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13. ABSTRACT

A strong feedback of density fluctuations to electron density-density correlations leads to a nonlinear integral equation for the memory function in the magnetoconductivity of 2D electron systems. It is shown that this integral equation has a solution which represents the quantized Hall effect and that the solution is independent of impurity scattering. Thus, the quantized Hall effect is explained.
The quantized Hall effect has a very important practical application: It provides the best standard for resistivity.

Additional keywords: equations; Naval research.

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Quantized Hall effect in two-dimensional electron systems

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Abstract. The appearance of plateaus in the Hall conductivity of two-dimensional electron systems is explaned.

It has been discovered by von Klitzing et al (1980) and studied (Kawaji et al 1981, Wakabayashi and Kawaji 1980, Paalanen et al 1982, Ebert et al 1982) that the Hall conductivity of two-dimensional electron systems shows plateaus when plotted against electron density n or magnetic field H. The plateau values are found to be integral multiples of e^2/h at the accuracy of 1 part in 10^7. More recently, Tsui et al (1982) found that similar plateaus appear even when the lowest Landau level is partially occupied. For distinction, the former case is called the "Integral" quantized Hall effect, while the latter the "Fractional" quantized Hall effect.

These two cases may be related with each other, but it is convenient to treat them separately because they appear in different regions of electron density. In what follows in the present article, a new treatment of the Integral Hall effect is given under the assumption that there is a locarized region between two adjacent Landau levels, as often assumed in explaining the effect.

The quantized Hall effect has attracted considerable theoretical attention. Based on an ideal system, Laughlin (1981) proposed a gauge invariance theory. Joynt and Prange (1984) investigated impurity effects. Halperin (1982) and others (Heinonen et al 1983, Guiliani et al 1983) emphasized the role played by the edge current.

Our approach to the Hall effect is somewhat different. Let us start with the dynamic conductivity of two-dimensional electrons given in the following form (Ting et al 1977, Götze and Hajdu 1978, Shiwa and Isihara 1983):

$$\sigma_{xx} + i\sigma_{xy} = \frac{(ine^2/m)}{\omega - \omega_c + M(\omega)} \tag{1}$$

where σ_{xx} and σ_{xy} are the magnetoconductivity and Hall conductivity respectively, ω_c is the cyclotron frequency, n is the electron density, m is the effective mass, and $M(\omega)$ is given to the lowest order in impurity scaatering by

$$M(\omega) = \frac{n_i}{4\pi nm} \int_0^\infty dq\, q^3 \, |v(q)|^2 C(q,\omega) \tag{2}$$

where n_i is the impurity concentration, v(q) is the impurity potential, and $C(q,\omega)$ is the density-density correlation function defined by

$$C(q,\omega) = i\int_0^\infty dt\, e^{i\omega t}(\rho_q(t),\rho_{-q}(0)) \tag{3}$$

ρ_q is the Fourier transform of the density operator.

The function $M(\omega)$ is called the memory function. Its calculation is important in determining the magnetoconductivity and Hall conductivity. We note that in the RPA approximation, which can be considered as the first approximation, the density-density correlation function becomes

$$C_o(q,\omega) = \frac{1}{\omega}[\chi_o(q,\omega) - \chi_o(q, 0)] \tag{4}$$

where

$$\chi_o(q,\omega) = \frac{\lambda(q,\omega)}{1+u(q)\lambda(q,\omega)} \tag{5}$$

Here, $u(q) = 2\pi e^2/q$ is the Coulomb potential. The function $\lambda(q,\omega)$ has been evaluated elsewhere (Isihara et al 1982). It represents the eigenvalues of the effective electron propagator. In strong magnetic field, it is given by

$$\lambda(q,\omega) = \frac{e^{-Q}}{4\pi\gamma_o}[(-Q)^{-\Omega}\gamma(\omega,-Q) + (-Q)^{\Omega}\gamma(-\Omega, -Q)] \tag{7}$$

where for simplicity we have used dimensionless variables:

$$1/\gamma_o = 2n/(eH/ch), \quad \Omega = \omega/\omega_c, \quad Q = \hbar q^2/2m\omega_c \tag{8}$$

$\gamma(\Omega,-Q)$ is the incomplete gamma function of the first kind. Equation (7) represents the RPA polarization function. Since it diverges at cyclotron frequencies, Isihara et al (1982) introduced phenomenologically a broadening parameter to each Landau level in their treatments of cyclotron resonance which is determined by the magnetoconductivity.

Such a phenomenological elimination of the divergence is undesirable. On the other hand, in the Götze's truncation method (Götze 1978, Gold et al 1981, 1982) which has been used for some other phenomena, such a divergence is eliminated in a self consistent way by improving the RPA through the density correlation function given by

$$C(q,\omega) = \frac{C_o(q,\omega+M(\omega))}{1 + M(\omega)C_o(q,\omega+M(\omega))/\chi_o(q,0)} \tag{9}$$

Note here that the frequency in the RPA correlation function is shifted by the memory function itself. Gold et al (1982) used for $C_o(q,\omega)$ the RPA expression in the absence of magnetic field, while Shiwa and Isihara (1984) took the field dependence into consideration for cyclotron resona

Equation (9) represents the second approximation for the correlation function in which $\chi_o(q, 0)$ is used. When eqauation (9) is introduced into equation (2) we obtain a nonlinear integral equation for the memory function $M(\omega)$. This function is generally complex so that by writing

$$M(\omega) = M'(\omega) + iM''(\omega) \tag{10}$$

we obtain from equation (1) the following expressions:

$$\sigma_{xx}(\omega) = \frac{(ne^2/m)M''}{(\omega-\omega_c+M')^2 + M''^2} \tag{11}$$

$$\sigma_{xy}(\omega) = \frac{(ne^2/m)(\omega-\omega_c + M'')}{(\omega-\omega_c+M')^2+M''^2} \tag{12}$$

Let us now apply these formulae to the quantized Hall effect. We try to explain that in a locarized region where the magnetoconductivity vanishes, the Hall conductivity takes on a plateau value which is an integral multiple of e^2/h. From the form of equation (11), we assume that the imaginary part of the memory function vanishes in the locarized region.

In order to make use of the coupled equations (2) and (9), we introduce:

$$\lambda(q,\omega+M(\omega)) = \frac{1}{2\pi\gamma_o}[\phi'(q,\Omega) + i\phi''(Q,\Omega)] \tag{13}$$

where $\phi'(Q,\Omega)$ and $\phi''(Q,\Omega)$ are the real and imaginary parts. We note that $\lambda(q,\omega)$ of equation (7) can be expressed as

$$\lambda(q,\omega) = \frac{1}{2\pi\gamma_o}\frac{e^{-Q}}{2\sin(\pi\Omega)}\int_o^{\pi} dt\, e^{-Q\cos t}[\cos(Q\sin t - \Omega t) - \cos(Q\sin t+\Omega t)] \tag{14}$$

Therefore, we arrive at

$$\phi'(Q,\Omega) = \frac{e^{-Q}}{\sin[\pi(\Omega+\tilde{M}'(\Omega)]}\int_o^{\pi} dt\, e^{-Q\cos t}\sin(Q\sin t)\sin[(\Omega+\tilde{M}'(\Omega))t] \tag{15}$$

Especially, in a locarized region and for the static case, we obtain

$$\phi' = \frac{e^{-Q}}{\sin\pi\tilde{M}'}\int_o^{\infty} dt\, e^{-Q\cos t}\sin(Q\sin t)\sin(\tilde{M}'t) \tag{16}$$

where we have used the notations such that $\tilde{M}' = \tilde{M}'(0)$ amd $\phi' = \phi'(Q,0)$.

On the other hand, equation (2) becomes

$$\tilde{M}' = \frac{n_i}{4\pi nm\omega_c^2}\int_o^{\infty} dq\, q^3|v(q)|^2\chi_o\frac{Q^{1/2}(\phi' e^{Q} - \Lambda)}{\phi'(\Lambda\,\tilde{r}_s + Q^{1/2}e^{Q})} \tag{17}$$

where $\tilde{r}_s = r_s/\gamma_o^{3/2}$; r_s being the usual dimensionless density parameter, and

$$\chi_o(q) = \frac{1}{2\pi\gamma}\frac{1}{e^{Q} + \tilde{r}_s\Lambda(Q)/Q^{1/2}} \tag{18}$$

$$\Lambda = \int_o^{Q} dt(e^{t} - 1)/t$$

Equation (17) represents a nonlinear integral equation from which a solution which improves the RPA result is expected. However, it is very difficult, if not impossible, to solve this equation because of its nonlinearity. Fortunately, for our purpose we need only a particular solution which corresponds to the static case.

In order to find such a solution, we first assume that $\pi\tilde{M}'<<1$. Then, equation (16) yields

$$\phi' = \frac{e^{-Q}}{\pi}\int_0^\pi dt\; e^{-Q\cos t}\; t\,\sin(Q\sin t)$$

$$= e^{-Q}\Lambda$$

From equation (17), we learn that $\tilde{M}'$ actually vanishes, in consistence with our original assumption.

Equation (12) then yields in the static limit:

$$\sigma_{xy} = \frac{(ne^2/m)}{-\omega_c}$$

$$= -\frac{e^2}{h}\frac{1}{2\gamma_o} \qquad (20)$$

Thus, it becomes necessary to examine the behavior of $1/\gamma_o$.

Let us consider changing the chemical potential μ of the system. If there is no spin degeneracy, the Landau levels are given by $(2j+1)\mu_B H$, and each level can take up to eH/ch electrons per unit area. If the chemical potential is between the jth and (j+1)th levels, $\mu/\mu_B H = 1/\gamma = 2(j+1)$. About this point, we consider that the state is locarized. On the ohther hand, the quantity $1/\gamma_o$ is stationary as illustrated in figure 1. Here, the dotted lines represent the ideal case for 0 K, the solid curve illustrates a more realistic case with elliptic broadening correponding to the density of states which varies as $\{1-(\epsilon-\epsilon_j)^2/\Gamma^2\}^{1/2}$ with a broadening parameter $\Gamma = 0.2\mu_B H/\pi$ for H = 2T and 0 K. The chain line is the high temperature limit. For the case with spin degeneracy, a similar graph has been given by Shiwa and Isihara (1983, 1984). Since $1/2\gamma_o$ is stationary until μ moves out of the locarized region towards the center of the next Landau level, we arrive at

$$\sigma_{xy} = -\frac{e^2}{h} j \qquad \text{(locarized region)} \qquad (21)$$

Formula (20) is related to the one derived by Streda and Smrcka (1983). For a locarized region, they note that the edge-:: current dominates. Based on a Maxwell relation, they obtain

$$\sigma_{xy} = -ec\left(\frac{\partial n}{\partial H}\right)_\mu \qquad \text{(locarized region)} \qquad (22)$$

It is important to note that the derivative in the above formula is to be obtained at constant chemical potential. For absolute zero, if the chemical potential is in a locarized region, $1/\gamma_o = (\hbar^2/2m)(4\pi n)/\mu_B H$ is flat about an even integer. Hence, n and H are proportional so that one can rewrite equation (22) such that

$$\sigma_{xy} = -ec\frac{n}{H} = -\frac{e^2}{h}\frac{1}{2\gamma_o} \qquad (23)$$

in agreement with equation (20).

We remark also that our derivation of equation (20) is independent of the form of the impurity potential. Therefore, the plateau values will not be affected by impurities. On the other hand, the locarized region is expected to widen if the impurity concentration is increased. Our basic formula (2) is correct to the lowest order in impurity scattering. In principle, therefore, no assumption on electron correlations is made, although in practice we needed a certain approximation. In the present case, we used the formalism which avoids the cyclotron divergence inherent to the RPA approximation in a self-consistent way. Thereotically, the unnatural divergence should be avoided. However, theoretically once such a divergence is eliminated, impurity scattering is unimportant for the plateau values but affects only the width of the plateaus of the quantized Hall conductivity.

We have discussed only the case of the Integral Quantized Hall effect. The case of the Fractional Hall effect requires a different approach, because we used the high density result for the polarization factor.

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References

Ebert G and von Klitzing K 1982 Solid State Commun. 44 95-98

Gold A and Götze 1981 J. Phys. C: Solid State Phys. 14 4049-4066

Gold A, Allen S J, Wilson B A and Tsui D C 1982 Phys. Rev. B 25 3519-3528

Götze W 1978 Solid State Commun 27 1393-1395

Götze W and Hajdu J 1978 J. Phys. C: Solid State Phys. 11 3993-4008

Guiliani G F, Quinn J J and Ying S C 1983 Phys. Rev. B 28 2969-2978

Halperin B I 1982 Phys. Rev. B 25 2185-2190

Heinonen O and Taylor P L 1983 Phys. Rev. B 28 6119-6122

Isihara A, Lee S J, Mukai M and Ioriatti L 1982 Physica B 112 L416-418

Isihara A and Mukai M and Lee S J 1982 Phys. Rev. B 26 3279-3284

Kawaji S, Wakabayashi J and Moriyama J 1981 J. Phys. Soc. Jpn 50 3839-3840

Kawaji S and Moriyama J 1983 Solicd State Commun. 45 511-513

Laughlin R B 1981 Phys. Rev. B 23 5632-5633

Paalanen M A, Tsui D C and Gossard A C 1982 Phys. Rev. B 25 5566-5569

Joynt R and Prange R E 1984 Phys. Rev. B 29 3303-3317

Shiwa Y and Isihara A 1983 J. Phys. C: Solid State Phys. 16 4853-4864

Shiwa Y and Isihara A 1983 Phys. Rev. B 27 4743-4753

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Streda P and Smrcka L 1983 J. Phys. C: Solid State Phys. 16 L895-899

Ting C S, Ying S C and Quinn J J 1977 Phys. Rev. B 16 5394-5404

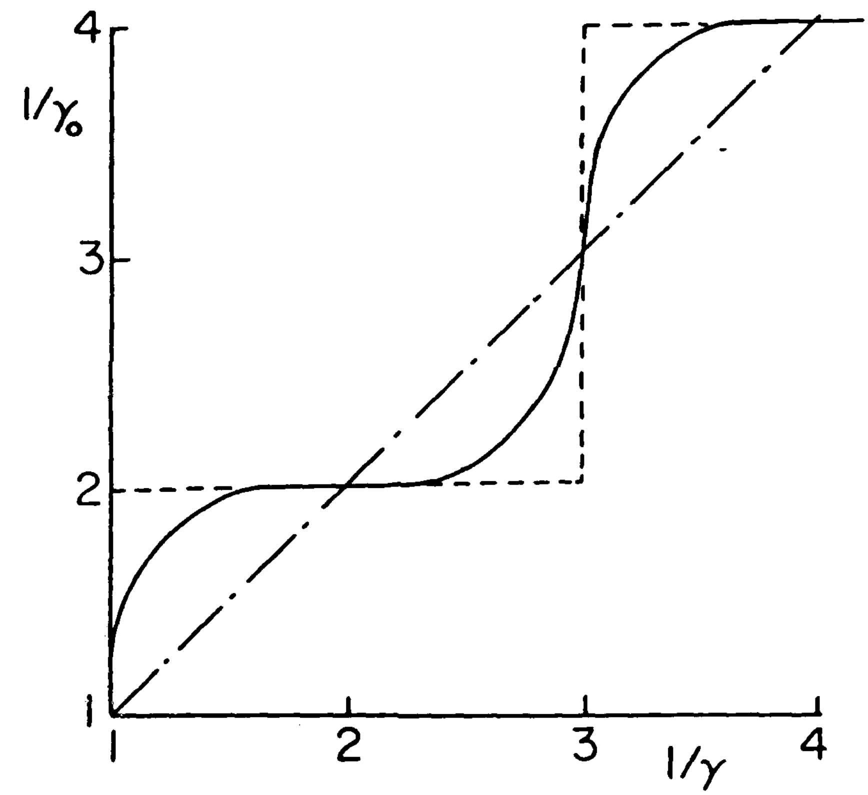
Tsui D C, Störmer H L and Gossard A C 1982 Phys. Rev. L 48 1559-1962

Wakabayashi J and Kawaji S 1980 J Phys. Soc. Jpn L 48 333-334

Figure captions

Figure 1. Variation of $1/\gamma_o = (\hbar^2/2m)(4\pi n)/\mu_B H$ with the reduced chemical potential $1/\gamma = \mu/\mu_B H$ for the case in which spin degeneracy is lifted. Solid curve: Elliptic denisty of states of the form $[1 - (\epsilon-\epsilon_j)^2/\Gamma^2]^{1/2}$ with a broadening parameter $\Gamma = 0.2\mu_B H/\pi$ for H = 2T. Dotted curve: ideal case at 0 K. Chain line: high temperature limit.

Figure 2. Variation of the Hall conductivity with n/(eH/ch). The bottom graph illustrates the density of states schematically.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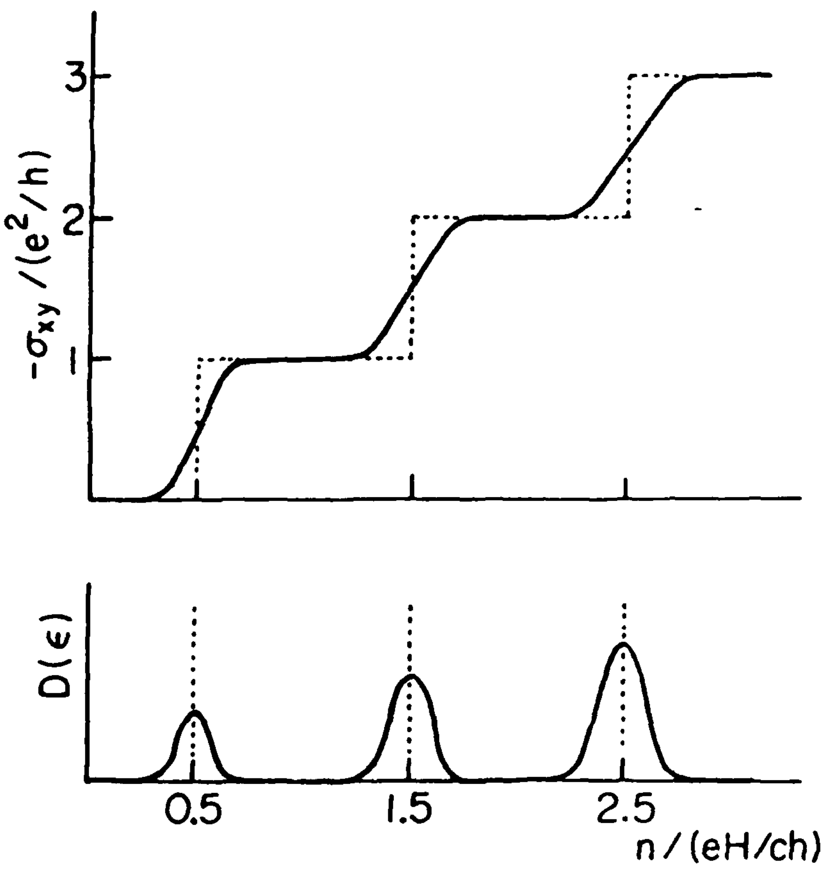

$-\sigma_{xy}/(e^2/h)$
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2
1
$D(\epsilon)$
0.5
1.5
2.5
n / (eH/ch)

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