

AD-A153 754

TENTH-ORDER EXPANSION FORMULAE FOR THE ORBITAL  
ECCENTRICITY FUNCTION  $K$  (S. (U) NAVAL SURFACE WEAPONS  
CENTER DAHLGREN VA S C MATNEY ET AL. NOV 84

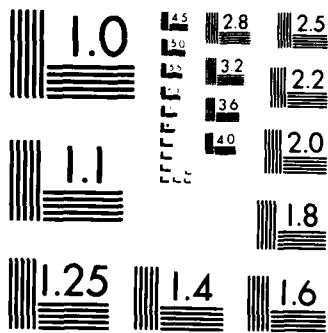
1/1

UNCLASSIFIED

F/G 12/1

NL

							END						
							FILED						
							DEC						



MICROCOPY RESOLUTION TEST CHART  
NATIONAL BUREAU OF STANDARDS 1963-A

AD-A153 754

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER NSWC TR 84-355	2. GOVT ACCESSION NO	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) TENTH-ORDER EXPANSION FORMULAE FOR THE ORBITAL ECCENTRICITY FUNCTION $K_{\ell pq}(e)$ OBTAINED USING A SYMBOLIC ALGEBRAIC MANIPULATION SYSTEM		5. TYPE OF REPORT & PERIOD COVERED Final
		6. PERFORMING ORG REPORT NUMBER
7. AUTHOR(S) S. C. Matney A. D. Parks		8. CONTRACT OR GRANT NUMBER(S)
9. PERFORMING ORGANIZATION NAME AND ADDRESS Naval Surface Weapons Center (K13) Dahlgren, VA 22448		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
11. CONTROLLING OFFICE NAME AND ADDRESS		12. REPORT DATE November 1984
		13. NUMBER OF PAGES 35
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)		15. SECURITY CLASS. (of this report) UNCLASSIFIED
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE
16. DISTRIBUTION STATEMENT (of this Report)  Approved for public release; distribution unlimited.		
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)		
18. SUPPLEMENTARY NOTES		
19. KEY WORDS (Continue on reverse side if necessary and identify by block number) eccentricity function formulae MACSYMA		
20. ABSTRACT (Continue on reverse side if necessary and identify by block number) Extensive tables of formulae for the orbital eccentricity function $K_{\ell pq}(e)$ have been generated using the MACSYMA* symbolic algebraic manipulation system. These formulae are presented through tenth order in the orbital eccentricity for $2 \leq \ell \leq 10$ , $0 \leq p \leq 5$ , and $-2 \leq q \leq 2$ . Comparisons are made with tabulations obtained by other workers.		
*MACSYMA is a large symbolic manipulation program developed at MIT and supported during 1975-1983 by NASA, ONR, DOE, and the USAF, and since 1982 by Symbolics, Inc. of Cambridge, Mass. MACSYMA is a trademark of Symbolics, Inc.		

DD FORM 1473  
1 JAN 73EDITION OF 1 NOV 65 IS OBSOLETE  
S. N. O102-LF-014-6601

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)



CONTENTS

	Page
INTRODUCTION .....	1
THE $K_{\ell pq}(e)$ ECCENTRICITY FUNCTION .....	1
DISCUSSION .....	2
REFERENCES .....	3
APPENDIX	
TABLE OF $K_{\ell pq}(e)$ ECCENTRICITY FUNCTION FORMULAE FOR $2 \leq \ell \leq 10, 0 \leq p \leq 5, \text{ AND }  q  \leq 2$ .....	A-1
DISTRIBUTION .....	(1)

## INTRODUCTION

Although digital computers have been performing complex numerical computations for several decades, their application to the solution of nonnumeric problems is a relatively recent development.<sup>1,2</sup> Several sophisticated symbolic computer algebra systems currently exist and can serve as a most valuable asset to the scientific researcher.<sup>3,4</sup> Such systems can provide general solutions to a wide variety of problems in a manner that eliminates the need for tedious, manually performed algebraic operations, while simultaneously minimizing the likelihood of introducing errors into the solution.

One such "state of the art" system is the MACSYMA computer algebra system developed by the Matlab group at the Massachusetts Institute of Technology.<sup>5</sup> In this work, MACSYMA has been used to generate expansion formulae for the eccentricity function through tenth order in the orbital eccentricity. These formulae are presented in tabular form for  $2 \leq \ell \leq 10$ ,  $0 \leq p \leq 5$ , and  $|q| \leq 2$  and may be used to verify numerically generated values for  $K_{\ell pq}(e)$  or applied directly to the analytical resolution of orbital dynamics problems. For the sake of completeness, the following sections also include a mathematical description of the function  $K_{\ell pq}(e)$  as well as a comparison of results with those obtained by other workers.

THE  $K_{\ell pq}(e)$  ECCENTRICITY FUNCTION

The  $K_{\ell pq}(e)$  eccentricity function is related to the  $G_{\ell pq}(e)$  function used by Kaula<sup>6</sup> through the expression

$$G_{\ell pq}(e) = e^{1q} K_{\ell pq}(e) \quad (1)$$

where,<sup>7</sup> for  $q > 0$ :

$$K_{\ell p q}(e) = (-1)^{|q|} 2^{\ell} (1+\gamma)^{-\ell-|q|} \sum_{k=0}^{\infty} \sum_{r=0}^{|q|+k} \sum_{t=0}^k \frac{(-1)^t}{r!t!} \binom{2p-2\ell}{|q|+k-r} \binom{-2p}{k-t} \left( \frac{\ell-2p+q}{2} \right)^{r+t} (1+\gamma)^{r+t-k} (1-\gamma)^k \quad (2)$$

for  $q < 0$ :

$$K_{\ell p q}(e) = (-1)^{|q|} 2^{\ell} (1+\gamma)^{-\ell-|q|} \sum_{k=0}^{\infty} \sum_{r=0}^{|q|+k} \sum_{t=0}^k \frac{(-1)^t}{r!t!} \binom{-2p}{|q|+k-r} \binom{2p-2\ell}{k-t} \left( \frac{\ell-2p+q}{2} \right)^{r+t} (1+\gamma)^{r+t-k} (1-\gamma)^k \quad (3)$$

and for  $q = 2p - \ell$ :

$$K_{\ell p(2p-\ell)}(e) = \gamma^{-2\ell+1} \sum_{k=0}^{p'-1} \binom{\ell-1}{2k+|2p-\ell|} \binom{2k+|2p-\ell|}{k} (1-\gamma^2)^k 2^{-2k-|2p-\ell|} \quad (4)$$

where

$$\gamma = \sqrt{1-e^2} \quad (5)$$

and

$$p' = \frac{\ell-|2p-\ell|}{2} \quad (6)$$

## DISCUSSION

Equations 2 through 4 of the previous section were coded in the MACSYMA programming language and processed by the MACSYMA system to generate the tables of expansion formulae provided in the attached appendix. Since the majority of satellite orbits to which these results might be applied are of low eccentricity, it was found to be adequate to set the upper limit for the summation index  $k$  in Equations 2 and 3 to 5 and constrain the  $q$  subscript to  $|q| \leq 2$ .<sup>8</sup>

The MACSYMA generated results were compared with those obtained by other workers and agreed identically with those fourth-order formulae presented by Giacaglia<sup>7</sup> for  $2 \leq \ell \leq 4$ ,  $0 \leq p \leq 4$ , and  $|q| \leq 2$ . The MACSYMA formulae also compared identically with Kaula's<sup>6</sup> fourth-order expressions for  $G_{\ell pq}(e)$  for  $2 \leq \ell \leq 4$ ,  $0 \leq p \leq 4$ , and  $|q| \leq 2$ , except for the cases  $(\ell, p, q) = (4, 1, -1)$  and  $(4, 3, 1)$  where Kaula's second term should be  $33e^3/16$  instead of  $33e^2/16$ . A comparison was also made with fourth-order results for  $G_{2cp}(e)$  for  $0 \leq p \leq 2$  and  $|q| \leq 2$  given by Goldreich and Peale.<sup>9</sup> The agreement was identical except for the cases  $(\ell, p, q) = (2, 0, -1)$  and  $(2, 2, 1)$ , where their first term should be  $-e/2$  instead of  $e/2$ , and  $(2, 2, -1)$  where their second term should be  $-123e^3/16$  instead of  $-123e^3/15$ .

Consistency checks were also performed upon the MACSYMA formulae using the following relations:<sup>10</sup>

$$K_{\ell p, +1} = \frac{1}{2} (3\ell - 4p + 1) + O(e^2) \quad (7)$$

and

$$K_{\ell p, -1} = \frac{1}{2} (4p - \ell + 1) + O(e^2) \quad (8)$$

General overall agreement was attained to this order of accuracy.

#### REFERENCES

1. R. Pavelle, M. Rothstein, and J. Fitch, "Computer Algebra," *Scientific American*, 245, No. 6, 1981, pp. 136-152.
2. E. M. Gaposchkin, "Literal Algebra for Satellite Dynamics," *Satellite Dynamics, COSPAR-IAU-IUTAM Symposium São Paulo 1974*, Springer-Verlag, Berlin, Heidelberg, New York, 1975, pp. 170-179.
3. M. S. Davis, "Review of Nonnumerical Uses of Computers," *Recent Advances In Dynamical Astronomy*, D. Reidel Publishing Company, Dordrecht-Holland, 1973, pp. 351-391.
4. R. L. Randall, W. H. Jefferys, and R. A. Broucke, "A General Precompiler for Algebraic Manipulation," *Celestial Mechanics*, 29, No. 2, Feb. 1983, pp. 179-190.

5. Mathlab Group, *MACSYMA Reference Manual*, Version 20, Symbolics, Inc., 1983.
6. W. M. Kaula, *Theory of Satellite Geodesy*, Blaisdell Publishing Company, Waltham, Massachusetts, 1966.
7. G. E. O. Giacaglia, "The Equations of Motion of an Artificial Satellite in Nonsingular Variables," *Celestial Mechanics*, 15, 1977, pp. 191-215.
8. A. Szeto, and K. Lambeck, "On Eccentricity Functions for Eccentric Orbits," *Celestial Mechanics*, 27, 1982, pp. 325-337.
9. P. Goldreich, and S. J. Peale, "The Dynamics of Planetary Rotations," *Annual Review of Astronomy and Astrophysics*, 6, 1968, pp. 287-320.
10. R. R. Allan, "Change of Inclination in Passing Through Resonance," *Recent Advances in Dynamical Astronomy*, D. Reidel Publishing Company, Dordrecht-Holland, 1973, pp. 333-348.

APPENDIX A

TABLE OF  $K_{\ell pq}(e)$  ECCENTRICITY FUNCTION\*  
FORMULAE FOR  $2 \leq \ell \leq 10$ ,  $0 \leq p \leq 5$ ,  
AND  $|q| \leq 2$

---

\*For the sake of brevity use is made of the property  $K_{\ell pq} = K_{\ell, \ell-p, -q}$  in presenting the following tables.

$l p q l p q$	$K_{l p q}$
2 0 -2 2 2 2	0
2 0 -1 2 2 1	$1 \frac{e^2}{16} + \frac{5e^4}{384} + \frac{143e^6}{18432} + \frac{9097e^8}{1474560} + \frac{878959e^{10}}{176947200} + \dots$
2 0 0 2 2 0	$1 - \frac{e^2}{2} + \frac{13e^4}{16} - \frac{35e^6}{288} + \frac{5e^8}{576} - \frac{49e^{10}}{3600} + \dots$
2 0 1 2 2 -1	$7 \frac{123e^2}{16} + \frac{489e^4}{128} + \frac{1763e^6}{2048} + \frac{13527e^8}{163840} + \frac{180369e^{10}}{6553600} + \dots$
2 0 2 2 2 -2	$17 \frac{115e^2}{6} + \frac{601e^4}{48} + \frac{1423e^6}{360} + \frac{48619e^8}{69120} + \frac{275587e^{10}}{2419200} + \dots$
2 1 -2 2 1 2	$9 \frac{7e^2}{4} + \frac{141e^4}{64} + \frac{197e^6}{80} + \frac{62401e^8}{23040} + \frac{262841e^{10}}{89600} + \dots$
2 1 -1 2 1 1	$3 \frac{27e^2}{16} + \frac{261e^4}{128} + \frac{14309e^6}{6144} + \frac{423907e^8}{163840} + \frac{55489483e^{10}}{19660800} + \dots$
2 1 0 2 1 0	$1 - \sqrt{(1-e)(-1+e)}$
3 0 -2 3 3 2	$8 \frac{48e^2}{48} + \frac{3072e^4}{92160} + \frac{92160e^6}{17694720} + \frac{172981e^8}{1238630400} + \dots$
3 0 -1 3 3 1	$-1 + \frac{5e^2}{4} - \frac{7e^4}{48} + \frac{23e^6}{288} - \frac{61e^8}{1440} + \frac{23453e^{10}}{691200} + \dots$

$l_p q l p q$	$K_{l p q}$
3 0 0 3 3 0	$1 - 6e + \frac{423e^2}{64} + \frac{125e^4}{64} + \frac{7533e^6}{16384} + \frac{81e^{10}}{1600} + \dots$
3 0 1 3 3 -1	$5 - 22e + \frac{607e^2}{24} + \frac{98e^4}{9} + \frac{32753e^6}{11520} + \frac{4669e^{10}}{17280} + \dots$
3 0 2 3 3 -2	$127 - \frac{3065e^2}{8} + \frac{243805e^4}{3072} - \frac{798865e^6}{18432} + \frac{49537175e^8}{3538944} - \frac{130509569e^{10}}{49545216} + \dots$
3 1 -2 3 2 2	$11 - \frac{49e^2}{8} + \frac{15665e^4}{3072} - \frac{684503e^6}{92160} + \frac{3937739e^8}{393216} - \frac{15899369747e^{10}}{1238630400} + \dots$
3 1 -1 3 2 1	$\frac{1}{\sqrt{(1-e)(1-2e+e^2)}}$
3 1 0 3 2 0	$1 + 2e + \frac{239e^2}{64} + \frac{3323e^4}{576} + \frac{1193893e^6}{147456} + \frac{4922717e^8}{460800} + \dots$
3 1 1 3 2 -1	$3 + \frac{11e^2}{4} + \frac{245e^4}{48} + \frac{463e^6}{64} + \frac{27929e^8}{2880} + \frac{4286563e^{10}}{345600} + \dots$
3 1 2 3 2 -2	$53 - \frac{39e^2}{8} + \frac{7041e^4}{1024} - \frac{89317e^6}{10240} + \frac{7447867e^8}{655360} - \frac{650531047e^{10}}{45875200} + \dots$
4 0 -2 4 4 2	$\frac{1e^2}{2} - \frac{11e^6}{3} + \frac{1477e^8}{360} - \frac{6989e^{10}}{69120} + \dots$

$l_p q l_p q$	$K_{l_p q}$
$4 0 -1 4 4 1$	$\frac{3}{2} + \frac{75 e^2}{16} + \frac{393 e^4}{128} + \frac{1251 e^6}{2048} + \frac{25371 e^8}{163840} + \frac{275439 e^{10}}{6553600} + \dots$
$4 0 0 4 4 0$	$1 - 11 e^2 + \frac{199 e^4}{8} + \frac{655 e^6}{36} + \frac{13325 e^8}{2304} + \frac{72461 e^{10}}{57600} + \dots$
$4 0 1 4 4 -1$	$\frac{13}{2} + \frac{765 e^2}{16} + \frac{37925 e^4}{384} + \frac{1493005 e^6}{18432} + \frac{3289075 e^8}{98304} + \frac{62149637 e^{10}}{7077888} + \dots$
$4 0 2 4 4 -2$	$\frac{51}{2} + \frac{321 e^2}{2} + \frac{2613 e^4}{8} + \frac{47583 e^6}{160} + \frac{379419 e^8}{2560} + \frac{4247043 e^{10}}{89600} + \dots$
$4 1 -2 4 3 2$	$\frac{3}{\sqrt{1 - e^2} (-4 + 12 e^2 - 12 e^4 + 4 e^6)}$
$4 1 -1 4 3 1$	$\frac{1}{2} + \frac{33 e^2}{16} + \frac{1865 e^4}{384} + \frac{169229 e^6}{18432} + \frac{499765 e^8}{32768} + \frac{4116160969 e^{10}}{176947200} + \dots$
$4 1 0 4 3 0$	$1 + e^2 + \frac{65 e^4}{16} + \frac{139 e^6}{18} + \frac{30311 e^8}{2304} + \frac{588173 e^{10}}{28800} + \dots$
$4 1 1 4 3 -1$	$\frac{9}{2} + \frac{3 e^2}{16} + \frac{963 e^4}{128} + \frac{22737 e^6}{2048} + \frac{2927457 e^8}{163840} + \frac{172753887 e^{10}}{6553600} + \dots$
$4 1 2 4 3 -2$	$\frac{53}{4} + \frac{179 e^2}{24} + \frac{977 e^4}{64} + \frac{81553 e^6}{5760} + \frac{649547 e^8}{27648} + \frac{35552851 e^{10}}{1075200} + \dots$
$4 2 -2 4 2 2$	$\frac{5}{12} + \frac{155 e^2}{32} + \frac{835 e^4}{288} + \frac{12953 e^6}{3456} + \frac{242797 e^8}{53760} + \dots$

$I_{pq}$	$K_{I_{pq}}$
$93296^{-2}$	$\frac{301}{8} + \frac{3743 e^2}{24} + \frac{2016079 e^4}{3072} + \frac{4823461 e^6}{2304} + \frac{20321424821 e^8}{3538944} + \frac{346318732157 e^{10}}{24772608} + \dots$
$94^{-2}952$	$\frac{89}{8} + \frac{961 e^2}{8} + \frac{2072387 e^4}{3072} + \frac{30799309 e^6}{11520} + \frac{49909418551 e^8}{5890240} + \frac{14144477757409 e^{10}}{619315200} + \dots$
$94^{-1}951$	$(64 + 336 e^2 + 280 e^4 + 35 e^6)$ $/(\sqrt{(1 - e^2)(16 - 128 e^2 + 448 e^4 - 896 e^6 + 1120 e^8 - 896 e^{10} + 448 e^{12} - 128 e^{14} + 16 e^{16})})$
$940950$	$1 + \frac{43 e^2}{2} + \frac{11087 e^4}{64} / \left( \frac{994855 e^6}{1152} + \frac{473226805 e^8}{147456} + \frac{71957389547 e^{10}}{7372800} + \dots \right)$
$94195^{-1}$	$6 + \frac{145 e^2}{2} + \frac{10531 e^4}{24} + \frac{88091 e^6}{48} + \frac{34856761 e^8}{5760} + \frac{728862887 e^{10}}{43200} + \dots$
$94295^{-2}$	$\frac{179}{8} + \frac{1559 e^2}{8} + \frac{995215 e^4}{1024} + \frac{2284221 e^6}{640} + \frac{7022778909 e^8}{655360} + \frac{638446109013 e^{10}}{22937600} + \dots$

$l_p q l_p q$	$K_{l_p q}$
9 2 -1 9 7 1	$5 e + \frac{65 e}{3} + \frac{265 e}{3} + \frac{80915 e}{288} + \frac{5296655 e}{6912} + \dots$
9 2 0 9 7 0	$1 - \frac{5 e}{2} + \frac{1655 e}{64} + \frac{69055 e}{1152} + \frac{32227525 e}{147456} + \frac{177233315 e}{294912} + \dots$
9 2 1 9 7 -1	$10 - \frac{55 e}{2} + \frac{915 e}{8} + \frac{3775 e}{32} + \frac{62827 e}{128} + \frac{3005819 e}{2560} + \dots$
9 2 2 9 7 -2	$\frac{455}{8} - \frac{1365 e}{8} + \frac{1432025 e}{3072} + \frac{103243 e}{1152} + \frac{1264544029 e}{1179648} + \frac{37457929817 e}{17694720} + \dots$
9 3 -2 9 6 2	$\frac{31}{8} + \frac{1001 e}{24} + \frac{720481 e}{3072} + \frac{5369863 e}{5760} + \frac{10479640175 e}{3538944} + \frac{4968366161891 e}{619315200} + \dots$
9 3 -1 9 6 1	$2 + \frac{49 e}{2} + \frac{3589 e}{24} + \frac{181457 e}{288} + \frac{12045013 e}{5760} + \frac{2026653877 e}{345600} + \dots$
9 3 0 9 6 0	$1 + \frac{27 e}{2} + \frac{5895 e}{64} + \frac{53239 e}{128} + \frac{23783421 e}{16384} + \frac{3465568467 e}{819200} + \dots$
9 3 1 9 6 -1	$8 + 53 e + \frac{800 e}{3} + \frac{71039 e}{72} + \frac{27043 e}{9} + \frac{1369364531 e}{172800} + \dots$

$l_{pq}$	$K_{lpq}$						
$90299-2$	$\frac{859}{8}$	$\frac{64867 e^2}{24}$	$\frac{73661797 e^4}{3072}$	$\frac{605735603 e^6}{5760}$	$\frac{4671361372183 e^8}{17694720}$	$\frac{255917766969601 e^{10}}{619315200}$	+ ...
$91-2982$	$\frac{11}{8}$	$\frac{49 e^2}{8}$	$\frac{27509 e^4}{3072}$	$\frac{1751 e^6}{1152}$	$\frac{3831427 e^8}{393216}$	$\frac{556580551 e^{10}}{24772608}$	+ ...
$91-1981$	$-2$	$\frac{41 e^2}{2}$	$\frac{475 e^4}{8}$	$\frac{591 e^6}{8}$	$\frac{4509 e^8}{128}$	$\frac{107079 e^{10}}{3200}$	+ ...
$910980$	$1$	$\frac{53 e^2}{2}$	$\frac{10655 e^4}{64}$	$\frac{467369 e^6}{1152}$	$\frac{71447509 e^8}{147456}$	$\frac{2180918677 e^{10}}{7372800}$	+ ...
$91198-1$	$12 - 193$	$\frac{5851 e^2}{6}$	$\frac{35141 e^4}{16}$	$\frac{1897571 e^6}{720}$	$\frac{78885121 e^8}{43200}$		+ ...
$91298-2$	$641$	$\frac{8183 e^2}{8}$	$\frac{4719017 e^4}{1024}$	$\frac{12842891 e^6}{1280}$	$\frac{1616611371 e^8}{131072}$	$\frac{214509553077 e^{10}}{22937600}$	+ ...
$92-2972$	$5$	$\frac{45 e^2}{8}$	$\frac{31005 e^4}{1024}$	$\frac{29885 e^6}{256}$	$\frac{47673655 e^8}{131072}$	$\frac{4464073067 e^{10}}{4587520}$	+ ...

$l_p q l_p q$	$K_{l_p q}$
84-1841	$- + \frac{9 \cdot 783 e^2}{2 \cdot 16} + \frac{33795 e^4}{128} + \frac{2025143 e^6}{2048} + \frac{481921009 e^8}{163840} + \frac{48852422813 e^{10}}{6553600} + \dots$
840840	$- (16 + 168 e^2 + 210 e^4 + 35 e^6)$ $/(\sqrt{1 - e^2} (-16 + 112 e^2 - 336 e^4 + 560 e^6 - 560 e^8 + 336 e^{10} - 112 e^{12} + 16 e^{14}))$
90-2992	$\frac{49}{8} - \frac{1813 e^2}{24} + \frac{940555 e^4}{3072} - \frac{6417187 e^6}{11520} + \frac{9184555933 e^8}{17694720} - \frac{24711168097 e^{10}}{88473600} + \dots$
90-1991	$- 4 + 95 e^2 - \frac{4121 e^4}{6} + \frac{317263 e^6}{144} - \frac{2638483 e^8}{720} + \frac{149923637 e^{10}}{43200} + \dots$
900990	$1 - \frac{117 e^2}{2} + \frac{51327 e^4}{64} - \frac{573825 e^6}{128} + \frac{207360045 e^8}{16384} - \frac{16531322157 e^{10}}{819200} + \dots$
90199-1	$14 - \frac{935 e^2}{2} + \frac{117175 e^4}{24} - \frac{846385 e^6}{36} + \frac{70993175 e^8}{1152} - \frac{333965303 e^{10}}{3456} + \dots$

$l_p q l_p q$	$K_{l_p q}$
8 2 2 8 6 -2	$42 - \frac{149 e^2}{4} + \frac{7005 e^4}{32} + \frac{51513 e^6}{160} + \frac{223569 e^8}{256} + \frac{46990503 e^{10}}{25600} + \dots$
8 3 -2 8 5 2	$- (336 + 560 e^2 + 105 e^4) / (\text{sqrt}(1 - e^2))$ $(- 64 + 448 e^2 - 1344 e^4 + 2240 e^6 - 2240 e^8 + 1344 e^{10} - 448 e^{12} + 64 e^{14})$
8 3 -1 8 5 1	$5 - \frac{433 e^2}{2} + \frac{56877 e^4}{16} + \frac{10099021 e^6}{18432} + \frac{2408719349 e^8}{1474560} + \frac{146849722309 e^{10}}{35389440} + \dots$
8 3 0 8 5 0	$1 + 14 e^2 + \frac{1387 e^4}{16} + \frac{51035 e^6}{144} + \frac{1292339 e^8}{1152} + \frac{171620111 e^{10}}{57600} + \dots$
8 3 1 8 5 -1	$13 - \frac{765 e^2}{2} + \frac{28143 e^4}{128} + \frac{1534433 e^6}{2048} + \frac{343550181 e^8}{163840} + \frac{33392327199 e^{10}}{6553600} + \dots$
8 3 2 8 5 -2	$103 - \frac{3053 e^2}{4} + \frac{46325 e^4}{96} + \frac{4144921 e^6}{2880} + \frac{506333237 e^8}{138240} + \frac{60185662059 e^{10}}{9676800} + \dots$
8 4 -2 8 4 2	$27 - \frac{111 e^2}{2} + \frac{8127 e^4}{16} + \frac{272871 e^6}{160} + \frac{3603907 e^8}{768} + \frac{502522513 e^{10}}{44800} + \dots$

$l p q l p q$	$K_{l p q}$
8 1 0 8 7 0	$1 - 18 e + \frac{1251 e^2}{16} + \frac{1957 e^4}{16} + \frac{3261 e^6}{32} + \frac{126249 e^{10}}{6400} + \dots$
8 1 1 8 7 -1	$\frac{21}{2} - \frac{1911 e^2}{16} + \frac{55055 e^4}{128} + \frac{4070857 e^6}{6144} + \frac{275593591 e^8}{491520} - \frac{13561810053 e^{10}}{58982400} + \dots$
8 1 2 8 7 -2	$\frac{249}{4} - \frac{4643 e^2}{8} + \frac{122621 e^4}{64} - \frac{5731619 e^6}{1920} + \frac{30655519 e^8}{11520} - \frac{1122269891 e^{10}}{806400} + \dots$
8 2 -2 8 6 2	$1 + \frac{103 e^2}{12} + \frac{3899 e^4}{96} + \frac{50363 e^6}{360} + \frac{848807 e^8}{2160} + \frac{4623839569 e^{10}}{4838400} + \dots$
8 2 -1 8 6 1	$\frac{1}{2} + \frac{99 e^2}{16} + \frac{3795 e^4}{128} + \frac{216251 e^6}{2048} + \frac{49870857 e^8}{163840} + \frac{4956626553 e^{10}}{6553600} + \dots$
8 2 0 8 6 0	$1 + 2 e^2 + \frac{47 e^4}{2} + \frac{707 e^6}{9} + \frac{540877 e^8}{2304} + \frac{8501691 e^{10}}{14400} + \dots$
8 2 1 8 6 -1	$\frac{17}{2} - \frac{5 e^2}{16} + \frac{28345 e^4}{384} + \frac{3196075 e^6}{18432} + \frac{138380365 e^8}{294912} + \frac{7635333395 e^{10}}{7077888} + \frac{3579441729 e^{10}}{179200} + \dots$

$l p q l p q$	$K_{l p q}$					
$80-2882$	$9 \cdot 81 e$	$2 \cdot 1845 e$	$4 \cdot 11169 e$	$6 \cdot 106389 e$	$8 \cdot 1345311 e$	$10 \cdot$
	$2$	$2$	$16$	$80$	$1280$	$44800$
$80-1881$	$7 \cdot 1015 e$	$2 \cdot 131831 e$	$4 \cdot 14758807 e$	$6 \cdot 1379540071 e$	$8 \cdot 108056815223 e$	$10 \cdot$
	$2$	$16$	$384$	$18432$	$1474560$	$176947200$
$800880$	$1 - 46 e$	$2 \cdot 1963 e$	$4 \cdot 75595 e$	$6 \cdot 5113919 e$	$8 \cdot 37315013 e$	$10 \cdot$
		$4$	$36$	$1152$	$7200$	
$80188-1$	$25 \cdot 5337 e$	$2 \cdot 352899 e$	$4 \cdot 21222321 e$	$6 \cdot 3416816241 e$	$8 \cdot 6483396051 e$	$10 \cdot$
	$2$	$16$	$128$	$2048$	$163840$	$262144$
$80288-2$	$173 \cdot 10585 e$	$2 \cdot 603415 e$	$4 \cdot 3152105 e$	$6 \cdot 595640575 e$	$8 \cdot 5065352177 e$	$10 \cdot$
	$2$	$6$	$48$	$72$	$6912$	$48384$
$81-2872$	$3 \cdot 13 e$	$2 \cdot 5 e$	$4 \cdot 491 e$	$6 \cdot 115411 e$	$8 \cdot 98179673 e$	$10 \cdot$
	$4$	$8$	$2$	$120$	$9216$	$3225600$
$81-1871$	$3 \cdot 165 e$	$2 \cdot 2225 e$	$4 \cdot 99355 e$	$6 \cdot 542995 e$	$8 \cdot 61204775 e$	$10 \cdot$
	$2$	$16$	$128$	$6144$	$98304$	$2359296$

$l_p q l_p q$	$K_{l_p q}$
7 2 1 7 5 -1	$7 + \frac{27 e^2}{2} + \frac{385 e^4}{6} + \frac{24509 e^6}{144} + \frac{1548977 e^8}{3840} + \frac{294026953 e^{10}}{345600} + \dots$
7 2 2 7 5 -2	$\frac{235}{8} + \frac{1025 e^2}{48} + \frac{147075 e^4}{1024} + \frac{5723405 e^6}{10432} + \frac{2410472275 e^8}{3538944} + \frac{22138823275 e^{10}}{16515072} + \dots$
7 3 -2 7 4 2	$\frac{55}{8} + \frac{2405 e^2}{48} + \frac{207871 e^4}{1024} + \frac{56046421 e^6}{92160} + \frac{26655858011 e^8}{17694720} + \frac{269963728819 e^{10}}{82575360} + \dots$
7 3 -1 7 4 1	$\frac{24 + 60 e^2 + 15 e^4}{\text{sart}(1 - e^2)(8 - 48 e^2 + 120 e^4 - 160 e^6 + 120 e^8 - 48 e^{10} + 8 e^{12})}$
7 3 0 7 4 0	$1 + 13 e^2 + \frac{4535 e^4}{64} + \frac{18397 e^6}{72} + \frac{106003029 e^8}{147456} + \frac{6324397591 e^{10}}{3686400} + \dots$
7 3 1 7 4 -1	$5 + \frac{147 e^2}{4} + \frac{7369 e^4}{48} + \frac{136175 e^6}{288} + \frac{4604159 e^8}{3840} + \frac{45857329 e^{10}}{17280} + \dots$
7 3 2 7 4 -2	$\frac{129}{8} + \frac{1353 e^2}{16} + \frac{302853 e^4}{1024} + \frac{8305863 e^6}{10240} + \frac{1242373407 e^8}{655360} + \frac{181184734241 e^{10}}{45875200} + \dots$

$l_p q l_p q$	$K_{l_p q}$
$71-2762$	$  \begin{array}{r}  \begin{array}{cccccc}  & 2 & 4 & 6 & 8 & 10 \\  3 & 3 e & 2331 e & 68481 e & 11060469 e & 1706068467 e \\  - & + & + & + & + & + \dots  \end{array} \\  \begin{array}{cccccc}  8 & 16 & 1024 & 10240 & 655360 & 45875200  \end{array}  \end{array}  $
$71-1761$	$  \begin{array}{r}  \begin{array}{cccccc}  & 2 & 4 & 6 & 8 & 10 \\  9 e & 13 e & 241 e & 51577 e & 10650431 e & \\  - & 1 + & - & + & - & + \dots  \end{array} \\  \begin{array}{cccccc}  2 & 6 & 36 & 3840 & 345600 &  \end{array}  \end{array}  $
$710760$	$  \begin{array}{r}  \begin{array}{cccccc}  & 2 & 4 & 6 & 8 & 10 \\  1 - 11 e & + & 2015 e & - & 220 e & + & 3523685 e & + & 3230695 e & + \dots  \end{array} \\  \begin{array}{cccccc}  & 64 & 9 & 147456 & 147456 &  \end{array}  \end{array}  $
$71176-1$	$  \begin{array}{r}  \begin{array}{cccccc}  & 2 & 4 & 6 & 8 & 10 \\  267 e & 2607 e & 9651 e & 134619 e & 399951 e & \\  9 - & + & - & + & - & + \dots  \end{array} \\  \begin{array}{cccccc}  4 & 16 & 64 & 1280 & 25600 &  \end{array}  \end{array}  $
$71276-2$	$  \begin{array}{r}  \begin{array}{cccccc}  & 2 & 4 & 6 & 8 & 10 \\  373 & 14281 e & 703881 e & 65327381 e & 8398039433 e & 5944612901 e \\  - & + & - & + & - & + \dots  \end{array} \\  \begin{array}{cccccc}  8 & 48 & 1024 & 92160 & 17694720 & 58982400  \end{array}  \end{array}  $
$72-2752$	$  \begin{array}{r}  \begin{array}{cccccc}  & 2 & 4 & 6 & 8 & 10 \\  13 & 575 e & 50273 e & 13696627 e & 6574316609 e & 37289822893 e \\  - & + & + & + & + & + \dots  \end{array} \\  \begin{array}{cccccc}  8 & 48 & 1024 & 92160 & 17694720 & 45875200  \end{array}  \end{array}  $
$72-1751$	$  \begin{array}{r}  \begin{array}{cccccc}  & 2 & 4 & 6 & 8 & 10 \\  1 + & 33 e & 1709 e & 64559 e & 1109107 e & 447496337 e \\  + & + & + & + & + & + \dots  \end{array} \\  \begin{array}{cccccc}  4 & 48 & 576 & 3840 & 691200 &  \end{array}  \end{array}  $
$720750$	$  \begin{array}{r}  \begin{array}{cccccc}  & 2 & 4 & 6 & 8 & 10 \\  1 + 5 e & + & 1647 e & + & 335 e & + & 3652765 e & + & 209965191 e & + \dots  \end{array} \\  \begin{array}{cccccc}  & 64 & 4 & 16384 & 409600 &  \end{array}  \end{array}  $

$I_{pq}$	$K_{I_{pq}}$
63-1631	$7 \frac{371 e^2}{16} + \frac{32515 e^4}{384} + \frac{4225655 e^6}{18432} + \frac{761278399 e^8}{1474560} + \frac{181404893243 e^{10}}{176947200} + \dots$
630630	$\frac{8 + 40 e^2 + 15 e^4}{\text{sort}(1 - e) (-8 + 40 e^2 - 80 e^4 + 80 e^6 - 40 e^8 + 8 e^{10})}$
70-2772	$25 \frac{925 e^2}{8} + \frac{36125 e^4}{48} + \frac{463525 e^6}{1024} + \frac{31680025 e^8}{18432} + \frac{11921725 e^{10}}{3538944} + \dots$
70-1771	$-3 + \frac{159 e^2}{4} + \frac{2445 e^4}{16} + \frac{969 e^6}{4} + \frac{235401 e^8}{1280} + \frac{505257 e^{10}}{6400} + \dots$
700770	$1 - 35 e^2 + \frac{17927 e^4}{64} + \frac{63245 e^6}{72} + \frac{194651765 e^8}{147456} + \frac{3940556921 e^{10}}{3686400} + \dots$
70177-1	$11 - 228 e^2 + \frac{17341 e^4}{12} + \frac{36794 e^6}{9} + \frac{3880157 e^8}{640} + \frac{56415557 e^{10}}{10800} + \dots$
70277-2	$543 \frac{17505 e^2}{8} + \frac{6238503 e^4}{16} + \frac{167296347 e^6}{1024} + \frac{15988224537 e^8}{10240} + \frac{1022767782417 e^{10}}{655360} + \dots$

$l_p q l_p q$	$K_{l_p q}$
6 1 1 6 5 -1	$\frac{15}{2} \frac{505 e^2}{16} + \frac{19795 e^4}{384} + \frac{32125 e^6}{2048} + \frac{9717515 e^8}{294912} + \frac{383515015 e^{10}}{7077888} + \dots$
6 1 2 6 5 -2	$\frac{133}{4} - \frac{131 e^2}{4} + \frac{13105 e^4}{64} - \frac{36219 e^6}{320} + \frac{93879 e^8}{1024} - \frac{676209 e^{10}}{12800} + \dots$
6 2 -2 6 4 2	$\frac{10 + 5 e^2}{\text{sqrt}(1 - e^2) (-4 + 20 e^2 - 40 e^4 + 40 e^6 - 20 e^8 + 4 e^{10})}$
6 2 -1 6 4 1	$\frac{3}{2} + \frac{161 e^2}{16} + \frac{14239 e^4}{384} + \frac{621595 e^6}{6144} + \frac{67628063 e^8}{294912} + \frac{81019919057 e^{10}}{176947200} + \dots$
6 2 0 6 4 0	$1 + \frac{13 e^2}{2} + \frac{419 e^4}{16} + \frac{21697 e^6}{288} + \frac{408937 e^8}{2304} + \frac{42045317 e^{10}}{115200} + \dots$
6 2 1 6 4 -1	$\frac{11}{2} + \frac{277 e^2}{16} + \frac{7093 e^4}{128} + \frac{278093 e^6}{2048} + \frac{47240923 e^8}{163840} + \frac{3609377983 e^{10}}{6553600} + \dots$
6 2 2 6 4 -2	$19 + \frac{35 e^2}{48} + \frac{4997 e^4}{48} + \frac{40543 e^6}{180} + \frac{5098781 e^8}{11520} + \frac{964570001 e^{10}}{1209600} + \dots$
6 3 -2 6 3 2	$\frac{35}{4} + \frac{133 e^2}{3} + \frac{6727 e^4}{48} + \frac{498547 e^6}{1440} + \frac{10115105 e^8}{13824} + \frac{47930237 e^{10}}{34560} + \dots$

$l p q / p q$	$K_{l p q}$				
	2	4	6	8	10
$6 0 -2 6 6 2$	$2 - \frac{23 e}{3} + \frac{185 e}{24} - \frac{1909 e}{720} + \frac{2399 e}{3456} - \frac{13423 e}{1209600} + \dots$				
$6 0 -1 6 6 1$	$5 - \frac{365 e}{2} + \frac{22145 e}{16} - \frac{1030925 e}{384} + \frac{7379545 e}{18432} - \frac{50465285 e}{294912} + \dots$				
$6 0 0 6 6 0$	$1 - \frac{51 e}{2} + \frac{2331 e}{16} - \frac{10079 e}{32} + \frac{80043 e}{256} - \frac{2114073 e}{12800} + \dots$				
$6 0 1 6 6 -1$	$19 - \frac{2359 e}{2} + \frac{263935 e}{16} - \frac{25687879 e}{384} + \frac{2128113547 e}{18432} - \frac{152987554951 e}{1474560} + \dots$				
$6 0 2 6 6 -2$	$103 - \frac{1907 e}{2} + \frac{64079 e}{3} - \frac{64079 e}{24} + \frac{1907333 e}{360} - \frac{199062983 e}{34560} + \frac{1157658989 e}{302400} + \dots$				
$6 1 -2 6 5 2$	$1 - \frac{2}{4} + \frac{677 e}{192} + \frac{26671 e}{2880} + \frac{104891 e}{5120} - \frac{97317793 e}{2419200} + \dots$				
$6 1 -1 6 5 1$	$1 - \frac{31 e}{2} + \frac{305 e}{16} - \frac{15579 e}{128} + \frac{2779023 e}{2048} - \frac{221939397 e}{163840} + \dots$				
$6 1 0 6 5 0$	$1 - \frac{11 e}{2} + 11 e - \frac{17 e}{9} + \frac{34897 e}{2304} - \frac{3260153 e}{115200} + \dots$				

$I_{pq}$	$K_{I_{pq}}$
5 1 0 5 4 0	$1 - \frac{3e^2}{2} + \frac{303e^4}{64} - \frac{925e^6}{128} + \frac{240493e^8}{16384} - \frac{22151757e^{10}}{819200} + \dots$
5 1 1 5 4 -1	$6 - \frac{21e^2}{2} + \frac{31e^4}{2} - \frac{929e^6}{96} + \frac{45443e^8}{1920} - \frac{4444903e^{10}}{115200} + \dots$
5 1 2 5 4 -2	$\frac{177}{8} - \frac{177e^2}{4} + \frac{52945e^4}{1024} - \frac{7739e^6}{3072} + \frac{14847329e^8}{393216} - \frac{108103973e^{10}}{2064384} + \dots$
5 2 -2 5 3 2	$\frac{29}{8} + \frac{193e^2}{12} - \frac{135175e^4}{3072} + \frac{4381873e^6}{46080} - \frac{3150020521e^8}{17694720} + \frac{46806205751e^{10}}{154828800} + \dots$
5 2 -1 5 3 1	$\frac{4 + 3e^2}{\sqrt{(1 - e^2)(2 - 8e^2 + 12e^4 - 8e^6 + 2e^8)}}$
5 2 0 5 3 0	$1 + \frac{13e^2}{2} + \frac{1399e^4}{64} + \frac{61837e^6}{1152} + \frac{16171237e^8}{147456} + \frac{1463469397e^{10}}{7372800} + \dots$
5 2 1 5 3 -1	$4 + \frac{29e^2}{2} + \frac{467e^4}{12} + \frac{24181e^6}{288} + \frac{909551e^8}{5760} + \frac{46630727e^{10}}{172800} + \dots$
5 2 2 5 3 -2	$\frac{87}{8} + \frac{107e^2}{4} - \frac{64539e^4}{1024} + \frac{633603e^6}{5120} - \frac{143148297e^8}{655360} + \frac{127755531e^{10}}{358400} + \dots$

$l_p q l_p q$	$K_{l_p q}$
4 2 -1 4 2 1	$5 \frac{135 e^2}{2} + \frac{7285 e^4}{384} + \frac{643015 e^6}{18432} + \frac{5591339 e^8}{98304} + \frac{3020339579 e^{10}}{35389440} + \dots$
4 2 0 4 2 0	$\frac{2 + 3 e^2}{\text{sqrt}(1 - e^2) (-2 + 6 e^2 - 6 e^4 + 2 e^6)}$
5 0 -2 5 5 2	$9 \frac{9 e^2}{8} + \frac{873 e^4}{4} + \frac{891 e^6}{5120} + \frac{3807 e^8}{655360} + \frac{7029 e^{10}}{819200} + \dots$
5 0 -1 5 5 1	$-2 + \frac{23 e^2}{2} + \frac{101 e^4}{6} + \frac{2401 e^6}{288} + \frac{12799 e^8}{5760} + \frac{104137 e^{10}}{345600} + \dots$
5 0 0 5 5 0	$1 - \frac{35 e^2}{2} + \frac{4255 e^4}{64} + \frac{104075 e^6}{1152} + \frac{7873525 e^8}{147456} + \frac{5269187 e^{10}}{294912} + \dots$
5 0 1 5 5 -1	$8 \frac{177 e^2}{2} + \frac{573 e^4}{2} + \frac{3109 e^6}{8} + \frac{168009 e^8}{640} + \frac{1352919 e^{10}}{12800} + \dots$
5 0 2 5 5 -2	$299 \frac{4067 e^2}{8} + \frac{3146269 e^4}{3072} + \frac{65724449 e^6}{46080} + \frac{19022800279 e^8}{17694720} + \frac{1393243901 e^{10}}{2764800} + \dots$
5 1 -2 5 4 2	$3 \frac{7 e^2}{8} + \frac{5111 e^4}{4} + \frac{170717 e^6}{15360} + \frac{125537797 e^8}{5898240} + \frac{118680949 e^{10}}{3225600} + \dots$
5 1 -1 5 4 1	$3 \frac{e^2}{2} + \frac{4 e^4}{16} + \frac{147 e^6}{384} + \frac{6899 e^8}{23040} + \frac{727813 e^{10}}{23040} + \dots$

$l_p q l_p q$	$K_{l_p q}$											
$10\ 0\ -2\ 10\ 10\ 2$	$8 -$	$\frac{388\ e}{3}$	$+$	$\frac{1409\ e}{2}$	$+$	$\frac{321229\ e}{180}$	$+$	$\frac{2594831\ e}{1080}$	$+$	$\frac{47407249\ e}{25200}$	$+$	$\dots$
$10\ 0\ -1\ 10\ 10\ 1$	$9 -$	$\frac{2169\ e}{2}$	$+$	$\frac{161343\ e}{128}$	$+$	$\frac{10068481\ e}{2048}$	$+$	$\frac{1946663721\ e}{163840}$	$+$	$\frac{101493784107\ e}{6553600}$	$+$	$\dots$
$10\ 0\ 0\ 10\ 10\ 0$	$1 -$	$\frac{145\ e}{2}$	$+$	$\frac{19865\ e}{16}$	$+$	$\frac{2527475\ e}{288}$	$+$	$\frac{36684775\ e}{1152}$	$+$	$\frac{153124733\ e}{2304}$	$+$	$\dots$
$10\ 0\ 1\ 10\ 10\ -1$	$31 -$	$\frac{10131\ e}{2}$	$+$	$\frac{3129731\ e}{384}$	$+$	$\frac{901040371\ e}{18432}$	$+$	$\frac{79489366957\ e}{491520}$	$+$	$\frac{57385925962211\ e}{176947200}$	$+$	$\dots$
$10\ 0\ 2\ 10\ 10\ -2$	$261 -$	$\frac{7947\ e}{2}$	$+$	$\frac{686997\ e}{16}$	$+$	$\frac{1157274\ e}{5}$	$+$	$\frac{1847400507\ e}{2560}$	$+$	$\frac{126939453777\ e}{89600}$	$+$	$\dots$
$10\ 1\ -2\ 10\ 9\ 2$	$9 -$	$\frac{63\ e}{4}$	$+$	$\frac{2241\ e}{64}$	$+$	$\frac{4833\ e}{160}$	$+$	$\frac{12987\ e}{640}$	$+$	$\frac{495207\ e}{44800}$	$+$	$\dots$
$10\ 1\ -1\ 10\ 9\ 1$	$5 -$	$\frac{579\ e}{2}$	$+$	$\frac{59689\ e}{384}$	$+$	$\frac{5312279\ e}{18432}$	$+$	$\frac{25620931\ e}{98304}$	$+$	$\frac{5441780687\ e}{35389440}$	$+$	$\dots$

$l p q l p q$	$K_{l p q}$
$10\ 1\ 0\ 10\ 9\ 0$	$1 - \frac{73 e^2}{2} + \frac{2521 e^4}{8} - \frac{156895 e^6}{144} + \frac{67187 e^8}{36} - \frac{1582691 e^{10}}{900} + \dots$
$10\ 1\ 1\ 10\ 9\ -1$	$27 \frac{4653 e^2}{2} + \frac{252189 e^4}{128} - \frac{12448593 e^6}{2048} + \frac{1652177691 e^8}{163840} - \frac{64905473151 e^{10}}{6553600} + \dots$
$10\ 1\ 2\ 10\ 9\ -2$	$401 \frac{20101 e^2}{4} + \frac{633685 e^4}{64} - \frac{8264365 e^6}{288} + \frac{163340485 e^8}{3456} - \frac{780075869 e^{10}}{16128} + \dots$
$10\ 2\ -2\ 10\ 8\ 2$	$1 \frac{19 e^2}{2} + \frac{335 e^4}{16} + \frac{3992 e^6}{45} - \frac{20986301 e^8}{69120} + \frac{238193761 e^{10}}{268800} + \dots$
$10\ 2\ -1\ 10\ 8\ 1$	$1 \frac{93 e^2}{2} + \frac{4291 e^4}{16} + \frac{1274365 e^6}{18432} - \frac{7647523 e^8}{32768} + \frac{4919413781 e^{10}}{7077888} + \dots$
$10\ 2\ 0\ 10\ 8\ 0$	$1 \frac{17 e^2}{2} + \frac{681 e^4}{16} + \frac{437 e^6}{32} - \frac{25741 e^8}{128} + \frac{3430701 e^{10}}{6400} + \dots$
$10\ 2\ 1\ 10\ 8\ -1$	$23 \frac{1143 e^2}{2} + \frac{94339 e^4}{384} - \frac{1984871 e^6}{18432} + \frac{286202917 e^8}{491520} - \frac{191661701273 e^{10}}{176947200} + \dots$

$l p q l p q$	$K_{lpq}$
$10\ 2\ 2\ 10\ 8\ -2$	$74 - \frac{1246\ e^2}{3} + \frac{9563\ e^4}{8} - \frac{179773\ e^6}{180} + \frac{1998401\ e^8}{1080} + \frac{45645853\ e^{10}}{25200} + \dots$
$10\ 3\ -2\ 10\ 7\ 2$	$11 - \frac{401\ e^2}{4} + \frac{6747\ e^4}{12} + \frac{1343161\ e^6}{32} + \frac{452148163\ e^8}{1440} + \frac{15659474077\ e^{10}}{138240} + \dots$
$10\ 3\ -1\ 10\ 7\ 1$	$3 - \frac{327\ e^2}{2} + \frac{17613\ e^4}{16} + \frac{1308531\ e^6}{128} + \frac{381139059\ e^8}{2048} + \frac{46648204029\ e^{10}}{163840} + \dots$
$10\ 3\ 0\ 10\ 7\ 0$	$1 + \frac{23\ e^2}{2} + \frac{703\ e^4}{8} + \frac{61919\ e^6}{144} + \frac{3762383\ e^8}{2304} + \frac{594945653\ e^{10}}{115200} + \dots$
$10\ 3\ 1\ 10\ 7\ -1$	$19 - \frac{783\ e^2}{2} + \frac{109991\ e^4}{16} + \frac{20778515\ e^6}{384} + \frac{364006933\ e^8}{18432} + \frac{74380337167\ e^{10}}{98304} + \dots$
$10\ 3\ 2\ 10\ 7\ -2$	$207 - \frac{591\ e^2}{4} + \frac{6315\ e^4}{4} + \frac{417141\ e^6}{8} + \frac{39309261\ e^8}{160} + \frac{13098171383\ e^{10}}{5120} + \dots$

lpq pq	K <sub>lpq</sub>
10 4 -2 10 6 2	$-(288 + 1008 e^2 + 630 e^4 + 63 e^6)$
10 4 -1 10 6 1	$\frac{7}{2} + \frac{897 e^2}{16} + \frac{160547 e^4}{384} + \frac{38138513 e^6}{18432} + \frac{1288138311 e^8}{163840} + \frac{4385825119633 e^{10}}{176947200} + \dots$
10 4 0 10 6 0	$1 + \frac{47 e^2}{2} + \frac{3377 e^4}{16} + \frac{337285 e^6}{288} + \frac{1386901 e^8}{288} + \frac{463529633 e^{10}}{28800} + \dots$
10 4 1 10 6 -1	$\frac{15}{2} + \frac{1509 e^2}{16} + \frac{79137 e^4}{128} + \frac{5772381 e^6}{2048} + \frac{331616595 e^8}{32768} + \frac{40132348803 e^{10}}{1310720} + \dots$
10 4 2 10 6 -2	$\frac{67}{2} + \frac{1753 e^2}{6} + \frac{6191 e^4}{4} + \frac{274574 e^6}{45} + \frac{1362223271 e^8}{69120} + \frac{14785480307 e^{10}}{268800} + \dots$
10 5 -2 10 5 2	$\frac{77}{4} + \frac{2783 e^2}{12} + \frac{92763 e^4}{64} + \frac{9146203 e^6}{1440} + \frac{762793537 e^8}{34560} + \frac{13098171383 e^{10}}{201600} + \dots$

lpqpa	Klpq
$10\ 5\ -1\ 10\ 5\ 1$	$\frac{11}{2} + \frac{1419 e^2}{16} + \frac{254551 e^4}{384} + \frac{60493279 e^6}{18432} + \frac{6126632897 e^8}{491520}$ $+ \frac{6947140035539 e^{10}}{176947200} + \dots$
$10\ 5\ 0\ 10\ 5\ 0$	$- (128 + 2304 e^2 + 6048 e^4 + 3360 e^6 + 315 e^8)$ $/ ( \text{sart}(1 - e^2) (-128 + 1152 e^2 - 4608 e^4 + 10752 e^6 - 16128 e^8 + 16128 e^{10} - 10752 e^{12} + 4608 e^{14} - 1152 e^{16} + 128 e^{18}) )$

## DISTRIBUTION

	Copies		Copies
Naval Space Command Dahlgren, VA 22448 - 5170	5	Office of Naval Operations Navy Space Systems Division (NOP-943) Washington, DC 20350	2
Library of Congress ATTN: Gift and Exchange Division Washington, DC 20540	4	Naval Research Laboratory ATTN: Mr. Al Bartholomew Washington, DC 20375	2
Defense Mapping Agency ATTN: Mr. Jack Callander Washington, DC 20305	10	Naval Oceanographic Office Bay St. Louis, MS 39522	2
Defense Mapping Agency Hydrographic, Topographic Center ATTN: Dr. Patrick Fell Washington, DC 20390	10	Office of Naval Research Physical Sciences Division 800 N. Quincy St. Arlington, VA 22217	2
Defense Mapping Agency Acrospace Center ATTN: Dr. Robert Ballew St. Louis, MO 61118	10	Air Force Geophysics Laboratory Hanscom Field Bedford, MA 01731	2
Office of Chief of Naval Operations Naval Oceanography Division (NOP-952) Bldg. 1, U. S. Naval Observatory Washington, DC 20390	2	Goddard Space Flight Center ATTN: Dr. David Smith Greenbelt, MD 20771	1
Applied Research Laboratory University of Texas ATTN: Dr. Arnold Tucker Austin, TX 78712	1	Jet Propulsion Laboratory ATTN: Dr. William Melbourne Pasadena, CA 91103	1
Physical Sciences Laboratory New Mexico State University Box 3 - PSI ATTN: Dan Martin Las Cruces, NM 88003	1	The University of Texas at Austin ATTN: Dr. Byron Tapley Austin, TX 78712	1

## DISTRIBUTION (Continued)

	Copies		Copies
Applied Physics Laboratory Johns Hopkins University Johns Hopkins Road ATTN: Harold Black Laurel, MD 20707	1	Minor Planet Center Smithsonian Astrophysical Observatory ATTN: B. G. Marsden Cambridge, MA 02138	1
Institute for Laboratory Astrophysics University of Colorado ATTN: Dr. Peter Bender Boulder, CO 80309	1	F. Peters 1017 Derwydd Lane Berwyn, PA 19312	1
U. S. Naval Observatory ATTN: W. L. Klepczaski D. D. McCarthy P. K. Seidelmann Washington, DC 20360	1 1 2	Directorate of Astrodynamics Headquarters Space Command Peterson Air Force Base, CO 80914	1
Naval Space Surveillance System Dahlgren, VA 22448 - 5180	2	Department of Astronomy 525 Davey Laboratory Pennsylvania State University University Park, PA 16802	1
Headquarters Space Division (AFCS) Los Angeles Air Force Station Box 92960 Worldway Postal Center Los Angeles, CA 90009	2	University of Virginia Department of Mechanical and Aerospace Engineering ATTN: H. S. Morton Charlottesville, VA 22804	1
Mike Staley Mail Zone 602 IBM - Dept. ND4 18100 Frederick Pike Gaithersburg, MD 20879	1	Internal Distribution: E211 (Green) E211 (Wiggen) E231 E31 (GIDEP) F14 K05 K10 K12 K13 K14 K40 K107	1 1 9 1 4 1 2 5 35 5 1 1
Dr. Richard Pavelle MACSYMA Group Symbolics, Inc. 257 Vassar St. Cambridge, MA 02139	1		

**END**

**FILMED**

**6-85**

**DTIC**