PLANE
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SUMMARY

Analytical expressions for the stress and displacement fields in a half-plane containing a bonded interference-fit disc of the same material have been enunciated. Some representative geometric configurations have been evaluated, the results being given in both tabular and graphical forms.



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NOMENCLATURE

a	distance from hole centre to plate boundary (free edge)
A	$= a + x = 2a + r \cos \theta$
B	$= y = r \sin \theta$
E	Young's modulus
G	Shear modulus $= E/[2(1+\nu)]$
i	$\sqrt{-1}$
P	pressure to reduce disc to radius R [$= 4G\lambda/(\kappa-1)$]
q	$= P(\kappa-1)/(\kappa+1)$
r	radius from hole centre to point z
R	radius of hole
u_x, v_y	Cartesian displacements in x and y directions
u_r, v_θ	polar displacements, radial and circumferential
x, y	Cartesian coordinates
z	location of point under consideration ($= x + iy$)
\hat{x}_x, \hat{y}_y	Cartesian stresses
\hat{x}_y	Cartesian shear stress
θ	angle at hole centre between x axis and point z
κ	$= (3-\nu)/(1+\nu)$ for plane stress; $(3-4\nu)$ for plane strain
λ	radial interference/ R
ν	Poisson's ratio
π	pi
ϕ, ψ	stress functions
$\hat{\theta}\hat{\theta}, \hat{r}\hat{r}$	polar stresses, circumferential and radial
$\hat{r}\hat{\theta}$	polar shear stress

1. INTRODUCTION

This paper is concerned with the evaluation of the elastic stress and displacement fields arising from the bonding of an interference-fit disc in a hole near the free-edge of a large plate, Fig. 1. Such a situation is approximated quite closely in many applications where the use of interference-fit fasteners has been found highly beneficial in extending fatigue life.

The exact solutions for the title subject have been provided by Richardson¹ in the form of stress functions for both disc and plate together with two tantalizing graphs showing the influence of polar angle and geometry on the stresses. This Report presents explicitly the formulations for stress and displacement together with tabulations and graphs for several representative geometric configurations.

Both Cartesian and polar formulations are given, the derivations using the complex representation used by Muskhelishvili.² Non-dimensional presentations are adopted throughout.

It is to be noted that the analysis is valid only for disc and plate of the same material and where there is no failure of the bond nor relative movement around the interface.

2. STRESS FUNCTIONS

Stresses and displacements are found directly from the stress functions $\phi(z)$ and $\psi(z)$ as follows:²

$$\widehat{x}\widehat{x} + \widehat{y}\widehat{y} = \widehat{r}\widehat{r} + \widehat{\theta}\widehat{\theta} = 2[\phi'(z) + \phi'(\bar{z})] = 4 \operatorname{Re} [\phi'(z)] \quad (1)$$

$$\widehat{y}\widehat{y} - \widehat{x}\widehat{x} + 2i \widehat{x}\widehat{y} = 2[\bar{z} \phi''(z) + \psi'(z)] \quad (2)$$

$$\widehat{\theta}\widehat{\theta} - \widehat{r}\widehat{r} + 2i \widehat{r}\widehat{\theta} = (\widehat{y}\widehat{y} - \widehat{x}\widehat{x} + 2i \widehat{x}\widehat{y}) e^{2i\theta} \quad (3)$$

and

$$2G(u_x + iv_y) = \kappa \phi(z) - z \bar{\phi}'(\bar{z}) - \psi(\bar{z}) \quad (4)$$

$$2G(u_r + iv_\theta) = 2G(u + iv)e^{-i\theta} \quad (5)$$

The stress functions for the present problem have been derived by Richardson using the superposition principle of Eshelby³ as follows (the notation is that of Muskhelishvili):

For the disc:

$$\Phi(z) = \frac{PR^2(\kappa-1)}{\kappa+1} \left\{ -\frac{1}{2R^2} + \frac{1}{(z+a)^2} \right\}$$

$$\Psi(z) = \frac{PR^2(\kappa-1)}{\kappa+1} \left\{ -\frac{1}{(z+a)^2} + \frac{2a}{(z+a)^3} \right\}$$

For the plate:

$$\Phi(z) = \frac{PR^2(\kappa-1)}{\kappa+1} \left\{ \frac{1}{(z+a)^2} \right\}$$

$$\Psi(z) = \frac{PR^2(\kappa-1)}{\kappa+1} \left\{ \frac{1}{(z-a)^2} - \frac{1}{(z+a)^2} + \frac{2a}{(z+a)^3} \right\}$$

For subsequent use here, the integrals of $\Phi(Z)$ and $\Psi(Z)$ are also required. Substituting the dummy variable q for $P(\kappa-1)/(\kappa+1)$ above yields the stress functions below:

For the disc:

$$\phi(z) = qR^2 \left\{ -\frac{z}{2R^2} - \frac{1}{z+a} \right\} \quad (6)$$

$$\psi(z) = qR^2 \left\{ \frac{1}{z+a} - \frac{a}{(z+a)^2} \right\} \quad (7)$$

For the plate:

$$\phi(z) = qR^2 \left\{ -\frac{1}{z+a} \right\} \quad (8)$$

$$\psi(z) = qR^2 \left\{ -\frac{1}{z-a} + \frac{1}{z+a} - \frac{a}{(z+a)^2} \right\} \quad (9)$$

These functions, together with their derivatives are now used with (1), (2) and (3) to determine disc and plate stresses, and with (4) and (5) to determine displacements.

3. STRESSES

Substitution of (6), (7), (8) and (9) into (1), (2) and (3) yield the following expressions for the stresses in disc and plate:

For the disc* ($0 \leq r \leq R$):

$$\left(\frac{\hat{H}\hat{H}/q}{\hat{r}\hat{r}/q} \right) = -1 + 2R^2 \frac{A^2 - B^2}{(A^2 + B^2)^2} \mp \frac{LR^2}{(A^2 + B^2)^3} \quad (10)$$

$$\hat{r}\hat{H}/q = \frac{MR^2}{(A^2 + B^2)^3} \quad (11)$$

* Notice that, at the interface ($r = R$), the circumferential disc stress (10, upper) is equal to that for the plate (12, upper) minus 2: radial and shear stresses for disc and plate are identical at all points on the interface.

For the plate ($r \geq R$):

$$\begin{pmatrix} \widehat{\theta\theta}/q \\ \widehat{r r}/q \end{pmatrix} = \pm \frac{R^2}{r^2} + 2R^2 \frac{A^2 - B^2}{(A^2 + B^2)^2} \mp \frac{LR^2}{(A^2 + B^2)^3} \quad (12)$$

$$\widehat{r\theta}/q = \frac{MR^2}{(A^2 + B^2)^3} \quad (13)$$

Cartesian stresses for the plate are

$$\begin{pmatrix} \widehat{x x}/q \\ \widehat{y y}/q \end{pmatrix} = 2R^2 \frac{A^2 - B^2}{(A^2 + B^2)^2} \mp NR^2 \quad (14)$$

$$\widehat{x y}/q = -\frac{2R^2 B(A - 2a)}{[(A - 2a)^2 + B^2]^2} + \frac{2R^2 B[A(5A^2 - 3B^2) - 2a(3A^2 - B^2)]}{(A^2 + B^2)^3} \quad (15)$$

where

$$\begin{pmatrix} L \\ M \end{pmatrix} = \left\{ 2[4AB(A^2 - B^2) - aB(3A^2 - B^2)] \begin{pmatrix} \sin 2\theta \\ \cos 2\theta \end{pmatrix} + rB(3A^2 - B^2) \begin{pmatrix} \sin 3\theta \\ \cos 3\theta \end{pmatrix} \right. \\ \left. \pm 2[A^4 - 6A^2B^2 + B^4 - aA(A^2 - 3B^2)] \begin{pmatrix} \cos 2\theta \\ \sin 2\theta \end{pmatrix} \pm rA(A^2 - 3B^2) \begin{pmatrix} \cos 3\theta \\ \sin 3\theta \end{pmatrix} \right\}$$

and

$$N = \left\{ \frac{(A - 2a)^2 - B^2}{[(A - 2a)^2 + B^2]^2} - \frac{3A^4 - 12A^2B^2 + B^4 - 4aA(A^2 - 3B^2)}{(A^2 + B^2)^3} \right\}$$

Some useful simplifications in these expressions occur at particular points. It is convenient to use the polar expressions for stress in the plate around the hole and Cartesian stresses along the free edge.

Around the hole $r = R$, but only inconsequential simplifications occur from this substitution. An expression for the maximum circumferential stress is impractical, as is that for the angular position at which it occurs: recourse must therefore be made to numerical solutions. Expressions for the stresses at $\theta = 0$ (on the x axis remote from the free edge) and $\theta = \pi$ (close to free edge) are, however, tractable:

$$(\widehat{r r}/q)_0^\pi = 4 \frac{2\left(\frac{a}{R}\right)^3 \mp 3\left(\frac{a}{R}\right)^2 \pm 1}{\left(2\frac{a}{R} \mp 1\right)^3}$$

$$(\widehat{\theta\theta}/q)_0^\pi = 4\left(\frac{a}{R}\right) \frac{2\left(\frac{a}{R}\right)^2 \mp 3\left(\frac{a}{R}\right) + 2}{\left(2\frac{a}{R} \mp 1\right)^3}$$

$$(\widehat{r\theta}/q)_0^\pi = 0.$$

Along the x axis $B = 0$, $A = a + x$ and (14) and (15) become

$$\widehat{xx}/q = \frac{\frac{a}{R} + \frac{a}{R}}{\left(\frac{a}{R} + \frac{x}{R}\right)^3} - \frac{1}{\left(\frac{a}{R} - \frac{x}{R}\right)^2}$$

$$\widehat{yy}/q = \frac{3\frac{a}{R} - \frac{x}{R}}{\left(\frac{a}{R} + \frac{x}{R}\right)^3} + \frac{1}{\left(\frac{a}{R} - \frac{x}{R}\right)^2}$$

and

$$\widehat{xy}/q = 0$$

where

$$0 \leq x \leq a - R$$

and

$$a + R \leq x.$$

The minimum value of \widehat{yy}/q occurs at

$$x = 0.2232a.$$

Along the free edge $x = 0$ and (14) and (15) become

$$\widehat{xx}/q = 0$$

$$\widehat{yy}/q = 4 \frac{\left(\frac{a}{R}\right)^2 - \left(\frac{y}{R}\right)^2}{\left[\left(\frac{a}{R}\right)^2 + \left(\frac{y}{R}\right)^2\right]^2}$$

and

$$\widehat{xy}/q = 0.$$

\widehat{yy}/q has its maximum at the origin, where

$$\widehat{yy}/q = \frac{4}{(a/R)^2}.$$

It becomes zero when $y = \pm a$ beyond which it remains negative, reaching a minimum of

$$\widehat{yy}/q = -\frac{1}{2} \frac{1}{(a/R)^2}$$

at $y = \pm \sqrt{3}a$ and asymptoting to zero at infinity.

Plate stresses calculated from (12) to (15) are given in Tables 1 and 2 for representative values of a/R from one to infinity. This covers the full range, from a hole whose surface just touches the free edge to a hole in an infinite plate. Circumferential, radial and shear stresses around the hole are given along with Cartesian stresses along the axis of symmetry (x axis) and the free edge (y axis). Selected data are plotted in Figs 2, 3 and 4. These reveal the following:

Along the free edge both normal and shear stresses are, of course, zero. Fig. 2 shows the variation in tangential stress with distance from the origin as quantified in Table 1. The maximum non-dimensional tangential stress occurs always at the origin and has a value of 4 when $a/R = 1$, decreasing rapidly as a/R increases. The general behaviour away from the origin has already been described and may be seen readily in Fig. 2.

Around the hole and along the axis of symmetry the plate stresses for several values of a/R are shown in Fig. 3. Here the location of the hole is fixed, the free edge being sited progressively further to the left with increasing a/R . The scale across the hole diameter is linear in θ . The higher peak circumferential stresses clearly correspond to lower a/R , the peak non-dimensional value of 4 occurring at $a/R = 1$ where the hole and free edge meet. The maximum circumferential stress remains on the free edge at the origin for low a/R until, with increasing a/R , it falls below that at the hole. The minimum stress at the hole occurs always at the point most remote from the free edge, $\theta = 0$, from which point the circumferential stresses along the x axis diminish in a remarkably similar manner for all a/R .

Radial stresses behave in an understandable manner except perhaps for the small region near $\theta = \pi$ where, for very low a/R , positive values are developed: the bonding at the interface allows this as a sensible outcome. Except for the lowest a/R , shear stresses are comparatively quite small and undergo a sign change at points asymptoting from $\theta = \pi$ towards $\theta = \pi/2$ with increasing a/R : they are zero along the axis of symmetry.

The behaviour of significant non-dimensional circumferential stresses described above can be seen more clearly in Fig. 4. As a function of a/R , this shows the variation in tangential stress at the free edge at the origin, and circumferential stress at the hole nearest the origin ($\theta = \pi$),* most remote from the origin ($\theta = 0$), and in terms of its maximum value. Fig. 4 also shows how the location of this maximum changes with a/R . Up to $a/R = 1.724$ it is fixed at the position $\theta = \pi$, but beyond this value its location moves rapidly around the hole towards its final $\pi/2$ asymptote for large a/R . The very small difference between the maximum stress around the hole and that at $\theta = \pi$ is to be noted. For a/R up to 1.767 the maximum circumferential stress in the plate occurs at the free edge at the origin: beyond this value it occurs at the hole. Beyond $a/R = 1.770$ the hole stress at $\theta = \pi$ exceeds that at the origin and beyond $a/R = 1.975$ stresses at all points around the hole exceed those at the free edge.

Contours of the stress field in the plate in terms of maximum principal stress are shown in Figs 5(a) to 5(d) for several values of a/R . These confirm the influence of a/R on both location and magnitude of maximum principal stress. The behaviour of these latter is shown more clearly in Fig. 6.

* For the disc, there is a range of a/R from 1 to 1.261 where the circumferential stress at the interface near the free edge is tensile, corresponding to that region where the non-dimensional plate stress exceeds 2.

4. DISPLACEMENTS

Substitution of (6), (7), (8) and (9) into (4) and (5) yield the required expressions for displacements. They are expressed here *relative to that of the centre of the disc*: i.e. relative to a stationary disc centre.*

For the disc:

$$\frac{2G}{q} \frac{u_{rd}}{R} = \frac{(1-\kappa)r}{2R} + UR \quad (16)$$

$$\frac{2G}{q} \frac{v_{\theta d}}{R} = VR \quad (17)$$

For the plate:

$$\frac{2G}{q} \frac{u_{rp}}{R} = \frac{R}{r} + UR \quad (18)$$

$$\frac{2G}{q} \frac{v_{\theta p}}{R} = VR \quad (19)$$

In Cartesian coordinates, plate displacements are

$$\frac{2G}{q} \frac{u_x}{R} = R \left\{ \frac{1+\kappa}{2a} + \frac{A-2a}{(A-2a)^2 + B^2} - W_1 \right\} \quad (20)$$

$$\frac{2G}{q} \frac{v_y}{R} = R \left\{ \frac{B}{(a-2a)^2 + B^2} - W_2 \right\} \quad (21)$$

where the denominator terms $(A-2a)^2 + B^2 \equiv r^2$,

$$\begin{pmatrix} U \\ V \end{pmatrix} = \pm \frac{1+\kappa}{2a} \begin{pmatrix} \cos \theta \\ \sin \theta \end{pmatrix} \mp \frac{(1+\kappa)A \begin{pmatrix} \cos \theta \\ \sin \theta \end{pmatrix} \pm (1-\kappa)B \begin{pmatrix} \sin \theta \\ \cos \theta \end{pmatrix}}{A^2 + B^2} \\ \mp \frac{[A(A^2 - 3B^2) - 2a(A^2 - B^2)] \begin{pmatrix} \cos \theta \\ \sin \theta \end{pmatrix} \pm [2AB(A-2a) + B(A^2 - B^2)] \begin{pmatrix} \sin \theta \\ \cos \theta \end{pmatrix}}{(A^2 + B^2)^2}$$

$$W_1 = \frac{(1+\kappa)A}{A^2 + B^2} + \frac{A(A^2 - 3B^2) - 2a(A^2 - B^2)}{(A^2 + B^2)^2}$$

and

$$W_2 = \frac{(1-\kappa)B}{A^2 + B^2} + \frac{B(3A^2 - B^2) - 4aAB}{(A^2 + B^2)^2}$$

* The non-dimensional rigid body displacement of the plate is

$$\frac{(1+\kappa)R}{2a}$$

in the positive x direction.

At all points around the interface the sum of the magnitudes of the displacements of disc and plate must equal the radial interference. In terms of the above non-dimensional presentation, subtracting (16) from (18),

$$\frac{2G}{q} \left(\frac{u_{rp}}{R} - \frac{u_{rd}}{R} \right) = 1 - \frac{1-\kappa}{2}$$

$$= \frac{1+\kappa}{2}$$

$(1+\kappa)/2$ is the non-dimensional radial interference between disc and plate.

As for the stresses, some simplifications occur at particular points. Radial displacements at the interface at $\theta = 0$ and $\theta = \pi$ follow from (16) and (18), and from (20) and (21) displacements of the free edge in the x and y directions are found:

Around the interface $r = R$ and (16) and (18) give, for the disc

$$\left(\frac{2G}{q} \frac{u_{rd}}{R} \right)_0 = \frac{1-\kappa}{2} + \frac{(1+\kappa)R^2}{2a(2a \mp R)} - \frac{R^2}{(2a \mp R)^2}$$

and for the plate

$$\left(\frac{2G}{q} \frac{u_{rp}}{R} \right)_0 = \mp 1 \mp \frac{(1+\kappa)R^2}{2a(2a \mp R)} \pm \frac{R^2}{(2a \mp R)^2}$$

Along the free edge $x = 0$ and (20) and (21) become

$$\frac{2G}{q} \frac{u_x}{R} = \frac{(1+\kappa)R}{2a} - \frac{(1+\kappa)aR}{a^2+y^2}$$

and

$$\frac{2G}{q} \frac{v_y}{R} = \frac{(1+\kappa)yR}{a^2+y^2}$$

At the origin and at infinity

$$\left(\frac{2G}{q} \frac{u_x}{R} \right)_0 = \frac{(1+\kappa)R}{2a} - \left(\frac{2G}{q} \frac{u_x}{R} \right)_\infty$$

and

$$\left(\frac{2G}{q} \frac{v_y}{R} \right)_0 = 0 = \left(\frac{2G}{q} \frac{v_y}{R} \right)_\infty$$

At $y = \pm a$, $u_x = 0$ and v_y has its maximum value given by

$$\frac{2G}{q} \frac{v_y}{R} = \frac{(1+\kappa)R}{2a}$$

The appropriate equations have been used to evaluate plate displacements for representative values of a/R from 1 to infinity for plane stress and plane strain conditions along the axis of symmetry (x axis) and the free edge (y axis), Tables 3 and 4, and around the hole, Tables 5 and 6. Figures 7 and 8 show the effect of a/R on the transverse displacement of the free edge and on the radial displacement of the hole respectively. Fig. 9 shows the displacement fields for some low value a/R (a bipolar grid is used for its orthogonality to *both* boundaries). The Tables and Figures show the following:

*Along the free edge,** the transverse displacements show the expected maximum bulge at the origin for the lowest a/R , reducing to zero at $y = a$ and asymptoting towards the rigid body displacement position at infinity, Fig. 7. As previously noted, origin and remote transverse free edge displacements are of equal magnitude but opposite sign.

Around the interface, where the sum of the magnitudes of the radial displacements (i.e. the interference) is constant at all points, the maximum radial expansion of the plate occurs always at the minimum section between hole and edge: this also corresponds to the minimum compression of the disc and reference to Fig. 8 aids in understanding the existence of tensile circumferential *disc* stresses in this region at low a/R . The minimum radial expansion of the plate occurs at $\theta \sim 105^\circ$ for $a/R = 1$ and asymptotes towards $\theta = \pi/2$ with increasing a/R †: its location is therefore remarkably stable. Circumferential displacements of both disc and plate are identical at all points around the interface.

5. CONCLUSIONS

For a bonded interference-fit disc in an elastic half plane of the same material, the present analysis reveals the following:

- (a) The maximum tangential stress along the free edge occurs always at the point nearest the hole.
- (b) The maximum circumferential plate stress at the hole occurs at the point nearest the free edge up to an edge distance ratio of 1.724. Beyond this value the location of the maximum stress around the hole changes monotonically.
- (c) The maximum tangential stress at the free edge exceeds the maximum circumferential stress at the hole only up to an edge distance ratio of 1.767. Beyond a ratio of 1.975 the stresses everywhere around the hole exceed the maximum stress at the free edge.
- (d) The insertion of an interference-fit disc results in a relative displacement of disc and plate.
- (e) Considering the disc centre as fixed, the maximum displacement of the free edge at the point nearest the hole equals the rigid body displacement of the plate in the opposite direction.
- (f) The maximum out-of-round distortion of the interface occurs in the region nearest the free edge. It is a maximum when the hole and free edge just meet.

* All displacements for both plate and disc are with respect to a fixed *disc* centre.

† Note that the abscissa is drawn through the ordinate value unity, not zero: all radial displacements are positive, i.e. outwards.

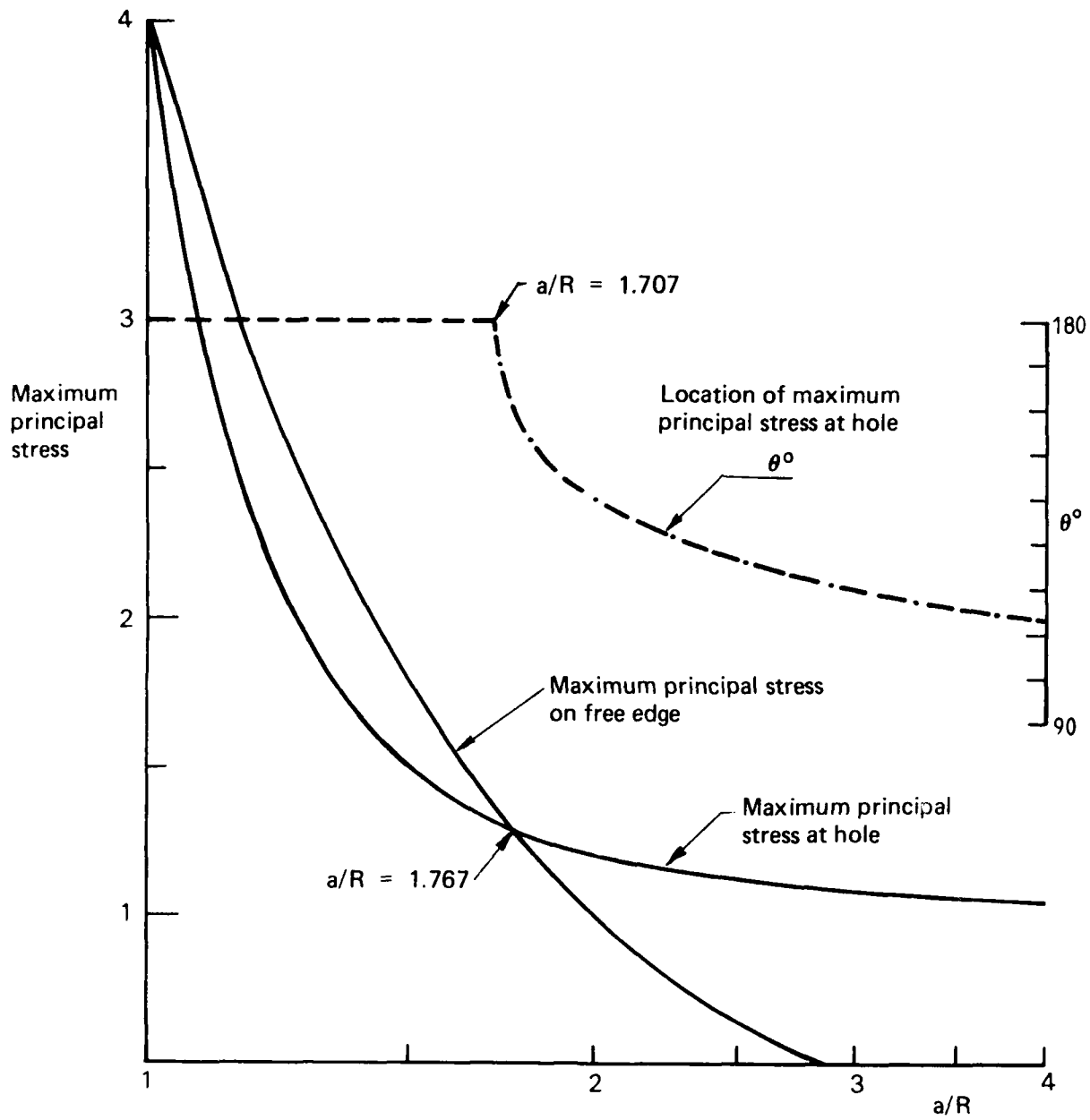
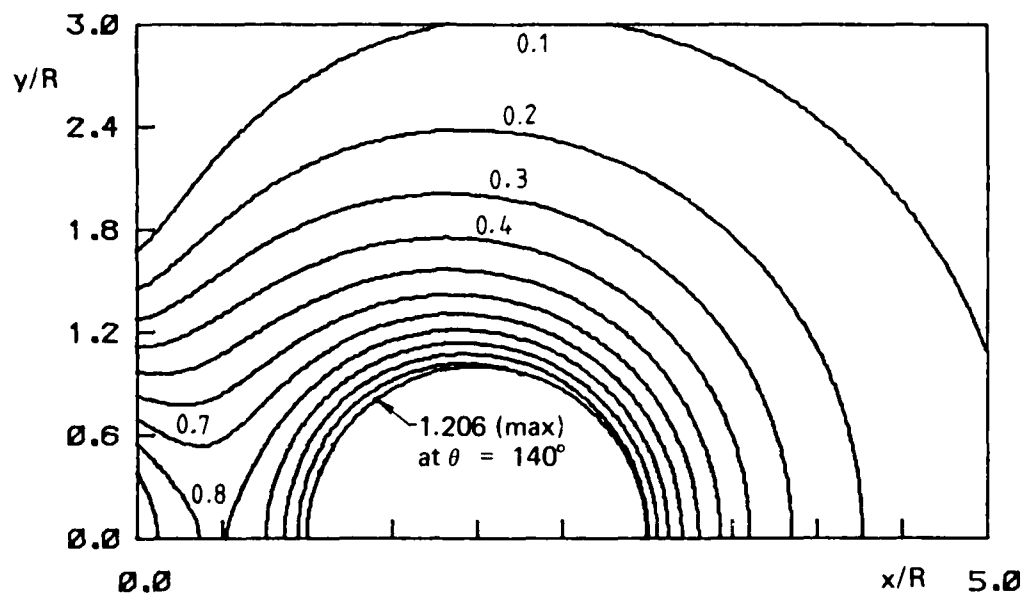
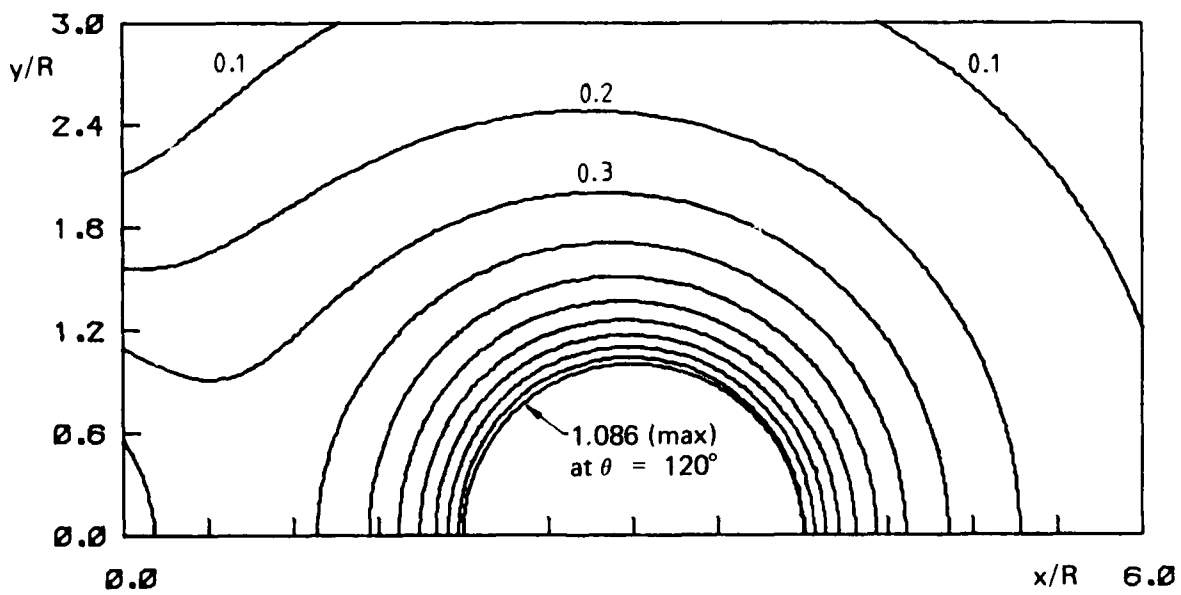


FIG. 6 MAXIMUM PRINCIPAL STRESSES AT HOLE AND FREE EDGE



(c) $a/R = 2.0$

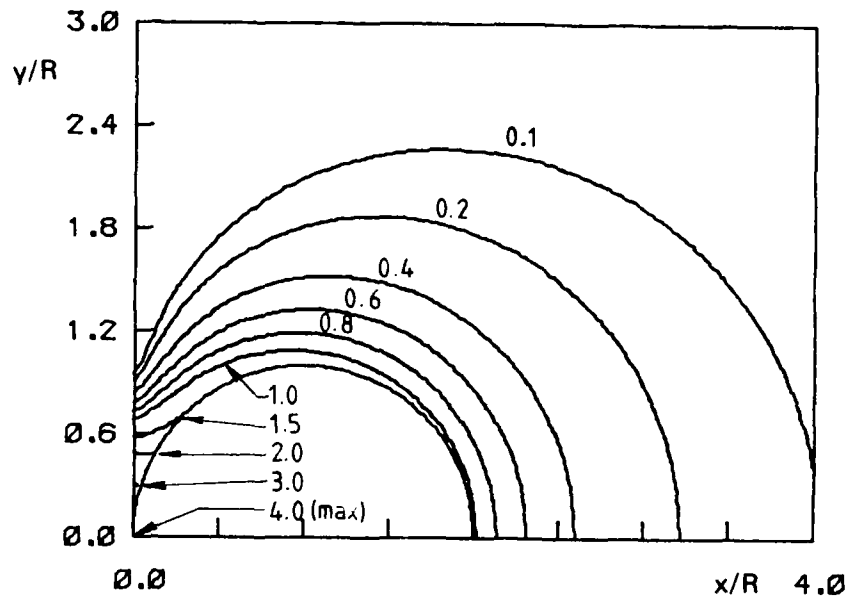
(contour interval = 0.1)



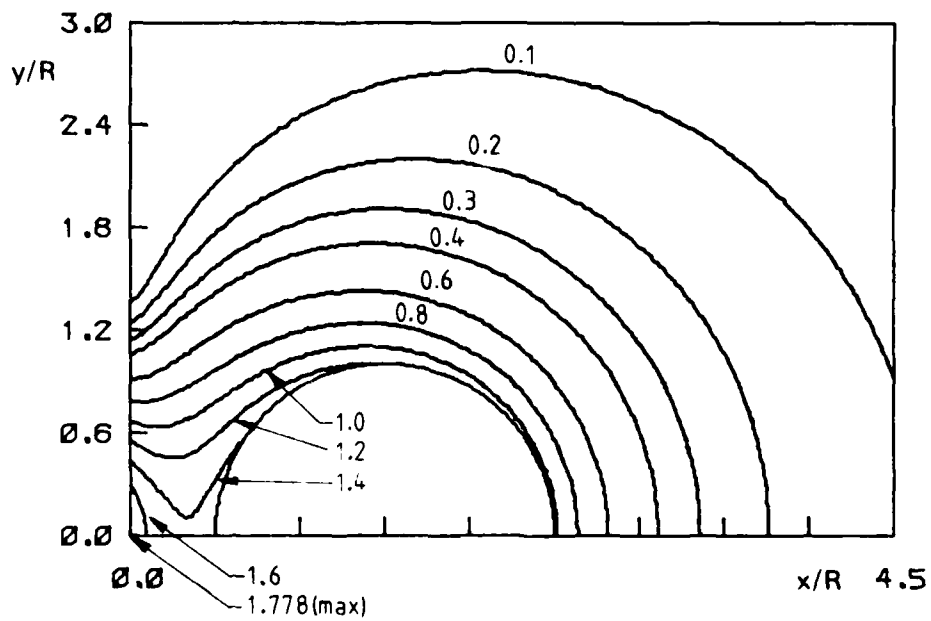
(d) $a/R = 3.0$

(contour interval = 0.1)

FIG. 5(c), (d) CONTOURS OF MAXIMUM NON-DIMENSIONAL PRINCIPAL STRESS



(a) $a/R = 1.0$



(b) $a/R = 1.5$

FIG. 5(a), (b) CONTOURS OF MAXIMUM NON-DIMENSIONAL PRINCIPAL STRESS

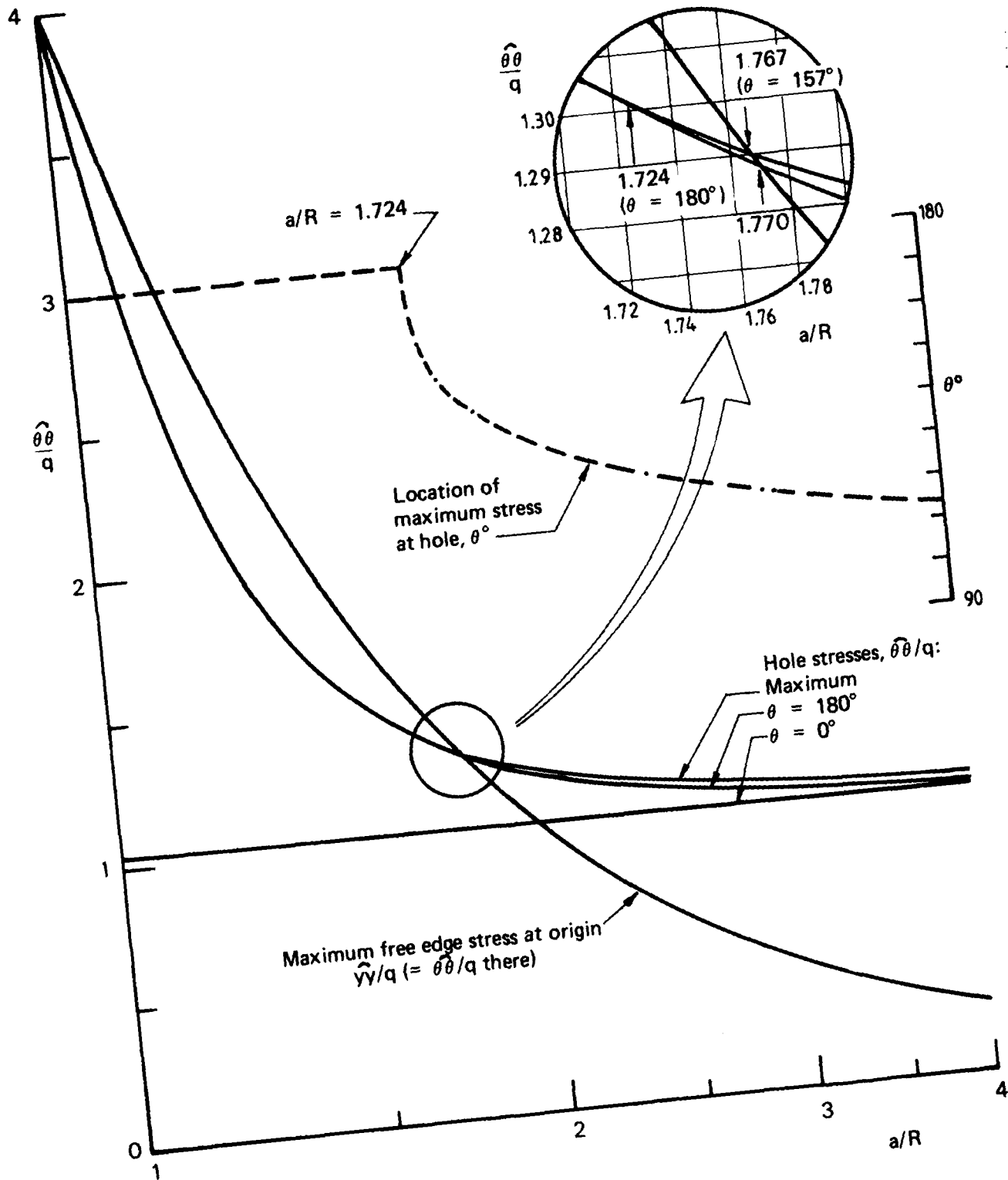


FIG. 4 INFLUENCE OF a/R ON STRESSES AT HOLE AND FREE EDGE, AND LOCATION OF MAXIMUM HOLE STRESS

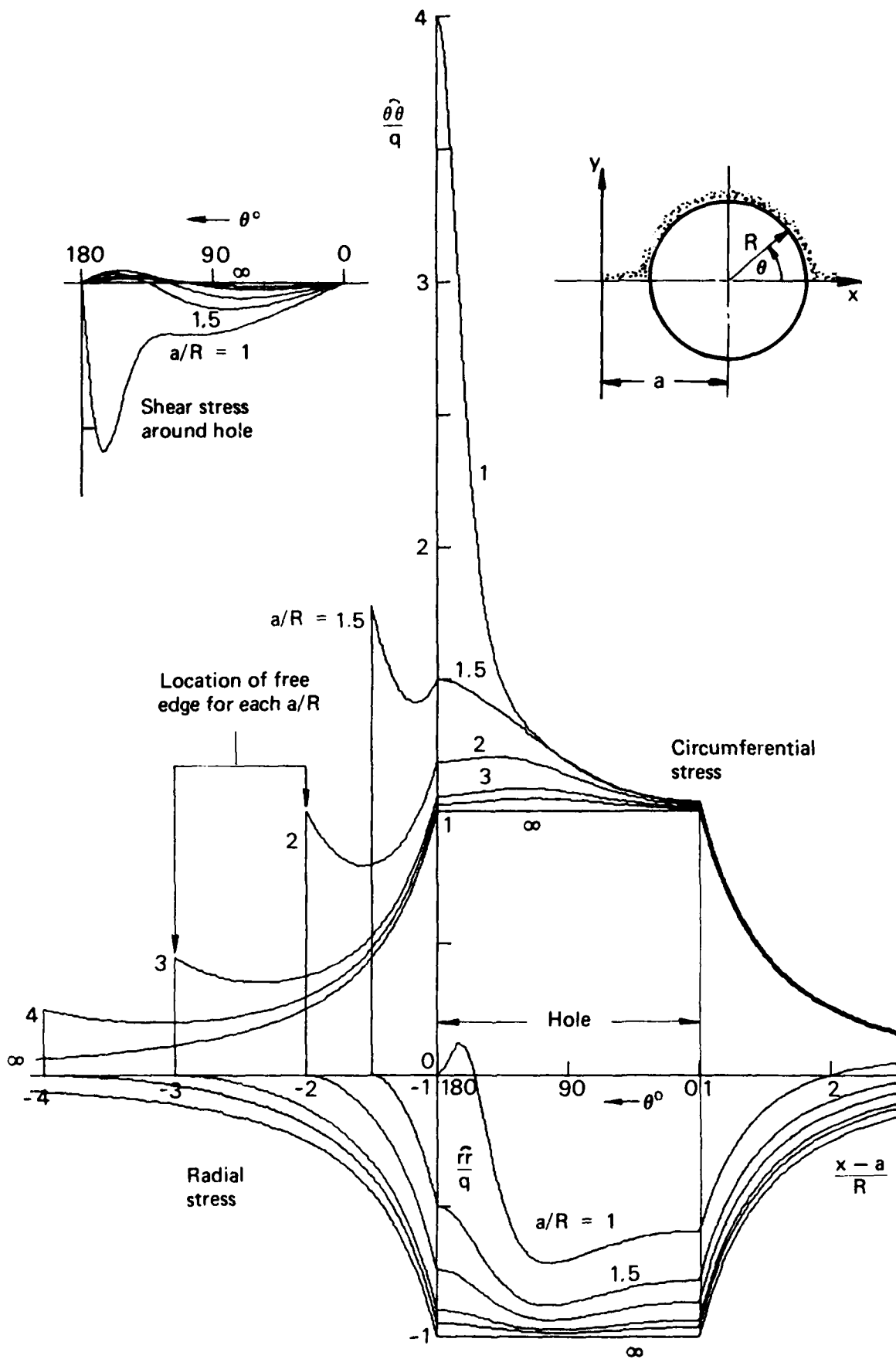


FIG. 3 STRESSES AROUND HOLE AND ALONG AXIS OF SYMMETRY

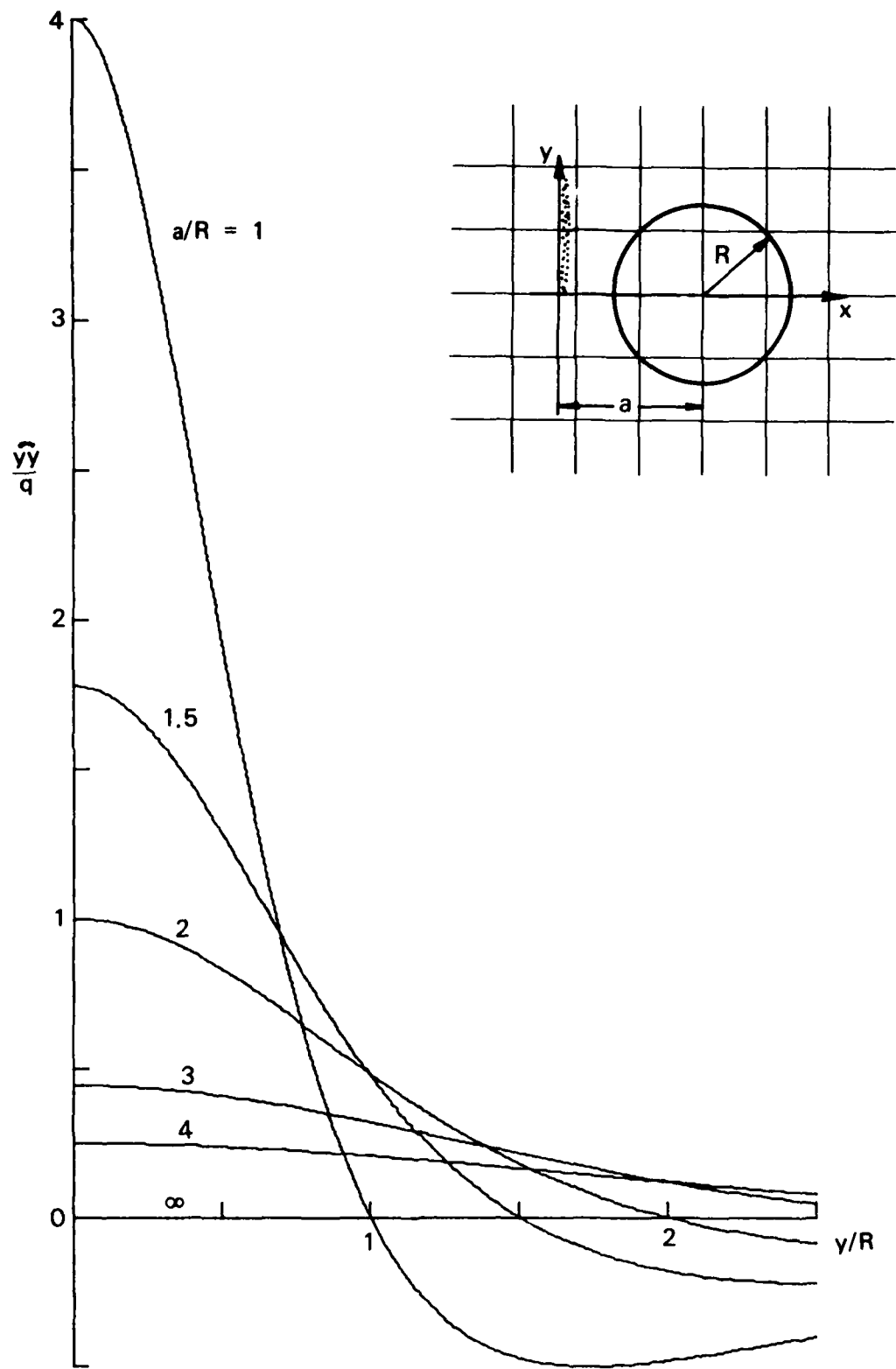


FIG. 2 TANGENTIAL STRESS ALONG FREE EDGE

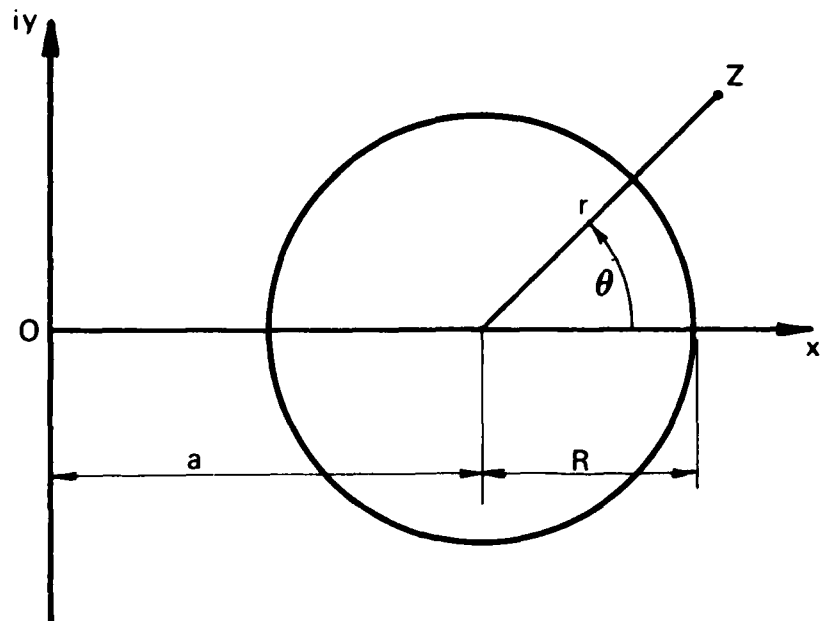


FIG. 1 CARTESIAN AND POLAR COORDINATE SYSTEM IN HALF PLANE CONTAINING HOLE

Table 6 NON-DIMENSIONAL PLATE DISPLACEMENTS - PLANE STRAIN ($\nu=0.3$)

(a) Axis of Symmetry

a/R=1.0		a/R=1.5		a/R=2.0		a/R=3.0		a/R=4.0		a/R=∞	
x/R	x/R	x/R	x/R	x/R	x/R	x/R	x/R	x/R	x/R	(x-a)/R	(x-a)/R
0.0	-1.4000	.00	-0.9333	0.0	-0.7000	0.0	-0.4667	0.0	-0.3500	-3.0	-0.3333
0.0	-1.4000	.05	-0.9592	.1	-0.7288	.2	-0.4920	.3	-0.3713	-2.8	-0.3571
0.0	-1.4000	.10	-0.9841	.2	-0.7564	.4	-0.5166	.6	-0.3921	-2.6	-0.3846
0.0	-1.4000	.15	-1.0085	.3	-0.7843	.6	-0.5426	.9	-0.4149	-2.4	-0.4167
0.0	-1.4000	.20	-1.0331	.4	-0.8139	.8	-0.5724	1.2	-0.4421	-2.2	-0.4545
0.0	-1.4000	.25	-1.0585	.5	-0.8467	1.0	-0.6083	1.5	-0.4764	-2.0	-0.5000
0.0	-1.4000	.30	-1.0852	.6	-0.8841	1.2	-0.6535	1.8	-0.5219	-1.8	-0.5556
0.0	-1.4000	.35	-1.1137	.7	-0.9279	1.4	-0.7121	2.1	-0.5843	-1.6	-0.6250
0.0	-1.4000	.40	-1.1447	.8	-0.9803	1.6	-0.7902	2.4	-0.6734	-1.4	-0.7143
0.0	-1.4000	.45	-1.1788	.9	-1.0438	1.8	-0.8979	2.7	-0.8082	-1.2	-0.8333
0.0	-1.4000	.50	-1.2167	1.0	-1.1222	2.0	-1.0533	3.0	-1.0296	-1.0	-1.0000
2.0	1.3556	2.50	1.1708	3.0	1.1000	4.0	1.0463	5.0	1.0265	1.0	1.0000
2.2	1.2411	2.70	1.0320	3.2	0.9505	4.2	0.8880	5.2	0.8648	1.2	0.8333
2.4	1.1696	2.90	0.9389	3.4	0.8478	4.4	0.7770	5.4	0.7506	1.4	0.7143
2.6	1.1238	3.10	0.8740	3.6	0.7740	4.6	0.6955	5.6	0.6660	1.6	0.6250
2.8	1.0941	3.30	0.8274	3.8	0.7193	4.8	0.6337	5.8	0.6011	1.8	0.5556
3.0	1.0750	3.50	0.7933	4.0	0.6778	5.0	0.5854	6.0	0.5500	2.0	0.5000
3.2	1.0632	3.70	0.7681	4.2	0.6457	5.2	0.5470	6.2	0.5089	2.2	0.4545
3.4	1.0563	3.90	0.7492	4.4	0.6206	5.4	0.5160	6.4	0.4752	2.4	0.4167
3.6	1.0530	4.10	0.7350	4.6	0.6007	5.6	0.4905	6.6	0.4473	2.6	0.3846
3.8	1.0523	4.30	0.7245	4.8	0.5848	5.8	0.4695	6.8	0.4239	2.8	0.3571
4.0	1.0533	4.50	0.7167	5.0	0.5721	6.0	0.4519	7.0	0.4040	3.0	0.3333
∞	1.4000		0.9333		0.7000		0.4667		0.3500		0.0000

(b) Free Edge

a/R	1.0	1.5	2.0	3.0	4.0
y/R 0.0	-1.4000	-0.9333	-0.7000	-0.4667	-0.3500
.2	-1.2923	-0.9007	-0.6861	-0.4625	-0.3483
.4	-1.0138	-0.8094	-0.6462	-0.4504	-0.3431
.6	-0.6588	-0.6759	-0.5844	-0.4308	-0.3346
.8	-0.3073	-0.5200	-0.5069	-0.4047	-0.3231
1.0	0.0000	-0.3590	-0.4200	-0.3733	-0.3088
1.2	0.2525	-0.2049	-0.3294	-0.3379	-0.2922
1.4	0.4541	-0.0643	-0.2396	-0.2998	-0.2736
1.6	0.6135	0.0602	-0.1537	-0.2600	-0.2534
1.8	0.7396	0.1683	-0.0735	-0.2196	-0.2321
2.0	0.8400	0.2613	0.0000	-0.1795	-0.2100
∞	1.4000	0.9333	0.7000	0.4667	0.3500

Table 5 NON-DIMENSIONAL PLATE DISPLACEMENTS - PLANE STRESS ($\nu=0.3$)

(a) Axis of Symmetry

a/R=1.0	a/R=1.5	a/R=2.0	a/R=3.0	a/R=4.0	a/R=∞						
x/R	x/R	x/R	x/R	x/R	(x-a)/R						
0.0	-1.5385	.00	-1.0256	0.0	-0.7692	0.0	-0.5128	0.0	-0.3846	-3.0	-0.3333
0.0	-1.5385	.05	-1.0456	.1	-0.7914	.2	-0.5324	.3	-0.4011	-2.8	-0.3571
0.0	-1.5385	.10	-1.0648	.2	-0.8130	.4	-0.5519	.6	-0.4177	-2.6	-0.3846
0.0	-1.5385	.15	-1.0840	.3	-0.8354	.6	-0.5734	.9	-0.4368	-2.4	-0.4167
0.0	-1.5385	.20	-1.1037	.4	-0.8600	.8	-0.5991	1.2	-0.4607	-2.2	-0.4545
0.0	-1.5385	.25	-1.1244	.5	-0.8882	1.0	-0.6314	1.5	-0.4922	-2.0	-0.5000
0.0	-1.5385	.30	-1.1467	.6	-0.9214	1.2	-0.6733	1.8	-0.5350	-1.8	-0.5556
0.0	-1.5385	.35	-1.1711	.7	-0.9613	1.4	-0.7288	2.1	-0.5951	-1.6	-0.6250
0.0	-1.5385	.40	-1.1982	.8	-1.0099	1.6	-0.8042	2.4	-0.6821	-1.4	-0.7143
0.0	-1.5385	.45	-1.2285	.9	-1.0701	1.8	-0.9095	2.7	-0.8149	-1.2	-0.8333
0.0	-1.5385	.50	-1.2628	1.0	-1.1453	2.0	-1.0626	3.0	-1.0345	-1.0	-1.0000
2.0	1.4017	2.50	1.1939	3.0	1.1138	4.0	1.0529	5.0	1.0304	1.0	1.0000
2.2	1.2931	2.70	1.0583	3.2	0.9665	4.2	0.8957	5.2	0.8693	1.2	0.8333
2.4	1.2267	2.90	0.9683	3.4	0.8657	4.4	0.7857	5.4	0.7557	1.4	0.7143
2.6	1.1853	3.10	0.9061	3.6	0.7938	4.6	0.7053	5.6	0.6717	1.6	0.6250
2.8	1.1596	3.30	0.8620	3.8	0.7408	4.8	0.6443	5.8	0.6075	1.8	0.5556
3.0	1.1442	3.50	0.8303	4.0	0.7009	5.0	0.5970	6.0	0.5569	2.0	0.5000
3.2	1.1357	3.70	0.8071	4.2	0.6703	5.2	0.5594	6.2	0.5164	2.2	0.4545
3.4	1.1319	3.90	0.7902	4.4	0.6465	5.4	0.5292	6.4	0.4832	2.4	0.4167
3.6	1.1313	4.10	0.7779	4.6	0.6280	5.6	0.5045	6.6	0.4558	2.6	0.3846
3.8	1.1331	4.30	0.7690	4.8	0.6133	5.8	0.4842	6.8	0.4329	2.8	0.3571
4.0	1.1364	4.50	0.7628	5.0	0.6018	6.0	0.4672	7.0	0.4134	3.0	0.3333
∞	1.5385	1.0256	0.7692	0.5128	0.3846	0.0000					

(b) Free Edge

a/R	1.0	1.5	2.0	3.0	4.0
y/R 0.0	-1.5385	-1.0256	-0.7692	-0.5128	-0.3846
.2	-1.4201	-0.9898	-0.7540	-0.5083	-0.3827
.4	-1.1141	-0.8895	-0.7101	-0.4949	-0.3770
.6	-0.7240	-0.7427	-0.6422	-0.4734	-0.3677
.8	-0.3377	-0.5714	-0.5570	-0.4447	-0.3550
1.0	0.0000	-0.3945	-0.4615	-0.4103	-0.3394
1.2	0.2774	-0.2251	-0.3620	-0.3714	-0.3211
1.4	0.4990	-0.0706	-0.2633	-0.3294	-0.3007
1.6	0.6742	0.0661	-0.1689	-0.2857	-0.2785
1.8	0.8128	0.1850	-0.0807	-0.2413	-0.2551
2.0	0.9231	0.2872	0.0000	-0.1972	-0.2308
∞	1.5385	1.0256	0.7692	0.5128	0.3846

Table 4 NON-DIMENSIONAL POLAR DISPLACEMENTS AROUND INTERFACE
FOR PLATE AND DISC-PLANE STRAIN ($\nu=0.3$)

a/R=1.0				a/R=1.5			
ANGLE	PLATE		DISC	ANGLE	PLATE		DISC
	RADIAL	CIRCUM	RADIAL		RADIAL	CIRCUM	RADIAL
0	1.3556	0.0000	-0.0444	0	1.1708	0.0000	-0.2292
10	1.3496	-0.0784	-0.0504	10	1.1674	-0.0372	-0.2326
20	1.3320	-0.1543	-0.0680	20	1.1572	-0.0725	-0.2428
30	1.3038	-0.2250	-0.0962	30	1.1409	-0.1044	-0.2591
40	1.2664	-0.2884	-0.1336	40	1.1196	-0.1313	-0.2804
50	1.2221	-0.3424	-0.1779	50	1.0946	-0.1517	-0.3054
60	1.1735	-0.3853	-0.2265	60	1.0679	-0.1647	-0.3321
70	1.1241	-0.4161	-0.2759	70	1.0415	-0.1693	-0.3585
80	1.0780	-0.4342	-0.3220	80	1.0179	-0.1655	-0.3821
90	1.0400	-0.4400	-0.3600	90	1.0000	-0.1533	-0.4000
100	1.0157	-0.4349	-0.3843	100	0.9905	-0.1338	-0.4095
110	1.0114	-0.4215	-0.3886	110	0.9921	-0.1086	-0.4079
120	1.0333	-0.4041	-0.3667	120	1.0068	-0.0801	-0.3932
130	1.0863	-0.3886	-0.3137	130	1.0354	-0.0518	-0.3646
140	1.1695	-0.3795	-0.2305	140	1.0762	-0.0271	-0.3238
150	1.2690	-0.3726	-0.1310	150	1.1240	-0.0093	-0.2760
160	1.3537	-0.3387	-0.0463	160	1.1703	0.0001	-0.2297
170	1.3939	-0.2201	-0.0061	170	1.2042	0.0022	-0.1958
180	1.4000	0.0000	0.0000	180	1.2167	0.0000	-0.1833

a/R=2.0				a/R=3.0			
ANGLE	PLATE		DISC	ANGLE	PLATE		DISC
	RADIAL	CIRCUM	RADIAL		RADIAL	CIRCUM	RADIAL
0	1.1000	0.0000	-0.3000	0	1.0463	0.0000	-0.3537
10	1.0977	-0.0214	-0.3023	10	1.0451	-0.0097	-0.3549
20	1.0911	-0.0416	-0.3089	20	1.0416	-0.0187	-0.3584
30	1.0805	-0.0593	-0.3195	30	1.0362	-0.0263	-0.3638
40	1.0668	-0.0735	-0.3332	40	1.0292	-0.0319	-0.3708
50	1.0510	-0.0832	-0.3490	50	1.0213	-0.0351	-0.3787
60	1.0344	-0.0878	-0.3656	60	1.0132	-0.0356	-0.3868
70	1.0184	-0.0868	-0.3816	70	1.0058	-0.0334	-0.3942
80	1.0047	-0.0804	-0.3953	80	0.9998	-0.0285	-0.4002
90	0.9952	-0.0689	-0.4048	90	0.9961	-0.0214	-0.4039
100	0.9912	-0.0532	-0.4088	100	0.9953	-0.0128	-0.4047
110	0.9943	-0.0348	-0.4057	110	0.9978	-0.0036	-0.4022
120	1.0050	-0.0159	-0.3950	120	1.0037	0.0050	-0.3963
130	1.0232	0.0012	-0.3768	130	1.0125	0.0119	-0.3875
140	1.0472	0.0140	-0.3528	140	1.0232	0.0159	-0.3768
150	1.0738	0.0203	-0.3262	150	1.0344	0.0165	-0.3656
160	1.0984	0.0193	-0.3016	160	1.0442	0.0136	-0.3558
170	1.1159	0.0116	-0.2841	170	1.0509	0.0076	-0.3491
180	1.1222	0.0000	-0.2778	180	1.0533	0.0000	-0.3467

a/R=4.0				a/R=∞			
ANGLE	PLATE		DISC	ANGLE	PLATE		DISC
	RADIAL	CIRCUM	RADIAL		RADIAL	CIRCUM	RADIAL
0	1.0265	0.0000	-0.3735	0	1.0000	0.0000	-0.4000
10	1.0258	-0.0055	-0.3742	10	1.0000	-0.0000	-0.4000
20	1.0237	-0.0105	-0.3763	20	1.0000	-0.0000	-0.4000
30	1.0204	-0.0146	-0.3796	30	1.0000	-0.0000	-0.4000
40	1.0162	-0.0175	-0.3838	40	1.0000	-0.0000	-0.4000
50	1.0115	-0.0190	-0.3885	50	1.0000	-0.0000	-0.4000
60	1.0068	-0.0188	-0.3932	60	1.0000	-0.0000	-0.4000
70	1.0025	-0.0169	-0.3975	70	1.0000	-0.0000	-0.4000
80	0.9993	-0.0136	-0.4007	80	1.0000	-0.0000	-0.4000
90	0.9974	-0.0092	-0.4026	90	1.0000	-0.0000	-0.4000
100	0.9973	-0.0040	-0.4027	100	1.0000	0.0000	-0.4000
110	0.9991	0.0012	-0.4009	110	1.0000	0.0000	-0.4000
120	1.0027	0.0059	-0.3973	120	1.0000	0.0000	-0.4000
130	1.0077	0.0093	-0.3923	130	1.0000	0.0000	-0.4000
140	1.0137	0.0111	-0.3863	140	1.0000	0.0000	-0.4000
150	1.0197	0.0108	-0.3803	150	1.0000	0.0000	-0.4000
160	1.0249	0.0085	-0.3751	160	1.0000	0.0000	-0.4000
170	1.0284	0.0047	-0.3716	170	1.0000	0.0000	-0.4000
180	1.0296	0.0000	-0.3704	180	1.0000	0.0000	-0.4000

Table 3 NON-DIMENSIONAL POLAR DISPLACEMENTS AROUND INTERFACE
FOR PLATE AND DISC-PLANE STRESS ($\nu=0.3$)

a/R=1.0				a/R=1.5			
ANGLE	PLATE		DISC	ANGLE	PLATE		DISC
	RADIAL	CIRCUM	RADIAL		RADIAL	CIRCUM	RADIAL
0	1.4017	0.0000	-0.1368	0	1.1939	0.0000	-0.3446
10	1.3958	-0.0811	-0.1426	10	1.1905	-0.0382	-0.3480
20	1.3785	-0.1597	-0.1599	20	1.1804	-0.0746	-0.3580
30	1.3507	-0.2332	-0.1878	30	1.1644	-0.1075	-0.3741
40	1.3139	-0.2994	-0.2245	40	1.1434	-0.1354	-0.3951
50	1.2704	-0.3564	-0.2681	50	1.1189	-0.1568	-0.4196
60	1.2229	-0.4024	-0.3155	60	1.0927	-0.1708	-0.4458
70	1.1750	-0.4365	-0.3635	70	1.0671	-0.1765	-0.4714
80	1.1308	-0.4581	-0.4076	80	1.0445	-0.1737	-0.4940
90	1.0954	-0.4677	-0.4431	90	1.0277	-0.1626	-0.5108
100	1.0745	-0.4665	-0.4640	100	1.0196	-0.1439	-0.5189
110	1.0746	-0.4573	-0.4639	110	1.0229	-0.1195	-0.5155
120	1.1026	-0.4441	-0.4359	120	1.0398	-0.0915	-0.4987
130	1.1637	-0.4322	-0.3748	130	1.0708	-0.0633	-0.4676
140	1.2577	-0.4254	-0.2807	140	1.1143	-0.0381	-0.4241
150	1.3712	-0.4177	-0.1673	150	1.1650	-0.0189	-0.3734
160	1.4719	-0.3768	-0.0665	160	1.2139	-0.0071	-0.3246
170	1.5264	-0.2428	-0.0121	170	1.2496	-0.0017	-0.2888
180	1.5385	0.0000	0.0000	180	1.2628	0.0000	-0.2756

a/R=2.0				a/R=3.0			
ANGLE	PLATE		DISC	ANGLE	PLATE		DISC
	RADIAL	CIRCUM	RADIAL		RADIAL	CIRCUM	RADIAL
0	1.1138	0.0000	-0.4246	0	1.0529	0.0000	-0.4856
10	1.1116	-0.0219	-0.4269	10	1.0517	-0.0098	-0.4868
20	1.1050	-0.0426	-0.4334	20	1.0483	-0.0190	-0.4902
30	1.0946	-0.0608	-0.4438	30	1.0429	-0.0267	-0.4956
40	1.0811	-0.0754	-0.4574	40	1.0360	-0.0325	-0.5025
50	1.0655	-0.0856	-0.4729	50	1.0282	-0.0359	-0.5103
60	1.0492	-0.0906	-0.4893	60	1.0202	-0.0366	-0.5183
70	1.0336	-0.0901	-0.5048	70	1.0129	-0.0344	-0.5256
80	1.0205	-0.0841	-0.5180	80	1.0071	-0.0296	-0.5314
90	1.0114	-0.0729	-0.5270	90	1.0035	-0.0226	-0.5349
100	1.0082	-0.0575	-0.5303	100	1.0030	-0.0141	-0.5355
110	1.0120	-0.0394	-0.5264	110	1.0058	-0.0049	-0.5327
120	1.0237	-0.0205	-0.5148	120	1.0119	0.0037	-0.5266
130	1.0428	-0.0032	-0.4956	130	1.0210	0.0106	-0.5175
140	1.0678	0.0099	-0.4706	140	1.0319	0.0149	-0.5065
150	1.0954	0.0169	-0.4431	150	1.0433	0.0157	-0.4952
160	1.1207	0.0168	-0.4177	160	1.0533	0.0129	-0.4852
170	1.1388	0.0103	-0.3997	170	1.0601	0.0073	-0.4783
180	1.1453	0.0000	-0.3932	180	1.0626	0.0000	-0.4759

a/R=4.0				a/R=∞			
ANGLE	PLATE		DISC	ANGLE	PLATE		DISC
	RADIAL	CIRCUM	RADIAL		RADIAL	CIRCUM	RADIAL
0	1.0304	0.0000	-0.5081	0	1.0000	0.0000	-0.5385
10	1.0297	-0.0055	-0.5088	10	1.0000	-0.0000	-0.5385
20	1.0276	-0.0106	-0.5109	20	1.0000	-0.0000	-0.5385
30	1.0243	-0.0148	-0.5142	30	1.0000	-0.0000	-0.5385
40	1.0201	-0.0178	-0.5183	40	1.0000	-0.0000	-0.5385
50	1.0155	-0.0193	-0.5230	50	1.0000	-0.0000	-0.5385
60	1.0108	-0.0192	-0.5276	60	1.0000	-0.0000	-0.5385
70	1.0066	-0.0174	-0.5318	70	1.0000	-0.0000	-0.5385
80	1.0034	-0.0141	-0.5350	80	1.0000	-0.0000	-0.5385
90	1.0017	-0.0097	-0.5368	90	1.0000	-0.0000	-0.5385
100	1.0016	-0.0046	-0.5368	100	1.0000	0.0000	-0.5385
110	1.0035	0.0007	-0.5350	110	1.0000	0.0000	-0.5385
120	1.0072	0.0054	-0.5313	120	1.0000	0.0000	-0.5385
130	1.0124	0.0088	-0.5261	130	1.0000	0.0000	-0.5385
140	1.0184	0.0106	-0.5200	140	1.0000	0.0000	-0.5385
150	1.0245	0.0104	-0.5139	150	1.0000	0.0000	-0.5385
160	1.0297	0.0083	-0.5087	160	1.0000	0.0000	-0.5385
170	1.0333	0.0046	-0.5052	170	1.0000	0.0000	-0.5385
180	1.0345	0.0000	-0.5039	180	1.0000	0.0000	-0.5385

Table 2 NON-DIMENSIONAL PLATE STRESSES AROUND HOLE

a/R=1.0				a/R=1.5			
ANGLE	CIRCUM	RADIAL	SHEAR	ANGLE	CIRCUM	RADIAL	SHEAR
0	1.0370	-0.5926	0.0000	0	1.0313	-0.7813	0.0000
10	1.0385	-0.5941	-0.0258	10	1.0330	-0.7826	-0.0177
20	1.0431	-0.5986	-0.0515	20	1.0384	-0.7865	-0.0349
30	1.0509	-0.6062	-0.0767	30	1.0475	-0.7930	-0.0514
40	1.0621	-0.6169	-0.1013	40	1.0604	-0.8019	-0.0665
50	1.0771	-0.6306	-0.1247	50	1.0774	-0.8132	-0.0797
60	1.0962	-0.6472	-0.1464	60	1.0985	-0.8264	-0.0903
70	1.1200	-0.6661	-0.1657	70	1.1240	-0.8408	-0.0972
80	1.1491	-0.6860	-0.1813	80	1.1539	-0.8553	-0.0995
90	1.1840	-0.7040	-0.1920	90	1.1880	-0.8680	-0.0960
100	1.2253	-0.7148	-0.1966	100	1.2257	-0.8759	-0.0855
110	1.2740	-0.7082	-0.1953	110	1.2661	-0.8746	-0.0676
120	1.3333	-0.6667	-0.1925	120	1.3076	-0.8586	-0.0429
130	1.4150	-0.5639	-0.2030	130	1.3487	-0.8219	-0.0144
140	1.5581	-0.3739	-0.2609	140	1.3883	-0.7612	0.0123
150	1.8676	-0.1111	-0.4094	150	1.4262	-0.6802	0.0298
160	2.5169	0.0983	-0.6068	160	1.4615	-0.5936	0.0323
170	3.4702	0.0863	-0.5565	170	1.4891	-0.5258	0.0204
180	4.0000	0.0000	0.0000	180	1.5000	-0.5000	0.0000

a/R=2.0				a/R=3.0			
ANGLE	CIRCUM	RADIAL	SHEAR	ANGLE	CIRCUM	RADIAL	SHEAR
0	1.0240	-0.8640	0.0000	0	1.0146	-0.9329	0.0000
10	1.0255	-0.8651	-0.0122	10	1.0155	-0.9337	-0.0066
20	1.0294	-0.8683	-0.0240	20	1.0183	-0.9358	-0.0129
30	1.0373	-0.8737	-0.0349	30	1.0228	-0.9393	-0.0184
40	1.0477	-0.8809	-0.0443	40	1.0290	-0.9439	-0.0228
50	1.0609	-0.8898	-0.0518	50	1.0365	-0.9494	-0.0257
60	1.0768	-0.8999	-0.0567	60	1.0452	-0.9554	-0.0268
70	1.0951	-0.9106	-0.0582	70	1.0545	-0.9613	-0.0257
80	1.1153	-0.9207	-0.0558	80	1.0638	-0.9665	-0.0223
90	1.1366	-0.9290	-0.0488	90	1.0725	-0.9702	-0.0166
100	1.1577	-0.9333	-0.0370	100	1.0796	-0.9714	-0.0088
110	1.1770	-0.9313	-0.0205	110	1.0844	-0.9693	0.0005
120	1.1925	-0.9203	-0.0008	120	1.0860	-0.9632	0.0101
130	1.2024	-0.8985	0.0195	130	1.0841	-0.9530	0.0185
140	1.2055	-0.8655	0.0362	140	1.0789	-0.9393	0.0239
150	1.2022	-0.8248	0.0446	150	1.0716	-0.9241	0.0250
160	1.1950	-0.7836	0.0407	160	1.0640	-0.9099	0.0207
170	1.1880	-0.7524	0.0244	170	1.0582	-0.8997	0.0118
180	1.1852	-0.7407	0.0000	180	1.0560	-0.8960	0.0000

a/R=4.0				a/R=∞			
ANGLE	CIRCUM	RADIAL	SHEAR	ANGLE	CIRCUM	RADIAL	SHEAR
0	1.0096	-0.9602	0.0000	0	1.0000	-1.0000	0.0000
10	1.0102	-0.9607	-0.0041	10	1.0000	-1.0000	-0.0000
20	1.0120	-0.9622	-0.0079	20	1.0000	-1.0000	-0.0000
30	1.0150	-0.9646	-0.0112	30	1.0000	-1.0000	-0.0000
40	1.0189	-0.9677	-0.0136	40	1.0000	-1.0000	-0.0000
50	1.0236	-0.9713	-0.0150	50	1.0000	-1.0000	-0.0000
60	1.0288	-0.9751	-0.0152	60	1.0000	-1.0000	-0.0000
70	1.0341	-0.9788	-0.0139	70	1.0000	-1.0000	-0.0000
80	1.0391	-0.9818	-0.0113	80	1.0000	-1.0000	-0.0000
90	1.0433	-0.9837	-0.0073	90	1.0000	-1.0000	-0.0000
100	1.0463	-0.9840	-0.0024	100	1.0000	-1.0000	0.0000
110	1.0476	-0.9824	0.0030	110	1.0000	-1.0000	0.0000
120	1.0470	-0.9787	0.0082	120	1.0000	-1.0000	0.0000
130	1.0444	-0.9729	0.0124	130	1.0000	-1.0000	0.0000
140	1.0404	-0.9657	0.0147	140	1.0000	-1.0000	0.0000
150	1.0355	-0.9580	0.0145	150	1.0000	-1.0000	0.0000
160	1.0308	-0.9511	0.0116	160	1.0000	-1.0000	0.0000
170	1.0275	-0.9463	0.0065	170	1.0000	-1.0000	0.0000
180	1.0262	-0.9446	0.0000	180	1.0000	-1.0000	0.0000

Table 1 NON-DIMENSIONAL PLATE STRESSES-AXIS OF SYMMETRY AND FREE EDGE

(a) Axis of Symmetry

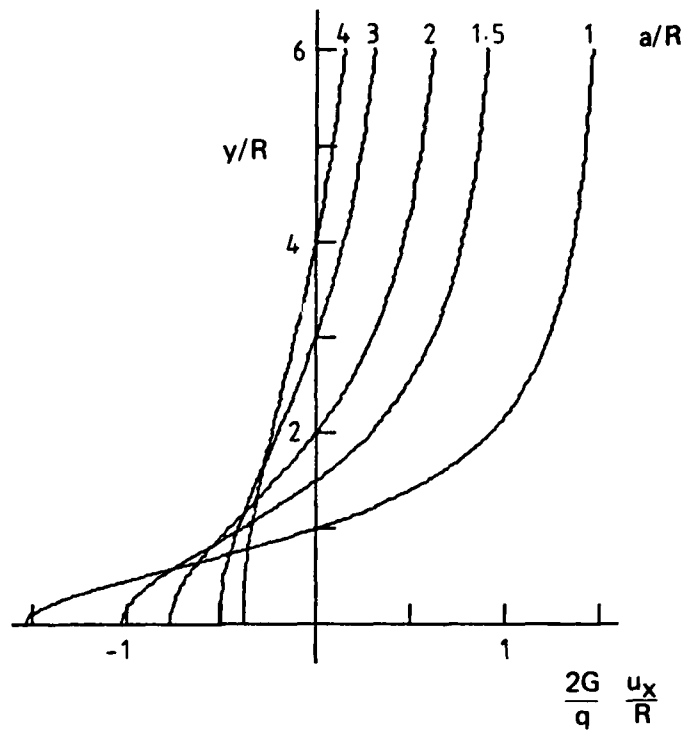
a/R=1.0			a/R=1.5			a/R=2.0		
x/R	PERP	PARALLEL	x/R	PERP	PARALLEL	x/R	PERP	PARALLEL
0.0	4.0000	0.0000	.00	1.7778	-0.0000	0.0	1.0000	0.0000
0.0	4.0000	0.0000	.05	1.6706	-0.0057	.1	0.9141	-0.0071
0.0	4.0000	0.0000	.10	1.5844	-0.0219	.2	0.8533	-0.0269
0.0	4.0000	0.0000	.15	1.5171	-0.0478	.3	0.8145	-0.0584
0.0	4.0000	0.0000	.20	1.4669	-0.0829	.4	0.7957	-0.1013
0.0	4.0000	0.0000	.25	1.4330	-0.1269		0.7964	-0.1564
0.0	4.0000	0.0000	.30	1.4146	-0.1800	.6	0.8174	-0.2257
0.0	4.0000	0.0000	.35	1.4116	-0.2428	.7	0.8610	-0.3123
0.0	4.0000	0.0000	.40	1.4242	-0.3162	.8	0.9313	-0.4211
0.0	4.0000	0.0000	.45	1.4532	-0.4013	.9	1.0356	-0.5599
0.0	4.0000	0.0000	.50	1.5000	-0.5000	1.0	1.1852	-0.7407
2.0	1.0370	-0.5926	2.50	1.0313	-0.7813	3.0	1.0240	-0.8640
2.2	0.7189	-0.3282	2.70	0.7187	-0.4920	3.2	0.7144	-0.5664
2.4	0.5255	-0.1794	2.90	0.5290	-0.3224	3.4	0.5267	-0.3895
2.6	0.3992	-0.0906	3.10	0.4050	-0.2160	3.6	0.4043	-0.2767
2.8	0.3123	-0.0353	3.30	0.3195	-0.1459	3.8	0.3199	-0.2010
3.0	0.2500	0.0000	3.50	0.2580	-0.0980	4.0	0.2593	-0.1481
3.2	0.2039	0.0228	3.70	0.2123	-0.0644	4.2	0.2142	-0.1101
3.4	0.1689	0.0377	3.90	0.1774	-0.0402	4.4	0.1797	-0.0821
3.6	0.1418	0.0473	4.10	0.1502	-0.0227	4.6	0.1528	-0.0610
3.8	0.1203	0.0533	4.30	0.1286	-0.0097	4.8	0.1314	-0.0449
4.0	0.1031	0.0569	4.50	0.1111	-0.0000	5.0	0.1140	-0.0324
a/R=3.0			a/R=4.0			a/R=∞		
x/R	PERP	PARALLEL	x/R	PERP	PARALLEL	(x-a)/R	PERP	PARALLEL
0.0	0.4444	-0.0000	0.0	0.2500	0.0000	-3.0	0.1111	-0.1111
.2	0.3961	-0.0055	0.3	0.2202	-0.0039	-2.8	0.1275	-0.1275
.4	0.3667	-0.0207	0.6	0.2036	-0.0146	-2.6	0.1479	-0.1479
.6	0.3537	-0.0450	0.9	0.1984	-0.0318	-2.4	0.1736	-0.1736
.8	0.3561	-0.0790	1.2	0.2044	-0.0564	-2.2	0.2066	-0.2066
1.0	0.3750	-0.1250	1.5	0.2231	-0.0909	-2.0	0.2500	-0.2500
1.2	0.4139	-0.1872	1.8	0.2589	-0.1400	-1.8	0.3086	-0.3086
1.4	0.4798	-0.2732	2.1	0.3206	-0.2131	-1.6	0.3906	-0.3906
1.6	0.5862	-0.3972	2.4	0.4272	-0.3296	-1.4	0.5102	-0.5102
1.8	0.7595	-0.5859	2.7	0.6226	-0.5335	-1.2	0.6945	-0.6945
2.0	1.0560	-0.8960	3.0	1.0262	-0.9446	-1.0	1.0000	-1.0000
4.0	1.0146	-0.9329	5.0	1.0096	-0.9602	1.0	1.0000	-1.0000
4.2	0.7073	-0.6301	5.2	0.7032	-0.6559	1.2	0.6945	-0.6945
4.4	0.5216	-0.4485	5.4	0.5182	-0.4729	1.4	0.5102	-0.5102
4.6	0.4006	-0.3314	5.6	0.3979	-0.3545	1.6	0.3906	-0.3906
4.8	0.3175	-0.2517	5.8	0.3152	-0.2736	1.8	0.3086	-0.3086
5.0	0.2578	-0.1953	6.0	0.2560	-0.2160	2.0	0.2500	-0.2500
5.2	0.2135	-0.1540	6.2	0.2121	-0.1736	2.2	0.2066	-0.2066
5.4	0.1797	-0.1230	6.4	0.1786	-0.1416	2.4	0.1736	-0.1736
5.6	0.1533	-0.0992	6.6	0.1525	-0.1169	2.6	0.1479	-0.1479
5.8	0.1322	-0.0806	6.8	0.1317	-0.0974	2.8	0.1275	-0.1275
6.0	0.1152	-0.0658	7.0	0.1149	-0.0818	3.0	0.1111	-0.1111

(b) Free Edge

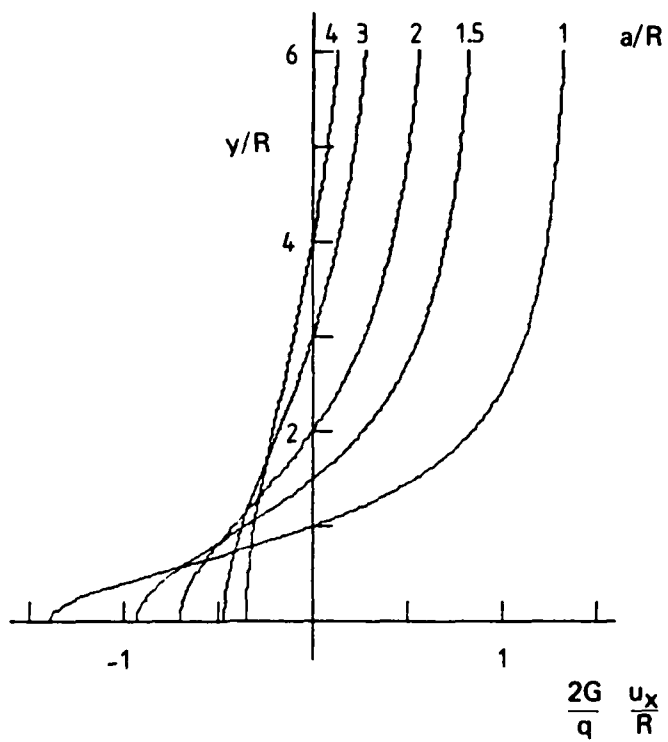
a/R	1.0	1.5	2.0	3.0	4.0
y/R 0.0	4.0000	1.7778	1.0000	0.4444	0.2500
.2	3.5503	1.6857	0.9705	0.4386	0.2481
.4	2.4970	1.4394	0.8876	0.4214	0.2426
.6	1.3841	1.1098	0.7659	0.3945	0.2337
.8	0.5354	0.7711	0.6243	0.3598	0.2219
1.0	0.0000	0.4734	0.4800	0.3200	0.2076
1.2	-0.2956	0.2380	0.3460	0.2774	0.1915
1.4	-0.4381	0.0654	0.2297	0.2344	0.1741
1.6	-0.4924	-0.0536	0.1338	0.1928	0.1561
1.8	-0.4984	-0.1314	0.0580	0.1538	0.1379
2.0	-0.4800	-0.1792	0.0000	0.1183	0.1200

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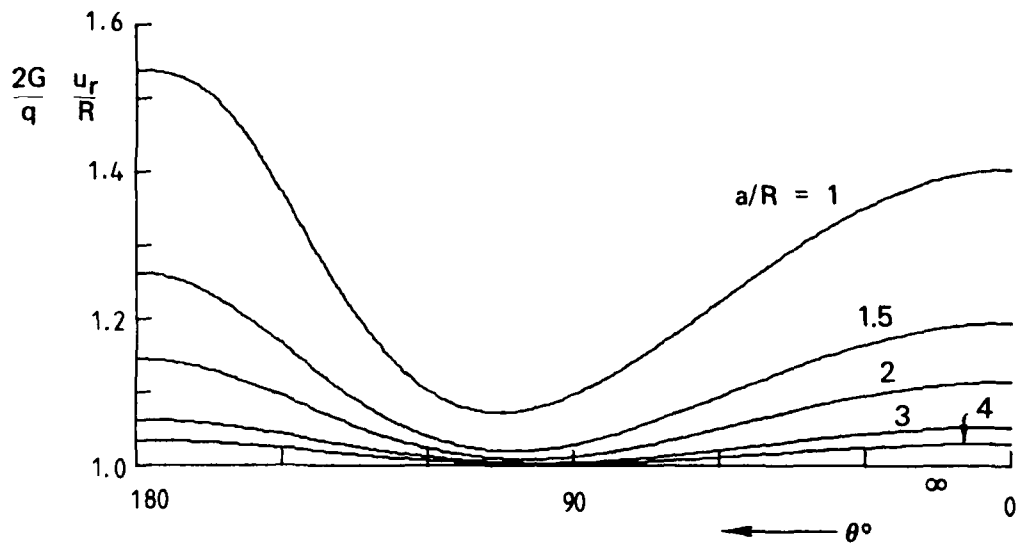


(a) Plane Stress

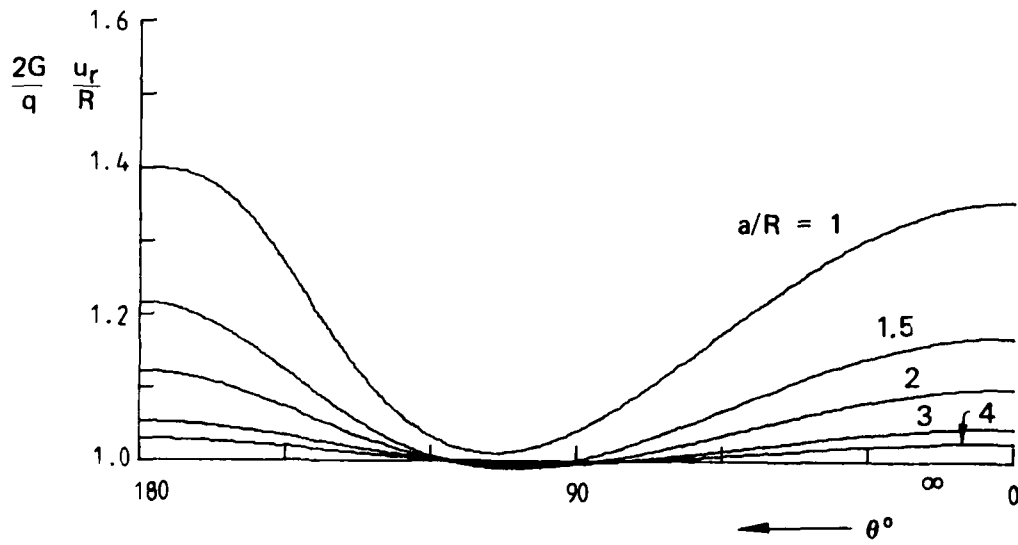


(b) Plane Strain

FIG. 7 TRANSVERSE DISPLACEMENT OF FREE EDGE ($\nu = 0.3$)

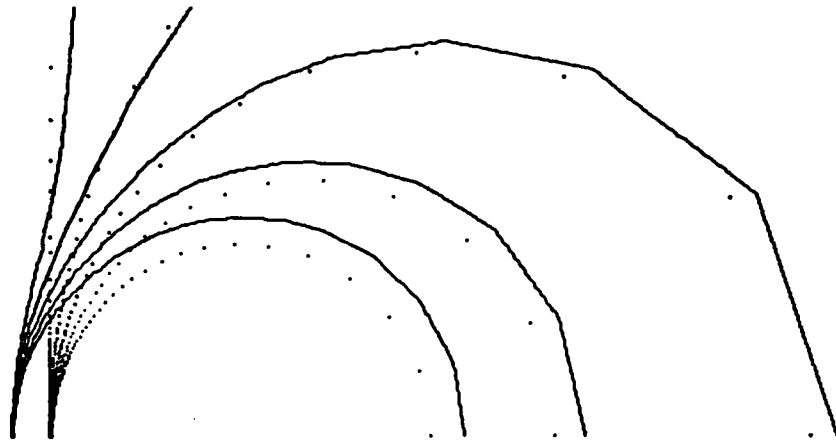


(a) Plane Stress

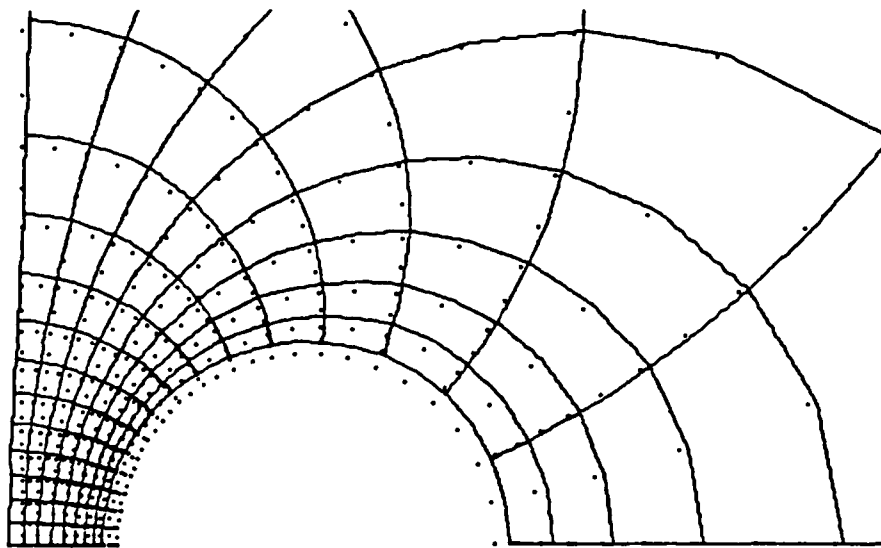


(b) Plane Strain

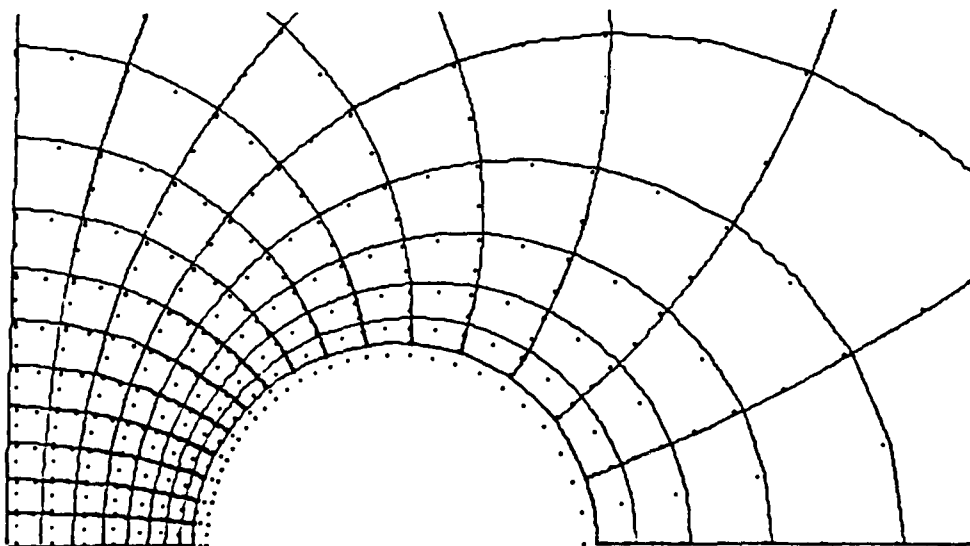
FIG. 8 RADIAL DISPLACEMENT OF HOLE ($\nu = 0.3$)



(a) $a/R = 1.0$ ($\lambda = 0.2$)



(b) $a/R = 1.5$ ($\lambda = 0.1$)



(c) $a/R = 2.0$ ($\lambda = 0.1$)

FIG. 9 DEFORMATION PLOTS - PLANE STRESS ($\nu = 0.3$)

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