AD-A 154 459

US ARMY
MATERIEL
COMMAND

AD A154459

MEMORANDUM REPORT BRL-MR-3437

THE INITIATION AND GROWTH OF ADIABATIC SHEAR BANDS

Thomas W. Wright
Romesh C. Batra

April 1985

US ARMY BALLISTIC RESEARCH LABORATORY
ABERDEEN PROVING GROUND, MARYLAND

Unclassified

SECURITY CLASSIFICATION OF THIS PAGE (*When Data Entered*)

REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER: MEMORANDUM REPORT BRL-MR-3437
2. GOVT ACCESSION NO.
3. RECIPIENT'S CATALOG NUMBER
4. TITLE (*and Subtitle*): The Initiation and Growth of Adiabatic Shear Bands
5. TYPE OF REPORT & PERIOD COVERED
6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s): Thomas W. Wright and Romesh C. Batra*
8. CONTRACT OR GRANT NUMBER(s)
9. PERFORMING ORGANIZATION NAME AND ADDRESS:
US Army Ballistic Research Laboratory
ATTN: AMXBR-TBD
Aberdeen Proving Ground, MD 21005-5066
10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
11. CONTROLLING OFFICE NAME AND ADDRESS:
US Army Ballistic Research Laboratory
ATTN: AMXBR-OD-ST
Aberdeen Proving Ground, MD 21005-5066
12. REPORT DATE: April 1985
13. NUMBER OF PAGES: 26
14. MONITORING AGENCY NAME & ADDRESS (*if different from Controlling Office*)
15. SECURITY CLASS. (*of this report*): Unclassified
15a. DECLASSIFICATION/DOWNGRADING SCHEDULE
16. DISTRIBUTION STATEMENT (*of this Report*): Approved for public release; distribution is unlimited.
17. DISTRIBUTION STATEMENT (*of the abstract entered in Block 20, if different from Report*)
18. SUPPLEMENTARY NOTES: *U of Missouri - Rolla
19. KEY WORDS (*Continue on reverse side if necessary and identify by block number*):

Finite Element Analysis	Viscoelasticity
Stress Strain Relations	Shear Stresses
Thermal Properties	Adiabatic Conditions
Plastic Properties	Plastic Deformation

20. ABSTRACT (*Continue on reverse side if necessary and identify by block number*)

A simple version of thermo/viscoplasticity theory is used to model the formation of adiabatic shear bands in high rate deformation of solids. The one dimensional shearing deformation of a finite slab is considered. For the constitutive assumptions made in this paper, homogeneous shearing produces a stress/strain response curve that always has a maximum when strain and rate hardening, plastic heating, and thermal softening are taken into account. Shear bands form if a perturbation is added to the homogeneous fields just before peak stress is obtained with these new fields being used as initial conditions. The resulting initial/boundary value problem is solved by the finite element method for one set of material parameters. The shear band grows slowly at first, then accelerates sharply, until finally the plastic strain rate in the center reaches a maximum, followed by a slow decline. Stress drops rapidly throughout the slab, and the central temperature increases rapidly as the peak in strain rate develops.

TABLE OF CONTENTS

LIST OF FIGURES

I. INTRODUCTION AND FORMULATION OF THE PROBLEM

Adiabatic shear is the name given to a localization phenomena that is important in many problems involving high rate deformation of solids. In the last five years or so there has been strong interest in the theoretical aspects of the subject. Clifton, et al.[1] have listed and briefly described more than a dozen papers on rapid shearing deformation in the recent literature, as well as several of the earlier pioneering works. However, there still appears to be a need to define a theoretically complete framework for the phenomenon and to find and examine dynamic solutions within such a framework. This paper summarizes our work to date in attempting to fill that need.

A general theory of thermoplasticity, due to Green and Naghdi,[2] has been taken as the starting point. In their theory, which is rate independent, plastic strain and work hardening are modeled as internal variables controlled by evolutionary equations. In this paper those general features are retained, but in addition the yield function is taken to depend on plastic strain rates, as well as stress and temperature, in a manner similar to that used by Rubin[3] and Drysdale.[4] This combination is completely self consistent in a thermodynamic sense and allows for smooth and continuous transitions between elastic and viscoplastic states. Details will be given elsewhere, Wright and Batra.[5] and only a summary of the equations are given here.

Figure 1 shows a block of material lying between Y = -H and Y = +H and undergoing only horizontal motion in the X direction. This motion is volume preserving and may be written as

$$\begin{aligned} x &= X + u(Y,t) \ , \\ y &= Y \ , \\ z &= Z \ . \end{aligned} \tag{1}$$

---

[1] *R. J. Clifton, J. Duffy, K. A. Hartley, and T. G. Shawki, "On Critical Conditions for Shear Band Formation at High Strain Rates,"* *Scripta Metallurgica* *18, p. 443-448 (1984).*

[2] *A. E. Green and P. M. Naghdi, "A General Theory of an Elastic-Plastic Continuum,"* *Arch. Rat. Mech. Anal.* *18, p. 252-281 (1965).*

[3] *M. B. Rubin, "A Thermoelastic-Viscoplastic Model with a Rate-Dependent Yield Strength,"* *J. Appl. Mech.* *49, p. 305-311 (1982).*

[4] *W. H. Drysdale, "The Theory of Plasticity with Rate Effects," ARBRL-TR-02559, ADA 142 102, May 1984, Ballistic Research Laboratory, Aberdeen Proving Ground, MD.*

[5] *T. W. Wright and R. C. Batra, "Adiabatic Shear in Simple and Dipolar Materials," to appear (1985).*

The balance relations for momentum, energy, and entropy in the absence of body forces and external sources of heat may be written

$$
\begin{aligned}
&s,_Y = \rho\ddot{u} \;, \\
&\rho\dot{U} = s\dot{u},_Y - q,_Y \;, \qquad (2) \\
&\rho\, T\, \dot{\eta} - \frac{q}{T}\, T,_Y + q,_Y \geq 0 .
\end{aligned}
$$

In these equations s is the shear stress on the planes of constant Y, U is internal energy, q is heat flux due to conduction, T is temperature, η is specific entropy, and ρ is density, which is constant. The dot and the comma indicate differentiation with respect to time t and the material coordinate Y respectively, and it is assumed in the usual way that shear strain may be decomposed into elastic and plastic parts

$$\gamma = u,_Y = \gamma_e + \gamma_p \;. \qquad (3)$$

With κ taken to be a measure of work hardening, it is assumed that a yield or loading function f exists such that

$$f(s,T,\dot{\gamma}_p) = \kappa \;, \qquad (4)$$

where f is monotonically decreasing in $\dot{\gamma}_p$, and the criterion for elastic or plastic loading is simply

$$
\begin{aligned}
&f(s,T,0) \leq \kappa \;, \text{ elastic } , \\
&f(s,T,0) > \kappa \;, \text{ plastic } .
\end{aligned}
\qquad (5)
$$

If plastic deformation is occurring, the sign of $\dot{\gamma}_p$ is taken to be the same as that of s, and its absolute value may be found uniquely from (4) because of the assumed monotonicity of f. If plastic deformation is not occurring, then of course $\dot{\gamma}_p = 0$. The situation is shown schematically in Figure 2. When the stress and temperature lie in the cross hatched region, deformation is wholly elastic, but when they lie outside, the plastic strain rate is nonzero. Furthermore, the farther outside that the point (s,T) lies, the larger the absolute value of $\dot{\gamma}_p$. Equation (4) is similar to the yield functions used by Rubin[3] and Drysdale.[4] but the treatment that follows here is somewhat different from either of theirs. The work hardening parameter is assumed to obey the following evolutionary equation,

$$\dot{\kappa} = M\,\dot{\gamma}_p \,. \tag{6}$$

M is a constitutive function that depends only on s, T and κ.*

II. CONSTITUTIVE FUNCTIONS

For computational purposes specific constitutive functions have been chosen as follows.

$$\begin{aligned} \rho U &= \frac{1}{2}\mu\,\gamma_e^2 + \rho\,T_0 c_v \left(e^{(\eta-\eta_0)/c_v} - 1\right) , \\ q &= -\,k\,T_{,y} \,, \end{aligned} \tag{7}$$

where μ is a constant shear modulus, T_0 is a reference temperature, c_v is the specific heat at constant volume, and k is the thermal conductivity. Standard thermodynamic arguments show that

$$s = \mu\,\gamma_e \,, \quad T = T_0 e^{(\eta-\eta_0)/c_v} \,, \tag{8}$$

so that the elastic response is linear, and there is no thermoelastic effect.

It is further assumed for a slow isothermal reference test at temperature T_0 that $s = \kappa = \hat{\kappa}(\varepsilon)$, where ε is the plastic strain in that case, and that whatever the rate of deformation, κ depends only on the plastic work done. Thus it follows that

$$\begin{aligned} \dot{W}_p &= \kappa\,\dot{\varepsilon} = s\dot{\gamma}_p \,, \\ M &= \frac{1}{\kappa}\,\frac{d\hat{\kappa}}{d\varepsilon}\,s \,, \end{aligned} \tag{9}$$

where $\frac{d\hat{\kappa}}{d\varepsilon}$ may be expressed as a function of κ. To complete the constitutive assumptions, the yield function and $\hat{\kappa}(\varepsilon)$ were chosen as follows.

**This scheme may be readily generalized to multidimensional states or to the case of dipolar stresses, Wright and Batra.[5] since it turns out that all plastic rates may be related by a single proportional factor, as in Green and Naghdi[2] and Green, McInnis, and Naghdi.[6] Then it is the proportional factor that is determined from the analog of (4) rather than $\dot{\gamma}_p$ itself.*

[6] *A. E. Green, B. C. McInnis, and P. M. Naghdi, "Elastic-Plastic Continua with Simple Force Dipole,"* Int. J. Eng. Sci. *6, p. 373-394 (1968).*

$$|s| = \kappa(1 - a\theta)(1 + b\,\dot{\gamma}_p)^m ,$$

$$\hat{\kappa}(\varepsilon) = \kappa_0(1 + \frac{\varepsilon}{\varepsilon_0})^n , \tag{10}$$

where $\theta = T - T_0$. It will be recognized that the viscoplastic effect in the present case comes from a multiplicative overstress factor, although (4) is sufficiently general to include an additive overstress or many other possibilities.

III. NONDIMENSIONAL VARIABLES AND HOMOGENEOUS SOLUTIONS

With nondimensional variables defined by

$$Y = H\overline{Y} , \ u = H\overline{u} , \ s = \kappa_0\overline{s} , \ \theta = \frac{\kappa_0}{\rho c_v}\overline{\theta}$$

$$t = \frac{1}{\dot{\gamma}_0}\overline{t} , \ \gamma = \overline{\gamma} , \ \kappa = \kappa_0\overline{\kappa} , \ \varepsilon = \overline{\varepsilon} , \tag{11}$$

where $\dot{\gamma}_0$ is the average strain rate imposed in the problem, the complete equations in nondimensional form become

Momentum: $$s,_Y = \frac{\rho H^2 \dot{\gamma}_0^2}{\kappa_0}\ddot{u} ,$$

Energy: $$\dot{\theta} = \frac{k}{\rho c_v \dot{\gamma}_0 H^2}\theta,_{YY} + \kappa\dot{\varepsilon} ,$$

Constitutive: $$\dot{s} = \frac{\mu}{\kappa_0}(\dot{\gamma} - \dot{\gamma}_p) , \tag{12}$$

$$\kappa = (1 + \frac{\varepsilon}{\varepsilon_0})^n ,$$

$$\kappa\dot{\varepsilon} = s\,\dot{\gamma}_p (= \dot{W}_p) ,$$

Yield Surface: $$|s| = (1 - \frac{a\kappa_0}{\rho c_v}\theta)(1 + b\dot{\gamma}_0\dot{\gamma}_p)^m \kappa,$$

where the overbars have been dropped, and $(12)_6$ is subject to (5). There are two relative length scales implicit in (12), namely a thermal length $\sqrt{\frac{k}{\rho c_v \dot{\gamma}_0 H^2}}$, and a viscous length $\frac{b}{H}\sqrt{\frac{\kappa_0}{\rho}}$. In addition there are seven other nondimensional parameters in (12) which are required to define the mass, elastic modulus, thermal softening, work hardening, and rate hardening of the material.

In a homogeneous deformation the true displacement field has the form $u = \dot{\gamma}_0 Yt$, where $\dot{\gamma}_0$ is a constant strain rate, or with nondimensional variables $u = Yt$, and nondimensional values of s, θ, γ_p, κ, and ε depend only on time. For this case the equations become ordinary differential equations with initial values

$$s(0) = 1, \ \theta(0) = 0, \ \varepsilon(0) = 0 \ , \tag{13}$$

where time is counted from the first onset of plastic flow, and $\dot{\gamma}_p$ is to be found from $(12)_6$. Equation $(12)_1$ is satisfied identically, and $(12)_2$ with $(12)_4$ substituted for κ can be integrated immediately to give $\theta(\varepsilon)$. The remaining two equations may now be written as the autonomous pair

$$\begin{aligned} \dot{s} &= \frac{\mu}{\kappa_0}(1 - \dot{\gamma}_p) \ , \quad s(0) = 1 \ , \\ \dot{\varepsilon} &= \frac{s\dot{\gamma}_p}{(1 + \frac{\varepsilon}{\varepsilon_0})^n} \ , \quad \varepsilon(0) = 0 \ , \end{aligned} \tag{14}$$

where

$$\begin{aligned} \theta(\varepsilon) &= \frac{\varepsilon_0}{1+n} \ (1 + \frac{\varepsilon}{\varepsilon_0})^{1+n} - 1 \ , \\ \dot{\gamma}_p &= \frac{1}{b\dot{\gamma}_0}\left[\left(\frac{s}{(1+\frac{\varepsilon}{\varepsilon_0})^n(1-\frac{a\kappa_0}{\rho c_v}\theta(\varepsilon))}\right)^{\frac{1}{m}} - 1\right] . \end{aligned} \tag{15}$$

Although solutions to (14) cannot be given explicitly, some of their features can be described qualitatively.[5] In particular, for the constitutive and yield functions chosen here, s always has a simple maximum at a critical value of γ, the exact value of which is influenced by work hardening, heat capacity, rate sensitivity, thermal softening, and yield strength, the first three tending to retard the peak, the last two to advance it. Figure 3 shows the homogeneous stress strain response for one particular choice of nondimensional parameters, as follows:

$$\frac{\rho\dot{\gamma}_0^2 H^2}{\kappa_0} = 3.928 \times 10^{-5} \ , \quad \frac{k}{\rho c_v H^2 \dot{\gamma}_0} = 3.978 \times 10^{-3} \ , \quad \frac{a\kappa_0}{\rho c_v} = 0.4973 \ ,$$

$$\frac{\mu}{\kappa_0} = 240.3 \ , \ n = 0.09 \ , \ \varepsilon_0 = 0.017 \ , \ \dot{\gamma}_0 b = 5 \times 10^5 \ , \ m = 0.02.$$

IV. RESPONSE TO PERTURBATIONS

Other analyses (e.g., Burns[7] or Shawki, et al.[8])have indicated that if a small perturbation is added to the homogeneous response, its amplitude will begin to grow once the peak stress for homogeneous deformation has been passed. The perturbation could be applied to any of the field variables, but in this paper a small symmetric temperature bump was added at the center of the slab just before the peak stress, and the problem was restarted as an initial/boundary value problem, the material parameter remaining exactly as before. The boundary values are

$$v(\pm 1,t) = \pm 1, \quad \theta_{,Y} (\pm 1,t) = 0 \ , \tag{16}$$

so that the average strain rate in the strip [-1,+1] is maintained, and the strip is adiabatic overall. The problem was solved by the finite element method; for details, see Wright and Batra.[5] Some of the principal results are shown in Figures 4, 5, and 6.

Figure 4 shows a cross section of the temperature at various times after introduction of the perturbation. On this scale the peak in the initial temperature itself does not show since it is only 0.02 higher than the surrounding ambient value, which is reached at $Y = \pm 0.1$ on either side of the central peak. Only half of the central part is shown since the profile is symmetric and remains flat on out to ± 1. Cross sections of plastic strain rate also show a strong central peak at late times.

Figure 5 shows the stress, plastic strain rate, and temperature as functions of time at a point very near to the center of the band. The plastic rate begins a slow increase, which is actually exponential at

---

[7] *T. J. Burns, "Approximate Linear Stability Analysis of a Model of Adiabatic Shear Band Formation," Sandia Report SAND83-1907, October 1983, Sandia National Laboratories, Albuquerque, NM.*

[8] *T. G. Shawki, R. J. Clifton, and G. Majda, "Analysis of Shear Strain Localization in Thermal Visco-Plastic Materials," Brown University Report ARO DAAG29-81-K-0121/3, October 1983, Providence, RI.*

first, and then after a fairly long run-in time, it accelerates rapidly, goes over an abrupt peak, and finally begins a slow decline. The temperature begins with a slow but steady increase and then rises very rapidly at the end, whereas the stress begins with a slow decrease and then drops rapidly at the end. It is during the period of most rapid change that the shear band takes recognizable shape.

Figure 6 shows the same three functions as in Figure 5, but at a point near the boundary. Here the plastic strain rate decreases slowly at first, and after the run-in time, it drops sharply and finally makes a rapid but smooth transition to zero. Since plastic work ceases towards the end, the temperature arrives at a plateau, but the stress continues to drop on into the elastic region. Comparison of the stress curves shows that, although central and edge stresses are nearly equal during the run-in time, the edge stress actually drops later than the central stress. Thus the curves indicate that the stress in the center drops rapidly because of thermal softening, but the stress at the edge drops because of momentum transfer. Since the average strain rate is constant, the stress/time plot may be interpreted as a stress/average strain plot. This is shown in Figure 3.

REFERENCES

1. R. J. Clifton, J. Duffy, K. A. Hartley, and T. G. Shawki, "On Critical Conditions for Shear Band Formation at High Strain Rates," Scripta Metallurgica 18, p.443-448 (1984).

2. A. E. Green and P. M. Naghdi, "A General Theory of an Elastic-Plastic Continuum," Arch. Rat. Mech. Anal. 18, p. 252-281 (1965).

3. M. B. Rubin, "A Thermoelastic-Viscoplastic Model with a Rate-Dependent Yield Strength," J. Appl. Mech. 49, p. 305-311 (1982).

4. W. H. Drysdale, "The Theory of Plasticity with Rate Effects," ARBRL-TR-02559, ADA 142 102, May 1984, Ballistic Research Laboratory, Aberdeen Proving Ground, MD.

5. T. W. Wright and R. C. Batra, "Adiabatic Shear in Simple and Dipolar Materials," to appear (1985).

6. A. E. Green, B. C. McInnis, and P. M. Naghdi, "Elastic-Plastic Continua with Simple Force Dipole," Int. J. Eng. Sci. 6, p. 373-394 (1968).

7. T. J. Burns, "Approximate Linear Stability Analysis of a Model of Adiabatic Shear Band Formation," Sandia Report SAND83-1907, October 1983, Sandia National Laboratories, Albuquerque, NM.

8. T. G. Shawki, R. J. Clifton, and G. Majda, "Analysis of Shear Strain Localization in Thermal Visco-Plastic Materials," Brown University Report ARO DAAG29-81-K-0121/3, October 1983, Providence, RI.

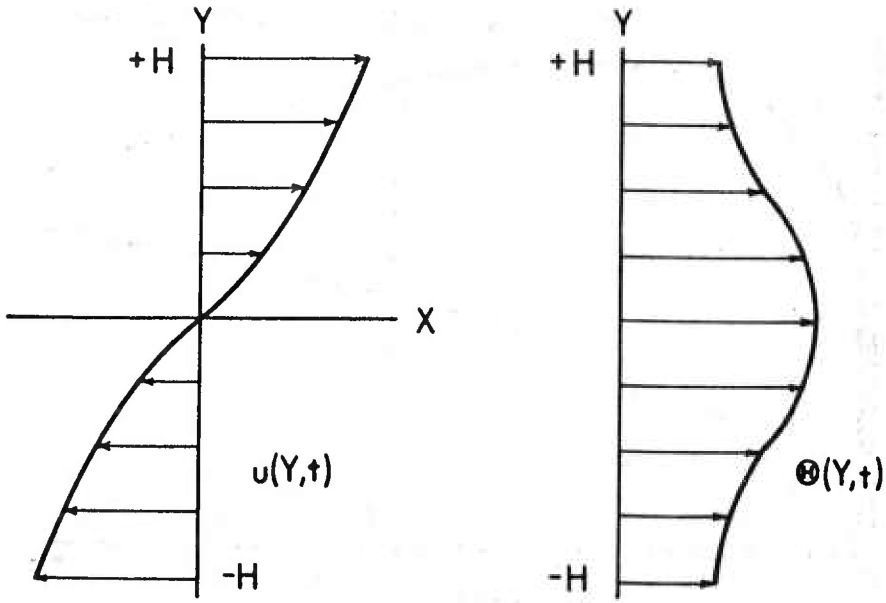

Figure 1. Shearing of a finite block of material with displacement u and temperature change θ.

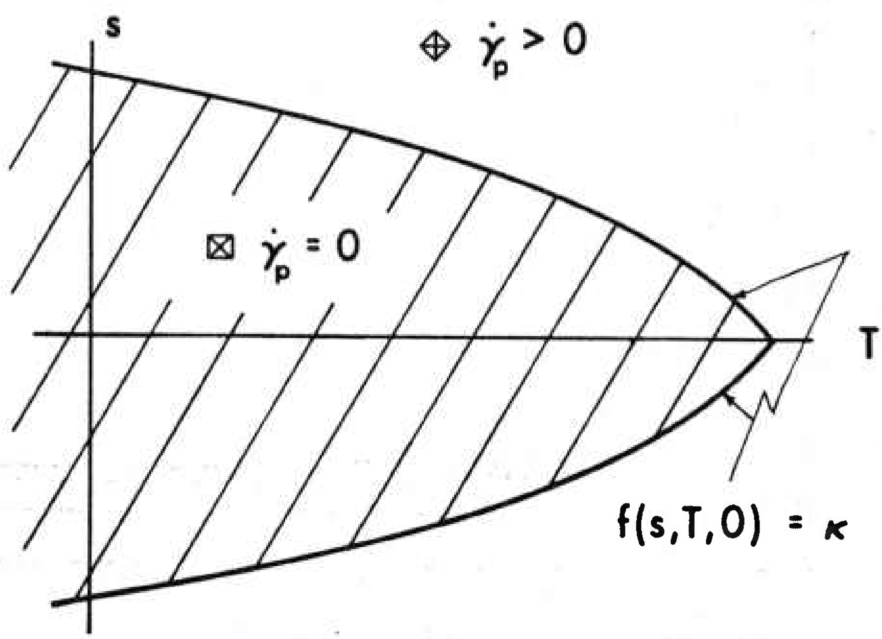

Figure 2. Yield surface in stress-temperature space.

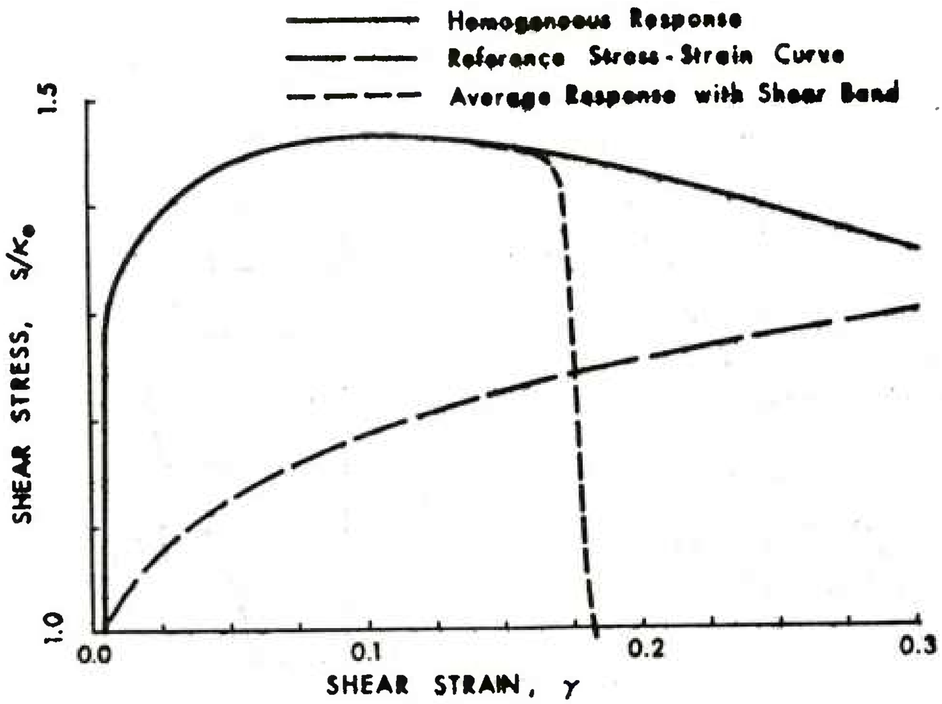

Figure 3. Stress-strain response curves for a block of material in shear.

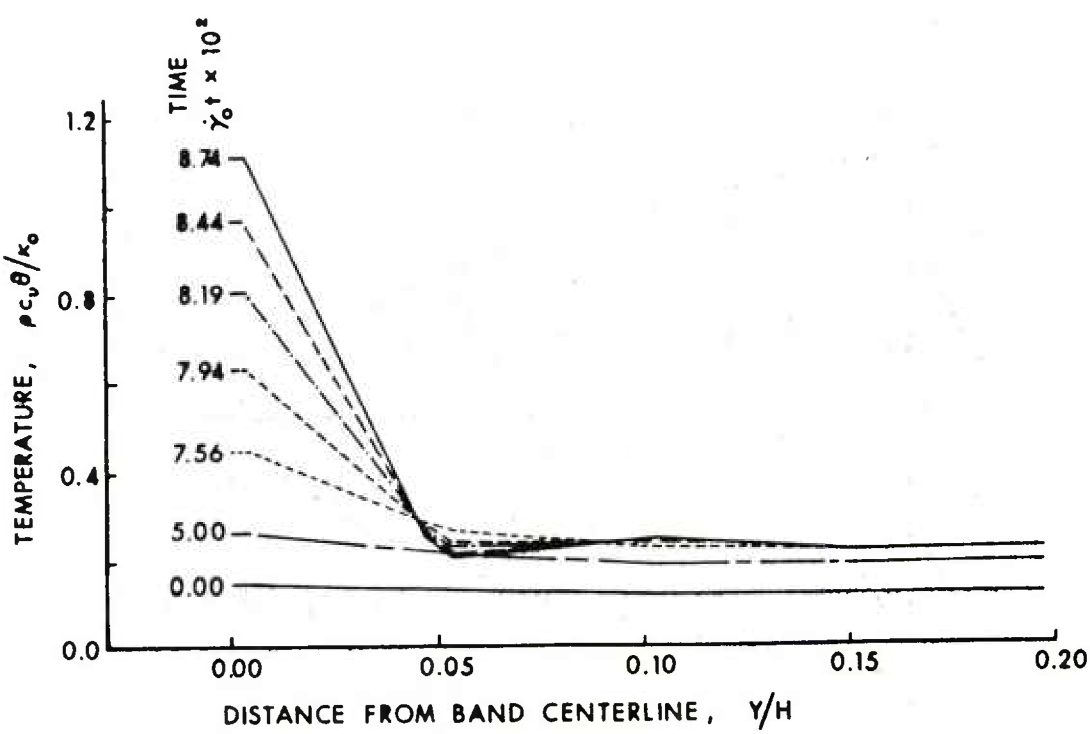

Figure 4. Temperature distribution in a finite block of material at various times as a shear band forms.

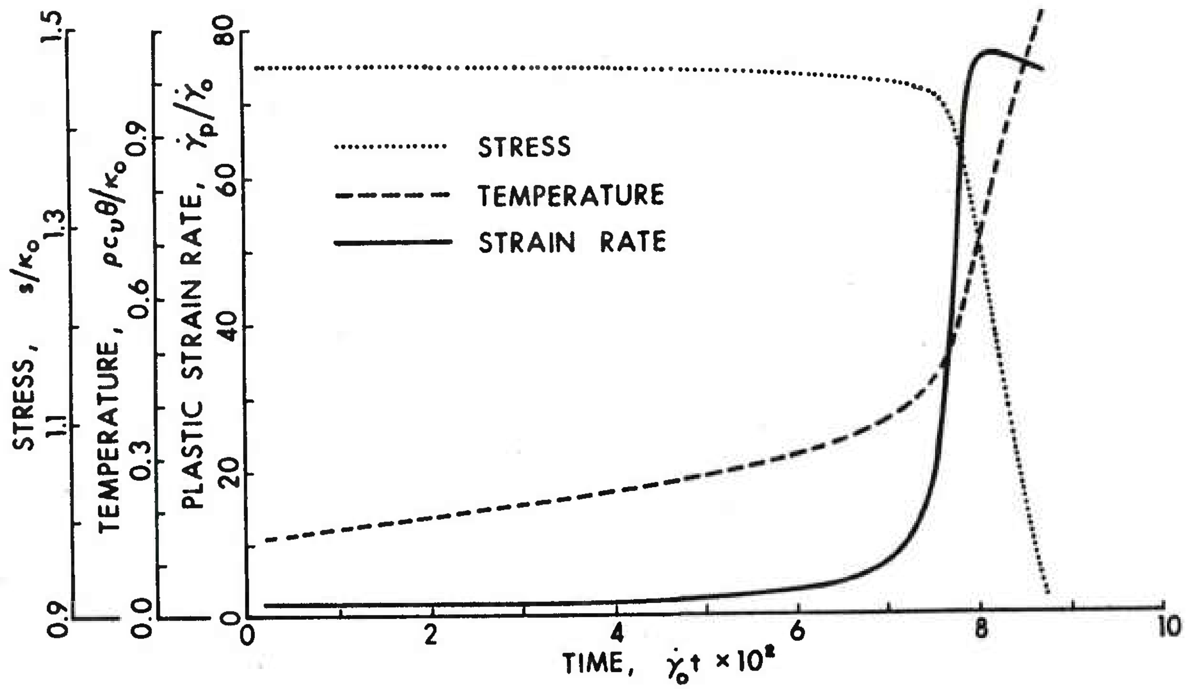

Figure 5. Stress, temperature, and plastic strain rate vs. time near the center of a block of material as a shear band forms.

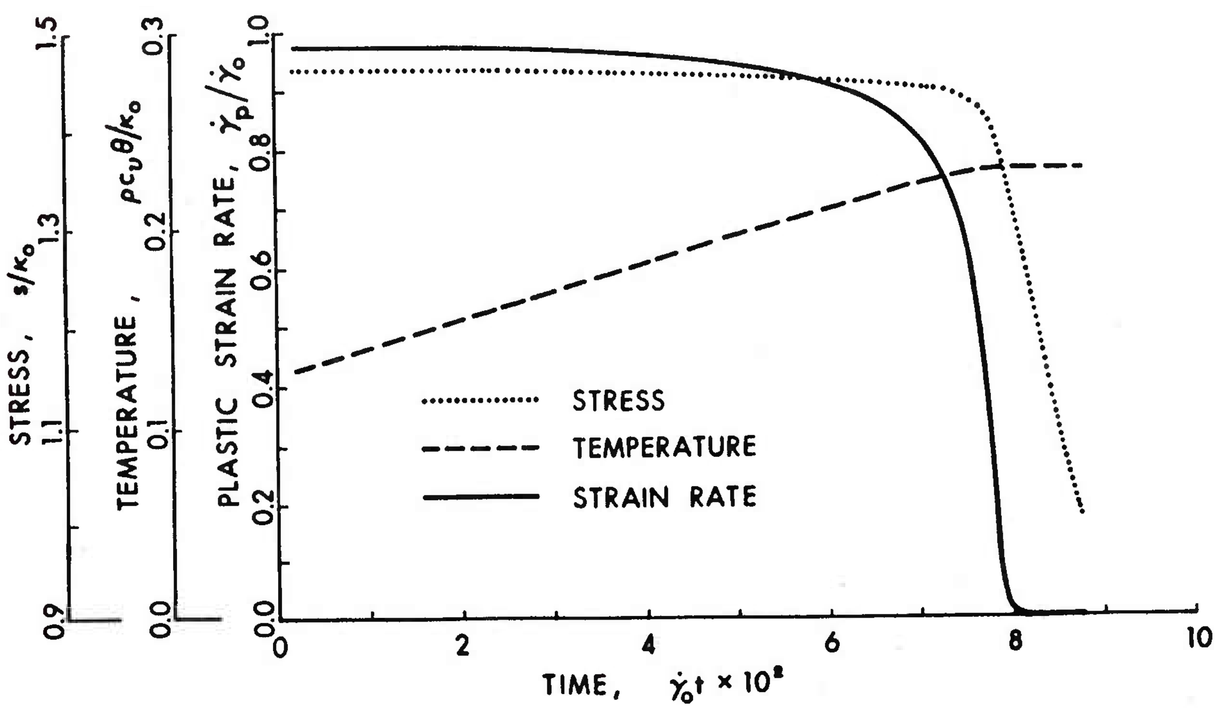

Figure 6. Stress, temperature, and plastic strain rate vs. time near the edge of a block of material as a shear band forms.

No. of Copies	Organization
12	Administrator Defense Technical Info Center ATTN: DTIC-DDA Cameron Station Alexandria, VA 22304-6145
1	Director Defense Advanced Research Projects Agency ATTN: Tech Info Dr. E. Van Reuth 1400 Wilson Boulevard Arlington, VA 22209
1	Deputy Assistant Secretary of the Army (R&D) Department of the Army Washington, DC 20310
1	HQDA DAMA-ART-M Washington, DC 20310
1	Commandant Command and General Staff College ATTN: Archives Fort Leavenworth, KS 66027
1	Commander US Army War College ATTN: Lib Carlisle Barracks, PA 17013
1	Commander US Army Materiel Command ATTN: AMCDRA-ST 5001 Eisenhower Avenue Alexandria, VA 22333-0001

No. of Copies	Organization
7	Commander Armament R&D Center US Army AMCCOM ATTN: SMCAR-TSS SMCAR-TDC SMCAR-LC, J. Frasier SMCAR-LCA, T. Davidson SMCAR-SC, J. D. Corrie J. Beetle E. Bloore Dover, NJ 07801
1	Commander US Army Armament, Munitions and Chemical Command ATTN: SMCAR-ESP-L Rock Island, IL 61299
1	Director Benet Weapons Laboratory Armament R&D Center US Army AMCCOM ATTN: SMCAR-LCB-TL Watervliet, NY 12189
6	Commander Benet Weapons Laboratory ATTN: Dr. M. A. Hussain Dr. Julian Wu Dr. John Underwood Mr. D. P. Kindall Dr. J. Throup Dr. E. Schneider Watervliet, NY 12189
1	Commander US Army Aviation Research and Development Command ATTN: AMSAV-E 4300 Goodfellow Boulevard St. Louis, MO 63120

DISTRIBUTION LIST

No. of Copies	Organization
1	Commander US Army Communications - Electronics Command Command ATTN: AMSEL-ED Fort Monmouth, NJ 07703
1	Commander US Army Electronics Research and Development command Technical Support Activity ATTN: DELSD-L Fort Monmouth, NJ 07703-5301
1	Commander US Army Harry Diamond Laboratory ATTN: DELHD-TA-L 2800 Powder Mill Road Adelphi, MD 20783
1	Commander US Army Development and Employment Agency ATTN: MODE-TED-SAB Fort Lewis, WA 98433
1	Commander US Army Missile Command ATTN: AMSMI-R Redstone Arsenal, AL 35898
1	Commander US Army Missile Command ATTN: AMSMI-YDL Redstone Arsenal, AL 35898
1	Commander US Army Belvoir R&D Center ATTN: STRBE-WC Tech Library (Vault), Bldg 315 Ft. Belvoir, VA 22060-5606
1	Commander US Army Natick Research and Development Center ATTN: DRDNA-DT, Dr. D. Sieling Natick, MA 01762
1	Commander US Army Tank Automotive Command ATTN: AMSTA-TSL Warren, MI 48090
1	Director US Army TRADOC Systems Analysis Activity ATTN: ATAA-SL White Sands Missile Range, NM 88002
1	Commander US Army Electronics Proving Ground ATTN: Tech Lib Fort Huachuca, AZ 85613
1	Commandant US Army Infantry School ATTN: ATSH-CD-CSO-OR Fort Benning, GA 31905
1	Director US Army Advanced BMD Technology Center ATTN: CRDABH-5, W. Loomis P. O. Box 1500, West Station Huntsville, AL 35807
3	Commander US Army Materiel and Mechanics Research Center ATTN: AMXMR-T, J. Mescall AMXMR-T, R. Shea AMXMR-H, S. C. Chou Watertown, MA 02172

DISTRIBUTION LIST

No. of Copies	Organization
5	Commander US Army Research Office Durham ATTN: Dr. R. Weigle Dr. E. Saibel Dr. G. Mayer Dr. F. Smeideshoff Dr. J. Chandra P. O. Box 12211 Research Triangle Park, NC 27709-2211
2	Commander US Army Standarization Group (Europe) ATTN: Dr. B. Steverding Dr. F. Rothwarf Box 65 FPO NY 09510
1	Office of Naval Research Department of the Navy ATTN: Code 402 Washington, DC 20360
1	Commander US Naval Air Systems Command ATTN: AIR-604 Washington, DC 20360
1	AUL (3T-AUL-60-118) Maxwell AFB, AL 36112
3	Commander Naval Surface Weapons Center ATTN: Dr. W. H. Holt Dr. W. Mock Tech Lib Dahlgren, VA 22448
3	Commander Naval Surface Weapons Center ATTN: Dr. R. Crowe Code R32, Dr. S. Fishman Tech lib Silver Spring, MD 20910
1	AFELM, The Rand Corporation ATTN: Library-D 1700 Main Street Santa Monica, CA 90406
5	Commander US Naval Research Laboratory ATTN: C. Sanday R. J. Weimer Code 5270, F. MacDonald Code 2020, Tech Lib Code 7786, J. Baker Washington, DC 20375
7	Commander US Naval Research Laboratory Engineering Materials Division ATTN: E. A. Lange G. R. Yoder C. A. Griffis R. J. Goode R. W. Judy, Jr. A. M. Sullivan R. W. Crooker Washington, DC 20375
5	Air Force Armament Laboratory ATTN: AFATL/DLODL J. Foster John Collins Joe Smith Guy Spitale Eglin AFB, FL 32542-5000
1	RADC (EMTLD, Lib) Griffiss AFB, NY 13441

DISTRIBUTION LIST

No. of Copies	Organization
2	Air Force Wright Aeronautical Laboratories Air Force Systems Command Materials Laboratory ATTN: Dr. Theodore Nicholas Dr. John P. Henderson Wright-Patterson AFB, OH 45433
1	Director Environmental Science Service Administration US Department of Commerce Boulder, CO 80302
1	Director Lawrence Livermore Laboratory ATTN: Dr. M. L. Wilkins P. O. Box 808 Livermore, CA 94550
6	Sandia Laboratories ATTN: Dr. L. Davison Dr. P. Chen Dr. L. Bertholf Dr. W. Herrmann Dr. J. Nunziato Dr. S. Passman Albuquerque, NM 87115
1	Director US Army Air Mobility Research and Development Laboratory Ames Research Center Moffett Field, CA 94035
1	Director Jet Propulsion Laboratory ATTN: Lib (TDS) 4800 Oak Grove Drive Pasadena, CA 91109
1	Aeronautical Research Associates of Princeton, Incorporated ATTN: Ray Gogolewski 1800 Old Meadow Rd., #114 McLean, VA 22102
1	Honeywell, Inc. Defense Systems Division ATTN: Dr. Gordon Johnson 600 Second Street, NE Hopkins, MN 55343
1	AFWL/SUL Kirtland AFB, NM 87117
2	Orlando Technology, Inc. ATTN: Dr. Daniel Matuska Dr. John J. Osborn P. O. Box 855 Shalimar, FL 32579
7	SRI International ATTN: Dr. George Abrahamson Dr. Donald R. Curran Dr. Donald A. Shockey Dr. Lynn Seaman Mr. D. Erlich Dr. A. Florence Dr. R. Caligiuri 333 Ravenswood Avenue Menlo Park, CA 94025
1	System Planning Corporation ATTN: Mr. T. Hafer 1500 Wilson Boulevard Arlington, VA 22209
1	Terra-Tek, Inc. ATTN: Dr. Arfon Jones 420 Wakara Way University Research Park Salt Lake City, UT 84108
6	Brown University Division of Engineering ATTN: Prof. R. Clifton Prof. H. Kolsky Prof. L. B. Freund Prof. A. Needleman Prof. R. Asaro Prof. R. James Providence, RI 02912

DISTRIBUTION LIST

No. of Copies	Organization
3	California Institute of Technology Division of Engineering and Applied Science ATTN: Dr. J. Mikowitz Dr. E. Sternberg Dr. J. Knowles Pasadena, CA 91102
3	Carnegie-Mellon University Department of Mathematics ATTN: Dr. D. Owen Dr. M. E. Gurtin Dr. B. D. Coleman Pittsburgh, PA 15213
2	Catholic University of America School of Engineering and Architecture ATTN: Prof. A. Durelli Prof. J. McCoy Washington, DC 20017
6	Cornell University Department of Theoretical and Applied Mechanics ATTN: Dr. Y. H. Pao Dr. G. S. S. Ludford Dr. A. Ruoff Dr. J. Jenkins Dr. R. Lance Dr. F. Moon Ithaca, NY 14853
1	University of Denver Denver Research Institute ATTN: Dr. R. Recht P. O. Box 10127 Denver, CO 80210
2	Forrestal Research Center Aeronautical Engineering Lab. Princeton University ATTN: Dr. S. Lam Dr. A. Eringen Princeton, NY 08540
1	Harvard University Division of Engineering and Applied Physics ATTN: Prof. J. R. Rice Cambridge, MA 02138
2	Iowa State University Engineering Research Laboratory ATTN: Dr. G. Nariboli Dr. A. Sedov Ames, IA 50010
2	Lehigh University Center for the Application of Mathematics ATTN: Dr. E. Varley Dr. R. Rivlin Bethlehem, PA 18015
1	New York University Department of Mathematics ATTN: Dr. J. Keller University Heights New York, NY 10053
1	North Carolina State University Department of Civil Engineering ATTN: Prof. Y. Horie Raleigh, NC 27607
1	Pennsylvania State University Engineering Mechanical Dept. ATTN: Prof. N. Davids University Park, PA 16802
3	Rensselaer Polytechnic Institute ATTN: Prof. E. H. Lee Prof. E. Krempl Prof. J. Flaherty Troy, NY 12181

DISTRIBUTION LIST

No. of Copies	Organization
2	Rice University ATTN: Dr. R. Bowen Dr. C. C. Wang P. O. Box 1892 Houston, TX 77001
1	Southern Methodist University Solid Mechanics Division ATTN: Prof. H. Watson Dallas, TX 75222
1	Southwest Research Institute ATTN: Dr. Charles Anderson 6220 Culebra Road P. O. Box Drawer 28510 San Antonio, TX 78228
2	Southwest Research Institute Department of Mechanical Sciences ATTN: Dr. U. Kindholm Dr. W. Baker 8500 Culebra Road San Antonio, TX 78228
1	Temple University College of Engineering Technology ATTN: Dr. R. Haythornthwaite Dean Philadelphia, PA 19122
4	The Johns Hopkins University ATTN: Prof. R. B. Pond, Sr. Prof. R. Green Prof. W. Sharpe Prof. J. Bell 34th and Charles Streets Baltimore, MD 21218
1	Tulane University Dept of Mechanical Engineering ATTN: Dr. S. Cowin New Orleans, LA 70112
3	University of California ATTN: Dr. M. Carroll Dr. W. Goldsmith Dr. P. Naghdi Berkeley, CA 94720
1	University of California Dept of Aerospace and Mechanical Engineering Science ATTN: Dr. Y. C. Fung P. O. Box 109 La Jolla, CA 92037
1	University of California Department of Mechanics ATTN: Dr. R. Stern 504 Hilgard Avenue Los Angeles, CA 90024
1	University of California at Santa Barbara Dept of Mechanical Engineering ATTN: Prof. T. P. Mitchel Santa Barbara, CA 93106
1	University of Dayton Research Institute ATTN: Dr. S. J. Bless Dayton, OH 45469
1	University of Delaware Dept of Mechanical Engineering ATTN: Prof. J. Vinson Newark, DE 19711
1	University of Delaware Dept of Mechanical and Aerospace Engineering ATTN: Dr. Minoru Taya Newark, DE 19711

DISTRIBUTION LIST

No. of Copies	Organization
3	University of Florida Dept. of Engineering Science and Mechanics ATTN: Dr. C. A. Sciammarilla Dr. L. Malvern Dr. E. Walsh Gainesville, FL 32611
2	University of Houston Department of Mechanical Engineering ATTN: Dr. T. Wheeler Dr. R. Nachlinger Houston, TX 77004
1	University of Maryland Department of Mathematics ATTN: Prof. S. Antman College Park, MD 20740
1	University of Illinois Dept. of Theoretical and Applied Mechanics ATTN: Dr. D. Carlson Urbana, IL 61801
1	University of Illinois at Chicago Circle College of Engineering Dept. of Materials Engineering ATTN: Dr. T. C. T. Ting P. O. Box 4348 Chicago, IL 60680
2	University of Kentucky Dept. of Engineering Mechanics ATTN: Dr. M. Beatty Prof. O. Dillon, Jr. Lexington, KY 40506
1	University of Florida ATTN: Graduate Research Professor of Engr Sciences (Dr. Daniel C. Drucker) 231 Aerospace Engineering Bldg Gainesville, FL 32611

No. of Copies	Organization
1	University of Minnesota Dept. of Aerospace Engineering and Mechanics ATTN: Prof. J. L. Erickson 107 Akerman Hall Minneapolis, MN 55455
1	University of Pennsylvania Towne School of Civil and Mechanical Engineering ATTN: Prof. Z. Hashin Philadelphia, PA 19104
4	University of Texas Department of Engineering Mechanics ATTN: Dr. M. Stern Dr. M. Bedford Prof. Ripperger Dr. J. T. Oden Austin, TX 78712
1	University of Washington Dept. of Aeronautics and Astronautics ATTN: Dr. Ian M. Fyfe 206 Guggenheim Hall Seattle, WA 98105
2	Washington State University Department of Physics ATTN: Dr. R. Fowles Dr. G. Duvall Pullman, WA 99163
2	Yale University ATTN: Dr. B.-T. Chu Dr. E. Onat 400 Temple Street New Haven, CT 06520
1	University of Missouri- Rolla Department of Engineering Mechanics ATTN: Romesh C. Batra Rolla, MO 65401-0249

DISTRIBUTION LIST

No. of Copies	Organization	No. of Copies	Organization

Aberdeen Proving Ground

Dir, USAMSAA
  ATTN: AMXSY-D
        AMXSY-MP, H. Cohen
Cdr, USATECOM
  ATTN: AMSTE-TO-F
Cdr, CRDC, AMCCOM
  ATTN: SMCCR-RSP-A
        SMCCR-MU
        SMCCR-SPS-IL

USER EVALUATION SHEET/CHANGE OF ADDRESS

This Laboratory undertakes a continuing effort to improve the quality of the reports it publishes. Your comments/answers to the items/questions below will aid us in our efforts.

1. BRL Report Number __________ Date of Report __________

2. Date Report Received __________

3. Does this report satisfy a need? (Comment on purpose, related project, or other area of interest for which the report will be used.) __________

4. How specifically, is the report being used? (Information source, design data, procedure, source of ideas, etc.) __________

5. Has the information in this report led to any quantitative savings as far as man-hours or dollars saved, operating costs avoided or efficiencies achieved, etc? If so, please elaborate. __________

6. General Comments. What do you think should be changed to improve future reports? (Indicate changes to organization, technical content, format, etc.)

CURRENT ADDRESS

Name

Organization

Address

City, State, Zip

7. If indicating a Change of Address or Address Correction, please provide the New or Correct Address in Block 6 above and the Old or Incorrect address below.

OLD ADDRESS

Name

Organization

Address

City, State, Zip

(Remove this sheet along the perforation, fold as indicated, staple or tape closed, and mail.)