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# APPROXIMATE PLANETARY EPHEMERIDES FOR SLBM APPLICATIONS

BY PATRICIA BOUTCHYARD MICHAEL D. HARKINS

STRATEGIC SYSTEMS DEPARTMENT

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NAVAL SURFACE WEAPONS CENTER

Dahlgren, Virginia 22448 • Silver Spring, Maryland 20910

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

This report describes a procedure for estimating planetary orbital parameters which are used to generate approximate planetary positions based on a simple ephemeris model. The computer programs developed to implement this procedure are also documented, and an analysis of the residual errors arising from the approximation model is provided.

20. The method described is based on the classical method of Least Squares Differential Corrections which applies to stable nonlinear estimation problems. The actual estimation process was carried out in terms of equinoctal elements to avoid any possible numerical problems and the observations employed were obtained from the precise ephemeris model, DE-92, developed by NASA's Jet Propulsion Laboratory.

# FOREWORD

This report describes a procedure for estimating Keplerian orbital parameters used to generate planetary positions which approximate actual planetary positions to a desired accuracy. It also discusses the computer programs developed to implement the procedure and presents an analysis of the residual errors arising from the approximation model. The work was performed in the Fire Control Formulation Branch (K41), SLBM Research and Analysis Division, Strategic Systems Department, and was authorized under Strategic Systems Project Office Task Assignment 36801.

The authors would like to acknowledge the contribution made by Midge Vittoria of COMPRO, Inc. in developing the initial version of the PLANET computer program.

This report has been reviewed and approved by D. L. Owen, Head, Fire Control Formulation Branch; and S. D. Young, Head, SLBM Research and Analysis Division.

Approved by:

O. F. BRAXTON, Head
Strategic Systems Department

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## INTRODUCTION

Present and planned Submarine Launched Ballistic Missile (SLBM) weapons systems employ a stellar sensor to mitigate some of the unavoidable error sources (principally guidance system alignment). The star to be sighted is selected by the Fire Control (F/C) program onboard the submarine and one of the criteria used in this process is the angular proximity of five of the planets to the candidate stars. If one of these brighter planets is too close (within some specified tolerance) to a particular star, the observation will be adversely affected owing to the relative luminosities. In order to determine the proximity, F/C computes approximate planetary positions based on a simple Keplerian model for planetary motion and predetermined sets of orbital elements. Historically the planetary elements have been determined by least squares fits to sets of precise planetary positions. Because of supposed accuracy requirements, the fitting spans were 5 years in duration so that F/C updates for orbital elements were required at the same interval.

Recently owing to an improved assessment of required accuracies for planetary positions, it has been recognized that much longer fit spans may be employed with a consequent reduction in the frequency of required F/C updates. In order to facilitate the update process, a system of standalone computer programs has been developed to perform the least squares fits and provide quantitative assessments of the resulting accuracies attained. This report describes the formulation which served as a basis for these programs and presents the results of their initial exercise. In addition, documentation of the programs themselves is contained in this report. A listing of each program as well as sample input and output is contained in the appendixes. In addition, Appendix A contains an outline of the program sequence.

## ORBITAL MODELS

The model used in F/C for predicting a planet's position is based on the approximation that the planets are subject to only the central force gravitational attraction of the sun. Even with this appoximation, there is no closed form expression for planetary positions. Classically, one partitions the orbital problem into a description of the in-plane evolution of the planetary position together with an ancilliary description of the orbital ellipse orientation. For central force motion, the planets will describe elliptical motion with one focus of the ellipse at the location of the sun. The orbital ellipse is completely characterized by its semi-major axis (or mean radius) and its eccentricity (Figure 1). The eccentricity is related to the semi-major and semi-minor axes by

$$e^2 = \frac{a^2 - b^2}{a^2} \qquad (1)$$

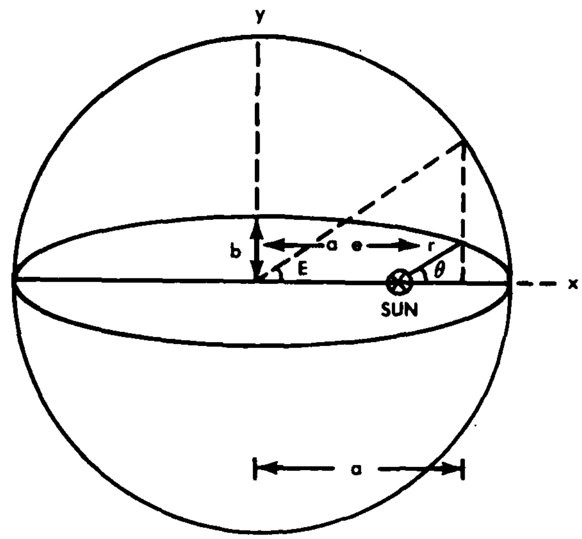


FIGURE 1. ORBITAL ELLIPSE

The position of the planet in this orbit is specified by its true anomaly, θ, which is the angular displacement of the planet from the point of closest approach (perihelion) to the sun. An elementary result (Reference 1) is that

$$r = \frac{a(1 - e^2)}{1 + e \cos \theta} \qquad (2)$$

from this it follows that

$$x = ae + r \cos \theta \qquad (3)$$

$$y = r \sin \theta \qquad (4)$$

Because there is no closed form expression for the true anomaly, its value is obtained from a somewhat circuitous procedure. One defines two intermediary angular quantities, the mean, M, and eccentric, E, anomalies. The mean anomaly is just the average angular displacement of the satellite from a specified epoch value, $M_o$, at $t_o$. If the orbit was circular (e = o), then we would have

$$M = \theta \qquad (5)$$

The mean anomaly in turn is specified by

$$M = M_o + n(t - t_o) \qquad (6)$$

where n, the mean motion, is just the average angular rate of motion and it is given in canonical units (see Reference 1 and Appendix B for definition and constants) by

$$n = a^{-3/2}\sqrt{1 + 1/m} \qquad (7)$$

Here a is again the semi-major axis and m is the reciprocal planet-to-sun mass ratio. (See Appendix C for planet ratios.) The current mean anomaly, M, is used to obtain the current eccentric anomaly, E, by an iterative solution of Kepler's equation

$$E = M + e \sin E \qquad (8)$$

The eccentric anomaly which has no particular interpretation aside from the trivial case for circular orbits can be used to obtain

$$r = a(1 - e \cos E) \qquad (9)$$

$$r \cos \theta = a \cos E - ae \qquad (10)$$

and

$$r \sin \theta = a\sqrt{1 - e^2} \sin E \qquad (11)$$

thus specifying the in-plane position of the planet. In order to specify the planet's position with respect to some chosen inertial system, one needs to describe the orientation of the orbital ellipse with respect to that frame. A *standard inertial system used in describing planetary* motion is a mean* heliocentric, ecliptic frame. This is a frame with its origin at the sun and a principle plane which approximates the Earth's orbital plane. The x-axis is specified by the intersection of the ecliptic plane and some mean equatorial plane with the positive direction pointing toward the vernal equinox which is in the general direction of the constellation Aries. Three angular variables are required to specify the orientation of the orbital ellipse with respect to this frame (Figure 2). These are the inclination, i, the right ascension of the ascending node, Ω, and the argument of perihelion, ω. The inclination is just the angle between the planet orbital plane and the fundamental plane. The intersection of these two planes is called the line of nodes, and the right ascension of the ascending node is just the angle between the x-axis and that part of the line of nodes corresponding to the planet ascending through the ecliptic. The argument of perihelion is the in-plane angle between the line of nodes and the direction of perihelion. Sometimes (as in the F/C selection program) one specifies the argument of latitude instead of the argument of perihelion, the former being defined as

$$\tilde{\omega} = \omega + \Omega \qquad (12)$$

*Various mean inertial frames corresponding to different epochs are related to each other by those transformations describing the precessional motion of the Earth's spin axis (Reference 2).

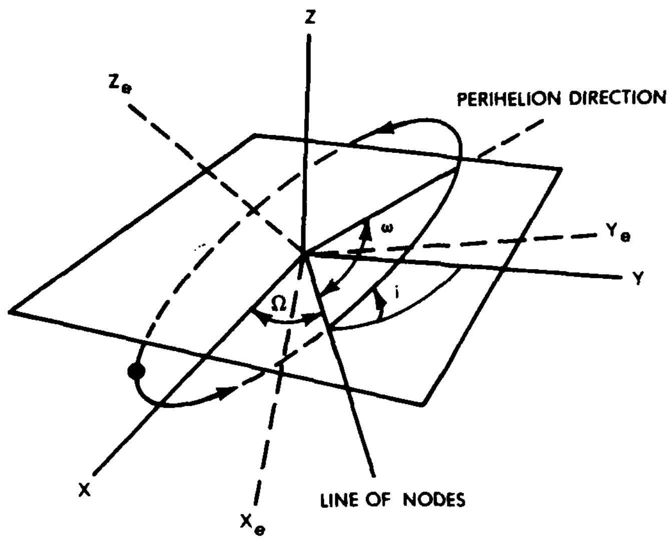


FIGURE 2. CLASSICAL ORBITAL ELEMENTS

While these classical elements have some appeal because of their direct geometrical significance and familiarity, they suffer a deficiency in that some of them lack definition for particular orbits, thus Ω for zero inclination orbits or ω for a zero eccentricity case. In the case of the planets, most have orbits which have rather small inclinations and most (save Mercury) have fairly small eccentricities.

In this analysis, we choose an alternate set of elements to completely eliminate the possibility of any numerical difficulties. Our choice is the well known equinoctal variables defined by (Reference 3) (for posigrade orbits).

$$\left.\begin{aligned} a &= \text{semi-major axis} \\ h &= e \sin \tilde{\omega} \\ k &= e \cos \tilde{\omega} \\ p &= \tan(i/2) \sin \Omega \\ q &= \tan(i/2) \cos \Omega \\ \lambda_o &= \omega + \Omega + M_o \end{aligned}\right\} \quad (13)$$

The geometrical significance of the elements follows: we first define the eccentricity vector, $\vec{e}$, as a vector pointing toward perihelion and having magnitude e together with the nodal vector $\vec{N}$ which points toward the ascending node and has magnitude tan(i/2). Then we define the equinoctal coordinate system (Figure 2) by choosing the origin of longitudes as our $x_e$-axis and the orbit plane normal as $z_e$. The origin of longitudes lies in the orbit plane and is displaced from the node $\vec{N}$ by the angle Ω. Of course, $y_e$ completes a right-handed system. With these definitions, we provide the following interpretations:

h = projection of $\vec{e}$ onto $y_e$ axis

k = projection of $\vec{e}$ onto $x_e$ axis

p = projection of $\vec{N}$ onto $y_e$ axis

q = projection of $\vec{N}$ onto $x_e$ axis

λ = the mean orbital longitude.

To obtain planetary coordinates given a set of equinoctal elements, we proceed as follows:

Calculate the current mean longitude using

$$\lambda = \lambda_o + n(t - t_o) \tag{14}$$

where the mean motion n is given by

$$n = a^{-3/2} \sqrt{1 + 1/m_i} \qquad \text{(canonical units)} \tag{15}$$

where

a = the semi-major axis

and

$m_i$ = the reciprocal planet-to-sun mass ratio.

The eccentric longitude is obtained iteratively from the Newton-Raphson method as

$$F_{i+1} = \frac{\lambda + (k - hF_i) \sin F_i - (h + kF_i) \cos F_i}{1 - h \sin F_i - k \cos F_i} \tag{16}$$

The orbit-plane coordinates are obtained from

$$XP = AX \cos F + B \sin F - ka \tag{17}$$

$$YP = AY \sin F + B \cos F - ha \tag{18}$$

where

$$AX = a(1 - h^2\beta) \tag{19}$$

$$B = ahk\beta \tag{20}$$

$$AY = a(1 - k^2\beta) \tag{21}$$

and

$$\beta = \frac{1}{1 + \sqrt{1 - h^2 - k^2}} \tag{22}$$

and the inertial cartesian components are given by

$$\bar{r} = XP\hat{f} + YP\hat{g} \tag{23}$$

with the unit vectors $\hat{f}$ and $\hat{g}$ given by

$$\hat{f} = \frac{1}{1 + p^2 + q^2} \begin{bmatrix} 1 + q^2 - p^2 \\ 2pq \\ -2p \end{bmatrix} \tag{24}$$

$$\hat{g} = \frac{1}{1 + p^2 + q^2} \begin{bmatrix} 2pq \\ 1 + p^2 - q^2 \\ 2q \end{bmatrix} \tag{25}$$

## LEAST SQUARES DIFFERENTIAL CORRECTIONS

Now the true motions of the planets are not governed by a simple inverse square central force motion so that our Kepler equations are only a crude approximation to the actual motion. In order to make this approximation accurate, we perform a least squares fit for the Kepler orbital parameters using precise orbital positions as "observations." It should be kept in mind that in our case the least squares fit represents a functionalization process and not a minimization of the effects of observation noise. Since the planetary coordinates are not linearly related to the Keplerian equinoctal elements, we employ the least squares differential correction method (Reference 4). Thus we assume that we have an initial approximation to our desired element set a, h, k, p, q, $\lambda$; and we develop a set of least squares normal equations for differential corrections $\delta a$, $\delta h$, $\delta k$, $\delta p$, $\delta q$, $\delta\lambda$ to this starting set as follows: let $\vec{R}(t_i)$ be our "observations," that is the coordinates of the planet at time $t_i$ obtained from some precise planetary orbital model and let $\vec{R}_c(t_i)$ be the corresponding coordinates computed from our starting element set using the equations presented above. Then the least squares "condition equations" for these observations are given by

$$A_i \, \delta\bar{e} = \vec{R}\,(t_i) - \vec{R}_c\,(t_i) \qquad i=1 \,.\,.\,.\,.\,.\,.\, N \tag{26}$$

where

$$\delta\bar{e} = \begin{pmatrix} \delta a \\ \delta h \\ \cdot \\ \cdot \\ \cdot \\ \delta\lambda \end{pmatrix} \tag{27}$$

is the vector of diffential corrections based on our N observations.

$$(A_i) = \frac{\partial \bar{R}_c(t_i)}{\partial \bar{e}} = \begin{pmatrix} \frac{\partial X_c(t_i)}{\partial a} & \frac{\partial X_c(t_i)}{\partial h} & \cdots & \frac{\partial X_c(t_i)}{\partial \lambda} \\ \cdots & \cdots & \cdots & \cdots \\ \frac{\partial Z_c(t_N)}{\partial a} & \frac{\partial Z_c(t_N)}{\partial h} & \cdots & \frac{\partial Z_c(t_N)}{\partial \lambda} \end{pmatrix} \tag{28}$$

is the matrix of observation partial derivatives. (The required partials are presented *in* Appendix D.) The least squares normal equations for the differential corrections which minimize

$$\sum_{i=1}^{N} (\bar{R}(t_i) - \bar{R}_c(t_i))^2 \tag{29}$$

are given by

$$A^{Tr} A \, \delta\bar{e} = A^{Tr} \Delta \tag{30}$$

(Tr denotes transpose)

where $\Delta$ is a column vector of "residuals" that is

$$\Delta = \begin{pmatrix} X(t_i) - X_c(t_i) \\ \cdot \\ \cdot \\ \cdot \\ Z(t_N) - Z_c(t_N) \end{pmatrix} \tag{31}$$

In standard notation, these equations are written as

$$B \, \delta\bar{e} = E \tag{32}$$

and the solution is obtained as

$$\delta\bar{e} = B^{-1} E \tag{33}$$

Now in practice one does not actually form normal equations for the complete set of observations at once since the dimensions of the A matrix would be prohibitively large. Instead, we make use of the well-known summability property of normal equations. In particular, if we let

$$B^{(L)} \delta\bar{e} = E^{(L)} \tag{34}$$

denote the set of normal equations developed for the subset of observations (L), then the complete set of normal equations for all observations is given by

$$\left[\sum_{L} B^{(L)}\right] \delta\bar{e} = \left[\sum_{L} E^{(L)}\right] \tag{35}$$

This allows us to partition our "observations" into some dimensionally convenient subsets. We remark that because our observations are not linearly related to our parameter set, one usually develops an iterative least squares solution to obtain refined improvements for our differential corrections.

## LEAST SQUARES FITS

In the current analysis, the refined elements were generated by a least squares fit using precise planetary positions obtained from the JPL DE-92 ephemeris (Reference 5) as "observations." Because these positions were given in heliocentric mean equatorial of 1950.0 coordinates, they were rotated into the mean ecliptic system of 1950.0 prior to the fitting. The observations used were Cartesian coordinates developed every 14 days from the period JD 2443690.5 (the epoch of the starting elements) through JD 2451544.5. No apriori observations were employed and an implicit standard error of 1 km was used for the observations. The starting and final elements are given in Table 1. The semi-major axis, a, is measured in km and the angular variables, i, ω, Ω, and $M_o$, are measured in radians.

### TABLE 1. STARTING AND FINAL ELEMENTS

Starting Elements

| | Earth | Mercury | Venus |
|---|---|---|---|
| a | 149598037.4642 | 57909134.07 | 108208902.1 |
| e | .01671416 | .20562040 | .00678807 |
| i | .00007099 | .12221041 | .05923274 |
| ω | 5.010136727 | .50664592 | .95464052 |
| Ω | 3.05571794 | .83248134 | 1.32905335 |
| $M_o$ | 3.07458491 | 1.24878002 | 1.16065148 |

TABLE 1. (Cont.)

| | Mars | Jupiter | Saturn |
|---|---|---|---|
| a | 227939292.2 | 778942518.0 | 1430861905.0 |
| e | .09339831 | .04749389 | .05448800 |
| i | .03224279 | .02281176 | .04342258 |
| ω | 4.99540506 | 4.789137097 | 5.954194377 |
| Ω | .85657063 | 1.74539404 | 1.97500760 |
| $M_o$ | 3.89962946 | 1.51508308 | .90983226 |

Refined Elements

| | Earth | Mercury | Venus |
|---|---|---|---|
| a | 149597871.9212 | 57909137.53653 | 108208961.4686 |
| e | .01671532949782 | .2056294264117 | .006775929468209 |
| i | .00008989551986890 | .1222021384653 | .05923236598127 |
| ω | 5.016964968452 | .5070493275543 | .9553692682276 |
| Ω | 3.049977190305 | .8323191351502 | 1.328556712384 |
| $M_o$ | 3.073477677683 | 1.248550488749 | 1.160433857370 |

| | Mars | Jupiter | Saturn |
|---|---|---|---|
| a | 227940894.4184 | 778368220.1753 | 1426940733.728 |
| e | .09339472415910 | .04825983079881 | .05315110813689 |
| i | .03223176504070 | .02280397821074 | .04342907360759 |
| ω | 4.996203334775 | 4.795474079402 | 5.908659409827 |
| Ω | .8562002543165 | 1.746015566160 | 1.974220936589 |
| $M_o$ | 3.899288800052 | 1.505867745775 | .9566213166208 |

## RESULTS AND CONCLUSIONS

In order to develop an assessment of the accuracy of our fitting process in terms of relevant quantities, we generated a set of position histories for each of the planets of interest at 2-week intervals. These histories were used to develop a set of right ascension and declination values at the same interval. These values were compared with a similar set based on the precise JPL coordinates. The aggregate statistics are given in Table 2 while Appendix E, Figures E-1 through E-10, shows the plotted residuals. For the sake of comparison, Table 3 shows similar statistics based on positions developed using the starting set of elements which are currently employed in the selection portion of the F/C program. The plotted residuals corresponding to these elements are shown in Figures E-11 through E-20. It should be noted that both sets of elements, that is, the starting set and the improved set were obtained by fitting the simple Kepler model to a set of precise independently generated planetary positions. In the first instance, the fit span was 5 years in duration, while in the latter instance, a span of approximately 20 years was employed. The shorter fit span was chosen because

TABLE 2. JPL AND KEPLER COMPARISON STATISTICS

Residual Error Statistics for Improved Elements (Arc Minutes)

| Planet | Error in Right Ascension | | | Error in Declination | | |
|---|---|---|---|---|---|---|
| | Mean | σ | Peak | Mean | σ | Peak |
| Mercury | .00 | .17 | .73 | .00 | .07 | -.25 |
| Venus | .01 | .24 | -1.33 | -.01 | .11 | -.66 |
| Mars | .09 | .53 | 2.09 | -.04 | .21 | -1.15 |
| Jupiter | -.01 | .64 | 2.18 | -.01 | .19 | .56 |
| Saturn | -.08 | 1.13 | -3.85 | -.04 | .35 | 1.11 |

TABLE 3. CURRENTLY USED COMPARISON STATISTICS

Residual Error Statistics for Starting Elements (Arc Minutes)

| Planet | Error in Right Ascension | | | Error in Declination | | |
|---|---|---|---|---|---|---|
| | Mean | σ | Peak | Mean | σ | Peak |
| Mercury | .29 | .14 | 1.30 | -.04 | .05 | -5.8 |
| Venus | .28 | .16 | 1.90 | -.01 | .09 | -1.1 |
| Mars | -.82 | .45 | -6.60 | -.05 | .18 | -2.96 |
| Jupiter | 13.7 | 5.5 | 45.8 | .33 | 2.6 | 17.5 |
| Saturn | 26.5 | 12.5 | 99.9 | 7.9 | 5.1 | 33.4 |

of supposed accuracy requirements and because of the knowledge that for Saturn and Jupiter there are long period perturbations of the order of a few minutes of arc in right ascension and declination. This of course is the reason why we see (Appendix E, Figures E-17 through E-20) the secular/periodic larger residuals for Jupiter, Saturn, and to some extent Mars once we go beyond the fit span used to develop the original set of elements. The periodic perturbations which were of concern in choosing the shorter span are evident (see Appendix E, Figures E-7 through E-10) in the angular residuals based on the 20 year fit span. The latter set of orbit parameters are considered superior. The original set of accuracy requirements have been recognized as overly stringent which in turn led to a requirement for a shorter term, higher accuracy model. Unfortunately, this model becomes unstable beyond the fit span. Using the longer fit span, we obtain a representation with slightly degraded accuracy but which exhibits a long-term stability in its error characteristics. Finally, it is recognized that if higher accuracy did become an issue, the residual errors shown could likely be modeled with a small augmentation of the original parameter set, principally with the addition of some periodic terms to the mean motion. In fact, if one is willing to represent a large number of the observed periodicities in the principal arguments of planetary motion, one can obtain a very precise (10 seconds of arc or better) analytical model for planetary motion (Reference 6).

## KEPLER ORBIT GENERATION PROGRAM PLANET

The program PLANET generates planetary positions based on a simple Keplerian model and has an option to generate partial derivatives. A flow chart of PLANET is given in Figure 3. The planetary positions are generated in the heliocentric ecliptic coordinate system.* The program PLANET consists of a driver program and subroutines NEWT, CONVRT, and PARTIAL. The main program accepts as an input file either classical or equinoctal elements. If classical elements are used as input, the equinoctal elements are calculated using equation (13). If equinoctal elements are used as input, the classical elements are calculated from the inverse transformations.

$$a = a \tag{36}$$

$$e = k/\cos\left(\tan^{-1}(h/k)\right) \tag{37}$$

$$i = 2\tan^{-1}\left[p/\sin\left(\tan^{-1}(p/q)\right)\right] \tag{38}$$

$$\omega = \tan^{-1}(h/k) - \tan^{-1}(p/q) \tag{39}$$

$$\Omega = \tan^{-1}(p/q) \tag{40}$$

$$M_o = \lambda_o - \tan^{-1}(h/k) \tag{41}$$

The input file also includes an epoch and ending time in Julian days and a delta time in days. The epoch and end times are not necessary for any calculations and are used for reference only. The delta time is the interval of time between successive calculations of the planetary ephemerides. Other input includes planet ID, interval between time lines on system's output, and the option for computing parital derivatives. The semi-major axis "a" and delta time are converted to canonical units. The mean motion is calculated using equation (7). The mean motion is then used to calculate current mean longitude used in subroutine NEWT.

As previously mentioned, the main program calls to a subroutine named NEWT. Subroutine NEWT calculates the eccentric longitude from the mean longitude at each given time step using Newton's method. For the epoch time, the mean longitude, $\lambda$, is equal to the original input value. A starting value is then computed for the eccentric longitude using

$$F_o = \lambda + k\sin\lambda - h\cos\lambda + \frac{(k^2 - h^2)}{2}\sin 2\lambda - \frac{kh}{2}\cos 2\lambda. \tag{42}$$

* Mean System of 1950.0

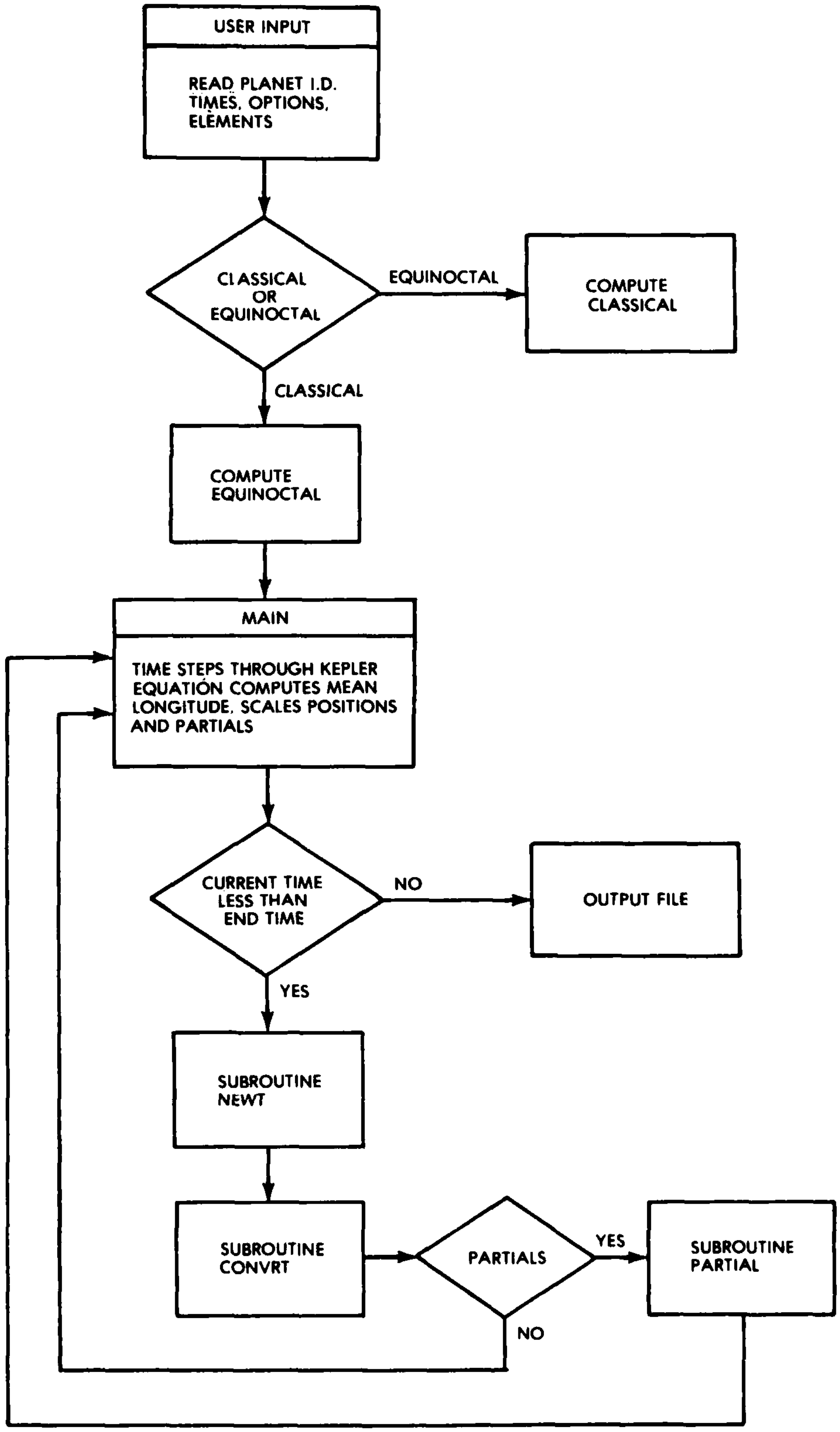


FIGURE 3. PLANET PROGRAM FLOW CHART

and successive approximations are computed iteratively using equation (16). The iteration terminates when the absolute value of the difference between the current eccentric longitude and the previously computed eccentric longitude is less than or equal to a tolerance (.000001745). Control is then returned to the main program.

Main immediately calls to subroutine CONVRT. This procedure converts elements and eccentric longitude to inertial Cartesian ecliptic positions using equations (17) through (25). Control again returns to the main program where the positions computed in CONVRT are scaled by

$$XS = X \cdot AU \tag{43}$$

$$YS = Y \cdot AU \tag{44}$$

$$ZS = Z \cdot AU \tag{45}$$

Time is stepped by adding the delta time. A new mean longitude is computed by

$$\lambda = \lambda + EN \cdot DT \tag{46}$$

where

$$DT = DELTA/TU \tag{47}$$

and EN is the previously computed mean motion. A test is made to see if current time is less than ending time. If current time exceeds ending time, the program returns to the beginning to input a new set of data for another planet. If current time is less than ending time, the entire process starting with subroutine NEWT is repeated using the new mean longitude. Termination of the program occurs when the input value for planet ID is greater than 100.

Subroutine PARTIAL is a procedure that may be invoked by the correct input option. PARTIAL computes analytical partial derivatives of positions with respect to the equinoctal elements at each time step. Here the eccentric longitude used is the current eccentric longitude at the current time. The current canonical time from epoch "CT" is accumulated in main in the mean longitude stepping loop

$$CT = CT + DT \tag{48}$$

The partial derivative equations used in this procedure are contained in Appendix D. The only quantities relating to the partial derivative computations which change with time are $X_p$, $Y_p$, F, and CT. Because of this, many of the factors appearing in the expressions need only be computed once. For example, $\frac{\partial \hat{f}}{\partial q}$, $\frac{\partial \hat{f}}{\partial r}$, $\frac{\partial \hat{g}}{\partial q}$, and $\frac{\partial \hat{g}}{\partial r}$, are all computed once.

Most of the partial derivatives computed need to be scaled. No scaling is required for $\frac{\partial \vec{r}}{\partial a}$.

The remaining partial derivatives, however, require scaling. This is accomplished by multiplying each one by a factor of one AU. A listing of the program and a sample of the input and output can be found in Appendix F.

## PROGRAM EXTRACT

In order to perform a least squares fit, a high accuracy or "true" position history is needed. This is provided by the JPL DE-92 ephemeris. Because these positions are given in heliocentric mean equatorial of 1950.0 coordinates, it is necessary to rotate them into the mean ecliptic system of 1950.0. This was accomplished in program EXTRACT. This program extracts position histories from JPL and rotates them by using the following formula:

$$\bar{E} = .4092061859 \text{ radians (The mean obliquity of 1950.0)} \qquad (49)$$

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix}_{\text{ecliptic}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\bar{E} & \sin\bar{E} \\ 0 & -\sin\bar{E} & \cos\bar{E} \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}_{\text{equatorial}} \qquad (50)$$

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \begin{pmatrix} X \\ y\cos\bar{E} + z\sin\bar{E} \\ -y\sin\bar{E} + z\cos\bar{E} \end{pmatrix} \qquad (51)$$

Refer to Appendix G for a listing of the program and sample input and output.

## LEAST SQUARES PROGRAM - LESTSQ

To improve the accuracy of the preliminary orbits, a set of improvements to the starting elements is obtained using the method of least squares differential corrections. This method is the basis for program LESTSQ. A flow chart of LESTSQ is given in Figure 4.

LESTSQ receives its input from two files. One file contains the high accuracy planet coordinates provided by DE-92. The other file contains the coordinates computed in the Kepler program as well as the partial derivatives. From this input, LESTSQ computes a column vector of residuals using equation (31). The residuals are then used in conjunction with the matrix of observation partial derivatives to compute the right-hand side of the least squares normal equations, equations (30) and (32). The left-hand side of the least squares normal equations or the normal matrix is then computed. The ultimate goal of program LESTSQ,

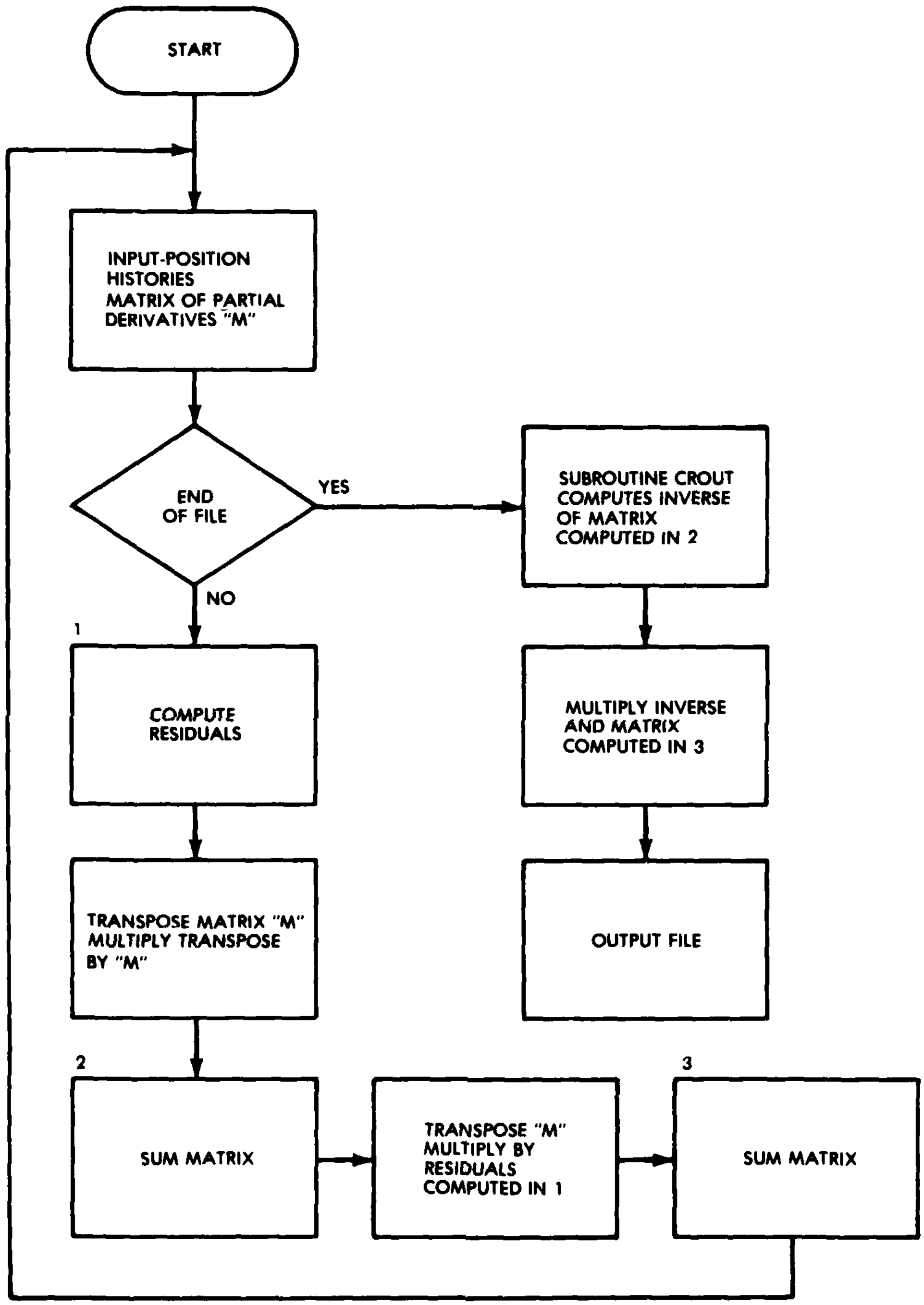


FIGURE 4. LEAST SQUARES PROGRAM FLOW CHART

which is to determine the vector of differential corrections based on the observations, is satisfied by rewriting the least squares normal equations into an equivalent expression given in equation (33). A full listing of the program as well as sample input and output is contained in Appendix H.

## PROGRAM GEOEQ

The refined elements obtained using the least squares method are used to generate a new set of improved position histories for each of the planets of interest. It is desired that these positions be in a geocentric equatorial coordinate frame. Program GEOEQ is written for this purpose. The Earth's heliocentric trajectory coordinates are subtracted from each of the planet's coordinates. The planet's coordinates are then rotated using the inverse transformation of equation (51).

The position histories provided by JPL are also rotated using this program. A listing of the program as well as sample input and output is contained in Appendix I.

## PROGRAM DELALP

In order to determine the accuracy of the Kepler orbit generation results, it is necessary to compare the model trajectories with those of JPL in terms of relevant quantities. Program DELALP, see Figure 5 for a flow chart, uses the rotated Kepler generated positions and develops a set of right ascension and declination values per trajectory using the following formulas:

$$\alpha = \arctan (y/x), \; 0 \leq \alpha \leq 2\pi \quad \alpha = \text{right ascension} \tag{52}$$

$$\delta = \arctan (Z/RO), \; RO = \sqrt{X^2 + Y^2}, \; -\frac{\pi}{2} \leq \delta \leq \frac{\pi}{2} \tag{53}$$

$\delta$ = declination

Right ascension and declination values are also generated for JPL using the same formulas. The angles are returned in arcminutes. This is accomplished by multiplying each angle by a constant where

$$\text{constant} = 10800.00/\text{PI} \tag{54}$$

and

$$\text{PI} = 3.1415926535898. \tag{55}$$

The angular differences are computed and written onto an output file. A set of statistics including maximum and minimum values, mean values, and standard deviation for both sets of angle differences is computed for plotting purposes. The

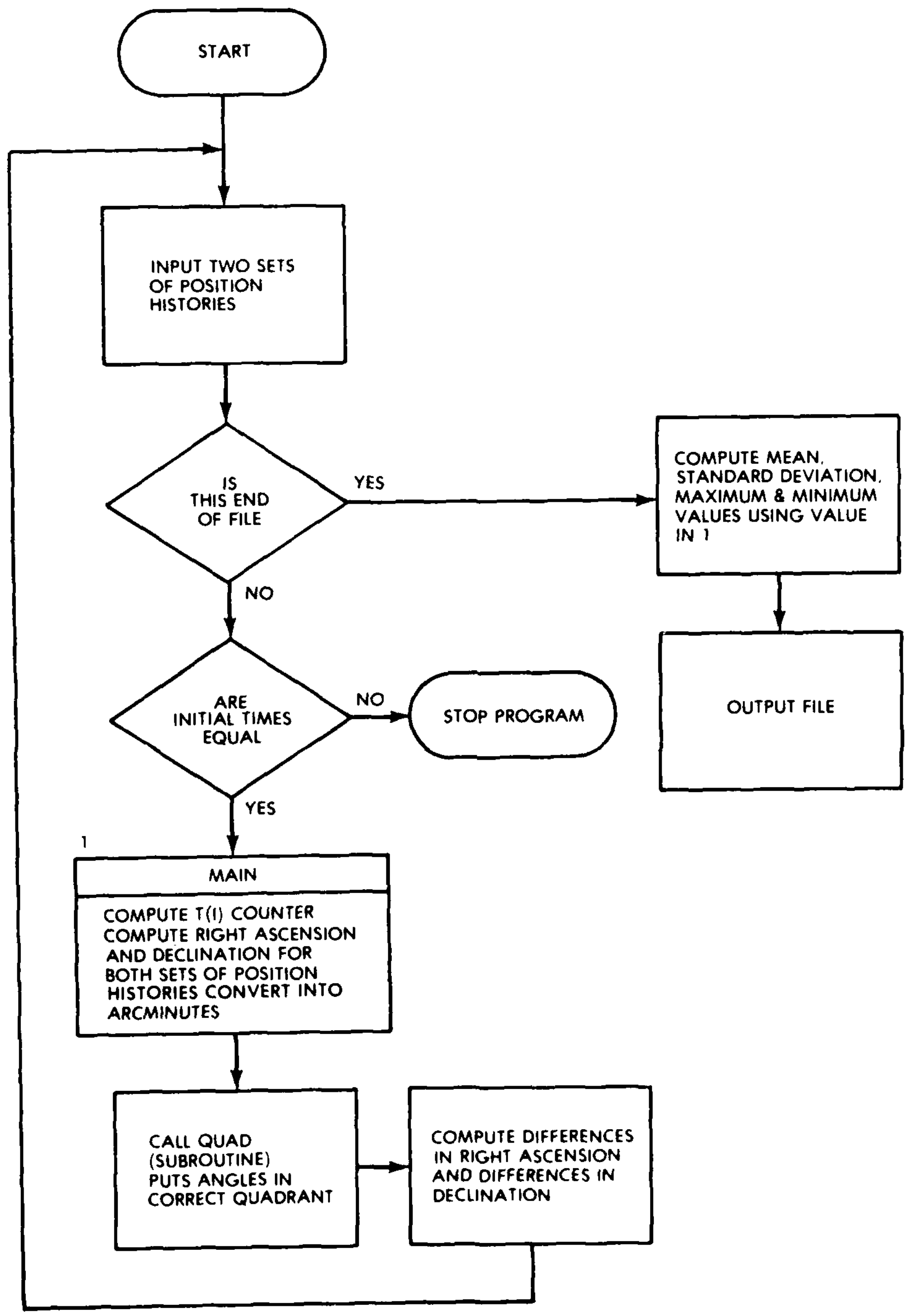


FIGURE 5. PROGRAM DELALP FLOW CHART

aphed results are included in Appendix E, Figures E-1 through E-10. A complete sting of the program as well as sample input and output is included in Appenx J.

## PROGRAM SELECT

In order to complete the accuracy test of the Kepler orbit generation reılts, a subroutine was obtained from a program called SELECTION which is the acıal program used in F/C on the submarine. Program SELECT is this subroutine. A .ow chart of SELECT is given in Figure 6. SELECT uses data statements as input ıd computes right ascension and declination for all the planets at one time. The ıput elements for SELECT are slightly different from the ones used in PLANET. :LECT inputs the classical elements e, i, Ω, and $M_o$ as does PLANET. The slightly .fferent input elements are semi-major axis in astronomical units, argument of ıtitude as defined in equation (12), and mean motion as defined in equation (7).

SELECT steps through time in the same manner as PLANET. It computes the mean ıomaly for each planet using equation (6). It then calls subroutine ELCORD. EL-)RD converts the time-evolved elements into Cartesian coordinates.

First ELCORD computes an eccentric anomaly iteratively. After computing a tarting value, successive approximations to eccentric anomaly are generated. A ɔlerance of .000001745 is used to end the iteration. The coordinates are compued using as input the sine and cosine values for the angle of ascending node (Ω), rue anomaly (V), and the angle of inclination (i) as well as values for semi-maɔr axis (a), eccentricity (e), the eccentric anomaly (E) and radius vector (R), ll at time "t." The following are the equations used for the transformations.

$$EO = Am + e \sin Am + \frac{e^2 \sin^2 Am}{2} \text{ - starting value} \quad (56)$$

$$E = EO + \frac{Am - EO + e \sin EO}{1 - e \cos EO} \text{ - eccentric anomaly} \quad (57)$$

$$V = 2 \arctan (\tan(E/2) \sqrt{1 + e}, \sqrt{1 - e}) \quad (58)$$

$$R = a(1 - e \cos E) \quad (59)$$

$$X = R\left[\cos \Omega \cdot \cos(\tilde{\omega} - \Omega + V) - \sin \Omega \cdot \sin(\tilde{\omega} - \Omega + V) \cdot \cos i\right] \quad (60)$$

$$Y = R\left[\sin \Omega \cdot \cos(\tilde{\omega} - \Omega + V) + \cos \Omega \cdot \sin(\tilde{\omega} - \Omega + V) \cdot \cos i\right] \quad (61)$$

$$Z = R\left[\sin(\tilde{\omega} - \Omega + V) \cdot \sin i\right] \quad (62)$$

The coordinates in a heliocentric ecliptic coordinate system are then rotated nto a heliocentric equatorial coordinate system. The coordinates are transmitted ack to the main program where they are changed from heliocentric trajectory cordinates to geocentric trajectory coordinates. Right ascension and declination ngles are then computed using equations (52) and (53).

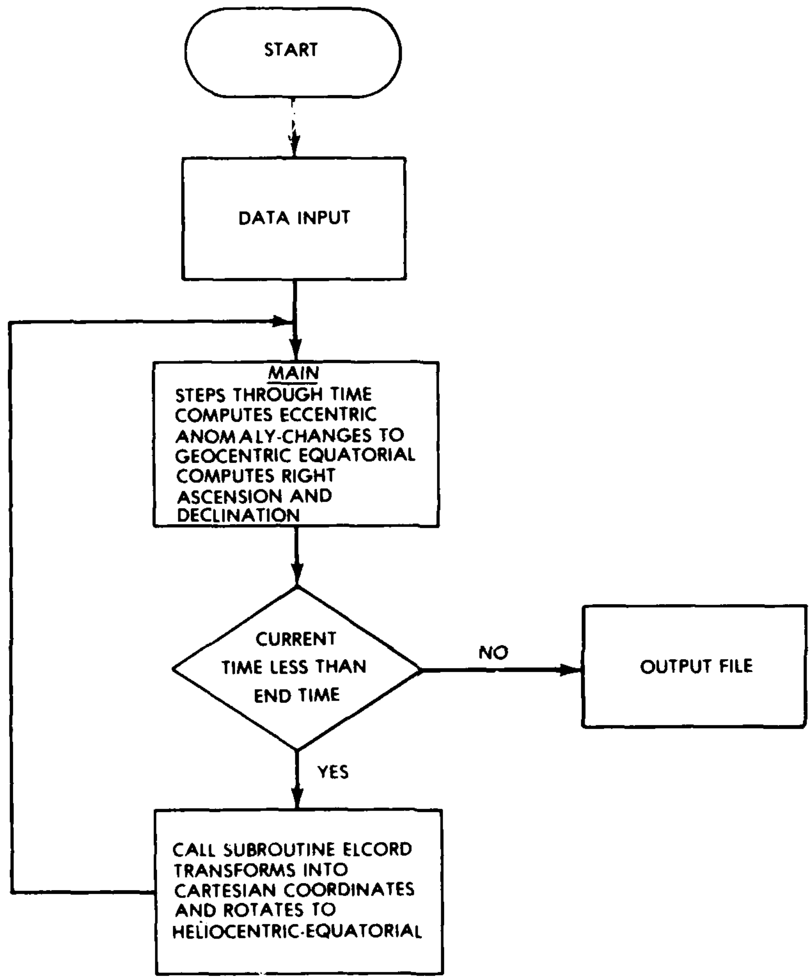


FIGURE 6. PROGRAM SELECT FLOW CHART

The angles are returned in arc minutes and written to five different output es keeping in mind that right ascension and declination are computed for all the planets of interest at one time. It is now necessary to compare the ults of SELECT and JPL using program DELALP. Since SELECT generates right ension and declination angles and JPL generates position histories, it is ne- sary to change DELALP slightly. An input statement is changed to allow angles input. The computations that changed Kepler's position histories into angles d to be deleted. DELALP, renamed DELAPM for clarity, generates a set of delta ht ascension and delta declinations and a set of statistics for SELECT and JPL, essary for plotting, just as previously done for Kepler and JPL. The statis- s are given in Table 4 and a listing of DELAPM can be found in Appendix J.

TABLE 4. RESIDUAL ERROR STATISTICS FOR IMPROVED ELEMENTS (ARC MINUTES) SELECT

| Planet | Error in Right Ascension | | | Error in Declination | | |
|---|---|---|---|---|---|---|
| | Mean | σ | Peak | Mean | σ | Peak |
| Mercury | .00 | .17 | .73 | .00 | .07 | -.25 |
| Venus | .01 | .24 | -1.33 | -.01 | .11 | -.66 |
| Mars | .09 | .53 | 2.09 | -.04 | .21 | -1.15 |
| Jupiter | -.01 | .64 | 2.18 | -.01 | .19 | .56 |
| Saturn | -.08 | 1.13 | -3.85 | -.04 | .35 | 1.11 |

SELECT was implemented to ensure that the F/C computer, using the refined ements produced by LESTSQ, generated the same results as the Kepler orbit gene- tion program. A comparison of Table 4 and Table 2 shows the two programs pro- ced the same results. For this reason, the graphed results of program SELECT e omitted as they would be a duplication of Figures E-1 through E-10. A com- ete listing of the program as well as sample input and output is given in Ap- ndix K.

## PROGRAM PLOT

Program PLOT graphs the differences in angular positions computed in Program LALP as a function of time. An initialization time of 2443690.5 and an ending ne of 2451544.5 are used. The trajectories themselves were computed every 14 ys. Each fifth trajectory point is plotted. The plotted results are contained Appendix E. A complete listing of the program and sample input is given in pendix L, and a sample output plot is shown in Figure L-1.

## REFERENCES

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# APPENDIX A

## OUTLINE OF PROGRAM SEQUENCE

I. Kepler Orbit Generation Program PLANET
   A. Input
      a. ID for planet
         1. 1 = Mercury
         2. 2 = Venus
         3. 3 = Earth
         4. 4 = Mars
         5. 5 = Jupiter
         6. 6 = Saturn
      b. Element option
         1. 0 = Classical
         2. 1 = Equinoctal
      c. Interval between time lines on system's output
      d. Option for computing partial derivatives
         1. 0 = No partials
         2. 1 = Compute partials
      e. Epoch time (Julian days)
      f. Delta time (days)
      g. End time (Julian days)
      h. Elements
         1. Classical
            a. $a$ = Semi-major axis
            b. $e$ = Eccentric.ty
            c. $i$ = Inclination
            d. $\omega$ = Angle of perigee
            e. $\Omega$ = Angle of ascending node
            f. $M_o$ = Initial mean anomaly
         2. Equinoctal
            a. $a$ = Semi-major axis
            b. $h$
            c. $k$
            d. $p$
            e. $q$
            f. $\lambda_o$ - Mean longitude
   B. Output
      a. Time (Julian days)
      b. Position vectors (heliocentric ecliptic)
      c. Partial derivatives of x, y, and z with respect to equinoctal elements.

II. Program EXTRACT
- A. Input
  - a. Time (Julian days) (binary)
  - b. Position vectors from JPL (heliocentric equatorial) (in binary)
- B. Output
  - a. Time (Julian days)
  - b. Position vectors (heliocentric ecliptic)

III. Program LESTSQ
- A. Input
  - a. Time (Julian days)
  - b. Position histories (heliocentric ecliptic) from Kepler and JPL
  - c. *Observation matrices "H" which are the partial derivatives of* X, Y, and Z with respect to the equinoctal elements (from Kepler)
- B. Output
  - a. Matrices

    1. $H_i^{Tr} H_i$

    2. $$\sum_{i=1}^{562} H_i^{Tr} H_i$$

    3. $H_i^{Tr} Z_i$ *where Z is a matrix of residuals for each position history*

    4. $$\sum_{i=1}^{562} H_i^{Tr} Z_i$$

  - b. Parameter improvements
    1. Improvement for a
    2. Improvement for h
    3. Improvement for k
    4. Improvement for λ
    5. *Improvement for* q
    6. Improvement for p

IV. Add parameter improvements to original elements and rerun PLANET.

V. Program GEOEQ
- A. Input
  - a. Time (Julian days)
  - b. Position histories for planets Mercury, Venus, Mars, Jupiter, and Saturn (these histories are in heliocentric ecliptic and are the

ones obtained after fit (Kepler). In second run, use JPL position histories.

c. Position histories for planet Earth (heliocentric ecliptic) from Kepler after fit and JPL.

B. Output

a. Time (Julian days)

b. Position histories for planets Mercury, Venus, Mars, Jupiter, and Saturn in geocentric equatorial.

VI. Program DELALP

A. Input

a. Time (Julian days)

b. Position histories in geocentric equatorial from Kepler and JPL

B. Output

a. Time (Julian days)

b. Counter

c. Delta right ascension (arcmin) Δ = JPL-KEPLER

d. Delta declination (arcmin) Δ = JPL-KEPLER

e. Mean for Δ right ascension and Δ declination

f. Standard deviation for Δ right ascension and Δ declination

g. The maximum values of Δ right ascension and Δ declination

h. Minimum values of Δ right ascension and Δ declination

VII. Program PLOT

A. Input

a. Title

b. KC = Number of characters in title

c. L = Column in which KC characters will be written

d. Option for graphing = N

1. 1 = plot Δ right ascension

2. 0 = plot Δ declination

B. Output

a. Graphs of Δ right ascension and Δ declination

VIII. Program SELECT

A. Input

a. Data Statements (elements)

1. Semi-major axis in astronomical units

2. Eccentricity

3. Inclination

4. Angle of ascending node

5. $\tilde{\omega}$ longitude of periapses $\omega + \Omega$

6. Initial mean anomaly

7. Mean motion

B. Output

a. Angle of right ascension (arcmin)

b. Angle of declination (arcmin)

IX. Change "read" statement in DELALP to read in angles from SELECT instead of position histories and rerun DELALP to obtain Δ right ascensions and Δ declinations: Δ = JPL-SELECT.

X. Rerun program PLOT to obtain graphs of Δ right ascensions and Δ declinations.

# APPENDIX B

## CANONICAL UNITS

Definition: A normalized system of units called "canonical units" was developed to help in mathematical calculations since certain fundamental quantities as the mean distance from the Earth to the sun, the mass and mean distance of the moon, and mass of the sun are not accurately known. It is assumed that the mass of the sun is one "mass unit" and the mean distance from the Earth to the sun is one "astronomical unit (AU)." We define the "distance unit (DU)" to be the radius of a hypothetical reference orbit where the sun is the central body. We define the "time unit (TU)" such that the speed of a satellite is 1 DU/TU.

In PLANET

1 AU = 149597871.41056 km
1 TU = 58.13244087 day (Reference 5)

# APPENDIX C

## RECIPROCAL PLANET-TO-SUN MASS RATIOS

The reciprocal planet-to-sun mass ratios (Reference 5) are listed as follows:

| | | M(i) |
|---|---|---|
| i=1 | Mercury | M(1)=6023600 |
| i=2 | Venus | M(2)=408523.5 |
| i=3 | Earth | M(3)=332480 |
| i=4 | Mars | M(4)=3098710 |
| i=5 | Jupiter | M(5)=1047.355 |
| i=6 | Saturn | M(6)=3498.5 |
| i=7 | Uranus | M(7)=22869 |
| i=8 | Neptune | M(8)=19314 |
| i=9 | Pluto | M(9)=3000000 |

# APPENDIX D

## PARTIAL DERIVATIVES

$$\frac{\partial \vec{r}}{\partial a} = \frac{\partial x_p}{\partial a} \hat{f} + \frac{\partial y_p}{\partial a} \hat{g} \tag{D-1}$$

where

$$\frac{\partial x_p}{\partial a} = \frac{x_p}{a} - \frac{3n}{2} (t - t_o) \quad \frac{[hk\beta \cos F - (1-h^2\beta) \sin F]}{1 - h \sin F - k \cos F} \tag{D-2}$$

and

$$\frac{\partial y_p}{\partial a} = \frac{y_p}{a} - \frac{3n}{2} (t - t_o) \; [(1 - k^2\beta) \cos F - hk\beta \sin F] \tag{D-3}$$

$$\frac{\partial \vec{r}}{\partial h} = \frac{\partial x_p}{\partial h} \hat{f} + \frac{\partial y_p}{\partial h} \hat{g} \tag{D-4}$$

where

$$\frac{\partial x_p}{\partial h} = a \left[ k\left(\beta + \frac{h^2\beta^2}{1 - \beta}\right) \sin F - \left(2h\beta + \frac{h^3\beta^3}{1 - \beta}\right) \cos F \right] - a \frac{\cos F \; (hk\beta \cos F - (1 - h^2\beta) \sin F)}{1 - h \sin F - k \cos F} \tag{D-5}$$

$$\frac{\partial y_p}{\partial h} = a \left[ \frac{-k^2h\beta^3}{1 - \beta} \sin F + k \left(\beta + \frac{h^2\beta^3}{1 - \beta}\right) \cos F - 1 \right] - a \frac{\cos F \; [(1 - k^2\beta) \cos F - hk\beta \sin F]}{1 - h \sin F - k \cos F} \tag{D-6}$$

$$\frac{\partial \vec{r}}{\partial k} = \frac{\partial x_p}{\partial k} \hat{f} + \frac{\partial y_p}{\partial k} \hat{g} \tag{D-7}$$

where

$$\frac{\partial x_p}{\partial k} = a \left[\frac{-h^2 k\beta^3}{1-\beta} \cos F + h\left(\beta + \frac{k^2\beta^3}{1-\beta}\right) \sin F - 1\right] \tag{D-8}$$

$$+ a \sin F \frac{(hk\beta \cos F - (1 - h^2\beta) \sin F)}{1 - h \sin F - k \cos F}$$

$$\frac{\partial y_p}{\partial k} = a \left[-\left(2k\beta + \frac{k^3\beta^3}{1-\beta}\right) \sin F + h\left(\beta + \frac{k^2\beta^3}{1-\beta}\right) \cos F\right] \tag{D-9}$$

$$+ a \sin F \frac{[(1 - k^2\beta) \cos F - hk\beta \sin F]}{1 - h \sin F - k \cos F}$$

$$\frac{\partial \vec{r}}{\partial \lambda} = \frac{\partial x_p}{\partial \lambda} \hat{f} + \frac{\partial y_p}{\partial \lambda} \hat{g} \tag{D-10}$$

where

$$\frac{\partial x_p}{\partial \lambda} = \frac{a(hk\beta \cos F - (1-h^2\beta) \sin F)}{1 - h \sin F - k \cos F} \tag{D-11}$$

and

$$\frac{\partial y_p}{\partial \lambda} = \frac{a[(1-k^2\beta) \cos F - hk\beta \sin F]}{1 - h \sin F - k \cos F} \tag{D-12}$$

For the p and q partials we use

$$\frac{\partial \vec{r}}{\partial (q,p)} = x_p \frac{\partial \hat{f}}{\partial (q,p)} + y_p \frac{\partial \hat{g}}{\partial (q,p)} \tag{D-13}$$

$$\frac{\partial \hat{f}_1}{\partial q} = \frac{4qp^2}{(1 + p^2 + q^2)^2} \tag{D-14}$$

$$\frac{\partial \hat{f}_2}{\partial q} = \frac{2p(1 + p^2 - q^2)}{(1 + p^2 + q^2)^2} \tag{D-15}$$

$$\frac{\partial \hat{f}_3}{\partial q} = \frac{4pq}{(1 + p^2 + q^2)^2} \tag{D-16}$$

$$\frac{\partial \hat{g}_1}{\partial q} = \frac{\partial \hat{f}_2}{\partial q} \tag{D-17}$$

$$\frac{\partial \hat{g}_2}{\partial q} = \frac{-4q(1 + p^2)}{(1 + p^2 + q^2)^2} \quad \text{(D-18)}$$

$$\frac{\partial \hat{g}_3}{\partial q} = \frac{2(1 + p^2 - q^2)}{(1 + p^2 + q^2)^2} \quad \text{(D-19)}$$

$$\frac{\partial \hat{f}_1}{\partial p} = \frac{-4p(1 + q^2)}{(1 + p^2 + q^2)^2} \quad \text{(D-20)}$$

$$\frac{\partial \hat{f}_2}{\partial p} = \frac{2q(1 + q^2 - p^2)}{(1 + p^2 + q^2)^2} \quad \text{(D-21)}$$

$$\frac{\partial \hat{f}_3}{\partial p} = \frac{-4qp}{(1 + p^2 + q^2)^2} \quad \text{(D-22)}$$

$$\frac{\partial \hat{g}_1}{\partial p} = \frac{\partial \hat{f}_2}{\partial p} \quad \text{(D-23)}$$

$$\frac{\partial \hat{g}_2}{\partial p} = \frac{4pq^2}{(1 + p^2 + q^2)^2} \quad \text{(D-24)}$$

$$\frac{\partial \hat{g}_3}{\partial p} = \frac{-\partial \hat{f}_1}{\partial q} \quad \text{(D-25)}$$

# APPENDIX E

## PLOTTED RESIDUALS

The graphs show plotted residuals in right ascension and declination angles based on precise JPL coordinates and coordinates obtained using fitted elements.

Figures E-1 through E-10 show plotted residuals using elements obtained after an approximate 20-year fit span. Figures E-11 through E-20 show residuals based on an old set of elements obtained after a 5-year fit span.

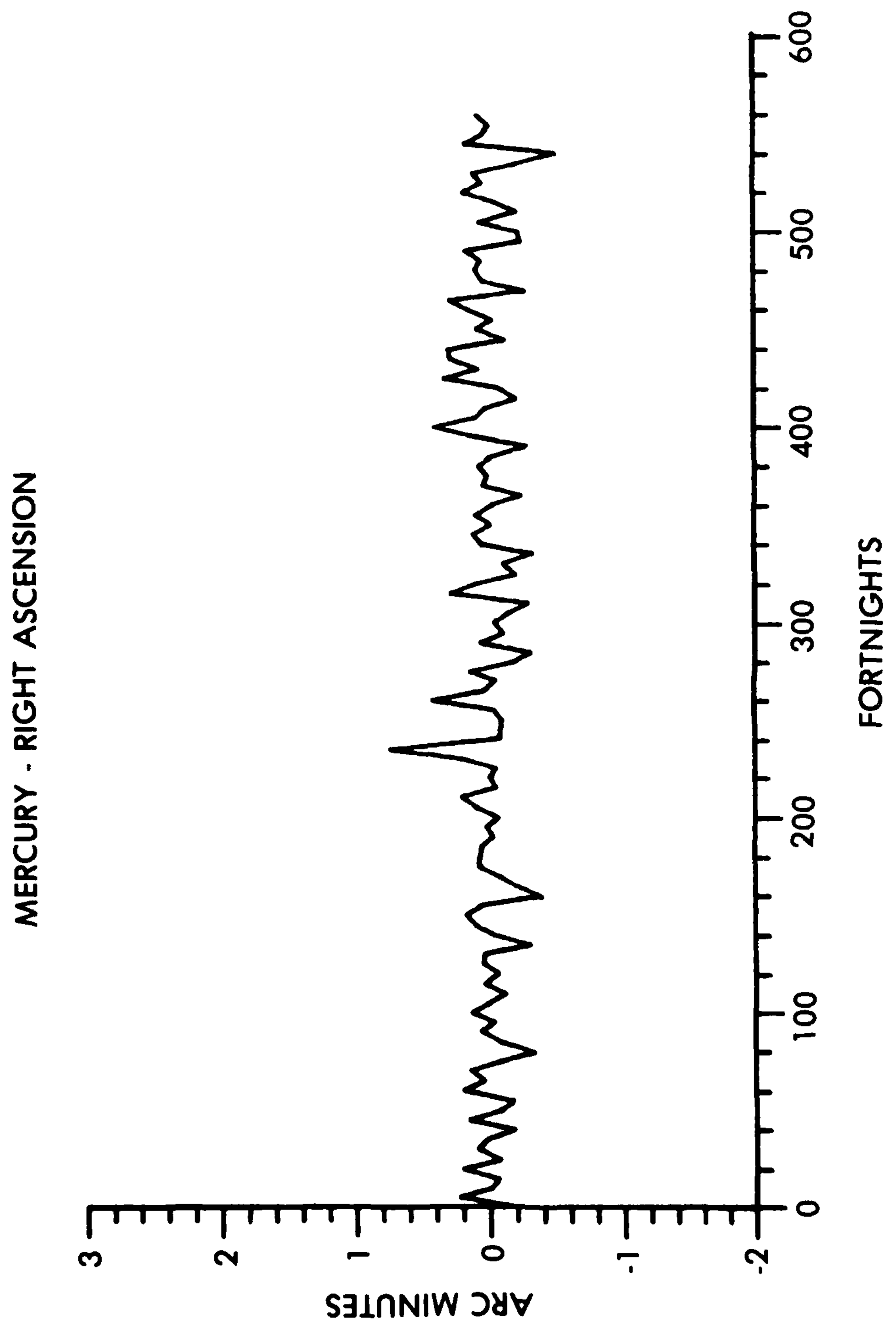


FIGURE E-1. MERCURY - RIGHT ASCENSION 20-YEAR FIT SPAN

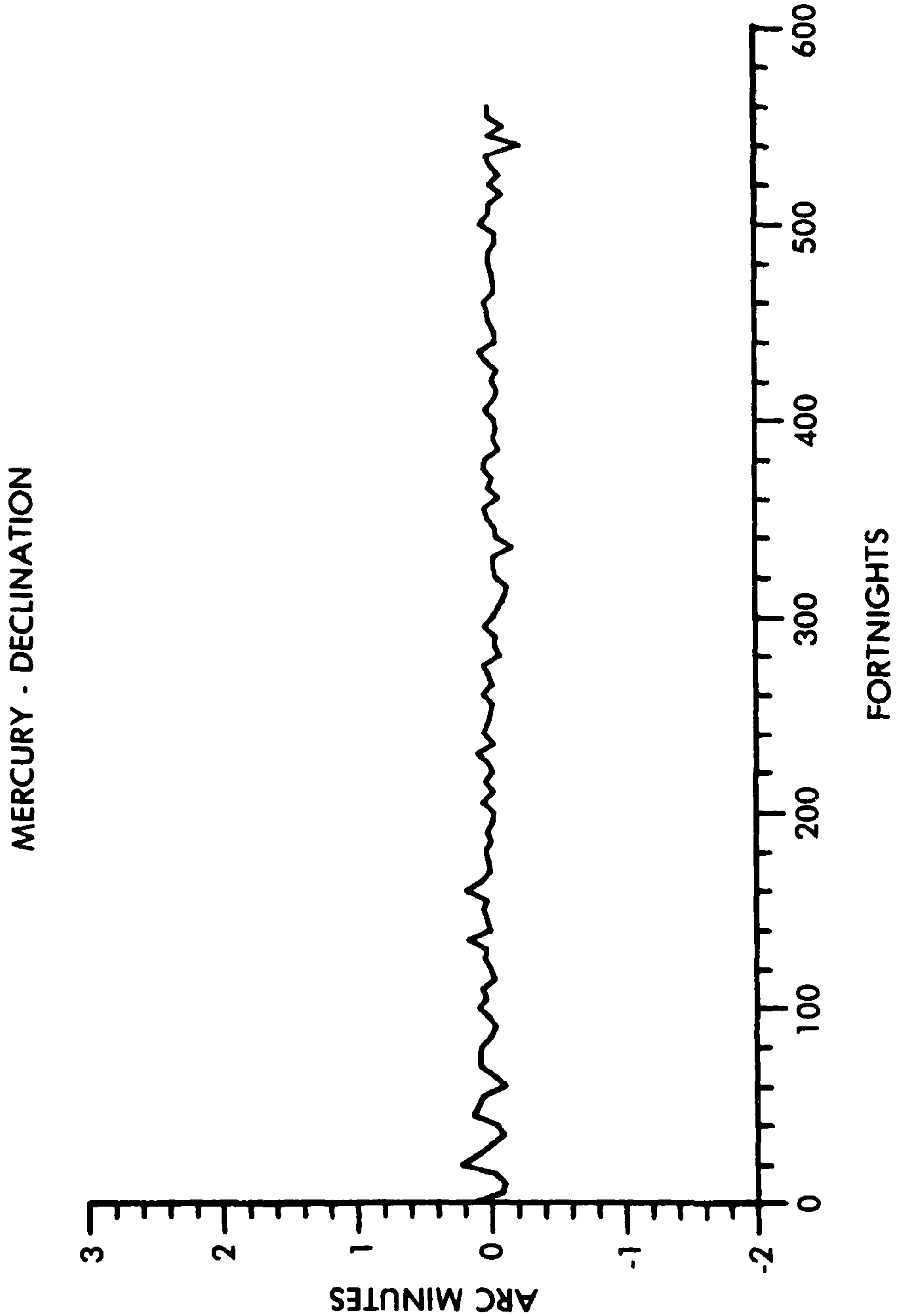


FIGURE E-2. MERCURY - DECLINATION 20-YEAR FIT SPAN

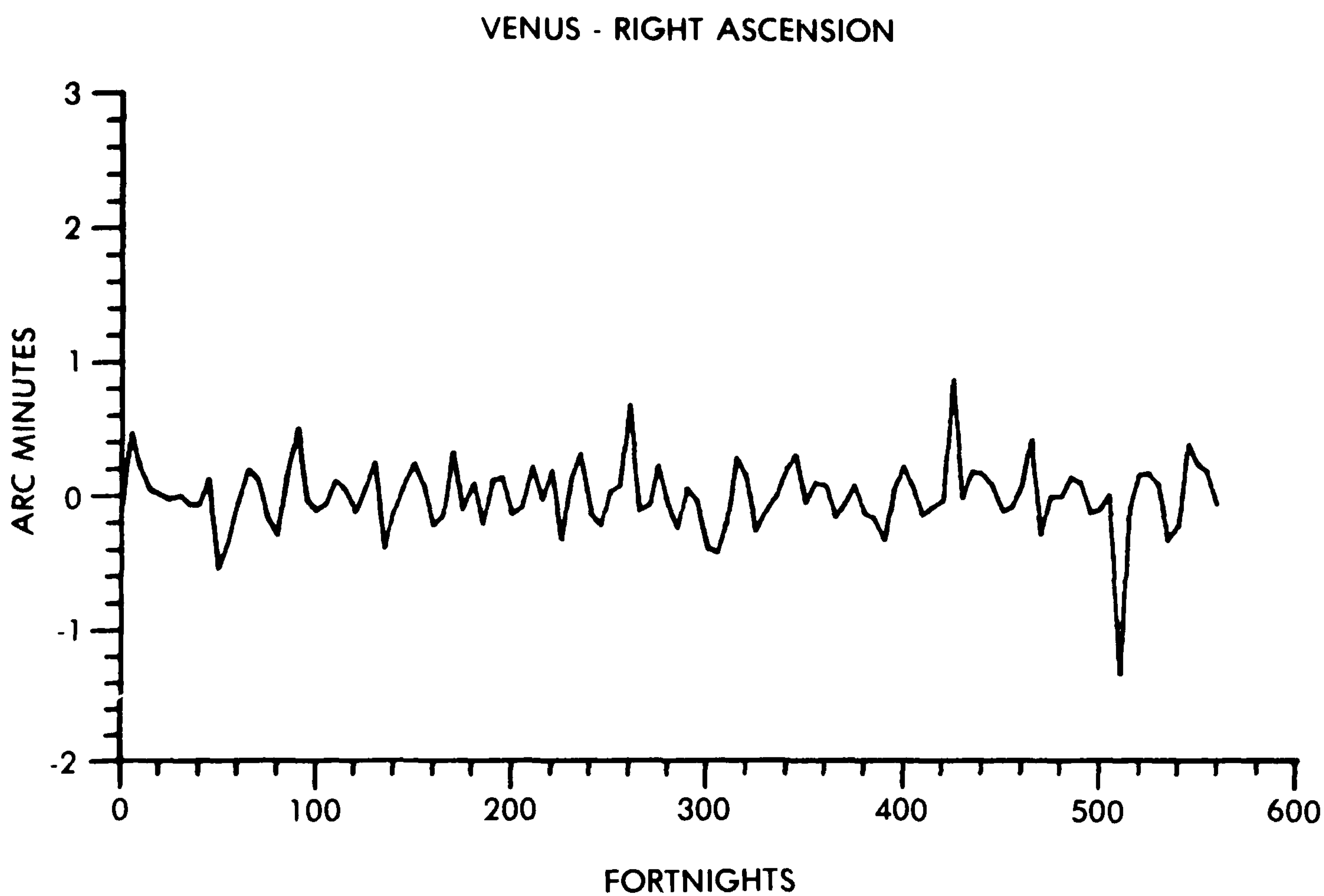


FIGURE E-3. VENUS - RIGHT ASCENSION 20-YEAR FIT SPAN

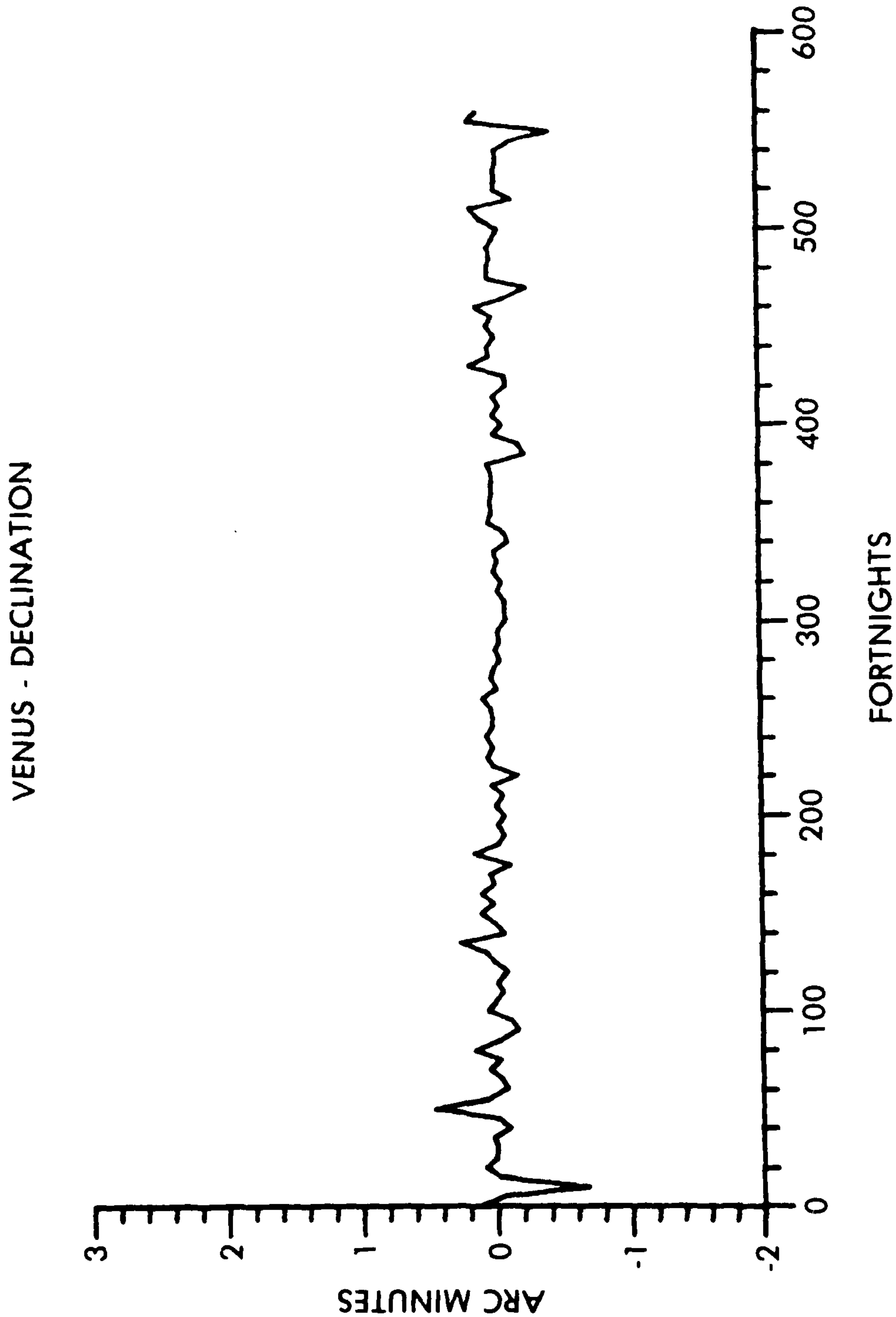


FIGURE E-4. VENUS - DECLINATION 20-YEAR FIT SPAN

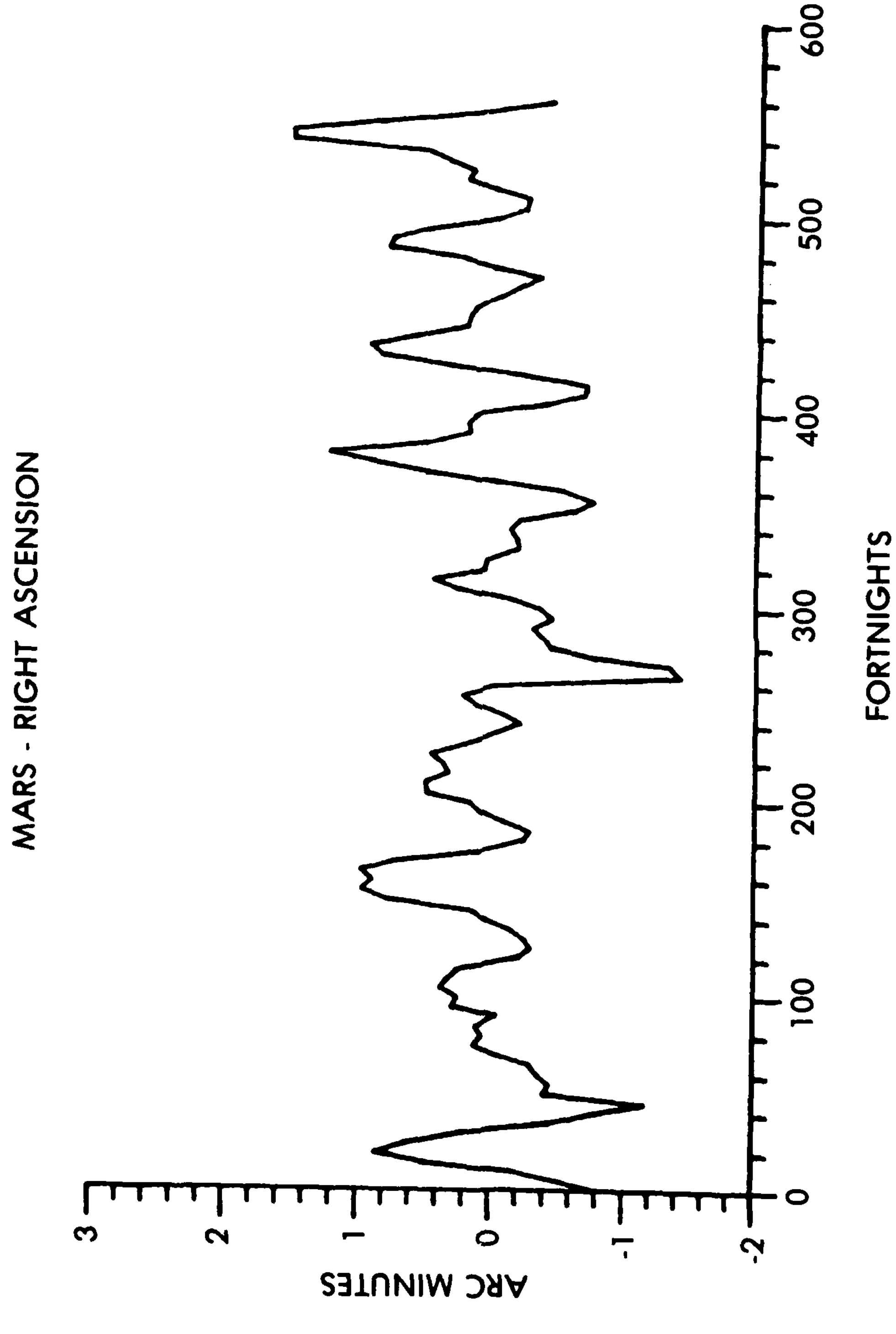


FIGURE E-5. MARS - RIGHT ASCENSION 20-YEAR FIT SPAN

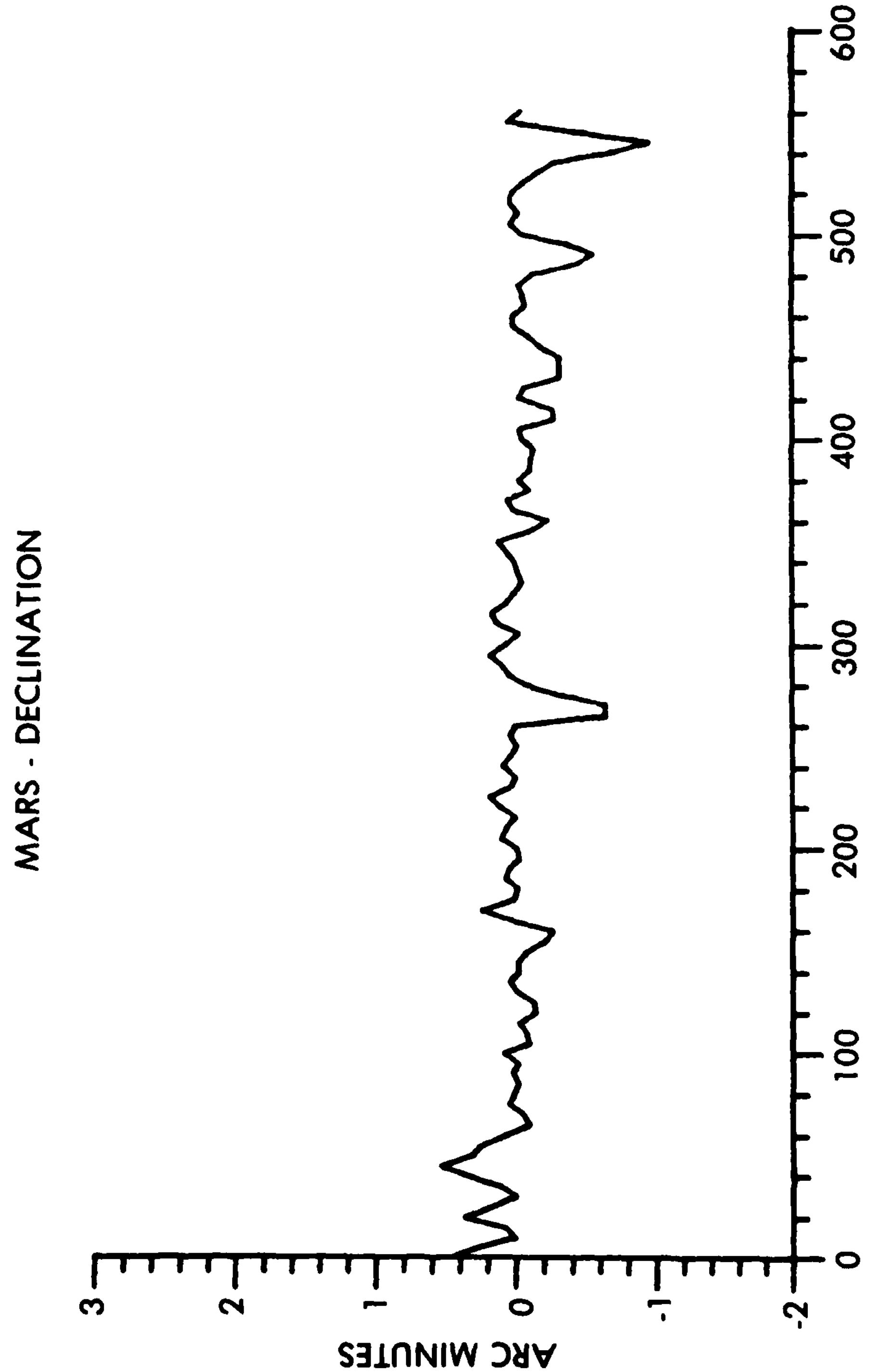


FIGURE E-6. MARS - DECLINATION 20-YEAR FIT SPAN

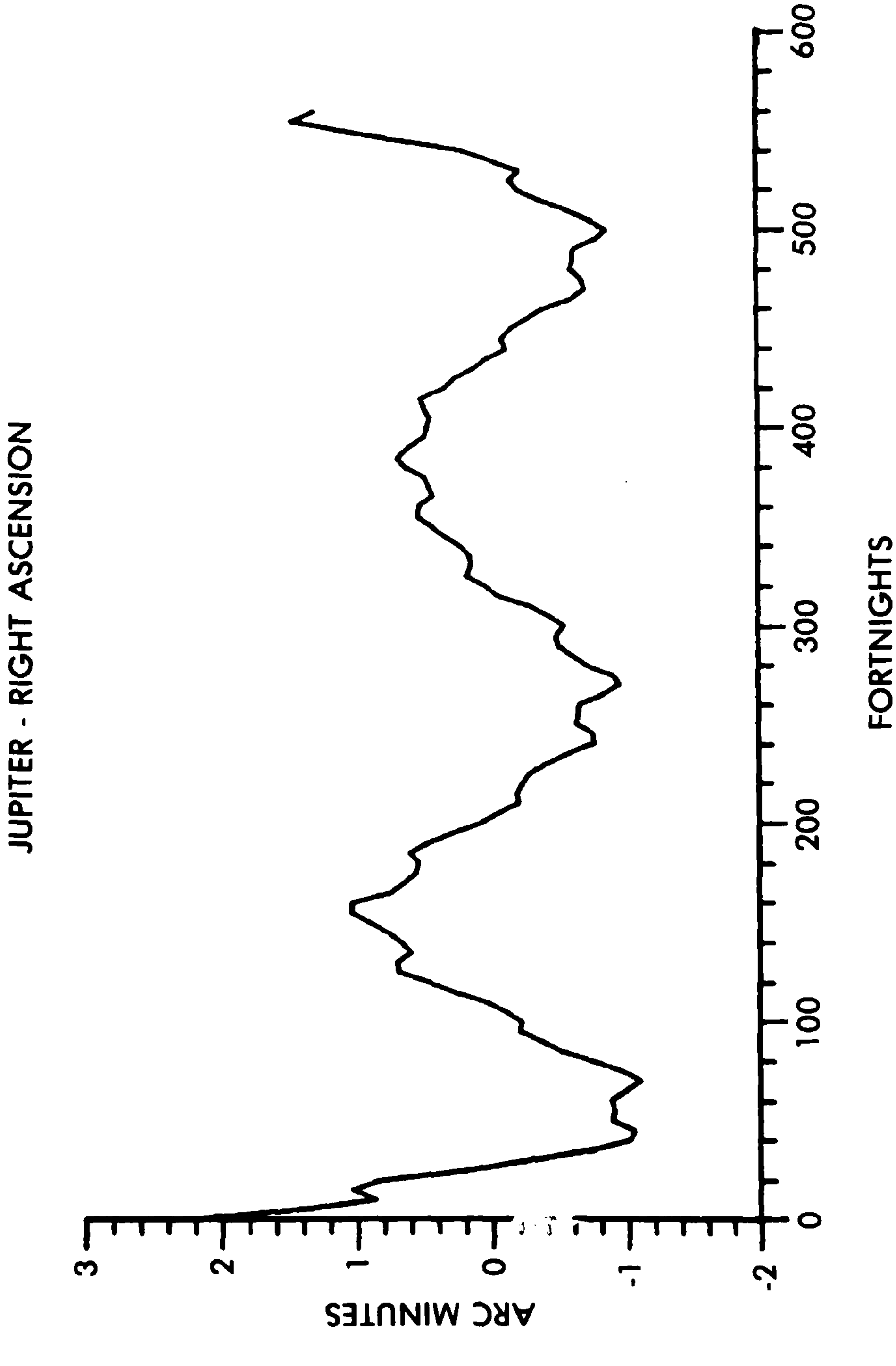


FIGURE E-7. JUPITER - RIGHT ASCENSION 20-YEAR FIT SPAN

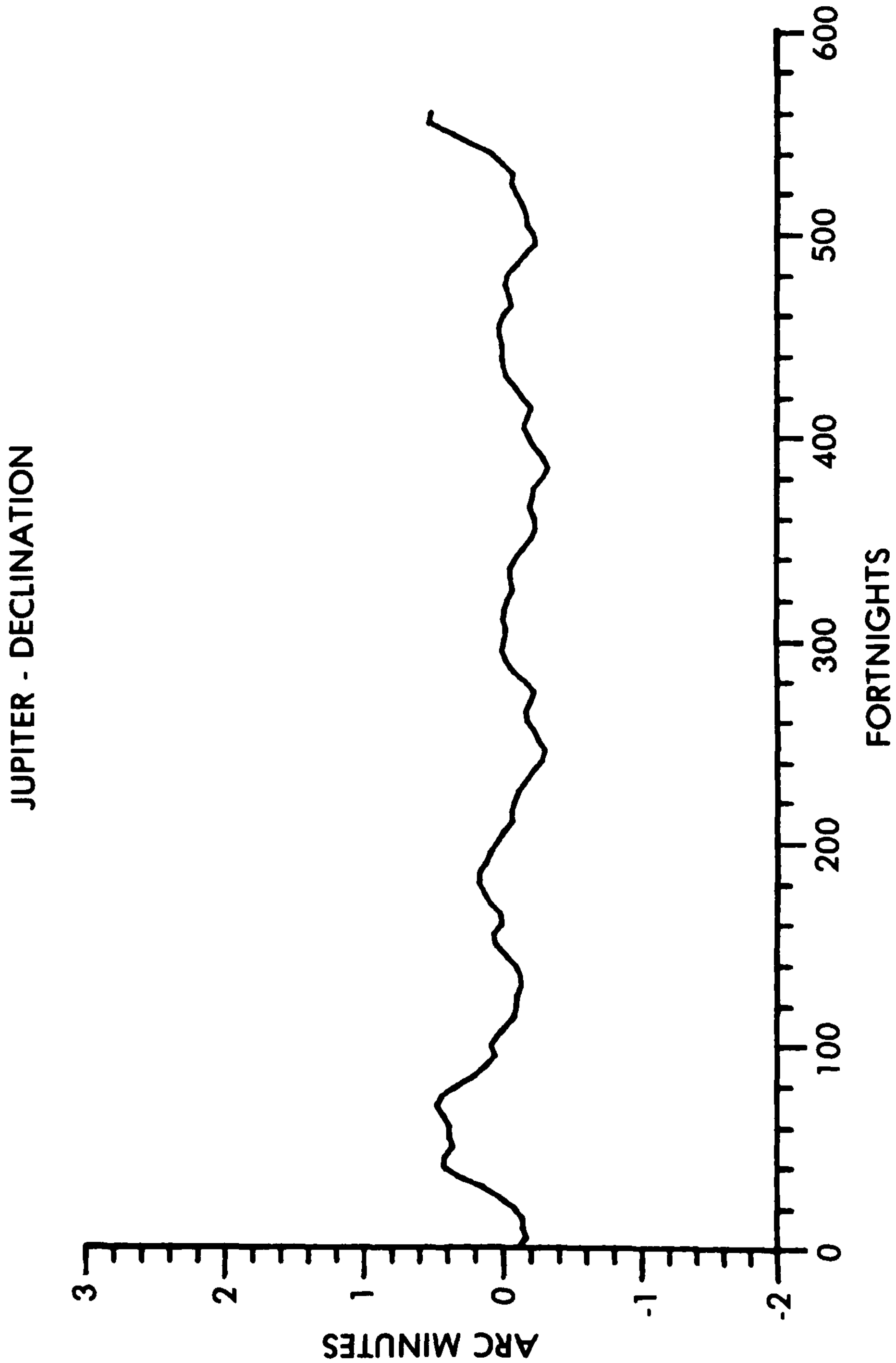


FIGURE E-8. JUPITER - DECLINATION 20-YEAR FIT SPAN

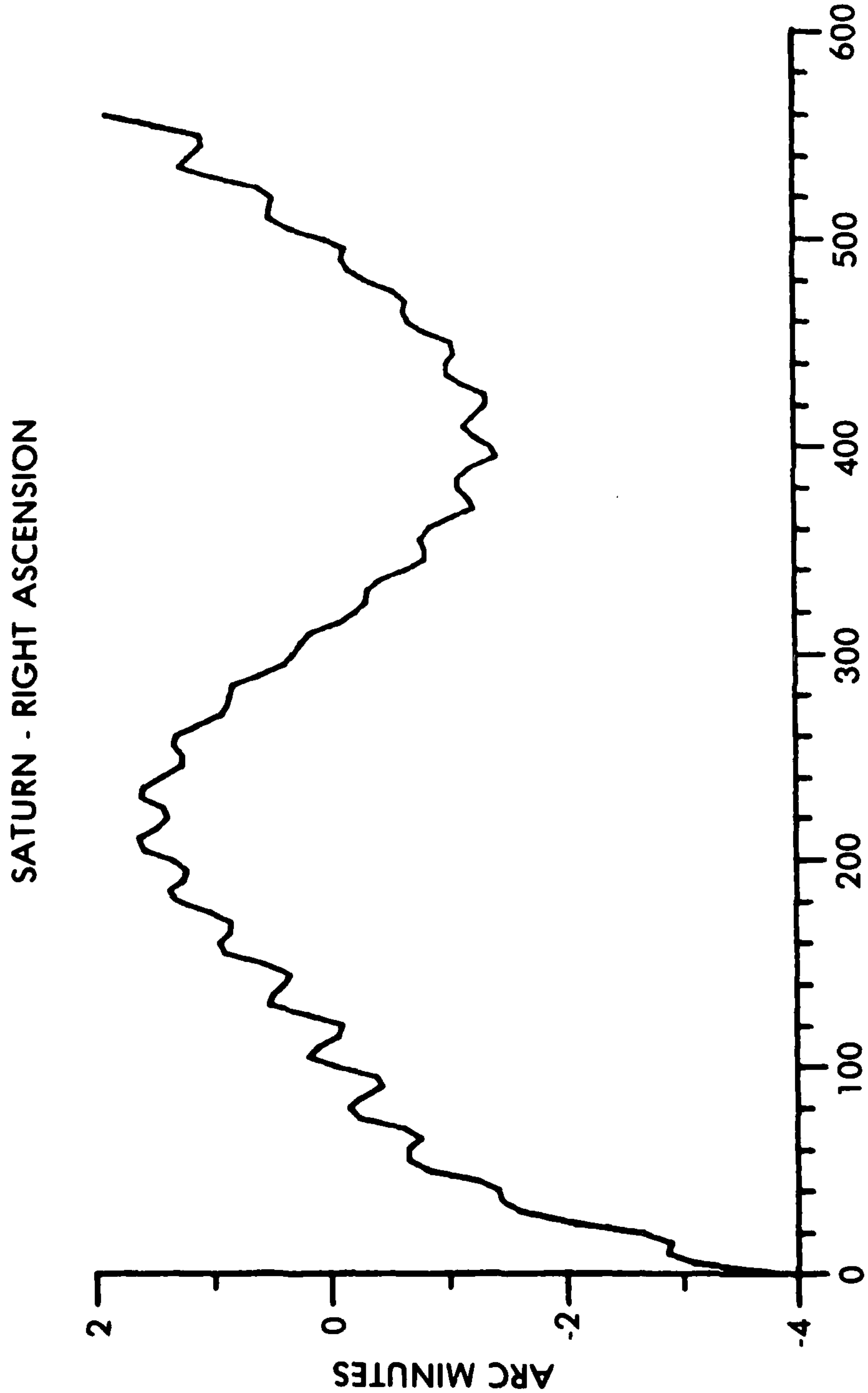


FIGURE E-9. SATURN - RIGHT ASCENSION 20-YEAR FIT SPAN

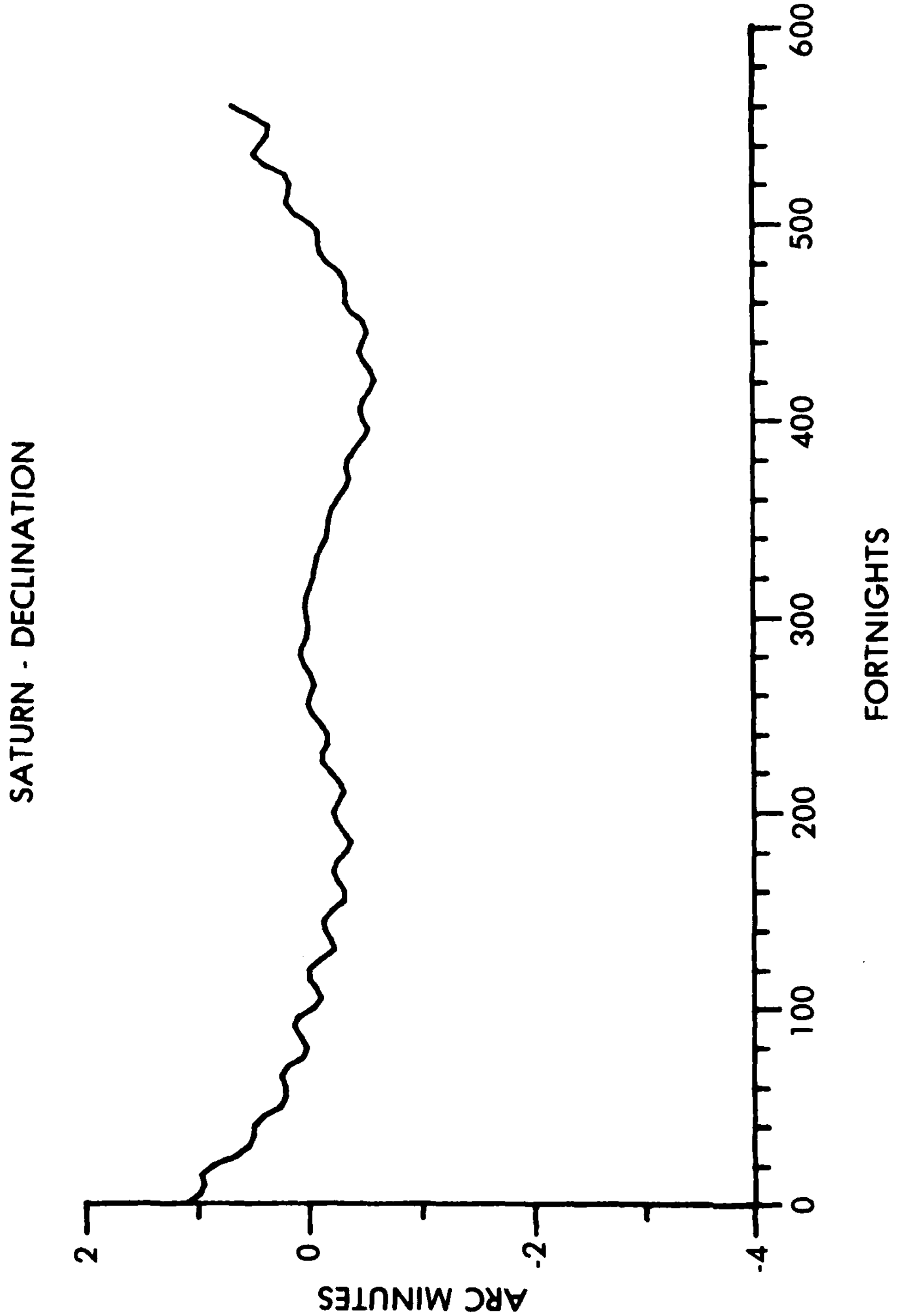


FIGURE E-10. SATURN - DECLINATION 20-YEAR FIT SPAN

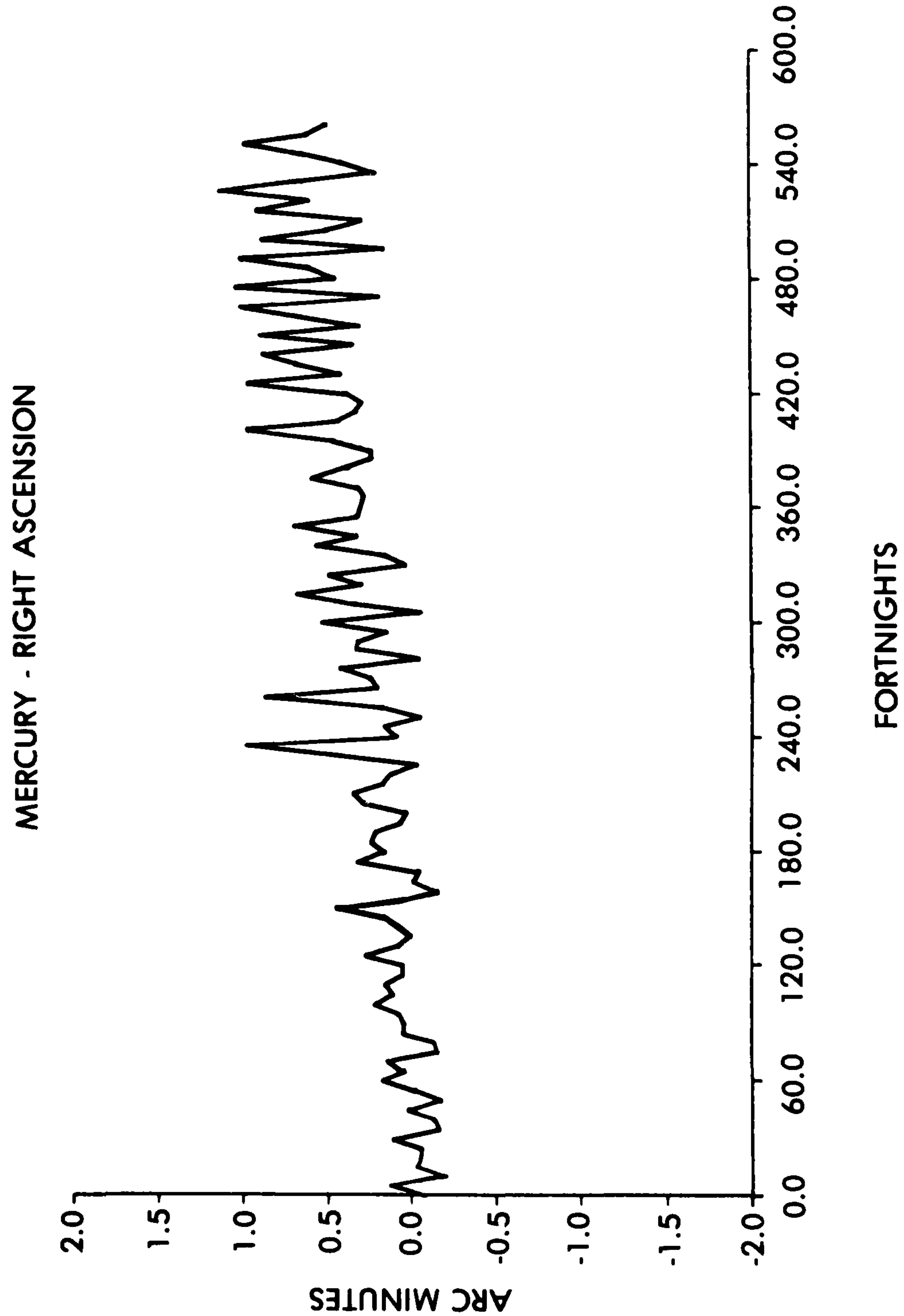


FIGURE E-11. MERCURY - RIGHT ASCENSION 5-YEAR FIT SPAN

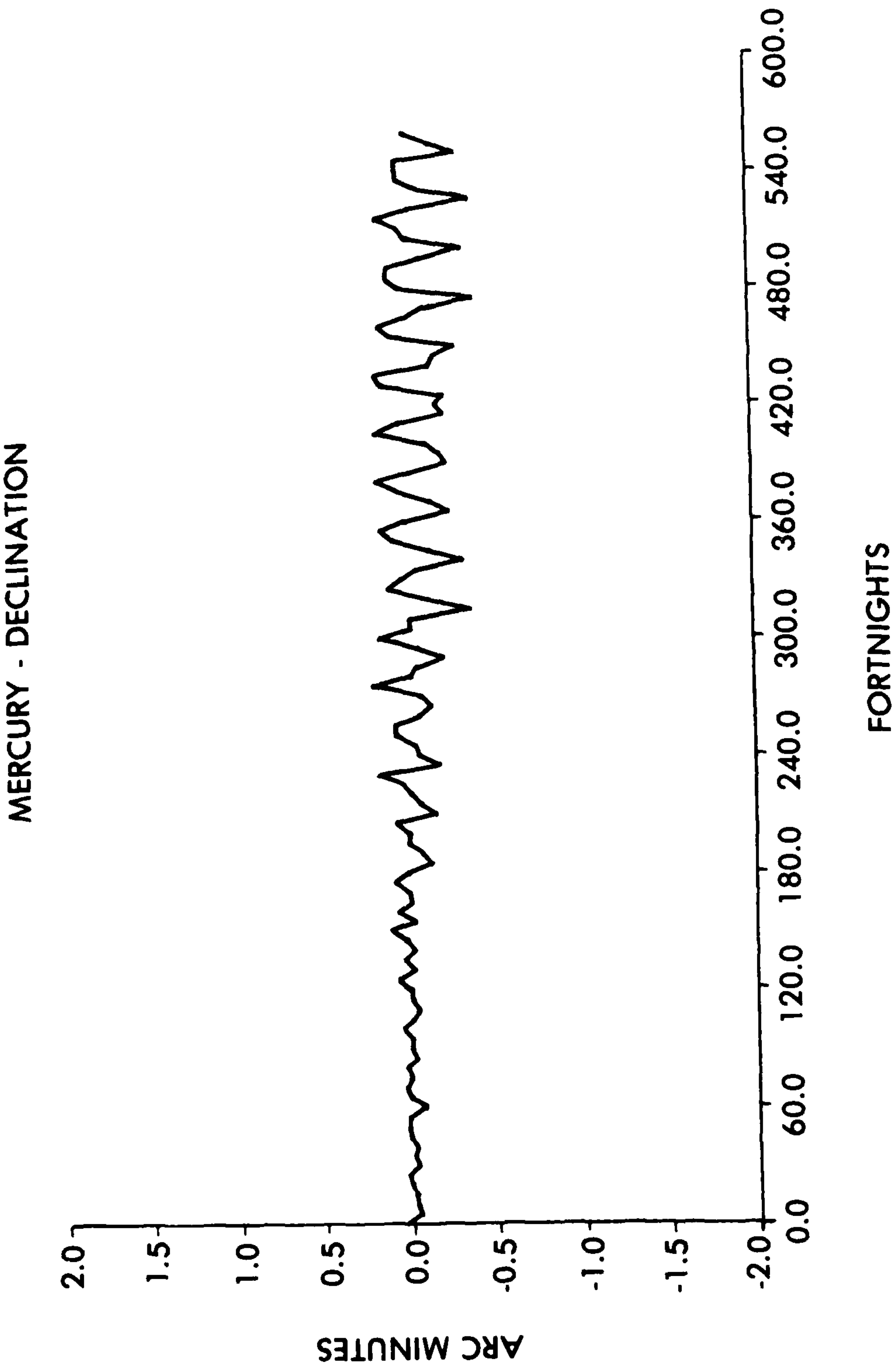


FIGURE E-12. MERCURY - DECLINATION 5-YEAR FIT SPAN

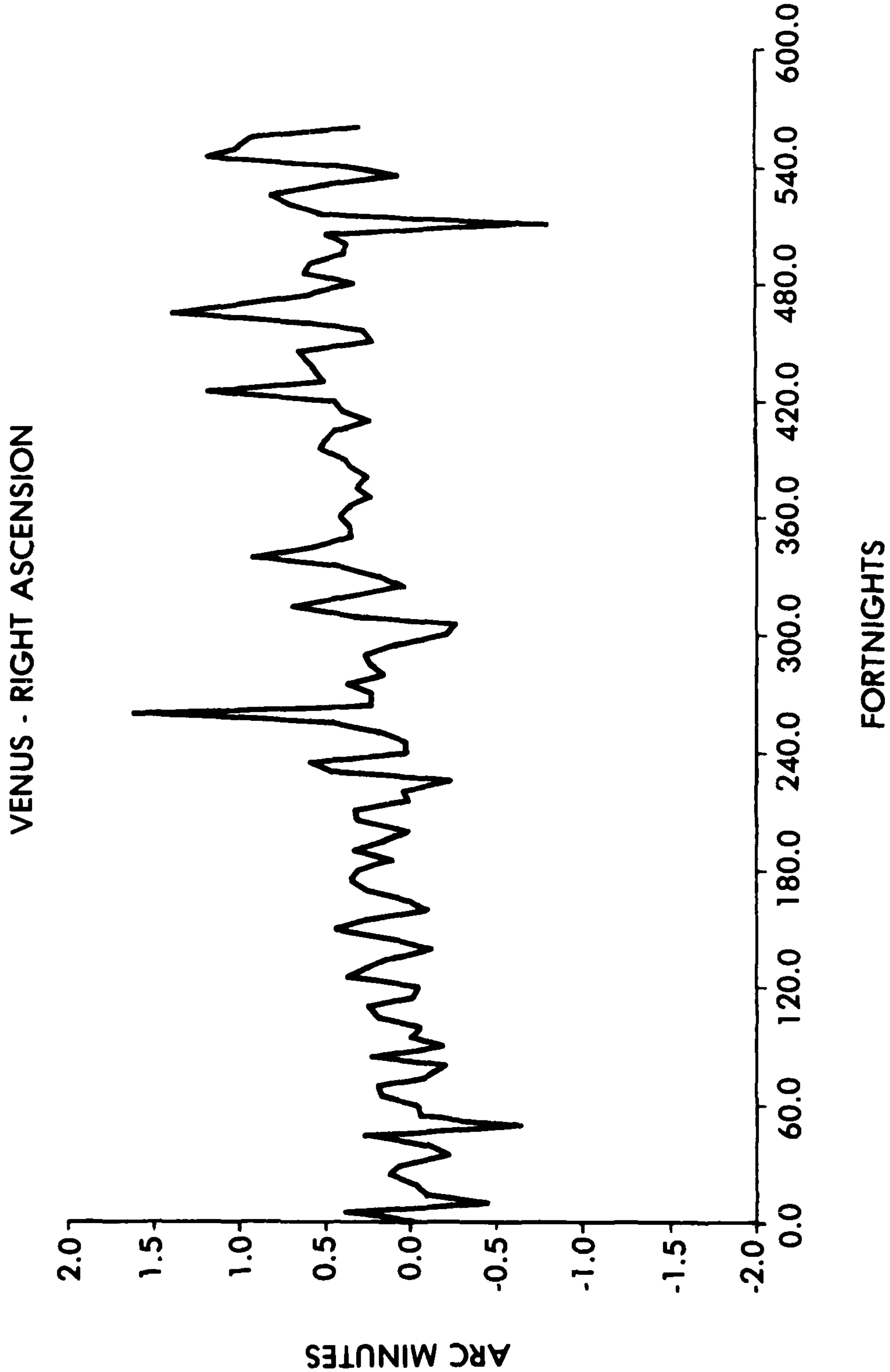


FIGURE E-13. VENUS - RIGHT ASCENSION 5-YEAR FIT SPAN

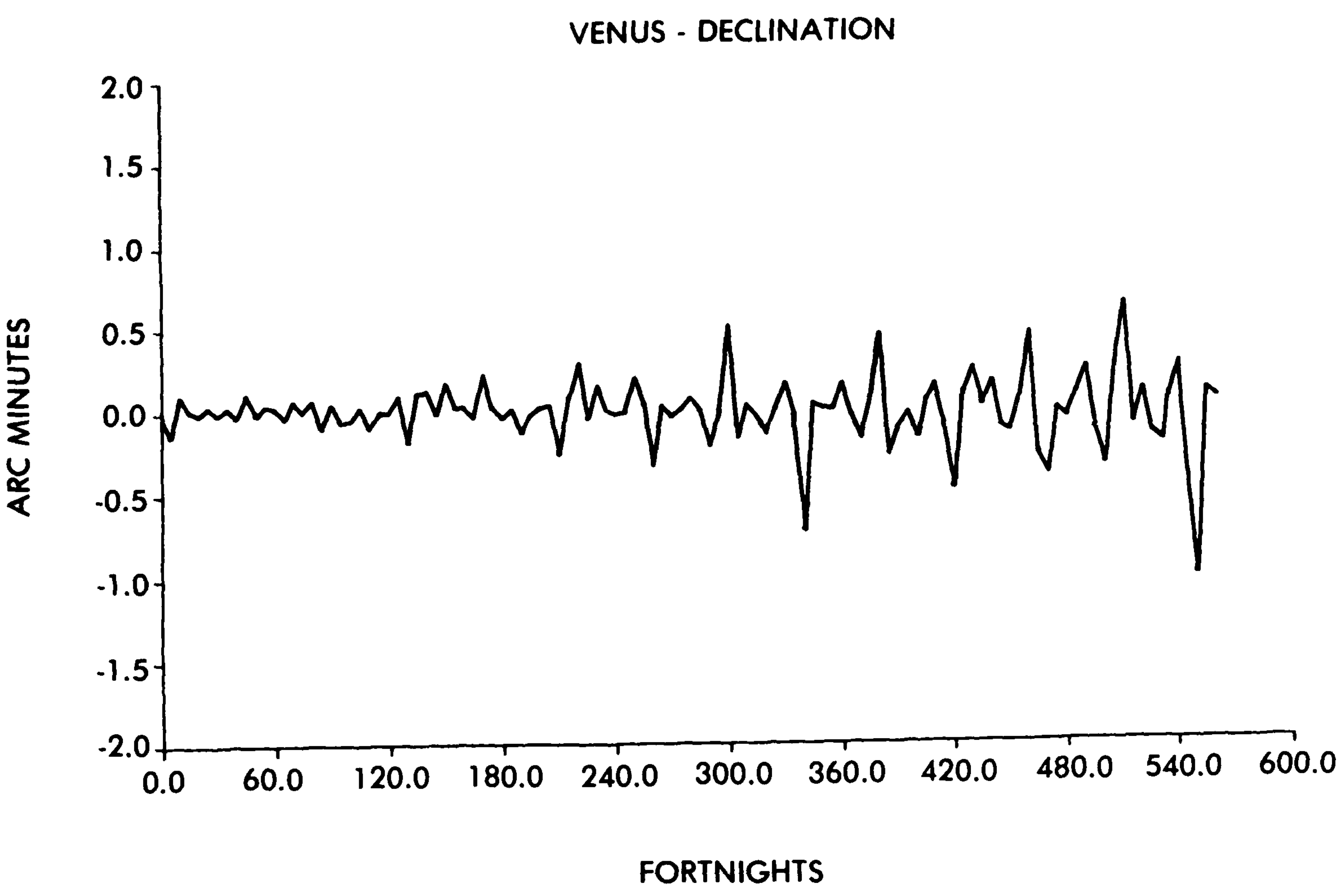


FIGURE E-14. VENUS - DECLINATION 5-YEAR FIT SPAN

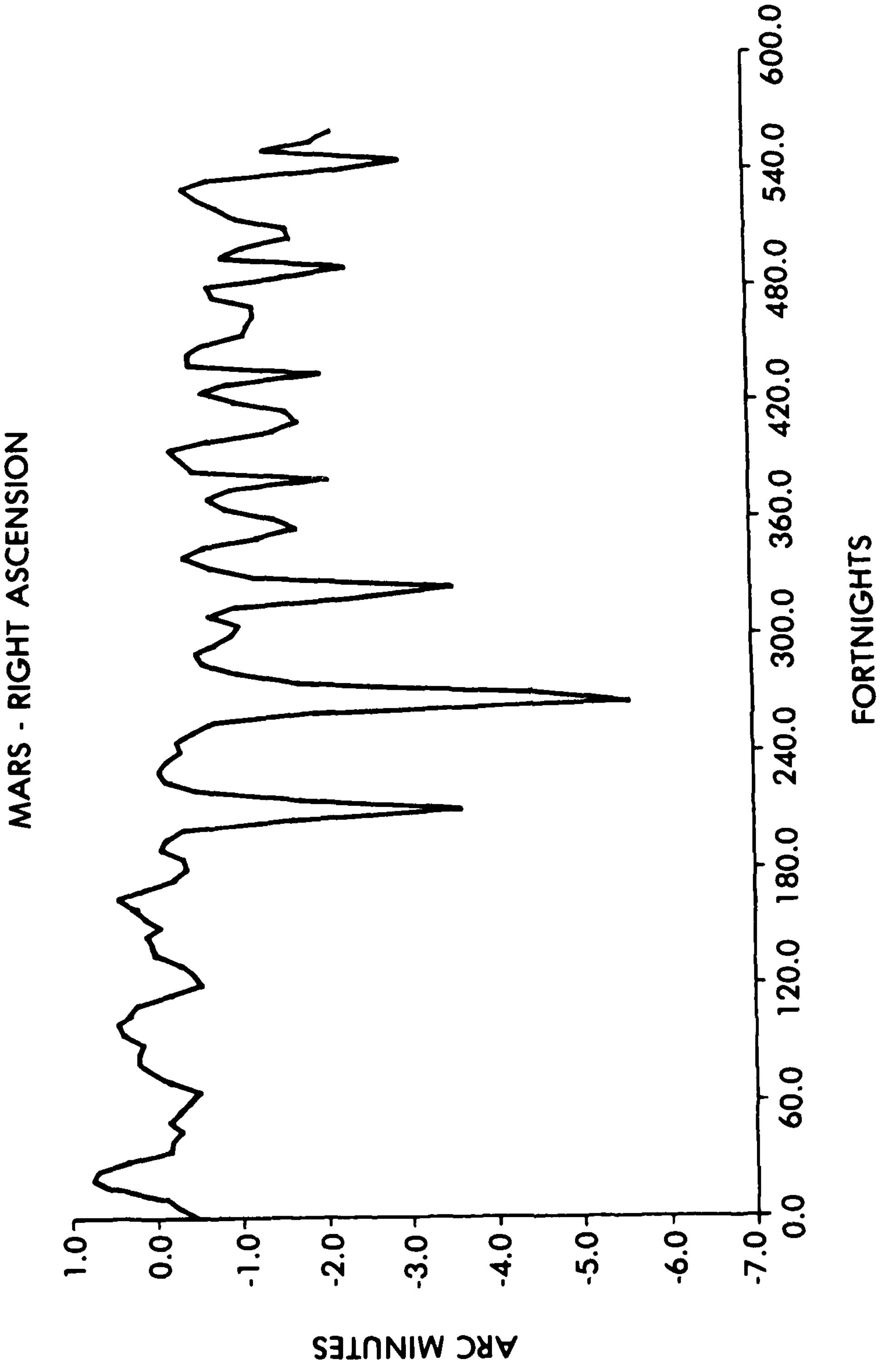


FIGURE E-15. MARS - RIGHT ASCENSION 5-YEAR FIT SPAN

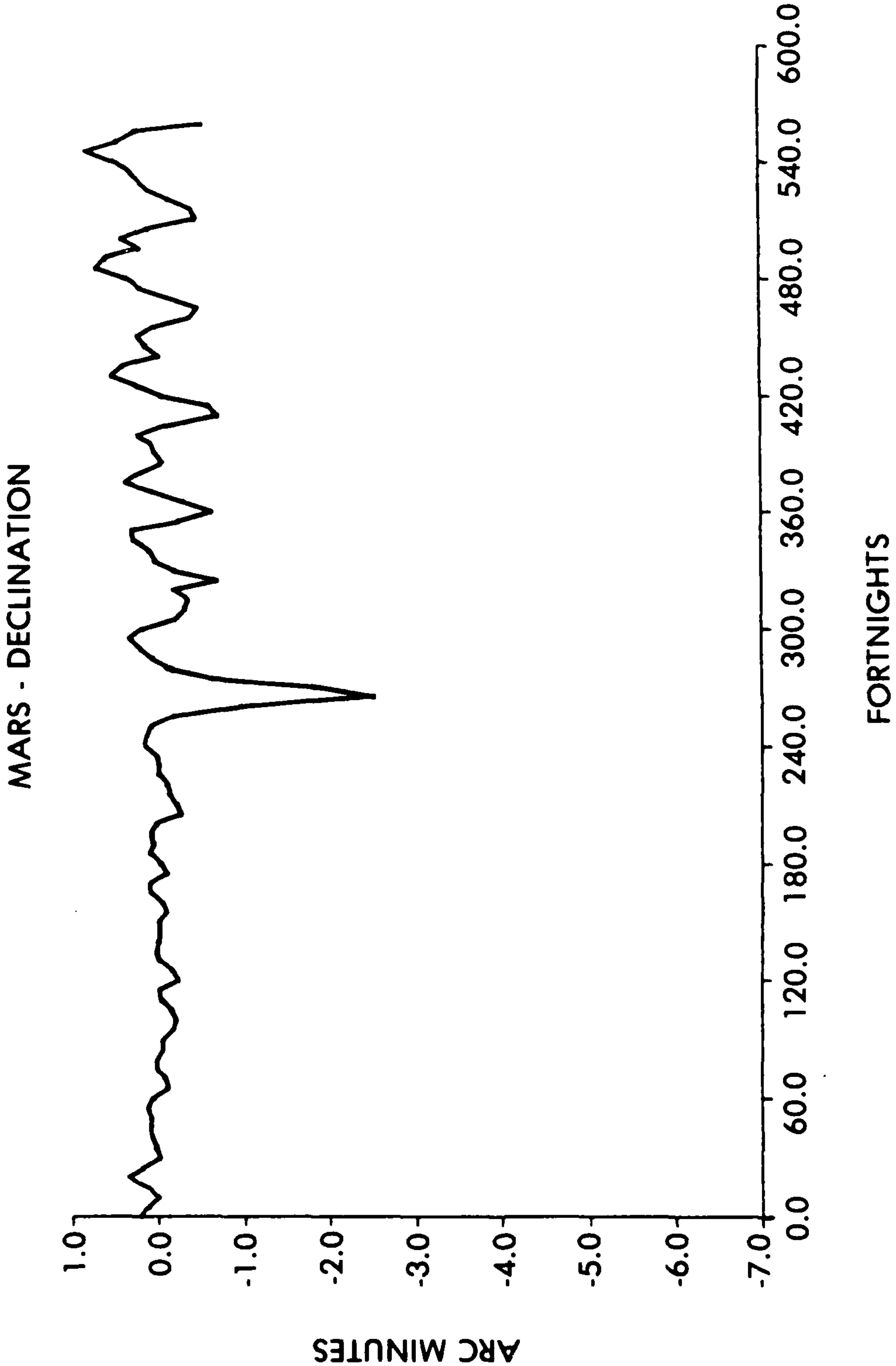


FIGURE E-16. MARS - DECLINATION 5-YEAR FIT SPAN

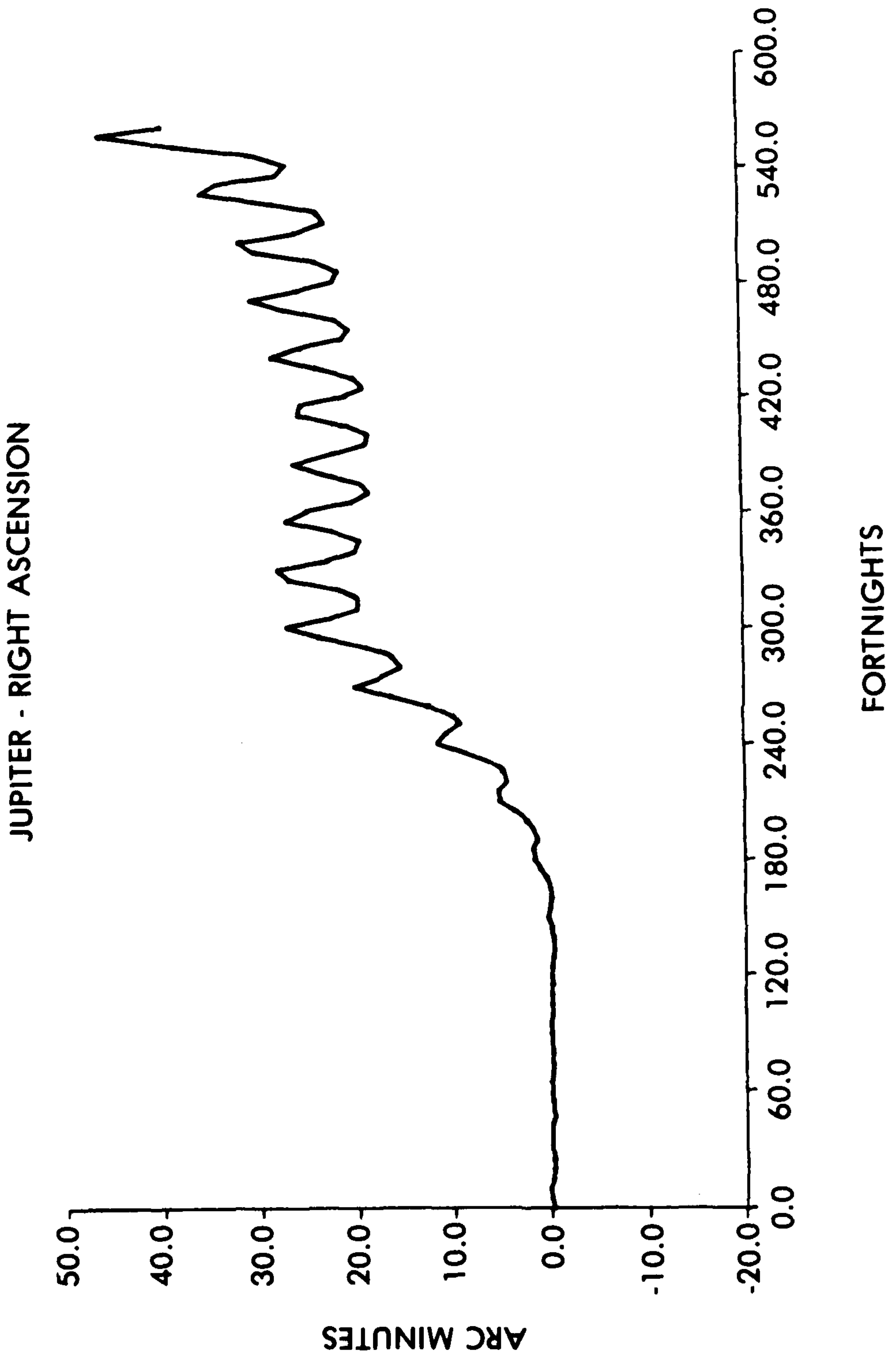


FIGURE E-17. JUPITER - RIGHT ASCENSION 5-YEAR FIT SPAN

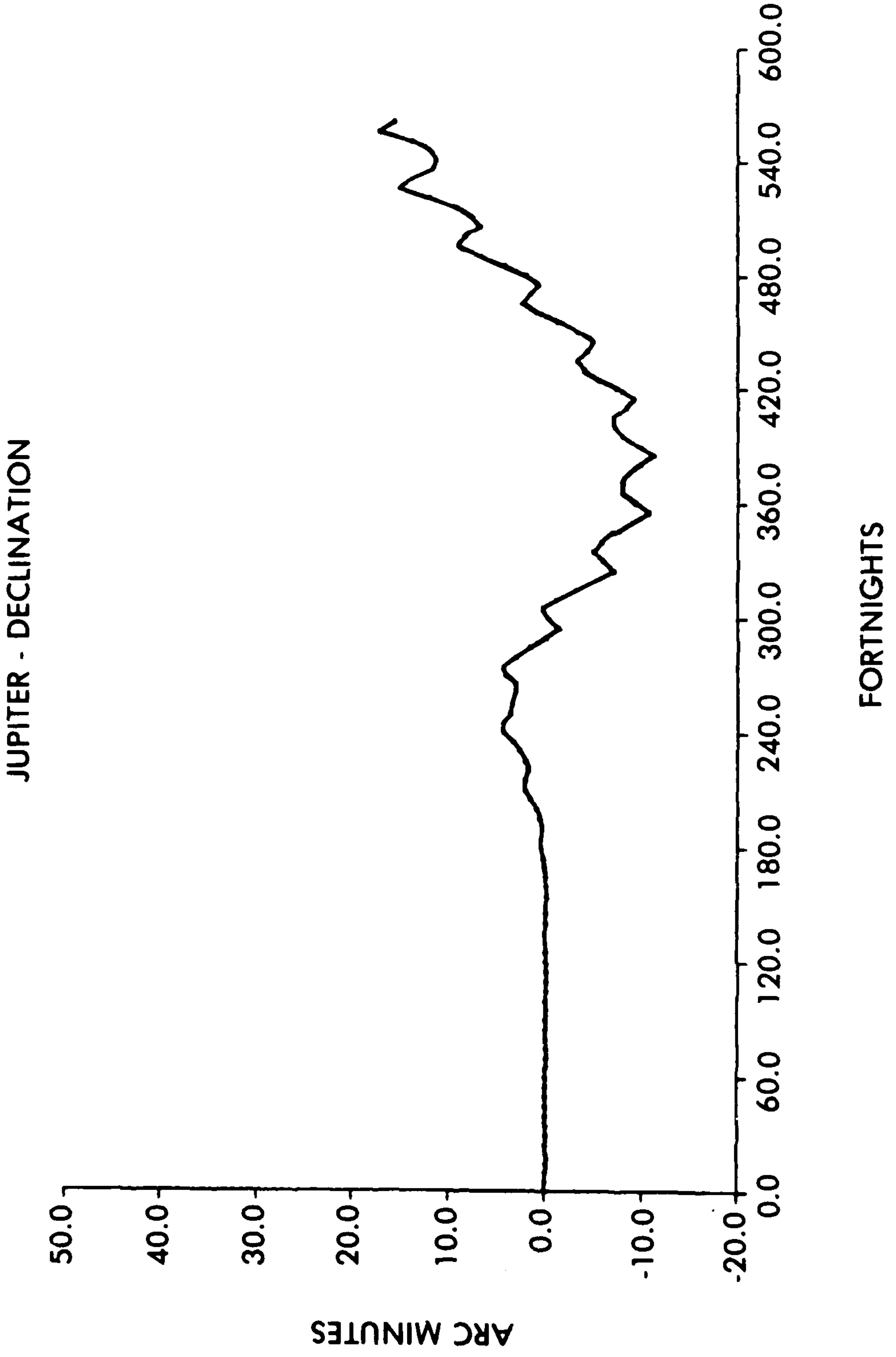


FIGURE E-18. JUPITER - DECLINATION 5-YEAR FIT SPAN

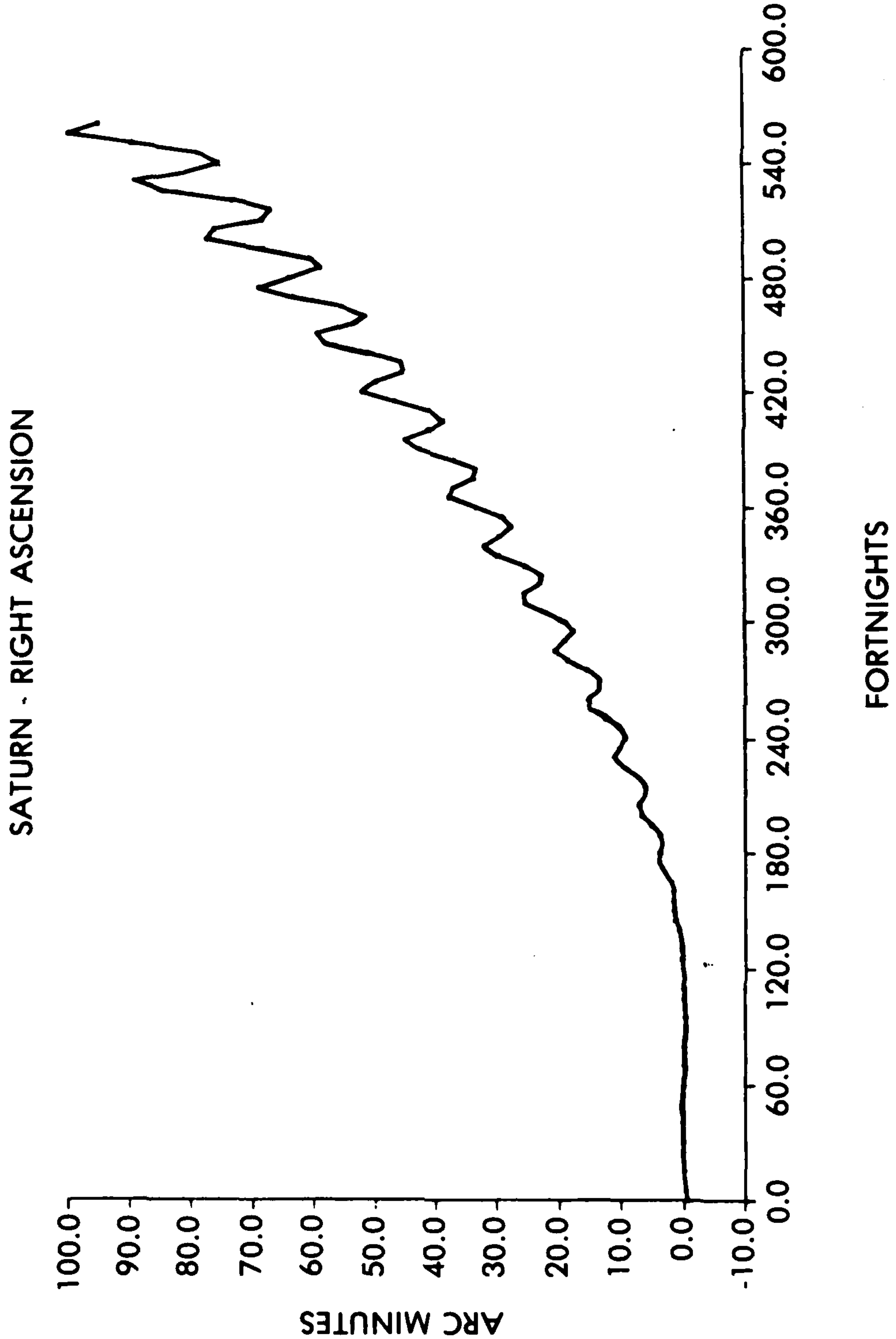


FIGURE E-19. SATURN - RIGHT ASCENSION 5-YEAR FIT SPAN

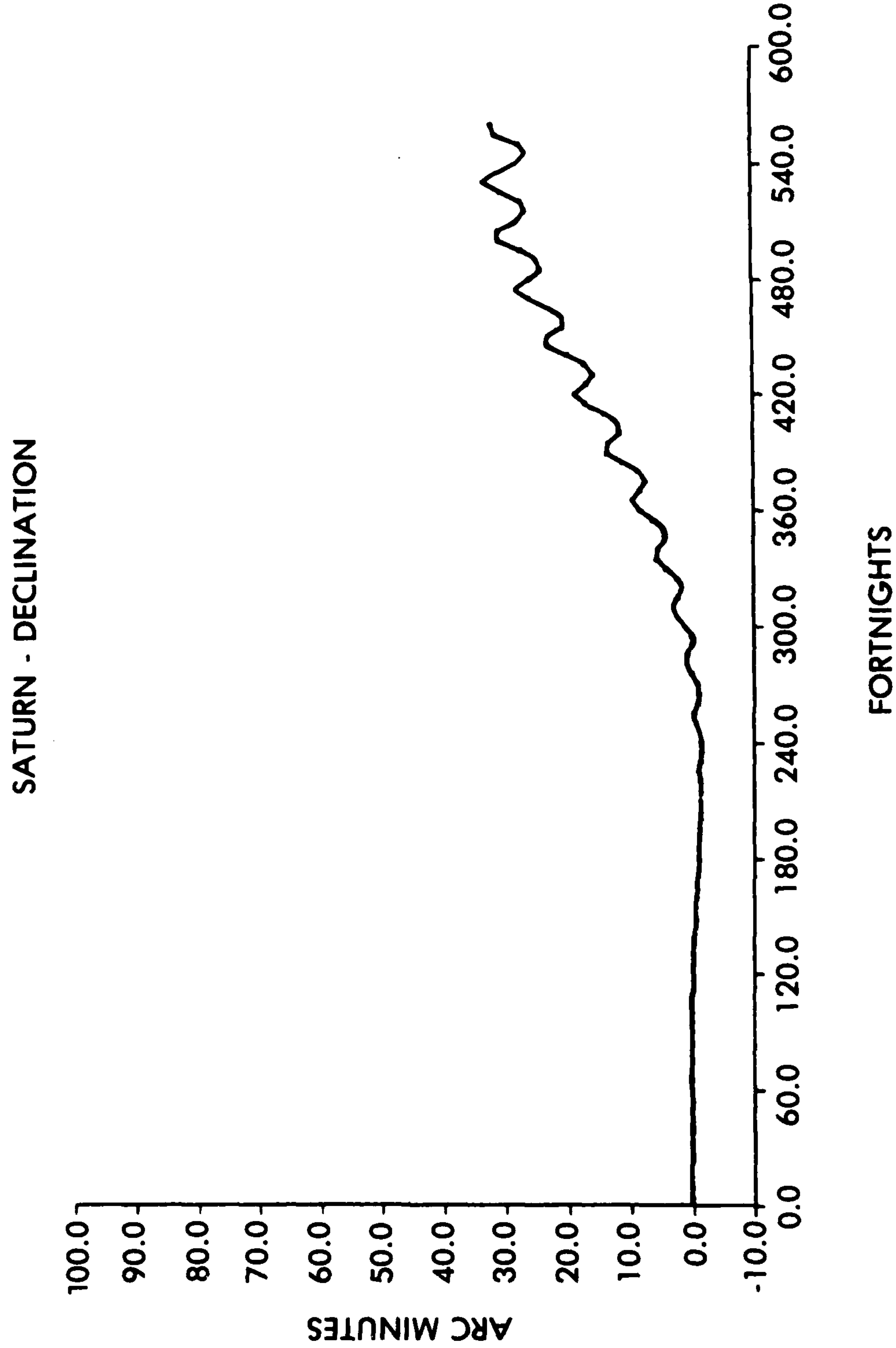


FIGURE E-20. SATURN - DECLINATION 5-YEAR FIT SPAN

# APPENDIX F

## PROGRAM PLANET

```
      PROGRAM PLANET(INPUT,OUTPUT,TAPE5,TAPE6)
C
C     PURPOSE:    TO COMPUTE PLANETARY POSITIONS AND PARTIAL DERIVATIVES
C                 BASED ON A SIMPLE KEPLERIAN MODEL
C                 POSITIONS AND PARTIALS FOR EACH TIME STEP
C     INPUT:     ID=PLANET ID
C                  1=MERCURY
C                  2=VENUS
C                  3=EARTH
C                  4=MARS
C                  5=JUPITER
C                  6=SATURN
C                  7=URANUS
C                  8=NEPTUNE
C                  9=PLUTO
C                 10=MOON
C                 11=SUN
C               INEL:  OPTION FOR INPUTTING PLANETARY ELEMENTS
C                  O=CLASSICAL ELEMENTS
C                  1=EQUINOCTAL ELEMENTS
C                INC:  INTERVAL BETWEEN TIME LINES ON SYSTEM'S OUTPUT
C              ITRAJ:  OPTION FOR COMPUTING PARTIAL DERIVATIVES
C                  O=NO PARTIALS
C                  1=COMPUTE PARTIALS
C             TEPOCH:  EPOCH TIME (JULIAN DAY)
C               DELT:  DELTA TIME (DAYS)
C               TEND:  END TIME (JULIAN DAY)
C            CLASSICAL PLANETARY ELEMENTS (INEL=O)
C                  A:  SEMI-MAJOR AXIS (KM)
C                  E:  ECCENTRICITY
C                 AI:  ANGLE OF INCLINATION (RAD)
C             COMEGA:  ANGLE OF ASCENDING NODE (RAD)
C             SOMEGA:  ANGLE OF PERIGEE (RAD)
C                AMO:  INITIAL MEAN ANOMOLY (RAD)
C            EQUINOCTAL ELEMENTS (INEL=1)
C                  A:  SEMI-MAJOR AXIS (KM)
C                  H:
C                 AK:
C                  P
C                  Q:
C              ALAMO:  LATITUDE (RAD)
C     OUTPUT:    TIME, POSITION (X,Y,Z), PARTIAL DERIVATIVES OF X,Y,Z
C                WRT EACH PLANETARY ELEMENT
C     SUBPROGRAMS:  NEWT, CONVRT, PARTIAL
C     LANGUAGE:  FORTRAN
C
      COMMON/CONV/BETA,XP,YP,FHAT(3),GHAT(3)
      COMMON/ELEMTS/A,H,AK,P,Q,ALAMO
      COMMON/PART/PAR(18)
C
      DIMENSION AM(9)
C
      DATA INTPE/5/,IOUT/6/
      DATA AU/149597871.41056/,TU/58.13244087/
      DATA (AM(I),I=1,9)/6023600.,408523.5,332480.,3098710.,1047.355,
     1 3498.5,22869.,19314.,3000000./
C
      REWIND INTPE
      REWIND IOUT
```

```
   90 IFIRST = 0
      READ(INTPE,*) ID,INEL,INC,ITRAJ
      IF(ID.GT.100) GO TO 170
      READ(INTPE,*) TEPOCH,DELT,TEND
      IF(INEL.NE.O) GO TO 100
      READ(INTPE,*) A,E,AI,SOMEGA,COMEGA,AMO
        TOMEGA=SOMEGA+COMEGA
          SINO=SIN(TOMEGA)
          COSO=COS(TOMEGA)
          TANI=TAN(AI/2.)
         SO = SIN(COMEGA)
         CO = COS(COMEGA)
        H=E*SINO
        AK=E*COSO
        P=TANI*SO
        Q=TANI*CO
        ALAMO=TOMEGA+AMO
        GO TO 110
  100 READ(INTPE,*) A,H,AK,P,Q,ALAMO
      EMEG=ARTNQ(H,AK)
      COMEGA=ARTNQ(P,Q)
      PI=3.14159265389 8
      IF(EMEG.LT.COMEGA)EMEG=EMEG+2*PI
      SOMEGA=EMEG-COMEGA
      AMO=ALAMO-EMEG
      E=AK/COS(EMEG)
      AI=2*ATAN(P/SIN(COMEGA))
  110 CONTINUE
      DT = DELT/TU
      TIME = TEPOCH
      PRINT 2000, ID,INEL,INC,ITRAJ,TEPOCH,DELT,TEND
      PRINT 2001,A,E,AI,COMEGA,SOMEGA,AMO
      PRINT 2002, A,H,AK,P,Q,ALAMO
      A=A/AU
      EN=A**(-1.5)*SQRT(1.+1./AM(ID))
      ALAM=ALAMO
      CT=0
      ITER=0
  130 CALL NEWT(ALAM,F)
      CALL CONVRT(F,X,Y,Z,IFIRST)
        XS=AU*X
        YS=AU*Y
        ZS=AU*Z
      WRITE (IOUT,2003)TIME,XS,YS,ZS
      IF (ITRAJ.EQ.0) GO TO 150
        CALL PARTIAL (CT,F,EN,IFIRST)
      IFIRST = 1
        DO 140 I=2,18
      IF(I.EQ.7.OR.I.EQ.13) GOTO 140
          PAR(I)=PAR(I)*AU
  140    CONTINUE
        WRITE(IOUT,2008) PAR
  150 ITER=ITER+1
      IF (ITER.NE.INC) GO TO 160
        ITER=0
  160 TIME = TIME+DELT
      ALAM=ALAM + EN*DT
      CT=CT + DT
      IF (TIME.LT.TEND) GO TO 130
      GO TO 90
  170 END FILE IOUT
C
 1000 FORMAT (4I5)
 1001 FORMAT (3E20.13)
C
 2000 FORMAT (*1PLANET ID          = *,I5/* ELEMENT OPTION   = *,I5/
     1         * OUTPUT INTERVAL   = *,I5/* TRAJECTORY OPT   = *,I5/
     1         * EPOCH TIME        = *,E20.13/* DELTA TIME        = *,E20.13/
     1         * END TIME          = *,E20.13)
 2001 FORMAT(*OSEMI-MAJOR AXIS         = *,E20.13/
     A       * ECCENTRICITY             = *,E20.13/
```

```
     1          * INCLINATION               = *,E20.13/
     2          * ANG. OF ASCENDING NODE = *,E20.13/
     3          * ANG. OF PERIGEE           = *,E20.13/
     4          * INITIAL MEAN ANOMOLY      = *,E20.13)
 2002 FORMAT (*OSEMI-MAJOR AXIS     = *,E20.13/
     1          * H    = *,E20.13/* K    = *,E20.13/* P    = *,E20.13/
     A          * Q    = *,E20.13/
     2          * LATITUDE    = *,E20.13)
 2003 FORMAT (1F12.3,3E30.14)
 2004 FORMAT(* *,4E18.11)
 2006 FORMAT(*        TIME                    X                    Y*,
     1 *                  Z*)
 2005 FORMAT(1H0,3E20.13)
 2007 FORMAT (1H )
2008  FORMAT (6E20.13)
C
      END
      SUBROUTINE NEWT(ALAM,F)
C
C    THIS SUBROUTINE COMPUTES THE ECCENTRIC LONGITUDE (F) FROM THE
C      MEAN LONGITUDE (ALAM) USING NEWTON'S METHOD
      COMMON/ELEMTS/A,H,AK,P,Q,ALAMO
      DATA TOL/.1745E-5/
C
      FO = ALAM + AK*SIN(ALAM) - H*COS(ALAM) + (AK*AK-H*H)/2.*SIN
     1      (2.*ALAM) - AK*H/2.*COS(2.*ALAM)
  100 F = (ALAM + (AK-H*FO)*SIN(FO) - (H+AK*FO)*COS(FO))/
     1      (1.-H*SIN(FO) - AK*COS(FO))
      IF (ABS(F-FO).LE.TOL) GO  TO 110
      FO=F
      GO TO 100
  110 RETURN
      END
      SUBROUTINE CONVRT (F,X,Y,Z,IFIRST)
C
C    THIS SUBROUTING CONVERTS THE EPOCH ELEMENTS AND ECCENTRIC
C          LONGITUDE TO INERTIAL CARTESIAN (ECLIPTIC) ELEMENTS
C
C    INPUT:  F,A,H,AK,P,K
C    OUTPUT:  X,Y,Z,XP,YP,FHAT(3),GHAT(3)
C
      COMMON/ELEMTS/A,H,AK,P,Q,ALAMO
      COMMON/CONV/BETA,XP,YP,FHAT(3),GHAT(3)
C
      DATA FIRST/0/
C
      IF (IFIRST.NE.0) GO TO  100
        BETA = 1./(1.+SQRT(1.-H*H-AK*AK))
        AX = A*(1.-H*H*BETA)
        B = A*H*AK*BETA
        AY = A*(1.-AK*AK*BETA)
        CONST = 1./(1.+P*P+Q*Q)
        FHAT(1) = CONST*(1.+Q*Q-P*P)
        FHAT(2) = CONST*(2.*P*Q)
        FHAT(3) = CONST*(-2.*P)
        GHAT(1) = FHAT(2)
        GHAT(2) = CONST*(1.+P*P-Q*Q)
        GHAT(3) = CONST*(2.*Q)
  100 XP=AX*COS(F) + B*SIN(F) - AK*A
      YP = AY*SIN(F) + B*COS(F) - H*A
      X = XP*FHAT(1) + YP*GHAT(1)
      Y= XP*FHAT(2) + YP*GHAT(2)
      Z = XP*FHAT(3) + YP*GHAT(3)
      RETURN
      END
      SUBROUTINE PARTIAL (CT,F,EN,IFIRST)
```

```
C
C     THIS SUBROUTINE COMPUTES AT EACH TIME STEP PARTIAL
C           DERIVATIVES OF POSITION WRT INPUT ELEMENTS
C
C     INPUT:  CT = TIME - TO
C              F = CURRENT ECCENTRIC LONGITUDE
C             A,H,AKK,P,Q,ALAMO,EN,B,FHAT,GHAT,XP,YP
C     OUTPUT:  PARTIAL DERIVATIVES OF POSITION WRT A,H,AK,P,Q,LAMBDA
C
      COMMON/CONV/BETA,XP,YP,FHAT(3),GHAT(3)
      COMMON/ELEMTS/ A,H,AK,P,Q,ALAMO
      COMMON/PART/ PAR(18)
C
      DIMENSION PARFQ(3),PARGQ(3),PARFP(3),PARGP(3)
C
      IF (IFIRST.NE.O) GO TO 100
        BETA1 = 1.-BETA
        BETA3 = BETA**3
        AK2 = AK*AK
        H2 = H*H
        AK3 = AK2*AK
        H3 = H2*H
        P2= P*P
        Q2 = Q*Q
      TERM=1.+P2+Q2
      TERM2=TERM**2
        PARFQ(1) = 4.*P2*Q/TERM2
        PARFQ(2) = 2.*P*(1.+P2-Q2)/TERM2
        PARFQ(3) = PARFQ(1)/P
        PARGQ(1) = PARFQ(2)
        PARGQ(2) = -PARFQ(1)*(1.+P2)/P2
        PARGQ(3) = PARFQ(2)/P
        PARFP(1) = -4.*P*(1.+Q2)/TERM2
        PARFP(2) = 2.*Q*(1.+Q2-P2)/TERM2
        PARFP(3) = -2.*FHAT(1)/TERM
        PARGP(1) = PARFP(2)
        PARGP(2) = Q*PARFQ(1)/P
        PARGP(3) = -PARFQ(1)/P
  100 COSF = COS(F)
      SINF = SIN(F)
      TERM1 = 1. - H*SINF - AK*COSF
      XPA = XP/A - 1.5*EN*CT*(H*AK*BETA*COSF - (1.-H2*BETA)*SINF)/TERM1
      YPA = YP/A - 1.5*EN*CT*((1.-AK2*BETA)*COSF - H*AK*BETA*SINF)/TERM1
      XPH = A*(AK*(BETA + H2*BETA3/BETA1)*SINF - (2.*H*BETA + H3*BETA3/
     1 BETA1)*COSF) - A*COSF*(H*AK*BETA*COSF - (1.-H2*BETA)*SINF)
     2 /TERM1
      YPH = A*(-AK2*H*BETA3*SINF/BETA1 + AK*(BETA+H2*BETA3/BETA1)*COSF-
     1 1.) - A*COSF*((1.-AK2*BETA)*COSF-H*AK*BETA*SINF)/TERM1
      XPK = A*(-H2*AK*BETA3*COSF/BETA1 + H*(BETA+AK2*BETA3/BETA1)*SINF
     1 -1.) + A*SINF*(H*AK*BETA*COSF - (1.-H2*BETA)*SINF)/TERM1
      YPK = A*(-(2.*AK*BETA + AK3*BETA3/BETA1)*SINF + H*(BETA +AK2
     1 *BETA3/BETA1)*COSF) + A*SINF*((1.-AK2*BETA1)*COSF - H*AK*BETA
     2 *SINF)/TERM1
      XPL = A*(H*AK*BETA*COSF - (1.-H2*BETA)*SINF)/TERM1
      YPL = A*((1.-AK2*BETA)*COSF - H*AK*BETA*SINF)/TERM1
      DO 110 I=1,3
      N = 6*(I-1)+1
        PAR(N) = XPA*FHAT(I) + YPA*GHAT(I)
        PAR(N+1) = XPH*FHAT(I) + YPH*GHAT(I)
        PAR(N+2) = XPK*FHAT(I) +YPK*GHAT(I)
        PAR(N+3) = XPL*FHAT(I) + YPL*GHAT(I)
        PAR(N+4) = XP*PARFQ(I) + YP*PARGQ(I)
        PAR(N+5) = XP*PARFP(I) + YP*PARGP(I)
  110 CONTINUE
      RETURN
      END
```

SAMPLE INPUT FOR PROGRAM PLANET

```
100=1 1 1 1
110=2443690.500000 14. 2443788.5
120=57909134.07000 .2001271542194 .04721092077279 .04524996816221
130=.04117767074064 2.587907280000
140=999
```

100= ID for planet, element option (equinoctal), time line interval on output, partial derivative option (compute partials)

110= Epoch time, (Julian day) Delta time, (days) end time (Julian days)

120= equinoctal elements a, h, k, and p

130= equinoctal elements q and $\lambda_o$

140= next ID element needed to terminate program

The input data items for program PLANET are in free-format.

SAMPLE OUTPUT FOR PROGRAM PLANET FROM PRINT STATEMENT

```
PLANET ID          =    1
ELEMENT OPTION     =    1
OUTPUT INTERVAL    =    1
TRAJECTORY OPT     =    1
EPOCH TIME         =    .2443690500000E+07
DELTA TIME         =    .1400000000000E+02
END TIME           =    .2443788500000E+07

SEMI-MAJOR AXIS         =    .5790913407000E+08
ECCENTRICITY            =    .2056204000000E+00
INCLINATION             =    .1222104100000E+00
ANG. OF ASCENDING NODE  =    .8324813400000E+00
ANG. OF PERIGEE         =    .5066459200000E+00
INITIAL MEAN ANOMOLY    =    .1248780020000E+01

SEMI-MAJOR AXIS         =    .5790913407000E+08
H    =    .2001271542194E+00
K    =    .4721092077279E-01
P    =    .4524996816221E-01
Q    =    .4117767074064E-01
LATITUDE  =    .2587907280000E+01
```

SAMPLE OUTPUT FOR FILE IOUT, PLANET MERCURY

```
1= 2443690.500           -.55688511360619E+08          .77232678787654E+07
          .56971845701018E+07
2= -.9616533255929E+00 -.7737235343969E+07 -.6262151893769E+08 -.2030009253
  437E+08  .6963487064908E+06  .1071819684898E+08
3=  .1333687336687E+00 -.1113041406191E+09 -.2570707954793E+08 -.5567665816
  912E+08 -.6330864966379E+07 -.4569143334274E+07
4=  .9838144986273E-01 -.8498080881027E+07  .3563466788752E+07 -.2758438391
  076E+07  .1538893251802E+08  .1109616753957E+09
5= 2443704.500           -.50091922068032E+08          -.43884329261104E+08
           .92266050160459E+06
6= -.1560402182744E+01  .3117557983358E+08 -.7576013329111E+08  .2684783149
  828E+08 -.3956718373087E+07  .5439070612617E+07
```

1= Time in Julian days, X (heliocentric ecliptic), Y (heliocentric ecliptic) Z (heliocentric ecliptic)

2= $\frac{\partial x}{\partial a}, \frac{\partial x}{\partial h}, \frac{\partial x}{\partial k}, \frac{\partial x}{\partial \lambda}, \frac{\partial x}{\partial q}, \frac{\partial x}{\partial p}$

3= $\frac{\partial y}{\partial a}, \frac{\partial y}{\partial h}, \frac{\partial y}{\partial k}, \frac{\partial y}{\partial \lambda}, \frac{\partial y}{\partial q}, \frac{\partial y}{\partial p}$

4= $\frac{\partial z}{\partial a}, \frac{\partial z}{\partial h}, \frac{\partial z}{\partial k}, \frac{\partial z}{\partial \lambda}, \frac{\partial z}{\partial q}, \frac{\partial z}{\partial p}$

5= Time in Julian days, X (heliocentric ecliptic), Y (heliocentric ecliptic), Z (heliocentric ecliptic)

6= $\frac{\partial x}{\partial a}, \frac{\partial x}{\partial h}, \frac{\partial x}{\partial k}, \frac{\partial x}{\partial \lambda}, \frac{\partial x}{\partial q}, \frac{\partial x}{\partial p}$

The output generated from the print statement in program PLANET has the following formats:

```
     Format (*1 Planet ID      = *,I5/* Element Option  = *,I5/
1            *Output interval = *,I5/* Trajectory OPT  = *,I5/
1            *Epoch Time      = *,E20.13/*Delta Time   = *,E20.13/
1            *End Time        = *,E20.13)

     Format (*0Semi-major Axis          =*,E20.13/
A            *Eccentricity             =*,E20.13/
1            *Inclination              =*,E20.13/
2            *Ang. of Ascending Node   =*,E20.13/
3            *Ang. of Perigee          =*,E20.13/
4            *Initial Mean Anomaly     =*,E20.13)

     Format (*0Semi-major Axis = *,E20.13/
1            *H     = *,E20.13/*K  =*,E20.13/*P   = *,E20.13/
A            *Q     = *,E20.13/
2            *Latitude = *,E20.13)
```

The output generated from the write statement to file IOUT has the following formats. Format (1F12.3, 3E30.14) for records one and five and Format (6E20.13) for records two through four and record six. Records one through four are repeated computing new trajectories and partials until ending time is met. An end of file is written to file IOUT terminating the program.

# APPENDIX G

## PROGRAM EXTRACT

```
      PROGRAM EXTRACT(INPUT,OUTPUT,TAPE1,TAPE2,TAPE3,
     &TAPE4,TAPE5,TAPE6,TAPE7)
      DIMENSION EOUT(3,7),EOUTT(2,7)
      EPOCH=2443690.5
      EPS=.4092061859
      M1=1
      M2=2
      M3=3
      M4=4
      M5=5
      M6=6
4     CONTINUE
      READ(1)DA,((EOUT(I,J),I=1,3),J=1,7)
      IF(EOF(1))5,2
2     CONTINUE
      IF(DA.LT.EPOCH)GO TO 4
      DO 3 IK=1,7
3     EOUTT(M2,IK)=EOUT(M2,IK)
      DO 10 N=1,7
      EOUT(M2,N)=COS(EPS)*EOUTT(M2,N)+SIN(EPS)*EOUT(M3,N)
      EOUT(M3,N)=COS(EPS)*EOUT(M3,N)-SIN(EPS)*EOUTT(M2,N)
10    CONTINUE
      WRITE(2,40)(DA,EOUT(LM,M1),LM=1,3)
      WRITE(3,40)(DA,EOUT(LM,M2),LM=1,3)
      WRITE(4,40)(DA,EOUT(LM,M3),LM=1,3)
      WRITE(5,40)(DA,EOUT(LM,M4),LM=1,3)
      WRITE(6,40)(DA,EOUT(LM,M5),LM=1,3)
      WRITE(7,40)(DA,EOUT(LM,M6),LM=1,3)
      GO TO 4
5     CONTINUE
      ENDFILE 2
40    FORMAT(1F12.3,3E30.14)
      STOP
      END
```

The input file for program EXTRACT is a binary file. This is sample output for Tape 1, planet "Mercury." These are JPL rotated coordinates.

```
1= 2443690.500          -.55688472186030E+08      .77241282459283E+07
       .56976677220777E+07
2= 2443704.500          -.50093107182808E+08     -.43883503264652E+08
       .92347938546062E+06
3= 2443718.500          -.11296696038796E+08     -.68692200060998E+08
      -.46517635005862E+07
4= 2443732.500           .33658622960916E+08     -.54995137170457E+08
      -.7603644726349E+07
5= 2443746.500           .53448434105633E+08     -.47048408955032E+07
      -.52445205866421E+07
6= 2443760.500      .13070481298358E+08           .44048817675283E+08
       .24536453695890E+07
```

1= time (Julian days), X (heliocentric ecliptic), Y (heliocentric ecliptic), Z (heliocentric ecliptic)

2= time (Julian days), X (heliocentric ecliptic), Y (heliocentric ecliptic), Z (heliocentric ecliptic)

3= same

4= same

5= same

6= same

The output format for EXTRACT is Format (1F12.3, 3E30.14) for each record.

# APPENDIX H

## PROGRAM LESTSQ

```
      PROGRAM LESTSQ(INPUT,OUTPUT,TAPE6,TAPE7,TAPE8)
C
C     TO IMPROVE THE ACCURACY OF PRELIMINARY ORBIT
C     USING LEAST SQUARES NORMAL EQUATIONS
      DIMENSION XS(1000),YS(1000),ZS(1000),XSS(1000)
      DIMENSION YSS(1000),ZSS(1000)
      DIMENSION RESIDX(3),E(6),H(3,6),BS(6,6),ES(6)
      DIMENSION DELTEL(6),INDEX(5),TEMP(6),B(6,6)
      DATA INTPE/6/,INPUT/7/
      DATA BS/36*0./,ES/6*0./
      DO 100 I=1,1000
      READ(INTPE,250)DA,XS(I),YS(I),ZS(I)
      IF(EOF(INTPE))110,5
5     N=I
      READ(INTPE,240)((H(K,J),J=1,6),K=1,3)
      READ(INPUT,250)DA,XSS(I),YSS(I),ZSS(I)
      RESIDX(1)=XSS(I)-XS(I)
      RESIDX(2)=YSS(I)-YS(I)
      RESIDX(3)=ZSS(I)-ZS(I)
      DO 10 K1=1,6
      DO 10 IM=1,6
10    B(K1,IM)=0.
      DO 50 K2=1,6
      DO 40 IN=1,6
      DO 30 J1=1,3
30    B(K2,IN)=B(K2,IN)+H(J1,K2)*H(J1,IN)
40    BS(K2,IN)=BS(K2,IN)+B(K2,IN)
50    CONTINUE
      WRITE(8,150)
      WRITE(8,240)((B(I6,J6),J6=1,6),I6=1,6)
      WRITE(8,210)
      WRITE(8,240)((BS(K3,IS),IS=1,6),K3=1,6)
      DO 60 IP=1,6
60    E(IP)=0.
      DO 80 IL=1,6
      DO 70 JO=1,3
70    E(IL)=E(IL)+RESIDX(JO)*H(JO,IL)
80    ES(IL)=ES(IL)+E(IL)
      WRITE(8,160)
      WRITE(8,220)(E(M3),M3=1,6)
      WRITE(8,200)
      WRITE(8,220)(ES(IB),IB=1,6)
100   CONTINUE
110   CONTINUE
      CALL CROUT(0,6,0,BS,6,BB,0,D,INDEX,TEMP)
      WRITE(8,180)
      WRITE(8,240)((BS(LM,NO),NO=1,6),LM=1,6)
      WRITE(8,190)
      WRITE(8,220)D
      DO 120 IO=1,6
120   DELTEL(IO)=0.
      DO 130 M=1,6
      DO 130 N=1,6
130   DELTEL(M)=DELTEL(M)+BS(M,N)*ES(N)
      WRITE(8,170)
      WRITE(8,220)(DELTEL(M),M=1,6)
150   FORMAT(* B=HXH TRANSPOSE=*)
```

```
160   FORMAT(* E=H TRANSPOSE X RESID.=*)
170   FORMAT(* THE PARAMETER IMPROVEMENTS ARE*)
180   FORMAT(* B INVERSE IS*)
190   FORMAT(* THE DETERMINANT IS*)
200   FORMAT(* THE E SUM MATRIX ES=*)
210   FORMAT(* THE B SUM MATRIX BS=*)
220   FORMAT(E20.13)
230   FORMAT(3E20.13)
240   FORMAT(6E20.13)
250   FORMAT(1F12.3,3E30.14)
      STOP
      END
```

MPLE INPUT FOR FILE INTPE, PLANET MERCURY

This input was generated by Kepler planet program.

```
 2443690.500          -.556885113606l9E+08        .77232678787654E+07
          .56971845701018E+07
 -.9616533255929E+00 -.7737235343969E+07 -.6262151893769E+08 -.2030009253
437E+08  .6963487064908E+06   .1071819684898E+08
  .1333687336687E+00 -.1113041406191E+0981  -.2570707954793E+08 -.5567665816
912E+08 -.6330864966379E+07 -.4569143334274E+07
  .9838144986273E-01 -.8498080881027E+07  .3563466788752E+07 -.2758438391
076E+07   .1538893251802E+08 .1109616753957E+09
 2443704.500          -.50091922068032E+08       -.43884329261104E+08
          .92266050160459E+06
 -.1560402182744E+01  .3117557983358E+08 -.7576013329111E+08 .2684783149
828E+08 -.3956718373087E+07 .5439070612617E+07
```

: time (Julian days), X (heliocentric ecliptic), Y (heliocentric ecliptic), Z (heliocentric ecliptic)

: $\frac{\partial x}{\partial a}, \frac{\partial x}{\partial h}, \frac{\partial x}{\partial k}, \frac{\partial x}{\partial \lambda}, \frac{\partial x}{\partial q}, \frac{\partial x}{\partial p}$

: $\frac{\partial y}{\partial a}, \frac{\partial y}{\partial h}, \frac{\partial y}{\partial k}, \frac{\partial y}{\partial \lambda}, \frac{\partial y}{\partial q}, \frac{\partial y}{\partial p}$

: $\frac{\partial z}{\partial a}, \frac{\partial z}{\partial h}, \frac{\partial z}{\partial k}, \frac{\partial z}{\partial \lambda}, \frac{\partial z}{\partial q}, \frac{\partial z}{\partial p}$

: time (Julian days), X (heliocentric ecliptic), Y (heliocentric ecliptic), Z (heliocentric ecliptic)

ıe input format for file INTPE in program LESTSQ is Format (1F12.3, 3E30.14) for :cords one and five and Format (6E20.13) for records two through four and record ıx.

LE INPUT FOR FILE INPUT, PLANET MERCURY

This input was obtained from JPL data.

```
443690.500           -.55688472186030E+08            .77241282459283E+07
        .56976677220777E+07
443704.500           -.50093107182808E+08           -.43883503264652E+08
        .92347838546062E+06
443718.500           -.11296696038796E+08           -.68692200060998E+08
       -.46517635005862E+07
443732.500            .33658622960916E+08           -.54995137170457E+08
       -.76036447262349E+07
443746.500            .53448434105633E+08           -.47048408955032E+07
       -.52445205866421E+07
443760.500            .13070481298358E+08            .44048817675283E+08
        .24536453695890E+07
```

time (Julian Day), X (heliocentric ecliptic), Y (heliocentric ecliptic),
Z (heliocentric ecliptic)

time (Julian Day), X (heliocentric ecliptic), Y (heliocentric ecliptic),
Z (heliocentric ecliptic)

time (Julian Day), X (heliocentric ecliptic), Y (heliocentric ecliptic),
Z (heliocentric ecliptic)

same

same

same

input format for file INPUT in program LESTSQ is Format (1F12.3, 3E30.14) for
h record.

PLE OUTPUT FOR PROGRAM LESTSQ

```
B=HXH TRANSPOSE=
 .9522432474214E+00 -.8240007705156E+67  .5714225032370E+08  .1182474693
33E+08 -.1713633537292E-06  .2801418304443E-05
-.8240007705156E+07  .1252069390838E+17  .3315539197539E+16  .6377550616
53E+16  .5684872774240E+15 -.5173259315036E+15
 .5714225032370E+08  .3315539197539E+16  .4595006868502E+16  .2692677305
99E+16  .1739795855295E+15  -.1583221906994E+15
 .1182474693233E+08  .6377550616053E+16  .2692677305999E+16  .3519593004
42E+16  .2958960392146E+15  -.2692667016374E+15
-.1713633537292E-06  .5684872774240E+15  .1739795855295E+15  .2958960392
46E+15  .2773839967877E+15  .1743971966724E+16
```

$E = H^{Tr} \cdot \hat{Z}$

```
100=  .1448119382968E+05
101=  .2751285514654E+12
102=  .2601902869096E+12
103= -.1730909741358E+12
104= -.2609827959312E+11
105=  .1312392486448E+11
106= THE E SUM MATRIX ES=
107=  .2318293753911E+05
108=  .5366959346823E+11
109=  .6037889790544E+12
110=    -.4007279949659E+12

300=    -.8297610391578E+10
301=     .4121374502542E+11
302= THE E SUM MATRIX ES=
303=  .2167388785937E+06
304=     .1006521905542E+13
305=  .3678583926613E+12
306=    -.1540834753985E+13
307= -.3845430331050E+12
308=  .4075121829653E+12
309=   B=HXH TRANSPOSE=
310=  .4028520717956E+03  .137572828796E+10 -.2033913069406E+10 -.1407023890
      994E+10 -.8430494574897E+08  .7671787271573E+08
```

The output format for the normal matrix and the sum of the normal matrices is Format (6E20.13) as in records two through six. The format for the right-hand side of the normal equations is Format (E20.13) as in records 100 through 105

$E = H^{Tr} \cdot Z$

```
1000=   -.5438037886500E+10
1001=    .2185069505625E+11
1002=   THE E SUM MATRIX ES=
1003=    .1071446905724E+07
1004=    .1972181680237E+13
1005=    .1028334456689E+12
1006=   -.2978376667171E+13
1007=   -.9268345093082E+12
1008=    .9422572290006E+12
1009=   B=HXH TRANSPOSE=
1010=    .4156773953167E+04  .5442250291893E+10 -.5665462231553E+10 -.4450849843
         865E+10 -.2759070951785E+09  .2510766743425E+09

15746=     THE PARAMETER IMPROVEMENTS ARE
15747=       .3466525731675E+01
15748=  .2016734654506E-04
15749= -.4620221768938E-04
15750=  .1167145388865E-04
15751=  .4544758284259E-05
15752= -.9749637635815E-05
15753:{EOR   }
```

15746= THE PARAMETER IMPROVEMENTS ARE
15747= δ improvement for "a"
15748= δ improvement for "h"
15749= δ improvement for "k"
15750= δ improvement for "λ"
15751= δ improvement for "q"
15752= δ improvement for "p"

The output format for the parameter improvements is Format (E20.13) as in records 15746 through 15752.

# APPENDIX I

## PROGRAM GEOEQ

```
      PROGRAM GEOEQ(INPUT,OUTPUT,TAPE6,TAPE9,TAPE10)
      EPS=.4092061859
4     CONTINUE
      READ(6,1)DA,X,Y,Z
      IF(EOF(6))40,10
10    READ(9,1)DA,XS,YS,ZS
      XX=X-XS
      YY=Y-YS
      ZZ=Z-ZS
      YN=COS(EPS)*YY-SIN(EPS)*ZZ
      ZN=SIN(EPS)*YY+COS(EPS)*ZZ
      WRITE(10,1)DA,XX,YN,ZN
      GO TO 4
40    CONTINUE
      ENDFILE 10
1     FORMAT(1F12.3,3E30.14)
      STOP
      END
```

SAMPLE INPUT FOR PROGRAM GEOEQ

This is input for Tape 6 planet "Mercury." Tape 6 will change according to the planet. The planets used are: Mercury, Venus, Mars, Jupiter, and Saturn.

```
1= 2443690.500          -.55686115732592E+08        .77215772477416E+07
          .56958048221782E+07
2= 2443704.500          -.50087555986015E+08       -.43887541095688E+08
          .92061715596236E+06
3= 2443718.500          -.11288687930139E+08       -.68693434188216E+08
         -.46539081624056E+07
4= 2443732.500           .33664975344491E+08       -.54992892344885E+08
         -.76039388682113E+07
5= 2443746.500           .53450776728722E+08       -.47027915944105E+07
         -.52432159266678E+07
6= 2443760.500           .13071876612774E+08        .44048605506134E+08
          .24544754679073E+07
```

1= time (Julian day), X (heliocentric ecliptic), Y (heliocentric ecliptic), Z (heliocentric ecliptic)

2= time (Julian day), X (heliocentric ecliptic), Y (heliocentric ecliptic), Z (heliocentric ecliptic)

3= same

4= same

5= same

6= same

The input format for Tape 6, GEOEQ is Format (1F12.3, 3E30.14) for each record.

Input Tape 9 is reserved for planet Earth only.

SAMPLE INPUT FOR TAPE 9 PLANET EARTH

```
1= 2443690.500            .22288409290518E+08        -.15045085103602E+09
            .13284830829530E+05
2= 2443704.500            .56409568937832E+08        -.14121242573654E+09
            .12177197900465E+05
3= 2443718.500            .87428563942007E+08        -.12420994559592E+09
            .10400051418654E+05
4= 2443732.500            .11362338791314E+09        -.10035637247911E+09
            .80492806819268E+04
5= 2443746.500            .13351211082354E+09        -.70936103646182E+08
            .52520508739849E+04
6= 2443760.500            .14593133469083E+09        -.37550648582708E+08
            .21612948913257E+04
```

1= time (Julian days), X (heliocentric ecliptic), Y (heliocentric ecliptic), Z (heliocentric ecliptic)

2= time (Julian days), X (heliocentric ecliptic), Y (heliocentric ecliptic), Z (heliocentric ecliptic)

3-6= same

The input format for planet Earth's GEOEQ is Format (1F12.3, 3E30:14) for each record.

SAMPLE OUTPUT FOR TAPE 10

The following are the rotated Kepler coordinates for planet Mercury.

```
1= 2443690.500            -.77974525023110E+08        .14285226329953E+09
            .68147186096238E+08
2= 2443704.500            -.10649712492385E+09        .88927994700686E+08
            .39557176239299E+08
3= 2443718.500            -.98717251872146E+08        .52788739659046E+08
            .17809766287200E+08
```

4= 2443732.500 -.79958412568649E+08 .44646799926066E+08
.11065755727868E+08
5= 2443746.500 -.80061334094818E+08 .62853154790499E+08
.21537849807726E+08
6= 2443760.500 -.13285945807806E+09 .73886441590330E+08
.34716651073760E+08

1= time (Julian day), X (geocentric-equatorial), Y (geocentric-equatorial), Z (geocentric-equatorial)

2= time (Julian day), X (geocentric-equatorial), Y (geocentric-equatorial), Z (geocentric-equatorial)

3-6= same

The output format for GEOEQ is Format (1F12.3, 3E30.14) for each record.

# APPENDIX J

## PROGRAM DELALP

```
      PROGRAM DELALP(INPUT,OUTPUT,TAPE1,TAPE2,TAPE3)
      DIMENSION T(700),ALPHA(700),DELTA(700),EPOCH(700)
      DIMENSION TTT(700)
      REAL MNA,MND,MENA,MEND
      COMMON/PLOTD/XMIN,XMAX,YMIN,YMAX,ICHAR,IAXIS,PLOTM(11,51)
      ICT=0
C     READ(1,91)EPOCH1
C     READ(2,91)EPOCH2
91    FORMAT(1F12.3)
      DO 50 I=1,1000
      READ(1,4)TT,X1,Y1,Z1
      IF(EOF(1).NE.0.0)GO TO 60
      READ(2,4)TT1,X2,Y2,Z2
      IF(EOF(2).NE.0.0)GO TO 60
4     FORMAT(1F12.3,3E30.14)
      IF(ICT.NE.0)GO TO 51
      IF(TT.NE.TT1)41,51
41    PRINT 42,TT,TT1
42    FORMAT(* INITIAL TIMES ARE NOT EQUIVALENT. TT= *,E30.14,
     1*TT1= *,E30.14)
      GO TO 90
51    CONTINUE
      EPOCH(I)=TT
      T(I)=EPOCH(I)-EPOCH(1)
      IF(I.EQ.2)15,14
15    TDEL=EPOCH(2)-EPOCH(1)
14    CONTINUE
C     ALPH1,ALPH2 ARE RIGHT ASCENSIONS
C     DELTA1,DELTA2 ARE DECLINATIONS
C     ALPH1,ALPH2,DELTA1,DELTA2 ARE RETURNED IN ARCMINUTES
      PI=3.1415926535898
      ARCMIN=10800.00/PI
      ALPH1=ATAN(Y1/X1)
      CALL QUAD(ALPH1,X1,Y1)
      ALPH1=ALPH1*ARCMIN
      ALPH2=ATAN(Y2/X2)
      CALL QUAD(ALPH2,X2,Y2)
      ALPH2=ALPH2*ARCMIN
      RO1=X1**2+Y1**2
      RO1=SQRT(RO1)
      RO2=X2**2+Y2**2
      RO2=SQRT(RO2)
      DELTA1=ATAN(Z1/RO1)
      DELTA1=DELTA1*ARCMIN
      DELTA2=ATAN(Z2/RO2)
      DELTA2=DELTA2*ARCMIN
      PRINT 18,ALPH1,ALPH2,DELTA1,DELTA2
18    FORMAT(4E30.14)
C     ALPHA IS DIFFERENC  IN RIGHT ASCENSIONS AT TIME T
C     DELTA IS DIFFERENC  IN DECLINATIONS AT TIME T
      ALPHA(I)=ALPH1-ALPH2
      DELTA(I)=DELTA1-DELTA2
      ICT=ICT+1
      TTT(I)=TT1
50    CONTINUE
60    CONTINUE
```

```
      IF(ICT.EQ.0)10,11
10    PRINT 12
12    FORMAT(* E-O-F ENCOUNTERED AT BEGINNING OF INPUT. NO INFORMATION
     1 ON AT LEAST ONE OF THE INPUT TAPES.*)
      GO TO 90
11    PRINT 7,ICT
7     FORMAT("     ICT= ",1I3)
      PRINT 13
13    FORMAT(6X,*TIME(JULIAN DAYS)*,12X,*T(I)*,13X
     1,*ALPHA(ARCMINUTES)*,13X,*DELTA(ARCMINUTES)*)
C     T(I) IS COMPUTED FOR PLOTTING PURPOSES
      MCT=0
      MNA=0.0
      MND=0.0
      VARA=0.0
      VARD=0.0
      DO 40 I=1,ICT
      MNA=MNA+ALPHA(I)
      MND=MND+DELTA(I)
      MCT=MCT+1
      T(I)=T(I)/TDEL
      PRINT 5,TTT(I),T(I),ALPHA(I),DELTA(I)
      WRITE(3,5)TTT(I),T(I),ALPHA(I),DELTA(I)
5     FORMAT(1F12.3,3E30.14)
40    CONTINUE
      RMT=MCT
      MENA=MNA/RMT
      MEND=MND/RMT
      DO1 JJ=1,ICT
      VARA=VARA+(ALPHA(JJ)-MENA)**2
      VARD=VARD+(DELTA(JJ)-MEND)**2
1     CONTINUE
      SIGA=SQRT(VARA/RMT)
      SIGD=SQRT(VARD/RMT)
      PRINT 8,MENA,MEND,SIGA,SIGD
      AMINV=AMINE(T,ICT)
      AMAXV=AMAXE(T,ICT)
      XMIN=AMINV
      XMAX=AMAXV
      AMINV=AMINE(ALPHA,ICT)
      AMAXV=AMAXE(ALPHA,ICT)
      YMIN=AMINV
      YMAX=AMAXV
      PRINT 9,YMIN,YMAX
8     FORMAT(1H,4E30.14)
      AMINV=AMINE(DELTA,ICT)
      AMAXV=AMAXE(DELTA,ICT)
      YMIN=AMINV
      YMAX=AMAXV
      PRINT 9,YMIN,YMAX
9     FORMAT(1H,2E30.14)
90    CONTINUE
      STOP
      END
      SUBROUTINE QUAD(ALPHA,X,Y)
      PI=3.1415926535898
      IF(ALPHA.GT.0.0)1,2
1     IF(Y.GT.0.0) ALPHA=ALPHA
      IF(Y.LT.0.0) ALPHA=ALPHA+PI
      RETURN
2     IF(ALPHA.LT.0.0) 3,4
3     IF(Y.GT.0.0) ALPHA=ALPHA+PI
      IF(Y.LT.0.0) ALPHA=ALPHA+2.0*PI
      RETURN
4     IF(ALPHA.EQ.0.0) 5,6
5     IF(X.GT.0.0) ALPHA=0.0
      IF(X.LT.0.0) ALPHA=PI
6     CONTINUE
      RETURN
      END
```

## PROGRAM DELAPM

```
      PROGRAM DELAPM(INPUT,OUTPUT,TAPE1,TAPE2,TAPE3)
      DIMENSION T(700),ALPHA(700),DELTA(700),EPOCH(700)
      DIMENSION TTT(700)
      REAL MNA,MND,MENA,MEND
      COMMON/PLOTD/XMIN,XMAX,YMIN,YMAX,ICHAR,IAXIS,PLOTM(11,51)
      ICT=0
C     READ(1,91)EPOCH1
C     READ(2,91)EPOCH2
91    FORMAT(1F12.3)
      DO 50 I=1,1000
      READ(1,4)TT,X1,Y1,Z1
      IF(EOF(1).NE.0.0)GO TO 60
      READ(2,6)ALPH2,DELTA2
      IF(EOF(2).NE.0.0)GO TO 60
4     FORMAT(1F12.3,3E30.14)
6     FORMAT(2E30.14)
      EPOCH(I)=TT
      T(I)=EPOCH(I)-EPOCH(1)
      IF(I.EQ.2)15,14
15    TDEL=EPOCH(2)-EPOCH(1)
14    CONTINUE
C     ALPH1,ALPH2 ARE RIGHT ASCENSIONS
C     DELTA1,DELTA2 ARE DECLINATIONS
C     ALPH1,ALPH2,DELTA1,DELTA2 ARE RETURNED IN ARCMINUTES
      PI=3.1415926535898
      ARCMIN=10800.00/PI
      ALPH1=ATAN(Y1/X1)
      CALL QUAD(ALPH1,X1,Y1)
      ALPH1=ALPH1*ARCMIN
      RO1=X1**2+Y1**2
      RO1=SQRT(RO1)
      DELTA1=ATAN(Z1/RO1)
      DELTA1=DELTA1*ARCMIN
18    FORMAT(4E30.14)
C     ALPHA IS DIFFERENCE IN RIGHT ASCENSIONS AT TIME T
C     DELTA IS DIFFERENCE IN DECLINATIONS AT TIME T
      ALPHA(I)=ALPH1-ALPH2
      DELTA(I)=DELTA1-DELTA2
      ICT=ICT+1
      TTT(I)=TT
50    CONTINUE
60    CONTINUE
      IF(ICT.EQ.0)10,11
10    PRINT 12
12    FORMAT(* E-O-F ENCOUNTERED AT BEGINNING OF INPUT. NO INFORMATION
     1 ON AT LEAST ONE OF THE INPUT TAPES.*)
      GO TO 90
11    PRINT 7,ICT
7     FORMAT("     ICT= ",1I3)
C     T(I) IS COMPUTED FOR PLOTTING PURPOSES
      MCT=0
      MNA=0.0
      MND=0.0
      VARA=0.0
      VARD=0.0
      DO 40 I=1,ICT
      MNA=MNA+ALPHA(I)
      MND=MND+DELTA(I)
      MCT=MCT+1
      T(I)=T(I)/TDEL
      WRITE(3,5)TTT(I),T(I),ALPHA(I),DELTA(I)
5     FORMAT(1F12.3,3E30.14)
40    CONTINUE
      RMT=MCT
```

```
      MENA=MNA/RMT
      MEND=MND/RMT
      DO1 JJ=1,ICT
      VARA=VARA+(ALPHA(JJ)-MENA)**2
      VARD=VARD+(DELTA(JJ)-MEND)**2
1     CONTINUE
      SIGA=SQRT(VARA/RMT)
      SIGD=SQRT(VARD/RMT)
      PRINT 8,MENA,MEND,SIGA,SIGD
      AMINV=AMINE(T,ICT)
      AMAXV=AMAXE(T,ICT)
      XMIN=AMINV
      XMAX=AMAXV
      AMINV=AMINE(ALPHA,ICT)
      AMAXV=AMAXE(ALPHA,ICT)
      YMIN=AMINV
      YMAX=AMAXV
      PRINT 9,YMIN,YMAX
8     FORMAT(1H,4E30.14)
      AMINV=AMINE(DELTA,ICT)
      AMAXV=AMAXE(DELTA,ICT)
      YMIN=AMINV
      YMAX=AMAXV
      PRINT 9,YMIN,YMAX
9     FORMAT(1H,2E30.14)
90    CONTINUE
      STOP
      END
      SUBROUTINE QUAD(ALPHA,X,Y)
      PI=3.1415926535898
      IF(ALPHA.GT.0.0)1,2
1     IF(Y.GT.0.0) ALPHA=ALPHA
      IF(Y.LT.0.0) ALPHA=ALPHA+PI
      RETURN
2     IF(ALPHA.LT.0.0) 3,4
3     IF(Y.GT.0.0) ALPHA=ALPHA+PI
      IF(Y.LT.0.0) ALPHA=ALPHA+2.0*PI
      RETURN
4     IF(ALPHA.EQ.0.0) 5,6
5     IF(X.GT.0.0) ALPHA=0.0
      IF(X.LT.0.0) ALPHA=PI
6     CONTINUE
      RETURN
      END
```

SAMPLE INPUT FOR TAPE 1

These are JPL coordinates for planet "Mercury."

```
     Time

1= 2443690.500          -.77964575183693E+08      .14285139938627E+09
        .68152401023670E+08
2= 2443704.500          -.10649895546677E+09      .88922413933842E+08
        .39561718564281E+08
3= 2443718.500          -.98717650891779E+08      .52787733471752E+08
        .17814249153166E+08
4= 2443732.500          -.79964288575854E+08      .44635600380581E+08
        .11063858710573E+08
5= 2443746.500          -.80059614384767E+08      .62849725451473E+08
        .21535914238598E+08
6= 2443760.500          -.13286136735694E+09      .73876199041347E+08
        .34712225207949E+08
```

1= time (Julian days), X (geocentric-equatorial), Y (geocentric-equatorial),
Z (geocentric-equatorial)

2-6= same

SAMPLE INPUT FOR TAPE 2

These are Kepler generated coordinates for planet "Mercury."

```
     Time

1= 2443690.500          -.77974525023110E+08          .14285226329953E+09
        .68147186096238E+08
2= 2443704.500          -.10649712492385E+09          .88927994700686E+08
        .39557176239299E+08
3= 2443718.500          -.98717251872146E+08          .52788739659046E+08
        .17809766287200E+08
4= 2443732.500          -.79958412568649E+08          .44646799926066E+08
        .11065755727868E+08
5= 2443746.500          -.80061334094818E+08          .62853154790499E+08
        .21537849807726E+08
6= 2443760.500          -.13285956807806E+09          .73886441590330E+08
        .3471665107376E+08
```

1= time (Julian days), X (geocentric-equatorial), Y (geocentric-equatorial),
Z (geocentric-equatorial)

2-6= same

The input format for the original program DELALP is Format (1F12.3, 3E30.14) as in records one through six for both JPL and Kepler generated coordinates, Tape 1 and 2. The input formats for the revised program DELAPM are Format (1F12.3, 3E30.14)

›r the time and position histories generated by JPL, Tape 1 and Format (2E30.14)
›r the right ascension and declination angles generated by program select, Tape

## ΛMPLE OUTPUT FOR PROGRAM DELALP

Statistical output generated by the print statement for planet "Mercury."

```
:T= 562
        .47901140479242E-03      .91930569345719E-03      .17183945
l7541E+00          .71253067327114E-01
           -.71675558801508E+00          .73359012082801E+00
           -.25254465324997E+00          .23343114920863E+00
```

ICT = 562 → number of position histories
mean - right ascension, mean declination, standard deviation -
ight ascension, standard deviation - declination
minimum value - right ascension, maximum value - right ascension
minimum value - declination, maximum value - declination

Sample output for Tape 3 which reflects the changes in angles.

```
= 2443690.500          0.                              -.17574213532498E+00
           .13530473608989E+00
= 2443704.500          .10000000000000E+01             .13521342672175E+00
           .11827692166844E+00
= 2443718.500          .20000000000000E+01             .33026276330929E-01
           .13485020775261E+00
= 2443732.500          .30000000000000E+01              .47460765269352E+00
          -.68717342963282E-01
= 2443746.500          .40000000000000E+01              .55239058216102E-01
          -.38834355153085E-01
= 2443760.500          .50000000000000E+01              .22341148892883E+00
          -.78883304417104E-01
```

1= time (Julian days), counter, Δ right ascension
Δ declination

2= time (Julian days), counter, Δ right ascension
Δ declination

3-6= same

he output formats for the printed statistics are Format ("ICT=", 1I3) for the umber of position histories in the first line, Format (1H, 4E30.14) for mean nd standard deviation in lines 2 and 3 and Format (1H, 2E30.14) for minimum nd maximum values in lines 4 and 5. The output format for the tape is Format 1F12.3, 3E30.14). This format is repeated for each record.

# APPENDIX K

## PROGRAM SELECT

```
      PROGRAM SELECT(INPUT,OUTPUT,TAPE1,TAPE2,TAPE3,TAPE4,TAPE5,
     &TAPE6,TAPE11)
C
C         THESE FACTORS CONVERT THE GIVEN VALUES FROM
C              FACT    - ARC SECONDS TO RADIANS
C              FACT1   - TIME SECONDS TO RADIANS
C              FACT2      - DEGREES TO RADIANS
C              FACT3      - TIME HOURS TO RADIANS
      REAL IN,LN
      DIMENSION RA(8),DEC(8),DXP(8),DYP(8),DZP(8),RMAG(8)
      DIMENSION AD(6),EC(6),IN(6),LN(6),OMEGA(6),AMO(6),AM1(6),AM(6)
      DATA FACT/.48481368110953E-05/,FACT1/.72722052166429E-04/
      DATA FACT2/.17453292519943E-01/,FACT3/.26179938779914/
      DATA AD/1.000000003,.3870986732,.7233322265,1.52369076,5.203070156
     A ,9.538509603/
      DATA EC/.01671532949782,.2056294264117,.006775929468209,
     B .09339472415910,.04825983079881,.05315110813689/
      DATA IN/.0000898955198689O,.1222021384653,.05923236598127,
     C .03223176504070,.02280397821074,.04342907360759/
      DATA OMEGA/1.783756851,1.339368463,2.28392598,5.852403589,
     D .2583043378,1.599695040/
      DATA LN/3.049977190305,.8323191351502,1.328556712384,
     E .8562002543165,1.746015566160,1.974220936589/
      DATA AMO/3.073477677683,1.248550488749,1.160433857370,
     F 3.899288800052,1.505867745775,.9566213166208/
      DATA AM1/.0172021247,.0714247869,.027962452,.0091461066,
     G .001450104,.0005840133/
      DATA TEPOCH/2443690.5/,DELT/14.0/
      DATA TEND/2451558.5/
      DATA PI/3.1415926535898/
      DATA TU/58.13244087/
      TIME=TEPOCH
 16   EDAY=TIME-TEPOCH
      DO 30 K=1,6
 30   AM(K)=AMO(K)+AM1(K)*EDAY
      CONTINUE
      DO 40 KZ=1,6
      CALL ELCORD(KZ,AD(KZ),EC(KZ),IN(KZ),AM(KZ),OMEGA(KZ),LN(KZ),X,Y,Z)
      IF(KZ.EQ.1) GO TO 20
      DXP(KZ)=DX=X-XE
      DYP(KZ)=DY=Y-YE
      DZP(KZ)=DZ=Z-ZE
      ARCMIN=10800.00/PI
                                RMAG(KZ)=SQRT(DX*DX+DY*DY+DZ*DZ)
      RA(KZ)=ARTNQ(DY,DX)*ARCMIN
      DEC(KZ)=ASIN(DZ/RMAG(KZ))*ARCMIN
      WRITE(KZ,7202)RA(KZ),DEC(KZ)
      RA(KZ)=ARTNQ(DY,DX)/FACT3
      DEC(KZ)=ASIN(DZ/RMAG(KZ))/FACT2
      GO TO 40
   20 DXP(1)=XE=X
      DYP(1)=YE=Y
      DZP(1)=ZE=Z
      DXP(8)=-XE
      DYP(8)=-YE
      DZP(8)=-ZE
                               RMAG(8)=SQRT(XE*XE+YE*YE+ZE*ZE)
      RA(8)=ARTNQ(-YE,-XE)/FACT3
      DEC(8)=ASIN(-ZE/RMAG(8))/FACT2
```

```
   40 CONTINUE
      A=(350.737486+12.1907491914*DAYS)*FACT2
      B=(296.104608+13.0649924465*DAYS)*FACT2
      C=(358.475833+0.9856002669*DAYS)*FACT2
      D=(11.250889+13.229350449*DAYS)*FACT2
      TA=2.*A
      CB=COS(B) $ CB2A=COS(B-TA) $ C2A=COS(TA)
      SB=SIN(B) $ SB2A=SIN(B-TA) $ S2A=SIN(TA) $S2B=SIN(2.*B)
      SC=SIN(C) $ SD=SIN(D) $ SBPD=SIN(B+D) $ SD2A=SIN(D-TA)
      SDB=SIN(D-B)
      RL  =RMLM+(22639.5*SB-4586.426*SB2A+2369.902*S2A+769.016*S2B-668.1
     111*SC)*FACT
      RP =(18461.48*SD+1010.18*SBPD-999.695*SDB-623.658*SD2A)*FACT
      PARM=(3422.7+186.5398*CB+34.3117*CB2A+28.2333*C2A)*FACT
      CRL=COS(RL) $ SRL=SIN(RL)
      CRP=COS(RP) $ SRP=SIN(RP) $ SPARM=SIN(PARM)
      A1=CRP*SRL*CEPS-SRP*SEPS $ A2=CRP*CRL
      ALPHAM=ARTNQ(A1,A2)
      SDELTM=CRP*SRL*SEPS+SRP*CEPS
      CDELTM=SQRT(1.-SDELTM*SDELTM)
      HRANG=TS-ALPHAM
      X=CDELTM*SIN(HRANG)
      Y=CDELTM*COS(HRANG)-CLA*SPARM
      Z=SDELTM-SLA*SPARM
      RHO=SQRT(X*X+Y*Y+Z*Z)
      SD1=Z/RHO $ CD1=SQRT(1.-SD1*SD1)
      E=ARTNQ(X,Y) $ TSE=TS-E
      RA(7)=TSE/FACT3 $ DEC(7)=ASIN(SD1)/FACT2
      IF(RA(7).LT.O.) RA(7)=RA(7)+24.
      IF (RA(7).GE.24.) RA(7)=RA(7)-24.
      DXP(7)=CD1*COS(TSE)
      DYP(7)=CD1*SIN(TSE)
      DZP(7)=SD1
      RMAG(7)=SQRT(DXP(7)*DXP(7)+DYP(7)*DYP(7)+DZP(7)*DZP(7))
      PRINT 50,SPARM,TS,E,X,Y,Z
      PRINT 50,TA,B,C,D,RL,RP
      PRINT 50,CRL,SRL,CRP,SRP,ALPHAM,A1,A2,TSE,SD1,CD1
   50 FORMAT(6E19.12)
 7202 FORMAT(2E30.14)
      Y=Y+CLA*SPARM $ Z=Z+SLA*SPARM
      PRINT 50,X,Y,Z
      TIME=TIME+DELT
      IF(TIME.LT.TEND)GO TO 16
      END
      SUBROUTINE ELCORD(KPL,A,E,RI,RL,RO,RH,X,Y,Z)
C
C         METHOD IN SMART USING LONGITUDE OF PERIHELION AND TRUE ANOMALY
C              INSTEAD OF ARGUMENT OF PERIHELION AS IN DR O[S PROGRAM
C
      DIMENSION RNEW(3),ROLD(3)
      DATA SQRQ/-1.720210850E-02/,TOL/1.745E-06/
      DATA EPS/.4092061859/
      EO=RL+E*SIN(RL)+.5*E*E*SIN(2.*RL)
   15 DELEO=(RL-EO+E*SIN(EO))/(1.-E*COS(EO))
      EO=EO+DELEO
      IF (ABS(DELEO).LT.TOL) GO TO 25
      GO TO 15
   25 TE=TAN(EO*.5)
      V=2.*ARTNQ(TE*SQRT(1.+E),SQRT(1.-E))
      R=A*(1.-E*COS(EO))
      SH=SIN(RH) $ CH=COS(RH) $ ANG=RO-RH+V $ SA=SIN(ANG) $ CA=COS(ANG)
      CI=COS(RI) $ A11=CH*CA-SH*SA*CI $ A21=SH*CA+CH*SA*CI
      ROLD(1)=R*A11 $ ROLD(2)=R*A21 $ ROLD(3)=R*SA*SIN(RI)
      RNEW(1)=ROLD(1)
      RNEW(2)=ROLD(2)*COS(EPS)-SIN(EPS)*ROLD(3)
      RNEW(3)=SIN(EPS)*ROLD(2)+COS(EPS)*ROLD(3)
      X=RNEW(1) $ Y=RNEW(2) $ Z=RNEW(3)
      RETURN
      END
```

This subroutine was extracted from a F/C program. In our exercise, not all ·iables referenced in this routine were utilized. Therefore several variables ' appear undefined.

The input items for program SELECT are contained in data statements within : program.

1PLE OUTPUT FOR TAPE 2 PLANET MERCURY

| | Right Ascension | Declination |
|---|---|---|
| = | .71176495109891E+04 | .13632289664085E+04 |
| = | .84082320444799E+04 | .95480166004741E+03 |
| = | .91118697934113E+04 | .54237762899285E+03 |
| = | .90493253525415E+04 | .41338946820392E+03 |
| = | .85119521362692E+04 | .71685273096590E+03 |
| = | .90552286856794E+04 | .77182633642678E+03 |
| = | .10445917609449E+05 | .26981732572514E+03 |
| = | .11797161582502E+05 | -.37956019556662E+03 |
| = | .13048641098172E+05 | -.94346874995080E+03 |
| = | .14272139430453E+05 | -.13475293043596E+04 |
| = | .15350209710689E+05 | -.15233571132286E+04 |

1= right ascension-angle (arc minutes), declination-angle (arc minutes)
11= right ascension-angle (arc minutes), declination-angle (arc minutes)
- after a change in time - - after a change in time -

e output format for program SELECT is Format (2E30.14). This format is repeated r each record.

# APPENDIX L

## PROGRAM PLOT

```
      PROGRAM PLOT(INPUT,OUTPUT,TAPE1,
      DIMENSION T(600),C(600),ALPHA(600)
      DIMENSION DELTA(700),DUM(16),TITLE(3),EXXLAB(2)
      DATA EXXLAB/11HARC MINUTES/
      DATA EXLABB/10HFORTNIGHTS/
      READ 15,TITLE
      READ *,KC
      READ *,L
      READ *,N
      DO 10 I=1,562,5
      J=(I+4)/5
      READ(1,5)T(J),C(J),ALPHA(J),DELTA(J)
      IF(EOF(1))20,8
 8    READ (1,5) DUM
      K=J
 10   CONTINUE
 20   CALL XRANGE(0.0,600.0)
      CALL YRANGE(-2.0,2.5)
      IF(N.EQ.1)30,40
 30   CALL PLOTT(C,ALPHA,K)
      GO TO 50
 40   CALL PLOTT(C,DELTA,K)
 50   CALL LABELL(30,L,KC,TITLE)
      CALL LABELL(20,5,-11,EXXLAB)
      CALL LABELL(2,31,10,EXLABB)
      CALL PAUSEE(ICH)
 5    FORMAT(1F12.3,3E30.14)
 15   FORMAT(3A10)
      END
```

SAMPLE INPUT FOR TAPE 1 PLANET MERCURY

This input represents the differences between Kepler and JPL right ascension and declination angles.

```
1= 2443690.500          0.                           -.17574213532498E+00
        .13530473608989E+00
2= 2443704.500          .10000000000000E+01           .13521342672175E+00
        .11827692166844E+00
3= 2443718.500          .20000000000000E+01           .33026276330929E-01
        .13485020775261E+00
4= 2443732.500          .30000000000000E+01           .47460765269352E+00
       -.68717342963282E-01
5= 2443746.500          .40000000000000E+01           .55239058216102E-01
       -.38834355153085E-01
6= 2443760.500          .50000000000000E+01            22341148892883E+00
       -.78883304417104E-01
```

1= time (Julian days), Counter t(I), Δ right ascension
Δ declination

2= time (Julian days), Counter t(I), Δ right ascension
Δ declination

3-6 = same

The input format for the tape is Format (1F12.3, 3E30.14). This format is repeated for each record.

The following is sample input for program PLOT to be inserted during the time of execution at the terminal.

| Read | Input |
|---|---|
| Title | Mercury - Right Ascension |
| KC | 25 |
| L | 24 |
| N | 1 |

Title is the title of the plot and is formatted Format (3A10). KC is the number of characters in the title. L is the column in which the title should start to be written. N is the control parameter that decides if right ascension or declination is to be plotted. If N equals one, right ascension is plotted. For any other number, declination is plotted. KC, L, and N are all free-formatted input items.

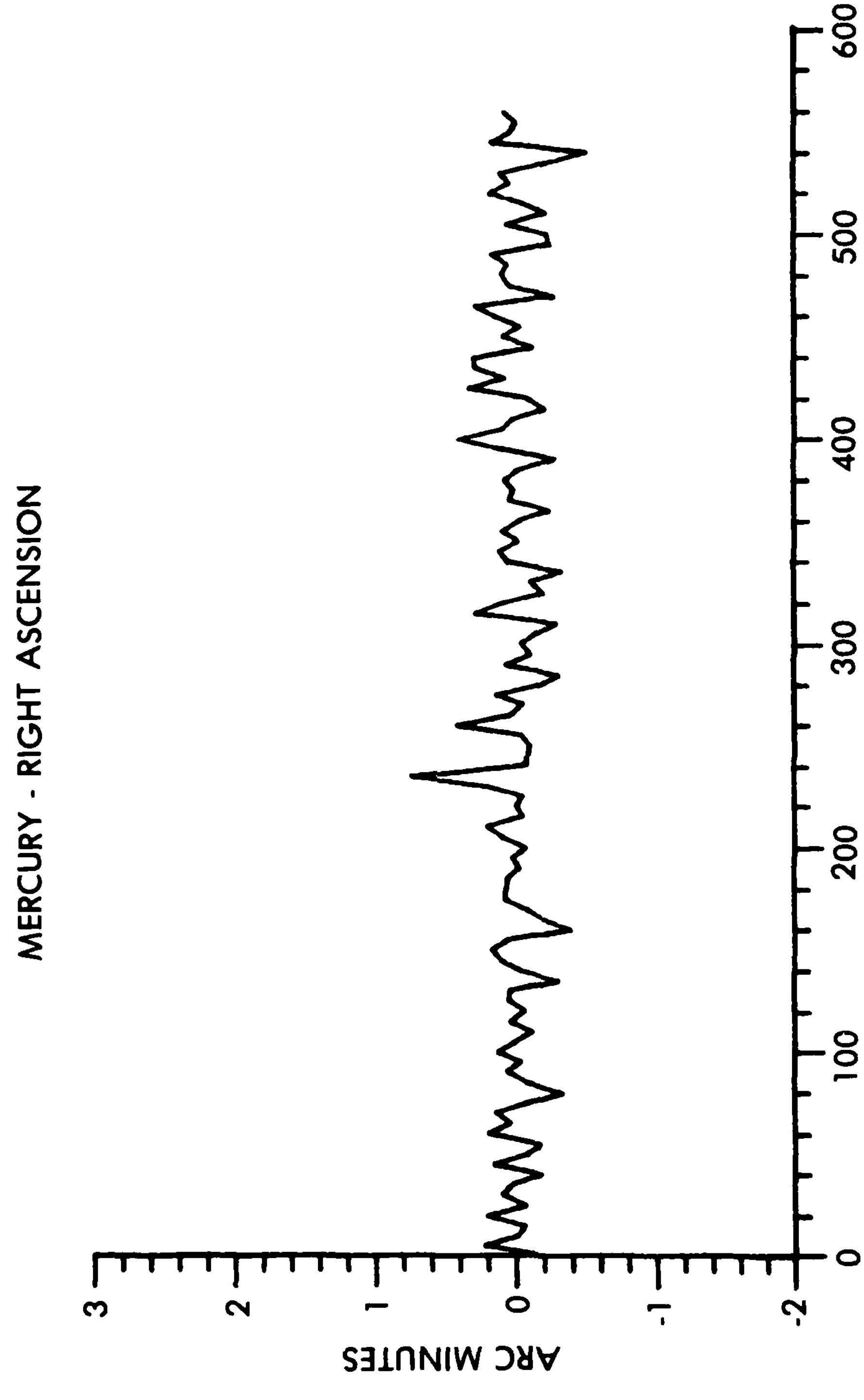


FIGURE L-1. SAMPLE OUTPUT FOR PROGRAM PLOT

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