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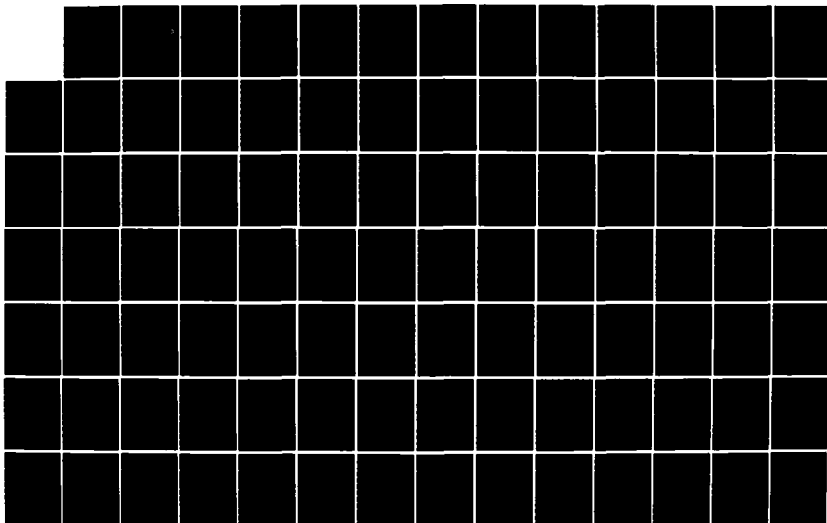
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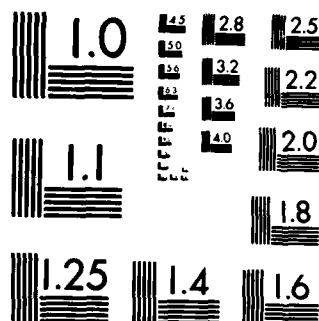
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THESIS

A Comparison of Methods
of
Least Squares Adjustment of Traverses

by

Saman Aumchantr

December 1984

Thesis Advisor:

Rolland L. Hardy

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Transverse Mercator grid coordinates. Closed traverse station positions were computed and adjusted using the Approximate Method and by the Least Squares Method. The adjusted coordinates of both methods were transformed from the Universal Transverse Mercator grid coordinates to geographic coordinates and compared with the coordinates which were adjusted by the U.S. National Ocean Service.

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A Comparison of Methods
of
Least Squares Adjustment of Traverses

by

Saman Aumchantr
Lieutenant, Royal Thai Navy
B.S., Royal Thai Naval Academy, 1976

Submitted in partial fulfillment of the
requirements for the degree of

MASTER OF SCIENCE IN HYDROGRAPHIC SCIENCES

from the

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December 1984

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ABSTRACT

Traverse is a method of surveying in which a sequence of lengths and directions of lines between points on the Earth are measured and used in determining positions of the points. This method is one of several used to find the accurate geodetic positions which various agencies use. Traversing is a convenient, rapid method for establishing horizontal control.

The theoretical background is provided here to explain the method of traverse station position computations and adjustments in the Universal Transverse Mercator grid coordinates. Closed traverse station positions were computed and adjusted using the Approximate Method and by the Least Squares Method. The adjusted coordinates of both methods were transformed from the Universal Transverse Mercator grid coordinates to geographic coordinates and compared with the coordinates which were adjusted by the U.S. National Ocean Service.

Handwritten notes:
The first part of the traverse is a closed traverse. The second part is an open traverse. The third part is a closed traverse. The fourth part is an open traverse. The fifth part is a closed traverse. The sixth part is an open traverse. The seventh part is a closed traverse. The eighth part is an open traverse. The ninth part is a closed traverse. The tenth part is an open traverse. The eleventh part is a closed traverse. The twelfth part is an open traverse. The thirteenth part is a closed traverse. The fourteenth part is an open traverse. The fifteenth part is a closed traverse. The sixteenth part is an open traverse. The seventeenth part is a closed traverse. The eighteenth part is an open traverse. The nineteenth part is a closed traverse. The twentieth part is an open traverse. The twenty-first part is a closed traverse. The twenty-second part is an open traverse. The twenty-third part is a closed traverse. The twenty-fourth part is an open traverse. The twenty-fifth part is a closed traverse. The twenty-sixth part is an open traverse. The twenty-seventh part is a closed traverse. The twenty-eighth part is an open traverse. The twenty-ninth part is a closed traverse. The thirtieth part is an open traverse. The thirty-first part is a closed traverse. The thirty-second part is an open traverse. The thirty-third part is a closed traverse. The thirty-fourth part is an open traverse. The thirty-fifth part is a closed traverse. The thirty-sixth part is an open traverse. The thirty-seventh part is a closed traverse. The thirty-eighth part is an open traverse. The thirty-ninth part is a closed traverse. The fortieth part is an open traverse. The forty-first part is a closed traverse. The forty-second part is an open traverse. The forty-third part is a closed traverse. The forty-fourth part is an open traverse. The forty-fifth part is a closed traverse. The forty-sixth part is an open traverse. The forty-seventh part is a closed traverse. The forty-eighth part is an open traverse. The forty-ninth part is a closed traverse. The fiftieth part is an open traverse. The fifty-first part is a closed traverse. The fifty-second part is an open traverse. The fifty-third part is a closed traverse. The fifty-fourth part is an open traverse. The fifty-fifth part is a closed traverse. The fifty-sixth part is an open traverse. The fifty-seventh part is a closed traverse. The fifty-eighth part is an open traverse. The fifty-ninth part is a closed traverse. The sixtieth part is an open traverse. The sixty-first part is a closed traverse. The sixty-second part is an open traverse. The sixty-third part is a closed traverse. The sixty-fourth part is an open traverse. The sixty-fifth part is a closed traverse. The sixty-sixth part is an open traverse. The sixty-seventh part is a closed traverse. The sixty-eighth part is an open traverse. The sixty-ninth part is a closed traverse. The seventieth part is an open traverse. The seventy-first part is a closed traverse. The seventy-second part is an open traverse. The seventy-third part is a closed traverse. The seventy-fourth part is an open traverse. The seventy-fifth part is a closed traverse. The seventy-sixth part is an open traverse. The seventy-seventh part is a closed traverse. The seventy-eighth part is an open traverse. The seventy-ninth part is a closed traverse. The eightieth part is an open traverse. The eighty-first part is a closed traverse. The eighty-second part is an open traverse. The eighty-third part is a closed traverse. The eighty-fourth part is an open traverse. The eighty-fifth part is a closed traverse. The eighty-sixth part is an open traverse. The eighty-seventh part is a closed traverse. The eighty-eighth part is an open traverse. The eighty-ninth part is a closed traverse. The ninetieth part is an open traverse. The ninety-first part is a closed traverse. The ninety-second part is an open traverse. The ninety-third part is a closed traverse. The ninety-fourth part is an open traverse. The ninety-fifth part is a closed traverse. The ninety-sixth part is an open traverse. The ninety-seventh part is a closed traverse. The ninety-eighth part is an open traverse. The ninety-ninth part is a closed traverse. The hundredth part is an open traverse.

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I. INTRODUCTION

Hydrographic surveying includes many branches of science for the purpose of the production of nautical charts specially designed for use by the mariner. The determination of position in hydrographic surveying is as important as the measurement of depth. Before determining an accurate hydrographic position, accurate geodetic positions¹ for shore control must be established. There are many methods available to establish geodetic control in the survey area. These methods are:

1. Triangulation
2. Trilateration
3. Traverse
4. Intersection
5. Resection

The process of making a proper nautical chart consists, first of all, in setting up a framework of marks on the ground. Before 1950 the main framework of a geodetic survey almost always consisted of triangulation, which was replaced by traverse if the topography made triangulation impracticable. During the last decade, the introduction of electronic distance measuring (EDM) equipment has made both trilateration and traverse economical, and an acceptable substitute for triangulation. In fact, it appears probable that these new methods will replace triangulation as the main framework for new geodetic surveys [Ref. 1]. It is evident that triangulation and traverse are the main methods used for establishing control. There seems to be no

¹A position of a point on the surface of the Earth expressed in terms of geodetic latitude, geodetic longitude, and geodetic height. A geodetic position implies an adopted geodetic datum.

agreement among the various agencies as to which of these two methods is mostly used. The U.S. National Ocean Service (NOS) does the majority of its horizontal control surveys for hydrography with traverse (about 90%) [Ref. 2]. The main factor for the selection of one or the other method depends on the geographical configuration of the survey project area and the availability of good EDM equipment. Traversing is a convenient, rapid method for establishing horizontal control. It is particularly useful in densely built areas, when the coastline tends to be even, along a railroad track, and in heavily forested areas where lengths of sight are short so that neither triangulation nor trilateration is suitable.

The objectives of this thesis are to show methods of traverse station position computation and a comparison of methods of least squares adjustment. Closed traverse station positions were computed and adjusted using the Approximate Method and by the Least Squares Method in the Universal Transverse Mercator (UTM) grid coordinates. The computer programs were written to calculate the adjusted traverse station positions. Test data included those data obtained during the Geodetic Survey Field Experience course at the Naval Postgraduate School (NPS) in October 1983.

The results of comparative computations are shown to more significant figures, in this thesis, than are normally considered desirable in production work. The same observed data are used with each of three computational methods. It is important to recognize that what is being compared is not observational precision but computational precision. Hence, it is considered necessary for a rigorous comparison of computational precision, including round off error, to show results to several more decimal places than is justifiable based on observational precision alone.

II. TRAVERSE

A. GENERAL

Traverse is a method of surveying in which a series of straight lines connect successive established points along the route of a survey. An angular measurement is taken using a theodolite at each point where the traverse changes direction. Distances along the line between successive traverse points are determined by EDM equipment. The points defining the ends of the traverse lines are called turning points, traverse points, or traverse stations. Each straight section of a traverse is called a leg or a traverse line.

A closed traverse originates at a point of known position and is closed on another point of known horizontal position. Traverse I-1-2-3-F originates at point I with a backsight along line IA of known azimuth and closes on point F, with a foresight along line FB also of known azimuth (Figure 2.1). This type of traverse is preferable to all

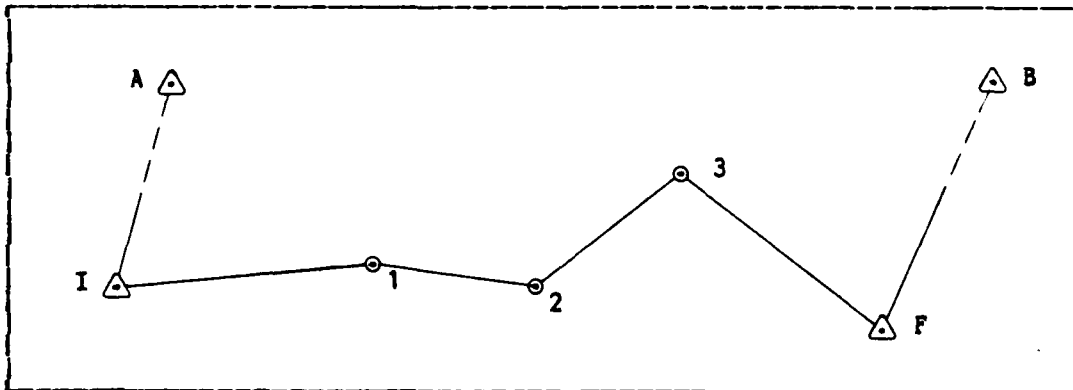


Figure 2.1 Closed Traverse.

others since computational checks are possible which allow detection of systematic errors² in both distance and direction.

A closed-loop (closed polygon) traverse is a special case of a closed traverse in which the originating and terminating points are the same point with a known horizontal position. Traverse I-1-2-3-4-I, originates and terminates on point I (Figure 2.2). This type of traverse permits an internal check on the angles, but there is no check on the linear measurements. Therefore, there is a possibility that an error proportional to distance may occur and not be detected.

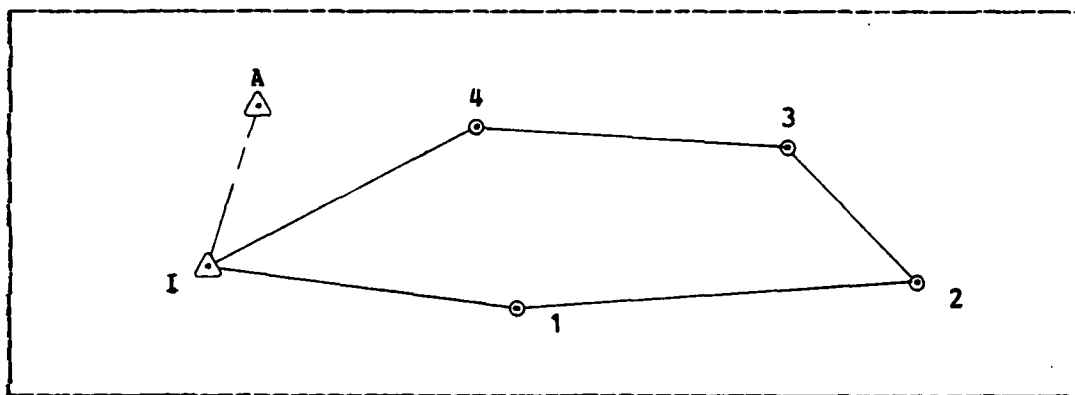


Figure 2.2 Closed-loop Traverse.

B. ANGLE AND DIRECTION MEASUREMENT

Angles and directions may be defined by means of bearings, azimuths, deflection angles, angles to the right, or interior angles. These quantities are said to be observed

²Systematic errors may be caused by faulty instruments or factors such as temperature or humidity changes which affect the performance of measuring instruments.

when obtained directly in the field and calculated when obtained indirectly by computation.

A theodolite is an instrument designed to observe horizontal directions and measure vertical angles. It consists of a telescope mounted to rotate vertically on a horizontal axis supported by a pair of vertical standards attached to a revolvable circular plate containing a graduated circle for observing horizontal directions. Another graduated arc is attached to one standard so that vertical angles can be measured.

Repeating and direction theodolites have features that are common to both types of instruments. Repeating theodolites are read directly to 20" or 01' and by estimation to one-tenth the corresponding direct reading. Direction theodolites are usually read directly to 01" and can be estimated to tenths of seconds [Ref. 3, p. 215]. In general, direction theodolites are more precise than are repeating theodolites.

The direction theodolite observes directions only and angles are computed by subtracting one direction from another. Assume that the horizontal angle AIB (Figure 2.3) is to be measured with a direction theodolite. The theodolite is set over point I, leveled and centered, and a sight is taken on point A. The horizontal circle is then viewed through the optical-viewing system and the circle reading is observed and recorded. Assume the reading is 45° 02' 40". The telescope is then sighted on point B. The horizontal circle is then viewed through the optical-viewing system and the circle reading is observed and recorded. Assume the reading is 124° 11' 59". These two observations constitute directions which have a common reference direction that is completely arbitrary. The clockwise horizontal angle is

$$i = (124^{\circ} 11' 59") - (45^{\circ} 02' 40") = 79^{\circ} 09' 19"$$

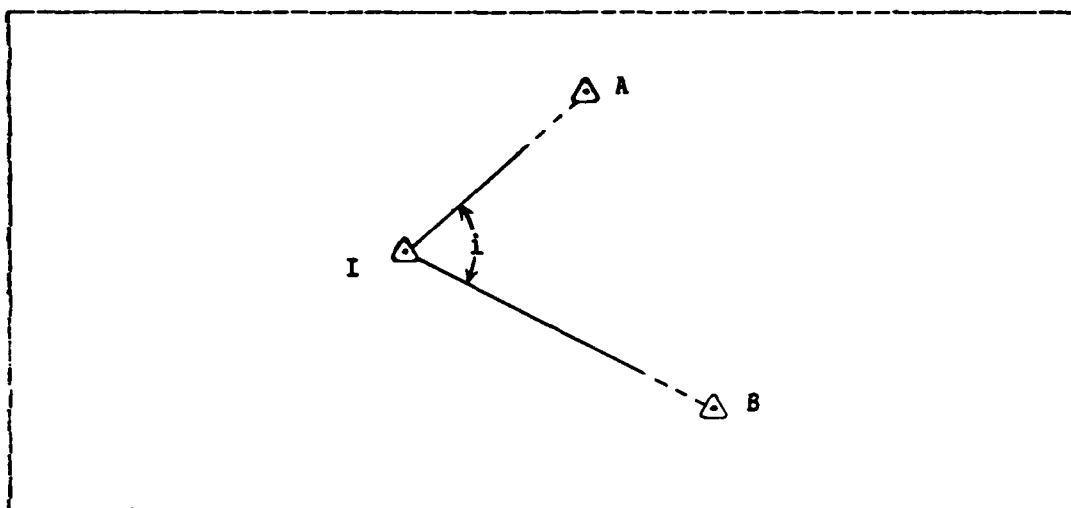


Figure 2.3 Horizontal Angle.

C. DISTANCE MEASUREMENTS

There are several methods of determining distance, the choice of which depends on the accuracy required and the cost. For example, tacheometry, taping, and EDM equipment can be used. The general availability of EDM equipment has practically eliminated the use of taping for the measurement of traverse lengths. Accuracies are comparable, or superior, to those obtained with invar tapes.

Distances measured using EDM equipment are subject to errors arising from the instrumental components, calibration of the equipment, inaccuracies in the meteorological data, elevation discrepancies, and centering of the instruments or reflectors. The reduction of measured distances involves converting the slope distance to a horizontal distance, converting the chord distance to an equivalent arc distance, and reducing the arc distance to the ellipsoid. The reduction of slope distance to horizontal distance is necessary to compensate for the difference in elevation of the end points of the measured line. The horizontal distance is

reduced to the geodetic distance by applying a sea level corrector and a chord-arc corrector to the horizontal distance [Ref. 4, pp. 124-125].

D. ACCURACY

In general, the accuracy of a traverse is judged on the basis of the resultant error of closure of the traverse. This resultant closing error is a function of the accuracies in the measurement of directions and distances. The classification, standards of accuracy, and general specifications for horizontal control have been prepared by the Federal Geodetic Control Committee (FGCC) [Ref. 5] and have been reviewed by the American Society of Civil Engineers, The American Congress on Surveying and Mapping, and the American Geophysical Union (Table I). The third-order traverse class I and class II are of particular interest because these are the orders of accuracy the hydrographer is usually required to accomplish.

E. DATA ACQUISITION

The data for this thesis were acquired at Moss Landing, California, by NPS Hydrographic students during the Geodetic Survey Field Experience course in October 1983. All of the known station positions and azimuths (Table II) were adjusted by the Coast and Geodetic Survey by third-order methods [Ref. 6].

The distances were observed by using a Kern DM102 and a Tellurometer MRA5. The Kern DM102 is an electro-optical distance meter. The measuring accuracy was $\pm(5 \text{ mm} + 5 \text{ ppm})$. The Tellurometer MRA5 is a microwave instrument. Precision in terms of probable error, in the temperature range of -32°C to $+44^{\circ}\text{C}$, is $\pm(5 \text{ cm} + 100 \text{ ppm})$ for a single

TABLE I
Classification, Standards of Accuracy, and General Specifications
for Horizontal Control

Classification	TRAVERSE			
	First-Order	Second-Order		Third-Order
		Class I	Class II	Class I
Recommended spacing of principal stations	Network stations 10-15 km Other surveys seldom less than 3 km.	Principal stations seldom less than 4 km except in metropolitan area surveys where the limitation is 0.3 km.	Principal stations seldom less than 2 km except in metropolitan area surveys where the limitation is 0.2 km.	Seldom less than 0.1 km in tertiary surveys in metropolitan area surveys. As required for other surveys.
Horizontal directions or angles*				
Instrument	0"-2	0"-2 } { 1"-0 8 } or { 12" 4" } { 5"	0"-2 } { 1"-0 6 } or { 8" 4" } { 5"	1"-0 4 5
Number of observations	16			
Rejection limit from mean	4"			
Length measurements				
Standard error ¹	1 part in 600,000	1 part in 300,000	1 part in 120,000	1 part in 60,000
Reciprocal vertical angle observations*				
Number of and spread between observations				
Number of stations between known elevations	3 D/R=10"	3 D/R=10"	2 D/R=10"	2 D/R=10"
Astro azimuths	4-6	6-8	8-10	10-15
Number of courses between azimuth checks*	5-6	10-12	15-20	20-25
No. of obs./night	16	16	12	8
No. of nights	2	2	1	1
Standard error	0"-45	0"-45	1"-5	3"-0
Azimuth closure at azimuth check point not to exceed*	1"-0 per station or 2"-√N	1"-5 per station or 3"-√N Metropolitan area surveys seldom to exceed 2"-0 per station or 3"-√N	2"-0 per station or 6"-√N Metropolitan area surveys seldom to exceed 4"-0 per station or 8"-√N	3"-0 per station or 10"-√N Metropolitan area surveys seldom to exceed 6"-0 per station or 15"-√N
Position closure** after azimuth adjustment	0.04m √K or 1:100,000	0.08 √K or 1:50,000	0.2m √K or 1:20,000	0.4m √K or 1:10,000
				0.8m √K or 1:5,000

* May be reduced to 8 and 4, respectively, in metropolitan areas.

TABLE I
(Continued)

NOTE (1)

The standard error is to be estimated by

$$\sigma_m = \sqrt{\frac{\sum v^2}{n(n-1)}} \quad \text{where } \sigma_m \text{ is the standard error of the mean, } v \text{ is a residual (that is, the difference between a measured length and the mean of all measured lengths of a line), and } n \text{ is the number of measurements.}$$

The term "standard error" used here is computed under the assumption that all errors are strictly random in nature. The true or actual error is a quantity that cannot be obtained exactly. It is the difference between the true value and the measured value. By correcting each measurement for every known source of systematic error, however, one may approach the true error. It is mandatory for any practitioner using these tables to reduce to a minimum the effect of all systematic and constant errors so that real accuracy may be obtained. (See page 267 of Coast and Geodetic Survey Special Publication No. 247, "Manual of Geodetic Triangulation," Revised edition, 1959, for definition of "actual error.")

NOTE (2)

The figure for "Instrument" describes the theodolite recommended in terms of the smallest reading of the horizontal circle. A position is one measure, with the telescope both direct and reversed, of the horizontal direction from the initial station to each of the other stations. See FGCC "Detailed Specifications" for number of observations and rejection limits when using transits.

NOTE (3)

The standard error for astronomic azimuths is computed with all observations considered equal in weight (with 75 percent of the total number of observations required on a single night) after application of a 5 second rejection limit from the mean for First- and Second Order observations.

NOTE (4)

See FGCC "Detailed Specifications" on "Elevation of Horizontal Control Points" for further details. These elevations are intended to suffice for computations, adjustments, and broad mapping and control projects, not necessarily for vertical network elevations.

NOTE (5)

Unless the survey is in the form of a loop closing on itself, the position closures would depend largely on the constraints or established control in the adjustment. The extent of constraints and the actual relationship of the surveys can be obtained through either a review of the computations, or a minimally constrained adjustment of all work involved. The proportional accuracy or closure (i.e. 1/100,000) can be obtained by computing the difference between the computed value and the fixed value, and dividing this quantity by the length of the loop connecting the two points.

NOTE (6)

See FGCC "Detailed Specifications" on "Trilateration" for further details.

NOTE (7)

The number of azimuth courses for First Order traverses are between Laplace azimuths. For other survey accuracies, the number of courses may be between Laplace azimuths and/or adjusted azimuths.

NOTE (8)

The expressions for closing errors in traverses are given in two forms. The expression containing the square root is designed for longer lines where higher proportional accuracy is required.

The formula that gives the smallest permissible closure should be used.

N is the number of stations for carrying azimuth.

K is the distance in kilometers.

TABLE II
Coordinates of Known Stations and Azimuths

Station	UTM grid coordinates	
	Grid Northing (m.)	Grid Easting (m.)
Moss 2	4,072,555.85206	608,279.04404
Holm	4,079,258.31754	612,238.85256
Grid azimuth clockwise from North		Azimuth
From Moss 2	to Pipher stations	100° 16' 23.778"
From Holm	to Moran stations	136° 33' 26.334"

determination. The distances were observed in the field, corrected by temperature and pressure for propagation error. Unfortunately, meteorological data are generally acquired only at the end points of a measured line. Using the mean of these meteorological values only approximates the actual conditions of the entire measured line and does not completely correct for errors in the propagation velocity of electromagnetic radiation. By applying the elevation, sea level, and scale factor corrections they were reduced to UTM grid distances [Ref. 4, pp. 124-125]. The UTM grid distances and standard deviations of the distances were determined in meters (Table III).

The angles were observed by using a Wild T-2 theodolite. To ensure the correctness of the beginning and ending azimuths, a check azimuth to a second station of known position was observed. The angles were observed at stations Moss 2, Mossback, Dune Temp, and Holm (Table IV). All observations were made by NPS students and conform to specification for a third-order class I traverse.

TABLE III
Grid Distances and Standard Deviations

Grid distances between stations			Distances (m.)	Standard deviation (m.)
Moss 2	and	Mossback	1,424.004	0.001
Mossback	and	Dune Temp	365.744	0.001
Dune Temp	and	Holm	6,476.271	0.003

TABLE IV
The Observed Angles and Standard Deviations

Backsight station	Center station	Foresight station	Observed angles			Standard deviation
			⁰	[']	["]	["]
Pipher	Moss 2	Mossback	246	05	43.200	01.984
Moss 2	Mossback	Dune Temp	222	51	08.600	01.405
Mossback	Dune Temp	Holm	190	15	02.600	01.203
Dune Temp	Holm	Moran	277	05	17.000	01.614

III. TRAVERSE COMPUTATIONS AND ADJUSTMENTS

A. INITIAL TRAVERSE COMPUTATIONS

A traverse (Figure 3.1) originates at station 1 (known position) and terminates at station 4 (also known position) (Table V). To compute the forward azimuth (Table VI) of an unknown leg, the angle is added to the back azimuth of the previous leg (Equation 3.1).

$$Az_i = Az_F + \sum_{j=1}^i \alpha_j - (i-1) 180^\circ \quad (3.1)$$

Where: i = the number of legs,

Az_i = the forward azimuth of an unknown leg, and

Az_F = the fixed initial azimuth.

For the computation of UTM grid coordinates, let ΔE_i and ΔN_i be designated as the departure and latitude for leg i (Figure 3.1) so that the general formulas to compute the departure and latitude are

$$\Delta E_i = d_i \sin Az_i \quad (3.2)$$

$$\Delta N_i = d_i \cos Az_i \quad (3.3)$$

where $i = 1, 2, 3, \dots, n$,

n = the number of the observed angles.

The algebraic signs of the departure and latitude for a traverse leg depend on the signs of the sine and cosine of the azimuth of that leg. The algebraic sign of the departure and latitude is determined by the following rules:

1. UTM grid azimuths are referred to grid north.
2. For azimuths between 0° and 180° , the departure is plus; for all other azimuths, the departure is minus.
3. For azimuths between 90° and 270° , the latitude is minus; for all other azimuths, the latitude is plus.

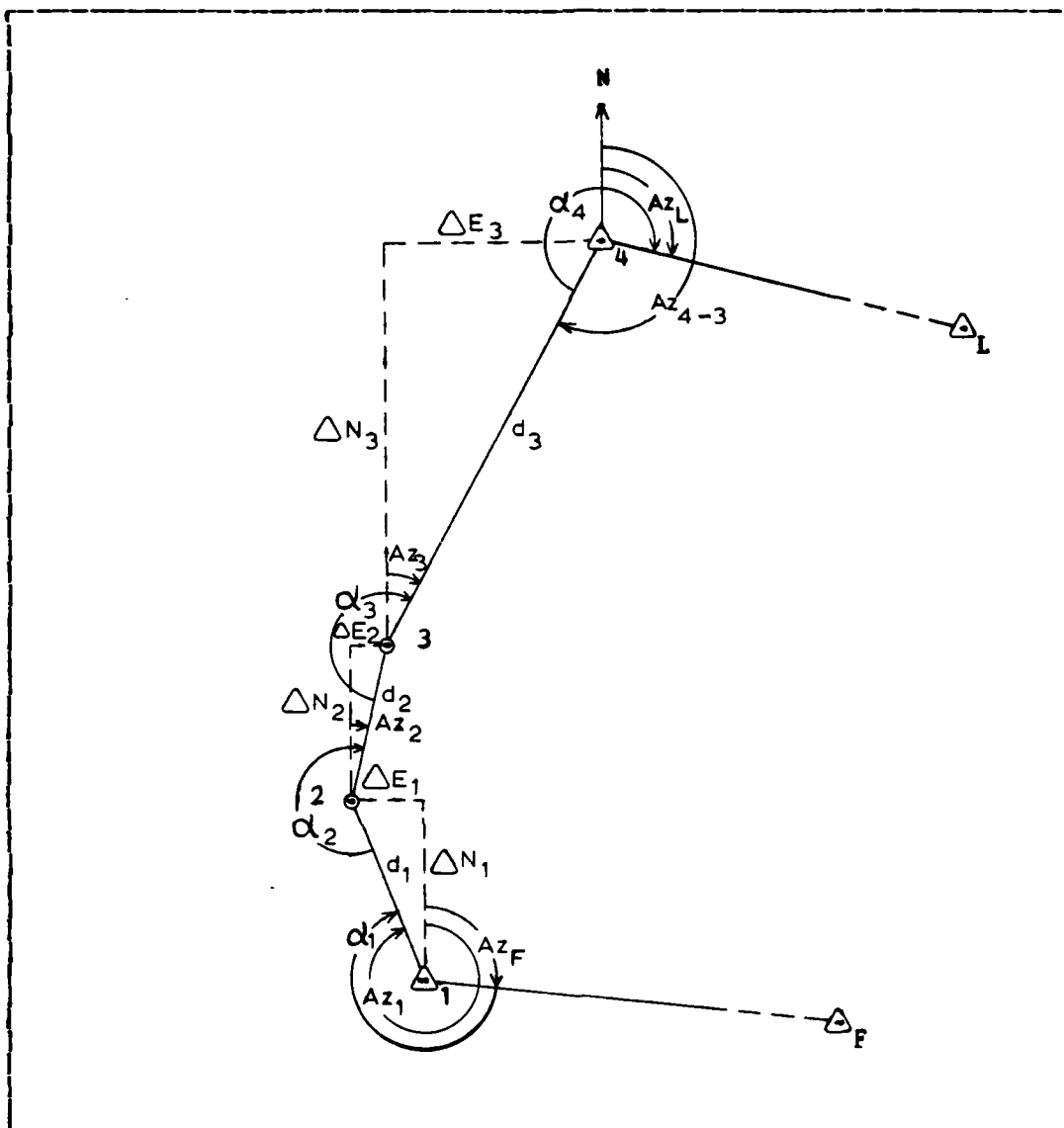


Figure 3.1 A Closed Traverse.

The calculators and computers with internal routines for trigonometric functions yield the proper sign automatically given the azimuth. The major portion of traverse computations consists of calculating departures and latitudes for successive legs and cumulating the values to determine the coordinates for consecutive traverse stations.

TABLE V
Data for Initial Traverse Computations

Leg	Distance (m.)	Angle
1 - 2	$d_1 = 1,424.004$	$\alpha_1 = 246^{\circ} 05' 43.200''$
2 - 3	$d_2 = 365.744$	$\alpha_2 = 222 51 08.600$
3 - 4	$d_3 = 6,476.271$	$\alpha_3 = 190 15 02.600$
The initial UTM grid azimuth		$Az_F = 100 16 23.778$
Station 1	UTM grid Northing	$N_1 = 4,072,555.85206 \text{ m.}$
	UTM grid Easting	$E_1 = 608,279.04404 \text{ m.}$

The coordinates of station j ($j = i + 1$) are E_j and N_j , the coordinates of station j are

$$E_j = E_i + \Delta E_i \quad (3.4)$$

$$N_j = N_i + \Delta N_i \quad (3.5)$$

where $i = 1, 2, 3, \dots, n$.

The values of departures, latitudes, and coordinates are shown in Table VII and were computed from the data in Tables V and VI by application of Equations (3.2) through (3.5).

B. COMPUTATION OF DISCREPANCIES

A closed traverse at Moss Landing (Figure 3.1) originated at station 1 (Moss 2) and closed at station 4 (Holm). The closing errors for a traverse are caused by observation errors in the observed angles and the measured distances. The closing errors may be computed by applying Equations (3.1) through (3.6).

TABLE VI
The Azimuth Calculation

Forward Azimuth	Back Azimuth	Observed Angle			
	Az_F		100°	$16'$	$23.778''$
		α_1	<u>246</u>	<u>05</u>	<u>43.200</u>
Az_1			346	22	06.978
			- 180	00	00.000
	Az_{2-1}		166	22	06.978
		α_2	<u>222</u>	<u>51</u>	<u>08.600</u>
Az_2			389	13	15.578
			- 180	00	00.000
	Az_{3-2}		209	13	15.578
		α_3	<u>190</u>	<u>15</u>	<u>02.600</u>
Az_3			399	28	18.178
			- 180	00	00.000
	Az_{4-3}		<u>219</u>	<u>28</u>	<u>18.178</u>

The angular error of closure may be computed by applying Equation (3.1) becomes

$$Az_F + \sum_{i=1}^n \alpha_i - (n-1)180^{\circ} - Az_L = W_1 \quad (3.6)$$

where Az_L is the fixed closing azimuth and W_1 is the angular error of closure. Az_L is the grid azimuth from stations Holm to Moran ($Az_L = 136^{\circ} 33' 26.334''$) (Table II). The computed closing azimuth is equal to Az_{4-3} plus the observed angle at station Holm = $(219^{\circ} 28' 18.178'') + (277^{\circ} 05' 17'') = 496^{\circ} 33' 35.178'' = 136^{\circ} 33' 35.178''$. From Equation (3.6), the angular error of closure is $(136^{\circ} 33' 35.178'') - (136^{\circ} 33' 26.334'') = + 08.844''$.

TABLE VII
Initial Traverse Computations

Values in meters		Values in meters	
	N ₁ 4,072,555.85206	E ₁ 608,279.04404	
△ N ₁	<u>1,383.89267</u>	△ E ₁	<u>- 335.60166</u>
	N ₂ 4,073,939.74473	E ₂ 607,943.44238	
△ N ₂	<u>319.20061</u>	△ E ₂	<u>178.54871</u>
	N ₃ 4,074,258.94534	E ₃ 608,121.99109	
△ N ₃	<u>4,999.28284</u>	△ E ₃	<u>4,116.94755</u>
	N ₄ <u><u>4,079,258.22818</u></u>	E ₄ <u><u>612,238.93864</u></u>	

To compute the errors in closure in position for a closed traverse. Equations (3.2) to (3.5) are applied to obtain:

$$W_2 = \sum_{i=1}^{n-1} \Delta E_i - (E_T - E_1) \quad (3.7)$$

$$W_3 = \sum_{i=1}^{n-1} \Delta N_i - (N_T - N_1) \quad (3.8)$$

or

$$W_2 = E_n - E_T \quad (3.7a)$$

$$W_3 = N_n - N_T \quad (3.8a)$$

Where: n = the number of observed angles,
 W₂ , W₃ = precalculated discrepancies,
 E₁ , N₁ = the fixed initial coordinates,
 E_T , N_T = the fixed closing coordinates, and
 E_n , N_n = the computed closing coordinates.

The result of the errors in closure in position of a traverse is

$$\Delta d = (W_2^2 + W_3^2)^{1/2} \quad (3.9)$$

The computed closing coordinates of this traverse are $E_4 = 612,238.93864$ m and $N_4 = 4,079,258.22818$ m. The fixed closing coordinates are $E_T = 612,238.85256$ m and $N_T = 4,079,258.31754$ m. The precalculated discrepancies can be computed by using Equations (3.7a) and (3.8a). The results are $W_2 = 0.08608$ m and $W_3 = -0.08936$ m. By applying Equation (3.9), Δd is 0.12408 m. From Table III, the total measured distance is 8,266.019 m. The ratio of the distance error of closure to the total distance is $0.12408/8,266.019$ or 1 part in 66,618. The ratio is an indication of the goodness of this traverse.

C. ADJUSTMENT OF A TRAVERSE BY AN APPROXIMATE METHOD

The angular error of closure of traverse may be distributed equally among the observed angles. The angular error of closure of this traverse is $+8.844''$, which corresponds to a correction for each angle of $-2.211''$. The observed angles were corrected by this value. The adjusted azimuth was then computed. The departures, latitudes, and coordinates were recomputed by using the adjusted azimuth (Table VIII). The closure corrections in E and N coordinates are $+0.09635$ m and -0.04326 m, respectively. The resultant closure is 0.10562 m. The ratio of the distance error of closure to the total distance is $0.10562/8,266.019$ or 1 part in 78,262.

The adjustments of a closed traverse by the Approximate Method are completed by the compass rule which proportions the errors in E and N coordinates according to the distance of the course [Ref. 4, p. 354]. The corrections are applied to the departures and latitudes prior to computation of coordinates.

TABLE VIII
Initial Traverse Computations (Approximate Method with
Adjusted Azimuth)

Observed angle		Correction	Adjusted azimuth		
				⁰	['] ["]
α_1	246° 05' 43.2"	- 2.211"	Az _F	100	16 23.778
				<u>246</u>	<u>05 40.989</u>
			Az ₁	346	22 04.767
α_2	222° 51' 08.6"	- 2.211"	-	180	00 00.000
			Az ₂₋₁	166	22 04.767
				<u>222</u>	<u>51 06.389</u>
α_3	190° 15' 02.6"	- 2.211"	Az ₂	389	13 11.156
			-	180	00 00.000
			Az ₃₋₂	209	13 11.156
α_4	277° 05' 17.0"	- 2.211"		<u>190</u>	<u>15 00.389</u>
			Az ₃	399	28 11.545
			-	180	00 00.000
			Az ₄₋₃	219	28 11.545
				<u>277</u>	<u>05 14.789</u>
			Az _{4-L}	<u>136</u>	<u>33 26.334</u>

Values in meters		Values in meters	
N ₁	4,072,555.85206	E ₁	608,279.04404
ΔN_1	<u>1,383.88907</u>	ΔE_1	<u>- 335.61649</u>
N ₂	4,073,939.74113	E ₂	607,943.42755
ΔN_2	<u>319.20444</u>	ΔE_2	<u>178.54187</u>
N ₃	4,074,258.94557	E ₃	608,121.96942
ΔN_3	<u>4,999.41523</u>	ΔE_3	<u>4,116.78679</u>
N ₄	4,079,258.36080	E ₄	612,238.75621
N _T	<u>4,079,258.31754</u>	E _T	<u>612,238.85256</u>
dN	<u>0.04326</u>	dE	<u>- 0.09635</u>

$$\delta E_i = (dE / D) \cdot d_i \quad (3.10)$$

$$\delta N_i = (dN / D) \cdot d_i \quad (3.11)$$

Where: δE_i = correction to ΔE_i ,
 δN_i = correction to ΔN_i ,
 dE = total closure correction in the E coordinate,
 dN = total closure correction in the N coordinate,
and
 D = total distance.

The adjusted E and N coordinates (Table IX) were different from the fixed E and N coordinates (Table II) by 0.00001 m due to round off error. The calculations and adjustments were illustrated in sections A, B, and C by using the hand calculator (TI-59) and rounded off at 5 decimal places. The computer program was written to calculate the adjusted traverse station positions by the Approximate Method by using 16 decimal places (Appendix A). The approximate traverse adjustment is based on an assumed condition that the angular precision equals the precision in linear distance.

D. ADJUSTMENT OF A TRAVERSE BY LEAST SQUARES METHODS

The method of least squares provides a rigorous adjustment and best estimates for positions of all traverse stations. The Least Squares Method is used to simultaneously eliminate closing errors in azimuths and coordinates of traverses.

1. The Principle of Least Squares

The fundamental condition of the least squares technique in surveying requires that the sum of the squares of the residuals be minimized. A residual is defined as the difference between the true and observed values. In making

TABLE IX
Adjusted Coordinates (Approximate Method)

Departure	Correction	Grid	Values in meters
		E ₁	608,279.04404
ΔE_1	-335.61649 + 0.01659		<u>- 335.59990</u>
		E ₂	607,943.44414
ΔE_2	178.54187 + 0.00426		<u>178.54613</u>
		E ₃	608,121.99027
ΔE_3	4,116.78679 + 0.07549		<u>4,116.86228</u>
		E ₄	<u>612,238.85255</u>
Latitude	Correction	Grid	Values in meters
		N ₁	4,072,555.85206
ΔN_1	1,383.88907 - 0.00745		<u>1,383.88162</u>
		N ₂	4,073,939.73368
ΔN_2	319.20444 - 0.00191		<u>319.20253</u>
		N ₃	4,074,258.93621
ΔN_3	4,999.41523 - 0.03389		<u>4,999.38134</u>
		N ₄	<u>4,079,258.31755</u>

physical measurements, the true values can never be determined. The least squares principle establishes a criterion for obtaining the best estimates of the true values.

If the best estimates of the true values are stated by x_i and observed values by $\bar{x}_i$, the residuals are expressed as

$$v_i = x_i - \bar{x}_i$$

The fundamental condition of least squares, for uncorrelated observations with equal precision are expressed as

$$\sum_{i=1}^n (v_i)^2 = (v_1)^2 + (v_2)^2 + (v_3)^2 + \dots + (v_n)^2 = \text{Minimum}$$

or in matrix form

$$v^T v = \text{Minimum} \quad (3.12)$$

In general, the observed values are of unequal precision. The observed value of high precision has a small variance. Conversely, a low precision of the observed value has a large variance. Since the value of the variance goes in opposite direction to that of the precision, the observation is assigned a value called weight corresponding to a quality that is inversely proportional to the observation's variance.

For uncorrelated measurements $x_1, x_2, x_3, \dots, x_n$, with variances $\sigma_1^2, \sigma_2^2, \sigma_3^2, \dots, \sigma_n^2$, respectively, the weights of these uncorrelated measurements are,

$$p_{11} = \sigma_0^2 / \sigma_1^2, p_{22} = \sigma_0^2 / \sigma_2^2, p_{33} = \sigma_0^2 / \sigma_3^2, \dots$$

$$p_{nn} = \sigma_0^2 / \sigma_n^2 \quad (3.13)$$

where σ_0^2 is the proportionally constant of an observation of unit weight [Ref. 7, p. 67]. These weights may be collected into a corresponding diagonal P matrix, called the weight matrix:

$$P = \begin{bmatrix} p_{11} & 0 & 0 & \dots & 0 \\ 0 & p_{22} & 0 & \dots & 0 \\ 0 & 0 & p_{33} & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & p_{nn} \end{bmatrix}$$

The weight matrix consists of weighted observations of a traverse with mixed kinds of measurements. They are distances and angles. The variance of a measured distance is expressed in meters and an angle in radians. For uncorrelated observations of unequal precision, the fundamental condition of least squares is expressed as

$$\sum_{i=1}^n p_{ii} v_i^2 = p_{11} v_1^2 + p_{22} v_2^2 + p_{33} v_3^2 + \dots + p_{nn} v_n^2$$

$$= \text{Minimum}$$

or in matrix form

$$v^T P v = \text{Minimum} \quad (3.14)$$

In the problems involving the adjustment of observed values, all observed quantities are expressed by functions of the quantities to be determined. In simple cases these relations are linear, but when this is not the case, the relations must be converted into the linear form by expanding them into Taylor's series. The terms of higher order are neglected, so as to obtain linear relations, solving the resulting linear equations, then iterating until the effect of the neglected higher order terms are minimized.

2. Least Squares Adjustment of Indirect Observations

The observed values are related to the desired unknown values through formulas or functions which are called observation equations. One observation equation is written for each measurement. To solve for the best value of each unknown parameter, at least one redundant observation equation must be written. That is, the number observations must be greater than the number of unknowns. The linear observation equations can be written in the general form as follows:

$$\begin{array}{ccccccc}
a_{10} + a_{11}x_1 + a_{12}x_2 + \dots + a_{1m}x_m & = & G_1 + v_1 \\
a_{20} + a_{21}x_1 + a_{22}x_2 + \dots + a_{2m}x_m & = & G_2 + v_2 \\
\cdot & & \cdot & & \cdot & & \cdot \\
\cdot & & \cdot & & \cdot & & \cdot \\
\cdot & & \cdot & & \cdot & & \cdot \\
\cdot & & \cdot & & \cdot & & \cdot \\
a_{n0} + a_{n1}x_1 + a_{n2}x_2 + \dots + a_{nm}x_m & = & G_n + v_n
\end{array}$$

where n is the number of observations; m is the number of the unknowns; $a_{10}, a_{20}, \dots, a_{n0}$ are constants; $a_{11}, a_{12}, \dots, a_{nm}$ are coefficients of the unknowns $x_1, x_2, \dots, x_m$; and $v_1, v_2, \dots, v_n$ are the residuals.

Because the observations G_i ($i = 1, 2, \dots, n$) are not free from random errors, each G_i must be corrected by a residual value, v_i . Let $b_i = G_i - a_{i0}$. Thus

$$\begin{array}{ccccccc}
a_{11}x_1 + a_{12}x_2 + \dots + a_{1m}x_m - b_1 & = & v_1 \\
a_{21}x_1 + a_{22}x_2 + \dots + a_{2m}x_m - b_2 & = & v_2 \\
\cdot & & \cdot & & \cdot & & \cdot \\
\cdot & & \cdot & & \cdot & & \cdot \\
\cdot & & \cdot & & \cdot & & \cdot \\
\cdot & & \cdot & & \cdot & & \cdot \\
a_{n1}x_1 + a_{n2}x_2 + \dots + a_{nm}x_m - b_n & = & v_n
\end{array}$$

or in the matrix form

$$V = AX - B \quad (3.15)$$

This equation is called the observation equation or the observation equation matrix.

For uncorrelated observations of unequal precision, substitute the value for the V matrix from the observation Equation (3.15) into Equation (3.14)

$$\begin{aligned}
V^T P V &= (AX - B)^T P (AX - B) \\
&= [(AX)^T - B^T] P (AX - B) \\
&= (X^T A^T - B^T) P (AX - B) \\
&= (X^T A^T P - B^T P) (AX - B) \\
&= X^T A^T P A X - X^T A^T P B - B^T P A X + B^T P B \\
&= X^T A^T P A X - X^T A^T P B - X^T A^T P B + B^T P B \\
&= X^T A^T P A X - 2X^T A^T P B + B^T P B \quad (3.16)
\end{aligned}$$

[from matrix algebra, $(AX)^T = X^T A^T$ and $B^T P A X = X^T A^T P B$]

The minimum of this function can be found by taking the partial derivatives of the function with respect to each unknown variable (i.e., with respect to the $x_1, x_2, \dots, x_m$) must equal zero. Hence,

$$\frac{\partial}{\partial x} (V^T P V) = 2A^T P A X - 2A^T P B = 0 \quad (3.17)$$

Dividing Equation (3.17) by 2, the following result is obtained:

$$A^T P A X = A^T P B$$

This represents the normal equations, and by multiplying this equation by $(A^T P A)^{-1}$, the solution is obtained:

$$X = (A^T P A)^{-1} A^T P B \quad (3.18)$$

For uncorrelated observations with equal precision, the weight matrix is an identity matrix, and equation (3.18) becomes

$$X = (A^T A)^{-1} A^T B \quad (3.19)$$

This equation can be derived similarly to the unequal precision case. Equations (3.18) and (3.19) are the basic least squares matrix equations.

If the relations are nonlinear, the relations must be converted into the linear form by using Taylor's series expansion [Ref. 3, p. 919]. Let $F = f(x_1, x_2, \dots, x_m)$ be the general observation equation that is a nonlinear function. The Taylor's series expansion is

$$F = f(x_1^0, x_2^0, \dots, x_m^0) + \frac{\partial f}{\partial x_1^0} \delta x_1 + \frac{\partial f}{\partial x_2^0} \delta x_2 + \dots + \frac{\partial f}{\partial x_m^0} \delta x_m + \text{Higher order terms}$$

where $x_1^0, x_2^0, \dots, x_m^0$ are the approximate values of the variables at which the function is evaluated. For the traverse problems, an approximate value can be the precalculated value, and $\delta x_1, \delta x_2, \dots, \delta x_m$ can be the corrections. The higher order terms in the series are neglected and only the zero and first order terms are maintained, the approximate value must be improved by successive iterations until the effect of the neglected higher order terms is minimized. After linearization, the observation equations become:

$$v_1 = f_1(x_1^0, x_2^0, \dots, x_m^0) + \frac{\partial f_1}{\partial x_1^0} \delta x_1 + \frac{\partial f_1}{\partial x_2^0} \delta x_2 + \dots + \frac{\partial f_1}{\partial x_m^0} \delta x_m - G_1$$

$$v_2 = f_2(x_1^0, x_2^0, \dots, x_m^0) + \frac{\partial f_2}{\partial x_1^0} \delta x_1 + \frac{\partial f_2}{\partial x_2^0} \delta x_2 + \dots + \frac{\partial f_2}{\partial x_m^0} \delta x_m - G_2$$

$$\begin{array}{ccccccc} \cdot & \cdot & & \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot & \cdot & & \cdot \end{array}$$

$$v_n = f_n(x_1^0, x_2^0, \dots, x_m^0) + \frac{\partial f_n}{\partial x_1^0} \delta x_1 + \frac{\partial f_n}{\partial x_2^0} \delta x_2 + \dots + \frac{\partial f_n}{\partial x_m^0} \delta x_m - G_n$$

or in the matrix form

$$V = AX - B \quad (3.20)$$

where

$$A = \begin{bmatrix} \frac{\partial f_1}{\partial x_1^0} & \frac{\partial f_1}{\partial x_2^0} & \cdot & \cdot & \cdot & \cdot & \frac{\partial f_1}{\partial x_m^0} \\ \frac{\partial f_2}{\partial x_1^0} & \frac{\partial f_2}{\partial x_2^0} & \cdot & \cdot & \cdot & \cdot & \frac{\partial f_2}{\partial x_m^0} \\ \cdot & \cdot & & & & & \cdot \\ \cdot & \cdot & & & & & \cdot \\ \cdot & \cdot & & & & & \cdot \\ \frac{\partial f_n}{\partial x_1^0} & \frac{\partial f_n}{\partial x_2^0} & \cdot & \cdot & \cdot & \cdot & \frac{\partial f_n}{\partial x_m^0} \end{bmatrix} \quad V = \begin{bmatrix} v_1 \\ v_2 \\ \cdot \\ \cdot \\ \cdot \\ v_n \end{bmatrix}$$

$$B = \begin{bmatrix} G_1 - f_1(x_1^0, x_2^0, \cdot \cdot \cdot \cdot x_m^0) \\ G_2 - f_2(x_1^0, x_2^0, \cdot \cdot \cdot \cdot x_m^0) \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ G_n - f_n(x_1^0, x_2^0, \cdot \cdot \cdot \cdot x_m^0) \end{bmatrix} \quad X = \begin{bmatrix} \delta x_1 \\ \delta x_2 \\ \cdot \\ \cdot \\ \cdot \\ \delta x_m \end{bmatrix}$$

The remainder of the least squares procedure is the same as indicated by Equations (3.16) to (3.19).

There are two types of observation equations for the adjustment of the traverse. They are the angle and distance observation equations. Both of them are nonlinear, and they must be linearized using the Taylor's series.

To derive the observation equation for the angle α_i (Figure 3.1), first write the two azimuth observation equations for the azimuth Az_{ik} and Az_{ij} , where Az_{ik} is the back azimuth and Az_{ij} is the forward azimuth.

$$Az_{ik} = \arctan \frac{E_k - E_i}{N_k - N_i}$$

$$Az_{ij} = \arctan \frac{E_j - E_i}{N_j - N_i}$$

Then, the angle observation equation for the angle α_i is

$$\begin{aligned} v_i &= Az_{ij} - Az_{ik} - \alpha_i \\ &= \arctan \frac{E_j - E_i}{N_j - N_i} - \arctan \frac{E_k - E_i}{N_k - N_i} - \alpha_i \end{aligned} \quad (3.21)$$

Equation (3.21) is nonlinear in the parameters.

Write this equation in function form:

$$v_i = f_i(E_i, N_i, E_j, N_j, E_k, N_k) - \alpha_i$$

Thus, the linearized form of the angle observation equation is

$$\begin{aligned} v_i &= f_i(E_i^0, N_i^0, E_j^0, N_j^0, E_k^0, N_k^0) + \frac{\partial f_i}{\partial E_i^0} \delta E_i + \frac{\partial f_i}{\partial N_i^0} \delta N_i \\ &\quad + \frac{\partial f_i}{\partial E_j^0} \delta E_j + \frac{\partial f_i}{\partial N_j^0} \delta N_j + \frac{\partial f_i}{\partial E_k^0} \delta E_k + \frac{\partial f_i}{\partial N_k^0} \delta N_k - \alpha_i \end{aligned}$$

The observation equation for the distance d_i between two points i and j is

$$v_i = [(E_i - E_j)^2 + (N_i - N_j)^2]^{1/2} - d_i$$

or in the functional form

$$v_i = f_i(E_i, N_i, E_j, N_j) - d_i$$

Thus, the linearized form of the distance observation equation is

$$\begin{aligned} v_i &= f_i(E_i^0, N_i^0, E_j^0, N_j^0) + \frac{\partial f_i}{\partial E_i^0} \delta E_i + \frac{\partial f_i}{\partial N_i^0} \delta N_i \\ &\quad + \frac{\partial f_i}{\partial E_j^0} \delta E_j + \frac{\partial f_i}{\partial N_j^0} \delta N_j - d_i \end{aligned}$$

For the adjustment of the traverse which was conducted at Moss Landing with the technique of adjustment of indirect observations, four angle and three distance

observation equations need to be written. The seven equations would include as unknown parameters the four coordinates of stations 2 (Mossback) and 3 (Dune Temp). Four normal equations are formed and solved for corrections to the approximate values (precalculated values) of the parameters. The corrections are added to the approximations to update their values, and the solution is repeated until the last set of corrections is insignificantly small.

The linearized equations of this problems are

$$\begin{aligned}
 v_1 &= a_{11} \delta E_2 + a_{12} \delta N_2 + a_{13} \delta E_3 + a_{14} \delta N_3 - b_1 \\
 v_2 &= a_{21} \delta E_2 + a_{22} \delta N_2 + a_{23} \delta E_3 + a_{24} \delta N_3 - b_2 \\
 &\cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \\
 &\cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \\
 &\cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \\
 &\cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \\
 v_7 &= a_{71} \delta E_2 + a_{72} \delta N_2 + a_{73} \delta E_3 + a_{74} \delta N_3 - b_7
 \end{aligned}$$

The zero order terms of the angle and distance functions are

$$\begin{aligned}
 f_1 &= \arctan \frac{E_2^\circ - E_1}{N_2^\circ - N_1} - Az_F \\
 f_2 &= \arctan \frac{E_3^\circ - E_2^\circ}{N_3^\circ - N_2^\circ} - \arctan \frac{E_1 - E_2^\circ}{N_1 - N_2^\circ} \\
 f_3 &= \arctan \frac{E_4^\circ - E_3^\circ}{N_4^\circ - N_3^\circ} - \arctan \frac{E_2^\circ - E_3^\circ}{N_2^\circ - N_3^\circ} \\
 f_4 &= Az_L - \arctan \frac{E_3^\circ - E_4}{N_3^\circ - N_4} \\
 f_5 &= [(E_1 - E_2^\circ)^2 + (N_1 - N_2^\circ)^2]^{1/2} \\
 f_6 &= [(E_2^\circ - E_3^\circ)^2 + (N_2^\circ - N_3^\circ)^2]^{1/2} \\
 f_7 &= [(E_3^\circ - E_4)^2 + (N_3^\circ - N_4)^2]^{1/2}
 \end{aligned}$$

By using the data of the known coordinates (Table II) and the approximate values (Table VII)

$$a_{11} = \frac{\partial f_1}{\partial E_2^\circ} = \left[\frac{N_2^\circ - N_1}{(N_2^\circ - N_1)^2 + (E_2^\circ - E_1)^2} \right] = 0.68246 \times 10^{-3}$$

$$a_{12} = \frac{\partial f_1}{\partial N_2^0} = - \left[\frac{E_2^0 - E_1}{(N_2^0 - N_1)^2 + (E_2^0 - E_1)^2} \right] = 0.16550 \times 10^{-3}$$

$$a_{13} = \frac{\partial f_1}{\partial E_3^0} = 0$$

$$a_{14} = \frac{\partial f_1}{\partial N_3^0} = 0$$

$$a_{21} = \frac{\partial f_2}{\partial E_2^0} = - \left[\frac{N_3^0 - N_2^0}{(N_3^0 - N_2^0)^2 + (E_3^0 - E_2^0)^2} \right] + \left[\frac{N_1 - N_2^0}{(N_1 - N_2^0)^2 + (E_1 - E_2^0)^2} \right]$$

$$= - 3.06868 \times 10^{-3}$$

$$a_{22} = \frac{\partial f_2}{\partial N_2^0} = \left[\frac{E_3^0 - E_2^0}{(N_3^0 - N_2^0)^2 + (E_3^0 - E_2^0)^2} \right] - \left[\frac{E_1 - E_2^0}{(N_1 - N_2^0)^2 + (E_1 - E_2^0)^2} \right]$$

$$= 1.16926 \times 10^{-3}$$

$$a_{23} = \frac{\partial f_2}{\partial E_3^0} = \left[\frac{N_3^0 - N_2^0}{(N_3^0 - N_2^0)^2 + (E_3^0 - E_2^0)^2} \right] = 2.38621 \times 10^{-3}$$

$$a_{24} = \frac{\partial f_2}{\partial N_3^0} = - \left[\frac{E_3^0 - E_2^0}{(N_3^0 - N_2^0)^2 + (E_3^0 - E_2^0)^2} \right] = - 1.33476 \times 10^{-3}$$

$$a_{31} = \frac{\partial f_3}{\partial E_2^0} = - \left[\frac{N_2^0 - N_3^0}{(N_2^0 - N_3^0)^2 + (E_2^0 - E_3^0)^2} \right] = 2.38621 \times 10^{-3}$$

$$a_{32} = \frac{\partial f_3}{\partial N_2^0} = \left[\frac{E_2^0 - E_3^0}{(N_2^0 - N_3^0)^2 + (E_2^0 - E_3^0)^2} \right] = - 1.33476 \times 10^{-3}$$

$$a_{33} = \frac{\partial f_3}{\partial E_3^0} = - \left[\frac{N_4 - N_3^0}{(N_4 - N_3^0)^2 + (E_4 - E_3^0)^2} \right] + \left[\frac{N_2^0 - N_3^0}{(N_2^0 - N_3^0)^2 + (E_2^0 - E_3^0)^2} \right]$$

$$= - 2.50541 \times 10^{-3}$$

$$a_{34} = \frac{\partial f_3}{\partial N_3^0} = \left[\frac{E_4 - E_3^0}{(N_4 - N_3^0)^2 + (E_4 - E_3^0)^2} \right] - \left[\frac{E_2^0 - E_3^0}{(N_2^0 - N_3^0)^2 + (E_2^0 - E_3^0)^2} \right]$$

$$= 1.43291 \times 10^{-3}$$

$$a_{41} = \frac{\partial f_4}{\partial E_2^0} = 0$$

$$a_{42} = \frac{\partial f_4}{\partial N_2^0} = 0$$

$$a_{43} = \frac{\partial f_4}{\partial E_3^0} = - \left[\frac{N_3^0 - N_4}{(N_3^0 - N_4)^2 + (E_3^0 - E_4)^2} \right] = 0.11920 \times 10^{-3}$$

$$a_{44} = \frac{\partial f_4}{\partial N_3^0} = \left[\frac{E_3^0 - E_4}{(N_3^0 - N_4)^2 + (E_3^0 - E_4)^2} \right] = - 0.09816 \times 10^{-3}$$

$$a_{51} = \frac{\partial f_5}{\partial E_2^0} = - \left[\frac{E_1 - E_2^0}{[(E_1 - E_2^0)^2 + (N_1 - N_2^0)^2]^{1/2}} \right] = - 235.67466 \times 10^{-3}$$

$$a_{52} = \frac{\partial f_5}{\partial N_2^0} = - \left[\frac{N_1 - N_2^0}{[(E_1 - E_2^0)^2 + (N_1 - N_2^0)^2]^{1/2}} \right] = 971.83201 \times 10^{-3}$$

$$a_{53} = \frac{\partial f_5}{\partial E_3^0} = 0$$

$$a_{54} = \frac{\partial f_5}{\partial N_3^0} = 0$$

$$a_{61} = \frac{\partial f_6}{\partial E_2^0} = \left[\frac{E_2^0 - E_3^0}{[(E_2^0 - E_3^0)^2 + (N_2^0 - N_3^0)^2]^{1/2}} \right] = - 488.17948 \times 10^{-3}$$

$$a_{62} = \frac{\partial f_6}{\partial N_2^0} = \left[\frac{N_2^0 - N_3^0}{[(E_2^0 - E_3^0)^2 + (N_2^0 - N_3^0)^2]^{1/2}} \right] = - 872.74326 \times 10^{-3}$$

$$a_{63} = \frac{\partial f_6}{\partial E_3^0} = - \left[\frac{E_2^0 - E_3^0}{[(E_2^0 - E_3^0)^2 + (N_2^0 - N_3^0)^2]^{1/2}} \right] = 488.17948 \times 10^{-3}$$

$$a_{64} = \frac{\partial f_6}{\partial N_3^0} = - \left[\frac{N_2^0 - N_3^0}{[(E_2^0 - E_3^0)^2 + (N_2^0 - N_3^0)^2]^{1/2}} \right] = 872.74326 \times 10^{-3}$$

$$a_{71} = \frac{\partial f_7}{\partial E_2^0} = 0$$

$$a_{72} = \frac{\partial f_7}{\partial N_2^0} = 0$$

$$a_{73} = \frac{\partial f_7}{\partial E_3^0} = \left[\frac{E_3^0 - E_4}{[(E_3^0 - E_4)^2 + (N_3^0 - N_4)^2]^{1/2}} \right] = - 635.68254 \times 10^{-3}$$

$$a_{74} = \frac{\partial f_7}{\partial N_3^0} = \left[\frac{N_3^0 - N_4}{[(E_3^0 - E_4)^2 + (N_3^0 - N_4)^2]^{1/2}} \right] = - 771.95059 \times 10^{-3}$$

or in the matrix form

$$A = 10^{-3} \begin{bmatrix} 0.68246 & 0.16550 & 0.00000 & 0.00000 \\ -3.06868 & 1.16926 & 2.38621 & -1.33476 \\ 2.38621 & -1.33476 & -2.50541 & 1.43291 \\ 0.00000 & 0.00000 & 0.11920 & -0.09816 \\ -235.67466 & 971.83201 & 0.00000 & 0.00000 \\ -448.17948 & -872.74326 & 488.17948 & 872.74326 \\ 0.00000 & 0.00000 & -635.68254 & -771.95059 \end{bmatrix}$$

The computation of B matrix, by using the data from Tables II, III, IV, and VII

$$\begin{aligned} b_1 &= \alpha_1 - f_1 = 0.00000 \text{ radians} \\ b_2 &= \alpha_2 - f_2 = 0.00000 \text{ radians} \\ b_3 &= \alpha_3 - f_3 = 0.00002 \text{ radians} \\ b_4 &= \alpha_4 - f_4 = 0.00002 \text{ radians} \\ b_5 &= d_1 - f_5 = 0.00000 \text{ meters} \\ b_6 &= d_2 - f_6 = 0.00000 \text{ meters} \\ b_7 &= d_3 - f_7 = -0.01426 \text{ meters} \end{aligned}$$

or in the matrix form

$$B^T = [0 \quad 0 \quad 0.00002 \quad 0.00002 \quad 0 \quad 0 \quad -0.01426]$$

The weight matrix can be computed by using the variance of the observed angles and distances from Tables III and IV by applying to Equation (3.13). For uncorrelated observations of unequal precision, the weight matrix is defined as the diagonal P matrix.

$$\begin{aligned}
p_{11} &= 1 / \sin^2(1.984'') = 10,808,537,426.79957 \\
p_{22} &= 1 / \sin^2(1.405'') = 21,552,498,219.02474 \\
p_{33} &= 1 / \sin^2(1.203'') = 29,398,082,997.43487 \\
p_{44} &= 1 / \sin^2(1.614'') = 16,332,144,194.08730 \\
p_{55} &= 1 / (0.001)^2 = 1,000,000.00000 \\
p_{66} &= 1 / (0.001)^2 = 1,000,000.00000 \\
p_{77} &= 1 / (0.003)^2 = 111,111.11111
\end{aligned}$$

For uncorrelated observations of unequal precision, the values of A, B, and P matrices are applied to Equation (3.18). The correction vector is found to be

$$X = \begin{bmatrix} \delta E_2 = 0.01284 \\ \delta N_2 = 0.00336 \\ \delta E_3 = 0.01079 \\ \delta N_3 = 0.00495 \end{bmatrix}$$

The approximation values were improved by adding the corrections to the first approximation values. The improved approximation values after the first iteration are

$$\begin{aligned}
E_2 &= 607,943.44238 + 0.01284 = 607,943.45522 \text{ m.} \\
N_2 &= 4,073,939.74473 + 0.00336 = 4,073,939.74809 \text{ m.} \\
E_3 &= 608,121.99110 + 0.01079 = 608,122.00189 \text{ m.} \\
N_3 &= 4,074,258.94534 + 0.00495 = 4,074,258.95029 \text{ m.}
\end{aligned}$$

Using these values, the solution is iterated. After the second iteration, the correction vector is zero to six decimal places, so the improved approximation values are the final estimates of the coordinates.

For uncorrelated observations of equal precision, the correction vector is found by solving Equation (3.19). The correction vector after the first iteration is

$$X = \begin{bmatrix} \delta E_2 = 0.01756 \\ \delta N_2 = 0.00426 \\ \delta E_3 = 0.01660 \\ \delta N_3 = 0.00479 \end{bmatrix}$$

The improved approximation values after the first iteration are

$$\begin{aligned} E_2 &= 607,943.44238 + 0.01756 = 607,943.45994 \text{ m.} \\ N_2 &= 4,073,939.74473 + 0.00426 = 4,073,939.74899 \text{ m.} \\ E_3 &= 608,121.99110 + 0.01660 = 608,122.00770 \text{ m.} \\ N_3 &= 4,074,258.94534 + 0.00479 = 4,074,258.95013 \text{ m.} \end{aligned}$$

The solution is iterated by using these adjusted values. The correction vector is zero to six decimal places after the second iteration. These values are the final estimates of the coordinates.

The standard deviation of an observation which has unit weight can be found by the following equations, for uncorrelated observations of unequal precision,

$$\sigma_o = \left[\frac{\mathbf{v}^T \mathbf{p} \mathbf{v}}{n-m} \right]^{1/2} \quad (3.22)$$

for uncorrelated observations of equal precision,

$$\sigma_o = \left[\frac{\mathbf{v}^T \mathbf{v}}{n-m} \right]^{1/2} \quad (3.23)$$

where n is the number of observation equations and m is the number of unknowns [Ref. 7, p. 249]. After calculating the best estimate values of the unknowns, the V matrix can be computed from Equation (3.20).

The standard deviations of the best estimate values for the unknowns are then given by the following equations:

$$\sigma_i = \sigma_o [S_{ii}]^{1/2} \quad (3.24)$$

where σ_i is the standard deviation of the i th adjusted quantity. The quantity in the i th row of the X matrix, S_{ii} is an element of the $(A^T P A)^{-1}$ matrix for uncorrelated observations of unequal precision. For uncorrelated observations of equal precision, S_{ii} is an element of the $(A^T A)^{-1}$ matrix [Ref. 7, p. 250].

$$(A^T P A)^{-1} \text{ or } (A^T A)^{-1} = \begin{bmatrix} S_{11} & S_{12} & . & . & . & . & S_{1m} \\ S_{21} & S_{22} & . & . & . & . & S_{2m} \\ . & . & . & . & . & . & . \\ . & . & . & . & . & . & . \\ . & . & . & . & . & . & . \\ S_{m1} & S_{m2} & . & . & . & . & S_{mm} \end{bmatrix}$$

The standard deviations of adjusted angles and distances can be computed by the following equation:

$$\sigma_i = \sigma_o [Q_{ii}]^{1/2} \quad (3.25)$$

where σ_i is the standard deviations of the i th adjusted quantity. The quantity in the i th row of the V matrix, Q_{ii} is an element of $A(A^T P A)^{-1} A^T$ matrix for uncorrelated observations of unequal precision. For uncorrelated observations of equal precision, Q_{ii} is an element of the $A(A^T A)^{-1} A^T$ matrix [Ref. 3, p. 912].

$$\begin{array}{l} A(A^T P A)^{-1} A^T \\ \text{or} \\ A(A^T A)^{-1} A^T \end{array} = \begin{bmatrix} Q_{11} & Q_{12} & . & . & . & . & Q_{1n} \\ Q_{21} & Q_{22} & . & . & . & . & Q_{2n} \\ . & . & . & . & . & . & . \\ . & . & . & . & . & . & . \\ . & . & . & . & . & . & . \\ . & . & . & . & . & . & . \\ Q_{n1} & Q_{n2} & . & . & . & . & Q_{nn} \end{bmatrix}$$

A closed traverse was conducted at Moss Landing. After the second iteration, the V matrix can be found by applying the values of A , B , and X matrices to Equation (3.20).

For uncorrelated observations of unequal precision,
the V matrix is found to be

$$V = \begin{bmatrix} v_{a1} = 0.93149 \times 10^{-5} \text{ radians or } 1.92134'' \\ v_{a2} = -1.63120 \times 10^{-5} \text{ radians or } -3.36459'' \\ v_{a3} = -1.28363 \times 10^{-5} \text{ radians or } -2.64768'' \\ v_{a4} = -2.30436 \times 10^{-5} \text{ radians or } -4.75308'' \\ v_{d1} = 0.00024 \text{ meters} \\ v_{d2} = 0.00039 \text{ meters} \\ v_{d3} = 0.00358 \text{ meters} \end{bmatrix}$$

The standard deviation of unit weight is found by solving
Equation (3.22), the result is

$$\sigma_u = \pm 2.69685$$

The values of $(A^T P A)^{-1}$ are

$$10^{-5} \begin{bmatrix} 1.25178 & 0.22019 & 1.25575 & 0.10788 \\ 0.22019 & 0.13819 & 0.21810 & 0.11127 \\ 1.25575 & 0.21810 & 1.46408 & 0.03815 \\ 0.10788 & 0.11127 & 0.03815 & 0.22515 \end{bmatrix}$$

The standard deviations of the best estimate values for
positions can be found by solving Equation (3.24), the
results are

$$\begin{aligned} \sigma_{E2} &= \pm 0.00954 \text{ meters} \\ \sigma_{N2} &= \pm 0.00317 \text{ meters} \\ \sigma_{E3} &= \pm 0.01032 \text{ meters} \\ \sigma_{N3} &= \pm 0.00405 \text{ meters} \end{aligned}$$

The values of $A(A^T P A)^{-1} A^T$ are

$$10^{-10} \begin{bmatrix} 0.064 & -0.052 & -0.021 & 0.010 & -4.165 & -6.963 & -63.88 \\ -0.052 & 0.225 & -0.175 & 0.002 & 3.415 & 4.690 & 41.59 \\ -0.021 & -0.175 & 0.211 & -0.014 & 2.563 & 3.574 & 31.79 \\ 0.010 & 0.002 & -0.014 & 0.002 & -1.813 & -1.301 & -9.501 \\ -4.165 & 3.415 & 2.563 & -1.813 & 9917 & -117.4 & -1047 \\ -6.963 & 4.690 & 3.574 & -1.301 & -117.4 & 9830 & -1525 \\ -63.88 & 41.59 & 31.79 & -9.501 & -1047 & -1525 & 76323 \end{bmatrix}$$

The standard deviations of adjusted angles and distances are found by solving Equation (3.25), the results are

$$\begin{aligned} \sigma_{a1} &= \pm 1.40346 \text{ seconds} \\ \sigma_{a2} &= \pm 2.64103 \text{ seconds} \\ \sigma_{a3} &= \pm 2.55260 \text{ seconds} \\ \sigma_{a4} &= \pm 0.26137 \text{ seconds} \\ \sigma_{d1} &= \pm 0.00269 \text{ meters} \\ \sigma_{d2} &= \pm 0.00267 \text{ meters} \\ \sigma_{d3} &= \pm 0.00745 \text{ meters} \end{aligned}$$

For uncorrelated observations of equal precision, the V matrix is found to be

$$V = \begin{bmatrix} v_{a1} = 1.26896 \times 10^{-5} \text{ radians or } 2.61742'' \\ v_{a2} = -1.56893 \times 10^{-5} \text{ radians or } -3.23615'' \\ v_{a3} = -1.75414 \times 10^{-5} \text{ radians or } -3.61817'' \\ v_{a4} = -2.23358 \times 10^{-5} \text{ radians or } -4.60710'' \\ v_{d1} = 0.14037 \times 10^{-7} \text{ meters} \\ v_{d2} = 0.23844 \times 10^{-7} \text{ meters} \\ v_{d3} = 0.24365 \times 10^{-7} \text{ meters} \end{bmatrix}$$

The standard deviation of unit weight is found by solving Equation (3.23), the result is

$$\sigma_0 = \pm 2.01144 \times 10^{-5}$$

The values of $(A^T A)^{-1}$ are

$$10^{-4} \begin{bmatrix} 0.28193 & 0.06827 & -0.85449 & 0.70380 \\ 0.06827 & 0.01664 & -0.20731 & 0.17075 \\ -0.85449 & -0.20731 & 2.59561 & -2.13724 \\ 0.70380 & 0.17075 & -2.13724 & 1.75999 \end{bmatrix}$$

The standard deviations of the best estimate values for positions can be found by solving Equation (3.24), the results are

$$\begin{aligned} \sigma_{E2} &= \pm 0.00107 \text{ meters} \\ \sigma_{N2} &= \pm 0.00026 \text{ meters} \\ \sigma_{E3} &= \pm 0.00324 \text{ meters} \\ \sigma_{N3} &= \pm 0.00267 \text{ meters} \end{aligned}$$

The values of $A(A^T A)^{-1}A^T$ are

$$10^{-3} \begin{bmatrix} 1.472 & -27.197 & 26.960 & -1.235 & -0.518 & -0.843 & -0.856 \\ -27.197 & 503.1 & -498.7 & 22.855 & 0.348 & 0.480 & 0.473 \\ 26.960 & -498.7 & 494.4 & -22.66 & 0.370 & 0.523 & 0.518 \\ -1.235 & 22.855 & -22.66 & 1.038 & -0.201 & -0.159 & -0.134 \\ -0.518 & 0.348 & 0.370 & -0.201 & 10000 & -0.001 & -0.001 \\ -0.843 & 0.480 & 0.523 & -0.159 & -0.001 & 10000 & -0.001 \\ -0.856 & 0.473 & 0.518 & -0.134 & -0.001 & -0.001 & 10000 \end{bmatrix}$$

The standard deviations of adjusted angles and distances are found by solving Equation (3.25), the results are

$$\begin{aligned} \sigma_{a1} &= \pm 0.15917 \text{ seconds} \\ \sigma_{a2} &= \pm 2.94270 \text{ seconds} \\ \sigma_{a3} &= \pm 2.91732 \text{ seconds} \\ \sigma_{a4} &= \pm 0.13370 \text{ seconds} \\ \sigma_{d1} &= \pm 0.00002 \text{ meters} \\ \sigma_{d2} &= \pm 0.00002 \text{ meters} \\ \sigma_{d3} &= \pm 0.00002 \text{ meters} \end{aligned}$$

The computations in this subsection were performed by the NPS IBM 3033 computer using 16 decimal places and were rounded off to 5 decimal places prior to output.

3. Least Squares Adjustment by the Condition Equation Method

The general principle of least squares adjustment by the condition equation method in surveying is

to minimize a function consisting of the sum of the squares of the corrections to the observations plus the necessary mathematical conditions involving some or all the corrections; each condition by itself is made equal to zero by adding the corrections to the discrepancy determined from a preadjusted calculation (i.e. calculated from the observed values, rather than from adjusted values); thus, the magnitude of the sum of the squares is not changed by adding conditions [Ref. 8].

For uncorrelated observations of unequal precision, the principle condition of least squares function may be expressed in matrix form as follows:

$$\begin{aligned}
 U &= V^T P V - 2K[BV + W] \\
 &= V^T P V - 2[BV + W]^T K \\
 &= V^T P V - 2[(BV)^T + W^T] K \\
 &= V^T P V - 2[V^T B^T + W^T] K \\
 &= V^T P V - 2V^T B^T K + 2W^T K \\
 &= \text{minimum}
 \end{aligned}$$

where U is the least squares function matrix, K is a matrix of Lagrange multipliers, P is the weight matrix, V is the correction vector, B is the constant coefficients of the corrections, and W is the precalculated discrepancy [Ref. 9]. It is numerically more convenient for later development to multiply by 2. Taking the partial derivatives of the U matrix with respect to each of the corrections and equating to zero leads to

$$\frac{\partial Q}{\partial V} = 2PV - 2B^T K = 0$$

or

$$V = P^{-1} B^T K \quad (3.26)$$

This equation is called a correlate equation or correlate equation matrix. The solution of the Lagrange multipliers matrix can be derived by multiplying Equation (3.26) by the B matrix then adding the W matrix

$$BV + W = BP^{-1} B^T K + W$$

$(BV + W)$ is the condition equation matrix which must equal zero. The solution of the Lagrange multiplier vector is

$$BP^{-1} B^T K = -W$$

$$K = (BP^{-1} B^T)^{-1} (-W) \quad (3.27)$$

The correction vector is derived by substituting Equation (3.27) into Equation (3.26)

$$V = P^{-1} B^T (BP^{-1} B^T)^{-1} (-W) \quad (3.28)$$

For uncorrelated observations of equal precision, the correction vector can be derived similarly to the unequal precision case or by using the inverse of the weight matrix which is the identity matrix. Equation (3.28) becomes

$$V = B^T (BB^T)^{-1} (-W) \quad (3.29)$$

Closed traverse station positions may be adjusted by using the technique of least squares adjustment by the Condition Equation Method. There are two condition equations. They are the azimuth and coordinate condition equations. The coordinate condition equations are divided into two parts, which are E and N coordinate condition equations.

The azimuth condition equation is the sum of the corrections to the angles in a traverse plus a precalculated discrepancy W_1 (Equation 3.6) which must equal zero. Where v_{ai} equals the corrections to the angles, the general equation can be written:

$$\begin{aligned} \text{or} \quad & \sum_{i=1}^n v_{ai} + W_1 = 0 \\ & \sum_{i=1}^n v_{ai} + \sum_{i=1}^{n-1} 0 \cdot v_{di} + W_1 = 0 \end{aligned} \quad (3.30)$$

where v_{di} is the correction to the distances. However, this term equals zero and does not effect the equation, but it does reserve space in matrix notation for distance corrections. A closed traverse which was conducted at Moss Landing can be written with an azimuth condition of

$$\begin{aligned} 1 \cdot v_{a1} + 1 \cdot v_{a2} + 1 \cdot v_{a3} + 1 \cdot v_{a4} + 0 \cdot v_{d1} + 0 \cdot v_{d2} + 0 \cdot v_{d3} \\ + W_1 = 0 \end{aligned} \quad (3.31)$$

where the precalculated discrepancy (W_1) is +8.844" or +0.00004288 radians.

The simple linearized form of an E coordinate condition equation is the sum of the adjusted departures and must equal zero. By applying Equations (3.2) and (3.7), the general formula for E coordinate condition equation is

$$\sum_{i=1}^{n-1} (d_i + v_{di}) \sin Az_{ai} - (E_T - E_1) = 0 \quad (3.32)$$

where

$$Az_{ai} = Az_F + \sum_{j=1}^i (\alpha_j + v_{aj}) - (i - 1) 180^\circ \quad (3.33)$$

Az_{ai} is the adjusted azimuth. By using Equations (3.32) and (3.33) with the data conducted at Moss Landing, these equations become

$$\begin{aligned} (d_1 + v_{d1}) \cdot \sin Az_{a1} + (d_2 + v_{d2}) \cdot \sin Az_{a2} \\ + (d_3 + v_{d3}) \cdot \sin Az_{a3} - (E_T - E_1) = 0 \end{aligned} \quad (3.34)$$

where

$$Az_{a1} = Az_F + \alpha_1 + v_{a1}$$

$$= Az_1 + v_{a1}$$

$$Az_{a2} = Az_F + \alpha_1 + v_{a1} + \alpha_2 + v_{a2} - 180^\circ$$

$$= Az_2 + v_{a1} + v_{a2}$$

$$Az_{a3} = Az_F + \alpha_1 + v_{a1} + \alpha_2 + v_{a2} + \alpha_3 + v_{a3} - 360^\circ$$

$$= Az_3 + v_{a1} + v_{a2} + v_{a3}$$

Az_i is the precalculated azimuth or unadjusted azimuth (Equation 3.1).

From trigonometry

$$\sin (A + \Delta A) = \sin A \cdot \cos \Delta A + \cos A \cdot \sin \Delta A$$

in which ΔA is a very small angle in radians, let

$$\cos \Delta A \doteq 1 \quad \text{and} \quad \sin \Delta A \doteq \Delta A$$

therefore,

$$\sin (A + \Delta A) = \sin A + \Delta A \cdot \cos A$$

from the first term of Equation (3.34)

$$(d_1 + v_{d1}) \sin Az_{a1}$$

$$= (d_1 + v_{d1}) \sin (Az_1 + v_{a1})$$

$$= (d_1 + v_{d1}) [\sin Az_1 + v_{a1} \cos Az_1]$$

$$= d_1 \sin Az_1 + v_{a1} d_1 \cos Az_1 + v_{d1} \sin Az_1 + v_{d1} v_{a1} \cos Az_1 \quad (3.35)$$

$(v_{d1} \cdot v_{a1})$ is very small, by letting this term equal zero, Equation (3.35) becomes

$$(d_1 + v_{d1}) \sin Az_{a1}$$

$$= d_1 \sin Az_1 + v_{a1} d_1 \cos Az_1 + v_{d1} \sin Az_1 \quad (3.36)$$

The second and third terms of Equation (3.34) can be derived similar to Equation (3.36), so

$$\begin{aligned} & (d_2 + v_{d2}) \sin Az_{a2} \\ &= d_2 \sin Az_2 + (v_{a1} + v_{a2}) d_2 \cos Az_2 + v_{d2} \sin Az_2 \end{aligned} \quad (3.37)$$

$$\begin{aligned} & (d_3 + v_{d3}) \sin Az_{a3} \\ &= d_3 \sin Az_3 + (v_{a1} + v_{a2} + v_{a3}) d_3 \cos Az_3 + v_{d3} \sin Az_3 \end{aligned} \quad (3.38)$$

Substituting Equations (3.36), (3.37), and (3.38) into Equation (3.34) and rearranging terms

$$\begin{aligned} & v_{a1} (d_1 \cos Az_1 + d_2 \cos Az_2 + d_3 \cos Az_3) \\ &+ v_{a2} (d_2 \cos Az_2 + d_3 \cos Az_3) + v_{a3} d_3 \cos Az_3 \\ &+ v_{d1} \sin Az_1 + v_{d2} \sin Az_2 + v_{d3} \sin Az_3 \\ &+ d_1 \sin Az_1 + d_2 \sin Az_2 + d_3 \sin Az_3 - (E_T - E_1) = 0 \end{aligned}$$

This equation can be written in the general formula as

$$\begin{aligned} & \sum_{i=1}^{n-1} (v_{ai} \sum_{k=i}^{n-1} d_k \cos Az_k) + \sum_{i=1}^{n-1} v_{di} \sin Az_i + \sum_{i=1}^{n-1} d_i \sin Az_i \\ & - (E_T - E_1) = 0 \end{aligned} \quad (3.39)$$

substituting Equations (3.2), (3.3), and (3.7) into Equation (3.39)

$$\sum_{i=1}^{n-1} (v_{ai} \sum_{k=i}^{n-1} \Delta N_k) + \sum_{i=1}^{n-1} v_{di} \sin Az_i + W_2 = 0 \quad (3.40)$$

For example, if the number of the observed angles (n) at Moss Landing is four, this equation can be expanded to

$$\begin{aligned} & v_{a1} (\Delta N_1 + \Delta N_2 + \Delta N_3) + v_{a2} (\Delta N_2 + \Delta N_3) + v_{a3} \Delta N_3 \\ & + v_{d1} \sin Az_1 + v_{d2} \sin Az_2 + v_{d3} \sin Az_3 + W_2 = 0 \end{aligned}$$

Substituting the numerical values from the precalculated of this traverse into this equation, the result is

$$(6702.37612) v_{a1} + (5318.48345) v_{a2} + (4999.28284) v_{a3} + 0 \cdot v_{a4} \\ - (0.23567) v_{d1} + (0.48818) v_{d2} + (0.63570) v_{d3} + W_2 = 0 \quad (3.41)$$

where the precalculated discrepancy (W_2) is 0.08608 m and $0 \cdot v_{a4}$ is equal to zero, which does not effect this equation, but space is reserved in matrix notation for the last angle correction.

The N coordinate condition equation is the sum of the adjusted latitudes and must equal zero. By applying Equations (3.3) and (3.7), the general condition equation is

$$\sum_{i=1}^{n-1} (d_i + v_{di}) \cos Az_{ai} - (N_T - N_1) = 0$$

This equation can be derived similarly to the E coordinate condition equation case, resulting in

$$-\sum_{i=1}^{n-1} (v_{ai} \sum_{k=1}^{n-1} \Delta E_k) + \sum_{i=1}^{n-1} v_{di} \cos Az_i + W_3 = 0 \quad (3.42)$$

From the data acquired at Moss Landing, this equation can be expanded to

$$-v_{a1} (\Delta E_1 + \Delta E_2 + \Delta E_3) - v_{a2} (\Delta E_2 + \Delta E_3) + v_{a3} \Delta E_3 \\ + v_{d1} \cos Az_1 + v_{d2} \cos Az_2 + v_{d3} \cos Az_3 + W_3 = 0$$

Substituting the numerical values from the calculation of this traverse into this equation, the result is

$$-(3959.89460) v_{a1} - (4295.49626) v_{a2} - (4116.94755) v_{a3} + 0 \cdot v_{a4} \\ + (0.97183) v_{d1} + (0.87274) v_{d2} + (0.77194) v_{d3} + W_3 = 0 \quad (3.43)$$

where the precalculated discrepancy (W_3) is -0.08936 m.

Equations (3.30), (3.40), and (3.42) can be written in the matrix form as $(BV + W)$, where B is the constant coefficients of the corrections, V is the correction vector which is composed of the angle and distance corrections, W is the precalculated discrepancy. Equations (3.31), (3.41), and (3.43) are written in matrix form as

$$B^T = \begin{bmatrix} 1 & 6702.37612 & -3959.89460 \\ 1 & 5318.48345 & -4295.49626 \\ 1 & 4999.28284 & -4116.94755 \\ 1 & 0.00000 & 0.00000 \\ 0 & -0.23567 & 0.97183 \\ 0 & 0.48818 & 0.87274 \\ 0 & 0.63570 & 0.77194 \end{bmatrix} \quad V = \begin{bmatrix} v_{a1} \\ v_{a2} \\ v_{a3} \\ v_{a4} \\ v_{d1} \\ v_{d2} \\ v_{d3} \end{bmatrix}$$

$$-W = \begin{bmatrix} -W_1 = -0.00004 \\ -W_2 = -0.08608 \\ -W_3 = 0.08936 \end{bmatrix}$$

For uncorrelated observations of unequal precision, the inverse of the weight matrix is the diagonal matrix, which is equal to the observation's variance. The diagonal elements of the inverse of the weight matrix is

$$\begin{aligned} p_{11}^{-1} &= \sin^2(1.984'') = 0.92519 \times 10^{-10} \\ p_{22}^{-1} &= \sin^2(1.405'') = 0.46398 \times 10^{-10} \\ p_{33}^{-1} &= \sin^2(1.203'') = 0.34016 \times 10^{-10} \\ p_{44}^{-1} &= \sin^2(1.614'') = 0.61229 \times 10^{-10} \\ p_{55}^{-1} &= (0.001)^2 = 0.10000 \times 10^{-5} \\ p_{66}^{-1} &= (0.001)^2 = 0.10000 \times 10^{-5} \\ p_{77}^{-1} &= (0.003)^2 = 0.90000 \times 10^{-5} \end{aligned}$$

For uncorrelated observations of unequal precision, the correction vector is found by solving Equation (3.28)

$$V = \begin{bmatrix} v_{a1} = 0.93141 \times 10^{-5} \text{ radians or } 1.92117'' \\ v_{a2} = -1.63114 \times 10^{-5} \text{ radians or } -3.36446'' \\ v_{a3} = -1.28358 \times 10^{-5} \text{ radians or } -2.64758'' \\ v_{a4} = -2.30438 \times 10^{-5} \text{ radians or } -4.75312'' \\ v_{d1} = 0.00024 \text{ meters} \\ v_{d2} = 0.00039 \text{ meters} \\ v_{d3} = 0.00357 \text{ meters} \end{bmatrix}$$

The corrected angles are

$$\begin{aligned} \alpha_1 &= 246^\circ 05' 45.12117'' \\ \alpha_2 &= 222^\circ 51' 05.23554'' \\ \alpha_3 &= 190^\circ 14' 59.95242'' \\ \alpha_4 &= 277^\circ 05' 12.24688'' \end{aligned}$$

The corrected distances are

$$\begin{aligned} d_1 &= 1424.00424 \text{ m.} \\ d_2 &= 365.74439 \text{ m.} \\ d_3 &= 6476.27457 \text{ m.} \end{aligned}$$

Using the corrected data to recompute the coordinates is similar to the initial traverse computations. The adjusted coordinates are

$$\begin{aligned} E_2 &= 607,943.45521 \text{ m.} \\ N_2 &= 4,073,939.74809 \text{ m.} \\ E_3 &= 608,122.00189 \text{ m.} \\ N_3 &= 4,074,258.95029 \text{ m.} \\ E_4 &= 612,238.85256 \text{ m.} \\ N_4 &= 4,079,258.31754 \text{ m.} \end{aligned}$$

The last adjusted coordinates are the same values as the fixed coordinates. Hence, it is not necessary to iterate. For uncorrelated observations with equal precision, the correction vector is found by solving Equation (3.29)

$$V = \left[\begin{array}{lll} v_{a1} & = & 1.26884 \times 10^{-5} \text{ radians or } 2.61717'' \\ v_{a2} & = & -1.56886 \times 10^{-5} \text{ radians or } -3.23601'' \\ v_{a3} & = & -1.75408 \times 10^{-5} \text{ radians or } -3.61804'' \\ v_{a4} & = & -2.23360 \times 10^{-5} \text{ radians or } -4.60713'' \\ v_{d1} & = & 0.14034 \times 10^{-7} \text{ meters} \\ v_{d2} & = & 0.23842 \times 10^{-7} \text{ meters} \\ v_{d3} & = & 0.24362 \times 10^{-7} \text{ meters} \end{array} \right]$$

The corrected angles are

$$\begin{aligned} \alpha_1 &= 246^\circ 05' 45.81717'' \\ \alpha_2 &= 222^\circ 51' 05.36399'' \\ \alpha_3 &= 190^\circ 14' 58.98196'' \\ \alpha_4 &= 277^\circ 05' 12.39287'' \end{aligned}$$

The corrected distances are

$$\begin{aligned} d_1 &= 1424.00400 \text{ m.} \\ d_2 &= 365.74400 \text{ m.} \\ d_3 &= 6476.27100 \text{ m.} \end{aligned}$$

By using the corrected data to recompute the coordinates, the adjusted coordinates are

$$\begin{aligned} E_2 &= 607,943.45994 \text{ m.} \\ N_2 &= 4,073,939.74899 \text{ m.} \\ E_3 &= 608,122.00770 \text{ m.} \\ N_3 &= 4,074,258.95013 \text{ m.} \\ E_4 &= 612,238.85256 \text{ m.} \\ N_4 &= 4,079,258.31754 \text{ m.} \end{aligned}$$

The last adjusted coordinates are the same values as the fixed coordinates. Hence, it is not necessary to iterate.

The standard deviation of an observation which has unit weight can be found by the following equations. For uncorrelated observations of unequal precision,

$$\sigma_o = \left[\frac{\mathbf{v}^T \mathbf{P} \mathbf{v}}{r} \right]^{1/2} \quad (3.44)$$

and, for uncorrelated observations of equal precision,

$$\sigma_o = \left[\frac{\mathbf{v}^T \mathbf{v}}{r} \right]^{1/2} \quad (3.45)$$

where r is the number of condition equations. There are three condition equations for a closed traverse ($r=3$). The standard deviations of adjusted angles and distances can be found by the following equation:

$$\sigma_i = \sigma_o \left[Q_{ii} \right]^{1/2} \quad (3.46)$$

where σ_i is the standard deviation of the i th adjusted quantity. The quantity in the i th row of the $\mathbf{V}$ matrix, Q_{ii} is an element of the $\mathbf{P}^{-1} - \mathbf{P}^{-1} \mathbf{B}^T (\mathbf{B} \mathbf{P}^{-1} \mathbf{B}^T)^{-1} \mathbf{B} \mathbf{P}^{-1}$ matrix for uncorrelated observations of unequal precision [Ref. 3, p. 918]. For uncorrelated observations of equal precision, Q_{ii} is the element of the $\mathbf{I} - \mathbf{B}^T (\mathbf{B} \mathbf{B}^T)^{-1} \mathbf{B}$ matrix. The matrix $\mathbf{P}^{-1} - \mathbf{P}^{-1} \mathbf{B}^T (\mathbf{B} \mathbf{P}^{-1} \mathbf{B}^T)^{-1} \mathbf{B} \mathbf{P}^{-1}$ or the matrix $\mathbf{I} - \mathbf{B}^T (\mathbf{B} \mathbf{B}^T)^{-1} \mathbf{B}$ is equal to

$$\begin{bmatrix} Q_{11} & Q_{12} & \cdot & \cdot & \cdot & \cdot & Q_{1n} \\ Q_{21} & Q_{22} & \cdot & \cdot & \cdot & \cdot & Q_{2n} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ Q_{n1} & Q_{n2} & \cdot & \cdot & \cdot & \cdot & Q_{nn} \end{bmatrix}$$

For uncorrelated observations of unequal precision, the standard deviation of unit weight is found by solving Equation (3.44), the result is

$$\sigma_0 = \pm 2.69679$$

The results of $P^{-1} - P^{-1}B^T(BP^{-1}B^T)^{-1}BP^{-1}$ are

$$10^{-10} \begin{bmatrix} 0.064 & -0.052 & -0.021 & 0.010 & -4.165 & -6.963 & -63.88 \\ -0.052 & 0.225 & -0.175 & 0.002 & 3.415 & 4.689 & 41.59 \\ -0.021 & -0.175 & 0.211 & -0.014 & 2.563 & 3.574 & 31.79 \\ 0.010 & 0.002 & -0.014 & 0.002 & -1.813 & -1.301 & -9.500 \\ -4.165 & 3.415 & 2.563 & -1.813 & 9917 & -117.4 & -1047 \\ -6.963 & 4.689 & 3.574 & -1.301 & -117.4 & 9830 & -1525 \\ -63.88 & 41.59 & 31.79 & -9.500 & -1047 & -1525 & 76323 \end{bmatrix}$$

The standard deviations of adjusted angles and distances are found by solving Equation (3.46); the results are

$$\begin{aligned} \sigma_{a1} &= \pm 1.40340 \text{ seconds} \\ \sigma_{a2} &= \pm 2.64096 \text{ seconds} \\ \sigma_{a3} &= \pm 2.55253 \text{ seconds} \\ \sigma_{a4} &= \pm 0.26136 \text{ seconds} \\ \sigma_{d1} &= \pm 0.00269 \text{ meters} \\ \sigma_{d2} &= \pm 0.00267 \text{ meters} \\ \sigma_{d3} &= \pm 0.00745 \text{ meters} \end{aligned}$$

For uncorrelated observations of equal precision, the standard deviation of unit weight is found by solving Equation (3.45), the result is

$$\sigma_0 = \pm 2.01139 \times 10^{-5}$$

The results of $I - B^T(BB^T)^{-1}B$ are

$$10^{-3} \begin{bmatrix} 1.472 & -27.199 & 26.962 & -1.235 & -0.518 & -0.843 & -0.856 \\ -27.199 & 503.1 & -498.7 & 22.855 & 0.348 & 0.480 & 0.473 \\ 26.962 & -498.7 & 494.4 & -22.66 & 0.370 & 0.523 & 0.518 \\ -1.235 & 22.855 & -22.66 & 1.038 & -0.201 & -0.159 & -0.134 \\ -0.518 & 0.348 & 0.370 & -0.201 & 10000 & -0.001 & -0.001 \\ -0.843 & 0.480 & 0.523 & -0.159 & -0.001 & 10000 & -0.001 \\ -0.856 & 0.473 & 0.518 & -0.134 & -0.001 & -0.001 & 10000 \end{bmatrix}$$

The standard deviations of adjusted angles and distances are found by solving Equation (3.46); the results are

$$\begin{aligned} \sigma_{a1} &= \pm 0.15918 \text{ seconds} \\ \sigma_{a2} &= \pm 2.94262 \text{ seconds} \\ \sigma_{a3} &= \pm 2.91722 \text{ seconds} \\ \sigma_{a4} &= \pm 0.13369 \text{ seconds} \\ \sigma_{d1} &= \pm 0.00002 \text{ meters} \\ \sigma_{d2} &= \pm 0.00002 \text{ meters} \\ \sigma_{d3} &= \pm 0.00002 \text{ meters} \end{aligned}$$

The computations in this subsection were also performed on the NPS IBM 3033 computer; they used 16 decimal places, and were rounded off to 5 decimal places in the final solutions. These are more decimal places than are normally used in practice because such precisions are not generally attainable by corresponding observations. However, in comparing two computational methods that are theoretically equal, one method may be more sensitive to round off error than the other. This will be commented on in the next chapter.

IV. DISCUSSIONS AND ANALYSIS OF RESULTS

Three programs were used to compute the initial traverse and adjust the closed traverse station positions. Programs 1, 2, and 3 were used to compute and adjust the traverse station positions by the Approximate Method, the Indirect Observations Method, and the Condition Equation Method, respectively. The programs were written in WATFIV language for implementation on the NPS IBM 3033 computer. The computer output and listings of Programs 1, 2, and 3 are provided in Appendices A, B, and C, respectively. The maximum number of intermediate traverse station positions that can be computed and adjusted by these programs is 30. The programs were tested using several fictitious data sets to ensure their performance in handling the various intermediate traverse station positions.

The computer storage area and CPU time of Programs 1, 2, and 3 has been compared (Table X). The adjustment of a closed traverse by the Approximate Method did not use a weight matrix in the matrix computations. The computer program for this method was written by using the variables in only one dimension for storage of both data and results. This method did not require any iterations. The least squares adjustment of a closed traverse characteristically is used to simultaneously eliminate closing errors in azimuths and distances, and Programs 2 and 3 both utilized such an adjustment. The computation by either Least Squares Method used matrices in two dimensions to compute the correction vector. Likewise the computer programs were written using variables in two dimensions. Included are all subroutines from Program 1 and extra subroutines for each individual program. Therefore, both Programs 2 and 3 used more computer storage area and CPU time than Program 1.

TABLE X

The Comparisons Between the Computer Storage Area and CPU Time of Programs 1, 2, and 3

	Programs			Unit
	1	2	3	
1. The number of statements	450	874	648	Bytes
2. The ratio of statements of Programs 1:2 and 1:3	1:1	1:1.9	1:1.4	
3. The ratio of statements of Programs 3:2	-	1:1.3	1:1	
4. The total computer storage area	25,536	294,856	129,728	
5. The ratio of computer storage area of Programs 1:2 and 1:3	1:1	1:11.5	1:5.1	
6. The ratio of computer storage area of Programs 3:2	-	1:2.3	1:1	
7. The CPU time for adjusted two stations	.62	1.56	1.10	
8. The ratio of CPU time of Programs 1:2 and 1:3	1:1	1:2.5	1:1.8	
9. The ratio of CPU time of Programs 3:2	-	1:1.4	1:1	
				Sec.

The Indirect Observations Method requires that the number of observation equations be equal to the sum of observation equations of both observed angles and distances. The number of observation equations is not constant. It changes depending on the number of the traverse stations-- this means that the row dimension of each matrix will change depending on the number of observation equations. Therefore, the subroutines affected by the number of traverse stations are difficult to write in the general program. The coefficients in the A matrix are computed by taking the partial derivatives of the functions with respect to each unknown variable, where the number of unknown variables is not constant. The number of unknown variables changes depending on the number of the traverse stations. Therefore, the subroutine for the computed A matrix is also difficult to write in the general program. The Condition Equations Method has only three equations which makes it easier to derive the general form and write the computer program. The adjustment using the Indirect Observations Method does not apply the corrections to the observed values directly; therefore, it requires the approximate values of the unknown coordinates be computed for the correction vector of the unknown coordinates. This method requires at least two iterations to check the insignificance of the correction vector when it is compared with an arbitrarily selected small number. The adjusted coordinates from the Condition Equations Method can be checked at the first iteration. Thus, Program 3 is more economical as it uses less computer storage area and CPU time than Program 2 (Table X).

The closed traverse at Moss Landing originated and terminated at control points with known positions which were determined by third-order methods. The azimuth closure was 2.211" per station and the position closure was 1:66,617. The classification, standards of accuracy, and general

specifications for a third-order class I traverse (Table I) indicate the azimuth closure is not to exceed 3" per station and position closure must be better than 1:10,000. This traverse met the specifications and standards of accuracy for a third-order class I traverse.

Using Programs 1, 2, and 3, the traverse was computed and adjusted in UTM grid coordinates. The final adjusted coordinates were transformed from UTM grid coordinates to geographic coordinates [Ref. 4, pp. 319-321]. The difference between the coordinates for each method and NOS [Ref. 10] have been computed (Table XI). The technique of Least Squares for both methods yields identical computational results to at least five decimal places (Table XI, Methods 3.1 and 4.1; 3.2 and 4.2). Least Squares provides the best estimates for positions of all traverse stations. Program 2 performs a statistical test, yielding the standard deviations of both adjusted positions and observed values. Conversely, Program 3 only yields the standard deviation of adjusted observed values. Comparisons of the standard deviations of adjusted observed angles and distances were made between these computed by the Indirect Observations Method and the Condition Equations Method (Table XII). The computed standard deviations of the adjusted angles differed significantly in the fourth decimal place in several cases, which is a slight indication that the Indirect Observations Method is more sensitive to round off error. The standard deviation estimates are larger for the Indirect Observations Method in every case of a significant difference in the fourth decimal place. However, a significant difference in the fourth decimal place is insignificant in terms of the observational precision. The standard deviations of the adjusted distances for both methods yield identical results (Table XII). Both Methods differ from the NOS positions in the fourth decimal place in seconds of arc for both latitude

and longitude. The horizontal control for third-order standards requires accuracy to three decimal places in seconds of arc for both latitude and longitude--thus, these methods can be used for computing third-order positions.

TABLE XI
Geographic Coordinates

Methods	At Mossback station			
	Latitude (N.)	Diff.	Longitude (W.)	Diff.
1. U.S. NOS	0 ° "	"	0 ° "	"
	36 48 25.09750	-	121 47 23.75903	-
2. Approximate	36 48 25.09713	+0.00037	121 47 23.75934	-0.00031
3. Indirect Observations				
3.1 Equal precision	36 48 25.09762	-0.00012	121 47 23.75869	+0.00034
3.2 Unequal precision	36 48 25.09759	-0.00009	121 47 23.75889	+0.00014
4. Condition Equation				
4.1 Equal precision	36 48 25.09762	-0.00012	121 47 23.75869	+0.00034
4.2 Unequal precision	36 48 25.09759	-0.00009	121 47 23.75889	+0.00014
Methods	At Dune Temp station			
	Latitude (N.)	Diff.	Longitude (W.)	Diff.
1. U.S. NOS	0 ° "	"	0 ° "	"
	36 48 35.38113	-	121 47 16.39167	-
2. Approximate	36 48 35.38077	+0.00036	121 47 16.39200	-0.00033
3. Indirect Observations				
3.1 Equal precision	36 48 35.38121	-0.00008	121 47 16.39129	+0.00038
3.2 Unequal precision	36 48 35.38122	-0.00009	121 47 16.39152	+0.00015
4. Condition Equation				
4.1 Equal precision	36 48 35.38121	-0.00008	121 47 16.39129	+0.00038
4.2 Unequal precision	36 48 35.38122	-0.00009	121 47 16.39152	+0.00015

TABLE XII
The Comparisons Between the Standard Deviations of Least Squares

Unequal Precision Observations									
		Standard Deviations of			Adjusted Distances				
		Adjusted Angles			Adjusted Distances				
Methods		1	2	3	4	1	2	3	m.
1. Condition Equation		"	"	"	"	m.	m.	m.	m.
	1.40340	2.64096	2.55253	0.26136	0.00269	0.00267	0.00745		
2. Indirect Observation	1.40346	2.64103	2.55260	0.26137	0.00269	0.00267	0.00745		
Difference		-0.00006	-0.00007	-0.00007	-0.00001	-	-	-	-
Equal Precision Observations									
		Standard Deviations of			Adjusted Distances				
		Adjusted Angles			Adjusted Distances				
Methods		1	2	3	4	1	2	3	m.
1. Condition Equation		"	"	"	"	m.	m.	m.	m.
	0.15918	2.94262	2.91722	0.13369	0.00002	0.00002	0.00002	0.00002	0.00002
2. Indirect Observation	0.15917	2.94270	2.91732	0.13370	0.00002	0.00002	0.00002	0.00002	0.00002
Difference		+0.00001	-0.00008	-0.00010	-0.00001	-	-	-	-

V. CONCLUSIONS AND RECOMMENDATION

A. CONCLUSIONS

The computer programs developed by the author and contained in this thesis have a variety of useful and practical hydrographic applications. In hydrography, geodetic field work plays a key role. Horizontal control is necessary in determining positions and, even in areas which appear to initially have adequate control, additional control stations often need to be established after arrival in the field. These field-generated control points must be adjusted to fit within the existing survey net and to minimize errors in the field measurements. The errors can be minimized by these computer programs. Direct application of these programs would be enormously beneficial, since no equivalent software exist at NPS to perform such adjustments at the present time.

In the Approximate Method, a traverse is adjusted by computations using a hand calculator. This method is suitable for field computation and the adjusted coordinates meet the specifications and standards of accuracy for a third-order class I traverse. Although the accuracy from this method is less than the Least Squares Method, it does not require a computer and it furnishes coordinates which can be checked in the field by the field party.

The Least Squares Method provides the best estimates for positions of all traverse stations. Least Squares does require more of a computational effort than the Approximate Method. However, the accuracy of Least Squares is better than the Approximate Method, and the Least Squares Method should be used for the final office computations. Both

Least Squares Methods yield identical results, but the Condition Equations Method is more economical and easier to derive than the Indirect Observations Method. Therefore, the Condition Equations Method should be applied at NPS.

B. RECOMMENDATION

Studies should be continued at NPS to compare the economy of adjustment methods used in this thesis with other methods of least squares adjustment.

APPENDIX A

PROGRAM 1

COMPUTER OUTPUT THE CALCULATION OF CLOSED TRAVERSE STATION POSITIONS BY APPROXIMATE METHOD

KNOWN HORIZONTAL STATION POSITIONS *****

NO	NAME OF STATIONS	UTM GRID COORDINATES		AZIMUTH FROM TO	GRID AZIMUTH CLOCKWISE
		GRID NORTHING	GRID EASTING		
1	PIPHER				
2	MOSS 2	4072555.85206	608279.04404	2 -> 1	100 16 23.77800
3	HOLM	4079258.31754	612238.85256	3 -> 4	136 33 26.33400
4	MORAN				

OBSERVED DATA *****

NAME OF STATIONS	OBSERVED ANGLE		STD. S.	GRID DISTANCE METERS	STD. M.
	D. M.	S.			
PIPHER					
MOSS 2	246	5 43.20000	1.98400	1424.00400	0.00100
MOSSBACK	222	51 8.60000	1.40500	365.74400	0.00100
DUNE TEMP	190	15 2.60000	1.20300	6476.27100	0.00300
HOLM	277	5 17.00000	1.61400		
MOPAN					

PRECALCULATED OF CLOSED TRAVERSE STATION POSITIONS

NO	NAME OF STATIONS	UTM GRID COORDINATES		AZIMUTH FROM TO	GRID AZIMUTH CLOCKWISE FROM NORTH
		GRID NORTHING	GRID EASTING		
1	MOSS 2	4072555.85206	608279.04404	1 -> 2	346 22 6.97800
2	MOSSBACK	4073939.74473	607943.44238	2 -> 1	166 22 6.97800
3	DUNE TEMP	4074258.94534	608121.99110	2 -> 3	29 13 15.57800
4	HOLM	4079258.22818	612238.93865	3 -> 2	209 13 15.57800
5	MORAN			3 -> 4	39 28 18.17800
				4 -> 3	219 28 18.17800
				4 -> 5	136 33 35.17800

TOTAL AZIMUTH ERROR = 0 D. 0 M. 8.84400 S.
 AZIMUTH ERROR PER STATIONS = 0 D. 0 M. 2.21100 S.
 TOTAL GRID DISTANCE = 8266.01900 METERS.
 DISTANCE ERROR = 0.12408 METERS.
 ACCURACY = 1: 66617

ADJUSTMENT OF CLOSED TRAVERSE STATION POSITIONS BY APPROXIMATE METHOD

NO	NAME OF STATIONS	UTM GRID COORDINATES		AZIMUTH		GRID AZIMUTH	
		GRID NORTHING	GRID EASTING	FROM TO	NO	NO	FROM NORTH
		METERS	METERS			D. M. S.	
1	MOSS 2	4072555.85206	608279.04404	1 -> 2	346 22	4.76700	
2	MOSSBACK	4073939.73368	607943.44415	2 -> 1	166 22	4.76700	
3	DUNE TEMP	4074258.93620	608121.99028	2 -> 3	29 13	11.15600	
4	HOLM	4079258.31754	612238.85256	3 -> 2	209 13	11.15600	
5	MORAN			3 -> 4	39 28	11.54500	
				4 -> 3	219 28	11.54500	
				4 -> 5	136 33	26.33400	

CHECK ROUND OFF ERROR

AZIMUTH = 0 D. 0 M. 0.00000 S.
DISTANCE = 0.00000 METERS.

```
C C TRAVAPP WATFIV A1
C C $JOB
C C SAM1,NOCHECK
C CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C C AUTHOR : LT. SAMAN AUMCHANTR RTN.
C C DATE : JULY 11 , 1984
C CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C C THIS IS A PROGRAM FOR CALCULATION OF TRAVERSE STATION
C C POSITIONS BY APPROXIMATE METHOD.
C C Ppppppppppppppppppppppppppppppppppppppppppppppppppppppppp
C C VARIABLE DEFINITIONS
C C OPT = OPTION FOR CALCULATION OF OPEN OR CLOSED TRAVERSE
C C CHOIC = OPTION FOR ENTER DATA BY DATA FILE OR INTERACTIVE
C C FROM TERMINAL KEYBOARD.
C C DUMMY1,DUMMY2,DUMID,DUMIM,DUMIS = THE DUMMY VARIABLES
C C NGRID(4)= GRID NORTHING OF THE KNOWN POSITIONS.
C C EGRID(4)= GRID EASTING OF THE KNOWN POSITIONS.
C C STNAME(4,5) = THE NAME OF KNOWN STATIONS.
C C TRNAME( 32,5) = THE NAME OF TRAVERSE STATIONS.
C C DIRECD(4),DIRECM(4),DIRECS(4) = THE GRID AZIMUTH OF KNOWN
C C POSITIONS.
C C STAT1(4),STAT2(4) = THE NUMBER OF KNOWN POSITIONS.
C C COUNT, COUNTA = THE NUMBER OF OBSERVED ANGLES.
C C ANG( 32)=THE OBSERVED ANGLE IN RADIAN.
C C ANGDI( 32),ANGMI( 32),ANGS( 32) = THE OBSERVED ANGLES IN DEGREE,
C C MINUTE AND SECOND.
C C COUNTN, COUNTD = THE NUMBER OF MEASURED DISTANCE.
C C DIST( 32) = THE MEASURED DISTANCE IN METERS.
C C TODIS = THE SUM OF MEASURED DISTANCE.
C C CFAZD( 32),CFAZM( 32),CFAZS( 32) = THE COMPUTED FORWARD AZIMUTH
C C CBABZD( 32),CBABZM( 32),CBABZS( 32) = THE COMPUTED BACK AZIMUTH
C C CNGRID( 32) = GRID NORTHING OF TRAVERSE STATION POSITIONS.
C C CNGRID( 32) = GRID EASTING OF TRAVERSE STATION POSITIONS.
C C ERGR = TOTAL DISTANCE ERROR.
C C DIFAZI = THE TOTAL ANGLE ERROR IN RADIAN.
C C EAZID, EAZIM, EAZIS = THE TOTAL ANGLE ERROR IN DEGREE,
C C MINUTE AND SECOND.
C C EAZ2D, EAZ2M, EAZ2S = THE ANGLE ERROR PER STATION.
```

```
C CC C
TOMS      = SUBROUTINE FOR CHANGE INPUT DATA FORM DD.MMSSSSSSS V
            TO DEGREE, MINUTE AND SECOND. V
C DMSR     = SUBROUTINE FOR CHANGE DEGREE, MINUTE AND V
            SECOND TO RADIAN. V
C CRDMS    = SUBROUTINE FOR CHANGE RADIANS TO DEGREE, MINUTE V
            AND SECOND. V
C CTANA    = SUBROUTINE FOR CALCULATION OF GRID AZIMUTH AND THE V
            THE DISTANCE BETWEEN POSITIONS #1 AND #2 V
C CXYC     = SUBROUTINE FOR COMPUTATION OF THE UTM GRID COORDINATES V
            V
VVVVVVVVVVVVVVVVVVVVVVVVVVVVVVVVVVVVVVVVVVVVVVVVVVVVVVVVVVV C
                VARIABLE DECLARATIONS C
CC DOUBLE PRECISION DUMMY1,CFAZS( 32),CBAAZS( 32),EAZIS,EAZ2S,CFGRID( C
   *32),CNGRID( 32),DIS21,AZ12,DIS34,AZ34,DUMID,AZ43,DUMIM,DUMIS, C
   *DUMMY2,ANG( 32),DIS1( 32),AZFIR,AZIMUT,NUM0,NUM360,AN36OR,DISTAN, C
   *AZC,DIFDIS,AZLAS,DIFAZI,CORAZ,TODIS,COAZR( 32),WPRERD(3),SUMDY C
   *DOUBLE PRECISION ANG$ ( 32),DIRECD$ (4),NGRID(4),CORDX1,CORD C
   *Y1,XGD,YGD,STDA( 32),STDD( 32),DELTAX( 32),DELTA( 32),SUMDX C
   INTEGER DES,OPT,COUNT,I,COUNTA,COUNTD,ERR,CHOIC,LOOP1,DIRECD(4),DI C
   *RECH(4),STAT1(4),STAT2(4),STNAME(4,5),K1,K2,ANGDI( 32),ANGM( 32),CO C
   *UNT1,TRNAME( 32,5),K4,K5,COUNTP,EALID,EAZIM,EAZ2D,EAZ2M,CFAZO( 32) C
   *,CBAAZD( 32),CBAAZM( 32),TEMD,TEMM,ADJL,CFAZM( 32),ITER1 C
DATA NUMO/O.OOO/,NUM360/360.OOO/,TODIS/O.OOO/,SUMDX/O.OOO/,SUMDY/O C
*.ODO/ C
CC ALGORITHM INPUT_DATA C
CC PRINT TITLE C
WRITE ( 6,1000 ) C
WRITE ( 6,1380 ) C
WRITE ( 6,1390 ) C
WRITE ( 6,1400 ) C
WRITE ( 6,1410 ) C
WRITE ( 6,1420 ) C
WRITE ( 6,1390 ) C
WRITE ( 6,1380 ) C
WRITE ( 6,1430 ) C
WRITE ( 6,1440 ) C
READ OPTIONS 1 = OPEN TRAVERSE C
```



```

*
WRITE ( 6,1680 ) ( STNAME(2,K1), K1=1,5), (STNAME(1,K1), K1=1
5)
WRITE ( 6,1690 )
READ ( DES,1360 ) DUMMY1
CALL TDMS ( DUMMY1, DUMID, DUMIM, DUMIS )
DIRECD ( 2 ) = IDINT ( DUMID )
DIRECM ( 2 ) = IDINT ( DUMIM )
DIRECS ( 2 ) = DUMIS
CALL CDMR ( DUMID, DUMIM, DUMIS, DUMMY1 )
AZ21 = DUMMY1
STAT1 ( 2 ) = 2
STAT2 ( 2 ) = 1
GO TO 60

```

```

40 CONTINUE
IF ( I .NE. 3 ) GO TO 50
WRITE ( 6,1530 )
READ ( DES,1370 ) ( STNAME ( I,K1 ) , K1 = 1,5 )
WRITE ( 6,1660 )
WRITE ( 6,1540 )
READ ( DES,1360 ) DUMMY1
NGRID ( I ) = DUMMY1
WRITE ( 6,1670 )
WRITE ( 6,1560 )
READ ( DES,1360 ) DUMMY1
EGRID ( I ) = DUMMY1
GO TO 60

```

```

50 CONTINUE
IF ( I .NE. 4 ) GO TO 60
WRITE ( 6,1510 )
READ ( DES,1370 ) ( STNAME ( I,K1 ) , K1 = 1,5 )
WRITE ( 6,1680 ) ( STNAME(3,K1), K1=1,5), (STNAME(4,K1), K1=1
5)
WRITE ( 6,1690 )
READ ( DES,1360 ) DUMMY1
CALL TDMS ( DUMMY1, DUMID, DUMIM, DUMIS )
DIRECD ( 3 ) = IDINT ( DUMID )
DIRECM ( 3 ) = IDINT ( DUMIM )
DIRECS ( 3 ) = DUMIS
CALL CDMR ( DUMID, DUMIM, DUMIS, DUMMY1 )
AZ34 = DUMMY1
STAT1 ( 3 ) = 3
STAT2 ( 3 ) = 4

```

```

60 CONTINUE
70 CONTINUE
CC
CC
CC

```

READ THE NUMBER OF OBSERVED ANGLES

```

C      WRITE ( 6,1580 )
      READ ( DES,1350 ) COUNT
      COUNTA = COUNT
C
C      READ THE OBSERVED ANGLES
C
      DO 100 I = 1, COUNTA
      IF ( I.GT.1 ) GO TO 80
      WRITE ( 6,1590 )
      GC TO 90
      CONTINUE
80     K2 = I - 1
      WRITE ( 6,1600 ) K2
      CONTINUE
      WRITE ( 6,1610 )
      READ ( DES,1360 ) DUMMY1
      CALL TDMS ( DUMMY1,DUMID,DUMIM,DUMIS )
      CALL CDMSR ( DUMID,DUMIS,DUMMY2 )
      ANG ( I ) = DUMMY2
      ANG2 ( I ) = IDINT ( DUMID )
      ANG3 ( I ) = IDINT ( DUMIM )
      ANG4 ( I ) = DUMIS
      WRITE ( 6,1550 )
      READ ( DES,1360 ) DUMMY1
      STDA ( I ) = DUMMY1
100    CONTINUE
C
C      READ THE GRID DISTANCE
C
      COUNTD = COUNT
      IF ( OPT.EQ.1 ) GO TO 110
      COUNTD = COUNT - 1
110    CONTINUE
      DO 140 I = 1, COUNTD
      IF ( I.GT.1 ) GO TO 120
      WRITE ( 6,1620 )
      GO TO 130
      CONTINUE
      K4 = I - 1
      K5 = I
      WRITE ( 6,1630 ) K4,K5
      CONTINUE
      WRITE ( 6,1640 )
      READ ( DES,1360 ) DUMMY1
      DIST ( I ) = DUMMY1
120
130

```

```

140 TODIS = TODIS + DUMMY1
    WRITE ( 6,1570 )
    READ ( DES,1360 ) DUMMY1
    STDD ( 1 ) = DUMMY1
    CONTINUE
    COUNTN = COUNTD
    IF ( OPT.EQ. 1 ) GO TO 150
    IF ( COUNTN = COUNTD - 1
150 CONTINUE
    DO 160 I = 1,COUNTN
        WRITE ( 6,1650 ) I
        READ ( DES,1370 ) ( TRNAME ( I,K1 ) , K1 = 1,5 )
160 CONTINUE
        CCCCCCCCCC
        END INPUT_DATA
        ALGORITHM PRINT_INPUT_DATA
        PRINT DETAILS OF THE KNOWN STATIONS

        WRITE ( 6,1000 )
        WRITE ( 6,1010 )
        WRITE ( 6,1020 )
        WRITE ( 6,1030 )
        WRITE ( 6,1040 )
        WRITE ( 6,1050 )
        WRITE ( 6,1060 )
        DO 180 I = 1,LOOP1
            IF ( I.EQ. 2 .OR. I.EQ. 3 ) GO TO 170
            WRITE ( 6,1230 ) I, ( STNAME ( I,K1 ) , K1 = 1,5 )
            GO TO 180
        CONTINUE
170 WRITE ( 6,1070 ) I,(STNAME(I,K1),K1=1,5),NGRID(I),EGRID(I),STA
        T1(I),STAT2(I),DIRECD(I),DIRECM(I),DIRECS(I)
180 * CONTINUE
        WRITE ( 6,1000 )
        WRITE ( 6,1080 )
        WRITE ( 6,1090 )
        WRITE ( 6,1100 )
        WRITE ( 6,1110 )
        WRITE ( 6,1120 )
        WRITE ( 6,1130 ) ( STNAME ( 1,K1 ) , K1 = 1,5 )
        WRITE ( 6,1140 ) ( STNAME(2,K1),K1 = 1,5),ANGD(1),ANGM(1),ANGS(1),S
        *TDA(1)
        WRITE ( 6,1150 ) DIST ( 1 ),STDD ( 1 )
        COUNTP = COUNTD - 1

```

```

190 IF ( COUNTP .NE. 0 ) GO TO 190
    WRITE ( 6,1160 ) ( TRNAME ( 1,K1 ) , K1 = 1,5 )
    GO TO 220
    CONTINUE
DO 200 I = 1,COUNTP
    K2 = I + 1
    WRITE ( 6,1140 ) ( TRNAME(I,K1),K1=1,5),ANGD(K2),ANGM(K2),ANGS(
    * K2),STDA(K2)
    WRITE ( 6,1150 ) DIST ( K2 ),STDD ( K2 )
200 CONTINUE
    IF ( OPT .NE. 1 ) GO TO 210
    WRITE ( 6,1160 ) ( TRNAME ( I,K1 ) , K1 = 1,5 )
    GO TO 220
    CONTINUE
210 WRITE ( 6,1140 ) ( STNAME(3,K1),K1=1,5),ANGD(COUNTA),ANGM(COUNTA),
    * ANGS(COUNTA),STDA(COUNTA)
    WRITE ( 6,1130 ) ( STNAME ( 4,K1 ) , K1 = 1,5 )
    CONTINUE
220 CONTINUE
    WRITE ( 6,1000 )

CCCCCCCCC

    END PRINT_INPUT_DATA
    ALGORITHM COMPUTE_ADJUST_APPROXIMATE
    THE CALCULATION OF OPEN TRAVERSE

    CALL CDMR ( NUM360,NUMO,NUMO,AN360R )
    ITER1 = 2
    IF ( OPT .NE. 1 ) GO TO 230
    ITER1 = 1
230 CONTINUE
    DO 390 ADJ1 = 1,ITER1
        XGD = EGRID ( 2 )
        YGD = NGRID ( 2 )
        AZFIR = AZ21
        DO 250 I = 1,COUNTD
            AZIMUT = AZFIR + ANG ( I )
            IF ( AZIMUT .LT. AN360R ) GO TO 240
            AZIMUT = AZIMUT - AN360R
        CONTINUE
        COAZR ( I ) = AZIMUT
        CALL CROMS ( AZIMUT,DUMID,DUMIM,DUMIS )
        CFAZD ( I ) = IDINT ( DUMID )
        CFAZM ( I ) = IDINT ( DUMIM )
        CFAZS ( I ) = DUMIS
        DISTAN = DIST ( I )
240

```

```

250 CALL CXYC ( XGD,YGD,DISTAN,AZIMUT,DUMMY1,DUMMY2 )
      DELTAX ( I ) = DUMMY1 - XGD
      DELTAY ( I ) = DUMMY2 - YGD
      SUMDX = SUMDX + DELTAX ( I )
      SUMDY = SUMDY + DELTAY ( I )
      CEGRID ( I ) = DUMMY1
      CNGRID ( I ) = DUMMY2
      CALL CTANA ( DUMMY1,DUMMY2,XGD,YGD,AZC,DUMIS )
      CALL CROMS ( AZC,DUMID,DUMIM,DUMIS )
      CBAZD ( I ) = IDINT ( DUMID )
      CBAZM ( I ) = IDINT ( DUMIM )
      CBAZS ( I ) = DUMIS
      AZFIR = AZC
      XGD = DUMMY1
      YGD = DUMMY2
      CONTINUE
C
C
C
C
      PRINT DETAILS OF PRECALCULATED OF TRAVERSE STATION POSITIONS

      IF ( OPT.EQ. 1 ) GO TO 280
      IF ( ADJ1.NE. 1 ) GO TO 260
      WRITE ( 6,1180 )
      WRITE ( 6,1190 )
      GO TO 290
260 CONTINUE
      CORDX1 = ( SUMDX - ( EGRID (3) - EGRID (2) ) ) / TOTDIS
      CORDY1 = ( SUMDY - ( NGRID (3) - NGRID (2) ) ) / TOTDIS
      XGD = EGRID ( 2 )
      YGD = NGRID ( 2 )
      DO 270 I = 1,COUNTD
        CEGRID ( I ) = XGD + ( DELTAX(I) - ( CORDX1*DIST(I)) )
        CNGRID ( I ) = YGD + ( DELTAY(I) - ( CORDY1*DIST(I)) )
        XGD = CEGRID ( I )
        YGD = CNGRID ( I )
      CONTINUE
      WRITE ( 6,1000 )
      WRITE ( 6,1290 )
      WRITE ( 6,1300 )
      GO TO 290
270 CONTINUE
      WRITE ( 6,1170 )
      WRITE ( 6,1200 )
280 CONTINUE
      WRITE ( 6,1030 )
      WRITE ( 6,1040 )
      WRITE ( 6,1050 )
290

```

```

WRITE ( 6,1060 ) (STNAME(2,K1),K1=1,5),NGRID(2),EGRID(2)
WRITE ( 6,1210 ) (STNAME(2,K1),K1=1,5),NGRID(2),EGRID(2)
DO 330 I = 1, COUNTD
  K4 = I + 1
  K5 = I + 1
  WRITE ( 6,1220 ) K4,K5,CFAZD(I),CFAZM(I),CFAZS(I)
  IF ( OPT.EQ.1 ) GO TO 310
  IF ( I.NE.COUNTD ) GO TO 300
  WRITE ( 6,1070 ) K5,(STNAME(3,K1),K1=1,5),CNGRID(I)
  ,CEGRID(I),K5,K4,CBAZD(I),CBAZM(I),CBAZS(I)
  CONTINUE
  CONTINUE
  IF ( I.EQ.COUNTD ) GO TO 320
  WRITE ( 6,1070 ) K5,(TRNAME(I,K1),K1=1,5),CNGRID(I),
  CEGRID(I),K5,K4,CBAZD(I),CBAZM(I),CBAZS(I)
  CONTINUE
  CONTINUE
  IF ( OPT.NE.1 ) GO TO 340
  WRITE ( 6,1070 ) K5,(TRNAME(COUNTD,K1),K1=1,5),CNGRID(COU
  NTD),CEGRID(COUNTD),K5,K4,CBAZD(COUNTD),CBAZM(COUNTD),CBA
  ZS(COUNTD)
  CONTINUE
  IF ( OPT.EQ.1 ) GO TO 380
  AZLAS = AZFIR + ANG ( COUNT )
  IF ( AZLAS.LT. AN360R ) GO TO 350
  AZLAS = AZLAS - AN360R
  CONTINUE
  K4 = I + 1
  K5 = I + 1
  CALL CROMS ( AZLAS,DUMID,DUMIM,DUMIS )
  TEMD = IDINT ( DUMID )
  TEMM = IDINT ( DUMIM )
  WRITE ( 6,1220 ) K4,K5,TEMD,TEMM,DUMIS
  WRITE ( 6,1230 ) K5,(STNAME(4,K1),K1=1,5)
  DIFAZI = AZLAS - AZ34
  CORAZ = DIFAZI / DFLOAT ( COUNTA )
  CALL CROMS ( DIFAZI,DUMID,DUMIM,DUMIS )
  EAZID = IDINT ( DABS ( DUMID ) )
  EAZIM = IDINT ( DABS ( DUMIM ) )
  EAZIS = DABS ( DUMIS )
  WPRED ( 1 ) = DIFAZI
  CALL CROMS ( CORAZ,DUMID,DUMIM,DUMIS )
  EAZ2D = IDINT ( DABS ( DUMID ) )
  EAZ2M = IDINT ( DABS ( DUMIM ) )
  EAZ2S = DABS ( DUMIS )
  WPRED ( 2 ) = SUMDX - ( EGRID(3) - EGRID(2) )
  WPRED ( 3 ) = SJMDY - ( NGRID(3) - NGRID(2) )
  DIFDIS = DSQRT ( WPRED(2)**2+WPRED(3)**2 )

```

```

ERR = IDINT ( TODIS / DIFDIS )
IF ( ADJL .NE. 1 ) GO TO 370
WRITE ( 6,1240 ) EAZ1D,EAZ1M,EAZ1S
WRITE ( 6,1250 ) EAZ2D,EAZ2M,EAZ2S
WRITE ( 6,1260 ) TODIS
WRITE ( 6,1270 ) DIFDIS
WRITE ( 6,1280 ) ERR

```

CORRECTED OBSERVED ANGLES

```

DO 360 I = 1,COUNTA
ANG ( I ) = ANG ( I ) - CORAZ
CONTINUE
SUMDX = 0.000
SUMDY = 0.000
CONTINUE

```

```

CONTINUE
IF ( OPT .EQ. 1 ) GO TO 400
WRITE ( 6,1310 ) EAZ1D,EAZ1M,EAZ1S
WRITE ( 6,1320 ) EAZ2D,EAZ2M,EAZ2S
SUMDX = EGRID ( 3 ) - CEGRID ( COUNTD )
SUMDY = NGRID ( 3 ) - CNGRID ( COUNTD )
DIFDIS = DSQRT ( SUMDX**2 + SUMDY**2 )
WRITE ( 6,1330 ) DIFDIS
CONTINUE
WRITE ( 6,1000 )

```

```

END COMPUTE_ADJUST_APPROXIMATE

```

STOP

```

1000 FORMAT ( '1' )
1010 FORMAT ( '//,23X,'KNOWN HORIZONTAL STATION POSITIONS', // )
1020 FORMAT ( '23X,'*****', // )
1030 FORMAT ( '31X,'UTM GRID COORDINATES',6X,'AZIMUTH GRID AZIMUTH', // )
1040 FORMAT ( '28X,'GRID NORTHING GRID EASTING FROM TO',4X,'CLOCKWISE', // )
1050 FORMAT ( '1X,'NO',3X,'NAME OF STATIONS',35X,'NO',3X,'NO',4X,'FROM N', // )
1060 FORMAT ( '32X,'METERS',8X,'METERS',14X,'D. M. S.', // )
1070 FORMAT ( '1X,12,1X,5A4,3X,F14.5,2X,F12.5,2X,I2,'->',12,1X,I3,1X,I2', // )
1080 FORMAT ( '///,31X,'OBSERVED DATA' )

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```

1090 FORMAT ( 31X, '*****', // )
1100 FORMAT ( 29X, 'OBSERVED', 19X, ' GRID ', // )
1110 FORMAT ( 6X, 'NAME OF STATIONS', 8X, 'ANGLE', 8X, 'STD.', 9X, 'DISTANCE',
*5X, 'STD.', // )
1120 FORMAT ( 26X, 'D. M. S.', 9X, 'S.', 11X, 'METERS', 7X, 'M.', )
1130 FORMAT ( /, 4X, 5A4, / )
1140 FORMAT ( 4X, 5A4, 1X, 13, 1X, 12, 1X, F8.5, 1X, F8.5 )
1150 FORMAT ( 52X, F14.5, 1X, F8.5 )
1160 FORMAT ( 4X, 5A4, / )
1170 FORMAT ( /, /, 17X, 'CALCULATION OF OPEN TRAVERSE STATION POSITIONS', )
1180 FORMAT ( /, /, 15X, 'PRECALCULATED OF CLOSED TRAVERSE STATION POSITI
*NS', )
1190 FORMAT ( 15X, '*****', // )
1200 FORMAT ( 17X, '*****', // )
1210 FORMAT ( 2X, 1, 1X, 5A4, 3X, F14.5, 2X, F12.5, / )
1220 FORMAT ( 57X, 12, ->, 12, 1X, 13, 1X, 12, 1X, F8.5, / )
1230 FORMAT ( 1X, 12, 1X, 5A4, / )
1240 FORMAT ( /, /, 2X, 'TOTAL AZIMUTH ERROR', 8X, '= ', 13, ' D. ', 12, ' M. ',
*F8.5, / )
1250 FORMAT ( /, /, 2X, 'AZIMUTH ERROR PER STATIONS = ', 13, ' D. ', 12, ' M. ',
*F8.5, / )
1260 FORMAT ( /, /, 'TOTAL GRID DISTANCE', 13X, '= ', F16.5, ' METERS.', )
1270 FORMAT ( /, 2X, 'DISTANCE ERROR', 13X, '= ', F10.5, ' METERS.', )
1280 FORMAT ( /, 2X, 'ACCURACY', 18X, '= ', 1, ' I12 )
1290 FORMAT ( /, /, 6X, 'ADJUSTMENT OF CLOSED TRAVERSE STATION POSITIONS B
*Y APPROXIMATE METHOD', )
1300 FORMAT ( 6X, '*****', // )
1310 FORMAT ( /, /, 2X, 'CHECK ROUND OFF ERROR', )
1320 FORMAT ( /, 2X, 'AZIMUTH = ', 13, ' D. ', 12, ' M. ', F8.5, ' S. ' )
1330 FORMAT ( /, 2X, 'DISTANCE = ', F10.5, ' METERS.', )
1340 FORMAT ( 11 )
1350 FORMAT ( 12 )
1360 FORMAT ( F20.12 )
1370 FORMAT ( 5A4 )
1380 FORMAT ( 10X, '*****', // )
1390 FORMAT ( 10X, '57X', // )
1400 FORMAT ( 10X, '9X, THIS IS A PROGRAM TO CALCULATION OF TRAVERSE',
*4X, 'STATION POSITIONS. THERE ARE TWO OPTIONS IN T
*HIS', 5X, '10X, '4X, 'PROGRAM', 43X, '#', )
1410 FORMAT ( 10X, '10X, '1 CALCULATION OF OPEN TRAVERSE STATION POSITI
*ONS', 4X, '10X, ' )
1420 FORMAT ( 10X, '4X, '2 CALCULATION OF CLOSED TRAVERSE STATION POSI
*TIONS', 3X, ' )
1430 FORMAT ( /, 28X, 'WHAT IS YOUR OPTION ?' )
1440 FORMAT ( /, 20X, 'PLEASE ENTER 1 OR 2 FOR YOUR OPTION', / )

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1450 FORMAT ( //,16X,'THERE ARE TWO OPTIONS TO ENTER DATA.',/ )
1460 FORMAT ( //,14X,'1. ENTER DATA BY DATA FILE.',/ )
1470 FORMAT ( //,14X,'2. ENTER DATA BY INTERACTIVE FROM TERMINAL KEYBOARD.',/ )
1480 FORMAT ( //,21X,'PLEASE ENTER 1 OR 2 FOR YOUR OPTION',/ )
1490 FORMAT ( //,16X,'WHAT IS THE NAME OF KNOWN POSITION #',11,'.?.')
1500 FORMAT ( //,12X,'THIS KNOWN POSITION WAS USED FOR THE FIRST BACKSIG
      *HT.',/ )
1510 FORMAT ( //,12X,'THIS KNOWN POSITION WAS USED FOR THE LAST FORESIG
      *HT.',/ )
1520 FORMAT ( //,12X,'THIS KNOWN POSITION IS THE FIRST POSITION AT WHICH
      *AN ANGLE WAS OBSERVED.',/ )
1530 FORMAT ( //,12X,'THIS KNOWN POSITION IS THE LAST POSITION AT WHICH'
      *AN ANGLE WAS OBSERVED.',/ )
1540 FORMAT ( //,12X,'FOR EXAMPLE: GRID NORTHING 4,072,555.85206 METERS
      *//,26X,'ENTER 4072555.85206000.',/ )
1550 FORMAT ( //,12X,'WHAT IS THE STANDARD DEVIATION OF THE OBSERVED AN
      *GLE',/ ,12X,'AT THIS STATION IN SECONDS?',/ ,12X,'FOR EXAMPLE STD.
      *= 3.9 SECONDS.',/ )
1560 FORMAT ( //,12X,'FOR EXAMPLE: GRID EASTING 608,279.04404 METERS.',/ )
1570 FORMAT ( //,26X,'ENTER 608279.04404000.',/ )
1580 FORMAT ( //,12X,'WHAT IS THE STANDARD DEVIATION IN METERS?',/ ,12X
      *//,26X,'ENTER 0.011000.',/ )
1590 FORMAT ( //,12X,'HOW MANY POSITIONS DID YOU OBSERVE ANGLES?',/ ,19
      *X,'PLEASE ENTER 01 TO 32.',/ )
1600 FORMAT ( //,12X,'ENTER THE OBSERVED ANGLE AT THE FIRST KNOWN POSITI
      *ON.',/ )
1610 FORMAT ( //,10X,'ENTER THE OBSERVED ANGLE AT TRAVERSE STATION POSIT
      *ION #',12,'.',/ )
1620 FORMAT ( //,10X,'FOR EXAMPLE: OBSERVED ANGLE CLOCKWISE FROM BACKSI
      *GHT TO FORESIGHT',/ ,24X,'IS 169 D. 32 M. 11.11 S.',/ ,24X,'ENTER 1
      *69.321111000.',/ )
1630 FORMAT ( //,12X,'ENTER GRID DISTANCE BETWEEN THE FIRST KNOWN POSITI
      *ON AND',/ ,12X,'TRAVERSE STATION POSITION #1.',/ )
1640 FORMAT ( //,12X,'ENTER GRID DISTANCE BETWEEN TRAVERSE STATION POSIT
      *ION #',12,'.',/ ,12X,'AND TRAVERSE STATION POSITION #',12,'.',/ )
1650 FORMAT ( //,12X,'FOR EXAMPLE: THE GRID DISTANCE BETWEEN THE STATIO
      *NS IS',/ ,26X,'399.052 METERS.',/ ,26X,'ENTER 399.052000.',/ )
1660 FORMAT ( //,12X,'WHAT IS THE NAME OF TRAVERSE STATION POSITION #',
      *12,'.',/ )
1670 FORMAT ( //,12X,'WHAT IS UTM GRID NORTHING AT THIS POSITION?',/ )
1680 FORMAT ( //,12X,'WHAT IS UTM GRID EASTING AT THIS POSITION?',/ )
1690 FORMAT ( //,16X,'WHAT IS UTM GRID AZIMUTH FROM',/ ,12X,'5A4',/ TO ',
      *5A4',/ )
1700 FORMAT ( //,12X,'FOR EXAMPLE: UTM GRID AZIMUTH FROM A. TO B.',
      *//,26X,'IS 139 D. 19 M. 29.3 S.: ENTER 139.19293000.',/ )

```

END

C C

```

C$EJECT          TD WATFIV A1
C  SUBROUTINE TDMS ( DUMMY1,DEG1,MIN1,SEC1 )
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C  AUTHOR : LT. SAMAN AUMCHANTR RTN
C  DATE : OCTOBER 23, 1983
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C  THIS IS A SUBROUTINE FOR COMPUTING DEGREE , MINUTE AND SECOND
C  FROM INPUT DATA OF THE FORM ODD.MMSSSSSS
C  INPUT : DUMMY1
C  OUTPUT : DEG1 , MIN1 AND SEC1
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C  VARIABLE DEFINITIONS
C  DUMMY1 = INPUT DATA IN DEGREE . MINUTE SECOND
C  DUMMY2 = INPUT DATA IN DEGREE . SECOND
C  DEG1 = INPUT DATA IN DEGREE
C  MIN1 = INPUT DATA IN MINUTE
C  SEC1 = INPUT DATA IN SECOND
C  SIG = THE SIGN OF THE INPUT DATA
C  NUM5 = 100000
C  NUM7 = 10000000
C  NUM100 = 100
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C  VARIABLE DECLARATIONS
C  DOUBLE PRECISION DUMMY1,DUMMY2,DEG1,MIN1,SEC1,NUM100,NUM7,NUM5,SIG
C  DATA NUM100/100.0D0/,NUM7/10000000.0D0/,NUM5/1000000.0D0/
C  CHANGE THE INPUT DATA TO DEGREE , MINUTE AND SECOND
C  SIG = ( DUMMY1 / DABS ( DUMMY1 ))*0.50D0
C  DEG1 = DFLOAT ( IDINT ( DUMMY1 ))
C  DUMMY2 = ( DUMMY1 - DEG1 ) * NUM100
C  MIN1 = OFLOAT ( IDINT ( DUMMY2 ))
C  SEC1 = ( DFLOAT ( IDINT ( ( ( DUMMY2 - MIN1 ) * NUM7 ) + SIG))) / NUM5
C  RETURN
C  END

```


[illegible]

```

CCCCC
THE CALCULATION OF THE DIFFERENCE IN DISTANCES IN X AND Y-AXES
DIFX = X2 - X1
DIFY = Y2 - Y1
CCCCC
THE CALCULATION OF THE GRID AZIMUTH
IF ( DIFX .NE. NUMO .OR. DIFY .NE. NUMO ) GO TO 10
  ANGLR = NUMO
  GO TO 50
10 CONTINUE
IF ( DIFX .NE. NUMO ) GO TO 30
  IF ( DIFY .LT. NUMO ) GO TO 20
    ANGLR = NUMO
    GO TO 50
  CONTINUE
  ANGLR = NU180R
  GO TO 50
20 CONTINUE
IF ( DIFX .LT. NUMO ) GO TO 40
  ANGLR = NU90R - DATAN ( DIFY / DIFX )
  GO TO 50
30 CONTINUE
ANGLR = NU270R - DATAN ( DIFY / DIFX )
40 CONTINUE
50
CCCCC
THE CALCULATION OF THE DISTANCE BETWEEN THE KNOWN POSITIONS
# 1 AND # 2
DIST12 = DSQRT ( DIFX**2 + DIFY**2 )
CCCCC
RETURN
CCCCC
END
CC

```

APPENDIX B

PROGRAM 2

COMPUTER OUTPUT

THE CALCULATION OF CLOSED TRAVERSE STATION POSITIONS
BY INDIRECT OBSERVATIONS METHOD
FOR UNCORRELATED OBSERVATIONS WITH EQUAL PRECISION

KNOWN HORIZONTAL STATION POSITIONS

NO	NAME OF STATIONS	UTM NORTH METERS	GRID NORTH METERS	UTM GRID COORDINATES GRID EASTING	AZIMUTH FROM TO NO NO	GRID AZIMUTH CLOCKWISE FROM NORTH D. M. S.
1	PIPHER					
2	MOSS 2	4072555.85206	608279.04404	2 -> 1	100 16	23.77800
3	HOLM	4079258.31754	612238.85256	3 -> 4	136 33	26.33400
4	MORAN					

89

OBSERVED DATA

NAME OF STATIONS	D. M. S.	OBSERVED ANGLE D. M. S.	STD. S.	GRID DISTANCE METERS	STD. M.
PIPHER					
MOSS 2	246 5	43.20000	1.98400	1424.00400	0.00100
MOSSBACK	222 51	8.60000	1.40500	365.74400	0.00100
DUNE TEMP	190 15	2.60000	1.20300		
HOLM	277 5	17.00000	1.61400	6476.27100	0.00300
MORAN					

PRECALCULATED OF CLOSED TRAVERSE STATION POSITIONS

NO	NAME OF STATIONS	UTM GRID COORDINATES		AZIMUTH		GRID AZIMUTH CLOCKWISE FROM NORTH
		GRID NORTHING	GRID EASTING	FROM TO	NO	
1	MOSS 2	4072555.85206	608279.04404			
2	MOSSBACK	4073939.74473	607943.44238	1 -> 2	346 22	6.97800
3	DUNE TEMP	4074258.94534	608121.99110	2 -> 1	166 22	6.97800
4	HOLM	4079258.22818	612238.93865	2 -> 3	29 13	15.57800
5	MORAN			3 -> 2	209 13	15.57800
				3 -> 4	39 28	18.17800
				4 -> 3	219 28	18.17800
				4 -> 5	136 33	35.17800

METERS Meters D. M. S.

TOTAL AZIMUTH ERROR = 0 D. 0 M. 8.84400 S.
 AZIMUTH ERROR PER STATIONS = 0 D. 0 M. 2.21100 S.
 TOTAL GRID DISTANCE = 8266.01900 METERS.
 DISTANCE ERROR = 0.12408 METERS.
 ACCURACY = 1: 666 17

 CALCULATION OF CLOSED TRAVERSE STATION POSITIONS BY LEAST-SQUARES ADJUSTMENT

BY INDIRECT OBSERVATIONS METHOD

UTM GRID COORDINATES					
NO	NAME OF STATIONS	GRID NORTHING		GRID EASTING	
		METERS	STD. M.	METERS	STD. M.
1	MOSSBACK	4073939.74899	0.00026	607943.45994	0.00137
2	DUNE TEMP	4074258.95013	0.00267	608122.00770	0.00324

THE STANDARD DEVIATION OF UNIT WEIGHT IS 0.0000201144

THE STANDARD DEVIATIONS OF ADJUSTED ANGLES ARE

ANGLE # 1 = 0.1591744830 SECONDS
 ANGLE # 2 = 2.9427035466 SECONDS
 ANGLE # 3 = 2.9173152616 SECONDS
 ANGLE # 4 = 0.1336986660 SECONDS

THE STANDARD DEVIATIONS OF ADJUSTED DISTANCES ARE

DISTANCE # 1 = 0.0000201144 METERS
 DISTANCE # 2 = 0.0000201144 METERS
 DISTANCE # 3 = 0.0000201144 METERS

THE CALCULATION OF CLOSED TRAVERSE STATION POSITIONS
BY INDIRECT OBSERVATIONS METHOD
FOR UNCORRELATED OBSERVATIONS WITH UNEQUAL PRECISION

KNOWN HORIZONTAL STATION POSITIONS

NO	NAME OF STATIONS	UTM GRID COORDINATES		AZIMUTH		GRID AZIMUTH	
		GRID NORTHING	GRID EASTING	FROM TO	CLOCKWISE	FROM NORTH	D. M. S.
1	PIPHER						
2	MOSS 2	4072555.85206	608279.04404	2 -> 1	100 16	23.77800	
3	HOLM	4079258.31754	612238.85256	3 -> 4	136 33	26.33400	
4	MORAN						

OBSERVED DATA

NAME OF STATIONS	OBSERVED		S/D.	GRID	
	ANGLE			DISTANCE	STD.
	D. M. S.	S.		METERS	M.
PIPHER					
MOSS 2	246	5 43.20000	1.98400	1424.00400	0.00100
MOSSBACK	222	51 8.60000	1.40500	365.74400	0.00100
DUNE TEMP	190	15 2.60000	1.20300	6476.27100	0.00300
HOLM	277	5 17.00000	1.61400		
MORAN					

PRECALCULATED OF CLOSED TRAVERSE STATION POSITIONS

NC	NAME OF STATIONS	UTM GRID COORDINATES		AZIMUTH		GRID AZIMUTH FROM NORTH
		GRID NORTHING	GRID EASTING	FROM TO	CLOCKWISE	
		METERS	METERS	NO	NO	D. M. S.
1	MOSS 2	4072555.85206	608279.04404	1 -> 2	346 22	6.97800
2	MOSSBACK	4073939.74473	607943.44238	2 -> 1	166 22	6.97800
3	DUNE TEMP	4074258.94534	608121.99110	2 -> 3	29 13	15.57800
4	HOLM	4079258.22818	612238.93865	3 -> 2	209 13	15.57800
5	MOFAN			3 -> 4	39 28	18.17800
				4 -> 3	219 28	18.17800
				4 -> 5	136 33	35.17800

TOTAL AZIMUTH ERROR = 0 D. 0 M. 8.84400 S.
 AZIMUTH ERROR PER STATIONS = 0 D. 0 M. 2.21100 S.
 TOTAL GRID DISTANCE = 8266.01900 METERS.
 DISTANCE ERROR = 0.12408 METERS.
 ACCURACY = 1: 66617

***** CALCULATION OF CLOSED TRAVERSE STATION POSITIONS BY LEAST-SQUARES ADJUSTMENT *****

BY INDIRECT OBSERVATIONS METHOD

		UTM GRID COORDINATES			
NO	NAME OF STATIONS	GRID NORTHING	STD.	GRID EASTING	STD.
		METERS	M.	METERS	M.
1	MOSSBACK	4073939.74809	0.00317	607943.45522	0.00954
2	DUNE TEMP	4074258.95029	0.00405	608122.00189	0.01032

THE STANDARD DEVIATION OF UNIT WEIGHT IS 2.6968502720

THE STANDARD DEVIATIONS OF ADJUSTED ANGLES ARE

ANGLE # 1 = 1.4034554964 SECONDS
 ANGLE # 2 = 2.6410292243 SECONDS
 ANGLE # 3 = 2.5525970615 SECONDS
 ANGLE # 4 = 0.2613735339 SECONDS

THE STANDARD DEVIATIONS OF ADJUSTED DISTANCES ARE

DISTANCE # 1 = 0.0026856614 METERS
 DISTANCE # 2 = 0.0026737945 METERS
 DISTANCE # 3 = 0.0074504772 METERS

AD-A153 612

A COMPARISON OF METHODS OF LEAST SQUARES ADJUSTMENT OF
TRAVERSES(U) NAVAL POSTGRADUATE SCHOOL MONTEREY CA
S AUMCHANTR DEC 84

2/2

UNCLASSIFIED

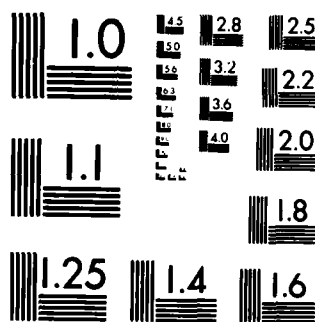
F/G 12/1

NL

END

FORMED

DTL



MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS-1963 A

ALGORITHM INPUT_DATA
PRINT TITLE

WRITE (6,1000)
WRITE (6,1380)
WRITE (6,1390)
WRITE (6,1400)
WRITE (6,1410)
WRITE (6,1420)
WRITE (6,1390)
WRITE (6,1380)
WRITE (6,1430)
WRITE (6,1440)

READ OPTIONS 1 = OPEN TRAVERSE
2 = CLOSED TRAVERSE

READ (4,1340) OPT
WRITE (6,1450)
WRITE (6,1460)
WRITE (6,1470)
WRITE (6,1430)
WRITE (6,1480)

READ OPTIONS 1 = ENTER DATA BY DATA FILE
2 = ENTER DATA BY INTERACTIVE

READ (4,1340) CHOIC

GET UTM GRID NORTHING, EASTING, AZIMUTH, AND THE NAME
OF KNOWN POSITIONS

DES = 04
IF (CHOIC .EQ. 2) GO TO 10
DES = 03
CONTINUE
LOOP1 = 2
IF (OPT .EQ. 1) GO TO 20

10

```

20      LOOP1 = 4
      CONTINUE
      DO 70 I = 1, LOOP1
        WRITE ( 6,1490 ) I
        IF ( I.NE. 1 ) GO TO 30
        WRITE ( 6,1500 )
        READ ( DES,1370 ) ( STNAME ( I,K1 ) , K1 = 1,5 )
        GO TO 60
30      CONTINUE
      IF ( I.NE. 2 ) GO TO 40
        WRITE ( 6,1520 )
        READ ( DES,1370 ) ( STNAME ( I,K1 ) , K1 = 1,5 )
        WRITE ( 6,1670 )
        WRITE ( 6,1540 )
        READ ( DES,1360 ) DUMMY1
        NGRID ( I ) = DUMMY1
        WRITE ( 6,1680 )
        WRITE ( 6,1560 )
        READ ( DES,1360 ) DUMMY1
        EGRID ( I ) = DUMMY1
        WRITE ( 6,1690 ) ( STNAME(2,K1),K1=1,5), (STNAME(1,K1),K1=1
          ,5)
        WRITE ( 6,1700 )
        READ ( DES,1360 ) DUMMY1
        CALL TDMS ( DUMMY1,DUMID,DUMIM,DUMIS )
        DIRECD ( 2 ) = IDINT ( DUMID )
        DIRECS ( 2 ) = IDINT ( DUMIM )
        CALL CDMSR ( DUMID,DUMIM,DUMIS,DUMMY1 )
        AZ21 = DUMMY1
        STAT1 ( 2 ) = 2
        STAT2 ( 2 ) = 1
        GO TO 60
40      CONTINUE
      IF ( I.NE. 3 ) GO TO 50
        WRITE ( 6,1530 )
        READ ( DES,1370 ) ( STNAME ( I,K1 ) , K1 = 1,5 )
        WRITE ( 6,1670 )
        WRITE ( 6,1540 )
        READ ( DES,1360 ) DUMMY1
        NGRID ( I ) = DUMMY1
        WRITE ( 6,1680 )
        WRITE ( 6,1560 )
        READ ( DES,1360 ) DUMMY1
        EGRID ( I ) = DUMMY1
        GO TO 60
50      CONTINUE
      IF ( I.NE. 4 ) GO TO 60

```

*

```

WRITE ( 6,1510 )
READ ( DES,1370 ) ( STNAME ( 1,K1 ) , K1 = 1,5 )
WRITE ( 6,1690 ) ( STNAME(3,K1),K1=1,5),(STNAME(4,K1),K1=1
,5)
WRITE ( 6,1700 )
READ ( DES,1360 ) DUMMY1
CALL TMS ( DUMMY1,DUMID,DUMIM,DUMIS )
DIRECD ( 3 ) = IDINT ( DUMID )
DIRECM ( 3 ) = IDINT ( DUMIM )
DIRECS ( 3 ) = DUMIS
CALL CDMR ( DUMID,DUMIM,DUMIS,DUMMY1 )
AZ34 = DUMMY1
STAT1 ( 3 ) = 3
STAT2 ( 3 ) = 4

```

```

CONTINUE
CONTINUE

```

```

READ THE NUMBER OF OBSERVED ANGLES .

```

```

WRITE ( 6,1580 )
READ ( DES,1350 ) COUNT
COUNTA = COUNT

```

```

READ THE OBSERVED ANGLES

```

```

DO 100 I = 1,COUNTA
IF ( 1.GT.1 ) GO TO 80
WRITE ( 6,1590 )
GO TO 90

```

```

CONTINUE
K2 = I - 1

```

```

WRITE ( 6,1600 ) K2

```

```

CONTINUE
WRITE ( 6,1610 )

```

```

READ ( DES,1360 ) DUMMY1
CALL TMS ( DUMMY1,DUMID,DUMIM,DUMIS )
CALL CDMR ( DUMID,DUMIM,DUMIS,DUMMY2 )

```

```

ANG ( 1 ) = DUMMY2
ANGD ( 1 ) = IDINT ( DUMID )
ANGM ( 1 ) = IDINT ( DUMIM )
ANGS ( 1 ) = DUMIS

```

```

WRITE ( 6,1550 )
READ ( DES,1360 ) DUMMY1
STDA ( 1 ) = DUMMY1

```

60
70

CCCCC

CCCCC

80

90

```

100 CONTINUE
CCCCCCCC
      READ THE GRID DISTANCE

COUNTD = COUNT
IF ( OPT .EQ. 1 ) GO TO 110
COUNTD = COUNT - 1
110 CONTINUE
DO 140 I = 1, COUNTD
  IF ( 1 .GT. 1 ) GO TO 120
  WRITE ( 6,1620 )
  GO TO 130
  CONTINUE
  K4 = I - 1
  K5 = I
  WRITE ( 6,1630 ) K4,K5
  CONTINUE
  WRITE ( 6,1640 )
  READ ( DES,1360 ) DUMMY1
  DIST ( I ) = DUMMY1
  TODIS = TODIS + DUMMY1
  WRITE ( 6,1570 )
  READ ( DES,1360 ) DUMMY1
  STDD ( I ) = DUMMY1
  CONTINUE
COUNTN = COUNTD
IF ( OPT .EQ. 1 ) GO TO 150
COUNTN = COUNTD - 1
140 CONTINUE
DO 160 I = 1, COUNTN
  WRITE ( 6,1650 ) I
  READ ( DES,1370 ) ( TRNAME ( I,K1 ) , K1 = 1,5 )
  CONTINUE
160 CONTINUE
CCCCCCCCCCCCCCCC
      END INPUT_DATA

      ALGORITHM PRINT_INPUT_DATA

      PRINT DETAILS OF THE KNOWN STATION POSITIONS

      WRITE ( 6,1000 )

```

```

WRITE ( 6,1010 )
WRITE ( 6,1020 )
WRITE ( 6,1030 )
WRITE ( 6,1040 )
WRITE ( 6,1050 )
WRITE ( 6,1060 )
DO 180 I = 1,LOOP1
  IF ( I.EQ.2 .OR. I.EQ.3 ) GO TO 170
  WRITE ( 6,1230 ) I, ( STNAME ( I,K1 ) , K1 = 1,5 )
  GC TO 180
170 CONTINUE
  WRITE ( 6,1070 ) I, (STNAME(I,K1),K1=1,5),NGRID(I),EGRID(I),STA
  T(I),STAT2(I),DIRECD(I),DIRECM(I),DIRECS(I)
180 * CONTINUE
  WRITE ( 6,1000 )
  WRITE ( 6,1080 )
  WRITE ( 6,1090 )
  WRITE ( 6,1100 )
  WRITE ( 6,1110 )
  WRITE ( 6,1120 )
  WRITE ( 6,1130 )
  WRITE ( 6,1140 ) ( STNAME ( I,K1 ) , K1 = 1,5 )
  *TDA(1) ( STNAME(2,K1),K1 = 1,5),ANGD(1),ANGM(1),ANGS(1),S
  WRITE ( 6,1150 ) DIST ( 1 ),STDD ( 1 )
  COUNTP = COUNTD - 1
  IF ( COUNTP.NE.0 ) GO TO 190
  WRITE ( 6,1160 ) ( TRNAME ( I,K1 ) , K1 = 1,5 )
  GO TO 220
190 CONTINUE
  DO 200 I = 1,COUNTP
    K2 = I + 1
    WRITE ( 6,1140 ) (TRNAME(I,K1),K1=1,5),ANGD(K2),ANGM(K2),ANGS(
    K2),STDA(K2)
    WRITE ( 6,1150 ) DIST ( K2 ),STDD ( K2 )
200 CONTINUE
  IF ( OPT.NE.1 ) GO TO 210
  WRITE ( 6,1160 ) ( TRNAME ( I,K1 ) , K1 = 1,5 )
  GO TO 220
210 CONTINUE
  WRITE ( 6,1140 ) (STNAME(3,K1),K1=1,5),ANGD(COUNTA),ANGM(COUNTA),
  *ANGS(COUNTA),STDA(COUNTA)
  WRITE ( 6,1130 ) ( STNAME ( 4,K1 ) , K1 = 1,5 )
220 CONTINUE
  WRITE ( 6,1000 )
C
C
C
END PRINT_INPUT_DATA

```

CCCCCCCC

ALGORITHM COMPUTE_ADJUST_BY_INDIRECT_OBSERVATIONS

THE CALCULATION OF OPEN TRAVERSE

```

CALL CDMSR ( NUM360,NUMO,NUMO,AN36OR )
XGD = EGRID ( 2 )
YGD = NGRID ( 2 )
AZFIR = AZ21
DO 240 I = 1,COUNTD
  AZIMUT = AZFIR + ANG ( I )
  IF ( AZIMUT .LT. AN36OR ) GO TO 230
  AZIMUT = AZIMUT - AN36OR
CONTINUE
COAZR ( I ) = AZIMUT
CALL CRDMS ( AZIMUT,DUMID,DUMIM,DUMIS )
CFAZD ( I ) = IDINT ( DUMID )
CFAZM ( I ) = IDINT ( DUMIM )
CFAZS ( I ) = DUMIS
DISTAN = DIST ( I )
CALL CXYC ( XGD,YGD,DISTAN,AZIMUT,DUMMY1,DUMMY2 )
DELTAX ( I ) = DUMMY1 - XGD
DELTAY ( I ) = DUMMY2 - YGD
SUMDX = SUMDX + DELTAX ( I )
SUMDY = SUMDY + DELTAY ( I )
CEGRID ( I ) = DUMMY1
CNGRID ( I ) = DUMMY2
CALL CTANA ( DUMMY1,DUMMY2,XGD,YGD,AZC,DUMIS )
CALL CRDMS ( AZC,DUMID,DUMIM,DUMIS )
CBAZD ( I ) = IDINT ( DUMID )
CBAZM ( I ) = IDINT ( DUMIM )
CBAZS ( I ) = DUMIS
AZFIR = AZC
XGD = DUMMY1
YGD = DUMMY2
240 CONTINUE

```

230

240

CCCCCC

PRINT DETAIL OF PRECALCULATED OF TRAVERSE STATION POSITIONS

```

IF ( OPT.EQ.1 ) GO TO 250
WRITE ( 6,1180 )
WRITE ( 6,1190 )
GO TO 260

```

```

250 CONTINUE
    WRITE ( 6,1170 )
    WRITE ( 6,1200 )

260 CONTINUE
    WRITE ( 6,1030 )
    WRITE ( 6,1040 )
    WRITE ( 6,1050 )
    WRITE ( 6,1060 )
    WRITE ( 6,1210 ) (SNAME(2,K1),K1=1,5),NGRID(2),EGRID(2)
    DO 300 I = 1,COUNTD
        K4 = I
        K5 = I + 1
        WRITE ( 6,1220 ) K4,K5,CFAZD(I),CFAZM(I),CFAZS(I)
        IF ( OPT.EQ.1 ) GO TO 280
        IF ( 1.NE.COUNTD ) GO TO 270
        WRITE ( 6,1070 ) K5,(SNAME(3,K1),K1=1,5),CNGRID(I),
            CEGRID(I),K5,K4,CBAZD(I),CBAZM(I),CBAZS(I)
        *
        CONTINUE
270 CONTINUE
        IF ( 1.EQ.COUNTD ) GO TO 290
        WRITE ( 6,1070 ) K5,(TRNAME(I,K1),K1=1,5),CNGRID(I),CEGRI
            D(I),K5,K4,CBAZD(I),CBAZM(I),CBAZS(I)
        *
        CONTINUE
290 CONTINUE
        IF ( OPT.NE.1 ) GO TO 310
        WRITE ( 6,1070 ) K5,(TRNAME(COUNTD,K1),K1=1,5),CNGRID(COUNTD),
            CEGRID(COUNTD),K5,K4,CBAZD(COUNTD),CBAZM(COUNTD),CBAZS(COUNT
            D)
        *
        CONTINUE
310 CONTINUE
        IF ( OPT.EQ.1 ) GO TO 370
        AZLAS = AZFIR + ANG ( COUNT )
        IF ( AZLAS.LT. AN360R ) GO TO 320
        AZLAS = AZLAS - AN360R
        CONTINUE
        K4 = I
        K5 = I + 1
        CALL CRDMS ( AZLAS,DUMID,DUMIM,DUMIS )
        TEMD = IDINT ( DUMID )
        TEMM = IDINT ( DUMIM )
        WRITE ( 6,1220 ) K4,K5,TEMD,TEMM,DUMIS
        DIFAZI = AZLAS - A234
        WPRED ( 1 ) = DIFAZI
        CORAZ = DIFAZI / DFLOAT ( COUNTA )
        CALL CRDMS ( DIFAZI,DUMID,DUMIM,DUMIS )
        EAZID = IDINT ( DUMID )
        EAZIM = IDINT ( DUMIM )
        EAZIS = DABS ( DUMIS )

```

```

CALL CRDMS ( CORAZ,DUMID,DUMIM,DUMIS )
EAZ2D = IDINT ( DABS ( DUMID ) )
EAZ2M = IDINT ( DABS ( DUMIM ) )
EAZ2S = DABS ( DUMIS )
WPRED ( 3 ) = SUMDY - ( EGRID(3) - EGRID(2) )
DIFDIS = DSORT ( WPRED(2)**2+WPRED(3)**2 )
ERR = IDINT ( TODIS / DIFDIS )
WRITE ( 6,1240 ) EAZ1D,EAZ1M,EAZ1S
WRITE ( 6,1250 ) EAZ2D,EAZ2M,EAZ2S
WRITE ( 6,1260 ) TODIS
WRITE ( 6,1270 ) DIFDIS
WRITE ( 6,1280 ) ERR
ASP = COUNTD - 1
DO 330 I = 1, ASP
  ASNY ( I ) = CNGRID ( I )
  ASEY ( I ) = CEGRID ( I )
CONTINUE

```

330
C
C
C
C

CALL SUBROUTINE LSTR TO ADJUSTED BY INDIRECT OBSERVATIONS METHOD

```

NT = COUNTA + COUNTD
NA = CCOUNTA
ND = COUNTD
NP = ASP
NP2 = ASP*2
NI = NP2**2 + 3*NP2
CALL LSTR ( NT,NA,ND,NP,NP2,NI,AZ21,AZ34,NGRID,EGRID,ANG,STDA,
DIST,SIDD,PRINTD,PW,WM,STDD,FM,RM,BMT,BMTWM,NM,IM,DELTA,NMI,
WK10,ASEY,ASEX,VM,STDA )
WRITE ( 6,1000 )
WRITE ( 6,1290 )
WRITE ( 6,1300 )
WRITE ( 6,1660 )
WRITE ( 6,1710 )
WRITE ( 6,1310 )
WRITE ( 6,1320 )
DO 340 I = 1,ASP
  K4 = I*2
  K5 = I + ( I-1 )
  WRITE ( 6,1330 ) I,(TRNAME(I,K1),K1=1,5),ASNY(I),TM(K4),
  ASEY(I),IM(K5)
CONTINUE
WRITE ( 6,1000 )
WRITE ( 6,1720 ) STDD
WRITE ( 6,1730 )

```

* *

*

340

```

DO 350 I = 1, NA
  CALL CROMS ( VM(1,I), DUM1D, DUM1M, DUM1S )
  DUMMY1 = DUM1D*3600.000 + DUM1M*60.000 + DUM1S
  WRITE ( 6,1740 ) I, DUMMY1
CONTINUE
WRITE ( 6,1750 )
DO 360 I = 1, ND
  K4 = NA + I
  WRITE ( 6,1760 ) I, VM ( 1,K4 )
CONTINUE
WRITE ( 6,1000 )
END COMPUTE_ADJUST_BY_INDIRECT_OBSERVATIONS

STOP

1000 FORMAT ( '1' )
1010 FORMAT ( //, 23X, 'KNOWN HORIZONTAL STATION POSITIONS: ' )
1020 FORMAT ( 23X, '*****' )
1030 FORMAT ( 31X, 'UTM GRID COORDINATES: ', 6X, 'AZIMUTH GRID, AZIMUTH: ', // )
1040 FORMAT ( 28X, 'GRID NORTHING GRID EASTING FROM TO: ', 4X, 'CLOCKWISE' )
1050 FORMAT ( '1' )
1060 FORMAT ( //, 23X, 'NAME OF STATIONS', 35X, 'NO', 3X, 'NO', 4X, 'FROM N' )
1070 FORMAT ( 32X, 'METERS', 8X, 'METERS', 14X, 'D. M. S: ', // )
1080 FORMAT ( 1X, 12, 1X, 5A4, 3X, F14.5, 2X, F12.5, 2X, 12, ' -> ', 12, 1X, 13, 1X, 12 )
1090 FORMAT ( //, 31X, 'OBSERVED DATA: ' )
1100 FORMAT ( 31X, '*****' )
1110 FORMAT ( 29X, 'OBSERVED: ', 19X, ' GRID: ', // )
1120 FORMAT ( 6X, 'NAME OF STATIONS', 8X, 'ANGLE', 8X, 'STD.', 9X, 'DISTANCE', )
1130 FORMAT ( 26X, 'D. M. S: ', 9X, 'S: ', 11X, 'METERS', 7X, 'M. ' )
1140 FORMAT ( //, 4X, 5A4, // )
1150 FORMAT ( 4X, 5A4, 1X, 13, 1X, 12, 1X, F8.5, 1X, F8.5 )
1160 FORMAT ( 52X, F14.5, 1X, F8.5 )
1170 FORMAT ( 4X, 5A4, // )
1180 FORMAT ( //, 17X, 'CALCULATION OF OPEN TRAVERSE STATION POSITIONS: ' )
1190 FORMAT ( //, 15X, 'PRECALCULATED OF CLOSED TRAVERSE STATION POSITIONS' )
1200 FORMAT ( '1' )
1210 FORMAT ( //, 17X, '*****' )
1220 FORMAT ( //, 17X, '*****' )

```



```

1540 FORMAT ( /,12X,'FOR EXAMPLE : GRID NORTHING 4,072,555.85206 METERS
1550 *//,26X,'ENTER 4072555.85206000',
1550 FORMAT ( //,12X,'WHAT IS THE STANDARD DEVIATION OF THE OBSERVED AN
*GLE', /,12X,'AT THIS STATION IN SECONDS?', /,12X,'FOR EXAMPLE STD.
*= 3.9 SECONDS. : ENTER 3.9000',
1560 FORMAT ( //,12X,'FOR EXAMPLE : GRID EASTING 608,279.04404 METERS.',
*//,26X,'ENTER 608279.04404000',
1570 FORMAT ( //,12X,'WHAT IS THE STANDARD DEVIATION IN METERS?', /,12X
*//,26X,'ENTER 0.011000',
1580 *//,12X,'HOW MANY POSITIONS DID YOU OBSERVE ANGLES?', /,12X,19
*//,12X,'PLEASE ENTER 01 TO 32',
1590 FORMAT ( //,12X,'ENTER THE OBSERVED ANGLE AT THE FIRST KNOWN POSITI
*ON.',
1600 FORMAT ( //,10X,'ENTER THE OBSERVED ANGLE AT TRAVERSE STATION POSIT
*ION #', /,12X,'',
1610 FORMAT ( //,10X,'FOR EXAMPLE : OBSERVED ANGLE CLOCKWISE FROM BACKSI
*GH TO FORESIGHT', /,24X,'IS 169 D. 32 M. 11.11 S.', /,24X,'ENTER 1
*69.32111000',
1620 FORMAT ( //,12X,'ENTER GRID DISTANCE BETWEEN THE FIRST KNOWN POSITI
*ON AND', /,12X,'TRAVERSE STATION POSITION #', /,12X,'',
1630 FORMAT ( //,12X,'ENTER GRID DISTANCE BETWEEN TRAVERSE STATION POSIT
*ION #', /,12X,'', /,12X,'AND TRAVERSE STATION POSITION #', /,12X,'',
1640 *//,12X,'',
1640 FORMAT ( //,12X,'FOR EXAMPLE : THE GRID DISTANCE BETWEEN THE STATIO
*NS IS', /,26X,'399.052 METERS.', /,26X,'ENTER 399.052000',
1650 FORMAT ( //,12X,'WHAT IS THE NAME OF TRAVERSE STATION POSITION #',
*//,12X,'',
1660 *//,12X,'',
1660 FORMAT ( //,12X,'BY INDIRECT OBSERVATIONS METHOD', /,12X,'',
1670 *//,12X,'',
1670 FORMAT ( //,12X,'WHAT IS UTM GRID NORTHING AT THIS POSITION?', /,12X,'',
1680 *//,12X,'',
1680 FORMAT ( //,12X,'WHAT IS UTM GRID EASTING AT THIS POSITION?', /,12X,'',
1690 *//,12X,'',
1690 FORMAT ( //,16X,'WHAT IS UTM GRID AZIMUTH FROM', /,12X,'5A4', /,12X,'TO',
*//,12X,'',
1700 *//,12X,'',
1700 FORMAT ( //,12X,'FOR EXAMPLE : UTM GRID AZIMUTH FROM A. TO B.',
*//,26X,'IS 139 D. 19 M. 29.3 S. : ENTER 139.19293000',
1710 *//,12X,'',
1710 FORMAT ( //,12X,'UTM GRID COORDINATES', /,12X,'',
1720 *//,12X,'',
1720 FORMAT ( //,12X,'THE STANDARD DEVIATION OF UNIT WEIGHT IS', /,12X,'F15.10
*//,12X,'',
1730 *//,12X,'',
1730 FORMAT ( //,12X,'THE STANDARD DEVIATIONS OF ADJUSTED ANGLES ARE', /,12X,'',
*//,12X,'',
1740 *//,12X,'',
1740 FORMAT ( //,12X,'ANGLE #', /,12X,' =', /,12X,'F17.10', /,12X,' SECONDS', /,12X,'',
1750 *//,12X,'',
1750 FORMAT ( //,12X,'THE STANDARD DEVIATIONS OF ADJUSTED DISTANCES ARE',
*//,12X,'',
1760 *//,12X,'',
1760 FORMAT ( //,12X,'DISTANCE #', /,12X,' =', /,12X,'F16.10', /,12X,' METERS', /,12X,'',

```

END

C C C

```

$EJECT      LS WATFIV A1
C
C
C
SUBROUTINE ISTR ( NT,NA,ND,NP,NP2,NI,AZ34,NGRID,EGRID,ANG,STD
* A,DIST,STDD,PRINTD,PW,WM,STDD,FM,8M,8MT,BMTWM,NM,TM,DELTA,NMI,WK10
* ,ASNG,ASEG,VM,STAND)
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C
C      AUTHOR : LT. SAMAN AUMCHANTR RTN.
C      DATE : AUGUST 11, 1984
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C      THIS IS A SUBROUTINE TO ADJUSTMENT OF CLOSED TRAVERSE BY
C      INDIRECT OBSERVATIONS METHOD
P
P
P
C
C      VARIABLE DEFINITIONS
V
V
V      NT = THE TOTAL NUMBER OF OBSERVATION EQUATIONS
V      NA = THE TOTAL NUMBER OF OBSERVED ANGLES
V      ND = THE TOTAL NUMBER OF MEASURED DISTANCES
V      NP = THE TOTAL NUMBER OF INTERMEDIATE TRAVERSE STATION
V      POSITION
V      AZ21 = FIXED INITIAL AZIMUTH
V      AZ34 = FIXED CLOSED AZIMUTH
V      NGRID(4), EGRID(4) = FIXED COORDINATES
V      ANG(NA) = OBSERVED ANGLES
V      STDA(HA) = STANDARD DEVIATION OF OBSERVED ANGLES
V      DIST(ND) = MEASURED DISTANCES
V      STDD(ND) = STANDARD DEVIATION OF MEASURED DISTANCES
V      FM(NT) = B MATRIX
V      WM(NT,NT) = WEIGHT MATRIX
V      BM(NT,NP2) = A MATRIX
V      DELTA{NP2} = X MATRIX
V      TRFM = A SUBROUTINE TO COMPUTE R MATRIX
V      VMULFF = A SUBROUTINE TO COMPUTE THE MULTIPLICATION OF MATRIX
V      LINV2F = A SUBROUTINE FOR INVERSION OF MATRIX
V      USWFEM = A SUBROUTINE FOR PRINT THE OUTPUT IN MATRIX FORM
V
V
V
C
C      VARIABLE DECLARATIONS
C
C      DOUBLE PRECISION DUMMY1,NGRID(4),EGRID(4),DUMID,DU4IM,DUMIS,ANG(NA

```

```

*) ,DIST(ND),WM(NT,NT),STDD(NT),ASNG(NP),ASEG(NP),FM(NT),BM(NT,NP2),
*TEST1,TEST2,BMT(NP2,NT),BMTWM(NP2,NT),NM(NP2,NP2),TM(NP2),DELTA(NP
*2),NMI(NP2,NP2),WK10(NI),STDA(NA),STDD(ND),DUMMY2,NUMO,AZ21,AZ34,
*VM(1,NT),STAND

```

```

C      INTEGER I,J,K1,IER,PRINTD,PW,CHECK

```

```

C      DATA K1/0/,TEST1/0.000001000000D0/,NUMO/0.0D0/

```

```

C      NC4 = 4

```

```

C      NP5 = 6

```

```

C      N1 = 1

```

```

C      SET WEIGHT MATRIX

```

```

C      DO 30 I = 1,NT
C        DO 20 J = 1,NT
C          WM ( I,J ) = 0.0D0
C          IF ( I.NE. J ) GO TO 10
C            WM ( I,J ) = 1.0D0
C          CONTINUE
C        CONTINUE
C      CONTINUE

```

```

C      IF ( PW.EQ. 1 ) GO TO 90

```

```

C      STANDARD DEVIATION OF THE OBSERVED ANGLES

```

```

C      DO 40 I = 1,NA
C        DUMMY1 = STDA ( I )
C        CALL CDMSR ( NUMO,NUMO,DUMMY1,DUMMY2 )
C        STDD ( I ) = 1.0D0 / ( ( DSIN ( DUMMY2 ) )**2 )
C      CONTINUE

```

```

C      STANDARD DEVIATION OF THE OBSERVED DISTANCE

```

```

C      DO 50 I = 1,ND
C        J = NA + I
C        DUMMY1 = STDD ( I )
C        STDD ( J ) = 1.0D0 / ( DUMMY1**2 )
C      CONTINUE
C      DO 80 I = 1,NT
C        DO 70 J = 1,NT
C          IF ( I.NE. J ) GO TO 60
C            WM ( I,J ) = STDD ( I )
C          CONTINUE
C        CONTINUE

```

```

80      CONTINUE
90      CONTINUE
C      PRINT THE RESULT OF WEIGHT MATRIX
C
C      IF ( PRINTD.EQ.0 ) GO TO 100
C      WRITE ( 6,1000 )
C      WRITE ( 6,1010 )
C      CALL USWFM ( 'R-C.',NC4,WM,NT,NT,NT,NP5 )
C      WRITE ( 6,1000 )
100     CONTINUE
C
C      110    CONTINUE
C      K1 = K1 + 1
C      CHECK = 0
C
C      CALL SUBROUTINE TRFM FOR COMPUTATION 8 MATRIX
C
C      *      CALL TRFM ( AZ21,AZ34,NA,ND,NT,NGRID,EGRID,ANG,DIST,NP,ASNG,
C      *      ASEG,FM )
C
C      CALL SUBROUTINE COEFAM FOR COMPUTATION A MATRIX
C
C      CALL COEFAM ( NT,NA,ND,NP,NP2,NGRID,EGRID,ASNG,ASEG,BM )
C
C      THE CALCULATION OF THE TRANSPOSE OF A MATRIX
C
C      DO 130 I = 1,NT
C      DO 120 J = 1,NP2
C      BMT ( J,I ) = BM ( I,J )
C      CONTINUE
120     CONTINUE
130
C
C      THE COMPUTATION OF A MATRIX TRANSPOSE * WEIGHT MATRIX
C
C      CALL VMULFF ( BMT,WM,NP2,NT,NT,NP2,NT,BMTWM,NP2,IER )
C
C      THE COMPUTATION OF A MATRIX TRANSPOSE * WEIGHT MATRIX * A MATRIX
C
C      CALL VMULFF ( BMTWM,BM,NP2,NT,NP2,NP2,NT,NM,NP2,IER )
C
C      THE COMPUTATION OF A MATRIX TRANSPOSE * WEIGHT MATRIX * 8 MATRIX

```

```

C      CALL VMULFF ( BMTWM,FM,NP2,NT,N1,NP2,NT,TM,NP2,IER )
C
C      THE CALCULATION OF DELTA MATRIX ( THE CORRECTION VECTOR )
C
C      CALL LINV2F ( NM,NP2,NP2,NMI,N1,WK10,IER )
C      CALL VMULFF ( NMI,TM,NP2,NP2,N1,NP2,DELTA,NP2,IER )
C
C      UP-DATE THE APPROXIMATE VALUES
C
140  DO 140 I = 1,NP
      ASEG ( I ) = ASEG ( I ) + DELTA ( ( I-1 ) * 2 + 1 )
      ASNG ( I ) = ASNG ( I ) + DELTA ( ( I-1 ) * 2 + 2 )
      CONTINUE
      IF ( PRINTD .EQ. 0 ) GO TO 160
        WRITE ( 6,1020 ) KI
        WRITE ( 6,1030 )
        CALL USWFM ( 'R-C.',NC4,FM,NT,NT,N1,NP5 )
        WRITE ( 6,1000 )
        WRITE ( 6,1040 )
        CALL USWFM ( 'R-C.',NC4,BM,NT,NT,NP2,NP5 )
        WRITE ( 6,1000 )
        WRITE ( 6,1050 )
        CALL USWFM ( 'R-C.',NC4,BMT,NP2,NP2,NT,NP5 )
        WRITE ( 6,1000 )
        WRITE ( 6,1060 )
        CALL USWFM ( 'R-C.',NC4,BMTWM,NP2,NP2,NT,NP5 )
        WRITE ( 6,1070 )
        CALL USWFM ( 'R-C.',NC4,NM,NP2,NP2,NP5 )
        WRITE ( 6,1000 )
        WRITE ( 6,1080 )
        CALL USWFM ( 'R-C.',NC4,TM,NP2,NP2,N1,NP5 )
        WRITE ( 6,1000 )
        WRITE ( 6,1090 )
        CALL USWFM ( 'R-C.',NC4,NMI,NP2,NP2,NP2,NP5 )
        WRITE ( 6,1100 )
        CALL USWFM ( 'R-C.',NC4,DELTA,NP2,NP2,N1,NP5 )
        WRITE ( 6,1110 )
        DO 150 I = 1,NP
          PRINT , I , , N = , , ASNG(I) , , E = , , ASEG(I)
          PRINT ,
        CONTINUE
      CONTINUE
150
160

```

```

C C C C
CHECK DELTA MATRIX
DO 180 I = 1,NUMD
TEST2 = DABS ( DELTA ( I ) )
IF ( TEST2 .LT. TEST1 ) GO TO 170
CHECK = I
CONTINUE
170 CONTINUE
180 IF ( CHECK .NE. 0 ) GO TO 110
THE CALCULATION OF A*X MATRIX
CALL VMULFF ( BM,DELTA,NT,NP2,N1,NT,NP2,STDO,NT,IER )
THE CALCULATION OF V MATRIX
DO 190 I = 1,NT
STDO ( I ) = STDO ( I ) - FM ( I )
VM ( I,I ) = STDO ( I )
CONTINUE
190 IF ( PRINTD .EQ. 0 ) GO TO 200
WRITE ( 6,1000 )
WRITE ( 6,1120 )
CALL USWFM ( 'R-C.',NC4,STDO,NT,NT,N1,NP5 )
WRITE ( 6,1000 )
WRITE ( 6,1130 )
CALL USWFM ( 'R-C.',NC4,VM,N1,N1,NT,NP5 )
CONTINUE
200 CONTINUE
THE CALCULATION OF V.T*P*V MATRIX
CALL VMULFF ( WM,STDO,NT,NT,N1,NT,NT,FM,NT,IER )
CALL VMULFF ( VM,FM,N1,NT,N1,NT,STAND,N1,IER )
IF ( PRINTD .EQ. 0 ) GO TO 210
WRITE ( 6,1000 )
WRITE ( 6,1140 )
CALL USWFM ( 'R-C.',NC4,FM,NT,NT,N1,NP5 )
WRITE ( 6,1150 )
WRITE ( 6,1160 ) STAND
CONTINUE
210 CALL VMULFF ( NMI,BMT,NP2,NP2,NT,NP2,NP2,BMTWM,NP2,IER )
STAND = DSQRT ( STAND / ( DFLOAT ( N1 - NP2 ) ) )
DO 240 I = 1,NP2
DO 230 J = 1,NP2
NMI ( I,J ) = STAND * ( DSQRT ( DABS ( NMI ( I,J ) ) ) )

```

```

220 IF ( I.NE. J ) GO TO 220
230   TM ( I ) = NMI ( I,J )
240   CONTINUE
C   CONTINUE
C   THE CALCULATION OF A*(INVERSE A.T*P*A)*A.T MATRIX
C   CALL VMULFF ( RM,BMTWM,NT,NP2,NT,NT,NP2,WM,NT,IER )
C   IF ( PRINTD.EQ.0 ) GO TO 250
C   WRITE ( 6,1000 )
C   WRITE ( 6,1170 )
C   CALL USWFM ( 'R-C.',NC4,WM,NT,NT,NT,NP5 )
250 CONTINUE
DO 280 I = 1,NT
DO 270 J = 1,NT
WM ( I,J ) = STAND * ( DSQRT ( DABS ( WM ( I,J ) ) ) )
IF ( I.NE. J ) GO TO 260
IF ( WM ( I,I ) = WM ( I,J ) )
CONTINUE
CONTINUE
260 CONTINUE
270 CONTINUE
280 CONTINUE
C   RETURN
C
1000 FORMAT ( '1' )
1010 FORMAT ( '///,5X,'WEIGHT MATRIX',// )
1020 FORMAT ( '///,5X,12,'ITERATED',// )
1030 FORMAT ( '///,5X,8,'MATRIX',// )
1040 FORMAT ( '///,5X,'A MATRIX',// )
1050 FORMAT ( '///,5X,'TRANSPOSE OF A MATRIX',// )
1060 FORMAT ( '///,5X,'A.T*P MATRIX',// )
1070 FORMAT ( '///,5X,'A.T*P*A MATRIX',// )
1080 FORMAT ( '///,5X,'A.T*P*8 MATRIX',// )
1090 FORMAT ( '///,5X,'THE INVERSION OF A.T*P*A MATRIX',// )
1100 FORMAT ( '///,5X,'X MATRIX OR DELTA MATRIX',// )
1110 FORMAT ( '///,5X,'UP-DATE THE APPROXIMATE VALUES',// )
1120 FORMAT ( '///,5X,'V MATRIX',// )
1130 FORMAT ( '///,5X,'V.T MATRIX',// )
1140 FORMAT ( '///,5X,'P*V MATRIX',// )
1150 FORMAT ( '///,5X,'V.T*P*V MATRIX',// )
1160 FORMAT ( '///,5X,F20.15,/' )
1170 FORMAT ( '///,5X,'A*(INVERSE A.T*P*A)*A.T MATRIX',// )
C
C   END
C

```



```

CALL CDMSR ( N360, NUMO, NUMO, A360R )
DO 120 I1 = 1, NNA
  IF ( I1.NE. 1 ) GO TO 20
  TEMP1 = XGR ( 2 )
  TEMP2 = YGR ( 2 )
  AZB = AZ21
  TEMP3 = ASX ( I1 )
  TEMP4 = ASY ( I1 )
  CALL CTANA ( TEMP1, TEMP2, TEMP3, TEMP4, AZF, DUM1 )
  CDIS ( I1 ) = DUM1
  IF ( AZF.LT. AZB ) GO TO 10
  F ( I1 ) = OANG ( I1 ) - ( AZF - AZB )
  GO TO 110
CONTINUE
F ( I1 ) = OANG ( I1 ) - ( A360R+AZF-AZB )
GO TO 110
CONTINUE
IF ( I1.NE. 2 ) GO TO 50
TEMP1 = ASX ( I1-1 )
TEMP2 = ASY ( I1-1 )
TEMP3 = XGR ( 2 )
TEMP4 = YGR ( 2 )
CALL CTANA ( TEMP1, TEMP2, TEMP3, TEMP4, AZB, DUM1 )
TEMP3 = XGR ( 3 )
TEMP4 = YGR ( 3 )
IF ( NNA.EQ. 3 ) GO TO 30
TEMP3 = ASX ( I1 )
TEMP4 = ASY ( I1 )
CONTINUE
CALL CTANA ( TEMP1, TEMP2, TEMP3, TEMP4, AZF, DUM1 )
CDIS ( I1 ) = DUM1
IF ( AZF.LT. AZB ) GO TO 40
F ( I1 ) = OANG ( I1 ) - ( AZF - AZB )
GO TO 110
CONTINUE
F ( I1 ) = OANG ( I1 ) - ( A360R+AZF-AZB )
GO TO 110
CONTINUE
IF ( I1.NE. NNA ) GO TO 70
TEMP1 = XGR ( 3 )
TEMP2 = YGR ( 3 )
AZF = AZ34
TEMP3 = ASX ( NOS )
TEMP4 = ASY ( NOS )
CALL CTANA ( TEMP1, TEMP2, TEMP3, TEMP4, AZB, DUM1 )
IF ( AZF.LT. AZB ) GO TO 60
F ( I1 ) = OANG ( I1 ) - ( AZF - AZB )
GO TO 110

```

```

60      CONTINUE
      F ( I1 ) = OANG ( I1 ) - ( A360R+AZF-AZB )
70      GO TO 110
      CONTINUE
      I2 = NNA - 1
      IF ( I1.NE. I2 ) GO TO 90
      TEMP1 = ASX ( I2-1 )
      TEMP2 = ASY ( I2-1 )
      TEMP3 = ASX ( I2-2 )
      TEMP4 = ASY ( I2-2 )
      CALL CTANA ( TEMP1,TEMP2,TEMP3,TEMP4,AZB,DUM1 )
      TEMP3 = XGR ( 3 )
      TEMP4 = YGR ( 3 )
      CALL CTANA ( TEMP1,TEMP2,TEMP3,TEMP4,AZF,DUM1 )
      CDIS ( I1 ) = DUM1
      IF ( AZF.LT. AZB ) GO TO 80
      F ( I1 ) = OANG ( I1 ) - ( AZF-AZB )
      GO TO 110
      CONTINUE
      F ( I1 ) = OANG ( I1 ) - ( A360R+AZF-AZB )
80      GO TO 110
      CONTINUE
      F ( I1 ) = OANG ( I1 ) - ( A360R+AZF-AZB )
90      GO TO 110
      CONTINUE
      TEMP1 = ASX ( I1-1 )
      TEMP2 = ASY ( I1-1 )
      TEMP3 = ASX ( I1-2 )
      TEMP4 = ASY ( I1-2 )
      CALL CTANA ( TEMP1,TEMP2,TEMP3,TEMP4,AZB,DUM1 )
      TEMP3 = ASX ( I1 )
      TEMP4 = ASY ( I1 )
      CALL CTANA ( TEMP1,TEMP2,TEMP3,TEMP4,AZF,DUM1 )
      CDIS ( I1 ) = DUM1
      IF ( AZF.LT. AZB ) GO TO 100
      F ( I1 ) = OANG ( I1 ) - ( AZF-AZB )
      GO TO 110
      CONTINUE
      F ( I1 ) = OANG ( I1 ) - ( A360R+AZF-AZB )
100     CONTINUE
110     CONTINUE
120     I1 = NNA
      DO 130 I2 = 1,NND
      I1 = I1 + 1
      F ( I1 ) = ODIS ( I2 ) - CDIS ( I2 )
130     CONTINUE
      C
      C
      C
      RETURN
      END

```



```

10 AM ( I1,J1 ) = 0.000
11 AM ( I1,J2 ) = 0.000
12 IF ( I1,NE. 1 ) GO TO 20
13 IF ( J1,NE. 1 ) GO TO 10
14 DUMMY1 = ASX ( 1 ) - KNY ( 2 )
15 DUMMY2 = ASX ( 1 ) - KNX ( 2 )
16 DUMMY3 = DUMMY1**2 + DUMMY2**2
17 AM ( I1,J1 ) = ( DUMMY1 / DUMMY3 )
18 AM ( I1,J2 ) = - ( DUMMY2 / DUMMY3 )
19 CONTINUE
20 GO TO 150
21 CONTINUE
22 IF ( I1,NE. NA ) GO TO 40
23 IF ( J1,NE. NP1 ) GO TO 30
24 DUMMY1 = ASX ( NP ) - KNY ( 3 )
25 DUMMY2 = ASX ( NP ) - KNX ( 3 )
26 DUMMY3 = DUMMY1**2 + DUMMY2**2
27 AM ( I1,J1 ) = - ( DUMMY1 / DUMMY3 )
28 AM ( I1,J2 ) = ( DUMMY2 / DUMMY3 )
29 CONTINUE
30 GO TO 150
31 CONTINUE
32 IF ( I1,NE. 2 ) GO TO 80
33 IF ( J1,NE. 1 ) GO TO 60
34 DUMMY1 = KNY ( 3 ) - ASX ( I3 )
35 DUMMY2 = KNX ( 3 ) - ASX ( I3 )
36 IF ( NA.EQ. 3 ) GO TO 50
37 DUMMY1 = ASX ( I1 ) - ASY ( I3 )
38 DUMMY2 = ASX ( I1 ) - ASX ( I3 )
39 CONTINUE
40 DUMMY3 = DUMMY1**2 + DUMMY2**2
41 DUMMY4 = KNY ( 2 ) - ASX ( I3 )
42 DUMMY5 = KNX ( 2 ) - ASX ( I3 )
43 DUMMY6 = DUMMY4**2 + DUMMY5**2
44 AM ( I1,J1 ) = - ( DUMMY1 / DUMMY3 ) + ( DUMMY4 / DUMMY6 )
45 AM ( I1,J2 ) = ( DUMMY2 / DUMMY3 ) - ( DUMMY5 / DUMMY6 )
46 GO TO 150
47 CONTINUE
48 IF ( J1,NE. 3 ) GO TO 70
49 AM ( I1,J1 ) = ( DUMMY1 / DUMMY3 )
50 AM ( I1,J2 ) = - ( DUMMY2 / DUMMY3 )
51 CONTINUE
52 GO TO 150
53 CONTINUE
54 IF ( I1,NE. K1 ) GO TO 110
55 IF ( J1,NE. I5 ) GO TO 90
56 DUMMY1 = ASX ( I4 ) - ASY ( I3 )
57 DUMMY2 = ASX ( I4 ) - ASX ( I3 )

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90      DUMMY3 = DUMMY1**2 + DUMMY2**2
      AM ( I1,J1 ) = - ( DUMMY1 / DUMMY3 )
      AM ( I1,J2 ) = ( DUMMY2 / DUMMY3 )
      GO TO 150
      CONTINUE
      IF ( J1.NE. I6 ) GO TO 100
      DUMMY1 = KNY ( 3 ) - ASY ( I3 )
      DUMMY2 = KNX ( 3 ) - ASX ( I3 )
      DUMMY3 = DUMMY1**2 + DUMMY2**2
      DUMMY4 = ASY ( I4 ) - ASY ( I3 )
      DUMMY5 = ASX ( I4 ) - ASX ( I3 )
      DUMMY6 = DUMMY4**2 + DUMMY5**2
      AM ( I1,J1 ) = - ( DUMMY1 / DUMMY3 ) + ( DUMMY4 / DUMMY6 )
      AM ( I1,J2 ) = ( DUMMY2 / DUMMY3 ) - ( DUMMY5 / DUMMY6 )
      CONTINUE
      GO TO 150
      CONTINUE
      IF ( J1.NE. I5 ) GO TO 120
      DUMMY1 = ASY ( I4 ) - ASY ( I3 )
      DUMMY2 = ASX ( I4 ) - ASX ( I3 )
      DUMMY3 = DUMMY1**2 + DUMMY2**2
      AM ( I1,J1 ) = - ( DUMMY1 / DUMMY3 )
      AM ( I1,J2 ) = ( DUMMY2 / DUMMY3 )
      GO TO 150
      CONTINUE
      IF ( J1.NE. I6 ) GO TO 130
      DUMMY1 = ASY ( I1 ) - ASY ( I3 )
      DUMMY2 = ASX ( I1 ) - ASX ( I3 )
      DUMMY3 = DUMMY1**2 + DUMMY2**2
      DUMMY4 = ASY ( I4 ) - ASY ( I3 )
      DUMMY5 = ASX ( I4 ) - ASX ( I3 )
      DUMMY6 = DUMMY4**2 + DUMMY5**2
      AM ( I1,J1 ) = - ( DUMMY1 / DUMMY3 ) + ( DUMMY4 / DUMMY6 )
      AM ( I1,J2 ) = ( DUMMY2 / DUMMY3 ) - ( DUMMY5 / DUMMY6 )
      GO TO 150
      CONTINUE
      IF ( J1.NE. I7 ) GO TO 140
      DUMMY1 = ASY ( I1 ) - ASY ( I3 )
      DUMMY2 = ASX ( I1 ) - ASX ( I3 )
      DUMMY3 = DUMMY1**2 + DUMMY2**2
      AM ( I1,J1 ) = ( DUMMY1 / DUMMY3 )
      AM ( I1,J2 ) = - ( DUMMY2 / DUMMY3 )
      CONTINUE
      CONTINUE
      DO 240 I1 = 1, ND
      I2 = NA + I1
      I3 = I1 - 1

```

```

170      I4 = I3 + ( I3 - 1 )
180      I5 = I3 + ( I3 + 1 )
      DO 230 J1 = 1, NP1, 2
      J2 = J1 + 1
      AM ( I2, J1 ) = 0.000
      IF ( I1, NE, 1 ) GO TO 180
      IF ( J1, NE, 1 ) GO TO 170
      DUMMY1 = KNY ( 2 ) - ASY ( 1 )
      DUMMY2 = KNY ( 2 ) - ASX ( 1 )
      DUMMY3 = DSQRT ( DUMMY1**2 + DUMMY2**2 )
      AM ( I2, J1 ) = - ( DUMMY2 / DUMMY3 )
      AM ( I2, J2 ) = - ( DUMMY1 / DUMMY3 )
      CONTINUE
      GO TO 230
      CONTINUE
      IF ( I1, NE, ND ) GO TO 200
      IF ( J1, NE, NP1 ) GO TO 190
      DUMMY1 = ASY ( NP ) - KNY ( 3 )
      DUMMY2 = ASX ( NP ) - KNY ( 3 )
      DUMMY3 = DSQRT ( DUMMY1**2 + DUMMY2**2 )
      AM ( I2, J1 ) = ( DUMMY2 / DUMMY3 )
      AM ( I2, J2 ) = ( DUMMY1 / DUMMY3 )
      CONTINUE
      GO TO 230
      CONTINUE
      IF ( I4, NE, J1 ) GO TO 210
      DUMMY1 = ASY ( I3 ) - ASY ( I1 )
      DUMMY2 = ASX ( I3 ) - ASX ( I1 )
      DUMMY3 = DSQRT ( DUMMY1**2 + DUMMY2**2 )
      AM ( I2, J1 ) = ( DUMMY2 / DUMMY3 )
      AM ( I2, J2 ) = ( DUMMY1 / DUMMY3 )
      GO TO 230
      CONTINUE
      IF ( I5, NE, J1 ) GO TO 220
      DUMMY1 = ASY ( I3 ) - ASY ( I1 )
      DUMMY2 = ASX ( I3 ) - ASX ( I1 )
      DUMMY3 = DSQRT ( DUMMY1**2 + DUMMY2**2 )
      AM ( I2, J1 ) = - ( DUMMY2 / DUMMY3 )
      AM ( I2, J2 ) = - ( DUMMY1 / DUMMY3 )
      CONTINUE
      CONTINUE
      CONTINUE
      RETURN
      END

```

APPENDIX C

PROGRAM 3

COMPUTER OUTPUT

THE CALCULATION OF CLOSED TRAVERSE STATION POSITIONS
BY CONDITION EQUATIONS METHOD
FOR UNCORRELATED OBSERVATIONS WITH EQUAL PRECISION

KNOWN HORIZONTAL STATION POSITIONS

NO	NAME OF STATIONS	UTM GRID COORDINATES GRID NORTHING METERS	UTM GRID COORDINATES EASTING METERS	AZIMUTH FROM TO NO NO	GRID AZIMUTH CLOCKWISE FROM NORTH D. M. S.
1	PIPER				
2	MOSS 2	4072555.85206	608279.04404	2 -> 1	100 16 23.77800
3	HOLM	4079258.31754	612238.85256	3 -> 4	136 33 26.33400
4	MORAN				

OBSERVED DATA

NAME OF STATIONS	OBSERVED ANGLE	D. M. S.	STD. S.	GRID DISTANCE METERS	STD. M.
PIPER					
MOSS 2	246 5	43.20000	1.98400	1424.00400	0.00100
MOSSBACK	222 51	8.60000	1.40500	365.74400	0.00100
DUNE TEMP	190 15	2.60000	1.20300		
HOLM	277 5	17.00000	1.61400	6476.27100	0.00300
MORAN					

PRECALCULATED OF CLOSED TRAVERSE STATION POSITIONS

NC	NAME OF STATIONS	UTM GRID COORDINATES		AZIMUTH FROM TO	GRID AZIMUTH CLOCKWISE FROM NORTH
		GRID NORTHING	GRID EASTING		
1	MOSS 2	4072555.85206	608279.04404	1 -> 2	346 22 6.97800
2	MOSSBACK	4073939.74473	607943.44238	2 -> 1	166 22 6.97800
3	DUNE TEMP	4074258.94534	608121.99110	2 -> 3	29 13 15.57800
4	HOLM	4079258.22818	612238.93865	3 -> 2	209 13 15.57800
5	MORAN			3 -> 4	39 28 18.17800
				4 -> 3	219 28 18.17800
				4 -> 5	136 33 35.17800

TOTAL AZIMUTH ERROR = 0 D. 0 M. 8.84400 S.
 AZIMUTH ERROR PER STATIONS = 0 D. 0 M. 2.21100 S.
 TOTAL GRID DISTANCE = 8266.01900 METERS.
 DISTANCE ERROR = 0.12408 METERS.
 ACCURACY = 1: 66617

CORRECTED DATA

NAME OF STATIONS	OBSERVED		GRID DISTANCE METERS
	ANGLE D. M. S.		
PIPHER			
MOSS 2	246 5 45.81717		1424.00400
MOSSBACK	222 51 5.36399		365.74400
DUNE TEMP	190 14 58.98196		6476.27100
HOLM	277 5 12.39287		
MORAN			

***** CALCULATION OF CLOSED TRAVERSE STATION POSITIONS BY LEAST-SQUARES ADJUSTMENT *****

BY CONDITION EQUATION METHOD

NO	NAME OF STATIONS	UTM GRID COORDINATES		AZIMUTH FROM TO	GRID AZIMUTH CLOCKWISE
		GRID NORTHING	GRID EASTING	NO	NO
		METERS		D. M. S.	
1	MOSS 2	4072555.85206	608279.04404	1 -> 2	346 22 9.59517
2	MOSSBACK	4073939.74899	607943.45994	2 -> 1	166 22 9.59517
3	DUNE TEMP	4074258.95013	608122.00770	2 -> 3	29 13 14.95917
4	HOLM	4079258.31754	612238.85256	3 -> 2	209 13 14.95917
5	MORAN			3 -> 4	39 28 13.94113
				4 -> 3	219 28 13.94113
				4 -> 5	136 33 26.33400

CHECK ROUND OFF ERROR
 AZIMUTH = 0 D. 0 M. 0.00000 S.
 DISTANCE = 0.00000 METERS.

THE STANDARD DEVIATION OF UNIT WEIGHT IS 0.0000201139

THE STANDARD DEVIATIONS OF ADJUSTED ANGLES ARE

ANGLE # 1 = 0.1591810531 SECONDS
 ANGLE # 2 = 2.9426247963 SECONDS
 ANGLE # 3 = 2.9172247526 SECONDS
 ANGLE # 4 = 0.1336934925 SECONDS

THE STANDARD DEVIATIONS OF ADJUSTED DISTANCES ARE

DISTANCE # 1 = 0.0000201139 METERS
 DISTANCE # 2 = 0.0000201139 METERS
 DISTANCE # 3 = 0.0000201139 METERS

THE CALCULATION OF CLOSED TRAVERSE STATION POSITIONS
BY CONDITION EQUATIONS METHOD
FOR UNCORRELATED OBSERVATIONS WITH UNEQUAL PRECISION

KNOWN HORIZONTAL STATION POSITIONS

NO	NAME OF STATIONS	UTM GRID COORDINATES		AZIMUTH		GRID AZIMUTH	
		GRID NORTHING	GRID EASTING	FROM TO	FROM TO	CLOCKWISE	
		METERS	METERS	NO	NO	FROM NORTH	D. M. S.
1	PIPHER						
2	MOSS 2	4072555.85206	608279.04404	2	-> 1	100 16	23.77800
3	HOLM	4079258.31754	612238.85256	3	-> 4	136 33	26.33400
4	MORAN						

OBSERVED DATA

NAME OF STATIONS	OBSERVED		GRID	
	ANGLE	STD.	DISTANCE	STD.
	D. M. S.	S.	METERS	M.
PIPHER				
MOSS 2	246 5	43.20000	1.98400	
MOSSBACK	222 51	8.60000	1.40500	
DUNE TEMP	190 15	2.60000	1.20300	
HOLM	277 5	17.00000	1.61400	
MORAN				
			1424.00400	0.00100
			365.74400	0.00100
			6476.27100	0.00300

PRECALCULATED OF CLOSED TRAVERSE STATION POSITIONS

NO	NAME OF STATIONS	UTM GRID COORDINATES		AZIMUTH FROM TO	GRID AZIMUTH CLOCKWISE FROM NORTH
		GRID NORTHING	GRID EASTING		
1	MOSS 2	4072555.85206	608279.04404	1 -> 2	346 22 6.97800
2	MOSSBACK	4073939.74473	607943.44238	2 -> 1	166 22 6.97800
3	DUNE TEMP	4074258.94534	608121.99110	2 -> 3	29 13 15.57800
4	HOLM	4079258.22818	612238.93865	3 -> 2	209 13 15.57800
5	MORAN			3 -> 4	39 28 18.17800
				4 -> 3	219 28 18.17800
				4 -> 5	136 33 35.17800

TOTAL AZIMUTH ERROR = 0 D. 0 M. 8.84400 S.
 AZIMUTH ERROR PER STATIONS = 0 D. 0 M. 2.21100 S.
 TOTAL GRID DISTANCE = 8266.01900 METERS.
 DISTANCE ERROR = 0.12408 METERS.
 ACCURACY = 1: 66617

CORRECTED DATA

NAME OF STATIONS	OBSERVED		GRID	
	D. M.	ANGLE	DISTANCE	METERS
PIPHER				
MOSS 2	246 5	45.12117	1424.00424	
MOSSBACK	222 51	5.23554	365.74439	
DUNE TEMP	190 14	59.95242	6476.27457	
HOLM	277 5	12.24688		
MORAN				

 CALCULATION OF CLOSED TRAVERSE STATION POSITIONS BY LEAST-SQUARES ADJUSTMENT

 BY CONDITION EQUATION METHOD

NC	NAME OF STATIONS	UTM GRID COORDINATES		AZIMUTH		GRID AZIMUTH FROM TO CLOCKWISE
		GRID NORTHING	GRID EASTING	NO	NO	
1	MOSS 2	4072555.85206	608279.04404	1 ->	2 346 22	8.89917
2	MOSSBACK	4073939.74809	607943.45521	2 ->	1 166 22	8.89917
3	DUNE TEMP	4074258.95029	608122.00189	2 ->	3 29 13	14.13470
4	HOLM	4079258.31754	612238.85256	3 ->	2 209 13	14.13470
5	MORAN			3 ->	4 39 28	14.08712
				4 ->	3 219 28	14.08712
				4 ->	5 136 33	26.33400

CHECK ROUND OFF ERROR
 AZIMUTH = 0 D. 0 M. 0.00000 S.
 DISTANCE = 0.00000 METERS.

THE STANDARD DEVIATION OF UNIT WEIGHT IS 2.6967855476

THE STANDARD DEVIATIONS OF ADJUSTED ANGLES ARE

ANGLE # 1 =	1.4033951177	SECONDS
ANGLE # 2 =	2.6409612857	SECONDS
ANGLE # 3 =	2.5525289050	SECONDS
ANGLE # 4 =	0.2613580638	SECONDS

THE STANDARD DEVIATIONS OF ADJUSTED DISTANCES ARE

DISTANCE # 1 =	0.0026855980	METERS
DISTANCE # 2 =	0.0026737113	METERS
DISTANCE # 3 =	0.0074503265	METERS

```
C C TRAVCON WATFIV A1
C C $JOB      SAM3,NOCHECK
C C CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C C AUTHOR : LT. SAMAN AUMCHANTR RTN.
C C DATE : JULY 11 , 1984
C C CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C C THIS IS A PROGRAM FOR CALCULATION OF TRAVERSE STATION
C C POSITIONS BY CONDITION EQUATION METHOD.
C C PPPPPPPPPPPPPPPPPPPPPPPPPPPPPPPPPPPPPPPPPPPPPPPPPPPPPPPPP
C C VARIABLE DEFINITIONS
C C OPT = OPTION FOR CALCULATION OF OPEN OR CLOSED TRAVERSE
C C CHOIC = OPTION FOR ENTER DATA BY DATA FILE OR INTERACTIVE
C C FROM TERMINAL KEYBOARD.
C C DUMMY1,DUMMY2,DUMID,DUMIM,DUMIS = THE DUMMY VARIABLES
C C NGRID(4)= GRID NORTHING OF THE KNOWN POSITIONS.
C C EGRID(4)= GRID EASTING OF THE KNOWN POSITIONS.
C C STNAME(4,5) = THE NAME OF KNOWN STATIONS.
C C TRNAME( 32,5) = THE NAME OF TRAVERSE STATIONS.
C C DIRECD(4),DIRECM(4),DIRECS(4) = THE GRID AZIMUTH OF KNOWN
C C POSITIONS.
C C STAT1(4),STAT2(4) = THE NUMBER OF KNOWN POSITIONS.
C C COUNT,COUNTA = THE NUMBER OF OBSERVED ANGLES.
C C ANGI( 32)= THE OBSERVED ANGLE IN RADIAN.
C C ANGDI( 32),ANGM( 32),ANGSI( 32) = THE OBSERVED ANGLES IN DEGREE,
C C MINUTE AND SECOND.
C C COUNTN,COUNTO = THE NUMBER OF MEASURED DISTANCE.
C C DISTI( 32) = THE MEASURED DISTANCE IN METERS.
C C TODIS = THE SUM OF MEASURED DISTANCE.
C C CFZAD( 32),CFAZM( 32),CFAZSI( 32) = THE COMPUTED FORWARD AZIMUTH
C C CBZAD( 32),CBZAM( 32),CBZASI( 32) = THE COMPUTED BACK AZIMUTH
C C CMGRIDI( 32) = GRID NORTHING OF TRAVERSE STATION POSITIONS.
C C CEGRID( 32) = GRID EASTING OF TRAVERSE STATION POSITIONS.
C C CFR = THE TOTAL DISTANCE ERROR.
C C DIFAZI = THE TOTAL ANGLE ERROR IN RADIAN.
C C EAZID, EAZIM, FAZIS = THE TOTAL ANGLE ERROR IN DEGREE,
C C MINUTE AND SECOND.
C C EAZ2D, EAZ2M, EAZ2S = THE ANGLE ERROR PER STATION.
```

[illegible]

```

CC
PRINT TITLE
WRITE ( 6,1000 )
WRITE ( 6,1380 )
WRITE ( 6,1390 )
WRITE ( 6,1400 )
WRITE ( 6,1410 )
WRITE ( 6,1420 )
WRITE ( 6,1390 )
WRITE ( 6,1380 )
WRITE ( 6,1430 )
WRITE ( 6,1440 )

CC
READ OPTIONS 1 = OPEN TRAVERSE
              2 = CLOSED TRAVERSE

CC
READ ( 4,1340 ) OPT

CC
WRITE ( 6,1450 )
WRITE ( 6,1460 )
WRITE ( 6,1470 )
WRITE ( 6,1430 )
WRITE ( 6,1480 )

CC
READ OPTIONS 1 = ENTER DATA BY DATA FILE
              2 = ENTER DATA BY INTERACTIVE

CC
READ ( 4,1340 ) CHOIC

CC
GET UTM GRID NORTHING, EASTING, AZIMUTH, AND THE NAME
      OF KNOWN POSITIONS

CC
DES = 04
IF ( CHOIC.EQ. 2 ) GO TO 10
DES = 03
CONTINUE
LOOP1 = 2
IF ( OPT.EQ. 1 ) GO TO 20
LOOP1 = 4
CONTINUE
10
20

```

```

DO 70 I = 1, LOOP1
WRITE ( 6, 1490 ) I
IF ( I.NE. 1 ) GO TO 30
WRITE ( 6, 1500 )
READ ( DES, 1370 ) ( STNAME ( I, K1 ) , K1 = 1, 5 )
GO TO 60
30 CONTINUE
IF ( I.NE. 2 ) GO TO 40
WRITE ( 6, 1520 )
READ ( DES, 1370 ) ( STNAME ( I, K1 ) , K1 = 1, 5 )
WRITE ( 6, 1750 )
WRITE ( 6, 1540 )
READ ( DES, 1360 ) DUMMY1
NGRID ( 1 ) = DUMMY1
WRITE ( 6, 1760 )
WRITE ( 6, 1560 )
READ ( DES, 1360 ) DUMMY1
EGRID ( 1 ) = DUMMY1
WRITE ( 6, 1770 ) ( STNAME(2,K1), K1=1,5), (STNAME(1,K1), K1=1
, 5)
WRITE ( 6, 1780 )
READ ( DES, 1360 ) DUMMY1
CALL TDMS ( DUMMY1, DUMID, DUMIM, DUMIS )
DIRECD ( 2 ) = IDINT ( DUMID )
DIRECM ( 2 ) = IDINT ( DUMIM )
DIRECS ( 2 ) = DUMIS
CALL CDMSR ( DUMID, DUMIM, DUMIS, DUMMY1 )
AZZ1 = DUMMY1
STAT1 ( 2 ) = 2
STAT2 ( 2 ) = 1
GO TO 60
40 CONTINUE
IF ( I.NE. 3 ) GO TO 50
WRITE ( 6, 1530 )
READ ( DES, 1370 ) ( STNAME ( I, K1 ) , K1 = 1, 5 )
WRITE ( 6, 1750 )
WRITE ( 6, 1540 )
READ ( DES, 1360 ) DUMMY1
NGRID ( 1 ) = DUMMY1
WRITE ( 6, 1760 )
WRITE ( 6, 1560 )
READ ( DES, 1360 ) DUMMY1
EGRID ( 1 ) = DUMMY1
GO TO 60
50 CONTINUE
IF ( I.NE. 4 ) GO TO 60
WRITE ( 6, 1510 )
READ ( DES, 1370 ) ( STNAME ( I, K1 ) , K1 = 1, 5 )

```

```

*
WRITE ( 6,1770 ) (STNAME(3,K1),K1=1,5),(STNAME(4,K1),K1=1
5),
WRITE ( 6,1780 )
READ ( DES,1360 ) DUMMY1
CALL TMS ( DUMMY1,DUMID,DUMIM,DUMIS )
DIRECD ( 3 ) = IDINT ( DUMID )
DIRECM ( 3 ) = IDINT ( DUMIM )
DIRECS ( 3 ) = DUMIS
CALL CDMR ( DUMID,DUMIM,DUMIS,DUMMY1 )
AZ34 = DUMMY1
STAT1 ( 3 ) = 3
STAT2 ( 3 ) = 4
CONTINUE
CONTINUE
60
70
CCCCC
READ THE NUMBER OF OBSERVED ANGLES
WRITE ( 6,1580 )
READ ( DES,1350 ) COUNT
COUNTA = COUNT
CCCCC
READ THE OBSERVED ANGLES
DO 100 I = 1,COUNTA
IF ( 1.GT.1 ) GO TO 80
WRITE ( 6,1590 )
GO TO 90
CONTINUE
80
K2 = I - 1
WRITE ( 6,1600 ) K2
CONTINUE
90
WRITE ( 6,1610 )
READ ( DES,1360 ) DUMMY1
CALL TMS ( DUMMY1,DUMID,DUMIM,DUMIS )
CALL CDMR ( DUMID,DUMIM,DUMIS,DUMMY2 )
ANG ( I ) = DUMMY2
ANGD ( I ) = IDINT ( DUMID )
ANGM ( I ) = IDINT ( DUMIM )
ANGS ( I ) = DUMIS
WRITE ( 6,1550 )
READ ( CES,1360 ) DUMMY1
STDA ( I ) = DUMMY1
100 CONTINUE
C

```

```

C
C
C
      READ THE GRID DISTANCE
      COUNTD = COUNT
      IF ( OPT.EQ. 1 ) GO TO 110
      COUNTD = COUNT - 1
110  CONTINUE
      DO 140 I = 1,COUNTD
      IF ( I.GT. 1 ) GO TO 120
      WRITE ( 6,1620 )
      GO TO 130
120  CONTINUE
      K4 = I - 1
      K5 = I
      WRITE ( 6,1630 ) K4,K5
      CONTINUE
      WRITE ( 6,1640 )
      READ ( DES,1360 ) DUMMY1
      DIST ( I ) = DUMMY1
      TODIS = TODIS + DUMMY1
      WRITE ( 6,1570 )
      READ ( DES,1360 ) DUMMY1
      STDD ( I ) = DUMMY1
140  CONTINUE
      COUNTN = COUNTD
      IF ( OPT.EQ. 1 ) GO TO 150
      COUNTN = COUNTD - 1
150  CONTINUE
      DO 160 I = 1,COUNTN
      WRITE ( 6,1650 ) I
      READ ( DES,1370 ) ( TRNAME ( I,K1 ) , K1 = 1,5 )
160  CONTINUE
      END INPUT_DATA
      ALGORITHM PRINT_INPUT_DATA
      PRINT DETAILS OF THE KNOWN STATION POSITIONS
      WRITE ( 6,1000 )
      WRITE ( 6,1010 )
      WRITE ( 6,1020 )
      WRITE ( 6,1030 )
      WRITE ( 6,1040 )
      WRITE ( 6,1050 )
C
C
C
C
C
C
C

```



```

CALL CDMR ( NUM360,NUMO,NUMO,AN36OR )
ITER1 = 2
IF ( OPT.NE. 1 ) GO TO 230
    CONTINUE
DO 450 ADJL = 1, ITER1
    XGD = EGRID ( 2 )
    YGD = NGRID ( 2 )
    AZFIR = AAZ21
DO 250 I = 1,COUNTD
    AZIMUT = AZFIR + ANG ( I )
    IF ( AZIMUT.LT. AN36OR ) GO TO 240
        AZIMUT = AZIMUT - AN36OR
    CONTINUE
    COAZR ( I ) = AZIMUT
    CALL CRDM ( AZIMUT,DUMID,DUMIM,DUMIS )
    CFAZD ( I ) = IDINT ( DUMID )
    CFAZM ( I ) = IDINT ( DUMIM )
    CFAZS ( I ) = DUMIS
    DISTAN = DIST ( I )
    CALL CXYC ( XGD,YGD,DISTAN,AZIMUT,DUMMY1,DUMMY2 )
    DELTAX ( I ) = DUMMY1 - XGD
    DELTAY ( I ) = DUMMY2 - YGD
    SUMDX = SUMDX + DELTAX ( I )
    SUMDY = SUMDY + DELTAY ( I )
    CEGRID ( I ) = DUMMY1
    CNGRID ( I ) = DUMMY2
    CALL CTANA ( DUMMY1,DUMMY2,XGD,YGD,AZC,DUMIS )
    CALL CRDM ( AZC,DUMID,DUMIM,DUMIS )
    CBABD ( I ) = IDINT ( DUMID )
    CBABM ( I ) = IDINT ( DUMIM )
    CBABS ( I ) = DUMIS
    AZFIR = AZC
    XGD = DUMMY1
    YGD = DUMMY2
CONTINUE
250 CC
CC
CC
CC
CC
>>> PRINT DETAIL OF PRECALCULATED OF TRAVERSE STATION POSITIONS <<<

IF ( OPT.EQ. 1 ) GO TO 270
    IF ( ADJL.NE. 1 ) GO TO 260
        WRITE ( 6,I180 )
        WRITE ( 6,I190 )
        GO TO 280
    CCNTINUE
    WRITE ( 6,I000 )
260

```

```

270      WRITE ( 6,1290 )
        WRITE ( 6,1300 )
        WRITE ( 6,1740 )
        GO TO 280
CONTINUE
280      WRITE ( 6,1170 )
        WRITE ( 6,1200 )
CONTINUE
        WRITE ( 6,1030 )
        WRITE ( 6,1040 )
        WRITE ( 6,1050 )
        WRITE ( 6,1060 )
        WRITE ( 6,1210 ) (STNAME(2,K1),K1=1,5),NGRID(2),EGRID(2)
        DO 320 I = 1,COUNTD
          K4 = I
          K5 = I + 1
          WRITE ( 6,1220 ) K4,K5,CFAZD(I),CFAZM(I),CFAZS(I)
          IF ( OPT .EQ. 1 ) GO TO 300
          IF ( I .NE. COUNTD ) GO TO 290
          WRITE ( 6,1070 ) K5,(STNAME(3,K1),K1=1,5),CNGRID(I),CNGRID(I),CBAZD(I),K5,K4,CBAZD(I),CBAZM(I),CBAZS(I)
          CONTINUE
290      CONTINUE
          GO TO 310
300      IF ( I .EQ. COUNTD ) GO TO 310
          WRITE ( 6,1070 ) K5,(TRNAME(I,K1),K1=1,5),CNGRID(I),CNGRID(I),K5,K4,CBAZD(I),CBAZM(I),CBAZS(I)
          CONTINUE
310      CONTINUE
320      IF ( OPT .NE. 1 ) GO TO 330
          WRITE ( 6,1070 ) K5,(TRNAME(COUNTD,K1),K1=1,5),CNGRID(COUNTD),CNGRID(COUNTD),K5,K4,CBAZD(COUNTD),CBAZM(COUNTD),CBAZS(COUNTD)
          CONTINUE
330      IF ( OPT .EQ. 1 ) GO TO 440
          AZLAS = AZFIR + ANG ( COUNT )
          IF ( AZLAS .LT. AN360R ) GO TO 340
          AZLAS = AZLAS - AN360R
          CONTINUE
          K4 = I
          K5 = I + 1
          CALL CPDMS ( AZLAS,DUMID,DUMIM,DUMIS )
          TEMD = IDINT ( DUMID )
          TEMM = IDINT ( DUMIM )
          WRITE ( 6,1220 ) K4,K5,TEMD,TEMM,DUMIS
          DIFAZI = AZLAS - AZ34
          WPRED ( 1 ) = DIFAZI
          CORAZ = DIFAZI / DFLOAT ( COUNTA )
340

```

```

350 CALL CRDMS ( DIFAZI, DUMID, DUMIM, DUMIS )
    EAZID = IDINT ( DABS ( DUMID ) )
    EAZIM = IDINT ( DABS ( DUMIM ) )
    EAZIS = DABS ( DUMIS )
    CALL CRDMS ( CORAZ, DUMID, DUMIM, DUMIS )
    EAZ2D = IDINT ( DABS ( DUMID ) )
    EAZ2M = IDINT ( DABS ( DUMIM ) )
    EAZ2S = DABS ( DUMIS )
    WPRED ( 2 ) = SUMDX - ( EGRID(3) - EGRID(2) )
    WPRED ( 3 ) = SUMDY - ( NGRID(3) - NGRID(2) )
    DIFDIS = DSQRT ( WPRED(2)**2 + WPRED(3)**2 )
    ERR = IDINT ( TODIS / DIFDIS )
    IF ( ADJL .NE. 1 ) GO TO 430
    WRITE ( 6,1240 ) EAZID, EAZIM, EAZIS
    WRITE ( 6,1250 ) EAZ2D, EAZ2M, EAZ2S
    WRITE ( 6,1260 ) TODIS
    WRITE ( 6,1270 ) DIFDIS
    WRITE ( 6,1280 ) ERR
    DO 380 I = 1, COUNTA
    COBM1 ( I ) = 1.000
    COBM2 ( I ) = 0.000
    COBM3 ( I ) = 0.000
    IF ( I .GT. COUNTD ) GO TO 370
    J = I
    CONTINUE
    IF ( J.EQ. COUNTA ) GO TO 360
    COBM2 ( I ) = COBM2(I)+DELTA1X(J)
    COBM3 ( I ) = COBM3(I)+DELTA1Y(J)
    J = J + 1
    GO TO 350
    CONTINUE
    COBM2 ( I ) = COBM2 ( I )
    COBM3 ( I ) = COBM3 ( I ) * (-1.000)
    CONTINUE
    DO 390 I = 1, COUNTD
    J = COUNTA + I
    COBM1 ( J ) = 0.000
    COBM2 ( J ) = DSIN ( COAZR ( I ) )
    COBM3 ( J ) = DCOS ( COAZR ( I ) )
    CONTINUE
    NT = COUNTA + COUNTD
    CALL CLSQ ( NT, COUNTA, COUNTD, STDA, STDD, WPRED, COBM1,
    COBM2, COBM3, BM, BMT, PMI, PI8T, VAR, CORAD, PW, PRINTD, DM,
    BPIM, QBPIM, STAND, VTM )

```

C C CORRECTED OBSERVED DATA


```

*GLE',/12X,'AT THIS STATION IN SECONDS ?',/12X,'FOR EXAMPLE STD.
*=3.9 SECONDS. : ENTER 3.9000',
1560 FORMAT ( /12X,'FOR EXAMPLE: GRID EASTING 608,279.04404 METERS.',
/26X,'ENTER 608279.04404000',
1570 FORMAT ( /12X,'WHAT IS THE STANDARD DEVIATION IN METERS.?',/12X
,'FOR EXAMPLE STD. = .011 METERS : ENTER 0.011000',
1580 FORMAT ( /12X,'HOW MANY POSITIONS DID YOU OBSERVE ANGLES ?',/12X
,'PLEASE ENTER 01 TO 32',
1590 FORMAT ( /12X,'ENTER THE OBSERVED ANGLE AT THE FIRST KNOWN POSITI
ON.',
1600 FORMAT ( /10X,'ENTER THE OBSERVED ANGLE AT TRAVERSE STATION POSIT
ION #',/12X,')
1610 FORMAT ( /10X,'FOR EXAMPLE : OBSERVED ANGLE CLOCKWISE FROM BACKSI
GHT TO FORESIGHT',/24X,'IS 169 D. 32 M. 11.11 S.',/24X,'ENTER 1
69.321111000',
1620 FORMAT ( /12X,'ENTER GRID DISTANCE BETWEEN THE FIRST KNOWN POSITI
ON AND',/12X,'TRAVERSE STATION POSITION #',/12X,')
1630 FORMAT ( /12X,'ENTER GRID DISTANCE BETWEEN TRAVERSE STATION POSIT
ION #',/12X,'AND TRAVERSE STATION POSITION #',/12X,')
1640 FORMAT ( /12X,'FOR EXAMPLE: THE GRID DISTANCE BETWEEN THE STATIO
NS IS',/26X,'399.052 METERS.',/26X,'ENTER 399.052000',
1650 FORMAT ( /12X,'WHAT IS THE NAME OF TRAVERSE STATION POSITION #',
/12X,')
1660 FORMAT ( /12X,'CORRECTED DATA',/12X,')
1670 FORMAT ( /25X,'*****',/12X,')
1680 FORMAT ( /29X,'OBSERVED',/9X,'GRID',/12X,')
1690 FORMAT ( /6X,'NAME OF STATIONS',/8X,'ANGLE',/11X,'DISTANCE',/12X
,')
1700 FORMAT ( /27X,'D. M. S.',/11X,'METERS',/12X,')
1710 FORMAT ( /4X,'5A4',/12X,')
1720 FORMAT ( /4X,'5A4',/1X,'13,1X,12,1X,F8.5',/12X,')
1730 FORMAT ( /42X,'F14.5',/12X,')
1740 FORMAT ( /27X,'BY CONDITION EQUATION METHOD',/12X,')
1750 FORMAT ( /12X,'WHAT IS UTM GRID NORTHING AT THIS POSITION?',/12X
,')
1760 FORMAT ( /12X,'WHAT IS UTM GRID EASTING AT THIS POSITION?',/12X
,')
1770 FORMAT ( /16X,'WHAT IS UTM GRID AZIMUTH FROM',/12X,'5A4',/12X
,')
1780 FORMAT ( /12X,'FOR EXAMPLE : UTM GRID AZIMUTH FROM A. TO B.',
/12X,'IS 139 D. 19 M. 29.3 S. : ENTER 139.19293000',
1790 FORMAT ( /12X,'THE STANDARD DEVIATION OF UNIT WEIGHT IS',/12X
,'F15.10',/12X,')
1800 FORMAT ( /12X,'THE STANDARD DEVIATIONS OF ADJUSTED ANGLES ARE',/12X
,')
1810 FORMAT ( /12X,'ANGLE #',/12X,'=',/12X,'F17.10',/12X,'SECONDS',/12X
,')
1820 FORMAT ( /12X,'THE STANDARD DEVIATIONS OF ADJUSTED DISTANCES ARE',
/12X,')
1830 FORMAT ( /12X,'DISTANCE #',/12X,'= ',/12X,'F16.10',/12X,' METERS',/12X
,')
C
END
C

```



```

C
C
C      COMPUTATION OF THE TRANSPOSE OF B MATRIX
      IO5 = 6
      N1 = 1
      N3 = 3
      DO 10 I = 1, NUMT
        BM ( 1, I ) = BMR1 ( I )
        BM ( 2, I ) = BMR2 ( I )
        BM ( 3, I ) = BMR3 ( I )
      CONTINUE
      DO 20 I = 1, 3
        WM ( I ) = WM ( I ) * ( -1.000 )
      CONTINUE
      DO 30 I = 1, 3
        DO 40 J = 1, NUMT
          BMT ( J, I ) = BM ( I, J )
        CONTINUE
      CONTINUE
C
C
C      INVERSION OF P MATRIX
      DO 70 I = 1, NUMT
        DO 60 J = 1, NUMT
          DM ( I, J ) = NUMO
          PM1 ( I, J ) = NUMO
          IF ( I.NE.J ) GO TO 50
          PM1 ( I, J ) = 1.000
          DM ( I, J ) = 1.000
        CONTINUE
      CONTINUE
      CONTINUE
      IF ( PW.EQ.1 ) GO TO 130
      DO 80 I = 1, NA
        DUMMY1 = STDA ( I )
        CALL COMSR ( NUMO, NUMO, DUMMY1, DUMMY2 )
        VAR ( I ) = ( DSIN ( DUMMY2 ) )**2
      CONTINUE
      DO 90 I = 1, ND
        J = NA + I
        VAR ( J ) = ( STDD ( I ) )**2
      CONTINUE
      DO 120 I = 1, NUMT
        DO 110 J = 1, NUMT
          IF ( I.NE.J ) GO TO 100
          PM1 ( I, J ) = VAR ( I )

```

```

100 DM ( I,J ) = 1.0D0 / VAR ( I )
110 CONTINUE
120 CONTINUE
130 CONTINUE

MULTIPLICATION OF INVERSION OF P MATRIX BY TRANSPOSE OF B MATRIX

CALL VMULFF ( PMI,BMT,NUMT,NUMT,N3,NUMT,NUMT,PIBT,NUMT,IER )

MULTIPLICATION OF B MATRIX BY P.I*B.T MATRIX

CALL VMULFF ( BM,PIBT,N3,NUMT,N3,N3,NUMT,BBT,N3,IER )

INVERSION OF B*B.T MATRIX

CALL LINV2F ( BBT,N3,N3,BBTI,N1,WK3,IER )

COMPUTATION OF K MATRIX

CALL VMULFF ( BBTI,WM,N3,N3,N1,N3,N3,KM,N3,IER )

COMPUTATION OF V MATRIX

CALL VMULFF ( PIBT,KM,NUMT,N3,N1,NUMT,N3,VM,NUMT,IER )

THE CALCULATION OF THE STANDARD ERROR OF AN OBSERVATION WHICH
HAS UNIT WEIGHT

DO 132 I = 1,NUMT
  VTM ( 1,I ) = VM ( I )
CONTINUE
CALL VMULFF ( DM,VM,NUMT,NUMT,N1,NUMT,NUMT,VAR,NUMT,IER )
CALL VMULFF ( VTM,V,N1,NUMT,N1,N1,NUMT,STAND,N1,IER )

THE CALCULATION OF B*P.I MATRIX

CALL VMULFF ( BM,PMI,N3,NUMT,NUMT,N3,NUMT,BPIM,N3,IER )

THE CALCULATION OF (INVERSE B*P.I*B.T)*B*P.I MATRIX

CALL VMULFF ( BBTI,BPIM,N3,N3,NUMT,N3,N3,QBPIM,N3,IER )

THE CALCULATION OF P.I*B.T*(INVERSE B*P.I*B.T)*B*P.I MATRIX

```

```

C      CALL VMULFF ( PIBT,QBPIM,NUMT,N3,NUMT,NUMT,N3,DM,NUMT,IER )
C
C      PRINT DETAIL OF THE COMPUTATION
C
      IF ( PRINTD.EQ.0 ) GO TO 150
      WRITE ( 6,1000 )
      WRITE ( 6,1010 )
      DO 140 I = 1,3
        WRITE ( 6,1020 ) WM ( I )
      CONTINUE
      WRITE ( 6,1030 )
      CALL USWFM ( 'R-C',N3,PMI,NUMT,NUMT,NUMT,I05 )
      WRITE ( 6,1040 )
      CALL USWFM ( 'R-C',N3,BM,N3,N3,NUMT,I05 )
      WRITE ( 6,1000 )
      WRITE ( 6,1050 )
      CALL USWFM ( 'R-C',N3,BMT,NUMT,NUMT,N3,I05 )
      WRITE ( 6,1060 )
      CALL USWFM ( 'R-C',N3,PIBT,NUMT,NUMT,N3,I05 )
      WRITE ( 6,1070 )
      CALL USWFM ( 'R-C',N3,BBT,N3,N3,N3,I05 )
      WRITE ( 6,1000 )
      WRITE ( 6,1080 )
      CALL USWFM ( 'R-C',N3,BBTI,N3,N3,N3,I05 )
      WRITE ( 6,1090 )
      CALL USWFM ( 'P-C',N3,KM,N3,N3,N1,I05 )
      WRITE ( 6,1100 )
      CALL USWFM ( 'R-C',N3,VM,NUMT,NUMT,N1,I05 )
      WRITE ( 6,1000 )
      WRITE ( 6,1110 )
      CALL USWFM ( 'R-C',N3,BPIM,N3,N3,NUMT,I05 )
      WRITE ( 6,1120 )
      CALL USWFM ( 'R-C',N3,QBPIM,N3,N3,NUMT,I05 )
      WRITE ( 6,1000 )
      WRITE ( 6,1130 )
      CALL USWFM ( 'R-C',N3,DM,NUMT,NUMT,NUMT,I05 )
      WRITE ( 6,1140 )
      WRITE ( 6,1020 ) STAND
      CONTINUE
140
C
C      150
C      THE CALCULATION OF P.I - P.I*B.T*(INVERSE B*P.I*B.T)*B*P.I MATRIX
C
      STAND = DSQRT ( DABS ( STAND /.3.000 ) )
      DO 190 I = 1,NUMT
        DO 180 J = 1,NUMT

```

```

170 DM ( I,J ) = PMI ( I,J ) - DM ( I,J )
180 IF ( I.NE.J ) GO TO 170
190 VAR ( I ) = STAND*(DSQRT(DABS(DM(I,J))))
    CONTINUE
    CONTINUE
    CONTINUE
    IF ( PRINTD.EQ.0 ) GO TO 200
    WRITE ( 6,1000 )
    WRITE ( 6,1150 )
    CALL USWFM ( 'R-C',N3,DM,NUMT,NUMT,NUMT,IO5 )
200 CONTINUE
    WRITE ( 6,1000 )
    RETURN
C
C
1000 FORMAT ( '1' )
1010 FORMAT ( '13X,-W MATRIX',/ )
1020 FORMAT ( '12X,F15.10 )
1030 FORMAT ( '13X,INVERSION OF P MATRIX',/ )
1040 FORMAT ( '13X,INVERSION OF B MATRIX',/ )
1050 FORMAT ( '13X,TRANSPOSE OF B MATRIX',/ )
1060 FORMAT ( '13X,P.I*B.T MATRIX',/ )
1070 FORMAT ( '13X,B*P.I*B.T MATRIX',/ )
1080 FORMAT ( '13X,INVERSION OF B*P.I*B.T MATRIX',/ )
1090 FORMAT ( '13X,K MATRIX',/ )
1100 FORMAT ( '13X,V MATRIX',/ )
1110 FORMAT ( '13X,B*P.I MATRIX',/ )
1120 FORMAT ( '13X,(INVERSE B*P.I*B.T)*B*P.I MATRIX',/ )
1130 FORMAT ( '13X,P.I*B.T*(INVERSE B*P.I*B.T)*B*P.I MATRIX',/ )
1140 FORMAT ( '13X,V.I*P*V MATRIX',/ )
1150 FORMAT ( '13X,P.I - P.I*B.T*(INVERSE B*P.I*B.T)*B*P.I MATRIX',
    */ )
C
C
    END
C
C

```

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