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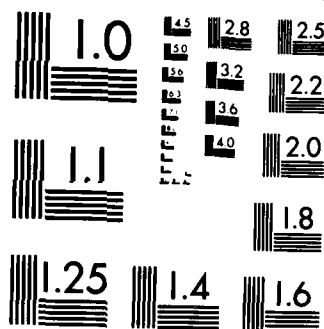
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Stephen R. Addison and Henry E. Bass

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Preface

This report is a reproduction of the Ph.D. dissertation prepared by Stephen R. Addison under the direction of Henry E. Bass. Certain pages in the dissertation required by the University but not contributing to the technical content here have been omitted. The dissertation was accepted by the faculty of the Department of Physics and Astronomy on 5 December 1984.

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STUDY OF SOUND ATTENUATION IN SEDIMENTS

BY

STEPHEN R. ADDISON

B.Sc., University College at Cardiff, UK, 1978
M.S., University of Mississippi, 1982

A Dissertation
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I. INTRODUCTION

The purpose of the research described in this dissertation is to contribute to the understanding of sound attenuation in ocean sediments. Although the measurements were performed in the laboratory on laboratory (i.e. artificial) sediments, they will aid in the interpretation of in-situ experiments as they were performed under controlled conditions. The specific goals of the experiments presented here were to increase understanding of the theories of sound attenuation in sediments, and to expand the data base of measurements made at low frequencies.

Sound waves are attenuated far less than light, or radio waves in the ocean,¹⁻² and have been used extensively for communication, navigation, and remote sensing. In deep water, the transmission loss for sound waves is well understood, as the ocean bottom plays no part in the propagation of acoustic signals. In shallow water, the transmission loss is dominated by sound absorption in the ocean floor. This transmission loss is the key to understanding the propagation of sound waves in shallow water.

When the frequency of the propagating waves is high, measurement of the attenuation of the waves is not difficult, in the ocean, or in the laboratory. However, at low frequencies, the long wavelength of compressional waves causes experimental difficulties. In the ocean, the sediment is inaccessible, the propagation path is inhomogeneous, and information on layering usually has to be derived from the same experiments as the attenuation data. In the laboratory, a large scale

apparatus is required, and care has to be exercised in sample preparation. The propagation velocity of shear waves is considerably smaller than the propagation velocity of compressional waves, and as a consequence, laboratory and in-situ shear-wave experiments are easier to perform than compressional wave experiments. This dissertation is mainly concerned with the propagation of compressional waves. Some investigators³ feel that shear wave measurements are sufficient to understand sound propagation in sediments, but the relationships between the propagation parameters of shear and compressional waves are based upon models which have not been extensively verified.

This dissertation describes a method for measuring the attenuation of compressional waves, at low frequencies, in laboratory sediments. The measurements presented here were performed using the tube method; this method has been widely used to determine sound attenuation in other media. The tube method is not without disadvantages, the effects of the tube wall on the propagating wave have to be taken into account, but the method does eliminate spreading losses and it is relatively easy to generate plane waves in a cylindrical tube.

Chapter II of this dissertation presents a review of the two most widely used models of sound attenuation in marine sediments. The simplest is an empirical model proposed by Hamilton,⁴⁻¹¹ the other model, to which most of the chapter is devoted, is a phenomenological model which was proposed by Biot¹²⁻¹⁸ as a general model of sound propagation in poro-elastic media, and which was adapted by Stoll^{3,19-21} for use on sediments.

In chapter III the physical properties of the sediments used in the experimental measurements are described. The calculations of thermal conductivity and specific heat for sediments, which are necessary for the calculation of the attenuation of sound at the tube walls, are also described.

Chapter IV is devoted to a description of the experimental system used to make the sound attenuation measurements.

Chapter V contains the analysis of the experimental data. The results of the experiments are compared with the predictions of the Hamilton and Biot models of sound attenuation in sediments which are described in chapter II. The final chapter summarizes conclusions.

The functions ber and bei, which derive their names from Bessel real and Bessel imaginary, are zero order Kelvin functions; they are related to Bessel functions through⁵⁶

$$\text{ber}(W) + i \text{bei}(W) = I_0(i^{1/2}W) = J_0(i^{3/2}W), \quad (23)$$

where W is real.

When the argument, W , of the viscosity function is much less than one, that is at low frequencies, the fluid flow in the pores is Poiseuille flow and the viscosity function may be set equal to one. The viscosity function may be neglected in the low-frequency regime of the Biot model. The changeover from low-frequency behavior to high-frequency behavior occurs when the viscous skin depth is approximately equal to the pore size a_p , that is to say Poiseuille flow breaks down when

$$\omega_c = 2v/d_f a_p^2. \quad (24)$$

This criterion determines if the viscosity correction factor is required.

(b) Permeability. The permeability k of a porous medium is related to the size and shape of the pores in the medium. It may be measured directly, calculated from measurements of the flow resistance, or estimated with one of several commonly used empirical equations. When experiments are carried out in the ocean, the Krumbein-Monk⁵⁷ or Hazen⁵⁸ relations are often used; these equations estimate the permeability of a sediment from its mean grain size. Care should be exercised in the use of these equations as they are only valid in regions of the ocean which

the oscillatory flow of the viscous fluid through the pores of the medium. The effects of oscillatory flow were first studied for a closed channel by Kirchhoff⁵³ in 1868, oscillatory viscosity having first been investigated by Stokes⁵⁴ in connection with the oscillation of pendulums a few years previously.

For porous media, Biot assumed that at low frequency the flow of the fluid is Poiseuille flow which obeys Darcy's law.^{12,55} As the frequency of oscillation is increased, inertial forces in the flow field become comparable with the viscous forces and Darcy's law no longer adequately describes the flow; eventually the flow becomes turbulent. Biot calculated a complex viscosity correction factor whose product with the fluid viscosity ν accounts for the deviation from Poiseuille flow.¹³

The complex viscosity correction factor was defined by Biot to be:

$$F(W) = (1/4)WT(W)/(1-2T(W)/iW) \quad (20)$$

where,

$$T(W) = [\text{ber}'(W) + i \text{bei}'(W)] / [\text{ber}(W) + i \text{bei}(W)] \quad (21)$$

and,

$$W = a_p (\omega d_f / \nu)^{1/2}. \quad (22)$$

The pore size parameter a_p is usually taken to be one-sixth to one-seventh of the mean size of the individual particles when the Biot model is applied to sediments.

0.25. The values of A and n for a wide variety of materials, may be found in the literature (see for example Ref. 50).

(c) Tortuosity in real and laboratory sediments. As has already been stated, tortuosity has been the most widely investigated parameter in the Biot model; in addition to its relationships with formation factor and added mass, tortuosity has also been related to the index of refraction of fourth sound in a consolidated (fused) matrix saturated with liquid Helium.⁵¹ Experimentally, the tortuosities measured by the electrical resistivity method and tortuosities measured by the acoustic index of refraction of superfluid ⁴He have been found to be in agreement with each other.⁵²

For real and artificial sediments, the most convenient quantity to measure is the low-frequency resistivity formation factor. The appropriate measurement techniques have been described by Wyllie⁴⁸ and Sweet.⁴⁵ If measurements of this quantity are not practical, the constants required for use of the Archie equation⁴¹ are tabulated for a wide variety of materials.⁵⁰ For closely sized spheres, the Slawinski equation⁴⁹ can be used to compute the formation factor with confidence. Use of the Maxwell expression should be avoided as it underestimates the formation factor for spheres in contact.⁴²⁻⁴⁴

3. Viscosity and permeability

(a) Viscosity. If the fluid independent frame moduli frame are real quantities, the only dissipation mechanism contained in the Biot model is

$$F=(3-f)/2f. \quad (17)$$

Measurements performed by Boyce⁴⁴ in the Bering sea of electrical conductivity showed an electrical conductivity 17% below that predicted by the Maxwell expression; this experience is not atypical.⁴² Boyce derived some empirical equations which fitted his data better than the Maxwell expression, as have many other investigators.^{42,45} The expressions in most common use are Archie's Law⁴¹ and developments of it, such as the Humble formula⁴⁵ used by Schlumberger in well logging.

It is not surprising that the resistivity formation factor is always underestimated by the Maxwell expression. A comparison of it with Berryman's calculation⁴⁶ of the Hashin-Shtrikman⁴⁷ lower bound on electrical conductivity shows that the Maxwell expression is identical with this lower bound on F.

Archie's law has the form,

$$F=Af^n, \quad (18)$$

where A and n are empirical constants. For an aggregate of unconsolidated spheres, Wyllie and Gregory⁴⁸ found that the formation factor could be accurately calculated by setting A = 1, and n = -1.3, for porosities between 0.1 and 0.56. They further found that the Slawinski equation,⁴⁹

$$F=(1.3219-0.3219f)^2/f, \quad (19)$$

fitted all available data for sphere packs with a porosity greater than

different values of the coefficient, but added mass is difficult to calculate for a sediment matrix because it is, in general, a tensor quantity.³⁶⁻³⁸ The use of resistivity formation factors seems to be the most promising approach for ocean acoustics of sediments.

(b) Resistivity formation factor. It has been shown by Brown⁴⁰ that the formation factor for electrical resistivity and tortuosity are related by

$$X = Ff, \quad (15)$$

where the resistivity formation factor for a porous medium was defined by Archie⁴¹ as the ratio of the resistivity of the porous sample saturated with a conducting fluid, R_o , to the resistivity of the conducting fluid in bulk R_w ;

$$F = R_o / R_w. \quad (16)$$

This equation holds only if the matrix material is an insulator and the saturating fluid is a conductor. The resistivity formation factor measures the influence of the pore structure on the conductivity of the medium, by definition it is always greater than one.

Little work has been done on the electrical conductivity of ocean floor sediments.⁴² It was long expected that the Maxwell⁴³ expression for the conductivity of an assembly of spheres embedded in a conductor could be adapted. The Maxwell expression leads to the following equation for the resistivity formation factor:

acceleration) of the paddle is greatly increased by the presence of the water. This increased inertia is known as the virtual mass of the paddle; the difference between the real and virtual mass of the paddle is the added, or induced mass. Thus, the added mass of a body immersed in a fluid is the difference between the inertia the body possesses while it is in the fluid, and the inertia it possesses in a vacuum. (The preceding illustration is due to Birkhoff.³⁶)

It is customary to define a coefficient of added mass by the ratio,

$$k = (\text{added mass of the body})/(\text{mass of fluid displaced}). \quad (13)$$

The added mass coefficient is a function only of the shape of the body; it is independent of the size of the body and its acceleration, and independent of the density of the fluid in which the body is immersed. A sphere oscillating in a fluid has an added mass coefficient equal to one-half. Chapter VI of Lamb's treatise on hydrodynamics³⁸ is devoted to the calculation of the added mass of nonspherical bodies.

For a porous medium, Berryman³⁹ has shown that the added mass of the frame is related to the tortuosity by

$$X = 1 + k(f^{-1} - 1), \quad (14)$$

where the induced mass per unit volume of the medium is equal to kd_f . For an aggregate of spheres, the added mass coefficient for an isolated sphere (1/2) is often used. This yields a value for the tortuosity which is too high, as the individual spheres are not as free to move as a single sphere. Some authors³³ have tried to compensate for this by using

compressibility of the gas, and to viscous drag at the pore walls, which, as discussed below, is the only attenuation mechanism originally included in the Biot model.

2. The fluid-solid coupling factor or the tortuosity

The density parameter which appears in Biot's coupled differential equations

$$d_c = X d_f / f, \quad (6)$$

where X is the fluid-solid coupling factor, or the electrical tortuosity of the frame, is the most investigated - and best understood - new parameter introduced into the acoustics of porous media by the Biot model. X is often referred to as the tortuosity of the frame, but it should not be confused with the hydraulic tortuosity which is used in the Kozeny-Carman³⁵ permeability model. The hydraulic and electric tortuosities are related to each other, but they are not equal. In this dissertation, when tortuosity is referred to, electric tortuosity is always implied. Tortuosity has been found to be related to the added mass of a body oscillating in a fluid, and to the low-frequency resistivity formation factor of the matrix.

(a) Added (or induced) mass. Added mass has been discussed extensively by Birkhoff³⁶ and by Landau and Lifshitz.³⁷ The effects of added mass are often experienced in the macroscopic world. For example, when paddling a canoe, the apparent inertia (i.e. resistance to

K_b is the bulk modulus of the free draining frame, and G is the shear modulus of the free draining frame. In specifying a free draining frame, we mean that the measurements of these moduli should be made in a system which allows the pore fluid to escape freely as the sediment is compressed.²⁰ In practice, this means that these moduli are usually defined in terms of a different composite; they are defined for a matrix from which the pore fluid has been removed. The moduli K_b and G are independent of the pore fluid that the matrix contains.

The dynamic parts of the moduli, K_b and G , are then defined in terms of the shear and compressional wave velocities through the drained matrix³²:

$$v_{sh} = \{G/[(1-f)d_s]\}^{1/2} \quad (11)$$

$$v_c = \{[K_b + (4/3)G]/[(1-f)d_s]\}^{1/2}. \quad (12)$$

For a liquid K_b and G are in general complex quantities. The real parts of K_b and G are defined through Eqs. (11) and (12); their imaginary parts may be defined through measurements of the logarithmic decrement of a drained frame,²¹ or from empirical relationships.³³⁻³⁴ K_b and G are allowed to be complex for a liquid saturated porous material in order to account for the various dissipation mechanisms which occur at grain boundaries (i.e. friction, relaxation etc.). If the pore fluid is a gas, then the frame moduli are assumed to be real quantities, and K_b and G are completely defined by Eqs. (11) and (12); for a gas saturated porous medium all the attenuation is assumed to be due to the complex

1. The Elastic Constants

H, C, and M are elastic constants, which are in general complex. The imaginary parts of H, C, and M are relevant to sound attenuation in the medium. H, C, and M may be defined in terms of more familiar moduli,^{20,28-30} some real and some complex:

$$H = (K_r - K_b)^2 / (D - K_b) + K_b + 4G/3 \quad (7)$$

$$C = K_r (K_r - K_b) / (D - K_b) \quad (8)$$

$$M = K_r^2 / (D - K_b) \quad (9)$$

where,

$$D = K_r [1 + f(K_r/K_f - 1)] \quad (10)$$

The most lucid description of the "Gedanken" experiments through which these quantities are defined is given by Stoll in Ref. 20. It will be sufficient for our purposes to define the terms appearing in Eqs. (7)-(10), and to explore their physical content.

K_r is the bulk modulus of the individual particles which make up the sediment matrix, this quantity is always a real quantity. It is assumed that this bulk modulus is for isothermal conditions. K_f is the bulk modulus of the pore fluid. The bulk modulus of the pore fluid is assumed to be a real quantity if the pore fluid is a liquid; if the pore fluid is a gas, its bulk modulus is sometimes regarded as a complex quantity³¹ to account for sound attenuation.

A change of variables is normally made when the Biot model is applied to marine sediments, so that the equations appear in terms of the dilatation of the frame e , and the relative dilatation between the frame and the fluid r , rather than in terms of the dilatation in the fluid E and the dilatation in the solid e ,

$$r = f \operatorname{div}(\underline{u} - \underline{U}) = f(e - E). \quad (3)$$

It is this set of transformed equations which will be considered in the following:

The coupled equations describing the propagation of compressional waves are:

$$\nabla^2(HE - Cr) = D_t^2(de - d_f r), \quad (4)$$

and

$$\nabla^2(Ce - Mr) = D_t^2(d_f e - d_c r) - (v/k)F(W)D_t r, \quad (5)$$

where

$$d_c = X d_f / f. \quad (6)$$

D_i^k denotes the k th partial derivative with respect to the variable i .

The differential equation for shear waves will not be presented here, but the dispersion equation for shear waves is given in section II.B.4. The various terms appearing in these equations will be defined, and discussed in some detail below.

to be homogeneous and isotropic. The size of the pores is required to be small compared to the wavelength of any disturbance. If this condition is not met scattering occurs and the model is not applicable.

The equations of motion for a poro-elastic medium are derived from the stress-strain equations. By requiring the theory to be linear, the potential energy can be expressed as a quadratic function of the strain components, and the kinetic energy can be expressed as a quadratic function of the fluid and solid velocities. The average stress acting on an arbitrary volume element is defined to be the sum of the forces acting on the fluid and solid components divided by the area of the volume element. Strain is defined in terms of the displacements of the fluid $\underline{U}$, and the solid $\underline{u}$. A Lagrangian analysis is then performed resulting in a pair of simultaneous coupled differential equations in terms of the fluid and solid displacements. These equations are then separated into a pair of simultaneous coupled differential equations containing only dilatations, this pair describes the propagation of compressional waves; and a pair of simultaneous coupled differential equations containing only rotations, describing the propagation of shear waves.

In these equations, the aggregate density, d , of the fluid-saturated porous medium is related to the density of the fluid, d_f , and the density of the grains, d_s , by the equation,

$$d = (1-f)d_s + fd_f, \quad (2)$$

where f is the porosity of the matrix.

applicable as the Biot model; the Biot model will reduce to the other models under the restrictive conditions to which they apply. Biot has, in subsequent years, further generalized the model, he has also simplified the derivations of the fundamental equations.¹⁵⁻¹⁸ The model was not applied to sediments until 1969, when it was done by Stoll¹⁹; the version of the theory presented in this dissertation is normally known as the Biot-Stoll model, although most of the necessary developments may in fact be found in the earlier work of Biot.²⁸

The principal prediction of the Biot model is that a fluid saturated porous medium will support three body waves. Two compressional waves, and one shear wave are predicted. The shear wave is essentially similar to shear waves which propagate in linear elastic solids; however it is modified by the presence of the fluid. The compressional waves are usually known as the fast wave (corresponding to in-phase motion of the fluid and the frame), and the slow wave (corresponding to out-of-phase motion of the fluid and the frame). These waves are also known as waves of types I and II, respectively. The fast wave is again essentially the compressional wave of a linear elastic solid, the slow wave is a new wave which is characteristic of wave propagation in fluid saturated porous media. The properties of these waves will be further discussed in section II.B.4, where the dispersion equations of three body waves are presented.

Biot treated a fluid-saturated porous medium as a solid matrix filled with arbitrarily shaped pores. The distribution of the pores is assumed

the frequency of the sound wave measured in kilohertz, and n is the exponent of frequency taken to be equal to one in Hamilton's model.

The attenuation of sound in a particular sediment may then be predicted, if the sediment type is known, that is to say that the density, porosity, or particle size of the sediment is known. Values of the proportionality constant, K , are tabulated and presented graphically as functions of the density, porosity, or mean particle size in many of Hamilton's papers⁶⁻⁸ for a wide variety of sediments.

The Hamilton model assumes that there is no significant velocity dispersion for compressional waves in sediments from frequencies of a few hertz to frequencies of several megahertz. Recent data casts doubt on the validity of this assumption,¹⁰⁻¹¹ and as this assumption is the basis of the Hamilton model, another model is clearly required.

B. The Biot Model

The Biot model, which is now the most widely accepted model for sound propagation in fluid-saturated porous media, was originally proposed by Biot in 1956, in two papers.¹²⁻¹³ The first paper dealt with wave propagation at low frequencies, and the second dealt with wave propagation at high frequencies. The terms low and high frequency as applied to the Biot model will be defined below in section II.B.3.a. The development of the model used some results from, and the notation of, Biot's earlier work on soil consolidation problems.²²

Similar theories had been developed in earlier years by other authors,²³⁻²⁷ but none of these other theories was as generally

II. THEORETICAL MODELS

In this chapter two widely used models of sound propagation in sediments are reviewed. The models are known as the Hamilton and Biot models, although the latter is sometimes called the Biot-Stoll model. The Biot model is presented in somewhat greater detail, both because of its greater complexity, and because the development of this model took place in a long series of papers, and no adequate review of the model exists. Although the primary concern of this dissertation is the attenuation of compressional (sound) waves, some attention will be given to the attenuation of shear waves.

Throughout this dissertation attenuations denoted by the symbol a are measured in dB/m, while those denoted by the symbol α are measured in nepers/m.

A. The Hamilton Model

Hamilton has been investigating the acoustic properties of marine sediments since the early 1950s.⁵⁻⁶ The Hamilton model predicts that the attenuation of compressional waves in sediments is proportional to the first power of frequency. That is in the equation,

$$a = Kf^n, \quad (1)$$

a , is the attenuation of compressional waves measured in dB/m, K is a proportionality constant dependent on the sediment classification, f is

have sediment types similar to the area in which the measurements were made from which the relations were extracted. They are not general and if measurements are possible they are to be preferred. In laboratory experiments using closely-sized spheres the Kozeny-Carman relation may be used with confidence.³⁵

4. Solution of Biot's equations

For an infinite medium, the dispersion equation for compressional waves in a porous medium may be readily found by assuming that Biot's equations admit plane wave solutions and substituting these solutions into the coupled differential equations. For wave propagation in bounded media, appropriate boundary conditions have been derived by Deresiewicz and Skalak.⁵⁹ Deresiewicz has investigated the properties of several different types of boundaries in a long series of papers.⁶⁰⁻⁶⁹

Biot's equations have been solved in cylindrical geometry by Gardner⁷⁰ and Berryman.⁷¹ Gardner's solution is valid only for very low frequencies while Berryman's solution is valid for the same range of frequencies for which the Biot model is valid. Both of these investigations dealt with the propagation of extensional waves in consolidated porous media; it is not immediately obvious that they are directly relevant to the experiments performed in this investigation as plane waves are generated. It is expected that if the cylindrical boundaries do have any effect on the attenuation that it could, in the first approximation, be accounted for by the classical solution of Pochhammer and Chree⁷³⁻⁷⁶ for wave propagation in elastic rods.

To obtain the dispersion equation for compressional waves, we assume plane wave solutions of the form:

$$e = A_1 \exp[i(\omega t - lx)] \quad (25)$$

and,

$$r = A_2 \exp[i(\omega t - lx)]. \quad (26)$$

Substitution of these solutions into Biot's equations leads to the determinantal frequency condition,

$$\begin{vmatrix} Hl^2 - d\omega^2 & d_f \omega^2 - Cl^2 \\ Cl^2 - d_f \omega^2 & d_c \omega^2 - Ml^2 - i\omega F(W)v/k \end{vmatrix} = 0. \quad (27)$$

The roots of this equation give the attenuation l_{Im} , and the phase velocity ω/l_{Re} as functions of frequency for compressional waves of the first and second kinds. A similar procedure is used to obtain the dispersion equation for shear waves.⁷⁶

The determinant is usually solved numerically, however the complex wavenumbers for shear and compressional waves will be given here for completeness

$$l_f^2 = \frac{\omega^2 \{ (Md - 2Cd_f + Hq) - [(Md - 2Cd_f + Hq) - 4(C^2 - HM)(d_f^2 - dq)]^{1/2} \}}{2(HM - C^2)} \quad (28)$$

$$l_s^2 = \frac{\omega^2 \{ (Md - 2Cd_f + Hq) + [(Md - 2Cd_f + Hq) - 4(C^2 - HM)(d_f^2 - dq)]^{1/2} \}}{2(HM - C^2)} \quad (29)$$

For shear waves,

$$1_{sh}^2 = \omega^2 (d - d_f^2/q)/G, \quad (30)$$

where,

$$q = d_c - iF(W)v/k. \quad (31)$$

In these equations f, s, sh on left hand sides denotes fast compressional, slow compressional, and shear waves respectively.

All three of the body waves are attenuated and dispersive. At low frequencies the slow compressional wave is not a propagational wave. The relative effects of the various parameters contained in these equations on wave velocity and attenuation have been widely¹²⁻²¹ reported and will not be repeated here.

5. Conclusions

It should not be too surprising that there are three propagational body waves in porous media as Biot treated the fluid and solid constituents of the medium on an equal footing. A fluid will support a compressional wave, and the matrix material will support a compressional wave and a shear wave. All three waves have been detected experimentally, although the compressional wave of the second kind was elusive, and was only detected in recent years by Plona.⁷⁷⁻⁸⁰

Biot's equations have been successfully applied to a variety of physical systems, including the propagation of fourth sound in a

superfluid/superleak system⁵¹, pressure diffusion through porous media,⁸¹⁻⁸², the elastodynamics of gels⁸³, and slow waves and the consolidation transition.⁸⁴

The main difficulty in applying the model to sediments is in the measurement or calculation of the fluid independent frame moduli K_b and G . For a consolidated matrix, Berryman⁸⁵⁻⁸⁶ has developed self-consistent schemes for their calculation, but these are unsuitable for the unconsolidated or partially consolidated matrix typical of surficial ocean sediments. The most commonly used method is to assume a value for Poisson's ratio for the sediment and to vary K_b and G until the Biot model produces the measured phase velocities. Future theoretical work will have to concentrate on, either the calculation of H, C , and M directly, or a satisfactory method of calculating the frame moduli for unconsolidated media.

III. The Sediment

A. Real and Laboratory Sediments

This investigation reports measurements performed on laboratory (artificial) sediments. The selection of an artificial sediment was not an easy one, as one of the long-range aims of this investigation is to provide a better understanding of the relationship between theoretical models of attenuation and in-situ attenuation measurements. However, the obvious short-term advantages of using a real sediment were outweighed by the requirement of a homogeneous, isotropic sample. The Biot model describes particles and pore spaces which are uniformly sized, although it has been applied to sediments containing a wide range of sizes. For this reason, a well characterized sample of closely sized spherical glass beads was chosen for the initial measurements. Real and laboratory sediments differ in some important ways, we shall now consider some of these differences.

In an in-situ sediment, the density, porosity,⁸⁷⁻⁸⁸ and bulk and shear moduli⁸⁹ of the sediment are all functions of depth within the sediment, and of the overburden pressure. The velocity and attenuation of shear and compressional waves are also functions of depth. There is a lot of data available on velocity gradients in sediments,^{7,88-89} but little available on the variation of attenuation with depth.^{10,88} If the porosity of a sediment is not a function of depth, the sediment is described as a non-compactive sediment.⁹⁰ Most in-situ sediments are

compactive, but an artificial sediment may be constructed so as to be non-compactive.

In this investigation it would clearly be advantageous to construct a non-compactive sediment. In a non-compactive sediment the density and frame bulk modulus of the sediment are not functions of depth; the relationship between aggregate density and porosity was given in the last chapter; the bulk modulus of the frame is also a function of porosity in the first approximation.⁹⁰ The shear modulus of the frame is still a function of depth in a non-compactive sediment; this is because the deeper a particle is within the body of the sediment, the more its movement is restricted by the weight of particles above it.⁹⁰ Thus, given the functional dependence of the complex wave numbers of shear and compressional waves on the shear modulus, the velocity and attenuation of shear and compressional waves should be expected to remain functions of depth. This effect is expected to be small in the case of compressional waves as the shear modulus is usually at least an order of magnitude smaller than the other material moduli in an unconsolidated sediment.

There are some other differences between real and laboratory sediments which should be noted. In natural sedimentary materials the sphericity of the particles decreases with decreasing size of the particles, so small spherical glass beads are not a good representation of similarly sized particles in a real sediment. In finely sized sediments the particles are often angular. Spherical glass beads will tend to have a smaller shear modulus as their smooth surfaces allow them to

move more easily. Compressional wave velocities are higher in laboratory sediments than they are in real sediments because the bulk modulus of glass is higher than the bulk modulus of quartz.

B. Physical Properties of the Sediment

1. The glass beads

The glass beads used in this experiment were obtained from the Ferro Corporation in Jackson, Mississippi. The manufacturer describes the beads as Class III Glas-Shot (sic), they are sold for use as abrasives. The sales literature gives the following description of the beads:

Glas-Shot (beads) are spheroidal in shape containing not more than fifteen (15) percent irregularly shaped particles. They are reasonably free of sharp angular particles, the quantity of which does not exceed three (3) percent, particles showing milkiness or surface scoring or scratching, the quantity of which does not exceed two (2) percent, and foreign matter, the quantity of which does not exceed one (1) percent including iron particles, the quantity of which does not exceed one-half (1/2) percent.

The beads selected are designated as type MS-L by the manufacturer. The beads range in size between 44 and 88 microns. A summary of the manufacturers specifications of the bulk properties of the glass is given in Table 1. The glass is a soda-lime-silica glass. A glass of this composition is usually called a soda-lime glass and will be so called throughout this dissertation.

There is some ambiguity over which modulus of elasticity is provided by the manufacturer, as only modulus of elasticity is specified. Consultation with a manual of glass properties⁹¹ suggests that the modulus of

TABLE 1

Manufacturer's Specifications of Glass Beads

Sound Velocity (glass in bulk)	5800 ms^{-1}
Modulus of Elasticity	$9.66 \times 10^6 \text{ psi}$
Density	2.5 g cm^{-3}
Thermal Conductivity	$2.5 \times 10^{-3} \text{ Cal s}^{-1} \text{ cm}^{-2}$
Coefficient of Thermal Expansion	8.47×10^{-6}
Mean Specific Heat	$0.27 \text{ Cal/g/}^{\circ}\text{C (40-800 }^{\circ}\text{C)}$

Properties of Glas-Shot Beads

Bulk Specific gravity	1.5 (average)
	1.55 (coarse sizes)
	1.45 (fine sizes)
Diameter of MS-L Beads	44-88 microns

elasticity quoted by the manufacturer is a typical value of Young's modulus for a soda-lime glass.

2. Porosity and laboratory sediments

The density and porosity of a regular packing of identical spheres does not change with grain diameter, but a regular packing of identical spheres does not have regularly sized pores, and its acoustic properties are aeolotropic. The porosity of a random packing of uniformly sized spheres is 0.38. Variations from this ideal porosity are caused by variations in grain size and shape, or in the packing.⁹²

Dullien⁹³ defines four modes of random packing. We need not concern ourselves with the details of how these packings may be achieved, but should note that variations in porosity from 0.36 to 0.44 are possible for random packings of spheres.

Some measurements of porosity were carried out on small samples in the laboratory using the liquid imbibition technique.⁹³ When water was added to the beads a porosity of 0.38 was measured. When beads were added to water a porosity of 0.41 was measured; when the samples were allowed to sit overnight the measured porosity decreased to 0.39. The standard deviation in all these porosity measurements was 0.01. A porosity of 0.40 is used in all subsequent calculations.

In the sound tube, beads were added to water. Although some settling takes place, fewer gas bubbles are trapped in this way. In the porosity measurements, when water was added to the beads, it took a long time to

permeate the column and the sample had to be stirred to remove some of the bubbles.

The container walls will have some effect on the packing of the beads, and therefore the porosity. The base plane of the pack will introduce local order into a random packing, but even if this first layer is completely regular, the second, and all subsequent layers will be built from multiple focal points and the pack will quickly assume the characteristics of a random pack. Near any external surface there will be a region of high porosity because of the difference in the radii of curvature of the beads and the container walls. The wall effects on bulk porosity are negligible if the diameter of the container is more than ten times the mean diameter of the particles in the pack.⁹³ That condition was met in the experiment described here.

3. Permeability

Permeability is the term used for the conductivity of a porous medium with respect to permeation by a Newtonian fluid. Permeability was not measured directly in this investigation; instead the flow resistance of the medium was measured using a Leonard⁹⁴ flow resistance apparatus modified as suggested by Rudnick.⁹⁵ The permeability of a porous medium is equal to the viscosity of the permeating fluid divided by the flow resistance of the medium. This method measures the gas permeability rather than the absolute permeability. The absolute permeability may be calculated by plotting gas permeability versus the inverse of the average pressure across the porous medium. In this investigation, only the

differential pressure across the sample was measured. The measured gas permeabilities are presented in table 2.

The permeability of a porous medium consisting of uniformly sized spherical particles may be calculated from the Kozeny-Carman equation³⁵

$$k = f^3 / [k_o S_o^2 (1 - f^2)] \quad (32)$$

where S_o is the specific surface of the particles, and k_o is an empirical constant. For uniform spheres,

$$S_o = 6/d, \quad (33)$$

where d is the diameter of the particles. The constant k_o may be is equal to five for a wide variety of porous media; thus the permeability of the sphere pack may be calculated from:

$$k = d^2 f^3 / [180 (1 - f^2)]. \quad (34)$$

The particles used in this investigation have diameters between 44 and 88 microns. These beads have been used previously by Bell.⁹⁶ He found that their mean diameter was 62.5 microns. Using this diameter, the Kozeny-Carman equation predicts a permeability of $3.86 \times 10^{-12} \text{ m}^2$ at a porosity of 0.40; this compares favorably with the measurements reported in table 2 and is used in all subsequent calculations. A summary of the physical properties of the sediment is provided in table 3.

TABLE 2

Permeability and Flow Resistance measured in Leonard's Apparatus

Differential Pressure (Nm ⁻²)	Flow Resistance (cgs Rayls)	Permeability (10 ¹² m ²)
96.18	5045	3.58
192.35	4742	3.82
384.71	4378	4.13
480.88	4242	4.27
673.24	4440	4.08
961.76	4348	4.16

TABLE 3

Summary of Sediment Physical Properties

Density of Grains	2500 kg/m^3
Density of Fluid	998 kg/m^3
Bulk Modulus of Grains	$3.83 \times 10^{10} \text{ Pa}$
Bulk Modulus of Fluid	$2.0 \times 10^9 \text{ Pa}$
Viscosity of Fluid	$1.0 \times 10^{-3} \text{ N.s/m}^2$
Mean Particle Size	$62.5 \times 10^{-6} \text{ m}$
Permeability of Frame	$3.86 \times 10^{-12} \text{ m}^2$
Porosity of Frame	40%

IV. EXPERIMENT

The velocity of compressional sound waves in sediments is more than 1500 meters per second;⁴⁻¹⁰ this means that laboratory measurements of the low-frequency attenuation of compressional sound waves in sediments require a large-scale apparatus so that the length of the sediment sample is longer than one wavelength of the compressional wave at the lowest desired frequency.

This chapter contains a description of the experimental apparatus used to perform low-frequency attenuation measurements and details of the experimental procedure employed.

A. Experimental Apparatus

The attenuation measurements reported in this dissertation were performed in a vertically mounted sound tube. The tube is made of aluminum, it is 4.35 m long, and has an inside diameter of 0.254 m with a wall thickness of 0.0095 m. The working length of the sound tube is 3.75 m, which enables the tube to hold a maximum of 0.19 m^3 (50 gallons) of sediment.

The sound tube is mounted on three wooden supports, and is surrounded by a scaffold to allow easy access to all parts of the tube. There are suitable systems which allow water to be added to or removed from the system. The system is diagrammed in Fig. 1.

It is permissible to treat a sediment as a fluid for the purpose of determining wall losses in a cylindrical tube, provided that the sediment shear modulus is small in comparison with the frame bulk modulus;¹⁰² this condition is met in an unconsolidated sediment which is by definition a sediment of low rigidity. Kirchhoff's wall loss expression was used to correct the attenuation data presented in Fig. 5 for the attenuation due to the walls of the sound tube.

In a sound tube of radius R for a sound wave of frequency ν propagating in a medium of viscosity η , thermal conductivity K , with a ratio of specific heats γ , and an average fluid pressure p , Kirchhoff wall losses may be calculated from:

$$\alpha_w = \frac{1}{R} \left[\frac{1}{\gamma^2} + \frac{(\gamma-1)}{\gamma} \left(\frac{K}{VC_v} \right) \right] \left(\frac{\pi V \nu}{p} \right)^{1/2} \quad (35)$$

where α_w is the sound attenuation due to the walls of the tube. The specific heat, C , of a sediment may be calculated from:¹⁰⁴

$$dC = (1-f)d_s C_s + f d_f C_f \quad (36)$$

where f is the frame porosity, and d , d_s and d_f are the densities of the aggregate the solid and the fluid. The ratio of specific heats for water is 1.004. The specific heat of water (C_f) is $4184 \text{ J/m}^3 \cdot ^\circ\text{C}$ and the specific heat of a soda-lime glass (C_s) is $840 \text{ J/m}^3 \cdot ^\circ\text{C}$; thus using the expression for the specific heat of a sediment the ratio of specific heats for the sediment is calculated to be 1.002.

The thermal conductivity K of a sediment may be estimated from its water content using:⁹⁰

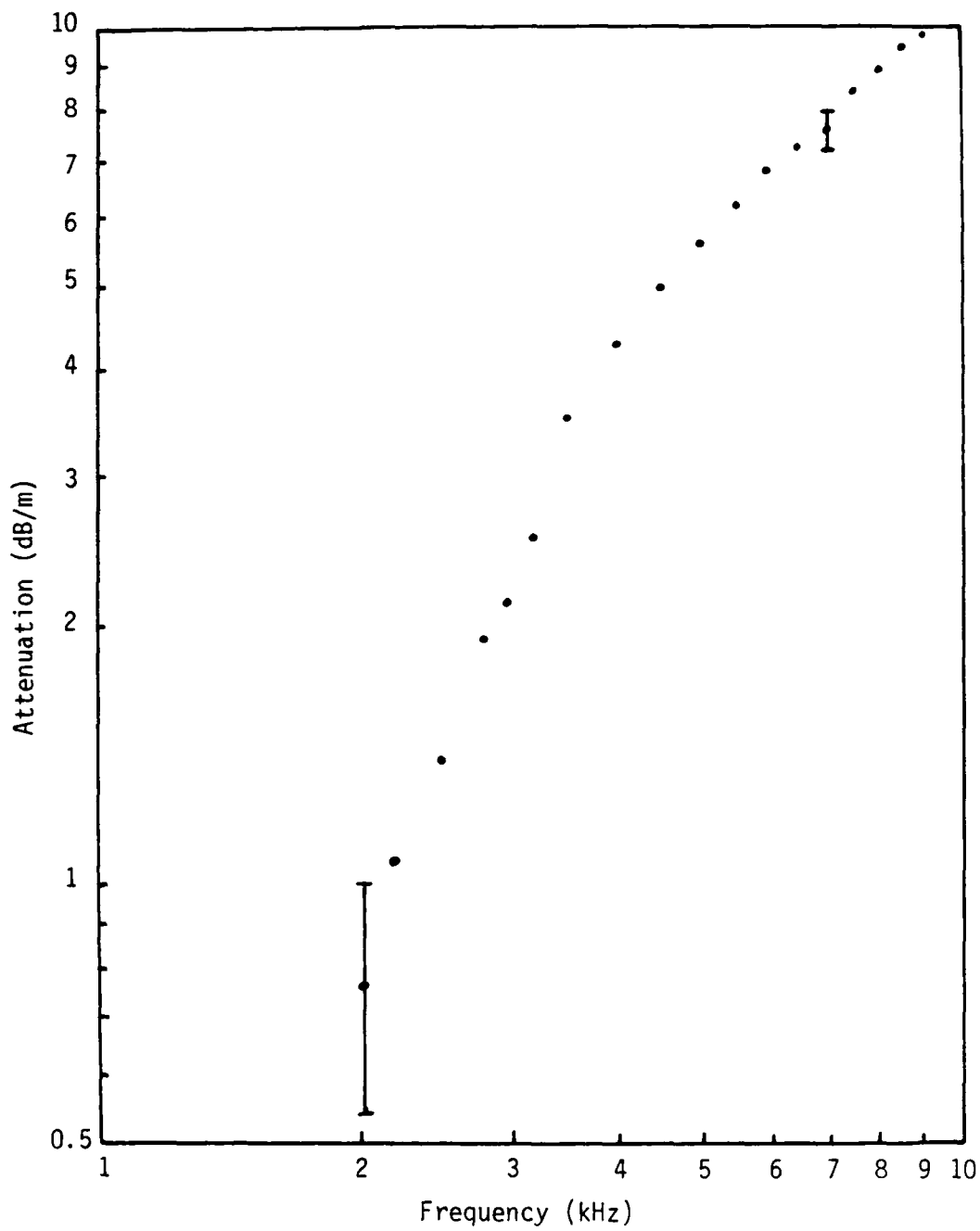


Figure 5

Attenuation Versus Frequency Uncorrected for Tube Absorption
(Bars indicate scatter between two measurements)

V. RESULTS AND DISCUSSION

The attenuation of compressional waves in a laboratory sediment was determined from the received signal level over different path lengths through the sediment. Path lengths of 2.114 m, 2.206 m, 2.679 m, and 3.073 m were used. The measurements at 2.114 m and 2.206 m were made with a different transmitter to the measurements made at 2.679 m and 3.073 m. The attenuation measurements reported in this dissertation were calculated by matching the gradients of received signal level versus path length for the two transmitters to determine the average attenuation. No systematic variation in the gradients produced by the different transmitters was noted. The resulting attenuation measurements are plotted versus frequency in Fig. 5.

When sound waves propagate in a cylindrical tube there is some attenuation at the walls produced by viscosity and thermal conduction. The theory of wall losses in a cylindrical tube containing a gas was worked out by Kirchhoff⁵³ in 1968. It is possible to demonstrate that Kirchhoff's expression for wall losses is valid for a cylinder containing any fluid.¹⁰⁰ When low-pressure gases are used it is necessary to introduce velocity slip and temperature jump at the walls into Kirchhoff's solution;¹⁰¹ these additional terms are unnecessary if the fluid is a water saturated porous medium because the mean free path in such a medium is on the order of atomic dimensions.

More sediment was then added to the tube using the methods previously outlined in this section.

The experimental technique used frequently caused damage to the transmitter. The damage occurred for two reasons. Glass beads stick to the walls of the sound tube. These glass beads are highly abrasive and can cause significant damage to the transmitter. The seal formed by the Teflon gasket surrounding the transmitter when it is in contact with the water can also cause problems. On one occasion it caused air to be forced behind the Mylar film as the transmitter was removed from the tube. The attenuation measurements reported in Chapter V were made with two different transducers; none of the transducers survived more than three immersions.

Since this transmitter was designed and constructed, better designs for underwater solid dielectric transducers have appeared in the literature.⁹⁹ Experience gained during the course of this investigation suggests that it might be advantageous to fill the tube with water above the level of the sediment and to correct the resulting measurements for the (small) attenuation due to the water.

sealed systems was then evacuated. The pressure in the system was maintained at the vapor pressure of water for several hours to allow any entrapped gas to escape from the surface of the sediment. The vacuum was then released and the length of the sediment sample was determined. The level of water above the glass beads was usually about six inches.

After the sample had been prepared, the transmitter was introduced into the sound tube. The transmitter was allowed to sink to the level of the water under its own weight. A continuous signal was broadcast by the transmitter so that by monitoring the signal received by the hydrophone we could determine when the transmitter was in contact with the water. No clear signals were observed until the transmitter was in contact with the surface of the water. With experience it became simple to recognize when the transmitter was in contact.

When the transmitter was in contact with the water, the electronics were adjusted so that tone-bursts were transmitted through the sediment and the received signal level was adequate. The same amplifier settings were then used in all subsequent experiment. Compressional waves with frequencies between 2 kHz and 10 kHz were transmitted through the sediment. The peak-to-peak output voltage of the received signal as amplified by the conditioning amplifier and the PAR pre-amplifier and displayed on the oscilloscope was recorded.

In order to determine sound attenuation, the path length through the sediment must be varied. After the completion of a data run the transmitter was removed from the sound tube using a block and tackle.

4. The water supply

Water is added to or removed from the system by a pair of water valves mounted to the bottom end-plate of the sound tube. The water is filtered by Cole-Palmer ion X-change cartridges in order to remove any impurities which might be present in the water. The water pressure in the laboratory is sufficient to fill the tube thus rendering the pumping of water into the tube unnecessary. The water flows readily around the transducer plate.

B. Experimental Procedure

The laboratory sediment was prepared by adding the glass beads to the water rather than by adding water to the glass beads. The sediment was prepared in this way because it has been reported⁹⁶ that glass beads have a tendency to trap large amounts of gas in their pores. Experiments carried out on small samples indicated that if the glass beads were added to the water no significant pockets of gas were trapped within the body of the sediment. When water was added to the glass beads large pockets of gas were formed which were extremely difficult to remove.

The sediment was prepared by introducing a quantity of water into the sound tube and then slowly pouring glass beads into the tube from the top. The walls of the sound tube were tapped periodically with a rubber mallet to help ensure an even packing of the glass beads.

When the required amount of sediment had been prepared, the sound tube was sealed by placing an end-plate at the top of the tube. The

3. The electronic equipment

The electronic equipment used for generating and detecting tone bursts is shown in Fig. 1. Sine waves were produced by a Rockland 5100 frequency synthesizer. The Rockland is a calibrated frequency source so it was unnecessary to monitor the frequency of the sine waves during data acquisition. Bursts of sine waves were produced by passing the output of the Rockland frequency synthesizer through a General Radio 1396-B tone-burst generator. The tone-burst generator allowed the duration of the bursts and the interval between the bursts to be controlled.

In the experiments the duration of the tone-bursts was maintained less than the time needed for the burst to travel twice the length of the tube in order to avoid the effect of standing waves. It was found that it was impossible to set up standing waves in the tube because of the high attenuation of compressional waves in sediments.

The tone-bursts were amplified by a Dvnaco Mark VI power amplifier which is capacitively coupled to the dc biased transmitter. The signal received by the hydrophone is amplified by a Bruel & Kjaer 2626 conditioning amplifier. The output of the conditioning amplifier is then further amplified by a Princeton Applied Research 113 low-noise pre-amplifier. The resulting waveform is displayed on a Nicolet digital oscilloscope.

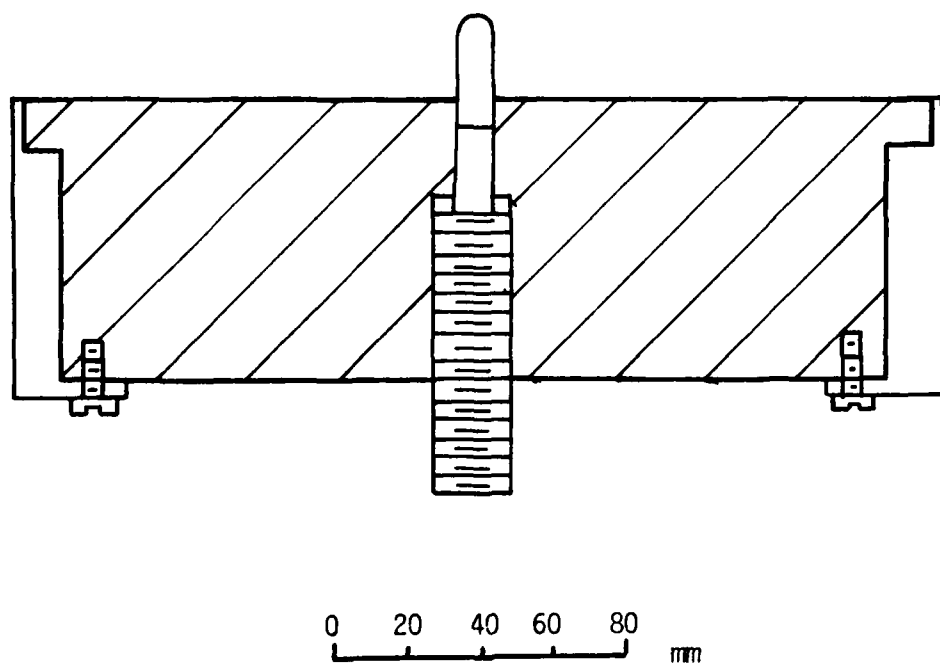


Figure 4
Hydrophone Plate

2. The receiver

The acoustic signal is received by a Bruel & Kjaer 8103 miniature hydrophone. The B&K 8103 is a small, high-sensitivity waterborne transducer designed for making absolute sound measurements over the frequency range 0.1 Hz to 200 kHz.

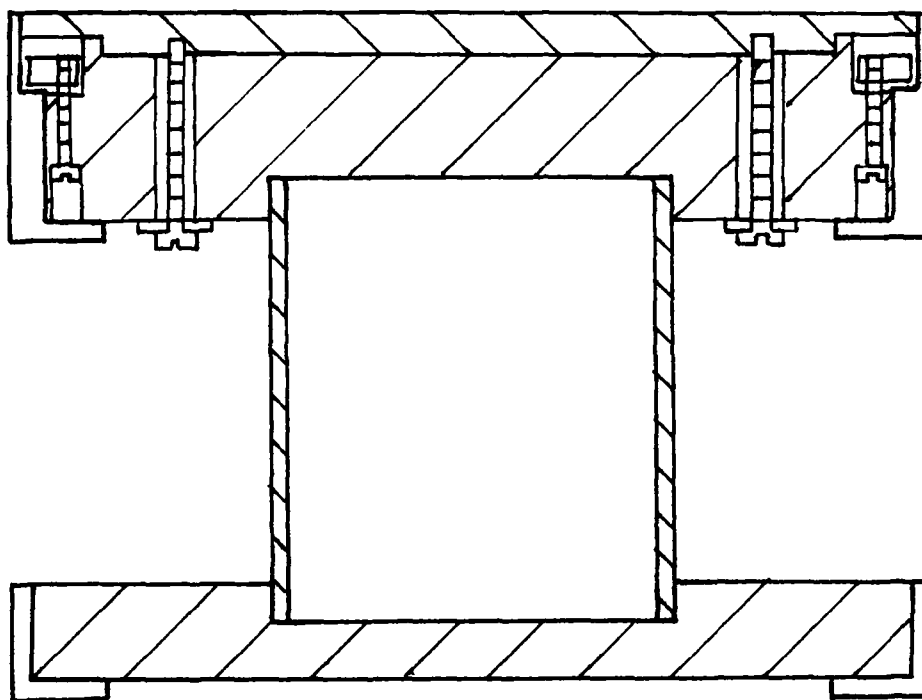
The hydrophone is mounted in a threaded brass sleeve. The entire assembly is bolted to a 0.076 m (3") thick stainless steel plate. The hydrophone plate is surrounded by a Teflon gasket to facilitate its movement in the sound tube. The hydrophone plate sits on three raising and lowering bolts which are threaded and emerge through the bottom end-plate of the sound tube. The raising and lowering bolts allow the hydrophone plate to be raised or lowered, to cover or expose a 0.0254 m (1") valve through which the sediment can be pumped out of the sound tube. The hydrophone assembly is diagrammed in Fig. 4.

Before the hydrophone plate was introduced into the sound tube an experiment was performed to determine the minimum distance that the hydrophone could protrude from the front of the hydrophone plate without affecting the received signal. It was found that it was necessary for the hydrophone to protrude 0.014 m from the hydrophone plate; this means that the acoustic center of the hydrophone is 0.004 m above the hydrophone plate.

caustic solution will etch aluminum off Mylar, the coating of paint prevented this.

The transmitter is connected to a second aluminum plate by means of an aluminum spacer. This second aluminum plate is also surrounded by a Teflon gasket. The transmitter when assembled has been described by Shields⁹⁸ as a 'spool-shaped plunger.' This arrangement reduces the tendency for the transmitter to twist in the tube. The transmitter is allowed to sink to the level of the sediment under its own weight. This process usually takes several minutes as air is forced around the edges of the Teflon gasket. The transducer is raised by a block and tackle suspended from the top of the scaffold.

The process of raising and lowering the transmitter caused most of the problems encountered in acquiring the attenuation data. When this research project was originated in 1979, one of its stated aims was to adapt expertise in determining sound attenuation in gases to laboratory sediments. That goal has been achieved, but problems became evident when the large-scale apparatus was used. This research has demonstrated the viability of using a solid dielectric transducer as the sound source, but the experience gained in using the system calls into question the advisability of using a moveable sound source. The transmitter had a tendency to become stuck in the tube because the glass beads tended to coat the walls of the sound tube. The transmitter was not very durable, several were used during the course of the experiment.



0 20 40 60 80 mm

Figure 3
The Transmitter

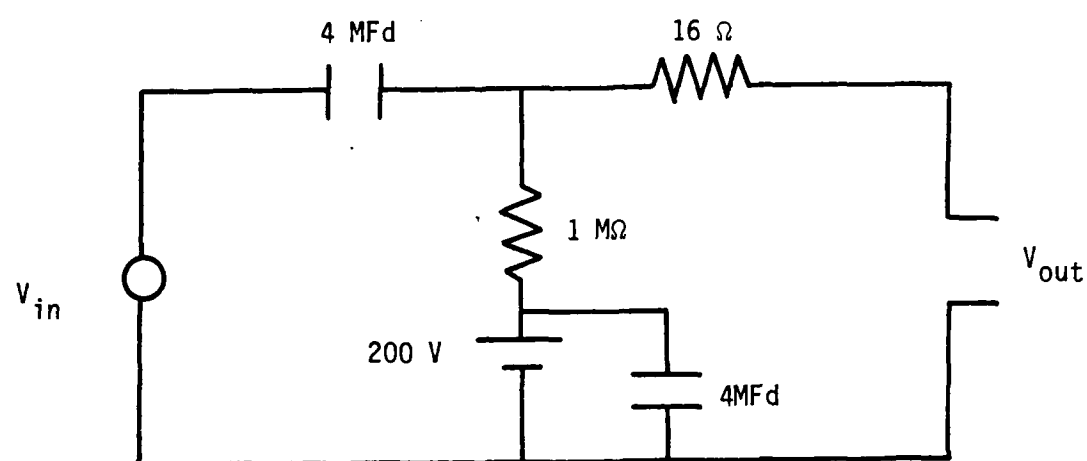


Figure 2
Voltage Adder Circuit

1. The sound transmitter

The sound transmitter used in this experiment is a larger version of the solid dielectric capacitor transducers designed by Kuhl;⁹⁷ it is a waterproof version of the large capacitor microphones used previously in this laboratory by Shields.⁹⁸

The transmitter is constructed using a sheet of 0.5-mil Mylar which is aluminized on one side. The Mylar is stretched over an aluminum backing plate with the aluminized surface on the outside electrically insulated from the backing plate. A dc polarizing voltage is applied between the aluminized Mylar and the backing plate and an ac voltage is applied in series with the polarizing voltage by means of the voltage adder circuit shown in Fig. 2.

The transducer was designed to completely fill the cross-sectional area of the tube. The construction details of the transducer are shown in Fig. 3. The Mylar film is attached to a floating tension ring, and the Mylar is stretched over the backing plate by means of three tension screws. The backing plate is coupled to, but is electrically insulated from, a 0.052 m (2") thick stainless steel loading plate. The transmitter is surrounded by a Teflon gasket which was designed to allow the transmitter to fit tightly in the sound tube and still allow it to move freely. The front surface of the aluminized Mylar was coated with an alkyl resin based paint; the paint water-sealed the transducer and also electrically insulated the aluminized Mylar from the water. When small glass beads are added to water, they form a caustic solution; a

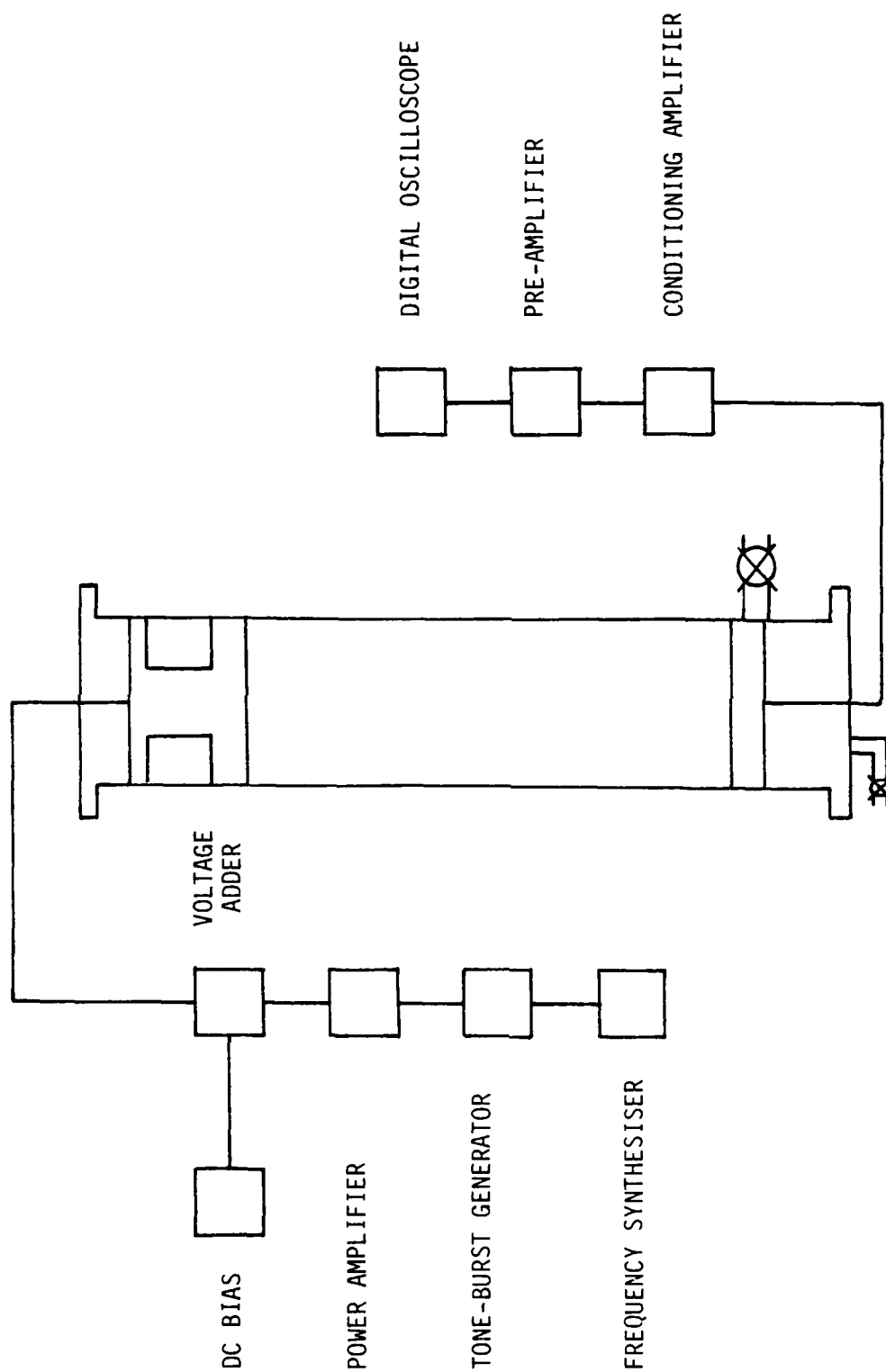


Figure 1
Block Diagram of System

$$K = \frac{d_f f + d_s (1-f)}{812d_f f + 161d_s - 161fd_s} (10^{-4} \text{ cal/cm}^{\circ}\text{C}). \quad (37)$$

The mean fluid pressure in the tube was calculated to be 1.3×10^5 Pa. Unless otherwise noted all data was obtained from the manufacturer's specifications for the glass beads or Kinsler and Frey.¹⁰⁶ When the wall losses were calculated for the sound tube containing the sediment it was found that the viscous losses dominated those due to thermal conduction at the walls.

The attenuation in the sediment with the wall losses subtracted is presented in Fig. 6. The attenuation measured in the laboratory sediment is not a linear function of frequency, as predicted by Hamilton;⁵⁻¹¹ it varies as the square of frequency at low frequency (below 4 kHz) and shows an approximately linear dependence on frequency above 4 kHz. The Biot model predicts that, at low frequencies, the attenuation of compressional waves in a sediment varies as the square of frequency, and at high frequencies the attenuation varies as the square root of frequency resulting in an approximately linear variation with frequency at intermediate frequencies.¹²⁻¹⁸ The attenuation measurements reported in this dissertation were performed at low and intermediate frequencies for the size of particles used in the laboratory sediment. As explained in Chapter II the terms low and high frequency are dependent on the size of the particles in the porous medium when the Biot model is used.

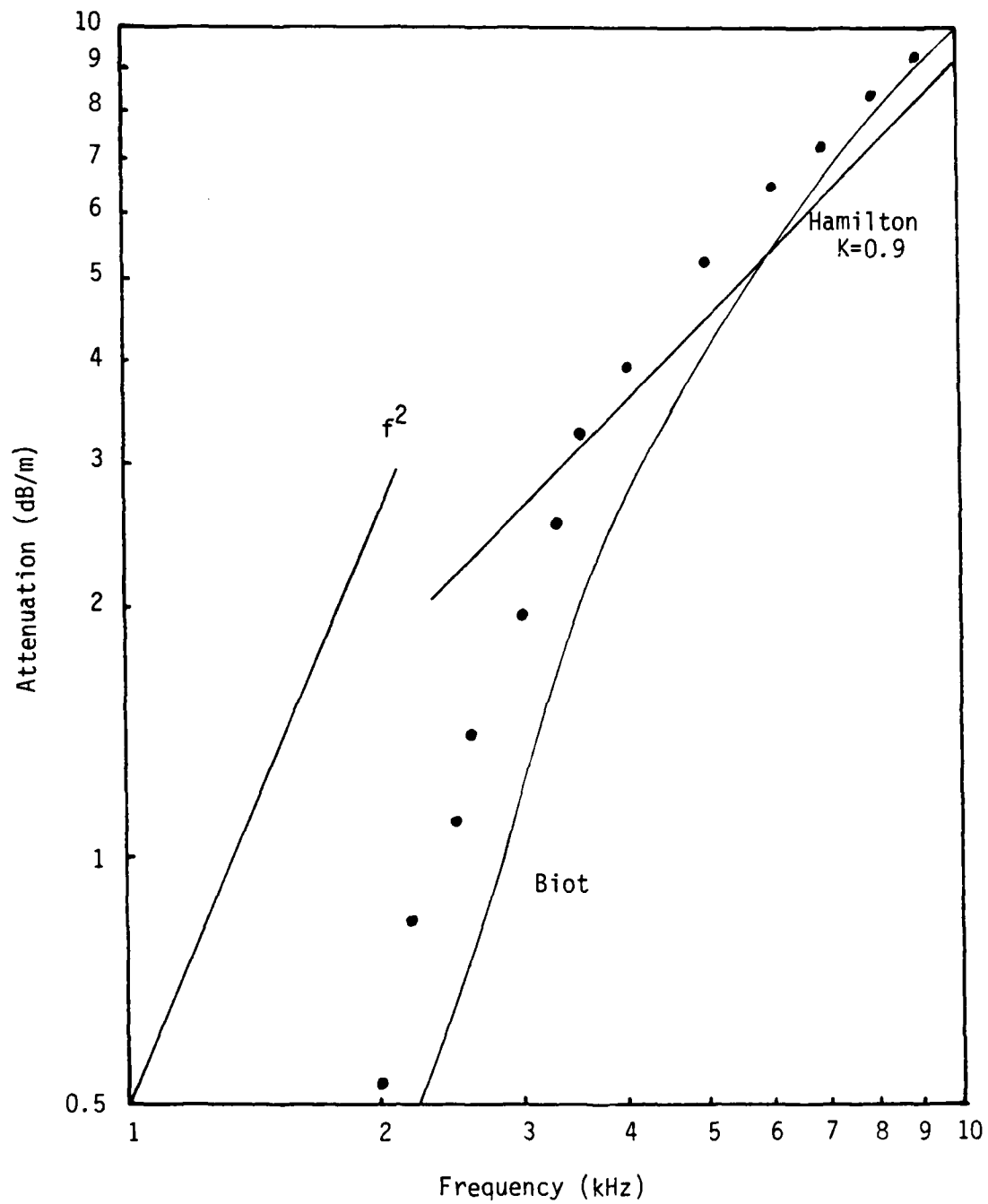


Figure 6

Attenuation Corrected for Wall Losses Compared to the Predictions of the Biot and Hamilton Models

VI. CONCLUSIONS

Although the functional dependence of the attenuation measurements (precisely in accord with the Biot model) is gratifying, the actual magnitude of the attenuation above 4 kHz is higher than expected. Figure 6 contains a plot of Eq. 1, the attenuation predicted by the Hamilton model using the maximum value of the parameter K (0.9) for particles on the order of 65 microns in size.⁶ Hamilton's model predicts an attenuation which is about 1 dB/m lower than measured here in the range from 5-10 kHz. This excess attenuation may be accounted for in one of two ways; the method of calculating the attenuation due to the tube walls may be inadequate due to the finite rigidity of the sediment, or the sediment may have contained some gas bubbles the presence of which will contribute to the measured attenuation, or a combination of both.

The relatively high attenuation in the system made it impossible to determine the propagation velocity of the compressional waves using the resonance method. The propagation velocity is useful to obtain an estimate of the magnitudes of the bulk and shear moduli of the frame. The absence of this estimate permits a considerable latitude in the choice of these quantities to fit the experimentally measured attenuation to the Biot model. The method used to calculate the attenuation predicted by the Biot model was to choose a value from Poisson's ratio (which relates the shear and bulk moduli of the frame)¹⁰⁷ and then to vary the bulk modulus of the frame until the experimental data fits the theoretical

curve. Fine adjustments are made by altering the value of Poisson's ratio when the experimental data lies close to the theoretical curve. In this way the experimental data can be made to lie as close as desired to the theoretical curve. The attenuation predicted by the Biot model contained in Fig. 6 was calculated using the computer program at Appendix A.

The functional dependence of the attenuation data displayed in Fig. 6 is a more convincing testament to the veracity of the Biot model than is the theoretical Biot curve in the absence of realistic estimates of the frame moduli.

Future attenuation measurements using the system designed for this experiment will be facilitated if the engineering improvements in the transmitter described in Chapter IV are made. Since the transmitter was designed and constructed, better designs for underwater solid dielectric transducers have appeared in the literature.⁹⁹ Experience gained during the course of this investigation suggests that it would be advantageous to fill the tube with water above the level of the sediment and to correct the resulting measurements for the (small) attenuation due to water. A system to measure the propagation velocity of the compressional waves would enable the experimental data to be more easily fitted to the Biot model, but it is recommended that methods be developed to directly determine the frame moduli to eliminate the free parameters in the Biot model. Future theoretical work should concentrate on either the calculation of H , C , and M directly, or a satisfactory method of calculating the frame moduli for unconsolidated media.

In conclusion it has been demonstrated that the Biot model successfully predicts the attenuation of low-frequency compressional waves in a laboratory sediment. The Biot model continues to pass every test to which it is put and it is recommended that it be adopted as the standard model for wave propagation through ocean sediments.

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APPENDIX
PROGRAM SOSED

This program was used to calculate the attenuation of sound in sediments using the Biot model.

```

C Computation of sound velocity and attenuation in sediments using the
C Biot Model
C Reconstructed version
C
C*****
C TYPE DECLARATIONS OF VARIABLES
C*****
C
  REAL KR,KF,PHI,PERM,VISC,NU,DIA
  REAL RHO,RHOF,RHOS,RHOC
  REAL PV1,PV2,ABS1,ABS2
  REAL FREQ,AF,AF2,AF3,AF4,PI
  REAL TORT,X,QDC1
  COMPLEX KB,MU,H,C,M
  COMPLEX KRB,DKB,D,I,F
  COMPLEX RPL,RMI,Q1,Q2
  COMPLEX QDA,QDB,QDB1,QDB2,QDC,QDC2
C
  I=(0.0,1.0)
  PI=3.1415926
  READ (10,*)KR,KF,KB,MU,VISC,PERM,TORT,DIA,RHOS,RHOF,PHI
C*****
C Calculation of H,C,M
C*****
C
  D=KR*(1.0+PHI*(KR-KF)/KF)
  KRB=KR-KB
  DKB=D-KB
  H=(KRB*KRB/DKB)+KB+(4.0/3.0)*MU
  C=KR*KRB/DKB
  M=KR*KR/DKB
C
C
C*****
C Calculation of densities
C*****
C
  RHO=(1.0-PHI)*RHOS+PHI*RHOF
  RHOC=TORT*RHOF/PHI
C
C

```

```

C
C*****
C Calculations using the Biot model
C*****
C
      QDA=(C*C)-(H*M)
      NU=VISC/PERM
      QDB1=M*RHO+H*RHOC-2.0*C*RHOF
      QDC1=RHOF*RHOF-RHO*RHOC
      DO 1000 IML=1,28
      READ(11,*)FREQ
      AF=(PI*FREQ)+(PI*FREQ)
      AF2=AF*AF
      AF3=AF*AF2
      AF4=AF2*AF2
      X=AF*RHOF/VISC
      X=SQRT(X)*DIA/7.0
      CALL DYNVIS(X,F)
      QDB2=I*H*NU*F
      QDC2=I*RHO*NU*F
      QDB=(QDB1*AF2)-(QDB2*AF)
      QDC=(QDC1*AF4)+(QDC2*AF3)
      CALL SOLVE(QDA,QDB,QDC,RPL,RMI)
      RPL=CSQRT(RPL)
      RMI=CSQRT(RMI)
C*****
C Calculation of phase velocity and attenuation in nepers/m
C*****
      PV1=AF/REAL(RMI)
      PV2=AF/REAL(RPL)
      ABS1=-IMAG(RMI)
      ABS2=-IMAG(RPL)
      WRITE(12,*)FREQ,PV1,ABS1
      WRITE(13,*)FREQ,PV2,ABS2
1000  CONTINUE
      END
C*****
C SUBROUTINE DYNVIS(X,F)
C Routine to calculate Biot's dynamic viscosity factor
C*****
      SUBROUTINE DYNVIS(X,F)
      COMPLEX I,T,F
      IF (X.LE.0.1) GO TO 2000
      BER=1.0
      BERP=0.0
      BEI=0.0
      BEIP=0.0
      I=(0.0,1.0)

```

C

C

```

DO 1001 K=1,5
N=K
A=(-1.0)**N
BRE2=2.0*N
BRE4=4.0*N
BBER=X**BRE4
CBRE=2.0*BRE4
DBRE=GAMMA(BRE2+1.0)
EBRE=DBRE*DBRE
TBER=(A*BBER)/(CBRE*EBRE)
BER=BER+TBER
BERP4M=4.0*N-1.0
BBERP=X**BERP4M
TBERP=(A*BBERP*BRE4)/(CBRE*EBRE)
BERP=BERP+TBERP
1001 CONTINUE
DO 1002 K=1,6
N=K-1
A=-1.0**N
BEI2=2.0*N+1.0
BEI4=4.0*N+2.0
BBEI=X**BEI4
CBEII=2.0*BEI4
DBEII=GAMMA(BEI2+1.0)
EBEII=DBEII*DBEII
TBEI=(A*BBEI)/(CBEII*EBEII)
BEI=BEI+TBEI
BEIP4M=4.0*N+1.0
BBEIP=X**BEIP4M
TBEIP=(A*BBEIP*BEI4)/(CBEII*EBEII)
BEIP=BEIP+TBEIP
1002 CONTINUE
C*****
C Computation of dynamic viscosity factor
C*****
T=(BERP+I*BEIP)/(BER+I*BEI)
F=(0.25*X*T)/(1.0-((2.0*T)/(I*X)))
RETURN
2000 F=(1.0,0.0)
RETURN
END
SUBROUTINE SOLVE(A,B,C,RPL,RMI)
COMPLEX A,B,C,RPL,RMI,DISCR
DISCR=CSQRT(B*B-4.0*A*C)
RPL=(-B+DISCR)/(2.0*A)
RMI=(-B-DISCR)/(2.0*A)
RETURN
END

```

END

FILMED

6-85

DTIC