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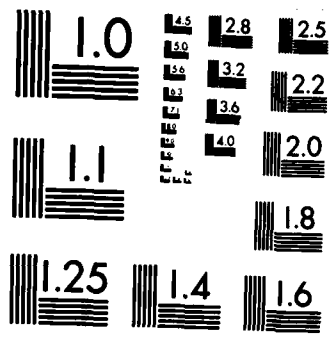
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TABLES FOR DISTRIBUTIONS OF QUADRATIC FORMS

BY

MARY ELLEN BOCK and HERBERT SOLOMON

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For the Office of Naval Research

Herbert Solomon, Project Director

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DEPARTMENT OF STATISTICS
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Tables For Distributions Of Quadratic Forms

by

Mary Ellen Bock and Herbert Solomon

1. Introduction.

The tables given here provide probabilities of the form

$$P\left[\sum_{i=1}^n a_i {}^{(i)}\chi_1^2 \leq t\right]$$

where $a_i > 0$, $\sum_{i=1}^n a_i = 1$ and $a_i {}^{(i)}\chi_1^2$ variables with one degree of freedom, for $n=2,3$.

The probabilities are needed for a variety of problems. Solomon (1960) illustrates their use in hitting probabilities. Moore and Spruill (1975) survey their use for the asymptotic distribution of certain chi-square goodness-of-fit tests. They also arise as the distribution of the trace of a Wishart matrix as noted by Pillai (1973). Bock (1984) discusses other applications.

Section 2 provides formulas for the missing table values as well as formulas that reduce probabilities of the form

$$P\left[\sum_{i=1}^n a_i {}^{(i)}\chi_1^2 < t\right]$$

for $n > 3$ to expressions involving probabilities found in this table (or in tables of Johnson and Kotz (1967, 1968)).

Some of the values from these tables are given in Owen (1962). Harter (1960) gives a table for a linear combination of two independent chi-square variables. Marsaglia (1960) gives tables for linear combinations of two or three independent chi-square variables each with one degree of freedom.

The extensive tables for $n = 2, 3$ given in this report were given in an unpublished Technical Report, Solomon (1960). A number of requests over the years plus the additional uses for it discussed in this paper and by Bock (1984) suggest that its republication is merited.

2. Formulas for Missing Values.

The following formulas enable one to fill in some of the missing values in the tables.

An unpublished result of David Kleinecke (see Ruben (1962)) proves that for $0 < p < 1$,

$$(1) \quad P[p^{(1)} \chi_1^2 + (1-p)^{(2)} \chi_1^2 < t] \\ = P[\chi_{2,A}^2 < B] - P[\chi_{2,B}^2 < A]$$

where

$$A = \frac{t}{4} (p^{-1/2} - (1-p)^{-1/2})^2$$

and

$$B = \frac{t}{4} (p^{-1/2} + (1-p)^{-1/2})^2$$



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where $(j) \chi_1^2$ are independent chi-square variables each with one degree of freedom for $j=1$ and 2 and $\chi_{2,\lambda}^2$ is a noncentral chi-square random variable with two degrees of freedom and noncentrality parameter λ .

(Thus for $X \sim N(\theta, I_2)$, we have

$$\|x\|^2 = x_1^2 + x_2^2 \sim \chi_{2,\lambda}^2$$

with

$$\lambda = \|\theta\|^2 = \theta_1^2 + \theta_2^2 .)$$

Haynam, Govindarajulu, Leone and Siefert (1982,1983) give extensive tables for noncentral chi-square distributions.

The distribution of a linear combination of three independent chi-square random variables each with one degree of freedom is simplified if exactly two of the three coefficients are equal by the results of Bock (1984). For $0 < p < 1/3$,

$$\begin{aligned} (2) \quad & P[p^{(1)} \chi_1^2 + p^{(2)} \chi_1^2 + (1-2p)^{(3)} \chi_1^2 > t] \\ &= P[\chi_1^2 > t/(1-2p)] + e^{-t/2(1-2p)} \left(\frac{4p}{\pi(1-3p)} \right)^{1/2} \mathcal{D}(x) \end{aligned}$$

where

$$x = \left[\frac{t}{2} (p^{-1} - (1-2p)^{-1}) \right]^{1/2}$$

and $\mathcal{D}(y)$ is Dawson's function tabled in Abramowitz and Stegun (1964), i.e.

$$D(y) = e^{-y^2} \int_0^y e^{u^2} du .$$

For $\frac{1}{3} < p < \frac{1}{2}$,

$$\begin{aligned} (3) \quad & P[p^{(1)}\chi_1^2 + p^{(2)}\chi_1^2 + (1-2p)^{(3)}\chi_1^2 < t] \\ & = P[\chi_1^2 > t/(1-2p)] + e^{-t/2p} \left(\frac{p}{3p-1}\right)^{1/2} P[\chi_1^2 < t((1-2p)^{-1} - p^{-1})] \end{aligned}$$

The values in these tables may also be used to fill in certain missing values in the tables of Johnson and Kotz (1967,1968) for the cumulative distribution functions of linear combination of four or five independent chi-square random variables each with one degree of freedom. In particular, when several of the coefficients in the linear combination are the same, the following formulas from Bock (1984) apply.

For a linear combination of size $n \geq 3$ independent chi-square variables, when the two largest coefficients c_0 are equal

(i.e. $c_0 > \max_{1 \leq i \leq n-2} \{c_i\}$) we have

$$\begin{aligned} (4) \quad & P\left[\sum_{i=1}^{n-2} c_i^{(1)}\chi_1^2 + c_0^{(n-1)}\chi_1^2 + c_0^{(n)}\chi_1^2 < t\right] \\ & = P\left[\sum_{i=1}^{n-2} c_i^{(1)}\chi_1^2 < t\right] - \alpha P\left[\sum_{i=1}^{n-2} c_i^{*(1)}\chi_1^2 < t\right] \end{aligned}$$

where

$$c_i^* = (c_i^{-1} - c_0^{-1})^{-1}$$

and

$$\alpha = e^{-t/2c_0} \prod_{i=1}^{n-2} \left(\frac{c_i^*}{c_i} \right)^{1/2}.$$

Thus if $n=4$ or 5 , values in the Solomon table may be used to fill in missing values in the Johnson and Kotz tables. (If $n=6$ or 7 , the formula (4) enables one to use the Johnson and Kotz tables to provide the missing probabilities.)

For a linear combination of five independent chi-square variables when $c_1 = c_2 = c_3$ and $c_4 = c_5$, the formula may be written as

$$\begin{aligned} (5) \quad & P[c_1^{(1)} \chi_1^2 + c_1^{(2)} \chi_1^2 + c_1^{(3)} \chi_1^2 + c_5^{(4)} \chi_1^2 + c_5^{(5)} \chi_1^2 < t] \\ &= \frac{c_1}{(c_1 - c_5)} P[\chi_3^2 < \frac{t}{c_1}] \\ &+ \frac{c_5}{(c_5 - c_1)} P[c_1^{(1)} \chi_1^2 + c_5^{(2)} \chi_1^2 + c_5^{(3)} \chi_1^2 < t] \end{aligned}$$

If in a linear combination of five independent chi-square variables there are exactly three distinct coefficients so that $c_1 = c_2$, $c_3 = c_4$, it can be shown that

$$\begin{aligned} (6) \quad & P[c_1^{(1)} \chi_1^2 + c_1^{(2)} \chi_1^2 + c_3^{(3)} \chi_1^2 + c_3^{(4)} \chi_1^2 + c_5^{(5)} \chi_1^2 > t] \\ &= \frac{c_1^2}{(c_1 - c_5)(c_1 - c_3)} P[c_1^{(1)} \chi_1^2 + c_1^{(2)} \chi_1^2 + c_5^{(3)} \chi_1^2 > t] \\ &+ \frac{c_3^2}{(c_3 - c_5)(c_3 - c_1)} P[c_3^{(1)} \chi_1^2 + c_3^{(2)} \chi_1^2 + c_5^{(3)} \chi_1^2 > t]. \end{aligned}$$

3. Examples.

Example 1. This example shows how one might fill in two missing values in these tables for a linear combination of three independent chi-square random variables when two of the coefficients are alike.

Using equation (3),

$$P[.45^{(1)}\chi_1^2 + .45^{(2)}\chi_1^2 + .1^{(3)}\chi_1^2 < .75]$$

$$= P[\chi_1^2 > 7.5] + .4928P[\chi_1^2 < 5.833]$$

$$= .4928.$$

Note that in the tables $P[.4^{(1)}\chi_1^2 + .4^{(2)}\chi_1^2 + .2^{(3)}\chi_1^2 < .75] = .4880$.

Using equation (2),

$$P[.25^{(1)}\chi_1^2 + .25^{(2)}\chi_1^2 + .5^{(3)}\chi_1^2 > 1.19]$$

$$= P[\chi_1^2 > 2.38] + .3433D(1.09)$$

$$= .3044, \text{ or}$$

$$P[.25^{(1)}\chi_1^2 + .25^{(2)}\chi_1^2 + .5^{(3)}\chi_1^2 < 1.19] = .6956 .$$

Note that in the Tables

$$P[.1^{(1)}\chi_1^2 + .4^{(2)}\chi_1^2 + .5^{(3)}\chi_1^2 < 1.19] = .6987 .$$

Example 2. This example shows how the tables in this report might be used to fill in a probability that is not in the Johnson and Kotz tables, using equation (4). (It shows that the formula is robust for substitution of approximate values of the probabilities in these tables.) The following probability

$$(*) = P[1.8^{(1)}\chi_1^2 + 1.8^{(2)}\chi_1^2 + .6^{(3)}\chi_1^2 + .5^{(4)}\chi_1^2 + .3^{(5)}\chi_1^2 < 9.930]$$

is tabled in Johnson and Kotz (1968) as .9.

According to equation (4), this may be written as

$$(*) = P[.429^{(1)}\chi_1^2 + .357^{(2)}\chi_1^2 + .214^{(3)}\chi_1^2 < 7.09] \\ - (.10)P[.461^{(1)}\chi_1^2 + .355^{(2)}\chi_1^2 + .184^{(3)}\chi_1^2 < 5.08] .$$

The particular coefficients given for the linear combinations of three independent chi-square variables do not appear in the tables of Solomon. However, rough approximations such as substituting

$$P[.4^{(1)}\chi_1^2 + .4^{(2)}\chi_1^2 + .2^{(3)}\chi_1^2 < 7]$$

for the first probability and

$$P[.5^{(1)}\chi_1^2 + .3^{(2)}\chi_1^2 + .2^{(3)}\chi_1^2 < 5]$$

for the second probability in the last representation for (*) give the value .9001.

The value .5 is given in Johnson and Kotz (1968) for

$$(***) = P[1.8^{(1)}\chi_1^2 + 1.8^{(2)}\chi_1^2 + .6^{(3)}\chi_1^2 + .5^{(4)}\chi_1^2 + .3^{(5)}\chi_1^2 < 4.016] .$$

By equation (4), this is

$$(***) = P[.429^{(1)}\chi_1^2 + .357^{(2)}\chi_1^2 + .214^{(3)}\chi_1^2 < 2.8686] \\ - (.5165)P[.461^{(1)}\chi_1^2 + .355^{(2)}\chi_1^2 + .184^{(3)}\chi_1^2 < 2.0562] .$$

Substituting $P[.4^{(1)}\chi_1^2 + .4^{(2)}\chi_1^2 + .2^{(3)}\chi_1^2 < 2.85]$ for the first probability in (***) and $P[.5^{(1)}\chi_1^2 + .3^{(2)}\chi_1^2 + .2^{(3)}\chi_1^2 < 2.05]$ for the second probability in (***), we have .4998 as an estimate of (***) .

For the probability

$$(**) = P[1.8^{(1)}\chi_1^2 + 1.8^{(2)}\chi_1^2 + .6^{(3)}\chi_1^2 + .5^{(4)}\chi_1^2 + .3^{(5)}\chi_1^2 < .936]$$

tabled in Johnson and Kotz as .05, we have

$$(**) = P[.429^{(1)}\chi_1^2 + .357^{(2)}\chi_1^2 + .214^{(3)}\chi_1^2 < .6683] \\ - (1.2152)P[.461^{(1)}\chi_1^2 + .355^{(2)}\chi_1^2 + .184^{(3)}\chi_1^2 < .4788] .$$

Substituting $P[.4 \chi_1^{(1)2} + .4 \chi_1^{(1)2} + .2 \chi_1^{(3)2} < .67]$ for the first probability in (**) and $P[.5 \chi_1^{(1)2} + .3 \chi_1^{(2)2} + .2 \chi_1^{(3)2} < .48]$ for the second probability in (**), we have .0510 as the estimate of (**).

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Table 1
 $P[a_1^{(1)} \chi_1^2 + a_2^{(2)} \chi_1^2 \leq t]$

a_2, a_1					
.8,.2	.875,.125	.9,.1	.95,.05	.99,.01	1,0
93149416	92433215	92179771	91663484	91246447	91142042
93279256	92564658	92312098	91797755	91382308	91278275
93406451	92693645	92442015	91929691	91515852	91412242
93531063	92820231	92569577	92059340	91647194	91543986
93653149	92944464	92694825	92186744	91776330	91673546
93772767	93066393	92817810	92311955	91903321	91800966
93889971	93186067	92938577	92435009	92028198	91926284
94074816	93303532	93057168	92555948	92150985	92049539
94117355	93418830	93173630	92674807	92271765	92170769
94227637	93532008	93288002	92791637	92390515	92290011
94335713	93643110	93400327	92906472	92507333	92407302
94441633	93752176	93510646	93019346	92622204	92522677
94545444	93859247	93618998	93130304	92735231	92636171
94647190	93964363	93725420	93239376	92846345	92747818
94746918	94067566	93829953	93346595	92955674	92857652
94844671	94168892	93932631	93452001	93063201	92965705
94940492	94268378	94033492	93555627	93168994	93072009
95034424	94366062	94132570	93657506	93273033	93176596
95126506	94461979	94229903	93757669	93375405	93279496
95216779	94556164	94325520	93856147	93476114	93380739
95305281	94648651	94419454	93952974	93575191	93480355
95392049	94739474	94511742	94048175	93672638	93578373
95477123	94828665	94602412	94141789	93768529	93674820
95560535	94916256	94691496	94233836	93862880	93769725
95642323	95002279	94779023	94324350	93955710	93863114
95801161	95169740	94949527	94500883	94136908	94045452
95953903	95331284	95114154	94671599	94312352	94222040
96100799	95487137	95273124	94836721	94482216	94393076
96242091	95637517	95426645	94996435	94646726	94558750
96378010	95782629	95574919	95150943	94806052	94719246
96508773	95922672	95718140	95300421	94960359	94874739
96634592	96057837	95856490	95445047	95109840	95025398
96755667	96188305	95990151	95584992	95254633	95171386
96872188	96314252	96119295	95720418	95394928	95312860
96984338	96435845	96244081	95851484	95530832	95449970
97092291	96553245	96364670	95978335	95662570	95582862
97196216	96666606	96481210	96101125	95790169	95711675
97296272	96776076	96593846	96219988	95913869	95836544
97392611	96881796	96702719	96335059	96033776	95957598
97485380	96983904	96807962	96446468	96149937	96074963

Table 1
 $P[a_1^{(1)} \chi_1^2 + a_2^{(2)} \chi_1^2 \leq t]$

a_2, a_1					
t	.5, .5	.6, .4	$\frac{2}{3}, \frac{1}{3}$	.7, .3	.75, .25
2.90	94497680	94352595	94088753	93904350	93562601
2.925	94633530	94486970	94221035	94035584	93692682
2.95	94766030	94618090	94350229	94163834	93819951
2.975	94895260	94746036	94476410	94289174	93944476
3.00	95021290	94870885	94599652	94411674	94066323
3.025	95144220	94992713	94720026	94531403	94185554
3.05	95264110	95111595	94837603	94648428	94302229
3.075	95381040	95227605	94952450	94762815	94416410
3.10	95495080	95340811	95064635	94874626	94528155
3.125	95606310	95451284	95174222	94983923	94637519
3.15	95714790	95559092	95281277	95090767	94744560
3.175	95820290	95664298	95385856	95195216	94849329
3.20	95923780	95766969	95488022	95297328	94951880
3.225	96024420	95867163	95587835	95397160	95052265
3.25	96122580	95964946	95685348	95494765	95150532
3.275	96218310	96060373	95780621	95590194	95246730
3.30	96311680	96153504	95873704	95683501	95340907
3.325	96402750	96244395	95964652	95774736	95433108
3.35	96491560	96333100	96053515	95863947	95523378
3.375	96578190	96419673	96140344	95951183	95611762
3.40	96662670	96504168	96225189	96036489	95698302
3.425	96745070	96586632	96308095	96119912	95783038
3.45	96825440	96667118	96389110	96201495	95866014
3.475	96903830	96745673	96468277	96281281	95947266
3.50	96980260	96822344	96545643	96359313	96026834
3.55	97127540	96970220	96695139	96510276	96181070
3.60	97267630	97111098	96837926	96654698	96329009
3.65	97400890	97245316	96974319	96792880	96470927
3.70	97527650	97373195	97104618	96925106	96607089
3.75	97648230	97495039	97229107	97051648	96737742
3.80	97762920	97611138	97348056	97172763	96863126
3.85	97872030	97721769	97461721	97288699	96983468
3.90	97975810	97827192	97570348	97399686	97098983
3.95	98074530	97927658	97674169	97505949	97209878
4.00	98168440	98023403	97773406	97607699	97316349
4.05	98257760	98114654	97868269	97705137	97418584
4.10	98342730	98201625	97958959	97798455	97516761
4.15	98423560	98284521	98045668	97887837	97611052
4.20	98500440	98363535	98128576	97973457	97701618
4.25	98573580	98438852	98207857	98055480	97788616

Table 1
 $P[a_1^{(1)} \chi_1^2 + a_2^{(2)} \chi_1^2 \leq t]$

a_2, a_1					
.8, .2	.875, .125	.9, .1	.95, .05	.99, .01	1, 0
86263825	85715512	85486633	84992446	84580226	84476086
86487318	85927381	85695932	85198058	84783662	84679036
86706725	86135706	85901829	85400485	84984077	84878984
86922137	86340555	86104390	85599797	85181463	85075983
87133636	86541999	86303684	85796069	85375952	85270087
87341308	86740105	86499768	85989320	85567584	85461346
87545233	86934944	86692708	86179617	85756377	85649811
87745491	87126572	86882566	86367028	85942409	85835531
87942156	87315059	87069397	86551594	86125695	86018554
88135309	87500467	87253256	86733368	86306329	86198926
88325019	87682853	87434203	86912408	86484302	86376694
88511360	87862277	87612299	87088751	86659725	86551902
88694400	88038792	87787583	87262453	86832593	86724594
88874208	88212460	87960120	87433549	87002924	86894813
89050849	88383329	88129951	87602100	87170834	87062600
89224389	88551453	88297131	87768138	87336360	87227997
89394886	88716891	88461708	87931716	87499465	87391044
89562410	88879684	88623732	88092884	87660302	87551780
89727016	89039889	88783243	88251664	87818801	87710244
89888763	89197547	88940289	88408108	87975068	87866474
90047710	89352710	89094918	88562252	88129100	88020506
90203910	89505421	89247168	88714136	88280942	88172376
90357419	89655726	89397085	88863803	88430684	88322120
90508291	89803669	89544707	89011300	88578273	88469773
90656574	89949296	89690075	89156641	88723804	88615368
90838368	90128131	89868684	89335384	88902873	88794520
91016302	90303487	90043908	89510840	89078777	88970573
91190461	90475443	90215821	89683169	89251645	89143588
91360939	90644073	90384498	89852399	89421536	89313627
91527827	90809451	90550009	90018595	89588445	89480748
91691206	90971652	90712419	90181825	89752501	89645009
91851164	91130742	90871795	90342149	89913734	89806466
92007780	91286789	91028199	90499623	90072174	89965174
92161132	91439862	91181701	90654310	90227944	90121186
92311297	91590019	91332356	90806259	90381007	90274555
92458351	91737328	91480227	90955527	90531486	90425332
92602368	91881848	91625366	91102168	90679403	90573567
92743413	92023642	91767837	91246231	90824821	90719308
92881560	92162762	91907692	91387777	90967772	90862604
93016873	92299268	92044984	91526846	91108291	91003500

Table 1
 $P[a_1^{(1)} \chi_1^2 + a_2^{(2)} \chi_1^2 \leq t]$

t	a_2, a_1				
	.5, .5	.6, .4	$\frac{2}{3}, \frac{1}{3}$	.7, .3	.75, .25
2.02	86734450	86723695	86677978	86626884	86492425
2.04	86997130	86981182	86926618	86869786	86725700
2.06	87254600	87233579	87170376	87107964	86954537
2.08	87506980	87480989	87409358	87341510	87179029
2.10	87754360	87723512	87643660	87570526	87399274
2.12	87996840	87961246	87873377	87795105	87615363
2.14	88234520	88194288	88098605	88015343	87827377
2.16	88467490	88422732	88319438	88231331	88035411
2.18	88695850	88646669	88535966	88443153	88239547
2.20	88919680	88866194	88748280	88650903	88439864
2.22	89139090	89081390	88956463	88854657	88636443
2.24	89354150	89292350	89160601	89054504	88829367
2.26	89564950	89499157	89360775	89250525	89018708
2.28	89771580	89701891	89557070	89442796	89204542
2.30	89974120	89900634	89749563	89631395	89386940
2.32	90172640	90095473	89938332	89816403	89565977
2.34	90367240	90286477	90123458	89997886	89741720
2.36	90557980	90473733	90305007	90175921	89914238
2.38	90744940	90657309	90483056	90350579	90083595
2.40	90928200	90837279	90657675	90521927	90249859
2.42	91107840	91013718	90828936	90690033	90413090
2.44	91283910	91186695	90996904	90854966	90573350
2.46	91456500	91356279	91161648	91016787	90730703
2.48	91625680	91522538	91323235	91175561	90885206
2.50	91791500	91685540	91481725	91331349	91036915
2.525	91994170	91884808	91675579	91521978	91222708
2.55	92191830	92079211	91864814	91708150	91404333
2.575	92384620	92268872	92049545	91889981	91581893
2.60	92572640	92453905	92229887	92067577	91755488
2.625	92756020	92634430	92405945	92241047	91925218
2.65	92934880	92810554	92577831	92410490	92091178
2.675	93109320	92982389	92745647	92576008	92253459
2.70	93279450	93150042	92909495	92737700	92412154
2.725	93445380	93313614	93069473	92895659	92567350
2.75	93607210	93473209	93225679	93049979	92719129
2.775	93765050	93628926	93378205	93200750	92867578
2.80	93918990	93780858	93527142	93348060	93012777
2.825	94069130	93929101	93672580	93491992	93154803
2.85	94215570	94073746	93814606	93632631	93293733
2.875	94358390	94214882	93953303	93770057	93429642

Table 1
 $P[a_1^{(1)} \chi_1^2 + a_2^{(2)} \chi_1^2 \leq t]$

a_2, a_1					
.8,.2	.875,.125	.9,.1	.95,.05	.99,.01	1,0
72577279	73168331	73253695	73235696	73105707	73063934
73075221	73615771	73685168	73642192	73497766	73452894
73562397	74053851	74107917	74041022	73882819	73834890
74039094	74482855	74522185	74432401	74260923	74210096
74505588	74903034	74928205	74816517	74632263	74578677
74962150	75314642	75326211	75193540	74997100	74940794
75409032	75717909	75716426	75563677	75355461	75296602
75846490	76113067	76099049	75927070	75707585	75646251
76274769	76500341	76474294	76283912	76053633	75989884
76694102	76879943	76842349	76634349	76393673	76327642
77104715	77252068	77203398	76978531	76727919	76659659
77506833	77616928	77557633	77316620	77056431	76986065
77900670	77974698	77905220	77648744	77379418	77306987
78286433	78325571	78246321	77975052	77696901	77622547
78664324	78669724	78581104	78295679	78009078	77932863
79034538	79007314	78909728	78610758	78316109	78238050
79397262	79338524	79232332	78920400	78617964	78538219
79752687	79663496	79549070	79224736	78914822	78833477
80100986	79982392	79860081	79523891	79206894	79123930
80442335	80295361	80165492	79817965	79494142	79409678
80776898	80602537	80465454	80107084	79776760	79690820
81104844	80904072	80760069	80391346	80054756	79967452
81426325	81200088	81049471	80670859	80328371	80239666
81741500	81490724	81333782	80945724	80597480	80507553
82050512	81776096	81613116	81216024	80862312	80771201
82353512	82056333	81887576	81481891	81123044	81030694
82650640	82331546	82157280	81743375	81379519	81286116
82942031	82601853	82422323	82000599	81631981	81537547
83227819	82867366	82682815	82253637	81880532	81785066
83508132	83128195	82938851	82502565	82125145	82028749
83783097	83384433	83190526	82747487	82365985	82268671
84052835	83636194	83437934	82988470	82603113	82504904
84317462	83883563	83681163	83225593	82836540	82737519
84577101	84126647	83920305	83458933	83066374	82966584
84831858	84365529	84155436	83688567	83292676	83192167
85081844	84600301	84386654	83914564	83515571	83414333
85327165	84831057	84614024	84136998	83735052	83633146
85567925	85057870	84837632	84355935	83951167	83848668
85804224	85280835	85057556	84571455	84164050	84060959
86036160	85500020	85273865	84783601	84373736	84270079

Table 1
 $P[a_1^{(1)} \chi_1^2 + a_2^{(2)} \chi_1^2 \leq t]$

a_2, a_1					
t	.5,.5	.6,.4	$\frac{2}{3}, \frac{1}{3}$	.7,.3	.75,.25
1.22	70476980	70754227	71226559	71533067	72049656
1.24	71061580	71330414	71786907	72081909	72576172
1.26	71634600	71895024	72335719	72619279	73091453
1.28	72196270	72448288	72873248	73145447	73595768
1.30	72746820	72990448	73399738	73660662	74089384
1.32	73286470	73521723	73915430	74165175	74572563
1.34	73815430	74042340	74420559	74659224	75045559
1.36	74333920	74552520	74915348	75143048	75508615
1.38	74842140	75052472	75400030	75616875	75961975
1.40	75340300	75542401	75874817	76080933	76405865
1.42	75828600	76022525	76339926	76535437	76840514
1.44	76307220	76493034	76795564	76980601	77266144
1.46	76776370	76954127	77241939	77416635	77682968
1.48	77236230	77405996	77679246	77843747	78091194
1.50	77686980	77848828	78107683	78262129	78491026
1.52	78128810	78282810	78527436	78671975	78882662
1.54	78561890	78708123	78938697	79073476	79266289
1.56	78986390	79124940	79341643	79466821	79642102
1.58	79402490	79533437	79736456	79852185	80010277
1.60	79810350	79933783	80123308	80229743	80370992
1.62	80210130	80326141	80502369	80599669	80724423
1.64	80602000	80710677	80873804	80962133	81070735
1.66	80986100	81087550	81237776	81317297	81410090
1.68	81362600	81456915	81594443	81665313	81742652
1.70	81731650	81818921	81943963	82006348	82068572
1.72	82093380	82173728	82286486	82340552	82388002
1.74	82447960	82521468	82622157	82668070	82701092
1.76	82795510	82862294	82951125	82989049	83007982
1.78	83136180	83196345	83273531	83303630	83308812
1.80	83470110	83523757	83589514	83611952	83603722
1.82	83797420	83844663	83899208	83914151	83892841
1.84	84118260	84159198	84202743	84210357	84176296
1.86	84432740	84467489	84500253	84500699	84454218
1.88	84740990	84769663	84791862	84785305	84726728
1.90	85043140	85065845	85077696	85064296	84993947
1.92	85339300	85356155	85357872	85337796	85255984
1.94	85629600	85640709	85632513	85605914	85512963
1.96	85914160	85919630	85901732	85868774	85764990
1.98	86193080	86193027	86165642	86126481	86012173
2.00	86466470	86461011	86424355	86379147	86254615

Table 1
 $P[a_1^{(1)} \chi_1^2 + a_2^{(2)} \chi_1^2 \leq t]$

a_2, a_1					
.8, .2	.875, .125	.9, .1	.95, .05	.99, .01	1, 0
55102125	57433026	58198335	59402056	59953979	60055616
55558374	57849613	58595843	59759395	60288544	60385606
56008901	58260451	58987820	60112108	60619062	60711678
56453791	58665659	59374400	60460296	60945684	61033909
56893125	59065349	59755675	60804068	61268437	61352374
57327002	59459636	60131783	61143512	61587340	61567145
57755490	59848613	60502819	61478735	61902579	61978292
58178686	60232390	60868894	61809796	62214194	62285883
58596660	60611074	61230108	62136802	62522178	62589984
59009495	60984754	61586565	62459837	62826666	62890660
59417270	61353535	61938367	62778975	63127699	63187972
59820057	61717500	62285592	63094292	63425382	63481980
60217935	62076750	62628347	63405862	63719716	63772744
60610982	62431363	62966711	63713759	64010765	64060320
60990262	62781431	63300780	64018055	64298636	64344763
61382849	63127047	63630636	64318819	64583375	64626128
61761819	63468277	63956352	64616121	64864979	64904465
62136228	63805214	64278021	64910004	65143545	65179829
62506156	64137927	64595719	65200559	65419122	65452266
62871662	64466503	64909512	65487830	65691777	65721826
63232815	64791008	65219476	65771882	65961515	65988555
63589680	65111514	65525687	66052762	66228373	66252500
63942315	65428101	65828208	66330523	66492526	66513705
64290785	65740829	66127114	66605238	66753859	66772213
64635150	66049773	66422458	66876951	67012465	67028067
64975468	66354991	66714319	67145708	67268424	67281309
65311803	66656554	67002750	67411556	67521744	67531979
65644209	66954522	67287822	67674546	67772541	67780116
65972741	67248956	67569585	67934731	68020732	68025759
66297459	67539921	67848092	68192149	68266400	68268946
66935659	68111662	68395592	68698878	68750506	68748096
67559241	68670195	68930747	69195060	69224888	69217850
68168613	69215956	69453970	69681036	69690054	69678479
68764179	69749361	69965662	70157111	70146079	70130242
69346319	70270810	70466188	70623584	70593364	70573387
69915406	70780665	70955902	71080728	71032144	71008152
70471804	71279303	71435146	71528829	71462574	71434765
71015860	71767061	71904246	71968141	71884896	71853444
71547907	72244282	72363508	72398886	72299423	72264399
72068275	72711270	72813234	72821341	72706277	72667831

Table 1
 $P[a_1^{(1)} \chi_1^2 + a_2^{(2)} \chi_1^2 \leq t]$

t	a_2, a_1				
	.5, .5	.6, .4	$\frac{2}{3}, \frac{1}{3}$	.7, .3	.75, .25
.71	50835580	51289400	52112827	52689866	53769827
.72	51324770	51776715	52595900	53169254	54240555
.73	51809100	52259063	53073809	53643359	54705777
.74	52288610	52736483	53546615	54112232	55165555
.75	52763340	53209034	54014369	54575935	55619969
.76	53233360	53676763	54477130	55034538	56069097
.77	53698690	54139720	54934963	55488099	56513000
.78	54159400	54597957	55387911	55936679	56951743
.79	54615520	55051520	55836034	56380332	57385407
.80	55067100	55500460	56279393	56819122	57814053
.81	55514190	55944826	56718032	57253104	58237747
.82	55956830	56384663	57152010	57682342	58656558
.83	56395070	56820024	57581379	58106885	59070552
.84	56828950	57250941	58006185	58526788	59479787
.85	57258510	57677482	58426487	58942112	59884324
.86	57683790	58099674	58842334	59352909	60284235
.87	58104840	58517566	59253773	59759235	60679570
.88	58521710	58931209	59660852	60161137	61070395
.89	58934420	59340645	60063627	60558673	61456771
.90	59343030	59745914	60462140	60951890	61838747
.91	59747580	60147061	60856442	61340844	62216390
.92	60148100	60544132	61246575	61725585	62589754
.93	60544630	60937163	61632593	62106156	62958891
.94	60937220	61326201	62014542	62482611	63323859
.95	61325900	61711281	62392459	62854998	63684715
.96	61710710	62092453	62766396	63223365	64041502
.97	62091700	62469755	63136395	63587757	64394281
.98	62468890	62843225	63502498	63948232	64743108
.99	62842330	63212901	63864753	64304817	65088019
1.00	63212050	63578827	64223204	64657571	65429077
1.02	63940510	64299572	64928846	65351750	66099821
1.04	64654530	65005767	65619763	66031122	66755724
1.06	65354420	65697714	66296278	66696035	67397154
1.08	66040450	66375701	66958708	67346814	68024493
1.10	66712890	67040023	67607364	67983789	68638083
1.12	67372020	67690954	68242551	68607278	69238271
1.14	68018100	68328775	68864566	69217597	69825400
1.16	68651380	68953749	69473692	69815043	70399789
1.18	69272130	69566150	70070215	70399912	70961757
1.20	69880580	70166223	70654414	70972491	71511613

Table 1
 $P[a_1^{(1)} \chi_1^2 + a_2^{(2)} \chi_1^2 \leq t]$

a_2, a_1					
.8,.2	.875,.125	.9,.1	.95,.05	.99,.01	1,0
30904502	34239293	35743308	39315313	41786435	42231980
31685367	35029841	36525848	40031800	42409534	42839234
32455196	35806035	37292634	40731147	43019519	43434090
33214184	36568234	38044135	41414136	43616996	44017077
33962503	37316794	38780816	42081493	44202482	44588686
34700338	38052044	39503091	42733888	44776487	45149375
35427873	38774324	40211416	43371984	45339455	45699572
36145269	39483949	40906163	43996360	45891858	46239675
36852712	40181223	41587734	44607603	46434061	46770058
37550361	40866457	42256508	45206236	46966461	47291073
38238377	41539937	42912860	45792751	47489418	47803048
38916929	42201945	43557130	46367632	48003309	48306294
39586171	42852762	44189641	46931320	48508324	48801104
40246264	43492654	44810751	47484234	49004850	49287753
40897349	44121872	45420749	48026772	49493156	49766503
41539584	44740684	46019964	48559293	49973544	50237601
42173110	45349308	46608668	49082175	50446177	50701281
42798071	45948013	47187152	49595741	50911327	51157766
43414615	46537002	47755681	50100311	51369210	51607267
44022877	47116520	48314543	50596169	51820139	52049985
44622989	47686773	48863968	51083638	52264164	52486112
45215085	48247972	49404211	51562946	52701554	52915829
45799296	48800333	49935511	52034385	53132518	53339311
46375751	49344041	50458097	52498171	53557193	53756724
46944579	49879304	50972193	52954540	53975761	54168227
47505897	50406297	51478013	53403730	54388367	54573972
48059830	50925210	51975758	53845947	54795125	54974105
48606497	51436227	52465636	54281385	55196324	55368765
49146014	51939511	52947840	54710236	55591942	55758085
49678494	52435235	53422559	55132676	55982198	56142194
50204051	52923565	53889967	55548897	56367209	56521216
50722794	53404657	54350254	55959060	56747129	56895268
51234832	53878663	54803578	56363317	57122019	57264465
51740273	54345740	55250106	56761835	57492014	57628917
52239218	54806037	55690006	57154739	57857225	57988729
52731775	55259681	56123415	57542196	58217808	58344003
53218036	55706832	56550501	57924309	58573795	58694837
53698109	56147607	56971391	58301246	58925300	59041325
54172086	56582151	57386241	58673088	59272426	59383559
54640059	57010581	57795170	59039994	59615320	59721628

Table 1
 $P[a_1^{(1)} \chi_1^2 + a_2^{(2)} \chi_1^2 \leq t]$

a_2, a_1					
t	.5, .5	.6, .4	$\frac{2}{3}, \frac{1}{3}$	.7, .3	.75, .25
.31	26655300	27047654	27787769	28331071	29413895
.32	27385100	27783585	28534544	29085189	30180977
.33	28107630	28511926	29273136	29830662	30938434
.34	28822970	29232774	30003619	30567580	31686416
.35	29531190	29946202	30726115	31296059	32425036
.36	30232370	30652299	31440701	32016201	33154433
.37	30926570	31351123	32147465	32728117	33874743
.38	31613860	32042763	32846513	33431897	34586103
.39	32294310	32727299	33537916	34127645	35288613
.40	32967990	33404790	34221785	34815461	35982426
.41	33634970	34075330	34898183	35495444	36667646
.42	34295320	34738976	35567209	36167696	37344407
.43	34949090	35395798	36228941	36832298	38012822
.44	35596360	36045884	36883469	37489363	38673006
.45	36237180	36689292	37530879	38138972	39325072
.46	36871630	37326089	38171249	38781216	39969147
.47	37499770	37956353	38804655	39416193	40605328
.48	38121660	38580154	39431182	40043989	41233735
.49	38737360	39197546	40050906	40664686	41854467
.50	39346930	39808616	40663912	41278378	42467636
.51	39950440	40413412	41270274	41885149	43073353
.52	40547940	41012007	41870059	42485085	43671717
.53	41139500	41604461	42463357	43078270	44262816
.54	41725170	42190852	43050230	43664773	44846772
.55	42305020	42771238	43630756	44244699	45423671
.56	42879090	43345664	44205016	44818116	45993616
.57	43447460	43914219	44773070	45385105	46556704
.58	44010160	44476942	45334998	45945736	47113021
.59	44567270	45033909	45890858	46500095	47662665
.60	45118840	45585173	46440731	47048263	48205725
.61	45664910	46130799	46984675	47590303	48742296
.62	46205560	46670841	47522767	48126296	49272466
.63	46740820	47205357	48055071	48656310	49796320
.64	47270760	47734407	48581649	49180432	50313947
.65	47795420	48258049	49102571	49698716	50825420
.66	48314870	48776333	49617898	50211238	51330837
.67	48829140	49289322	50127695	50718073	51830271
.68	49338300	49797072	50632022	51219288	52323811
.69	49842390	50299627	51130951	51714947	52811532
.70	50341470	50797057	51624531	52205117	53293512

Table 1
 $P[a_1^{(1)} \chi_1^2 + a_2^{(2)} \chi_1^2 \leq t]$

a_2, a_1					
.8, .2	.875, .125	.9, .1	.95, .05	.99, .01	1, 0
00622552	00751637	00827521	01132120	02363402	05637184
01240281	01494752	01643737	02235194	04462235	07965566
01853219	02229476	02448798	03310213	06340272	09747674
02461403	02955933	03242893	04358204	08033210	11246290
03064878	03674233	04026239	05380110	09570210	12563292
03663696	04384517	04799013	06376875	10975230	13750975
04257889	05086866	05561415	07349374	12267953	14840432
04847507	05781421	06313627	08298478	13464614	15851940
05432587	06468292	07055811	09224988	14578637	16799595
06013164	07147575	07788187	10129720	15621237	17693670
06589297	07819382	08510892	11013393	16601672	18541928
07161002	08483835	09224123	11876763	17527757	19350404
07728344	09141011	09928039	12720504	18406023	20123902
08291341	09791040	10622785	13545298	19241995	20866320
08850052	10434010	11308555	14351792	20040332	21580875
09404497	11070023	11985484	15140605	20805043	22270257
09954734	11699175	12653732	15912315	21539465	22936743
10500784	12321571	13313452	16667510	22246567	23582282
11042705	12937291	13964796	17406712	22928846	24208557
11580504	13546441	14607895	18130495	23588494	24817034
12643947	14745392	15869979	19533695	24847151	25985584
13691407	15919140	17100796	20880955	26035077	27096549
14723178	17068372	18301415	22175776	27161993	28156794
15739530	18193766	19472857	23421443	28235589	29171897
16740739	19295988	20616102	24620977	29262051	30146462
17727068	20375651	21732100	25777196	30246433	31084346
18698782	21433369	22821750	26892675	31192938	31988817
19656163	22469758	23885932	27969832	32105057	32862674
20599425	23485371	24925474	29010896	32985779	33708336
21528848	24480778	25941196	30017954	33837696	34527913
22444653	25456500	26933858	30992912	34662951	35323257
23347094	26413083	27904216	31937555	35463553	36096006
24236404	27351019	28852980	32853564	36241151	36847615
25112804	28270809	29780841	33742471	36997296	37579386
25976511	29172920	30688466	34605715	37733286	38292490
26827767	30057815	31576492	35444631	38450367	38987982
27666772	30925949	32445527	36260474	39149589	39666820
28493741	31777749	33296170	37054418	39831953	40329876
29308886	32613636	34128986	37827535	40498355	40977945
30112404	33434019	34944522	38580854	41149611	41611756

Table 1
 $P[a_1^{(1)} \chi_1^2 + a_2^{(2)} \chi_1^2 \leq t]$

t	a_2, a_1				
	.5,.5	.6,.4	$\frac{2}{3}, \frac{1}{3}$	.7,.3	.75,.25
.005	00498750	00509010	00528896	00543955	00575466
.01	00995020	01015346	01054773	01084654	01147062
.015	01488810	01519052	01577693	01622152	01714889
.02	01980130	02020149	02097692	02156449	02278938
.025	02469010	02518631	02614774	02687584	02839252
.03	02955450	03014547	03128965	03215582	03395869
.035	03439460	03507871	03640266	03740464	03948795
.04	03921060	03998643	04148721	04262225	04498082
.045	04400250	04486862	04654316	04780900	05043733
.05	04877060	04972544	05157095	05296521	05585790
.055	05351490	05455719	05657052	05809095	06124268
.06	05823550	05936372	06154223	06318651	06659209
.065	06293250	06414542	06648616	06825189	07190632
.07	06760620	06890232	07140245	07328737	07718554
.075	07225650	07363438	07629123	07829324	08243012
.08	07688370	07834209	08115273	08326950	08764033
.085	08148770	08302525	08598722	08821648	09281627
.09	08606880	08768418	09079462	09313447	09795844
.095	09062710	09231892	09557529	09802345	10306685
.10	09516260	09692971	10032922	10288367	10814191
.11	10416590	10607969	10975769	11251852	11819275
.12	11307960	11513484	11908151	12204059	12811291
.13	12190460	12409670	12830180	13145125	13790429
.14	13064180	13296574	13741973	14075183	14756880
.15	13929200	14174318	14643665	14994399	15710816
.16	14785620	15042992	15535349	15902883	16652434
.17	15633520	15902704	16417172	16800774	17581926
.18	16472980	16753526	17289226	17688203	18499442
.19	17304090	17595572	18151625	18565323	19405159
.20	18126920	18428925	19004502	19432245	20299268
.21	18941570	19253688	19847958	20289102	21181921
.22	19748120	20069921	20682098	21136022	22053292
.23	20546640	20877749	21507029	21973117	22913540
.24	21337210	21677242	22322870	22800534	23762830
.25	22119920	22468494	23129720	23618374	24601312
.26	22894840	23251585	23927687	24426771	25429148
.27	23662050	24026612	24716859	25225832	26246490
.28	24421620	24793631	25497364	26015685	27053487
.29	25173640	25552769	26269271	26796426	27850289
.30	25918180	26304081	27032711	27568185	28637046

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Table 1
 $P[a_1^{(1)}\chi_1^2 + a_2^{(2)}\chi_1^2 \leq t]$

t	a_2, a_1				
	.5,.5	.6,.4	$\frac{2}{3}, \frac{1}{3}$	.7,.3	.75,.25
4.30	98643140	98510648	98283675	98134065	97872195
4.35	98709320	98579092	98356188	98209364	97952495
4.40	98772270	98644341	98425545	98281518	98029651
4.45	98832140	98706548	98491890	98350667	98103796
4.50	98889100	98765857	98555356	98416942	98175050
4.55	98943280	98822405	98616075	98480465	98243533
4.60	98994820	98876323	98674168	98541358	98309359
4.65	99043840	98927735	98729755	98599733	98372634
4.70	99090470	98976760	98782946	98655699	98433463
4.75	99134830	99023510	98833849	98709360	98491945
4.80	99177030	99068092	98882565	98760814	98548174
4.85	99217160	99110609	98929192	98810156	98602240
4.90	99255340	99151158	98973821	98857476	98654232
4.95	99291660	99189832	99016543	98902860	98704232
5.00	99326210	99226718	99057440	98946390	98752318
5.25	99475250	99387139	99237254	99138867	98966622
5.50	99591320	99513898	99382084	99295420	99143279
5.75	99681720	99614149	99498904	99422940	99289104
6.00	99752120	99693502	99593255	99526952	99409627
6.50	99849660	99806207	99731320	99681339	99591948
7.00	99908810	99877154	99821985	99784728	99717245
8.00	99966450	99950295	99921266	99901068	99863371
9.00	99987660	99979729	99964897	99954195	99933531
10.00	99995460	99991677	99984251	99978669	99967489

Table 1
 $P[a_1^{(1)} \chi_1^2 + a_2^{(2)} \chi_1^2 \leq t]$

a_2, a_1					
.8,.2	.875,.125	.9,.1	.95,.05	.99,.01	1,0
97574718	97082531	96909701	96554339	96262571	96188759
97660761	97177801	97008064	96658789		96299102
97743636	97269838	97103169	96759938		96406104
97823466	97358755	97195128	96857895		96509872
97900369	97444663	97284054	96952765		96610511
97974459	97527674	97370051	97044648		96708122
98045842	97607887	97453221	97133653		96802801
98114624	97685402	97533662	97219872		96894642
98180904	97760316	97611469	97303396		96983735
98244776	97832720	97686729	97384313		97070167
98306333	97902702	97759536	97462710		97154023
98365662	97970346	97829970	97538670		97235383
98422848	98035736	97898114	97612272		97314327
98477971	98098950	97964045	97683593		97390930
98531109	98160062	98027839	97752701		97465265
98769535	98436583	98317194	98067610		97805320
98968346	98670471	98562960	98337119		98098350
99134333	98868527	98771937	98568037		98351132
99273073	99036418	98949818	98766083		98569410
99486379	99299858	99230612	99082173		98921253
99636234	99490152	99435105	99315858		99184901
99816454	99728130	99693816	99617861		99532225
99906792	99854147	99833048	99785304		99730018
99952424	99921369	99908526	99878809		99843457

Table 2

$$P[a_1^{(1)} \chi_1^2 + a_2^{(2)} \chi_1^2 + a_3^{(3)} \chi_1^2 \leq t]$$

a_3, a_2, a_1					
t	$\frac{1}{3}, \frac{1}{3}, \frac{1}{3}$	.4, .3, .3	.4, .4, .2	.5, .3, .2	.6, .2, .2
.01	00136861	00138896	00147199	00151975	00169689
.02	00383822	00389282	00412216	00425451	00474413
.03	00698916	00708662	00749801	00773620	00861521
.04	01066521	01081164	01143008	01178934	01311181
.05	01477295	01497298	01581685	01630868	01811473
.06	01924750	01950469	02058774	02122104	02354095
.07	02403997	02435709	02568953	02647123	02932783
.08	02911165	02949081	03108009	03201562	03542578
.09	03443076	03487356	03672485	03781823	04179415
.10	03997053	04047815	04259472	04384900	04839882
.11	04570803	04628124	04866481	05008211	05521059
.12	05162325	05226254	05491345	05649509	06220418
.13	05769859	05840416	06132161	06306819	06935737
.14	06391834	06469014	06787233	06978394	07665051
.15	07026844	07110622	07455048	07662663	08406610
.16	07673618	07763948	08134239	08358224	09158843
.17	08330998	08427822	08823571	09063786	09920331
.18	08997932	09101174	09521917	09778195	10689789
.19	09673450	09783022	10228248	10500393	11466051
.20	10356664	10472469	10941618	11229403	12248051
.21	11046750	11168677	11661158	11964331	13034814
.22	11742948	11870881	12386069	12704354	13825448
.23	12444552	12578362	13115609	13448712	14619134
.24	13150904	13290461	13849091	14196705	15415117
.25	13861393	14006559	14585878	14947680	16212698
.26	14575447	14726077	15325378	15701021	17011238
.27	15292531	15448476	16067037	16456175	17810142
.28	16012146	16173257	16810345	17212621	18608865
.29	16733822	16899941	17554818	17969861	19406896
.30	17457120	17628091	18300012	18727450	20203771
.31	18181627	18357289	19045508	19484964	20999051
.32	18906954	19087142	19790913	20241999	21792333
.33	19632737	19817290	20535861	20998188	22583251
.34	20358631	20547386	21280010	21753182	23371451
.35	21084313	21277103	22023037	22506667	24156619
.36	21809479	22006139	22764643	23258337	24938459
.37	22533841	22734205	23504547	24007906	25716692
.38	23257128	23461029	24242487	24755107	26491072
.39	23979085	24186362	24978208	25499707	27261362
.40	24699471	24909958	25711486	26241466	28027344

Table 2

$$P[a_1^{(1)}\chi_1^2 + a_2^{(2)}\chi_1^2 + a_3^{(3)}\chi_1^2 \leq t]$$

	a_3, a_2, a_1				
t	.5, .4, .1	.6, .3, .1	.7, .2, .1	.8, .1, .1	.9, .05, .05
.01	00185354	00195289	00221126	00291125	00538290
.02	00516804	00544222	00615346	00806304	01462629
.03	00935993	00985162	01112332	01450702	02583012
.04	01420799	01494686	01685248	02187747	03825329
.05	01957883	02058680	02317899	02995303	05145697
.06	02537983	02667320	02998998	03857970	06514772
.07	03154112	03313223	03720053	04764226	07911739
.08	03800801	03990564	04474383	05705063	09321344
.09	04473534	04694589	05256547	06673226	10732192
.10	05168606	05421351	06062030	07662755	12135693
.11	05882839	06167509	06887004	08668671	13525348
.12	06613505	06930171	07728204	09686777	14896260
.13	07358234	07706846	08582787	10713503	16244770
.14	08114973	08495343	09448288	11745788	17568187
.15	08881871	09293739	10322548	12781003	18864589
.16	09657328	10100322	11203638	13816871	20132649
.17	10439866	10913571	12089877	14851430	21371540
.18	11228206	11732121	12979758	15882976	22580803
.19	12021196	12554770	13871944	16910028	23760295
.20	12817774	13380420	14765233	17931307	24910102
.21	13616984	14208066	15658543	18945700	26030510
.22	14417978	15036826	16550928	19952248	27121957
.23	15219976	15865892	17441520	20950122	28184978
.24	16022284	16694549	18329545	21938606	29220211
.25	16824259	17522131	19214317	22917098	30228352
.26	17625297	18348014	20095202	23885075	31210143
.27	18424890	19171684	20971652	24842099	32166360
.28	19222553	19992645	21843161	25787804	33097788
.29	20017837	20810443	22709277	26721891	34005228
.30	20810355	21624664	23569605	27644112	34889491
.31	21599746	22434948	24423791	28554270	35751354
.32	22385658	23240964	25271508	29452221	36591620
.33	23167799	24042421	26112477	30337853	37411041
.34	23945886	24839025	26946454	31211092	38210384
.35	24719691	25630542	27773210	32071896	38990368
.36	25488984	26416751	28592548	32920256	39751714
.37	26253542	27197443	29404318	33756180	40495102
.38	27013176	27972458	30208360	34579702	41221199
.39	27767752	28741615	31004545	35390878	41930643
.40	28517088	29504792	31792795	36189780	42624046

Table 2

$$P[a_1^{(1)} \chi_1^2 + a_2^{(2)} \chi_1^2 + a_3^{(3)} \chi_1^2 \leq t]$$

t	a_3, a_2, a_1				
	$\frac{1}{3}, \frac{1}{3}, \frac{1}{3}$	.4, .3, .3	.4, .4, .2	.5, .3, .2	.6, .2, .2
.41	25418059	25631592	26442100	26980170	28788818
.42	26134635	26351053	27169847	27715618	29545604
.43	26848998	27068139	27894538	28447628	30297532
.44	27560957	27782664	28615995	29176018	31044447
.45	28270332	28494447	29334052	29900625	31786204
.46	28976955	29203319	30048550	30621297	32522675
.47	29680666	29909123	30759347	31337902	33253742
.48	30381315	30611717	31466309	32050290	33979296
.49	31078760	31310952	32169301	32758360	34699235
.50	31772869	32006709	32868215	33461981	35413480
.51	32463517	32698856	33562934	34161059	36121944
.52	33150586	33387275	34253362	34855500	36824553
.53	33833966	34071867	34939399	35545209	37521245
.54	34513553	34752528	35620959	36230089	38211970
.55	35189250	35429163	36297965	36910093	38896669
.56	35860966	36101682	36970337	37585143	39575308
.57	36528617	36770008	37638009	38255163	40247847
.58	37192123	37434055	38300917	38920105	40914255
.59	37851410	38093758	38959001	39579916	41574504
.60	38506409	38749054	39612215	40234547	42228573
.61	39157056	39399868	40260504	40883970	42876453
.62	39803292	40046160	40903827	41528126	43518126
.63	40445062	40687862	41542146	42167000	44153597
.64	41082315	41324942	42175426	42800557	44782850
.65	41715005	41957346	42803637	43428762	45405886
.66	42343088	42585032	43426748	44051610	46022716
.67	42966526	43207969	44044739	44669081	46633349
.68	43585282	43826122	44657591	45281153	47237788
.69	44199325	44439466	45265283	45887817	47836057
.70	44808626	45047967	45867801	46489066	48428163
.71	45413158	45651604	46465138	47084897	49014135
.72	46012899	46250362	47057286	47675313	49593991
.73	46607828	46844212	47644230	48260303	50167749
.74	47197928	47433154	48225979	48839880	50735444
.75	47783184	48017162	48802526	49414040	51297101
.76	48363583	48596230	49373875	49982798	51852746
.77	48939115	49170354	49940025	50546154	52402410
.78	49509772	49739529	50500985	51104139	52946141
.79	50075549	50303749	51056763	51656746	53483951
.80	50636441	50863009	51607364	52203992	54015889

Table 2

$$P[a_1^{(1)} \chi_1^2 + a_2^{(2)} \chi_1^2 + a_3^{(3)} \chi_1^2 \leq t]$$

a_3, a_2, a_1					
t	.5, .4, .1	.6, .3, .1	.7, .2, .1	.8, .1, .1	.9, .05, .05
.41	29261073	30261851	32572985	36976487	43302001
.42	29999565	31012688	33345058	37751116	43965086
.43	30732463	31757188	34108965	38513765	44613827
.44	31459680	32495286	34864654	39264561	45248755
.45	32181120	33226905	35612090	40003638	45870363
.46	32896701	33951978	36351254	40731134	46479134
.47	33606395	34670450	37082142	41447193	47075518
.48	34310096	35382276	37804743	42151970	47659955
.49	35007792	36087437	38519063	42845618	48232858
.50	35699433	36785892	39225136	43528299	48794624
.51	36384986	37477615	39922965	44200178	49345636
.52	37064424	38162604	40612586	44861419	49886251
.53	37737729	38840853	41294036	45512196	50416820
.54	38404870	39512356	41967352	46152668	50937673
.55	39065871	40177118	42632581	46783007	51449122
.56	39720705	40835144	43289764	47403388	51951467
.57	40369371	41486450	43938970	48013982	52444999
.58	41011878	42131064	44580247	48614954	52929994
.59	41648229	42768989	45213660	49206482	53406715
.60	42278443	43400255	45839277	49788719	53875408
.61	42902535	44024894	46457141	50361848	54336320
.62	43520511	44642936	47067346	50926033	54789670
.63	44132413	45254416	47669960	51481427	55235687
.64	44738246	45859357	48265042	52028203	55674577
.65	45338024	46457816	48852678	52566517	56106542
.66	45931807	47049823	49432942	53096535	56531774
.67	46519608	47635422	50005904	53618403	56950459
.68	47101464	48214645	50571648	54132287	57362769
.69	47677392	48787556	51130249	54638328	57768883
.70	48247433	49354193	51681795	55136683	58158950
.71	48811634	49914608	52226353	55627503	58563131
.72	49370033	50468847	52764018	56110922	58951592
.73	49922656	51016949	53294859	56587093	59334454
.74	50469548	51558987	53818961	57056144	59711860
.75	51010757	52095003	54336409	57518226	60083946
.76	51546305	52625046	54847242	57973458	60450845
.77	52076258	53149166	55351577	58421990	60812676
.78	52600650	53667421	55849568	58863939	61169552
.79	53119520	54179872	56341189	59299440	61521598
.80	53632902	54686576	56826519	59728614	61868910

Table 2

$$P[a_1^{(1)} \chi_1^2 + a_2^{(2)} \chi_1^2 + a_3^{(3)} \chi_1^2 \leq t]$$

t	a_3, a_2, a_1				
	$\frac{1}{3}, \frac{1}{3}, \frac{1}{3}$	.4, .3, .3	.4, .4, .2	.5, .3, .2	.6, .2, .2
.81	51192447	51417315	52152804	52745897	54541999
.82	51743567	51966667	52693093	53282484	55062306
.83	52289802	52511074	53228249	53813769	55576864
.84	52831156	53050534	53758278	54339775	56085699
.85	53367634	53585057	54283210	54860515	56588862
.86	53899243	54114654	54803052	55376028	57086394
.87	54425991	54639338	55317829	55886328	57578335
.88	54947888	55159123	55827560	56391446	58064730
.89	55464945	55674003	56332271	56891406	58545624
.90	55977173	56184008	56831984	57386236	59021066
.91	56484586	56689157	57326718	57875956	59491102
.92	56987199	57189460	57816495	58360619	59955772
.93	57485028	57684929	58301360	58840227	60415134
.94	57978089	58175596	58781313	59314832	60869226
.95	58466400	58661464	59256393	59784460	61318094
.96	58949979	59142579	59726633	60249150	61761789
.97	59428846	59618932	60192066	60708915	62200358
.98	59903022	60090569	60652695	61163810	62633758
.99	60372528	60557499	61108584	61613850	63062329
1.00	60837386	61019754	61559739	62059099	63485810
1.01	61297618	61477356	62006200	62499557	63904375
1.02	61753248	61930330	62448000	62935281	64318042
1.03	62204301	62378704	62885168	63366299	64726884
1.04	62650801	62822497	63317736	63792658	65130942
1.05	63092773	63261745	63745738	64214379	65530270
1.06	63530243	63696459	64169207	64631514	65924902
1.07	63963238	64126692	64588172	65044090	66314890
1.08	64391784	64552457	65002680	65452139	66700298
1.09	64815909	64973793	65412757	65855714	67081163
1.10	65235640	65390702	65818428	66254849	67457533
1.11	65651006	65803249	66219730	66649572	67829455
1.12	66062036	66211437	66616712	67039930	68196983
1.13	66468758	66615317	67009396	67425951	68560155
1.14	66871201	67014915	67397821	67807678	68919025
1.15	67269396	67410241	67782027	68185154	69273637
1.16	67663371	67801353	68162030	68558412	69624044
1.17	68053157	68188262	68537887	68927487	69970291
1.18	68438785	68571019	68909630	69292427	70312425
1.19	68820285	68949635	69277279	69653272	70650479
1.20	69197687	69324162	69640892	70010045	70984513

Table 2

$$P[a_1^{(1)}\chi_1^2 + a_2^{(2)}\chi_1^2 + a_3^{(3)}\chi_1^2 \leq t]$$

a_3, a_2, a_1					
t	.5, .4, .1	.6, .3, .1	.7, .2, .1	.8, .1, .1	.9, .05, .05
.81	54140868	55187554	57305772	60151583	62211604
.82	54643444	55682901	57778892	60568465	62549777
.83	55140679	56172653	58245896	60979382	62883533
.84	55632634	56656870	58707074	61384440	63212965
.85	56119326	57135607	59162413	61783765	63538158
.86	56600826	57608924	59611912	62177450	63859216
.87	57077162	58076866	60055737	62565623	64176207
.88	57548397	58539502	60493964	62948377	64489220
.89	58014572	58996870	60926613	63325800	64798344
.90	58475729	59449039	61353748	63698022	65103643
.91	58931909	59896051	61775540	64065132	65405211
.92	59383186	60337977	62191990	64427214	65703102
.93	59829572	60774863	62603195	64784372	65997395
.94	60271142	61206764	63009206	65136697	66288154
.95	60707920	61633735	63410077	65484279	66575457
.96	61139969	62055828	63805981	65827209	66859353
.97	61567324	62473101	64196876	66165563	67139916
.98	61990049	62885595	64582820	66499439	67417198
.99	62408166	63293377	64964018	66828909	67691264
1.00	62821748	63696493	65340437	67154057	67962173
1.01	63230807	64094997	65712017	67474961	68229982
1.02	63635410	64488869	66079097	67791702	68494737
1.03	64035595	64878287	66441622	68104347	68756496
1.04	64431419	65263296	66799599	68412974	69015306
1.05	64822923	65643885	67153107	68717655	69271226
1.06	65210129	66020017	67502269	69018455	69524288
1.07	65593103	66391921	67847022	69315451	69774558
1.08	65971891	66759610	68187652	69608708	70022082
1.09	66346531	67123063	68524080	69898292	70266891
1.10	66717059	67482267	68856148	70184249	70509030
1.11	67083524	67837362	69184379	70466675	70748562
1.12	67445970	68188485	69508614	70745599	70985500
1.13	67804431	68535599	69828629	71021105	71219911
1.14	68158967	68878695	70144915	71293239	71451816
1.15	68509593	69217818	70457365	71562055	71681263
1.16	68856384	69553117	70765817	71827622	71908298
1.17	69199339	69884618	71070698	72089978	72132935
1.18	69538530	70212270	71371788	72349200	72355232
1.19	69873995	70536169	71669267	72605313	72575202
1.20	70205769	70856355	71963213	72858382	72792905

Table 2

$$P[a_1^{(1)} \chi_1^2 + a_2^{(2)} \chi_1^2 + a_3^{(3)} \chi_1^2 \leq t]$$

t	a_3, a_2, a_1				
	$\frac{1}{3}, \frac{1}{3}, \frac{1}{3}$	.4, .3, .3	.4, .4, .2	.5, .3, .2	.6, .2, .2
1.21	69571023	69694619	70000484	70365789	71314574
1.22	69940324	70061043	70356096	70711546	71640691
1.23	70305620	70423462	70707774	71056355	71962931
1.24	70666944	70781910	71055527	71397255	72281321
1.25	71024327	71136421	71399431	71734275	72595914
1.26	71377800	71487020	71739485	72067449	72906744
1.27	71727395	71833758	72075741	72396762	73213874
1.28	72073144	72176657	72408240	72722352	73517336
1.29	72415078	72515739	72737005	73044247	73817166
1.30	72753230	72851054	73062058	73362498	74113411
1.31	73087631	73182620	73383457	73677080	74406123
1.32	73418313	73510484	73701229	73987963	74695335
1.33	73745308	73834667	74015418	74295283	74981088
1.34	74068648	74155214	74326055	74599120	75263419
1.35	74388365	74472143	74633155	74899504	75542378
1.36	74704491	74785494	74936779	75196389	75818003
1.37	75017057	75095296	75236940	75489757	76090334
1.38	75326096	75401590	75533683	75779685	76359401
1.39	75631639	75704400	75827034	76066306	76625256
1.40	75933717	76003764	76117034	76349675	76887933
1.41	76232363	76299706	76403723	76629769	77147469
1.42	76527608	76592263	76687112	76906546	77403900
1.43	76819483	76881469	76967249	77180080	77657268
1.44	77108020	77167356	77244188	77450402	77907614
1.45	77393250	77449958	77517922	77717642	78154958
1.46	77675204	77729294	77788500	77981784	78399357
1.47	77953913	78005408	78055954	78242762	78640840
1.48	78229408	78278322	78320332	78500724	78879429
1.49	78501720	78548084	78581647	78755611	79115182
1.50	78770879	78814713	78839920	79007533	79348122
1.51	79036916	79078232	79095202	79256549	79578279
1.52	79299862	79338666	79347529	79502584	79805689
1.53	79559746	79596081	79596924	79745751	80030400
1.54	79816599	79850480	79843406	79986097	80252428
1.55	80070451	80101902	80087025	80223569	80471816
1.56	80321332	80350375	80327786	80458267	80688587
1.57	80569271	80595926	80565752	80690151	80902779
1.58	80814298	80838582	80800940	80919309	81114425
1.59	81056442	81078395	81033363	81145831	81323569
1.60	81295732	81315367	81263071	81369648	81530212

Table 2

$$P[a_1^{(1)} \chi_1^2 + a_2^{(2)} \chi_1^2 + a_3^{(3)} \chi_1^2 \leq t]$$

t	a_3, a_2, a_1				
	.5, .4, .1	.6, .3, .1	.7, .2, .1	.8, .1, .1	.9, .05, .05
1.21	70533883	71172948	72253536	73108466	73008365
1.22	70858382	71485858	72540446	73355593	73221605
1.23	71179316	71795256	72823796	73599832	73432678
1.24	71496699	72101088	73103835	73841209	73641602
1.25	71810591	72403446	73380502	74079781	73848399
1.26	72121018	72702298	73653752	74315595	74053116
1.27	72427964	72997805	73923894	74548703	74255778
1.28	72731553	73289947	74190751	74779126	74456411
1.29	73031825	73578751	74454415	75006914	74655053
1.30	73328816	73864206	74715024	75232115	74851715
1.31	73622522	74146448	74972470	75454762	75046436
1.32	73912909	74425535	75227027	75674906	75239238
1.33	74200098	74701425	75478404	75892570	75430159
1.34	74484162	74974074	75726869	76107799	75619210
1.35	74765147	75243660	75972496	76320642	75806426
1.36	75042994	75510248	76215220	76531113	75991830
1.37	75317709	75773821	76455016	76739259	76175432
1.38	75589355	76034389	76692094	76945118	76357274
1.39	75858032	76291955	76926408	77148712	76537373
1.40	76123813	76546612	77157972	77350085	76715759
1.41	76386666	76798421	77386794	77549267	76892440
1.42	76646573	77047236	77613072	77746291	77067441
1.43	76903578	77293383	77836652	77941191	77240806
1.44	77157737	77536750	78057811	78133984	77412528
1.45	77409143	77777374	78276285	78324723	77582643
1.46	77657784	78015133	78492254	78513424	77751163
1.47	77903617	78250286	78705755	78700115	77918121
1.48	78146761	78482852	78916899	78884811	78083513
1.49	78387176	78712803	79125428	79067570	78247398
1.50	78624951	78940095	79331830	79248400	78409748
1.51	78860137	79164764	79535612	79427333	78570613
1.52	79092683	79387024	79737126	79604395	78730004
1.53	79322664	79606732	79936101	79779622	78887941
1.54	79550134	79824012	80132979	79953018	79044435
1.55	79775066	80038683	80327784	80124624	79199512
1.56	79997519	80251120	80520591	80294447	79353183
1.57	80217471	80461058	80711134	80462526	79505466
1.58	80435003	80668810	80899667	80628891	79656378
1.59	80650191	80873984	81086043	80793544	79805945
1.60	80862971	81077100	81270026	80956529	79954163

Table 2

$$P[a_1^{(1)}x_1^2 + a_2^{(2)}x_1^2 + a_3^{(3)}x_1^2 \leq t]$$

t	a_3, a_2, a_1				
	$\frac{1}{3}, \frac{1}{3}, \frac{1}{3}$	.4, .3, .3	.4, .4, .2	.5, .3, .2	.6, .2, .2
1.61	81532197	81549533	81490083	81590833	81734412
1.62	81765867	81780928	81714438	81809324	81936185
1.63	81996769	82009582	81936156	82025241	82135558
1.64	82224932	82235519	82155266	82238664	82332576
1.65	82450385	82458784	82371806	82449578	82527263
1.66	82673155	82679370	82585777	82657984	82719643
1.67	82893271	82897341	82797227	82863851	82909733
1.68	83110760	83112699	83006193	83067684	83097592
1.69	83325649	83325494	83212692	83269969	83283226
1.70	83537966	83535727	83416759	83468917	83466668
1.71	83747738	83743446	83618386	83663252	83647943
1.72	83954992	83948673	83817649	83857239	83827083
1.73	84159755	84151435	84014540	84048823	84004099
1.74	84362054	84351754	84209109	84238337	84179051
1.75	84561914	84549668	84401355	84426945	84351932
1.76	84759362	84745182	84591329	84610263	84522779
1.77	84954424	84938346	84779033	84792875	84691617
1.78	85147125	85179178	84964514	84973296	84858480
1.79	85337491	85317688	85147791	85151703	85023379
1.80	85525548	85503917	85328895	85327976	85186343
1.81	85711320	85687888	85507814	85502090	85347398
1.82	85894833	85869617	85684620	85674210	85506566
1.83	86076111	86049149	85859313	85843638	85663867
1.84	86255179	86226502	86031918	86014098	85819334
1.85	86432061	86401676	86202455	86179646	85972998
1.86	86606781	86574728	86370956	86342005	86124847
1.87	86779363	86745662	86537443	86504190	86274930
1.88	86949831	86914503	86701948	86664091	86423261
1.89	87118208	87081273	86864471	86822260	86569879
1.90	87284517	87246018	87025062	86978592	86714776
1.91	87448782	87408738	87183715	87132947	86857994
1.92	87611026	87569446	87340462	87285703	86999546
1.93	87771271	87728203	87495326	87436231	87139464
1.94	87929540	87884985	87648347	87585143	87277743
1.95	88085855	88039860	87799521	87732358	87414431
1.96	88240238	88192813	87948877	87877623	87549536
1.97	88392711	88343890	88096426	88021430	87683077
1.98	88543296	88493097	88242220	88163168	87815073
1.99	88692014	88640471	88386242	88303850	87945539
2.00	88838886	88786014	88528546	88443449	88074518

Table 2

$$P[a_1^{(1)} \chi_1^2 + a_2^{(2)} \chi_1^2 + a_3^{(3)} \chi_1^2 \leq t]$$

t	a_3, a_2, a_1				
	.5, .4, .1	.6, .3, .1	.7, .2, .1	.8, .1, .1	.9, .05, .05
1.61	81073402	81277787	81451888	81117860	80101069
1.62	81281451	81476331	81631960	81277553	80246662
1.63	81487204	81672555	81810163	81435630	80390965
1.64	81690733	81866634	81986641	81592115	80533993
1.65	81892026	82058548	82160807	81747043	80675772
1.66	82091087	82248281	82332852	81900407	80816288
1.67	82287891	82435974	82502978	82052244	80955572
1.68	82482476	82621446	82671175	82202570	81093651
1.69	82674960	82804954	82837616	82351404	81230517
1.70	82865372	82986353	83002509	82498769	81366200
1.71	83053673	83165869	83165788	82644687	81500706
1.72	83239883	83343203	83327490	82789158	81634042
1.73	83424004	83518624	83487218	82932213	81766231
1.74	83606069	83692153	83644868	83073879	81897291
1.75	83786146	83863723	83800576	83214150	82027220
1.76	83964248	84033475	83954681	83353051	82156033
1.77	84140324	84201119	84107192	83490619	82283757
1.78	84314427	84367084	84258325	83626847	82410391
1.79	84486725	84531141	84407850	83761764	82535938
1.80	84657100	84693473	84555904	83895373	82660438
1.81	84825558	84853915	84702224	84027703	82783870
1.82	84992225	85012605	84846703	84158766	82906275
1.83	85156821	85169525	84989189	84288571	83027648
1.84	85319805	85324735	85130352	84417142	83147998
1.85	85480977	85478216	85270146	84544499	83267353
1.86	85640331	85630073	85408664	84670634	83385701
1.87	85798071	85780142	85545903	84795575	83503067
1.88	85953772	85928554	85681580	84919331	83619455
1.89	86107911	86075426	85815658	85041921	83734874
1.90	86260387	86220659	85948316	85163370	83849352
1.91	86411088	86364219	86079479	85283674	83962879
1.92	86560340	86506365	86209492	85402845	84075466
1.93	86707591	86646781	86338175	85520917	84187139
1.94	86853388	86785748	86465742	85637874	84297877
1.95	86997647	86923169	86592052	85753748	84407732
1.96	87140152	87059041	86717102	85868546	84516676
1.97	87281325	87193479	86840790	85982284	84624747
1.98	87420765	87326444	86962614	86094967	84731925
1.99	87558612	87457842	87082913	86206610	84838250
2.00	87695129	87588007	87202116	86317224	84943710

Table 2

$$P[a_1^{(1)}\chi_1^2 + a_2^{(2)}\chi_1^2 + a_3^{(3)}\chi_1^2 \leq t]$$

t	a_3, a_2, a_1				
	$\frac{1}{3}, \frac{1}{3}, \frac{1}{3}$	.4, .3, .3	.4, .4, .2	.5, .3, .2	.6, .2, .2
2.05	89546295	89487135	89214681	89110327	88697394
2.10	90210643	90145786	89860380	89739430	89285361
2.15	90834371	90764360	90467927	90331769	89840482
2.20	91419800	91345190	91039533	90889539	90364747
2.25	91969136	91890458	91577261	91414828	90859986
2.30	92484474	92402208	92083048	91908707	91327957
2.35	92967802	92882431	92558763	92373968	91770256
2.40	93421003	93332957	93006156	92812182	92188405
2.45	93845861	93755588	93426875	93224614	92583837
2.50	94244063	94151949	93822474	93613050	92957875
2.55	94617206	94523627	94194441	93977778	93311771
2.60	94966798	94872101	94544162	94321818	93646700
2.65	95294264	95198782	94872954	94646642	93963753
2.70	95600949	95505000	95182042	94951306	94263964
2.75	95888121	95791965	95472596	95239517	94548283
2.80	96156976	96060882	95745714	95509422	94817634
2.85	96408642	96312841	96002432	95764748	95072858
2.90	96644181	96548899	96243723	96005972	95314747
2.95	96864593	96770036	96470511	96231416	95544065
3.00	97070819	96977158	96683656	96445409	95761501
3.05	97263745	97171137	96883975	96645543	95967718
3.10	97444205	97352811	97072230	96833972	96163346
3.15	97612981	97522933	97249151	97012452	96348947
3.20	97770810	97682231	97415415	97180218	96525098
3.25	97918383	97831356	97571641	97339631	96692286
3.30	98056350	97970971	97718459	97490258	96851029
3.35	98185322	98101658	97856418	97629034	97001756
3.40	98305871	98224009	97986047	97761128	97144911
3.45	98418535	98338519	98107864	97886202	97280905
3.50	98523819	98445692	98222320	98005301	97410088
3.55	98622195	98545992	98329858	98116926	97532880
3.60	98714108	98639871	98430910	98220848	97649560
3.65	98799974	98727721	98525846	98318904	97760466
3.70	98880183	98809895	98615053	98411043	97865924
3.75	98955101	98886818	98698870	98498553	97966192
3.80	99025071	98958777	98777615	98581928	98061552
3.85	99090414	99026104	98851600	98661118	98152250
3.90	99151430	99089087	98921121	98734631	98238527
3.95	99208401	99148025	98986428	98804327	98320624
4.00	99261591	99203157	99047792	98870820	98398744

Table 2

$$P[a_1^{(1)} \chi_1^2 + a_2^{(2)} \chi_1^2 + a_3^{(3)} \chi_1^2 \leq t]$$

t	a_3, a_2, a_1				
	.5, .4, .1	.6, .3, .1	.7, .2, .1	.8, .1, .1	.9, .05, .05
2.05	88354913	88216198	87780291	86855286	85458391
2.10	88979083	88812074	88331407	87369325	85952851
2.15	89569620	89375782	88853948	87860617	86428052
2.20	90128405	89910515	89349301	88330367	86884938
2.25	90656908	90415299	89820246	88779680	87324360
2.30	91157167	90895837	90270479	89209628	87747125
2.35	91629964	91349222	90697422	89621181	88154018
2.40	92077680	91780213	91104658	90015238	88545741
2.45	92501146	92186836	91491567	90392689	88922989
2.50	92902054	92573273	91860421	90754341	89286381
2.55	93279965	92939208	92211597	91100944	89636537
2.60	93638484	93287970	92547482	91433226	89974018
2.65	93978470	93617955	92866833	91751864	90299375
2.70	94298811	93929089	93171374	92057508	90613117
2.75	94603532	94225421	93461863	92350734	90915709
2.80	94889999	94508391	93739531	92632141	91207629
2.85	95162411	94776018	94003351	92902251	91489315
2.90	95421082	95027702	94255156	93161583	91761175
2.95	95663636	95266775	94496070	93410613	92023605
3.00	95895262	95494309	94726120	93649811	92276975
3.05	96113132	95712151	94950300	93879609	92521639
3.10	96317749	95918844	95160142	94100412	92757943
3.15	96513305	96114154	95362175	94312606	92986207
3.20	96697909	96301144	95554224	94516588	93206739
3.25	96874467	96477685	95737147	94712687	93419843
3.30	97041236	96645137	95914696	94901252	93625782
3.35	97196091	96804607	96081396	95082605	93824830
3.40	97343808	96956103	96241050	95257053	94017259
3.45	97484354	97101154	96393815	95424871	94203298
3.50	97619099	97238859	96540323	95586345	94383185
3.55	97745707	97369230	96680574	95741735	94557134
3.60	97863181	97493489	96815896	95891293	94725383
3.65	97974514	97611594	96944015	96035260	94888124
3.70	98079363	97723852	97066910	96173869	95045559
3.75	98179666	97830966	97184781	96307325	95197866
3.80	98275968	97931832	97297600	96435847	95345247
3.85	98368061	98027868	97405544	96559618	95487846
3.90	98453351	98119631	97508867	96678852	95625857
3.95	98534810	98207384	97608002	96793708	95759446
4.00	98613136	98290648	97703106	96904369	95888737

Table 2

$$P[a_1^{(1)} \chi_1^2 + a_2^{(2)} \chi_1^2 + a_3^{(3)} \chi_1^2 \leq t]$$

 a_3, a_2, a_1

t	$\frac{1}{3}, \frac{1}{3}, \frac{1}{3}$	.4, .3, .3	.4, .4, .2	.5, .3, .2	.6, .2, .2
4.05	99311248	99254732	99105433	98932982	98473077
4.10	99357602	99302982	99159601	98991239	98543841
4.15	99400869	99348104	99210483	99046538	98611203
4.20	99441253	99390313	99258288	99098587	98675338
4.25	99478942	99429791	99303197	99147863	98736382
4.30	99514114	99466723	99345394	99194833	98794540
4.35	99546935	99501260	99385042	99239175	98849921
4.40	99577560	99533570	99422282	99280954	98902675
4.45	99606134	99563793	99457265	99320027	98952918
4.50	99632793	99592049	99490132	99357214	99000788
4.55	99657663	99618472	99521016	99392335	99046401
4.60	99680864	99643194	99550021	99425331	99089876
4.65	99702505	99666317	99577279	99456826	99131309
4.70	99722691	99687942	99602881	99486196	99170783
4.75	99741518	99708164	99626935	99513776	99208411
4.80	99759077	99727063	99649530	99539851	99244281
4.85	99775452	99744759	99670762	99564584	99278494
4.90	99790723	99761298	99690705	99588132	99311099
4.95	99804963	99776739	99709438	99610412	99342184
5.00	99818241	99791215	99727037	99631387	99371845
5.50	99910366	99893092	99853888	99792144	99602675
6.00	99955921	99945210	99921776	99879548	99747362
6.50	99978361	99971879	99958121	99929510	99838653
7.00	99989379	99985542	99977534	99958993	99896550
7.50	99994774	99992557	99987873	99975736	99933430
8.00	99997409	99996155	99993285	99985744	99957027
8.50	99998693	99998012	99995956	99991490	99972177
9.00	99999318	99998970	99997001	99994914	99981946
9.50	99999622	99999469	99996967	99996929	99988256
10.00	99999769	99999727	99996025	99998157	99992336

Table 2

$$P[a_1^{(1)} \chi_1^2 + a_2^{(2)} \chi_1^2 + a_3^{(3)} \chi_1^2 \leq t]$$

t	a_3, a_2, a_1				
	.5, .4, .1	.6, .3, .1	.7, .2, .1	.8, .1, .1	.9, .05, .05
4.05	98686317	98380785	97796029	97011003	96013892
4.10	98754587	98455644	97883325	97113763	96135063
4.15	98819888	98526963	97966790	97212803	96252375
4.20	98881368	98598747	98046741	97308282	96365963
4.25	98939814	98663605	98123396	97400308	96475953
4.30	98995976	98725104	98196950	97489031	96582473
4.35	99049364	98783236	98267583	97574576	96685638
4.40	99099893	98845291	98335372	97657088	96785551
4.45	99146907	98897419	98400380	97736638	96882339
4.50	99192140	98947306	98462847	97813369	96976071
4.55	99234795	98994798	98522832	97887382	97066896
4.60	99274655	99040200	98580422	97958773	97154874
4.65	99312916	99083668	98635671	98027656	97240118
4.70	99348880	99125279	98689384	98094109	97322715
4.75	99382467	99164810	98740073	98158224	97402747
4.80	99414315	99204974	98788730	98220101	97480299
4.85	99444725	99240521	98835411	98279808	97555447
4.90	99473941	99274531	98880262	98337449	97628289
4.95	99501818	99307065	98923355	98393061	97698875
5.00	99527881	99338204	98964765	98446749	97767299
5.50	99750905	99583105	99298686	98891694	98346329
6.00	99856212	99733650	99523265	99206295	98771996
6.50	99914625	99830209	99674216	99429879	99086089
7.00	99951798	99890877	99776655	99589460	99318510
7.50	99971116	99929617	99846479	99703766	99490992
8.00	99983293	99954935	99894153	99785854	99619258
8.50	99989933	99970724	99926946	99844959	99714823
9.00	99993949	99980952	99949396	99887597	99786149
9.50	99996324	99987561	99964908	99918425	99839456
10.00	99997911	99991860	99975591	99940714	99879378

Table 3
 $P[a_1^{(1)}\chi_1^2 + a_2^{(2)}\chi_1^2 \leq t]$

a_2, a_1					
P	.5, .5	.6, .4	$\frac{2}{3}, \frac{1}{3}$	.7, .3	.75, .25
.05	.0512933	.0502834	.0484347	.0471210	.0445980
.10	.1053605	.1033439	.0996528	.0970299	.0919943
.15	.1625189	.1595026	.1539828	.1500613	.1425363
.20	.2231436	.2191393	.2118145	.2066130	.1966385
.25	.2876821	.2827083	.2736149	.2671620	.2548001
.30	.3566749	.3507583	.3399502	.3322877	.3176293
.35	.4307829	.4239617	.4115148	.4027026	.3858766
.40	.5108256	.5031533	.4891744	.4792956	.4604821
.45	.5978370	.5893882	.5740254	.5631956	.5426442
.50	.6931472	.6840256	.6674860	.6558649	.6339206
.55	.7985077	.7888589	.7714293	.7592430	.7363857
.60	.9162907	.9063221	.8884132	.8759775	.8528807
.65	1.0498221	1.0398349	1.0220402	1.0098117	.9874373
.70	1.2039728	1.1944190	1.1776240	1.1662788	1.1460333
.75	1.3862944	1.3778818	1.3634634	1.3540430	1.3380548
.80	1.6094379	1.6033472	1.5935807	1.5877814	1.5794361
.85	1.8971200	1.8955189	1.8945236	1.8953551	1.9004575
.90	2.3025851	2.3101499	2.3266188	2.3402352	2.3700817
.95	2.9957323	3.0265143	3.0855245	3.1287252	3.2119173

Table 3

$$P[a_1^{(1)}\chi_1^2 + a_2^{(2)}\chi_1^2 \leq t]$$

a_2, a_1					
.8, .2	.875, .125	.9, .1	.95, .05	.99, .01	1, 0
.0412995	0.0343785	0.0313116	0.0231255	0.0113775	0.0039321
.0854130	0.0716190	0.0655149	0.0492758	0.0264839	0.0157908
.1327088	0.1121513	0.1030800	0.0791007	0.0469824	0.0357658
.1836280	0.1564972	0.1445808	0.1134158	0.0747420	0.0641848
.2387046	0.2052967	0.1907259	0.1532382	0.1112559	0.1015311
.2985925	0.2593408	0.2423946	0.1998188	0.1574586	0.1484719
.3641019	0.3196192	0.3006879	0.2546648	0.2141737	0.2059002
.4362531	0.3873841	0.3669949	0.3195510	0.2824997	0.2749959
.5163519	0.4642405	0.4430833	0.3965315	0.3639473	0.3573172
.6061019	0.5522766	0.5312252	0.4879979	0.4605556	0.4549364
.7077789	0.6542579	0.6343803	0.5968432	0.5750892	0.5706519
.8245074	0.7739286	0.7564806	0.7268075	0.7113684	0.7083264
.9607254	0.9165067	0.9029067	0.8830853	0.8748335	0.8734573
1.1230118	1.0895570	1.0813581	1.0733542	1.0735506	1.0741943
1.3216774	1.3046746	1.3035784	1.3096868	1.3201595	1.3233038
1.5741595	1.5811170	1.5891177	1.6125527	1.6360350	1.6423745
1.9134105	1.9548644	1.9747334	2.0207290	2.0616020	2.0722508
2.4139603	2.5070381	2.5436905	2.6221806	2.6885579	2.7055437
3.3157800	3.4993314	3.5651446	3.7011350	3.8131053	3.8414598

Table 4

$$P[a_1^{(1)}\chi_1^2 + a_2^{(2)}\chi_1^2 + a_3^{(3)}\chi_1^2 \leq t]$$

a_3, a_2, a_1					
P	$\frac{1}{3}, \frac{1}{3}, \frac{1}{3}$	.4, .3, .3	.4, .4, .2	.5, .3, .2	.6, .2, .2
.05	.1172838	.1162503	.1121598	.1098702	.1023767
.10	.1947929	.1931584	.1867800	.1830821	.1710399
.15	.2659252	.2637962	.2556041	.2506954	.2347879
.20	.3350594	.3325023	.3228059	.3168023	.2974412
.25	.4041790	.4012462	.3902967	.3832849	.3607890
.30	.4745521	.4712908	.4593189	.4513756	.4260352
.35	.5471933	.5436504	.5308865	.5220894	.4942012
.40	.6230577	.6192834	.6059725	.5964080	.5663042
.45	.7031571	.6992088	.6856241	.6753956	.6434740
.50	.7886597	.7846056	.7710647	.7603039	.7270653
.55	.8810036	.8769286	.8638143	.8526958	.8187967
.60	.9820574	.9780719	.9658609	.9546252	.9209578
.65	1.0943731	1.0906255	1.0799350	1.0689260	1.0367462
.70	1.2216261	1.2183264	1.2099865	1.1997185	1.1708636
.75	1.3694513	1.3669116	1.3620965	1.3533718	1.3306658
.80	1.5472127	1.5459324	1.5464142	1.5405821	1.5286400
.85	1.7723537	1.7732174	1.7819266	1.7814896	1.7885747
.90	2.0838033	2.0886598	2.1112237	2.1216246	2.1649136
.95	2.6049232	2.6191934	2.6701791	2.7082268	2.8354503

Table 4

$$P[a_1^{(1)} \chi_1^2 + a_2^{(2)} \chi_1^2 + a_3^{(3)} \chi_1^2 \leq t]$$

a_3, a_2, a_1					
P	.5, .4, .1	.6, .3, .1	.7, .2, .1	.8, .1, .1	.9, .05, .05
.05	.0976010	.0942377	.0867553	.0725375	.0489181
.10	.1643891	.1587611	.1463208	.1230581	.0848072
.15	.2272579	.2195558	.2026281	.1714381	.1207631
.20	.2897753	.2800898	.2589169	.2204765	.1589431
.25	.3536368	.3420285	.3167887	.2716613	.2007928
.30	.4200059	.4065320	.3773992	.3261687	.2477116
.35	.4898878	.4746180	.4418026	.3851624	.3012680
.40	.5642940	.5473257	.5111112	.4499504	.3633136
.45	.6443518	.6258263	.5866130	.5221161	.4360567
.50	.7314079	.7115335	.6698963	.6036681	.5221283
.55	.8271599	.8062430	.7630155	.6972421	.6246956
.60	.9338467	.9123419	.8687372	.8063995	.7477316
.65	1.0545596	1.0331489	.9909504	.9341040	.8965913
.70	1.1937848	1.1735081	1.1354033	1.0935425	1.0791032
.75	1.3584452	1.3409569	1.3110758	1.2896947	1.3076064
.80	1.5601119	1.5481910	1.5332340	1.5427275	1.6031101
.85	1.8204696	1.8192014	1.8307625	1.8865691	2.0053720
.90	2.1882602	2.2086801	2.2696650	2.3980261	2.6039266
.95	2.8199090	2.8943630	3.0615976	3.3269842	3.6853594

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