AD-A153 633 FIXED WIDTH INTERVAL ESTIMATION IN LINEAR REGRESSION (U) STANFORD UNIV CA DEPT OF STATISTICS A Y KUK 07 MAR 85 TR-354 N00014-76-C-0475 1/1

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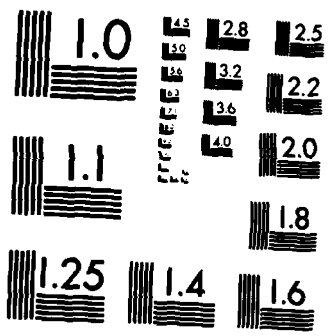

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AD-A153 633

FIXED WIDTH INTERVAL ESTIMATION IN LINEAR REGRESSION

BY

ANTHONY Y.C. KUK

TECHNICAL REPORT NO. 354
MARCH 7, 1985

Prepared Under Contract
N00014-76-C-0475 (NR-042-267)
For the Office of Naval Research

Herbert Solomon, Project Director

DEPARTMENT OF STATISTICS
STANFORD UNIVERSITY
STANFORD, CALIFORNIA

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1. INTRODUCTION

Stein (1945) describes a two-stage procedure to obtain a fixed-width confidence interval for the mean of a normal population when the variance is unknown. This is followed by works of Anscombe (1953) and Chow and Robbins (1965) who advocate sequential procedures. Hall (1981) suggests a three-stage sampling technique that combines the simplicity of Stein's procedure with the efficiency of the fully sequential method. For a linear model $Y_i = X_i\beta + \varepsilon_i$ where ε_i is $N(0,\sigma^2)$, the corresponding problem of obtaining a fixed-width confidence interval for one of the parameters is more difficult since the variance of the usual estimate depends not only on σ^2 but also on the X_i. To avoid this difficulty, Stein (1945) assumes that $X_1, \ldots, X_m$ are fixed and that they are repeated as a whole, as many times as is necessary. For example, $X_1, \ldots, X_m$ may correspond to an orthogonal design which we are replicating. Bishop (1978) continues to assume that the X_i are fixed.

In this paper, we consider simple linear regression $Y_i = \gamma + \beta X_i + \varepsilon_i$ where ε_i is $N(0,\sigma^2)$ and X_i is $N(\mu,\tau^2)$. In other words, we are sampling from a bivariate normal population. In section 2, we describe a two-stage procedure to obtain a fixed-width confidence interval for β and prove that the specified coverage probability is attained. Essentially, we estimate σ^2 and predict X_n, $n > m$ on the basis of a pilot sample $(X_1,Y_1), \ldots, (X_m,Y_m)$ to determine the size of the second sample. If we sample sequentially, then there is no need to predict X_n, $n > m$; such a procedure is described in section 3. We show that the corresponding confidence interval attains the specified coverage probability regardless of the distribution of the X_i. The procedure behaves like Stein's procedure for the estimation of a normal mean. By updating the estimate of σ^2 sequentially, we arrive at another procedure. Section 4 deals with the related problem of deriving a test procedure of $H : \beta = \beta_0$ at level α_0 which has power at least α_1 at $\beta = \beta_0 + \Delta$ independent of the values of the other parameters. One way to construct such a test makes use of fixed-width confidence intervals for β. A different approach which treats X_i and Y_i symmetrically is based on the distribution of the sample correlation coefficient. We show that the resulting test attains the specified level and power asymptotically.

2. A TWO-STAGE PROCEDURE

Suppose that σ^2 is known and the X_i are known constants, then $\hat{\beta}_n$ is $N(\beta, \sigma^2/\sum_1^n (X_i - \bar{X}_n)^2)$ where $\hat{\beta}_n = \sum_1^n (X_i - \bar{X}_n) Y_i / \sum_1^n (X_i - \bar{X}_n)^2$ is the least squares estimate of β based on $(X_1, Y_1), \ldots, (X_n, Y_n)$. It follows that $P(|\hat{\beta}_n - \beta| < d) \geq 1 - \alpha$ if

$$\sum_1^n (X_i - \bar{X}_n)^2 \geq Z_{1-\alpha/2}^2 \sigma^2/d^2 = S_0$$

where $Z_{1-\alpha/2}$ stands for the $(1-\alpha/2)$-percentile of the standard normal distribution. Since σ^2 is unknown and the X_i are stochastic, we need to estimate σ^2 and predict X_n, $n > m$ on the basis of the pilot sample $(X_1, Y_1), \ldots, (X_m, Y_m)$, $m \geq 3$. An obvious estimate of σ^2 is $\hat{\sigma}_m^2 = \sum_1^m (Y_i - \hat{\gamma}_m - \hat{\beta}_m X_i)^2/(m-2)$. To reduce the prediction problem, we note that we only need to predict $\sum_1^n (X_i - \bar{X}_n)^2$ for $n > m$. Since X_i is $N(\mu, \tau^2)$, we make the Helmert transformation to obtain $\sum_1^m (X_i - \bar{X}_m)^2 = \tau^2(U_2^2 + \ldots + U_m^2)$ and $\sum_1^n (X_i - \bar{X}_n)^2 = \tau^2(U_2^2 + \ldots + U_m^2 + \ldots + U_n^2)$ for $n > m$ where U_2, U_3, ... are independent standard normal variables. This allows us to make use of standard results of prediction for the gamma case. In particular, if $b_n = 1 + \chi_{1-c}^2(n-m)/\chi_g^2(m)$ where $\chi_{1-c}^2(n-m)$ and $\chi_g^2(m)$ are chi-square percentiles, then for each $n > m$, $(b_n \sum_1^m (X_i - \bar{X}_m)^2, \infty)$ is a (c,g) guaranteed coverage interval predictor of $\sum_1^n (X_i - \bar{X}_n)^2$ (Aitchison & Dunsmore 1975, Ch.6). Furthermore, we can guarantee coverage simultaneously so that with probability g, the pilot sample $X_1, \ldots, X_m$ is such that $P(\sum_1^n (X_i - \bar{X}_n)^2 > b_n \sum_1^m (X_i - \bar{X}_m)^2 | X_1, \ldots, X_m) \geq c$ for each $n > m$. We choose c, g so that $cg > 1 - \alpha$ and define α' by $1 - \alpha = cg(1 - \alpha')$. For conveience, we let $b_m = 1$. Consider the following two-stage sampling procedure.

Procedure 1. (i) Obtain a pilot sample $(X_1, Y_1), \ldots, (X_m, Y_m)$ and calculate $\hat{\gamma}_m$, $\hat{\beta}_m$ and $\hat{\sigma}_m^2$.

(ii) Draw a second sample of size $N_1 - m$ where N_1 is the smallest $n \geq m$ such that $b_n \sum_1^m (X_i - \bar{X}_m)^2 \geq t^2_{1-\alpha'/2}[m-2]\hat{\sigma}^2_m/d^2$ and $t_{1-\alpha'/2}[m-2]$ is the $(1-\alpha'/2)$-percentile of a t distribution with m-2 degrees of freedom.

The following theorem says that $(\hat{\beta}_{N_1} - d, \hat{\beta}_{N_1} + d)$ is a $(1-\alpha)$-level confidence interval for β.

Theorem 1. $P(|\hat{\beta}_{N_1} - \beta| < d) \geq 1 - \alpha$.

Before we prove theorem 1, we first state two lemmas.

Lemma 1. The conditional distribution of $\hat{\beta}_{N_1}$ *given* $\hat{\sigma}_m$ *and* $X_1, X_2, \ldots$ *is* $N(\beta, \sigma^2/S_1)$ *where* $S_1 = \sum_1^{N_1}(X_i - \bar{X}_{N_1})^2$.

Proof. Given $X_1, X_2, \ldots$, N_1 depends only on $\hat{\sigma}^2_m$ and $\hat{\beta}_{N_1}$ can be written as a linear combination of $\bar{Y}_m$, $\hat{\beta}_m$ and $Y_{m+1}, \ldots, Y_{N_1}$, all of which are independent of $\hat{\sigma}^2_m$.

Lemma 2. $P(\sum_1^{N_1}(X_i - \bar{X}_{N_1})^2 > b_{N_1}\sum_1^m (X_i - \bar{X}_m)^2 | \hat{\sigma}_m) \geq gc$.

Proof. Let $A = \{(x_1, \ldots, x_m) : \forall n > m,\ P(\sum_1^n (X_i - \bar{X}_m)^2 > b_n \sum_1^m (X_i - \bar{X}_m)^2 | X_1 = x_1, \ldots, X_m = x_m) \geq c\}$, then $P((X_1, \ldots, X_m) \in A) = g$ by our choice of b_n. Since $\hat{\sigma}_m$ is independent of the X_i, we also have $P((X_1, \ldots, X_m) \in A | \hat{\sigma}_m) = g$. If $(X_1, \ldots, X_m) = (x_1, \ldots, x_m) \in A$ and we write $n_1 = N_1(\hat{\sigma}_m, x_1, \ldots, x_m)$, then

$$\begin{aligned} &P(\sum_1^{N_1}(X_i - \bar{X}_{N_1})^2 > b_{N_1}\sum_1^m (X_i - \bar{X}_m)^2 | \hat{\sigma}_m, X_1 = x_1, \ldots, X_m = x_m) \\ &= P(\sum_1^{n_1}(X_i - \bar{X}_{N_1})^2 > b_{n_1}\sum_1^m (X_i - \bar{X}_m)^2 | \hat{\sigma}_m, X_1 = x_1, \ldots, X_m = x_m) \\ &= P(\sum_1^{n_1}(X_i - \bar{X}_{N_1})^2 > b_{n_1}\sum_1^m (X_i - \bar{X}_m)^2 | X_1 = x_1, \ldots, X_m = x_m) \\ &\geq c. \end{aligned}$$

Combining, we have the desired result.

Corollary 1. $P(\sum_1^{N_1}(X_i - \bar{X}_{N_1})^2 > t^2_{1-\alpha'/2}[m-2]\hat{\sigma}^2_m/d^2 | \hat{\sigma}_m) \geq gc$.

Proof. This follows from lemma 2 and the definition of N_1.

We now prove theorem 1.

$$P(|\hat\beta_{N_1}-\beta|<d|\hat\sigma_m)$$

$$= E_{X_1,X_2\ldots|\hat\sigma_m}\{P(|\hat\beta_{N_1}-\beta|<d|\hat\sigma_m,X_1,X_2,\ldots)\}$$

$$= E\{2\Phi(d\sqrt{S_1}/\sigma)-1|\hat\sigma_m\} \qquad \text{by lemma 1}$$

$$\geq gc\{2\Phi(t_{1-\alpha'/2}[m-2]\hat\sigma_m/\sigma)-1\} \qquad \text{by corollary 1.}$$

Thus $P(|\hat\beta_{N_1}-\beta|<d)\geq gc\ E\{2\Phi(t_{1-\alpha'/2}[m-2]\hat\sigma_m/\sigma)-1\}$

$$= gc(1-\alpha')$$

$$= 1-\alpha.$$

3. SEQUENTIAL PROCEDURES

If we sample sequentially, then prediction is no longer necessary.

Procedure 2. (i) Obtain a pilot sample of size m. (ii) Sample sequentially until $\sum_1^n(X_i-\bar X_n)^2\geq t^2_{1-\alpha/2}[m-2]\hat\sigma^2_m/d^2$.

Let N_2 be the sample size when we terminate sampling, our next theorem asserts that $(\hat\beta_{N_2}-d,\hat\beta_{N_2}+d)$ is a $(1-\alpha)$-level confidence interval for β.

Theorem 2. $P(|\hat\beta_{N_2}-\beta|<d)\geq 1-\alpha$.

We first state a lemma.

Lemma 3. The conditional distribution of $\hat\beta_{N_2}$ *given* $\hat\sigma_m$ *and* X_1, X_2, ... *is* $N(\beta,\sigma^2/S_2)$ where $S_2=\sum_1^{N_2}(X_i-\bar X_{N_2})^2$.

This is the analog of lemma 1 and can be proved using similar technique. We now prove theorem 2.

$$P(|\hat\beta_{N_2}-\beta|<d) = E\{P(|\hat\beta_{N_2}-\beta|<d|\hat\sigma_m,X_1,X_2,\ldots)\}$$

$$= E\{2\Phi(d\sqrt{S_2}/\sigma)-1\} \qquad \text{by lemma 3}$$

$$\geq E\{2\Phi(t_{1-\alpha/2}[m-2]\hat\sigma_m/\sigma)-1\}$$

$$= 1-\alpha.$$

We note that theorem 2 holds even when the X_i are not normally distributed.

Since the estimate of σ^2 is not updated as we sample sequentially, procedure 2 is inefficient. It behaves like Stein's procedure for the estimation of the mean of a normal population. In fact

$$\begin{aligned} E(S_2) &= E(\sum_1^{N_2}(X_i - \bar{X}_{N_2})^2) \\ &\geq E(t^2_{1-\alpha/2}[m-2]\hat{\sigma}^2_m/d^2) \\ &= S_0 t^2_{1-\alpha/2}[m-2]/Z^2_{1-\alpha/2} \end{aligned}$$

so that $E(S_2)/S_0 = t^2_{1-\alpha/2}[m-2]/Z^2_{1-\alpha/2} > 1$.

If the estimate of σ^2 is updated sequentially, we obtain the following procedure.

Procedure 3. (i) Obtain a pilot sample of size m. (ii) Sample sequentially until $\sum_1^n (X_i - \bar{X}_n)^2 \geq a_n^2 \hat{\sigma}_n^2/d^2$ where $\{a_n\}$ is a sequence of constants converging to $Z_{1-\alpha/2}$.

We expect procedure 3 to be the most efficient, but unlike procedures 2 and 3, the specified coverage probability is attained only asymptotically. Procedure 1 is least efficient since we have to deal with the additional problem of prediction, however, it has the advantage of requiring only two sampling operations.

4. A RELATED PROBLEM

A problem related to fixed-width interval estimation of β is that of deriving a test procedure of $H: \beta = \beta_0$ at level α_0 which has power at least α_1 at $\beta = \beta_0 + \Delta$, $\Delta > 0$. We can make use of our earlier results to solve this problem. For instance, we can use procedure 2 to obtain a $(1-\alpha)$-level confidence interval for β with width 2d, $d < \Delta$ and reject H if β_0 lies outside that interval. The resulting test has level α_0 and its power at $\beta = \beta_0 + \Delta$ is

$$\begin{aligned} P_{\beta_0+\Delta}(|\hat{\beta}_{N_2} - \beta_0| > d) &\geq P_{\beta_0+\Delta}(\hat{\beta}_{N_2} > \beta_0 + d) \\ &= E\{P_{\beta_0+\Delta}(\hat{\beta}_{N_2} > \beta_0 + d|\hat{\sigma}_m, X_1, \ldots)\} \end{aligned}$$

$$= E\{1 - \Phi((d - \Delta)\sqrt{S_2}/\sigma)\}$$

$$\geq E\{1 - \Phi((d - \Delta)t_{1-\alpha_0/2}[m-2]\hat{\sigma}_m/d\sigma)\}.$$

If we choose d such that $(\Delta - d)t_{1-\alpha_0/2}[m-2]/d = t_{\alpha_1}[m-2]$, then the power is at least α_1. As expected, if $d = \Delta$, then the power is at least $\frac{1}{2}$; as $d \to 0$, the power increases to 1.

The technique we employ so far is to condition on the X_i and then treat them as if they are fixed. An unconditional approach treating the X_i and Y_i symmetrically is described below. Without loss of generality, the hypothesis is $H : \beta = 0$. Assume that we are sampling from a bivariate normal population

$$\binom{X_i}{Y_i} \sim N_2\left(\binom{\mu}{\xi}\begin{pmatrix}\tau^2 & \rho\tau\nu \\ \rho\tau\nu & \nu^2\end{pmatrix}\right)$$

then H is equivalent to $\rho = 0$ and the usual t test rejects H if $|r|$ is too large where r is the sample correlation coefficient. Since the distribution of r depends on the parameters only through ρ, we can determine the sample size such that the level - α test of $\rho = 0$ has power α_1 at another ρ value. Bock (1977) makes use of Fisher Z-transformation to derive an approximate formula for the required sample size

$$Z_{1-\alpha_0/2} - (n - 3)^{\frac{1}{2}}\tanh^{-1}\rho = Z_{1-\alpha_1}.$$

Since $\rho = \theta/(1 + \theta^2)^{\frac{1}{2}}$ where $\theta = \beta\tau/\sigma$, the following procedure suggests itself.

Procedure 4. (i) Obtain a pilot sample of size m. (ii) Sample sequentially until $Z_{1-\alpha_0/2} - (n - 3)^{\frac{1}{2}}\tanh^{-1}\hat{\rho}_n(\Delta) \leq Z_{1-\alpha_1}$ where $\hat{\rho}_n(\Delta) = \hat{\theta}_n(\Delta)/(1 + \hat{\theta}_n(\Delta))^{\frac{1}{2}}$, $\hat{\theta}_n(\Delta) = \Delta\hat{\tau}_n/\hat{\sigma}_n$ and $\hat{\tau}_n^2 = \sum_1^n(X_i - \bar{X}_n)^2/(n - 1)$. (iii) Perform a two-sided t test treating the final sample size $N(\Delta)$ as if it is fixed. Thus if $T_n = \hat{\beta}_n(\sum_1^n(X_i - \bar{X}_n)^2)^{\frac{1}{2}}/\hat{\sigma}_n$, we reject H if $|T_{N(\Delta)}| > t_{1-\alpha_0/2}[N(\Delta) - 2]$.

The following theorem asserts that the test procedure attains the specified level and power asymptotically.

Theorem 3. $\lim_{\Delta\to 0} P_{\beta=0}(|T_{N(\Delta)}| < t_{1-\alpha_0/2}[N(\Delta) - 2]) = 1 - \alpha_0$,

$$\lim_{\Delta\to 0} P_{\beta=\Delta}(|T_{N(\Delta)}| > t_{1-\alpha_0/2}[N(\Delta)-2]) \geq \alpha_1.$$

Proof. (i) Since $r_n = T_n/(n-2+T_n^2)^{\frac{1}{2}}$ where r_n is the sample correlation coefficient computed from $(X_1,Y_1), \ldots, (X_n,Y_n)$,

$$1-\alpha_0 = P_{\beta=0}(|T_n| < t_{1-\alpha_0/2}[n-2])$$

$$= P_{\beta=0}(|(n-3)^{\frac{1}{2}}\tanh^{-1} r_n| < C_n)$$

where $C_n = (n-3)^{\frac{1}{2}}\tanh^{-1}(t_{1-\alpha_0/2}[n-2]/(n-2+t^2_{1-\alpha_0/2}[n-2])^{\frac{1}{2}})$. On the other hand, when $\beta = 0$

$$(n-3)^{\frac{1}{2}}\tanh^{-1} r_n \to_{\mathcal{D}} N(0,1) \text{ as } n \to \infty,$$

so we must have $\lim_{n\to\infty} C_n = Z_{1-\alpha_0/2}$. Since $N(\Delta) \to \infty$ a.s. as $\Delta \to 0$, $\lim_{\Delta\to 0} C_{N(\Delta)} = Z_{1-\alpha_0/2}$ a.s. and it follows from a theorem of Anscombe (1952) that when $\beta = 0$

$$(N(\Delta)-3)^{\frac{1}{2}}\tanh^{-1} r_{N(\Delta)} \to_{\mathcal{D}} N(0,1).$$

Thus $\lim_{\Delta\to 0} P_{\beta=0}(|T_{N(\Delta)}| < t_{1-\alpha_0/2}[N(\Delta)-2])$

$$= \lim_{\Delta\to 0} P_{\beta=0}(|(N(\Delta)-3)^{\frac{1}{2}}\tanh^{-1} r_{N(\Delta)}| < C_{N(\Delta)})$$

$$= 1-\alpha_0.$$

(ii) Assume for the time being that under $\beta = \Delta$

$$(N(\Delta)-3)^{\frac{1}{2}}\tanh^{-1} r_{N(\Delta)} \to_{\mathcal{D}} N(Z_{1-\alpha_0/2} - Z_{1-\alpha_1}, 1) \text{ as } \Delta \to 0, \quad (1)$$

then $\lim_{\Delta\to 0} P_{\beta=\Delta}(|T_{N(\Delta)}| > t_{1-\alpha_0/2}[N(\Delta)-2])$

$$\geq \lim_{\Delta\to 0} P_{\beta=\Delta}((N(\Delta)-3)^{\frac{1}{2}}\tanh^{-1} r_{N(\Delta)} > C_{N(\Delta)})$$

$$= \alpha_1.$$

To prove (1), we fix γ, σ, μ, τ and define $n(\Delta)-3$ to be the least integer greater than or equal to $(Z_{1-\alpha_0/2} - Z_{1-\alpha_1})^2/(\tanh^{-1}\rho(\Delta))^2$ where $\rho(\Delta) = \theta(\Delta)/(1+\theta(\Delta))^{\frac{1}{2}}$ and $\theta(\Delta) = \Delta\tau/\sigma$. Under $\beta = \Delta$

$$(n(\Delta)-3)^{\frac{1}{2}}(\tanh^{-1} r_{n(\Delta)} - \tanh^{-1}\rho(\Delta)) \to_{\mathcal{D}} N(0,1) \quad \text{as} \quad \Delta \to 0,$$

equivalently

$$(n(\Delta)-3)^{\frac{1}{2}}\tanh^{-1} r_{n(\vec{\Delta})} \xrightarrow{\mathcal{D}} N(Z_{1-\alpha_0/2} - Z_{1-\alpha_1}, 1) \quad \text{as} \quad \Delta \to 0 \qquad (2)$$

from which (1) follows if we can replace $n(\Delta)$ by $N(\Delta)$. To that end, we note that if X_i is $N(\mu,\tau^2)$ and Y_i is $N(\gamma,\sigma^2)$ independently of X_i, then the conditional distribution of $Y_i + \beta X_i$ given X_i is $N(\gamma + \beta X_i, \sigma^2)$. The advantage of this representation is that it enables us to deal with a single array of random variables rather than a double array. In particular, we can show $N(\Delta)/n(\Delta) \to 1$ a.s. as $\Delta \to 0$. A generalization of Anscombe's theorem enables us to replace $n(\Delta)$ by $N(\Delta)$ in (2), we omit the details.

ACKNOWLEDGEMENT

This research is supported in part by Office of Naval Research Contract N00014-76-C-0475.

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REPORT DOCUMENTATION PAGE

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1. REPORT NUMBER: 354	2. GOVT ACCESSION NO.: AD-A153 633
3. RECIPIENT'S CATALOG NUMBER	
4. TITLE (and Subtitle): Fixed Width Interval Estimation In Linear Regression	5. TYPE OF REPORT & PERIOD COVERED: TECHNICAL REPORT
	6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s): Anthony Y.C. Kuk	8. CONTRACT OR GRANT NUMBER(s): N00014-76-C-0475
9. PERFORMING ORGANIZATION NAME AND ADDRESS: Department of Statistics, Stanford University, Stanford, CA 94305	10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS: NR-042-267
11. CONTROLLING OFFICE NAME AND ADDRESS: Office of Naval Research, Statistics & Probability Program Code 411SP	12. REPORT DATE: March 7, 1985
	13. NUMBER OF PAGES: 11
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)	15. SECURITY CLASS. (of this report): UNCLASSIFIED
	15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

APPROVED FOR PUBLIC RELEASE: DISTRIBUTION UNLIMITED

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Asymptotic power; bivariate normal distribution; correlation coefficient; efficiency; guaranteed coverage prediction; sequential methods; two-stage sampling scheme.

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

We discuss fixed-width interval estimation for the slope parameter β in a simple linear regression $Y_i = \gamma + \beta X_i + \varepsilon_i$ when the X_i are also normally distributed. A two-stage procedure that combines prediction with estimation is described. In addition, we discuss two sequential procedures. The confidence intervals obtained are used to construct tests of H: $\beta = \beta_o$ with level α_0 and power at least α_1 at $\beta = \beta_o + \Delta$ independent of the values of the other parameters. We also consider a sequential procedure based on the distribution of the sample correlation coefficient; the resulting test attains the specified level and power asymptotically.

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