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Research Summary and a Proposal for Continuing Research on  
**ROUTING ALGORITHMS AND STOCHASTIC ANALYSIS FOR  
LARGE COMMUNICATION NETWORKS**

for the period August 21, 1985 - August 20, 1986

submitted to

The Office of Naval Research



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## I. PROGRESS UNDER THE CURRENT ONR RESEARCH CONTRACT

In this section we briefly summarize our research progress so far under the ONR Contract N00014-82-K-0359 which began on August 21, 1982, with an emphasis on recent developments.

### A. *Significant New Breakthrough*

As part of our ONR sponsored research in the subtopic "Linear-Time Stochastic Algorithms", we began investigating an optimization technique known as "simulated annealing". We have succeeded in giving a necessary and sufficient condition for the annealing algorithm to converge [2].

Annealing is the process of slowly cooling a physical system in order to obtain states with *globally* minimum energy. By simulating such a process, near globally-minimum-cost solutions can be found for very large optimization problems. The resulting simulated annealing algorithm has been applied to image restoration, combinatorial optimization (e.g. the traveling salesman and VLSI layout problems), code design for communication systems, and large artificial intelligence learning problems.

The level of monte carlo randomization in the algorithm is determined by a control parameter  $T$ , called temperature, which tends to zero according to a deterministic "cooling schedule". Our primary accomplishment so far has been to give a simple necessary and sufficient condition on the cooling schedule for the algorithm state to converge in probability to the set of globally minimum cost states [2]. In the special case that the cooling schedule has parametric form  $T(t) = c/\log(1+t)$ , the condition for convergence is that  $c$  be greater than or equal to the depth, suitably defined, of the deepest local minimum which is not a global minimum state.

### B. *Highlights of Research on Routing Algorithms*

New algorithms have been developed for open-loop computation of optimal state-dependent routing strategies for a fluid-approximation communication network model with a single destination [4,8,9,12]. One of the algorithms is an efficient combinatorial algorithm based on the solution of max-flow problems for networks the same size as the original network [4]. The other algorithms we have developed are based on a new scaling method and on nonlinear optimization techniques [9,12]. These

algorithms are much more efficient than previously known algorithms.

Our ONR sponsored research on the optimal dynamic routing problem is unique in the following respects:

- (1) Strategies are being produced which are computationally feasible for large networks.
- (2) Nonlinear optimization techniques are effectively introduced to handle linear optimization problems.
- (3) Both stochastic [3,6] and deterministic [4,8,9,12] flow models are being considered, with a resulting cross-fertilization of ideas.
- (4) Both combinatorial and nonlinear iterative optimization techniques are being applied, and we soon expect to see them fruitfully combined.

*C. Other Research Under the Contract*

A new theory of distributed resource allocation was proposed in [3]. The work addresses problems of route selection and scheduling in communication networks. In a large distributed communication network, the decisions that individual stations can make should sometimes be purposely limited a priori in order to facilitate the coordination of such decisions. Such limitations might be placed at one layer of protocol by mechanisms operating at a higher protocol level.

For example, the set of possible routes between each pair of stations might be restricted to be small. The question that we aim to answer is how to predict how well the demand on the network can be balanced within the constraints placed on the individual stations. We address this question in [3] by investigating a specific model with random resource constraints. Somewhat sophisticated methods are used to give close lower and upper bounds on how well global balance can be achieved under varying levels of local constraints.

A new elegant formulation of network flow problems is given in [7]. The work provides a solid theoretical foundation for network flow problems in which the set of nodes is a continuum. Such models arise, for example, by considering very large finite-node networks, or by considering continuous-time dynamic network flow problems. Network flows are given by finitely-additive measures and capacity constraints are given by subadditive set functions. A general decomposition result is

given for decomposing rather general min-cost maximum flow problems into separately solvable sub-problems of the same type. For finite networks with piecewise-constant time-varying capacity and buffer constraints, the theory yields an algorithm for optimal routing of traffic.

The structure of optimal dynamic controls was investigated in [3] for a detailed two-station stochastic model. The model consolidates many models used by previous authors. It was shown in [3] that optimal stochastic routing policies have a switch structure. A new method, based on stochastic coupling and policy iteration, was introduced to analyze control of a single station.

Completely new combinatorial arguments are given in [6] which establish the optimality of a simple strategy for deterministic splitting of packets in a communication network. The work promises to have applications to a variety of allocation problems such as the random access problem [6,11].

A new fundamental technique for comparing diffusions is given in [10]. Unlike classical comparison methods, this technique allows diffusions with nonconstant, nonidentical diffusion coefficients to be compared. The technique has many potential applications to stability and convergence analysis for adaptive control strategies, where diffusions often arise as solutions to nonlinear stochastic differential equations.

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**A. PUBLICATIONS OF WORK SPONSORED BY ONR CONTRACT** (For the period August 20, 1983-April 20, 1985, and works submitted as of April 20, 1985)

*Books or Chapters in Books:*

- [1] E. Wong and B. Hajek, *Stochastic Processes in Engineering Systems*, Springer-Verlag, New York, (1985). (ARO/JSEP/NSF/ONR)

*Journal Articles:*

- [2] B. Hajek, "Cooling schedules for optimal annealing," *Mathematics of Operations Research*, submitted March 1985. (ONR/NSF)
- [3] B. Hajek, "Optimal control of two interacting service stations," *IEEE Trans. on Automatic Control*, vol. 29, pp. 491-499, 1984. (ONR)
- [4] B. Hajek and R. Ogier, "Optimal dynamic routing in communication networks with continuous traffic," *Networks*, vol. 14, pp. 457-487, 1984. (ONR/NRL)
- [5] B. Hajek, "Mean stochastic comparison of diffusions," *Z. Wahrscheinlichkeitstheorie vrw. Geb.*, vol. 68, pp. 315-329, 1985. (ONR/ARO)
- [6] B. Hajek, "Extremal splittings of point processes," to appear in *Mathematics of Operations Research*, 1985. (ONR)

*Conference Publications:*

- [7] R. L. Cruz and B. Hajek, "Global load balancing by local adjustments," to appear in *Proceedings of the 1985 Conference on Information Sciences and Systems*, The Johns Hopkins University, Baltimore, MD, March 27-29, 1985. (ONR/NSF)
- [8] R.G. Ogier, "Optimal flows in generalized networks with submodular capacities," to appear in *Proceedings of the 1985 Conference on Information Sciences and Systems*, The Johns Hopkins University, Baltimore, MD, March 27-29, 1985. (ONR)
- [9] G.H. Sasaki and B. Hajek, "Optimal dynamic routing by iterative methods," to appear in *Proceedings of the 1985 Conference on Information Sciences and Systems*, The Johns Hopkins University, Baltimore, MD, March 27-29, 1985. (ONR)
- [10] B. Hajek, "Mean stochastic comparison of diffusions," *Proceedings of the IEEE Control and Decision Conference*, Las Vegas, NV, December 12-14, 1984, pp. 1490-1491. (ONR/ARO)

- [11] B. Hajek, "Extremal splittings of point processes - applications to service allocation problems," *ORSA-TIMS Bulletin*, vol. 18, pp. 152-153, 1984. (ONR)

*Technical Reports*

- [12] G.H. Sasaki, "Optimal dynamic routing by iterative methods," Report T-143, Coordinated Science Laboratory, University of Illinois at Urbana-Champaign, Urbana, IL, July 1984. (ONR)

**B. OTHER PUBLICATIONS**

*Journal Articles*

B. Hajek, "Stochastic approximation methods for decentralized control of multiaccess communications," to appear in *IEEE Transactions on Information Theory*, (Special Issue on Multi-Access Communications), vol. IT-31, March 1985. (JSEP)

B. Hajek, "Review of *Semimartingales: A Course on Stochastic Processes* by M. Metivier," *Bulletin of the American Mathematical Society*, vol. 11, pp. 198-203, July 1984.

G. Sasaki and B. Hajek, "Link scheduling in polynomial time," *IEEE Trans. on Info. Theory*, submitted February 1985. (JSEP)

B. Hajek and T. Berger, "A decomposition theorem for binary Markov random fields," *Annals of Probability*, to be submitted, April 1985. (NSF)

*Technical Reports*

J. Rossi, "Clustering algorithms for hierarchical routing in networks," Report R-1025, Coordinated Science Laboratory, University of Illinois at Urbana-Champaign, December 1984. Submitted to MILCOM '85. (JSEP)

W. F. Brady, "Correlation in coupled queues and simulation of a stochastic approximation procedure for multi-access communications," Report R-1103, Coordinated Science Laboratory, University of Illinois at Urbana-Champaign, Urbana, IL, March 1985. (JSEP)

Bruce Hajek, born in 1955, is currently an Associate Professor in the Department of Electrical and Computer Engineering and in the Coordinated Science Laboratory at the University of Illinois at Champaign-Urbana where he has been since he completed his graduate work in Electrical Engineering at Berkeley in August 1979. Dr. Hajek is currently the Associate Editor for Networks for the IEEE Transactions on Information Theory.

Dr. Hajek's research interests include multiple-user communication theory (e.g. random access, network routing problems and mobile radio networks), information theory, random fields and other stochastic processes, stochastic control and combinatorial optimization.

*Awards and Honors*

USA Math Olympiad Winner 1973, NSF Graduate Fellow '76 - '79

Xerox sponsored award for best research by an assistant professor in the UTUC College of Engineering during the year 1980-1981

Eckman Award of the American Automatic Control Council for Outstanding researcher in control under the age of 30 at the time of the award, 1982

Beckman Associate in the UTUC Center for Advanced Study, 1984-85 academic year

NSF Presidential Young Investigator Award, 1984

## II. PROPOSED RESEARCH

We propose to continue the research which was proposed in detail in the two year proposal with the same title as this one, which covered proposed research for the period August 21, 1984 - August 20, 1986. The main topics are:

- A. Dynamic routing algorithms
- B. Correlation inequalities for large random networks
- C. Linear-time stochastic algorithms
- D. Statistical analysis of a sample-path perturbation technique

## Recent Activities of Hajek

### Organized sessions at conferences:

"Distributed control in communication systems," for the IEEE Control and Decision Conference, December 1984.

"Networking in spread spectrum systems," for the ARO sponsored Workshop on Recent Trends in Spread Spectrum, August 1985.

"The foundations of simulated annealing," for the IEEE Control and Decision Conference, December 1985.

Conference Co-Chairman for the Twenty-Third Annual Allerton Conference, October 1985.

New Associate Editor for Networks for the *IEEE Trans. Information Theory*.

Elected to Board of Governors, 1985-1987, IEEE Information Theory Group.

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