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MRC-R-900

FINAL REPORT

EXPERIMENTAL AND ANALYTIC CHARACTERIZATION
OF NONLINEAR SEISMIC ATTENUATIONG. D. McCartor
W. R. Wortman

ARPA ORDER: 4397
PROGRAM CODE: 4A10
CONTRACTOR: MISSION RESEARCH CORPORATION
EFFECTIVE CONTRACT DATE: 21 MARCH 1984
CONTRACT EXPIRATION DATE: 20 MARCH 1985
AMOUNT OF CONTRACT: \$98,969
CONTRACT NUMBER: F49620-84-C-0049
PRINCIPAL INVESTIGATOR: GARY MCCARTOR
(805) 963-8761

Sponsored by
Advanced Research Projects Agency (DoD)
ARPA Order No. 4397
Monitored by AFOSR Under Contract #F49620-84-C-0049

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SECURITY CLASSIFICATION OF THIS PAGE

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REPORT DOCUMENTATION PAGE

1a. REPORT SECURITY CLASSIFICATION Unclassified			1b. RESTRICTIVE MARKINGS	
2a. SECURITY CLASSIFICATION AUTHORITY			3. DISTRIBUTION / AVAILABILITY OF REPORT APPROVED FOR PUBLIC RELEASE DISTRIBUTION UNLIMITED	
2b. DECLASSIFICATION / DOWNGRADING SCHEDULE			5. MONITORING ORGANIZATION REPORT NUMBER(S)	
4. PERFORMING ORGANIZATION REPORT NUMBER(S) MRC-R-900				
6a. NAME OF PERFORMING ORGANIZATION Mission Research Corporation		6b. OFFICE SYMBOL (if applicable) SB		7a. NAME OF MONITORING ORGANIZATION Air Force Office of Scientific Research
6c. ADDRESS (City, State, and ZIP Code) 735 State Street, P.O. Drawer 719 Santa Barbara, California 93102			7b. ADDRESS (City, State, and ZIP Code) Bolling Air Force Base Washington, D.C. 20332	
8a. NAME OF FUNDING / SPONSORING ORGANIZATION Defense Advanced Research Projects Agency		8b. OFFICE SYMBOL (if applicable)		9. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER
8c. ADDRESS (City, State, and ZIP Code) 1400 Wilson Boulevard Arlington, Virginia 22209			10. SOURCE OF FUNDING NUMBERS	
			PROGRAM ELEMENT NO.	PROJECT NO.
			TASK NO.	WORK UNIT ACCESSION NO.
11. TITLE (Include Security Classification) Experimental and Analytic Characterization of Nonlinear Seismic Attenuation				
12. PERSONAL AUTHOR(S) G. D. McCartor, W. R. Wortman				
13a. TYPE OF REPORT Final		13b. TIME COVERED FROM 3/21/84 TO 3/20/85		14. DATE OF REPORT (Year, Month, Day) 1985 March 20
15. PAGE COUNT 84				
16. SUPPLEMENTARY NOTATION				
17. COSATI CODES			18. SUBJECT TERMS (Continue on reverse if necessary and identify by block number)	
FIELD	GROUP	SUB-GROUP		
			Seismic Propagation, Nonlinear Constitutive Relation, Nonlinear Attenuation, SALMON Data.	
19. ABSTRACT (Continue on reverse if necessary and identify by block number)				
→ In order to assess the existence and impact of mild nonlinear contributions to attenuation of seismic signals from underground explosions, data from the test SALMON have been studied. It's found that the moderate strain regime ($\epsilon \sim 10^{-3}$) data are internally consistent with an attenuation function, Q, which is independent of amplitude over more than an order of magnitude. However the attenuation is much greater than that found from small strain measurements in different experiments. Nonlinear constitutive relations are considered which allow an analytic solution for the expected transition to linear small strain behavior. → and keywords include:				
20. DISTRIBUTION / AVAILABILITY OF ABSTRACT ☒ UNCLASSIFIED/UNLIMITED ☐ SAME AS RPT. ☐ DTIC USERS			21. ABSTRACT SECURITY CLASSIFICATION Unclassified	
22a. NAME OF RESPONSIBLE INDIVIDUAL			22b. TELEPHONE (Include Area Code) 22c. OFFICE SYMBOL	

ACKNOWLEDGMENT

During this work useful discussions were held with many colleagues. We wish to thank Steve Day and Jack Murphy of S-CUBED, Jack Trulio and Neil Perl of Applied Theory, Bernie Tittmann and Jim Bulau of Rockwell International, Bob Blandford of DARPA, and Gary Lundquist of Sierra Geophysics.

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Unannounced	☐	
Justification		
By _____		
Distribution/		
Availability Codes		
Dist	Avail and/or Special	
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SUMMARY

Teleseismic estimations of the yield of underground nuclear explosions are sometimes carried out by fitting the observed spectra and amplitudes of the teleseismic pulses to source functions taken from near-field data over a range of known events. The source functions can be parameterized according to yield, depth, and material type so that given some assumptions about depth, which may also be available from teleseismic data, and material, an estimate of yield can be achieved provided that properties of the propagation are known. It has usually been assumed that the attenuation of the propagating signal can be described by a Q function which implies that the mechanisms are linear in nature. The Q function is taken to be a function of location, especially depth, and it also may be a function of frequency. Establishment of the details of Q is the subject of considerable investigation which is largely based on teleseismic observations.

The assumption that propagation beyond the range of a few hundred meters per kiloton to the $1/3$ power is in fact linear, is the subject of some controversy. It is recognized from various experiments on attenuation of signals in earth solids that the apparent Q tends to decrease, giving more absorption, when the strain exceeds 10^{-6} . Since the strains at the radii where the linear source functions are taken to be valid are such that the strains are considerably greater than 10^{-6} , it is suggested that the mild nonlinear effects which may exist for a portion of the propagation path may significantly influence the yield estimates found, assuming a linear Q function description.

In this report we have studied the remarkably clean and complete data set taken from the free-field measurements following the nuclear event SALMON. This 5.3 kt event took place in a rather uniform salt dome and the data are the best available for the study of properties of attenuation in the moderate strain region. We have selected a set of six records from SALMON and attempted to establish whether or not they are consistent with the assumption of linear attenuation describable with a Q function. It appears that the data are internally consistent with a Q of about 10, which is independent of amplitude even though the amplitude changes over more than an order of magnitude over the records used and the strains are typically about 10^{-3} . However, it must be noted that other data from attenuation of oscillation of salt rods indicates that for small strains the Q is about 500. Thus, while the SALMON data are internally consistent with a constant Q , it is difficult to see how they can be made externally consistent with other data. Apparently for strains between 10^{-4} and 10^{-6} there must be a transition in behavior which is clearly indicative of nonlinear behavior. It may be possible to clarify this issue by studying the data from the COWBOY series of chemical explosions in salt since they cover a set of strains which include values down to about 10^{-5} .

Inclusion of nonlinear effects in attenuation due to physical mechanisms such as crack friction is generally accomplished by providing a description of the microscopic loss source and integrating it over a cycle to find an effective Q . However, this is not very satisfying since Q is inherently a linear concept. In anticipation of the need for a description of nonlinear attenuation, we have considered some constitutive relations which are of such a simple form that their effects can be studied analytically. The descriptions of nonlinear friction so obtained have been structured so as to enforce the conditions that the effective Q^{-1} increases in proportion to the strain and that the resulting waveform obeys the cube-root scaling which is observed in a wide variety of data

from explosions. It is found that a simple form can be found for a constitutive relation which is consistent with the experimental facts, including a tendency for the waveform to change shape only slowly during propagation over the range seen in the SALMON data. It is hoped that such a description will be useful in characterizing the expected transition to small strain behavior.

SECTION 1 INTRODUCTION

It is generally recognized that the near-field pulse from underground explosions exhibits a nonlinear behavior, at least out to a radius for which gross structural changes in the rock appears. Beyond this radius ($\approx 100 \text{ m/kt}^{1/3}$) there may be subtler nonlinear changes which influence propagation. For the purposes of establishing a seismic source function it is common to establish a range beyond which the pulse is taken as linear, as indicated by experimental data indicating a constant reduced displacement potential. Typically this "elastic radius" is taken at ≈ 200 or $300 \text{ m/kt}^{1/3}$ and using a model such as that from Mueller and Murphy¹, relations between teleseismic observations and implied source characteristics are deduced. In carrying out such calculations it is assumed that the signal propagates linearly with some attenuation imposed in the form of Q function. If there is significant contribution from nonlinear behavior the resulting yield estimates may be in error. In particular, there is evidence² that the pulse from explosions may have significant nonlinear features well beyond $300 \text{ m/kt}^{1/3}$.

Of the large body of data available for the propagation of seismic signals in the intermediate strain regime corresponding to strains varying from about 10^{-3} to about 10^{-6} the data for salt provides the best information. If we combine data from the SALMON nuclear explosion³, the COWBOY series⁴ of field chemical explosions and Larson's data on laboratory chemical explosions⁵ we have available propagation data in the relevant strain regime for a wide range of yields, distances and characteristic frequencies; the range of scaled distances ($\equiv \text{distance} \times \text{yield}^{-1/3}$) is

a factor of about 300 while the range of characteristic frequencies is about four orders of magnitude. In addition to the data from explosions we may add the laboratory data on the absorption of the energy in small oscillations of halite rods taken by Tittmann⁶.

When viewed in the large looking at the general trends of the data certain characteristics appear. In this section we shall describe these characteristics and give the elements of the data which support them. In Section 2 we give the results of a detailed analysis of data from event SALMON--the event for which the data best permits a data set analysis of the propagation of the seismic wave. We found in distinct contradiction to our prior expectations that the propagation is very well described by linear propagation in an inelastic medium. This observation is the most important result in the report. In Section 3 we give a non-linear constitutive relation which provides for propagation with properties thought characteristic of the overall data. The detailed studies of the propagation of the SALMON shock show that some features which appear to be characteristic of the entire data set are not characteristic of subsets of it; in particular the constitutive relation we shall give is not appropriate for calculating the propagation of the SALMON shock; whether it is useful for calculating the propagation of waves as observed in other parts of the data set is a question which answer will require further examination of the data.

PROPERTIES OF THE DATA

These properties appear to be characteristic of the propagation of waves in the intermediate strain region:

- 1) The absorption of energy increases with amplitude; one relation which has been proposed to express this effect is:

$$Q^{-1} = Q_0^{-1} + \alpha \epsilon$$

where ϵ is the peak strain; the quantity Q will be discussed below.

- 2) The propagation satisfies cube-root scaling²; that is, the propagation is invariant under the scale transformations:

$$\begin{aligned} t &\rightarrow y^{1/3} t_s \\ x &\rightarrow y^{1/3} x_s \\ u &\rightarrow y^{1/3} u_s \end{aligned}$$

where t_s , x_s and u_s are respectively the scaled time, scaled distance and scaled displacement while y is the event yield.

- 3) The propagation, although presumably nonlinear (property 1), nevertheless provides a waveform which changes only slowly with distance.

Let us first consider the justification for item 1. As indicated in the next section, the SALMON shock, with measured strains in the range $4 \cdot 10^{-3} - 3 \cdot 10^{-4}$, propagated very closely like a wave in an anelastic solid with a Q of ~ 10 -20; laboratory measurements by Tittmann⁶ give a value of Q of ~ 500 for strains greater than about 10^{-6} with the apparent Q beginning to increase with amplitude when the strain becomes greater than 10^{-6} . Tittmann's data were taken at a frequency of the order of 500 Hz, which is only somewhat higher than the frequency at which the SALMON data appears to be very good; thus the factor of ~ 50 difference in Q between the SALMON data and Tittman's data does not appear to be a frequency effect; presumably it is an amplitude effect. This argument provides the strongest reason for believing in item 1.

In addition to the argument just presented various workers have attempted to formulate arguments in favor of item 1 by using peak velocity data. Minster and Archambeau⁷ proposed a relationship of the form given in item 1, pointing out that such a relationship is consistent with the general trend of the SALMON-COWBOY data and also with Tittmann's laboratory data. Later, the present authors⁸ reached the same conclusions based on the same considerations but concluded that Q^{-1} proportional to $\sqrt{\epsilon}$ or ϵ^2 would work about as well as the linear relation given in item 1. (See Appendix A, where this paper is reproduced.) The fact is such arguments are very hard to make in any convincing way. The problem is that the decrease of peak amplitude depends on a combination of absorbed energy and charged waveform and the freedom this mix allows in possible propagation laws causes the peak amplitude data to be only a weak constraint on the underlying physics. In spite of these difficulties and reservations we shall, for the moment, adopt the attitude that item 1 is a characteristic of the propagation and shall attempt to formulate a constitutive relation which leads to solutions consistent with it.

The major data supporting item 2 is presented in Figure 1. In the figure the peak particle velocity is plotted against scaled range ($xy^{-1/3}$) for a number of laboratory experiments, field experiments with chemical explosives and the nuclear explosion SALMON. It appears that we can write

$$v(yx) = \bar{v}(x, y^{-1/3}) = \bar{v}(x_s) \quad (1)$$

In this relation we have suppressed the time dependence; the fact that, in the strain regime we are concerned with the signals travel with the sound speed of the medium independent of the yield implies:

$$v(y, x, t) = \bar{v}(xy^{-1/3}, ty^{-1/3}) = \bar{v}(x_s, t_s) \quad (2)$$

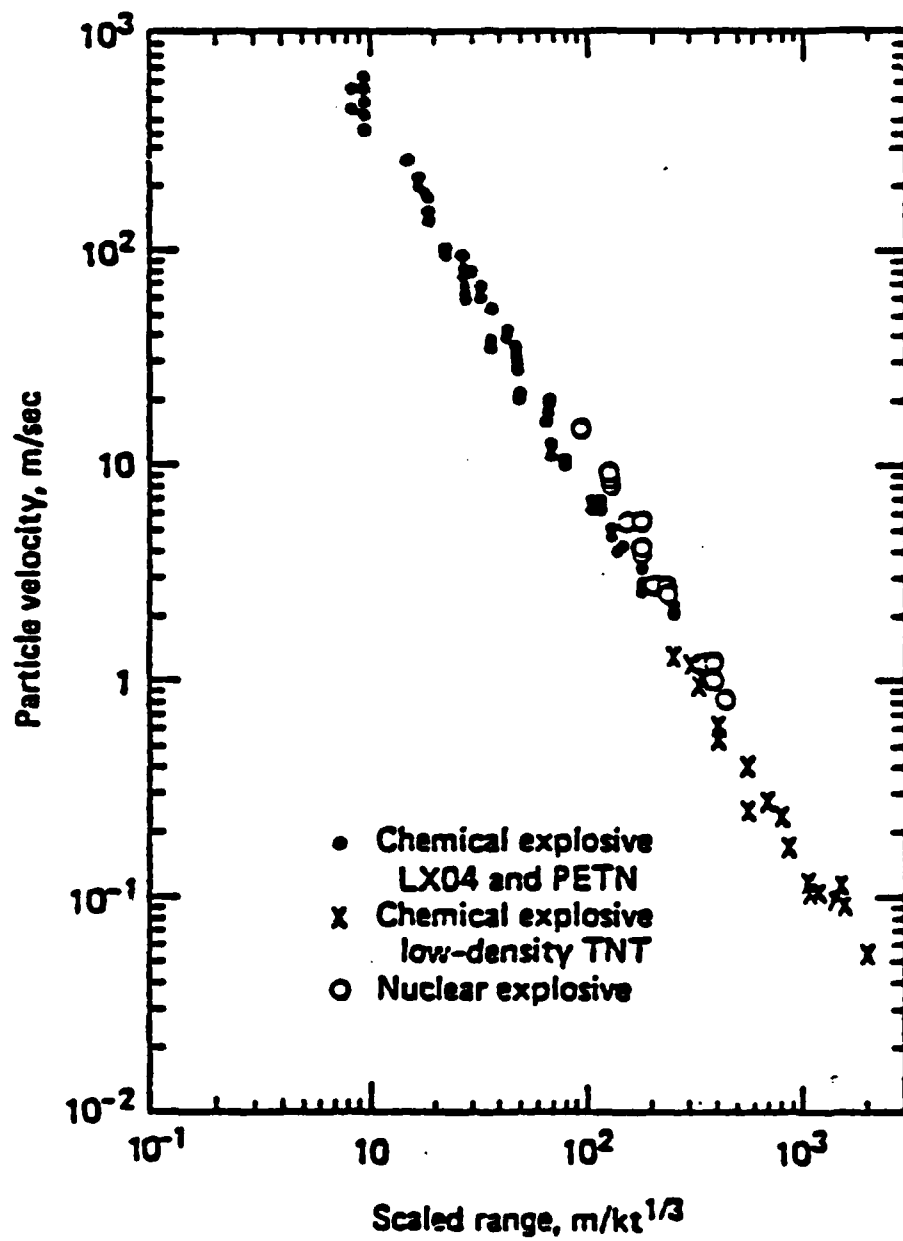


Figure 1. Peak particle velocities from explosions in salt (Reference 5).

Integrating equations (2) with respect to time we find

$$u(y,x,t) = y^{1/3} \int \nabla(x_s, t_s) dt_s = y^{1/3} u_s \quad (3)$$

where u is the particle displacement. Thus we obtain the scaling laws of item 2.

The argument just presented is again based on peak velocity data (Figure 1). In this case however we believe that the argument is much more persuasive than for item 1. The various experiments represented in Figure 1 corresponded to initial conditions which differed to at least some degree while the scaling rule appears to be very closely satisfied. It seems extremely likely that the scaling rules of item 2 are a property of the underlying dynamical equations rather than a combination of dynamics which does not satisfy scaling and particular initial conditions. We shall assume that our constitutive relations should lead to dynamical equations which satisfy scaling.

The scaling property of item 2 is often referred to as "frequency independence of Q "; let us understand why that is: If we Fourier transform the velocity we find:

$$\begin{aligned} \tilde{v}(y,x,t) &= \int e^{i\omega t} v(y,x,t) dt \\ &= y^{1/3} \int e^{i\omega_s t_s} \nabla(x_s, t_s) dt_s \\ &= y^{1/3} \tilde{v}(x_s, \omega_s) \end{aligned} \quad (4)$$

where

$$\omega_s \equiv y^{1/3} \omega$$

Let us now assume that the velocity propagates from one point to the next according to:

$$\bar{v}(y, x_2, \omega) = \bar{v}(y, x_1, \omega) e^{-\frac{\omega(x_2 - x_1)}{2Qc}} \quad (5)$$

which is:

$$\bar{v}(x_{s2}, \omega_s) = \bar{v}(x_{s1}, \omega_s) e^{-\frac{\omega_s(x_{s2} - x_{s1})}{2Qc}} \quad (6)$$

We assume that the product Qc is not a function of distance or yield; therefore, it cannot be a function of ω either, else, the R.H.S. would not be a function of scaled variables whereas the L.H.S. is. Observationally c is nearly independent of frequency; therefore Q must be also.

We thus see that for a linear theory scaling implies that Q is independent of frequency. For a nonlinear theory there are two problems with this statement. The first is that for general nonlinear propagation there is no obvious definition of Q . The second is that for special types of nonlinear propagation where there is an obvious definition of Q it may no longer be true that scaling implies that Q is frequency independent, as we shall see in an example given in Section 3. In thinking about nonlinear propagation we must remember that the data of Figure 1 implies scaling, not frequency independence of Q .

Item 3 is meant to be only a qualitative statement, the degree to which it is true can be judged by comparing Figures 2a and 2b which show the shock from the same event at two ranges for a representative case.

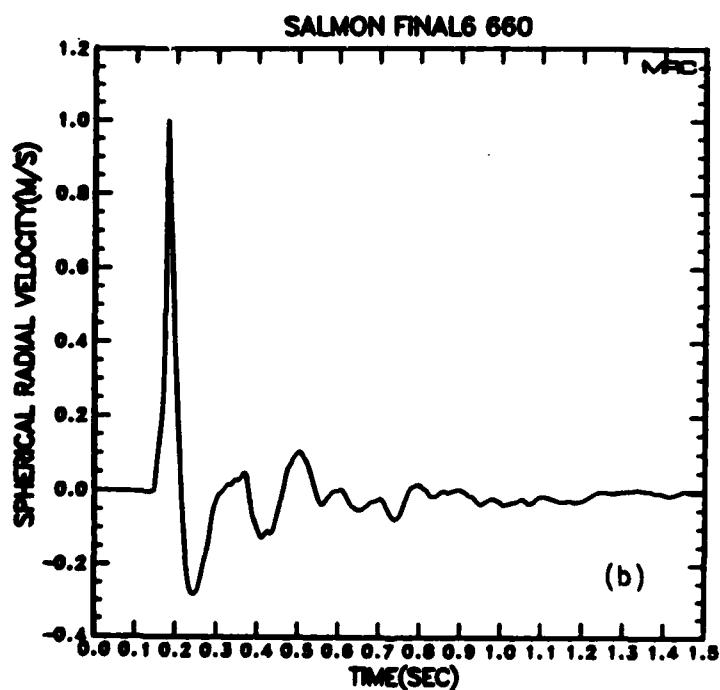
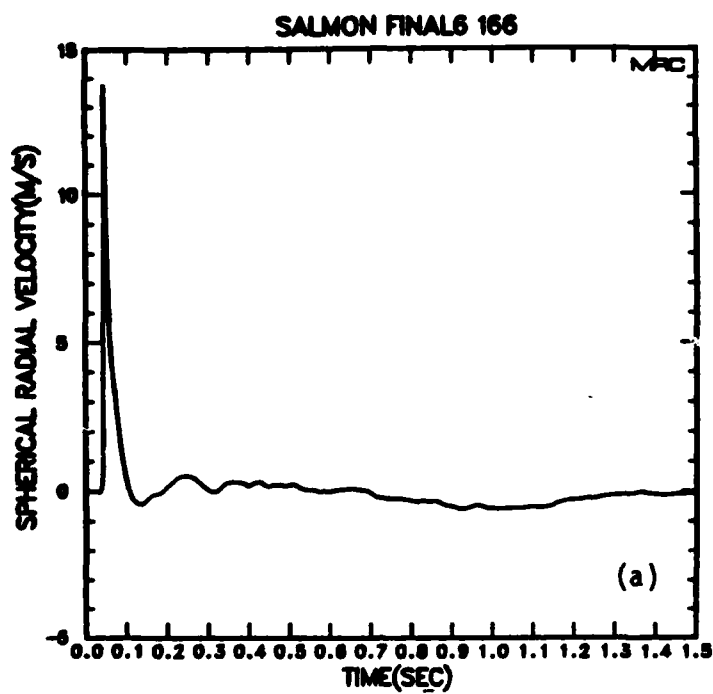


Figure 2. SALMON pulses at (a) 166 meters and (b) 660 meters from the explosion.

SECTION 2

SALMON DATA AND NONLINEAR BEHAVIOR

In this section we will examine some data from SALMON, which are among the cleanest available, in order to see if there is not definitive evidence for nonlinear features beyond the "elastic radius." This will be done by examination of a series of velocity records from subsurface points to see if they are consistent with strictly linear behavior since the range of linear effects is relatively limited. We shall attempt to determine if the records exhibit features which cannot be understood in terms of linear attenuation with an absorption, which is possibly a function of frequency. Since this is a form to which any linear mechanism can be reduced, it will serve to bound the range of linear behavior. Basically if the data are consistent with attenuation, which is independent of amplitude but possibly a function of frequency, it may allow the possibility that the propagation mechanism is linear without ruling out a nonlinear effect. However it can serve to rule out a wide range of possible nonlinear mechanisms. In fact this is exactly the outcome of this section.

NEAR-FIELD DATA

While there exists near-field data from shots in a variety of media⁹, by far the most consistent set are from explosions in salt^{2,3}. Salt is a relatively uniform medium for which a variety of experiments ranging from SALMON, through the COWBOY series⁴ to small laboratory explosions as reported by Larson⁵. These data provide the basis for many attitudes held concerning properties of near-field data so they will be reviewed at this time. The experiments of Larson for small chemical

explosions in pressed salt have provided pulses over a scaled range from $10 \text{ m/kt}^{1/3}$ to $200 \text{ m/kt}^{1/3}$. The range of frequencies covered was from about 10^4 to 10^6 Hz and the ratio of peak particle velocities to compressional sound speed (which is comparable with the strain) went from about 10^{-1} to less than 10^{-3} , which by most standards would suggest that the response was nonlinear. Yet, by performing a direct superposition experiment with a pair of simultaneous explosions, it was found that the resulting response was consistent with direct addition of the two pulses as would be expected from a linear medium. Still it is not clear just how nonlinear effects would be manifest in this experiment without knowing the character of any nonlinear behavior. That is, the apparent success of superposition for pulses with large strains may not directly negate the possibility of any sort of nonlinear behavior. The Cowboy series of chemical explosions had a range of yields from 10 to 2000 pounds of TNT, some of which were carried out in cavities for decoupling tests. The scaled ranges of these experiments were from $200 \text{ m/kt}^{1/3}$ to $3000 \text{ m/kt}^{1/3}$ and the corresponding peak strains were from a few times 10^{-4} to about 10^{-5} . The SALMON event, at 5.3 kt, took place in a natural salt dome and a comprehensive set of measurements were taken, both at surface and subsurface locations. These subsurface measurements included scaled ranges from $100 \text{ m/kt}^{1/3}$ to about $425 \text{ m/kt}^{1/3}$, which provided peak strains from about $4 \cdot 10^{-3}$ to about $3 \cdot 10^{-4}$ with a significant frequency content from about 1 Hz to more than 10^2 Hz. The SALMON data are well known for their internal consistency and their correspondence with the other salt data. Peak velocity data from the salt shots were shown in Figure 1 as a function of scaled range. The remarkable result is that the data from a huge range of yields tends to nearly fall on a straight line indicating a power law behavior with an exponent of about -1.9. This contrasts with a value of -1 which would be expected for elastic behavior. Furthermore, it is remarkable that the simple scaling with $\text{yield}^{1/3}$ seems to hold over the great range of strains. As Trulio² has noted, the nuclear event appears to scale slightly differently from chemical explosions but this is expected based on the different time scale of their source functions and can be

accounted for by simply altering yields by a multiplicative constant to convert to chemical yield. It should be noted that the range of yields over which simple scaling holds includes strains which are expected to give nonlinear behavior. However, it has not been clearly demonstrated that the deviation from r^{-1} behavior is necessarily a nonlinear effect. In particular, it has been observed that linear but inelastic (i.e., anelastic) behavior may provide such results. Trulio has noted that the SALMON data for decay of peak velocity with range are consistent with an effective Q of about 5. Here Q is found by considering amplitude decay with range, in addition to geometrical divergence, of the form

$$\exp(-\omega r/2cQ) \quad .$$

Note that simple scaling, in conjunction with linearity, indicates that Q must be independent of frequency and independent of amplitude. To some degree both of these suggestions are subject to experimental examination. The work of Larson on explosions in laboratory pressed salt indicate that while superposition appears to hold, values of Q of 12 and 25 with increasing range suggest that a constant Q cannot describe the results. Furthermore, laboratory experiments by Tittmann on decay of cyclic motion induced in salt samples indicate that for strains below 10^{-6} a value of Q of ~ 500 is appropriate but that for larger strains the value of Q (more properly, the effective Q) decreases indicating a nonlinear attenuation. While these various experiments took place in different frequency regimes, there is no known frequency dependence which can tie them together under a linear theory. The SALMON data provide a remarkably consistent set which may allow for drawing clean conclusions concerning the character of near-field propagation without the necessity for introduction of any assumptions about the nature of scaling among unrelated events. Perret³ has presented these data in comprehensive form and provided fits without regard to any particular theoretical assumptions. These data were further used by Trulio in his analyses of the degree to which they indicate scaling. In this report we will consider the extent to which the SALMON data

can shed light on the issue of linear versus nonlinear behavior by exploring the consequences of the assumption of linear behavior.

SALMON DATA REDUCTION

Perret has provided plots of the SALMON near-field pulses for accelerations as well as velocities and displacements. The original data, as digitized at Sandia, are available and they were kindly supplied to us by J. Trulio and N. Perl of Applied Theory, Inc. The use of the original data is preferred to tracing Perret's curves since Perret has made certain assumptions in correcting the data to force zero acceleration and velocity at late time. In his report, Perret did not indicate the sorts of corrections which are required and inasmuch as the character of the corrections may well influence the outcome of spectral methods we felt it best to return to the original data in order to have control over all operations on the data. The general nature of the nonphysical character of the raw data is indicated by Figure 3 which shows a once integrated acceleration record to give velocity.

It will be noted that two problems are evident. First, the velocity not only does not go to a constant but it increases at a constant rate. This is as result of a post-shot non-zero baseline for the acceleration instrument. Evidently the pulse from the explosion disturbed the instrument zero. This can be corrected for by altering the baseline at late times. There is a problem beyond this because even when the late time acceleration is taken as zero, the late time velocity, while constant, is not zero. This is generally thought to be a result of clipping of the acceleration peak due to inadequate instrument response or to inadequate bandwidth in recording. It is impossible to correct the difficulties in a unique manner. However, it appears to be possible to do so in such way that the effect on conclusions drawn from the result will not be important. We have chosen to correct the data by the following procedure as illustrated in Figure 4.

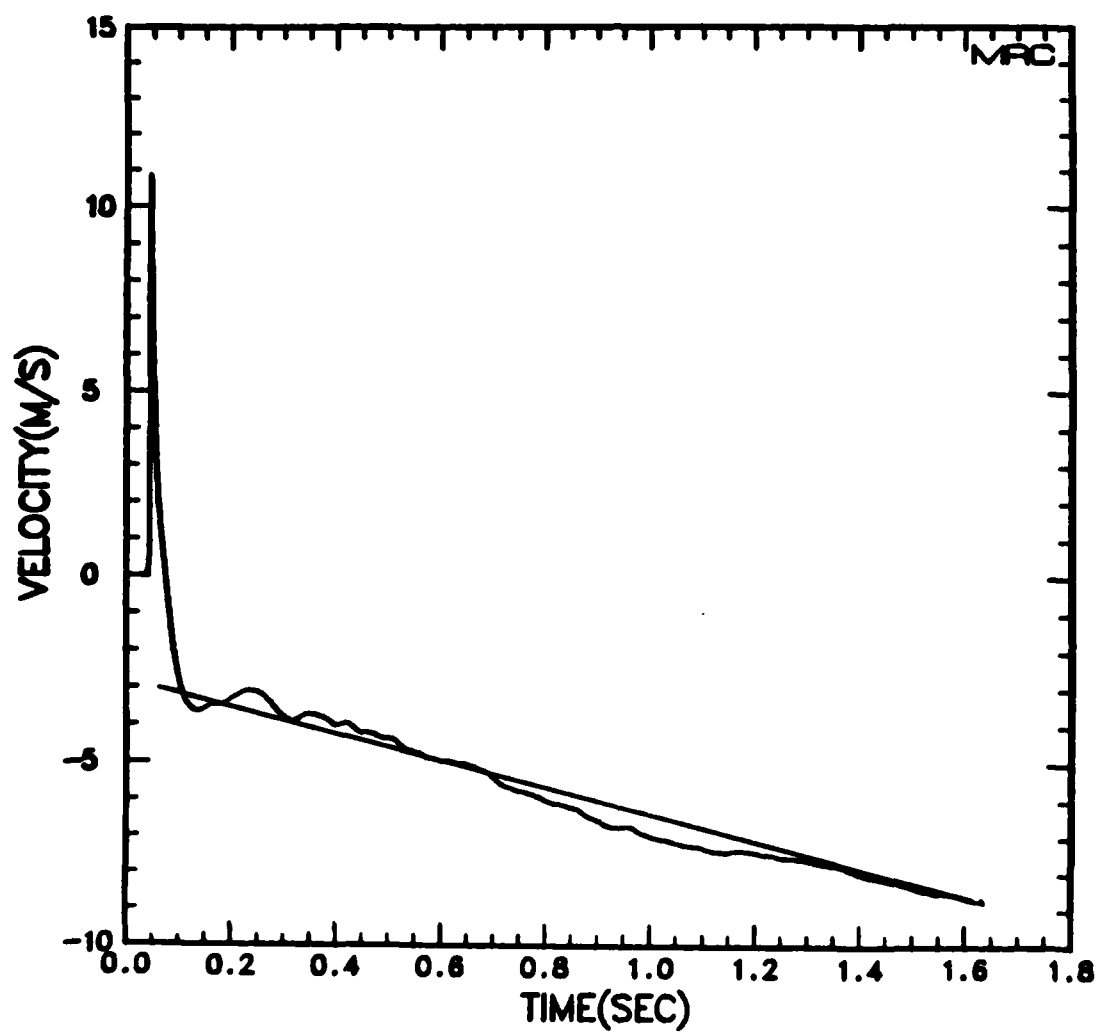


Figure 3. Raw velocity record from SALMON (E14C27AR) with straight line used to correct for baseline shifts.

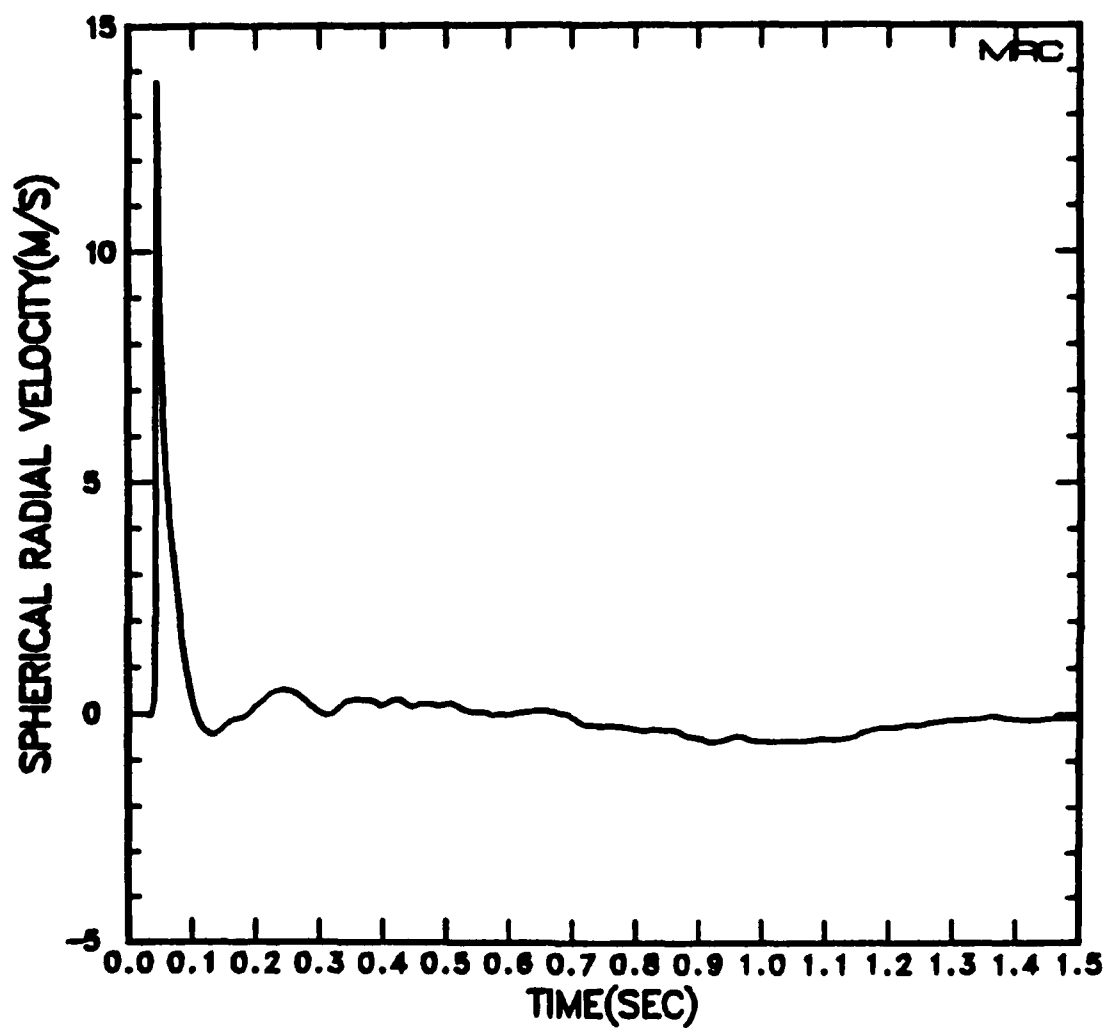


Figure 4. Corrected SALMON record to be compared with Figure 3.

First the raw velocity records, which are given in the appendix, were fit at late time with a straight line from the latest time shown (1.6 sec) back to the time of the peak in such a way as to give a behavior past the peak which was consistent with the results shown by Perret. The velocity before the peak was then altered by multiplication of each data value by the ratio of the new peak value to the old peak value. This assured that the velocity would be continuous although with a mild discontinuity in the first derivative exactly at the peak. It is impossible to detect this discontinuity by merely looking at the resulting plotted velocities. As a result we expect that the effects of this correction will be seen primarily in the high frequency end of the spectra and generally this will not be the region of interest. In addition to altering the raw data records for obvious instrumentation problems, we have at the same time introduced a geometrical factor which provides an estimate of the spherical radial velocity from each record. Since all six of the records chosen for manipulation are of horizontal radial motion, it is a straightforward matter to assume that the motion was strictly along a spherical radius and produce the spherical motion as

$$v(R)=v(r)/\cos(\theta)$$

where θ is the angle with respect to horizontal of the line through the instrument from the explosion. In several cases there do exist records of the vertical component of the motion at the same station and, in principle, one could either combine these with the radial component in a vector fashion or use them directly to estimate the spherical component. However, examination of a few cases indicated that nothing was gained in terms of consistency by using such methods so we have worked exclusively with the radial components.

The six records selected for study include those at ranges from 166 to 660 meters. The records were selected so as to have as large a

set of ranges as possible while also having them be as internally consistent as possible. The full set of original data records to which we had access is shown in Appendix B, along with indications of the locations of the instruments on which they were recorded. Consistency was established by taking records whose peak velocities fell along a smooth curve when plotted against range. While there is no real reason to believe that any one of the records is more desirable than some which were rejected, our use of spectral ratio methods require us to avoid examples which will obviously lead to unreasonable results when records are used in pairs. The records selected are indicated in Table 1 along with some of their properties.

Table 1. SALMON records used.

Record	Range (m)	V_R/V_r	$V_{R_{peak}}$ (m/s)	$V_{R_{peak}}^*$ (m/s)
E-14C-27-AR	166	1.0	13.8	14.0
E-14C-22-UR	225	1.36	8.0	8.2
E-14-20-AR	276	1.61	5.1	5.5
E-6-27-AR	318	1.0	3.75	3.9
E-14C-39-AR	402	2.40	2.5	2.7
E-11-34-AR	660	1.06	1.1	0.99

*Perret's values

The original records extend out to about five seconds past the explosion. It was found that after about 1.5 seconds the records showed no further significant contribution so we truncated the data at 1.6 seconds giving a convenient 8192 data points for each record. The corrected velocity records, some of which were taken from acceleration data, are shown on Figure 5.

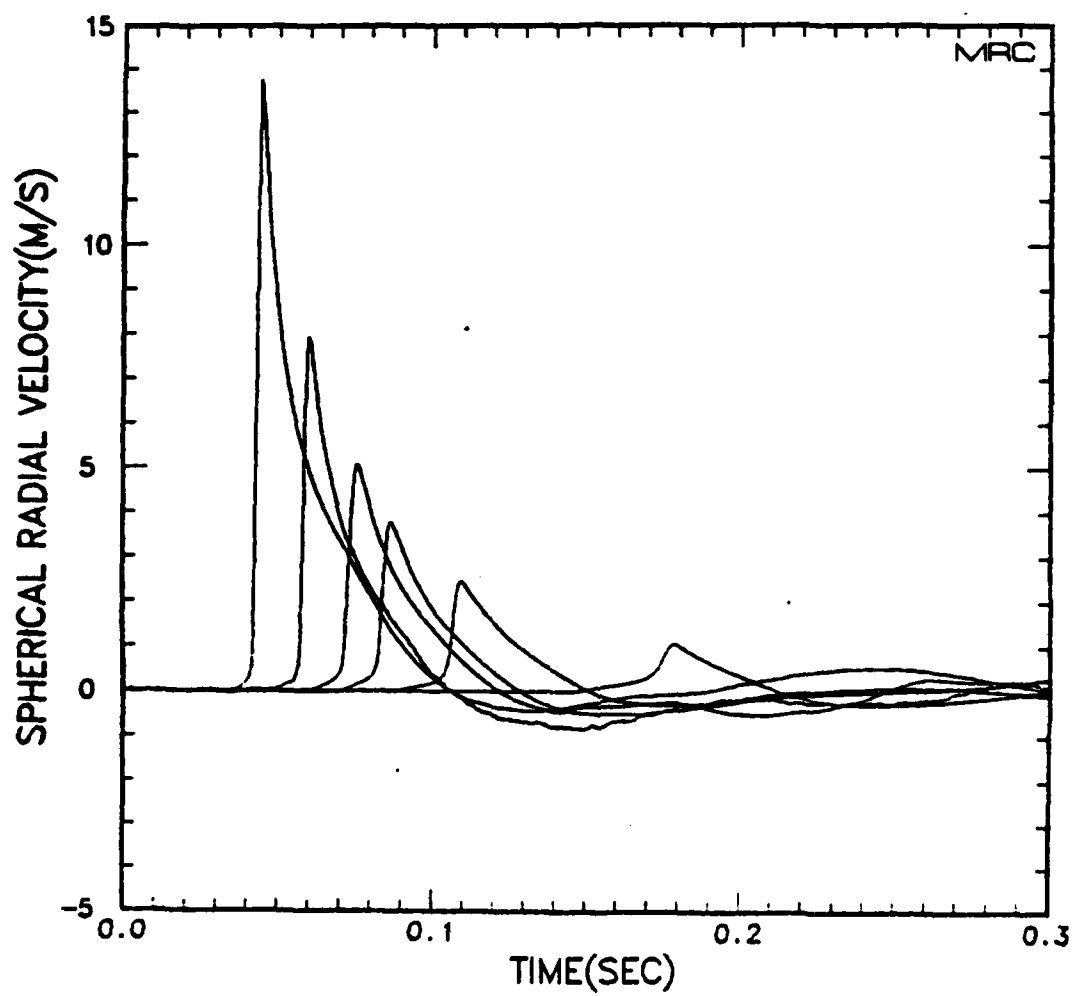


Figure 5. Corrected SALMON velocity records at 166, 225, 276, 402, and 660 meters.

When plotted on the same scale it becomes difficult to fully appreciate the features of the more distant examples so the same data are plotted again in Figure 6, suitably normalized to a unit peak amplitude.

It appears from the results in Figure 5 that the pulse progresses smoothly to large radii with a decrease in amplitude which, it will be shown, is approximately like radius to the -1.9 power. At the same time Figure 6 indicates that the pulse shape remains fairly stable but it shows a mild tendency to smooth out the sharp peak and to broaden the width. Both of these features suggest that the higher frequency components are being more rapidly attenuated than the low as would be appropriate, for example, with a constant Q should such a description be suitable. Beyond this we notice an initial ramp on each pulse which strengthens relative to the peak amplitude as the radius increases. This feature is generally attributed to shear failure, which occurs when the strain becomes sufficiently large, providing a reduced compressional velocity. Note that the leading edge of the pulses propagate at about 4.7 km/s while the peak propagates at about 3.7 km/s. In the following discussions we shall not make further reference to this nonlinear feature.

In order to establish the possible nonlinear character of the SALMON data we shall adopt the position of exploring the consequences of the assumption of anelastic behavior as expressed in terms of a Q function which we will attempt to evaluate. If this attempted description leads to contradictions, such as an effective Q which depends upon amplitude, we will attribute the effects to nonlinear behavior. Basically we shall use the data to make estimates of Q over a range of frequencies and amplitudes. Since we are dealing with data exclusively from a single event, no assumptions about the scaling behavior are necessary and no such information will be gained.

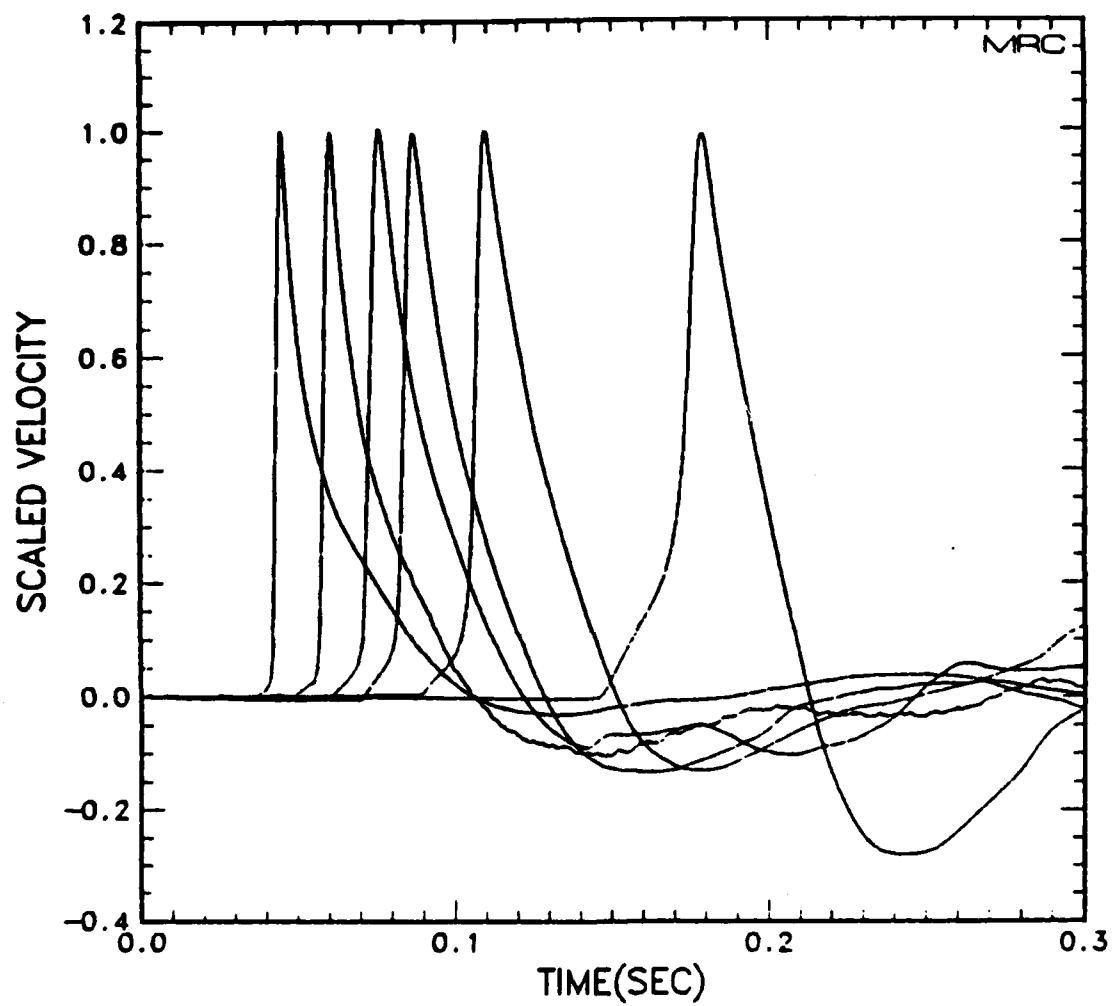


Figure 6. SALMON records from Figure 5 as plotted with common peak value to illustrate the waveforms.

A strict interpretation of anelastic behavior would require that we find a function Q , dependent generally upon frequency as well as a phase velocity c which is consistent with Q , so as to enforce the requirement of causality.¹¹ However, moderate values of Q are consistent with a nearly constant c , at least over the range of frequencies which are available to experimental verification. This feature can also be noted from experimental data by looking for dispersion in the implied p-wave phase velocities. The SALMON data cannot be made to establish the frequency dependence of c due to substantial scatter in the phase of the Fourier components of the signal.

The basic structure of the equations of motion for a spherical pulse propagating in an isotropic medium can be expressed as

$$\rho \frac{\partial^2 u}{\partial t^2} = \frac{1}{r^2} \left(\frac{\partial}{\partial r} (r^2 \sigma_{rr}) \right) - \frac{2}{r} \sigma_{\theta\theta} \quad (7)$$

where the stress components, σ , are related to the strain components, which are provided by the radial displacement u through

$$\epsilon_{rr} = \frac{\partial u}{\partial r}$$

and

$$\epsilon_{\theta\theta} = \frac{u}{r}$$

Of course it is necessary to have or find some constitutive relation between ϵ and σ in order to fix the propagation of a pulse. If a linear relation between ϵ and σ is taken, a general form which is convenient to work with is as follows

$$\begin{aligned} \sigma_{rr} = & (\lambda + 2\mu) \int m(t-\tau) \epsilon_{rr}(\tau) d\tau \\ & + 2\lambda \int m(t-\tau) \epsilon_{\theta\theta}(\tau) d\tau \end{aligned}$$

and

$$\begin{aligned}\sigma_{\theta\theta} = & (\lambda+2\mu) \int m(t-\tau) \epsilon_{\theta\theta}(\tau) d\tau \\ & + \lambda \int m(t-\tau) \epsilon_{rr}(\tau) d\tau\end{aligned}$$

where $m(t)$ is the derivative of the relaxation function. If $m(t)$ is a delta function the relations reduce to the standard elastic result with the Lamé constants λ and μ . If we introduce the definition of the reduced displacement potential, ϕ , as

$$u(r,t) = \frac{\partial}{\partial r} (\phi(r,t)/r) \quad (9)$$

the equation of motion can be written in compact form in the frequency domain as

$$\frac{\partial}{\partial r} \left(\frac{1}{r} (\rho \omega^2 \tilde{\phi} + \tilde{m}(\omega)(\lambda+2\mu) \frac{\partial^2}{\partial r^2} \tilde{\phi}) \right) = 0 \quad (10)$$

where $\tilde{\phi}$ will generally be a function of both r and ω . From this form we see that the advancement of ϕ can be expressed as

$$\phi(r,t) = \frac{1}{2\pi} \int \tilde{\phi}(0,\omega) e^{-i\omega t} e^{i\omega r/\alpha\sqrt{\tilde{m}}} d\omega \quad (11)$$

where the apparent p-wave velocity is given by

$$\alpha^2 = (\lambda+2\mu)/\rho \quad (12)$$

and $\tilde{\phi}(0,\omega)$ is the Fourier transform of the ϕ at $r=0$.

Kjartansson¹² has shown that there is a choice for $m(t)$ which will give a Q which is exactly constant and which provides a phase velocity

which is nearly constant over a wide range of frequency provided Q is not too small. This choice for m is

$$\bar{m}(\omega) = \left| \frac{\omega}{\omega_0} \right|^{2\gamma} \exp(i\pi\gamma \operatorname{sgn}(\omega)) \quad (13)$$

where γ and ω_0 are arbitrary real parameters. Thus $\omega/\bar{m}$ is the complex p-wave velocity. The observed pulse propagation velocity can be used to fix the value of ω_0 and the parameter γ can be expressed in terms of a constant Q by noting that the phase relation between e and s gives Q which can then written as

$$Q^{-1} = \tan(\pi\gamma) \quad (14)$$

so we can write

$$\phi(r,t) = \frac{1}{2\pi} \int d\omega \bar{\phi}(0,\omega) \exp(-i\omega(t - \frac{r}{c})) \cdot \exp(-\omega r \operatorname{sgn}(\omega)/(2cQ)) \quad (15)$$

where

$$c = c_0 \left| \frac{\omega}{\omega_0} \right|^\gamma / \cos(\pi\gamma/2) \quad (16)$$

so than the displacement will be advanced by

$$u(r,t) = \frac{1}{2\pi} \int d\omega u(a,\omega) \exp(-i\omega(t - \frac{r-a}{c}) - \omega(r-a) \operatorname{sgn}(\omega)/(2cQ)) \\ \times \frac{\frac{i\omega}{cr} - \frac{\omega \operatorname{sgn}(\omega)}{2cQr} - \frac{1}{r^2}}{\frac{i\omega}{ca} - \frac{\omega \operatorname{sgn}(\omega)}{2cQa} - \frac{1}{a^2}} \quad (17)$$

While Equation (15) is specific to the Kjartansson assumptions, it provides a generic form which is often taken to be an indication of

the attenuation which is associated with Q. That is, we see that Q forces an exponential decay with range of the RDP by a factor

$$\exp\left(-\frac{\omega r}{2cQ}\right),$$

and this provides a common definition of Q which is applied to experimental data by taking the phase velocity c to be constant, normally at the nominal speed of the pulse. Again, this is possible only because c is at most a slowly changing function of frequency so that the pulse retains a form which does not change greatly with range. The data from SALMON as shown in the Figure 2 do retain the pulse shape to such a degree that an initial presumption of constant c can be made. In fact in the following analysis we will estimate the product of cQ and then provide Q subject to particular values of c.

Manipulating the relations among the RDP, Q and the velocity in the frequency domain provides an operational definition of the experimental value of Q through the equation

$$\exp\left(-\frac{\omega(r-a)}{2cQ}\right) = \left| \frac{\tilde{v}(r, \omega)}{\tilde{v}(a, \omega)} \right| \left| \frac{\frac{i\omega}{ca} - \frac{\omega}{2cQa} - \frac{1}{a^2}}{\frac{i\omega}{cr} - \frac{\omega}{2cQr} - \frac{1}{r^2}} \right| \quad (18)$$

which provides an implicit expression for Q from the data. In practice it is found that the second term on the right hand side of (18) is nearly independent of Q for Q not too small so that one can write

$$\exp\left(-\frac{\omega(r-a)}{2cQ}\right) \approx \left| \frac{\tilde{v}(r, \omega)}{\tilde{v}(a, \omega)} \right| \left| \frac{\frac{i\omega}{a} - \frac{c}{a^2}}{\frac{i\omega}{r} - \frac{c}{r^2}} \right| \quad (19)$$

This is the usual "definition" of Q from experimental data although it does not exactly conform to the proper definition¹³. Note that at large ranges the $1/r^2$ term is negligible so the expression becomes yet simpler. However in the following we will take Equation (19) as the definition of Q from the data. One could solve more accurately using (18) but our calculations indicate that there is no significant difference between (18) and (19) for the SALMON case. Based on the motion of the peak of the SALMON pulse motion we take $c=3.7$ km/s as the constant sound speed.

Using the six data records given in Figure 5, one can form a set of five sequential estimates for Q at ever increasing ranges. The Q thus obtained is explicitly a function of frequency and based on a minimum sample spacing of 200 microseconds over 8192 samples, the frequency range so obtained is from .6 to 2500 Hz. However, we find that the high frequency end of the scale is not well defined by the data, due in part to the manipulations to avoid improper late-time behavior and in part to noise in the data. As a result we present only that portion of the frequency regime below 500 Hz with the suggestion that perhaps only the region below 100 or 200 Hz is meaningful. Figures 7 through 11 show the estimated $Q(\omega)$ found from successive records at 166, 225, 267, 318, 402, and 660 meters from the explosion. Figure 12 gives an average over the entire range by using only the closest and most distant records. It appears that the record at 318 meters is not very consistent with the others but when the results in Figures 9 and 10 are discarded some features of the Q estimates become clear. First comparing Figures 7 and 11 should indicate the extent of amplitude dependence of this effective Q and so point to nonlinear behavior. However these two estimates of Q are remarkably similar and they also are very much like the average estimate for Q from Figure 12, which also gives the RVP at the extreme radii.

In all cases the Q estimates at frequencies above about 200 Hz show such great scatter that they are probably meaningless. However below

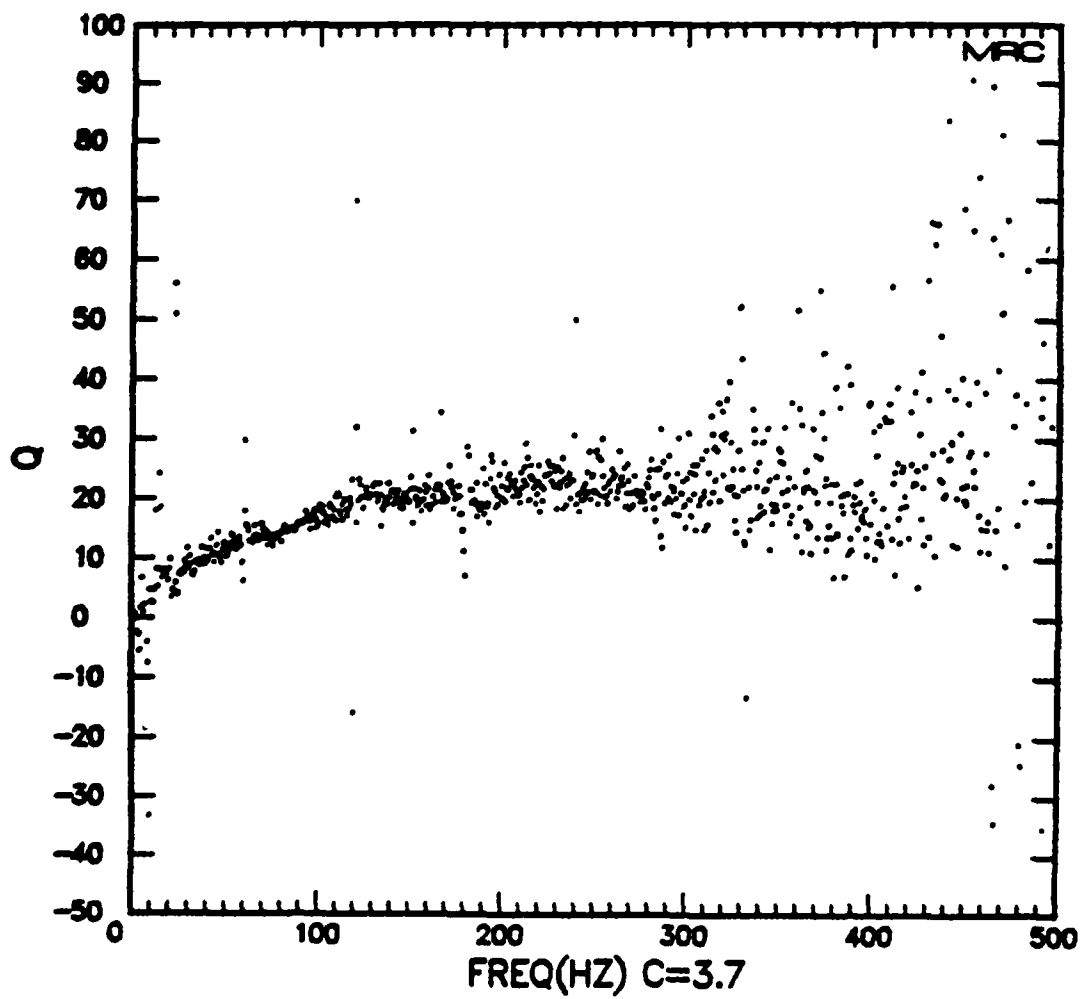


Figure 7. Estimate of $Q(\omega)$ from SALMON records at 166 and 225 meters.

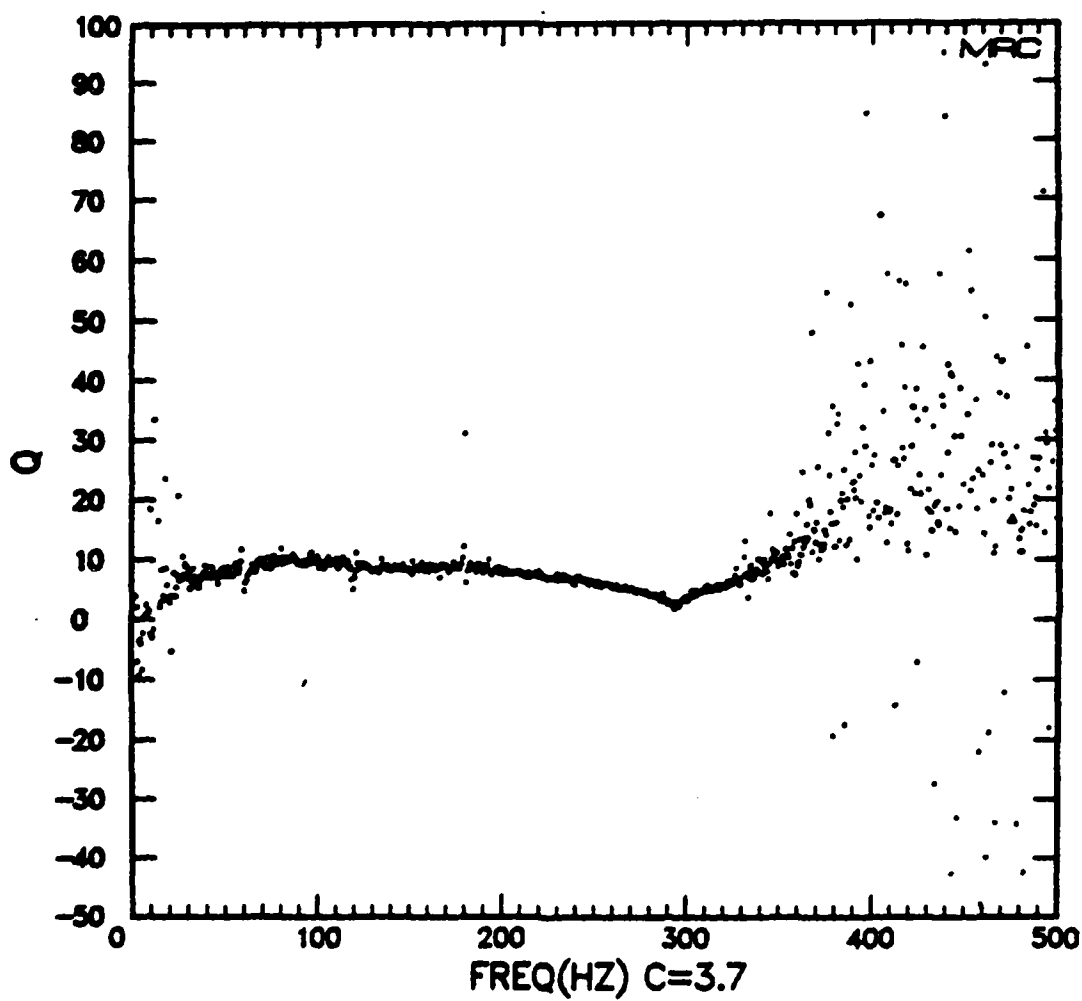


Figure 8. Estimate of $Q(\omega)$ from SALMON records at 225 and 276 meters.

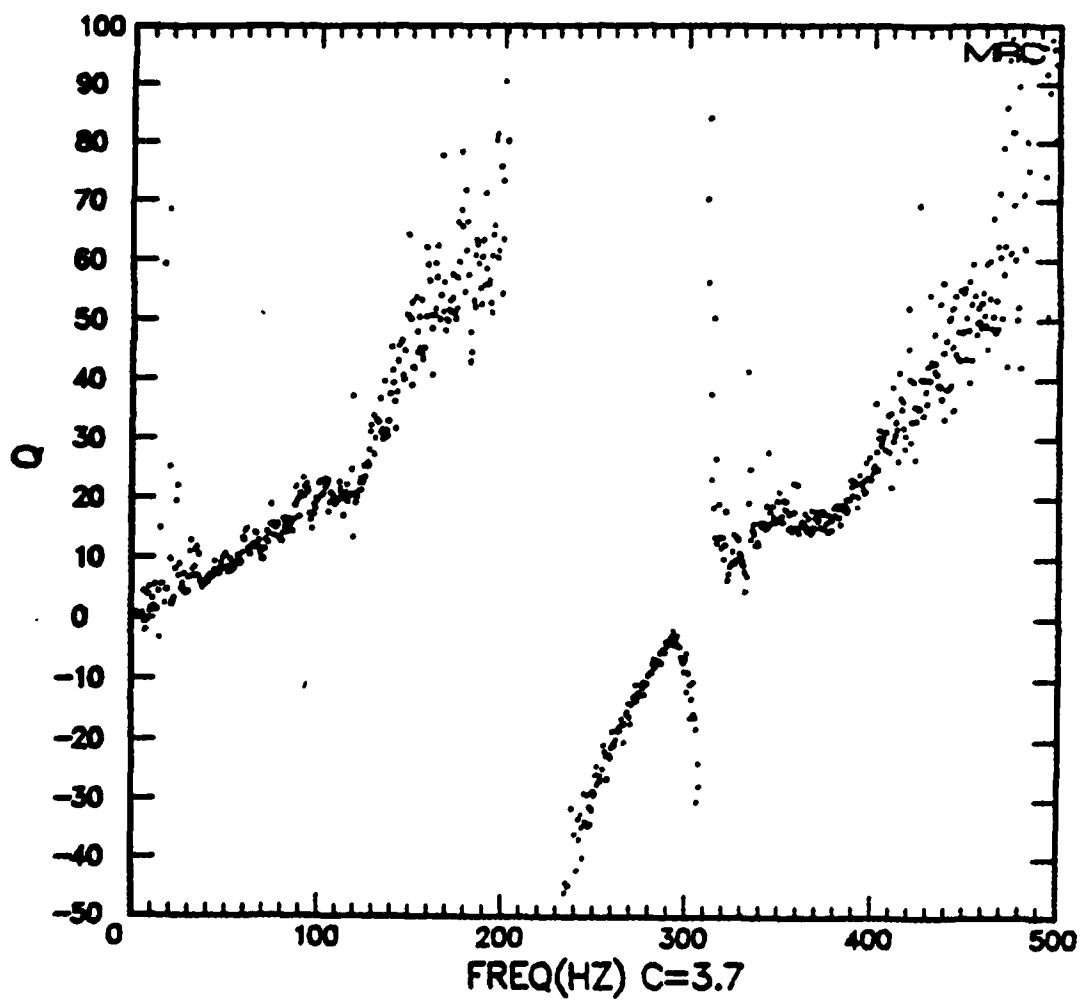


Figure 9. Estimate of $Q(\omega)$ from SALMON records at 276 and 318 meters.

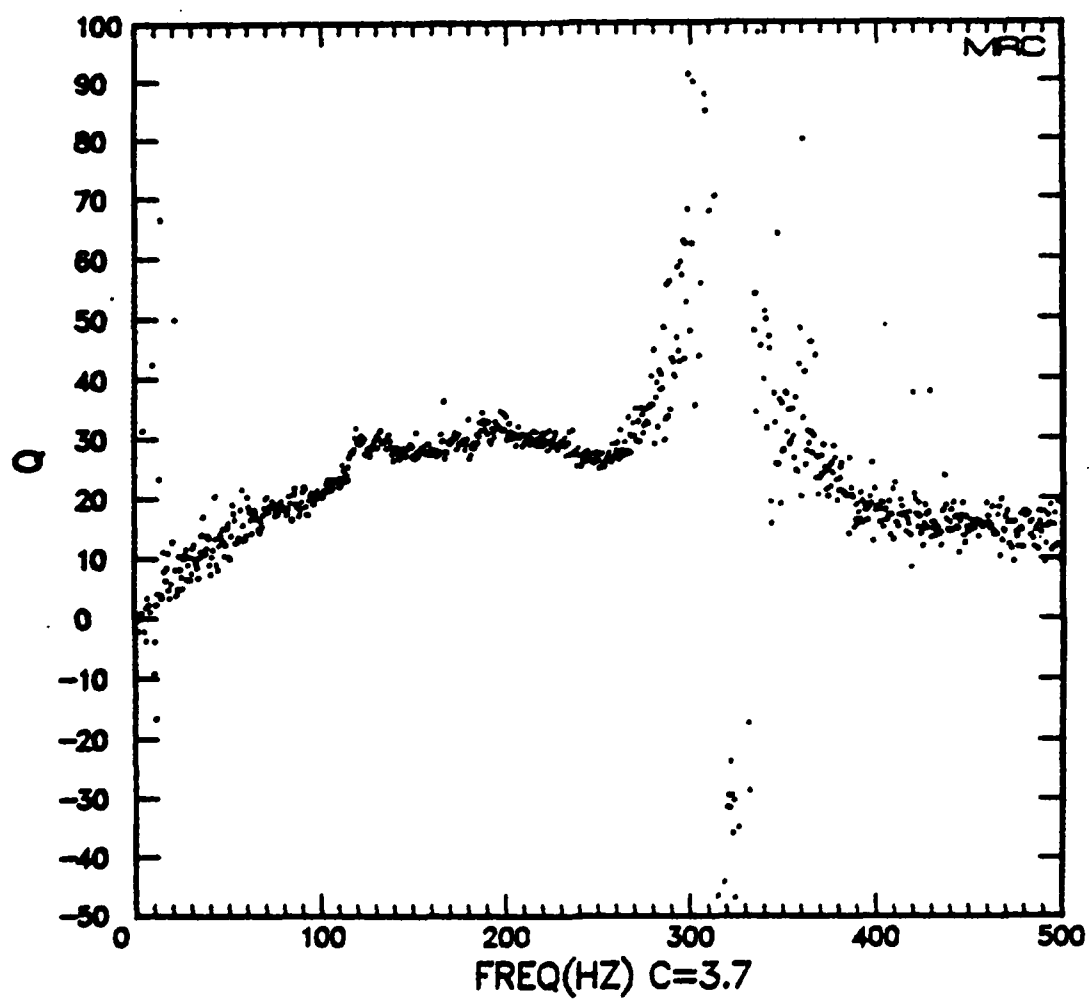


Figure 10. Estimate of $Q(\omega)$ from SALMON records at 318 and 402 meters.

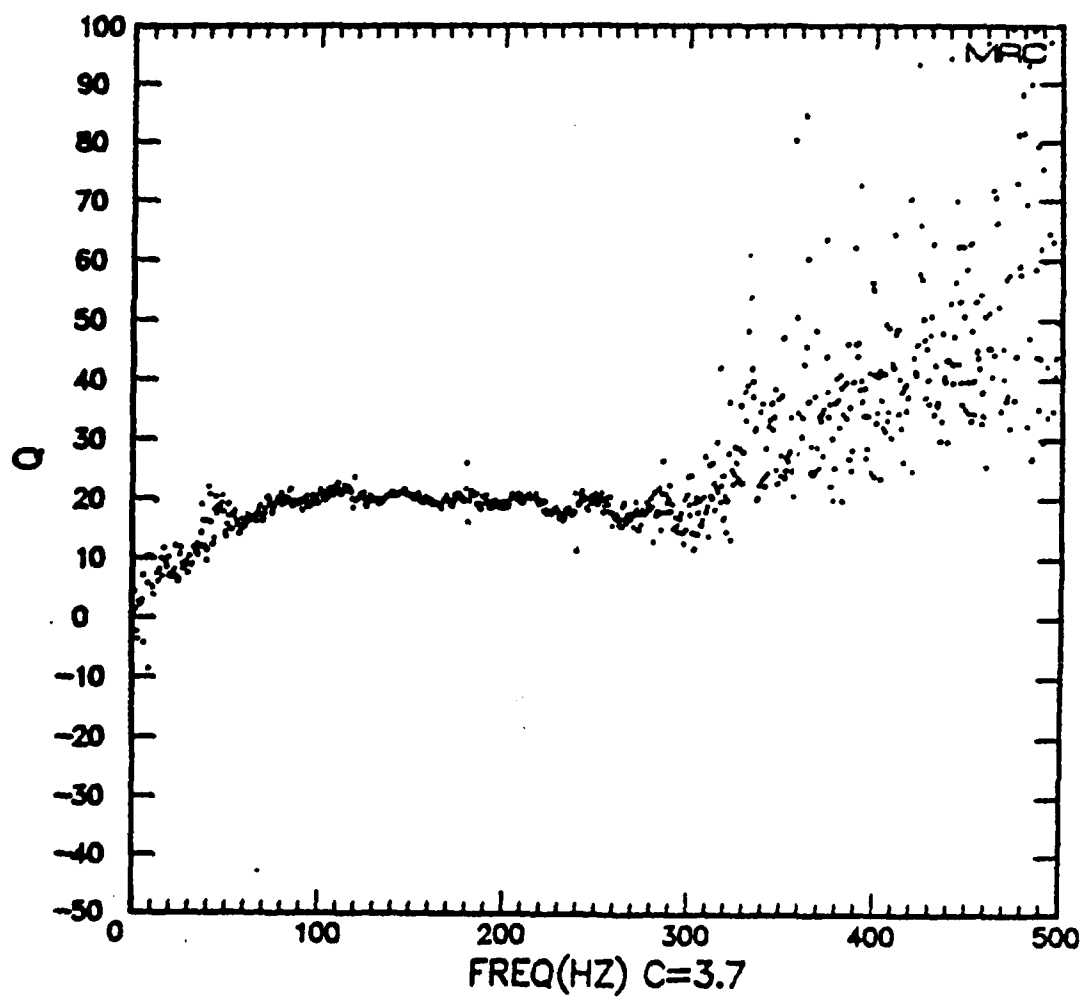


Figure 11. Estimate of $Q(\omega)$ from SALMON records at 402 and 660 meters.

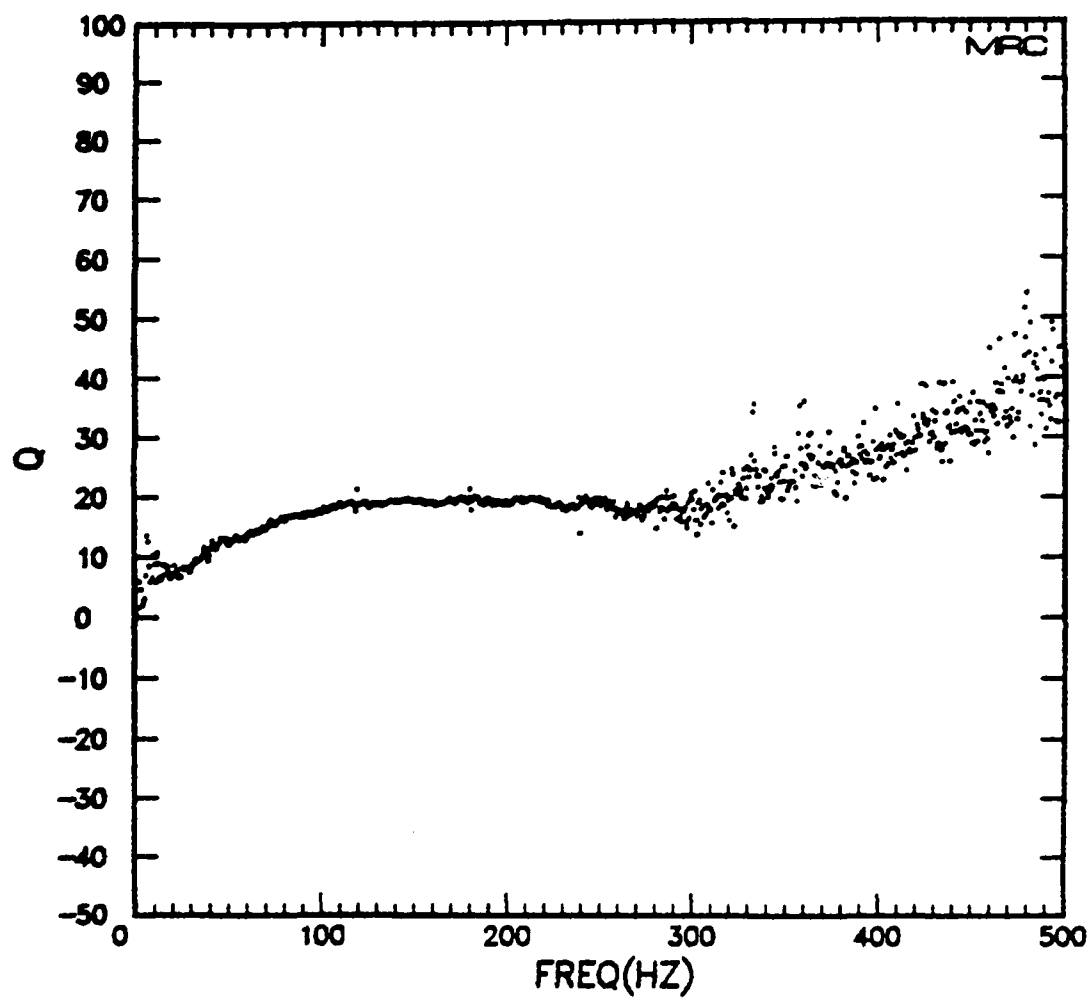


Figure 12a. Estimate of $Q(\omega)$ from SALMON records at 166 and 660 meters.

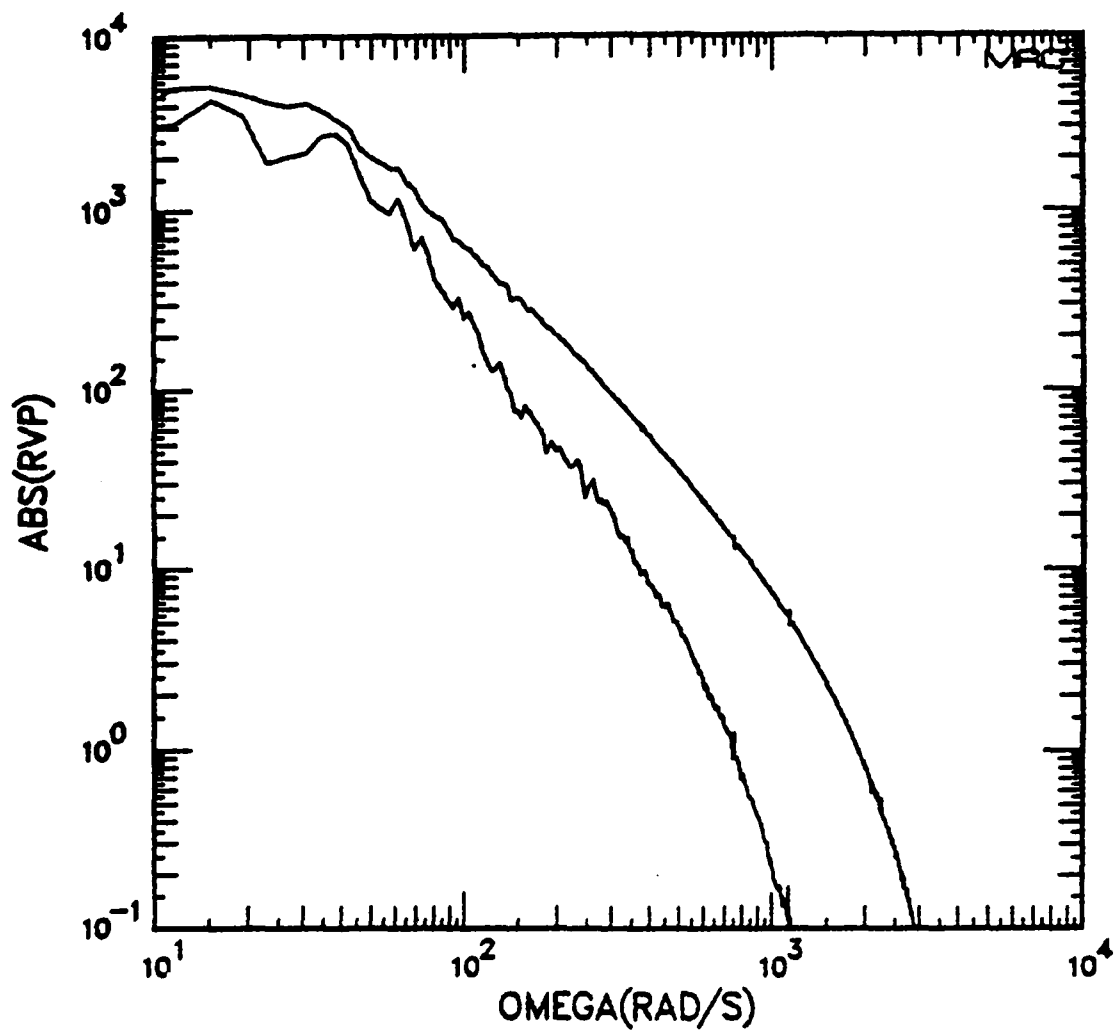


Figure 12b. The Fourier transform of the reduced velocity potential for SALMON records at 166 and 660 meters.

200 Hz the results are remarkably consistent in that they show a fairly constant Q down to a few tens of Hz, followed by a smooth rolloff to zero at low frequency. One should keep in mind that the pulses from SALMON have a typical period of perhaps 0.2 seconds with structure, which appears significant as small as 0.01 seconds. Thus the portion of the spectrum which is highly visible ranges from less than 5 to perhaps 100 Hz. It is thus of interest to see if the apparent frequency dependence of Q stems from features of the pulses which are evident in the time domain. We have checked this by reconstructing the outer five pulses from the inner one making two different assumptions about the behavior of Q . First consider the case of constant Q with c allowed to change with frequency according to the results from Kjartansson shown before. For this purpose we take a value of $Q=10$ and c at 1 Hz of 3.7 km/s. Now the result in Equation (15), when applied to velocity, can be used to propagate the observed at 166 meters subject to exactly constant Q . The results for the pulse shapes are shown in Figures 13 and 14, relative to the observed results, both unscaled and scaled. It appears that the peak values of the pulses are well reproduced but the peaks are smoothed out and broadened a bit so that it is easily possible to distinguish the two cases.

A second means of looking at a Q independent of amplitude was carried out by fitting the data in Figure 12 with an approximation to Q which is frequency dependent with the form

$$Q = \frac{20f}{f + 30} \quad (20)$$

although this has no corresponding c which we know of in the sense of a creep function. This ad hoc Q was then taken with a constant c to again propagate the initial SALMON pulse. The results are shown in Figures 15 and 16 for the unscaled and scaled cases compared to the original data.

In this case of a frequency dependent Q , the peak of the velocity pulses follow the data quite well and there is a strong correspondence

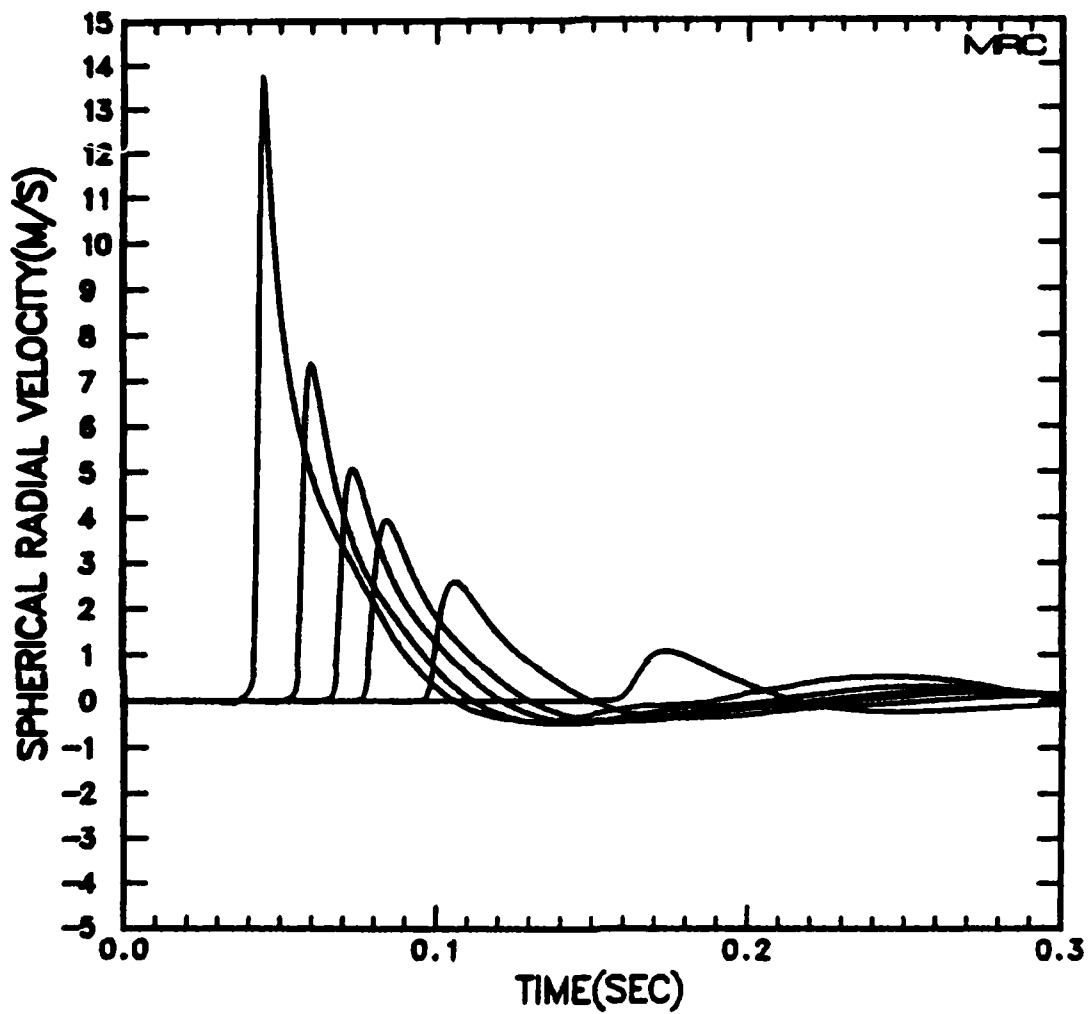


Figure 13. Estimates of pulses generated from the SALMON record at 166 meters and a Q of 10.

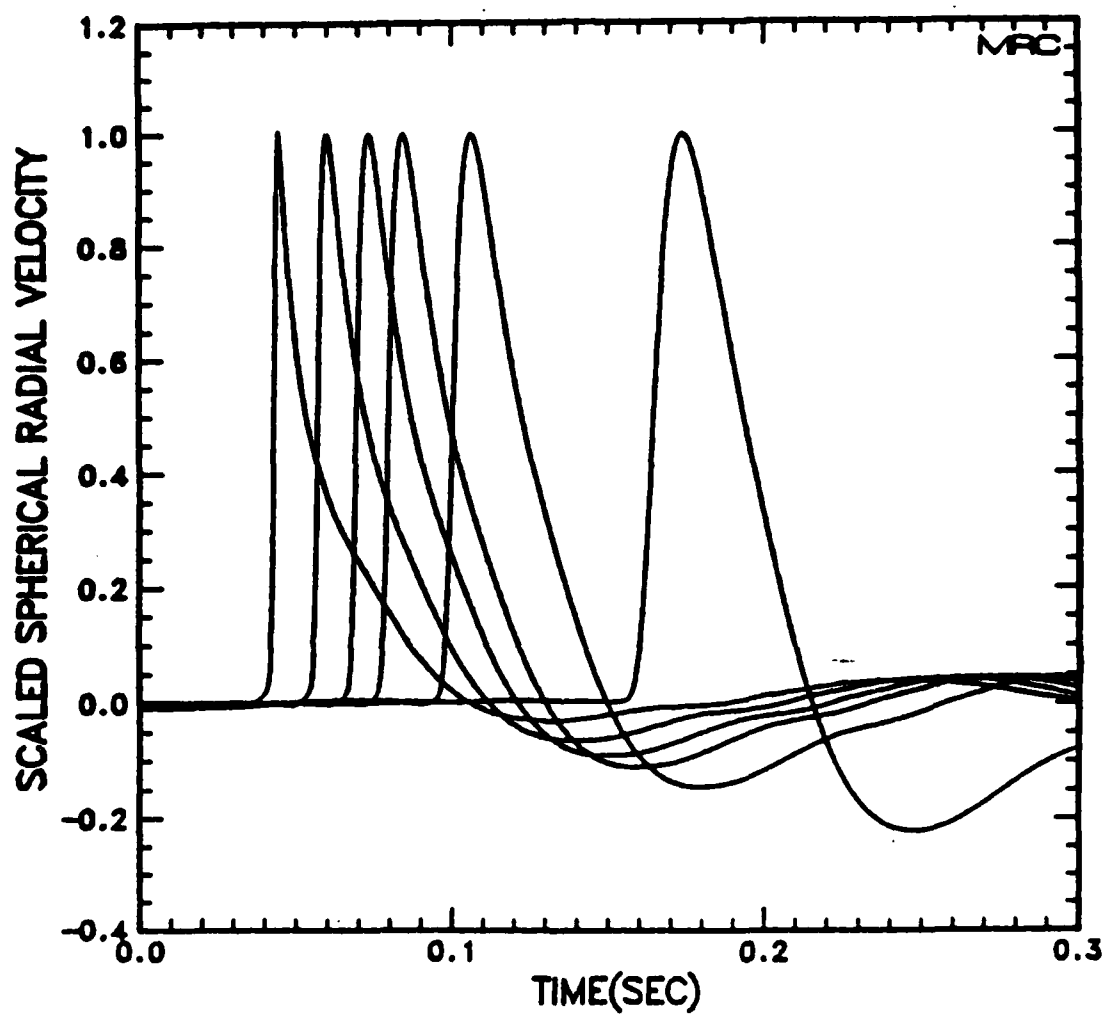


Figure 14. Reproduction of Figure 13 plotted with common peak value to illustrate waveform.

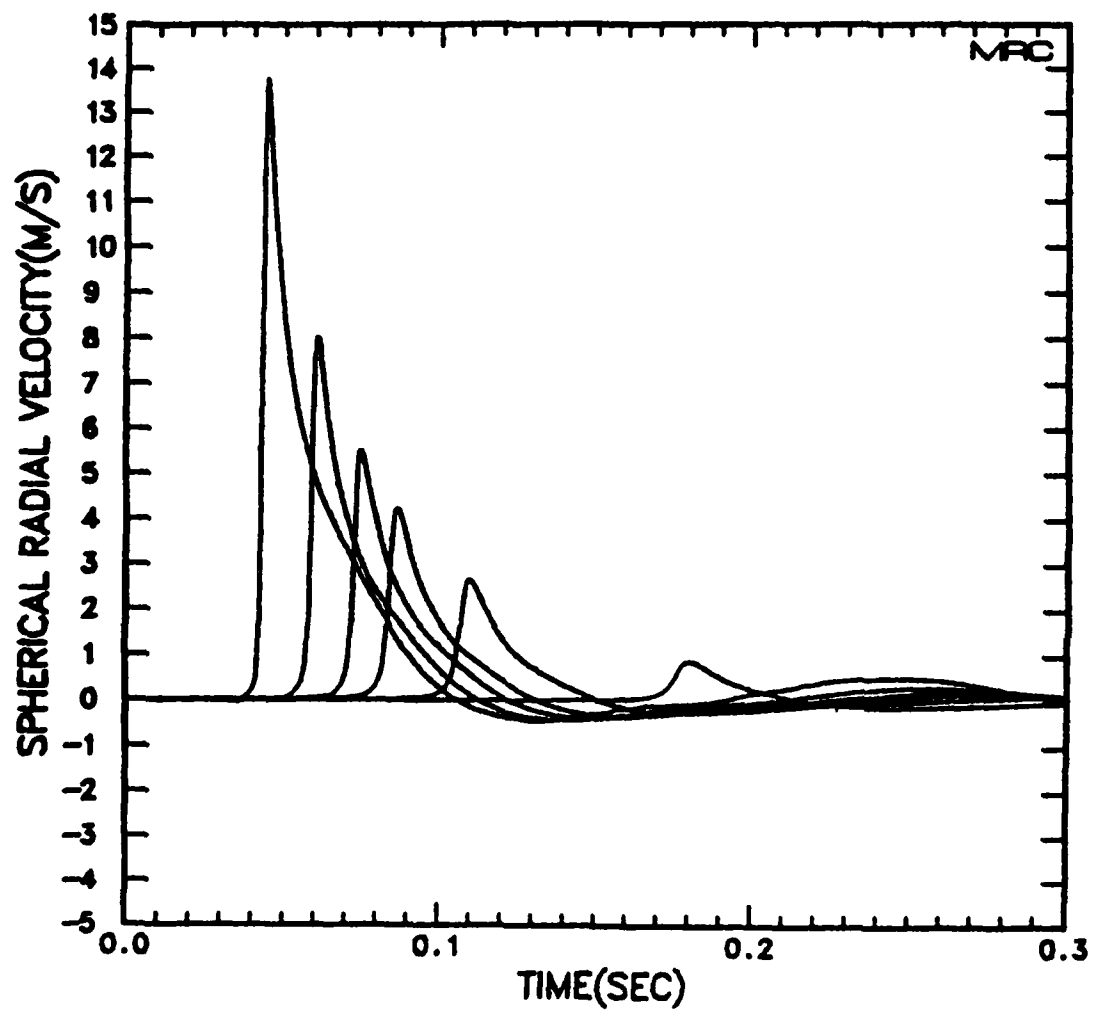


Figure 15. Estimates of pulses generated from the SALMON record at 166 meters using the fit to $Q(\omega)$.

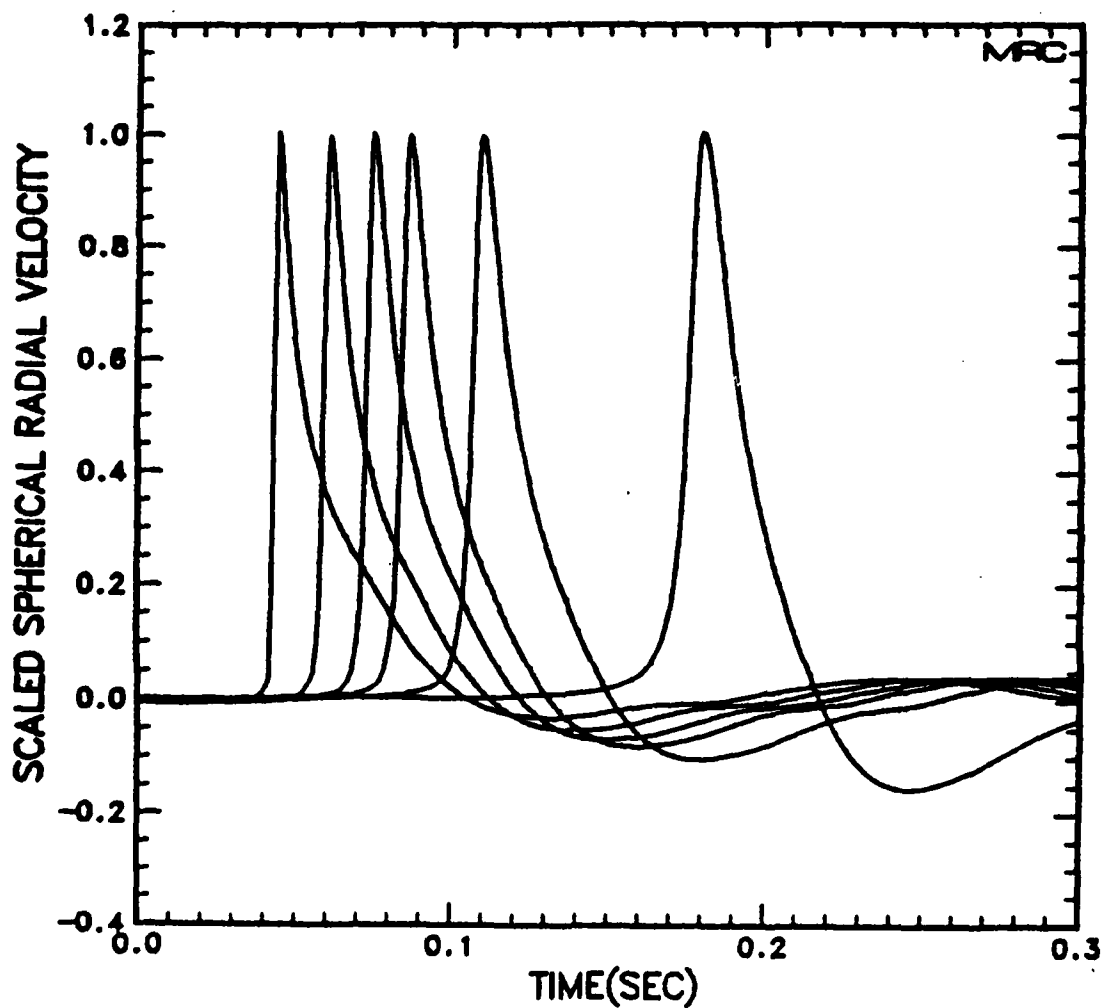


Figure 16. Reproduction of Figure 15 plotted with common peak value to illustrate waveform.

between the widths and sharpness of the peaks in the data and the linear fit. However, it is still not possible to provide a good fit to the initial portion of the experimental pulses which, as indicated before, are thought to be due to shear failure. Still the remainder of the pulses appear to be consistent with a Q which is independent of amplitude.

DISCUSSION

We have taken a set of six data records from the SALMON event and attempted to subject them to such an analysis that any indication of nonlinear behavior over the strain regime from 4×10^{-3} to 3×10^{-4} might be detected. Basically we find that the SALMON data, taken by themselves, are consistent with linear but inelastic behavior which can be described by a Q , which is independent of amplitude but which is at least a mild function of frequency. In spite of the fact that a strain variation of more than an order of magnitude is spanned by the data, the closest and most distant pairs of records do not show a significant difference in effective Q values. This Q approaches a value of about 20 at frequencies above 100 Hertz and declines to zero for lower frequencies.

It must be pointed out that these results cannot rule out the possibility that the same records may be consistent with some unspecified nonlinear mechanism which simply produces effects which, locally, are similar to linear mechanisms. In fact, this is a possibility which must be taken seriously since the experiments by Tittmann suggest that for strains of less than 10^{-6} a Q of about 500 is appropriate. Still the experiments are under conditions sufficiently different from explosions that one must be cautious in comparing the effects under somewhat different frequency regimes and different loading conditions.

SECTION 3

ANALYTIC APPROACH TO NONLINEAR FRICTION

In discussing the departure of a propagation medium from purely elastic behavior, it is common to focus on the quantity Q , the absorption per cycle. If one wishes to calculate the detailed propagation of a seismic wave, which is what we shall want to do in this section, one needs constitutive relations which can be combined with dynamical laws to specify the propagation. For a linear theory, Q and the phase velocity combine together to specify the propagation; they contain equivalent information to constitutive relations. Furthermore, Q and the phase velocity are related through dispersion relations. For a nonlinear theory, both concepts, Q and the phase velocity, lose precise meaning and we do not know how to translate information on Q into (nonlinear) constitutive relations. While not, in any way we currently understand, providing a direct determination of the constitutive relations, absorption measurements do provide a constraint on them by specifying results with which a correct rheology must agree. There have been a variety of efforts to relate Q to physical properties of materials, especially internal sliding friction due to cracks. Physically based models of attenuation such as those by Savage¹⁴, for thermoelastic losses, by Mavko and Nur¹⁵, for viscous attenuation by partially filled cracks, and by Walsh¹⁶ for sliding friction all provide a means of estimating the loss rate, and so Q , in terms of physical parameters, such as crack length, which are often not directly accessible to direct measurement. As a result these linear models provide Q which is only loosely tied to the observables of the problem and so they do not provide for direct verification of the source mechanism. As a result these models are not much more satisfying than a direct phenomenological specification for Q .

Many experimental studies^{6,17,18} of attenuation indicate that it is generally amplitude dependent with an onset of increased attenuation at strains of about 10^{-6} . As a result it has been recognized that friction models employing mechanisms which have a nonlinear basis must be used to account for the data. In this vein, Mavko¹⁹ has demonstrated that sliding friction on crack surfaces with irregularities and including partially closed cracks will product attenuation which increases with amplitude. Using a Q defined defined in terms of relative energy loss per cycle, Mavko finds Q^{-1} proportional to the peak strain over the cycle. In a similar fashion, Stewart et al.,²⁰ have proposed a physical attenuation model based on complex crack structure and friction which, like Mavko, provides a Q^{-1} proportional to peak strain and which is also proportional to the crack density and the $-4/3$ power of the confining pressure.

All the above physical models for friction have the feature that they isolate a portion of the material, impose a external loading and integrate the energy losses over a cycle. For the linear case, this can be translated into a Q function which can be included in constitutive relations which will apply to the equations of motion of the medium and determine, in principle, the effects of attenuation on seismic signals. However, when the nonlinear friction models indicate that the effective Q is a function of peak strain, this cannot be translated directly into a corresponding Q since Q is basically a linear concept. As a result there is currently no consistent way of including the nonlinear attenuation in calculations of seismic propagation.

In this section we wish to explore a possible means of accounting for nonlinear attenuation, at least in a phenomenological way which is consistent with the data and which is not confined to effective Q integrated over a cycle assumptions. This will be done by consideration of constitutive relations which are explicitly nonlinear and which allow approximate analytic solutions so that their consequences can be readily explored.

We shall specify constitutive relations which provides for propagation which satisfies Items 1, 2, and 3 given in the introduction.

- 1) The absorption of energy increases with amplitude; one relation which has been proposed to express this effect is:

$$Q^{-1} = Q_0^{-1} + \alpha \epsilon$$

where ϵ is the peak strain; the quantity Q will be discussed below.

- 2) The propagation satisfies cube-root scaling; that is, the propagation is invariant under the scale transformations:

$$\begin{aligned} t &\rightarrow y^{1/3} t_s \\ x &\rightarrow y^{1/3} x_s \\ u &\rightarrow y^{1/3} u_s \end{aligned}$$

where t_s , x_s and u_s are respectively the scaled time, scaled distance and scaled displacement while y is the event yield.

- 3) The propagation, although presumably nonlinear (property 1), nevertheless provides a waveform which changes only slowly with distance.

In addition to providing for propagation with these properties the relations we shall use are Galilean invariant and satisfy causality--in fact the relations are local in both space and time.

INTRODUCING FRICTION

We start from the elastic equation in one cartesian dimension:

$$\rho \frac{\partial^2 u}{\partial t^2} = \mu \frac{\partial u}{\partial x^2} \quad (21)$$

where u is the displacement and μ is an elastic modulus. We can introduce friction into the system by allowing the modulus to change with the strain and rate of change of strain in such a way that the increment in force always opposes changes in the state of the system:

$$\mu \rightarrow \mu(1 + \alpha \operatorname{sgn}(\frac{\partial u}{\partial x} \frac{\partial^2 u}{\partial x \partial t})) \quad (22)$$

A form similar to (22) is given by Knopoff and McDonald²¹; we prefer the form (22) given it is Galilean invariant whereas that of Reference 21 is not; for propagating disturbances - those for which the x and t derivatives are related by a constant - the two forms are equivalent. In Reference 21 it is argued by analogy with single oscillator models that (22) will lead to a frequency independent Q . By this same argument (22) will lead to an amplitude independent Q .

To satisfy our item 1 - that absorption increases with amplitude - we make the "coefficient of friction" α depend on the strain. One possibility is:

$$\alpha = \gamma + \beta \left| \frac{\partial u}{\partial x} \right| \quad (23)$$

in this case α increases with material distortion whether the material is under tension or under compression. Such need not be the case and we might equally well have:

$$\alpha = \begin{cases} \gamma + \beta \frac{\partial u}{\partial x} & \frac{\partial u}{\partial x} > 0 \\ \gamma & \frac{\partial u}{\partial x} < 0 \end{cases} \quad (24)$$

or

$$\alpha = \begin{cases} \gamma & \frac{\partial u}{\partial x} > 0 \\ \gamma - \beta \frac{\partial u}{\partial x} & \frac{\partial u}{\partial x} < 0 \end{cases} \quad (25)$$

Which, if any, of these relations is appropriate for materials in the earth is an experimental question. Lacking such guidance we shall consider relation (23); the qualitative conclusions reached here would apply to the other relations as well although the details of the propagation would be different.

We have arrived at the following dynamical equation:

$$\rho \frac{\partial^2 u}{\partial t^2} = u \left(1 + \left(\gamma + \beta \left| \frac{\partial u}{\partial x} \right| \right) \operatorname{sgn} \left(\frac{\partial u}{\partial x} \frac{\partial^2 u}{\partial x \partial t} \right) \right) \frac{\partial^2 u}{\partial x^2} \quad (26)$$

It is easy to see that this equation is yield^{1/3} scale invariant, that is, it is form invariant under the scale transformations of item 2.

APPROXIMATE SOLUTIONS

In this report we are primarily concerned with amplitude dependent effects. The number γ in relation (23) determines the amplitude independent value the absorption will approach in the limit of small amplitudes. To simplify future expressions and clarify the amplitude dependent effects predicted by equation (23) we shall set γ equal to zero. Furthermore we stated, in item 3, that we wished to have propagation of an approximately unchanging waveform. Thus we shall seek solutions of the form:

$$u = F(x) G(t - \frac{x}{c}) \quad (27)$$

where c is a constant and we shall normalize the functions F and G by specifying $\text{Max } |G| = 1$. Substitution of (27) into (26) does not, without further assumptions, yield a simple expression. If, however, we further assume that:

$$\frac{F''}{F} \lambda_G \ll 1 \quad (28)$$

where λ_G is the length scale of the waveform G , that is, we assume that the fractional change in the amplitude function F when the wave propagates through its length is small, then equation (26) reduces to:

$$\frac{F'}{F^2} = - \frac{\alpha \beta}{2c^2} |G''| \quad (29)$$

where we have chosen $c = \sqrt{\mu/\rho}$; the wave travels at the linear sound speed. The left hand side of (29) depends only on x whereas the right hand side depends only on the retarded time $\tau (\tau \equiv t - \frac{x}{c})$. Thus each side must be equal to a constant and G is a quadratic. Thus in somewhat of an analogy to soliton theory there is only one class of waveforms which will propagate according to (27).

The general solution for G is given by:

$$G = 4 \tau \frac{(T-\tau)}{T^2} \quad 0 < \tau < T \quad (30)$$

where T is a constant - the duration of the wave. From this expression we get:

$$|G''| = \frac{8}{T^2} \quad (31)$$

Combining this expression with (29) we find the general solution for F:

$$F = \frac{c^2 T}{4\alpha\beta} \frac{1}{a+x} \quad (32)$$

where a is an arbitrary constant. The displacement is thus given by:

$$u = \frac{c^2 T}{\alpha\beta} \frac{1}{a+x} \frac{\tau(T-\tau)}{T^2} \quad (33)$$

For a propagating linear wave of frequency ω a common definition of Q is in terms of the log-derivative of the amplitude; if A is the amplitude we have:

$$\frac{d \ln A}{dx} = - \frac{\omega}{2c} Q^{-1} \quad (34)$$

Since F is the amplitude of our nonlinear wave, the natural definition of Q is

$$Q^{-1} = - \frac{2c}{\omega} \frac{F'}{F} = \frac{8\alpha\beta}{\omega c T^2} F \quad (35)$$

Thus we see that for the current model Q^{-1} is a linear function of the amplitude as Minster and Archambeau suggested might be characteristic of the SALMON-COWBOY data. The characteristic frequency of our wave will be

$$\omega = \frac{\pi}{T} \quad (36)$$

which gives:

$$Q^{-1} = \frac{8\alpha\beta\omega}{\pi^2 c} F \quad (37)$$

This relation demonstrates our earlier statement that for nonlinear propagation cube-root scaling does not imply the frequency independence of Q: equation (26) is scale invariant but Q depends on ω .

From equation (33) we find that the peak velocity in the wave is given by:

$$v_p = \frac{c^2}{\alpha\beta} \frac{1}{a+x} \quad (38)$$

This relation, and all our work so far applies only to propagation in one cartesian dimension. Propagation in one spherical dimension will be more complicated and we have not yet carried out a proper analysis of that case. If we concern ourselves with radii which are not too small, so that the afterflow term is small, we expect that the peak velocity will behave like:

$$v_p \approx \frac{c^2}{\alpha\beta} \frac{1}{r(a+r)} \quad (39)$$

Recalling the peak velocity data from SALMON, we had $v_p = 14$ m/s at 166 meters and $v_p = 1$ m/s at 660 meters. Using these two data points to determine the constants in (39) we find $\frac{c^2}{\alpha\beta} = 4.6 \times 10^5$ m³/s and $a = 30$ meters. For other stations in the SALMON data we get

Table 2. Comparison of fit with SALMON data.

R	166	225	276	318	402	660
$V_{\max}$ (Formula)	14	8.1	5.6	4.4	2.8	1
$V_{\max}$ (Data)	14	8.0	5.1	3.8	2.5	1

It appears from the table that we get a pretty good fit to the SALMON peak velocity data using the nonlinear friction model. As we saw in the previous section however, the SALMON shock propagated not according to the nonlinear friction model but rather as a wave in an analastic solid. We do not know what to make of the apparently good fit shown in the table; probably one should view it as an example of the danger inherent in drawing too firm a conclusion about the validity of propagation models using only peak velocity data.

DISCUSSION

We have seen that the constitutive relation given in equation (23) provides for propagation having the properties set out at the beginning of this chapter. While we know from the detailed analysis of the SALMON data given in the previous chapter that the SALMON shock did not possess all these properties - in particular item 1 - it may be that the COWBOY data, taken at lower strains, will exhibit the properties and that the nonlinear friction model will be useful in modeling these results. We will not know whether or not this is the case until we complete a detailed study of the COWBOY data in the same way we have done for SALMON.

SECTION 4

CONCLUSIONS

From the studies presented in this report we draw the following principal conclusions:

1. The seismic wave from event SALMON propagated, in the range of strains from 10^{-3} to 10^{-4} , very much like a wave in an anelastic solid.
2. At some point in the propagation the attenuation must become less in order to join smoothly to small strain measurements.
3. By applying the procedures used in the present report for SALMON to the COWBOY data we should be able to study the nature of the transition in some detail.
4. Nonlinear friction can provide for propagation with properties much like those thought to be characteristic of the body of data for intermediate strains (though not for SALMON).

The arguments for item 2 are that Tittmann's laboratory data taken at strains of about 10^{-6} show much smaller absorption and that propagation through a medium as highly absorbing as we measure for SALMON, would not reach teleseismic distances; while it is true that teleseismic paths entirely through salt are not observed it is not generally thought that natural halite formations have low-amplitude absorption so much greater than other seismic media.

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APPENDIX A

ANALYTIC APPROACHES TO LINEAR AND NONLINEAR ATTENUATION*

SUMMARY

A review of near-field data from explosions indicates that nonlinear behavior extends well beyond a few hundred meters/ $kt^{1/3}$. Frequency dependent contributions to Q , perhaps from scattering, can lead to underestimates of magnitudes using spectral ratio methods.

INTRODUCTION

Until recently it has been assumed that the behavior of rock beyond the radius of cracking ($> 100 \text{ m}/kt^{1/3}$) due to a explosive source shows a linear but perhaps inelastic character. Beyond $\sim 300 \text{ m}/kt^{1/3}$, the "elastic radius," seismic propagation has been taken as nearly elastic with mild attenuation due to anelasticity which is generally described in terms of Q or Q^{-1} , the "internal friction." As a result a major topic for study for seismic detection is that of the behavior of path integrated Q^{-1} as a function of path, frequency and possibly amplitude. The approach to the study of Q^{-1} has been largely phenomenological and experimental with only moderate consideration of physical mechanisms or use of analytic techniques.

In this paper we shall review the data from explosions at ranges from about $10 \text{ m}/kt^{1/3}$ to $3000 \text{ m}/kt^{1/3}$ in order to establish the extent to

*This paper was submitted to the DARPA-sponsored meeting on BASIC RESEARCH IN THE VELA PROGRAM, May 1984, Santa Fe, New Mexico.

which the attenuation must be described as nonlinear and the extent to which Q^{-1} is dependent upon frequency. It will be found that these near-field data suggest a significant nonlinearity for which an empirical Q^{-1} is consistent with being linearly dependent upon amplitude at large strain but which becomes constant at small strains.

A. NEAR-FIELD DATA AND NONLINEAR ATTENUATION

Near-field data from explosive sources in salt provides the best available information on the attenuation of pulses in the moderate strain regime. These data are from: Larson's work in pressed salt with scaled ranges from $10 \text{ m/kt}^{1/3}$ to $200 \text{ m/kt}^{1/3}$ (mean frequency $\sim 10^4$ Hz); Salmon nuclear explosion in dome salt with ranges from $100 \text{ m/kt}^{1/3}$ to $400 \text{ m/kt}^{1/3}$ (mean frequency ~ 5 Hz); and Cowboy series tamped chemical explosions in dome salt with ranges from $200 \text{ m/kt}^{1/3}$ to $3000 \text{ m/kt}^{1/3}$ (mean frequency ~ 10 Hz or greater). A plot of peak velocity versus scaled range is shown in Figure A-1. This illustrates the well known result that simple cube root scaling applies with remarkable accuracy to data from events with a yield variation of ten orders of magnitude (and thus a range of frequencies spanning more than three orders of magnitude). This result is all the more interesting because it takes place in a regime where the peak values of both velocity and displacement fall off much more rapidly than $1/r$. That is, for this spherical geometry, the behavior cannot be simple elastic since the decay exceeds that for geometrical divergence alone. The residual attenuation may result from linear but inelastic (i.e., anelastic) behavior or from nonlinear attenuation. In either case, the existence of simple scaling demonstrates that the effective Q^{-1} must be nearly independent of frequency, at least for this frequency and strain regime. The peak strains ($\approx$ peak velocity/propagation speed) from Figure 1 range from about 10^{-1} to 10^{-5} .

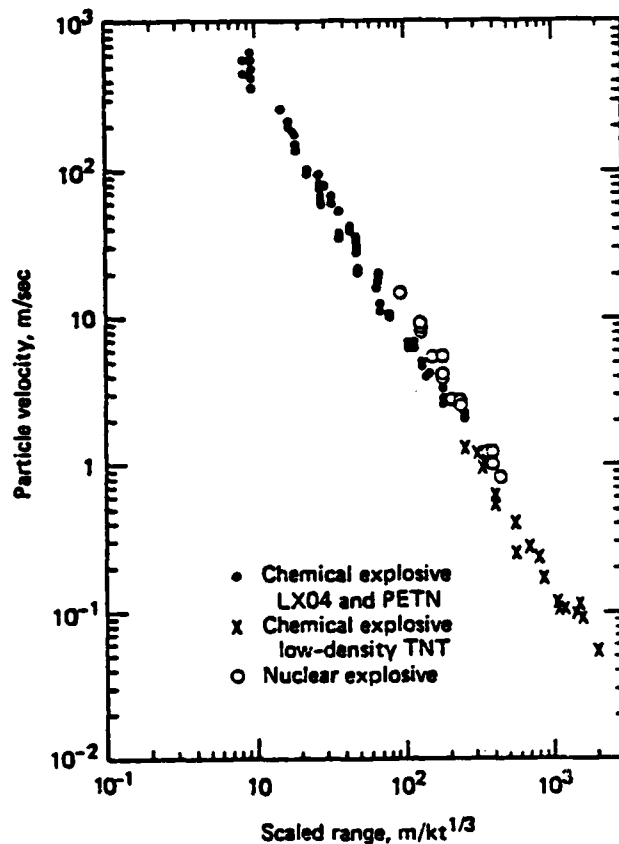


Figure A-1. Peak particle velocities from explosions in salt, from Larson (1982).

In order to address the question of linear or nonlinear attenuation, we now consider the implications for such possibilities. If a linear viscoelastic mechanism is to account for these data, the effect can be expressed in terms of a Q which is, at least approximately, independent of frequency over the frequency range of the experiments. This leaves some latitude in selection of an attenuation model but since the behavior in other frequency regimes has little effect, a reasonable attempt is to simply take Q to be completely independent of frequency. Such a model has been constructed by Kjartansson (1980) who provides rather simple expressions for attenuation and dispersion which can be written as a complex phase velocity

$$c(\omega) = e^{\frac{-i\pi\gamma}{2} \text{sgn}(\omega)} \left(\frac{\rho}{M_0} \left(\frac{\omega_0}{\omega} \right)^{2\gamma} \right)^{1/2}$$

where M_0 is a modulus, ρ the density, ω_0 an arbitrary frequency scale and γ is an arbitrary measure of the attenuation rate. In particular, $\gamma = Q^{-1}/\pi$ if $Q \gg 1/\pi$. For a spherical pulse with a velocity spectrum at radius 'a' of $\tilde{V}(a, \omega)$ the velocity pulse at any time is given by

$$V(r, t) = \frac{1}{2\pi} \int d\omega \tilde{V}(a, \omega) \left(\frac{\frac{i\omega}{cr} - \frac{\alpha}{r} - \frac{1}{r^2}}{\frac{i\omega}{ca} - \frac{\alpha}{a} - \frac{1}{a^2}} \right) \exp(-\alpha(r-a) + i\omega \left(\frac{r-a}{c} - t \right))$$

where $c = \left(\frac{M_0}{\rho} \right)^{1/2} \left| \frac{\omega}{\omega_0} \right|^\gamma (\cos(\pi\gamma/2))^{-1}$ is the phase velocity

and $\alpha = \omega c^{-1} \tan(\pi\gamma \text{sgn}(\omega)/2)$ is the spatial attenuation constant.

A velocity pulse from Salmon at a scaled range of $230 \text{ m/kt}^{1/3}$ is shown by Larson (1982) and this is used as the initial point for a study of effects of anelastic attenuation for constant Q . A sequence of pulses at increasing range is shown in Figure A-2 including frequency dependent attenuation (α is proportional to ω for constant Q) and the attendant dispersion. Attenuation and lengthening of the pulse are evident for this case using $Q=10$. The corresponding decay of peak velocity as compared to the Salmon and Cowboy data is shown in Figure A-3. The value of Q was chosen to match approximately the slope of the Salmon data ($v_{\max} \sim r^{-1.88}$) at the initial range and it is apparent that at larger ranges the decay is somewhat more rapid for constant Q . (It is easy to show that for sufficiently large ranges the peak velocity will fall off like r^{-2} if the spectrum is flat below the corner frequency. Smaller Q (greater attenuation)

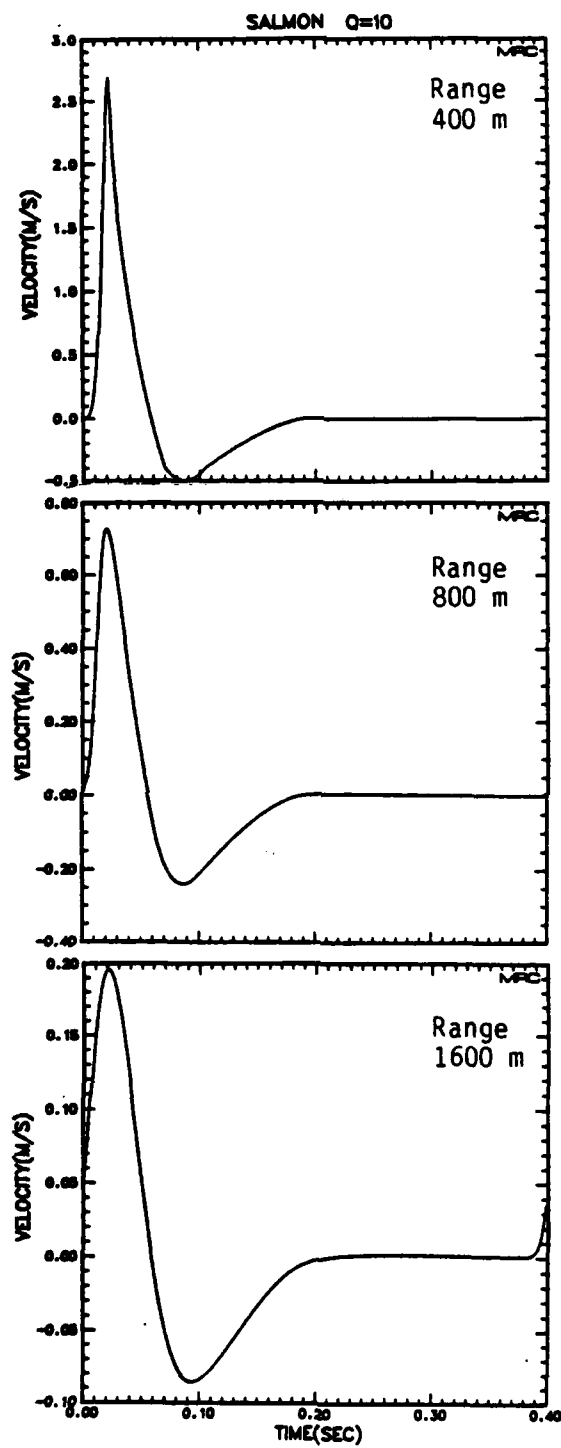


Figure A-2. Waveform of SALMON velocity pulse at three ranges for $Q = 10$.

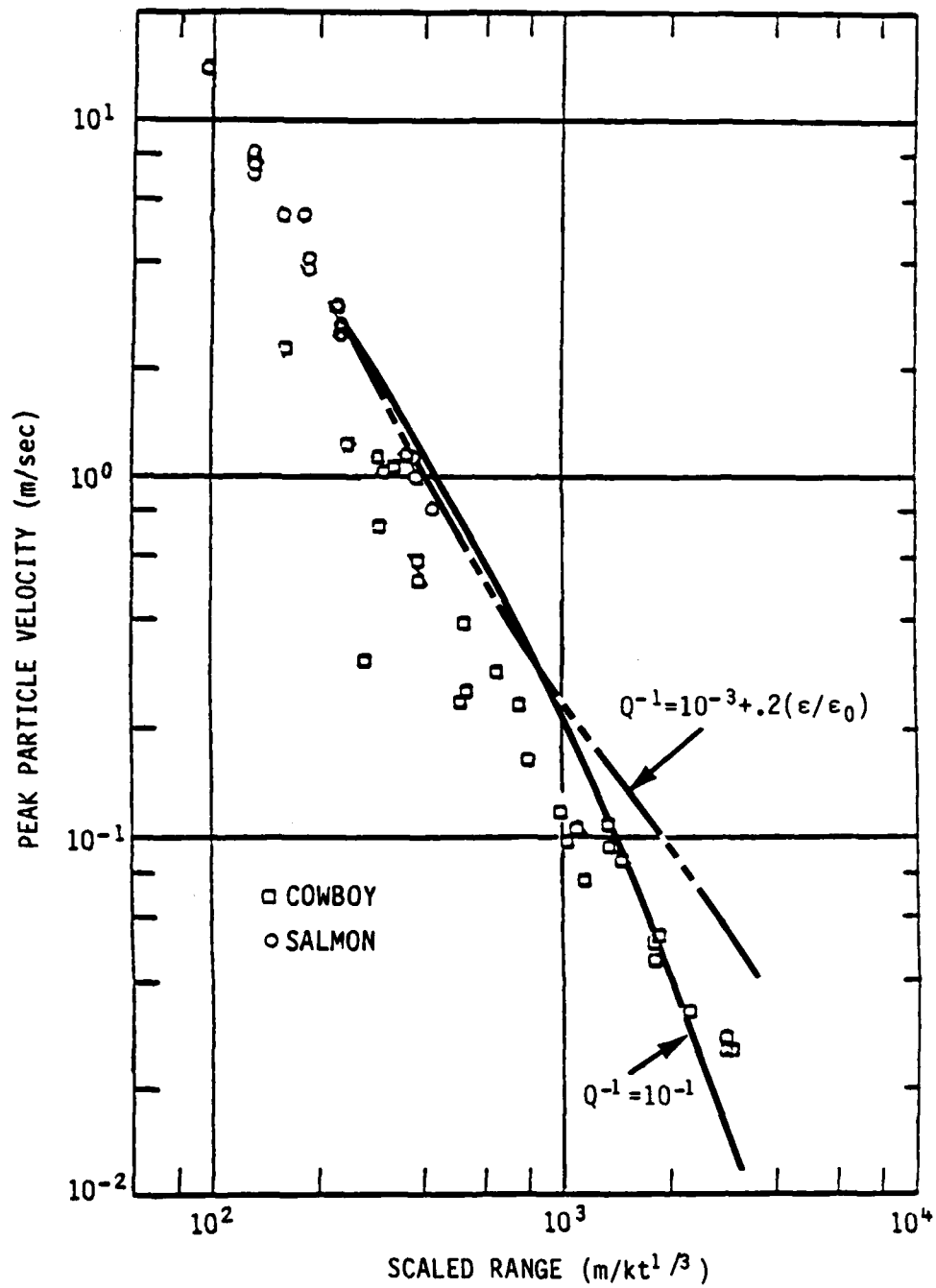


Figure A-3. SALMON and COWBOY velocity data compared for constant $Q = 10$ and nonlinear Q models (data from Trulio (1978)).

results in a more rapid onset of the r^{-2} behavior.) From this it seems that, in the absence of other constraints, a rather small $Q \sim 10$ from an anelastic model can give a rough fit to the Salmon and Cowboy data. However recent work at Rockwell (Tittmann, 1983) indicates that for strains from 10^{-8} to 10^{-6} , the Q from dome salt is approximately 500 as measured at ~ 500 Hz. Tittmann further notes that at strains greater than 10^{-6} Q^{-1} increases significantly at least up to strains of 10^{-5} which is the maximum available in the experiment. In short the experiments suggest that attenuation increases with amplitude (i.e., it is nonlinear) for strains in excess of 10^{-6} . The magnitude of the measured Q at small strains is much different from that required to fit the explosion data suggesting that a linear mechanism cannot account for these data. Furthermore the Rockwell data for nonlinear behavior indicate an onset at $\sim 10^{-6}$ suggesting that, since even the smallest strain from Cowboy is $\sim 10^{-5}$, nonlinear behavior is a vital element in accounting for the decay of the explosively generated pulses in the near-field. We intend to apply the spectral ratio method to Salmon and Cowboy data to verify nonlinearity as well as to explore any frequency dependence of Q .

Microscopic models of frictional attenuation due to sliding of crack surfaces such as Mavko (1980) and Stewart et al. (1982) generate a contribution to Q^{-1} which is proportional to the peak strain during a cycle but which is also independent of frequency. As a result it has been suggested that an approximate Q can be written as

$$Q^{-1} \approx Q_0^{-1} + (\epsilon/\epsilon_0) Q_{\epsilon_0}^{-1}$$

where Q_0 is the "intrinsic" Q and the second term is the frictional Q which is linear in strain (and normalized at ϵ_0). Such a form has been considered by Minster and Archambeau (1983) in a fashion similar to that which follows here. The above form for a nonlinear Q ignores the fact that Q is inherently a linear concept. Here it is to be interpreted to

mean that the spectral content will decay with range, r , according to a factor

$$\exp\left(-\frac{\omega r}{2cQ}\right)$$

where the strain, ϵ , which appears is taken as the peak strain during the pulse. Using such a model for nonlinear behavior, we have found the attenuation of the Salmon velocity pulse by a closely spaced series of Fourier synthesis advancements in which Q^{-1} is taken to be constant in each small step but altered (in fact decreased) according to the amplitude between steps. The limiting result for sufficiently small steps is shown in Figure 3. In the example shown $Q_0 = 10^3$ and Q at the smallest range (230 m/kt^{1/3}) was 5. These variable Q parameters have been chosen both to give a reasonable fit to the Salmon data (which are always a bit above the Cowboy data) and to provide an onset of nonlinear behavior which is consistent with the Rockwell experiments. However the result is rather insensitive to the value of Q_0 because the smallest strain in the explosion data is well above 10^{-6} .

It appears that a model of nonlinear Q^{-1} with a dominant term proportional to the strain gives a decay of the pulse from explosions in salt which is consistent with observation as well as consistent with small strain behavior of Q . However the data are not adequate to fix the exponent of the strain in Q^{-1} . For example $Q^{-1} \propto \epsilon^{1/2}$ or ϵ^2 can also give a fair fit to the data although the transition to linear behavior is moved somewhat from the 10^{-6} value. The strongest statement we can make is that the assumed form of Q is consistent with the explosion and laboratory data but the particular form is not tightly constrained. Still a nonlinear mechanism appears to be necessary to account for the two types of data.

B. IMPLICATIONS OF Q BEHAVIOR FOR TELESEISMIC SIGNALS

Customary calculations of teleseismic propagation of signals from explosions and earthquakes often treat a near-source region as giving the initial conditions for a subsequent linear propagation. In the case of explosions this "elastic radius" is often taken as $\sim 300 \text{ m/kt}^{1/3}$ and it is assumed that all nonlinear behavior occurs inside this radius. It is now generally thought that nonlinear effects extend out to larger radii, perhaps out to $\sim 10^4 \text{ m/kt}^{1/3}$. The behavior in this intermediate strain regime is a significant source of uncertainty in the ultimate problem of relating teleseismic signals to source characteristics.

From the point of view of linear calculations of seismic signals, nonlinear mechanisms provide anomalous attenuation which has the potential for altering the relation between teleseismic signals and the near-field pulse. However mere existence of nonlinear attenuation does not necessarily negate spectral ratio methods unless the nonlinear aspect has a different frequency dependence than the assumed Q. This is due to the fact that spectral ratio methods depend upon finding a path-integrated Q^{-1} , which is generally expressed at t^* , which may or may not be a function of frequency. If, for example, t^* is taken as independent of frequency and found by matching the high-frequency slope of the observed spectrum to an assumed source spectrum, the existence of a frequency independent but nonlinear contribution to t^* would not alter the result except to raise the value of t^* which is proportional to the mean value of Q^{-1} along the path. On the other hand, if the nonlinear contribution to t^* has a frequency dependence different from the assumed one, use of a suitable slope connecting t^* to find a magnitude at the source may be in error. This question has been raised by Der (1983) in a slightly different context. This arises from use of spectral shape changes with propagation distance to deduce information about spectral amplitude changes. In particular it is noted that if the true t^* , which describes the amplitude change, is related to the spectral slope apparent $\overline{t^*}$ by

$$\overline{t^*} = t^* + \omega \frac{\partial t^*}{\partial \omega}$$

If t^* is independent of ω the two are identical but if t^* decreases with frequency $\overline{t^*} < t^*$. According to the work of Lundquist and Samowitz (1983), such is in fact the case although this remains a question for discussion. For example, Der and Lundquist, working with exactly the same data set, have found significantly different values for t^* . Using an absorption band model for Q (for which $Q \propto \omega$ at large ω) Lundquist has fit the data using as variables the low-frequency t^* and a period below which r^* rolls off. Der, on the other hand, has fit the data assuming $\overline{t^*}$ is constant in various frequency bands and then inverted the above relation to find t^* . At 2Hz the difference between the fits at distant stations appear to be $\Delta t^* \sim .1$ to $.2$ sec so this translates into a source intensity ratio of ~ 10 . That is, attenuation will be underestimated if t^* is fit as constant and this will cause the magnitude of the source to be underestimated.

It should be noted that if Q^{-1} has a term which is proportional to ω^{-1} this will serve to provide an overall attenuation of the spectral amplitude but it will not alter the spectral shape so that the contribution to $\overline{t^*}$ will be zero. Thus it is very important to include any such effects which, for example, occur at high frequency in the absorption band model. Observations and analysis by Aki and Chouet (1975) of coda attenuation indicate that at least some portions of the coda result from single scatterings of the main pulse by inhomogeneities in the lithosphere. Earthquake coda have been analyzed to find an effective Q which is frequency dependent giving Q increasing with frequency from 1 to 25 Hz. Dainty (1983) has looked at other earthquakes finding similar effects showing Q , apparently due to scattering, which is approximately proportional to frequency in the 3 to 50 Hz regime.

Dainty, in a theoretical analysis, has found the contribution of scattering to Q by considering the Born approximation in the presence of randomly distributed inhomogeneities in propagation speed in the medium. It is found that if the autocorrelation function of the inhomogeneities has a nonzero first derivative at zero lag, the contribution to Q is proportional to frequency. It is suggested that discontinuities at the scattering structure edge lead to this result. Basically the Born scattering Q will be proportional to ω for wavelengths small compared to the overall dimensions of the structures but large compared to the thickness of the edges.

Independent of the exact nature of the physical mechanism which leads to the lithospheric contribution to Q , the fact that the observed character is $Q \propto \omega$ is of interest for application of spectral ratio methods to waves which pass through the lithosphere. Both Aki and Dainty indicate that the Q found from coda takes a value of ~ 200 at frequencies of a few Hertz. As was previously noted a $Q^{-1} \propto \omega^{-1}$ will provide an overall attenuation of the amplitude of the pulse spectrum but it will not alter the shape. An underestimate of source magnitude will result from propagation through the lithosphere and the degree can be found, given a path length. For example for a path of 100 km an additional Q^{-1} of 5×10^{-3} at 2 Hz will lead to an apparently decreased source magnitude at all frequencies of

$$\exp\left(\frac{2\pi fr}{cQ}\right) \approx 5$$

if the source of attenuation is not explicitly taken into account.

C. CONCLUSIONS AND RECOMMENDATIONS

A preliminary study of the attenuation observed for explosively generated near-field pulses in salt indicates that the behavior is dis-

tinctly nonlinear for the strains for 10^{-1} to 10^{-5} . The observed behavior is consistent with a Q^{-1} which is the sum of a constant (Q at small strains are found in Rockwell experiments) and a term linear in the strain (which is as found in frictional crack models). It is recommended that dynamical models be formulated which conform to these near-field data for inclusion in calculations of waveforms out to a proper elastic radius.

A frequency dependent Q will alter the relation between spectral amplitude and spectral shape leading possibly to substantial underestimates of source magnitudes using spectral ratio methods. This is especially true for lithospheric scattering which gives a contribution to Q which may not alter the spectral shape at all. It is important to establish the extent and character of scattering contributions to Q in order to refine magnitude estimation methods.

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APPENDIX B

SALMON DATA

SALMON was a 5.3 kT nuclear explosion which took place at a depth of 800 meters in a salt dome in Mississippi on October 22, 1964. It was part of a decoupling experiment in which explosion Sterling took place in the SALMON cavity at a later time. The SALMON explosion was one of the best instrumented cases ever carried out due to the unique character of the experiment. As a result, the data from SALMON have proved to be the best available for nuclear explosions in terms of the intermediate range ground motion recordings. Furthermore, by virtue of the rather uniform character of the salt dome, the data are internally quite consistent.

The data were originally recorded by analog methods by Sandia and they were later converted to digital form at that laboratory. The intermediate range data from 166 meters out to 660 meters were analyzed in great detail by Perret¹ as well as by Rogers² and Patterson³ shortly after the event. In more recent times some aspects of the data have been reexamined by Murphy⁴, Perl⁵, and Trulio⁶, and informally by others. The taped data records reported here were supplied to us by Neil Perl of Applied Theory, Inc., who originally received them from Sandia.

Figure B-1 indicates the arrangement of sensors for the SALMON event. Most of the instrument stations had several sensors so there is some redundancy in the records as well as information concerning, often, all three axes of motion. The records supplied to us are of two general types, acceleration (A) and velocity (U). The orientations are either horizontal radial (R), vertical (V) or horizontal transverse (T). The

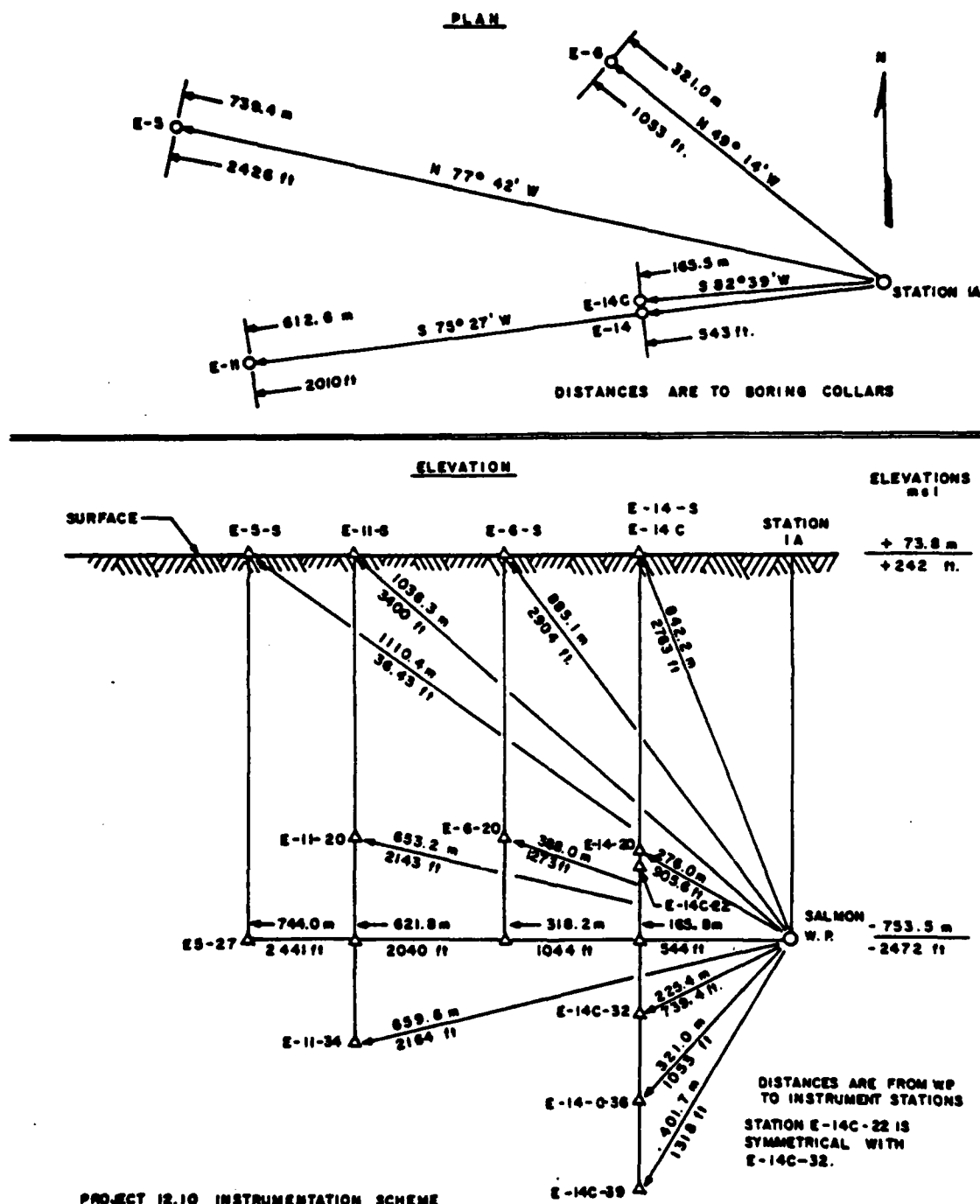


Figure B-1. Instrumentation locations for SALMON (Reference B-1).

records available are given in Table B-1 with the designation E#-D-A0 where # indicates the bore hole number, D the depth in hundreds of feet, A acceleration or velocity and 0 the orientation.

Table B-1. SALMON data records.

E5-S-AR	E6-27-UR	E11-34-AR	E14C-27-AR
-S-AT	-27-UT	-34-AV	-27-AV
-S-AV	-27-UV	-34-UR	-27-UV
-27-AT	E11-S-AR	-34-UV	-32-AR
-27-AV	-S-AT	E14-20-AR	-32-AV
-27-UT	-S-AV	-20-AV	-32-UR
-27-UV	-S-UR	-20-UR*	-32-UV
E6-S-AR	-S-UV	-34-AR	-36-AR
-S-AT	-20-AR	E14C-S-AR	-36-AV
-S-UR	-20-UR*	-S-AV	-36-UR
-S-UV	-20-UV	-S-UR	-36-UV
-20-AR	-27-AV	-22-AR	-39-AR
-20-AV	-27-UR	-22-AV	-39-UR
-27-AR	-27-UT	-22-UR	-39-UV
-27-AV	-27-UV	-22-UV	

* Instrument Mis-oriented

The records of interest in our study are the subsurface cases excluding transverse motion. The transverse motion measurements would be zero for perfect spherical symmetry and for perfect orientation. (Of course, they are not exactly zero.) We have looked only at the AR, AV, UR, and UV cases excluding those at the surface. The records consist of approximately 27,700 pairs of samples at intervals of 0.2 milliseconds. Each pair consists of a time in units of seconds and an acceleration (units of g, the acceleration of gravity) or a velocity (units of feet per second). The time ranges from about -0.544 seconds to 5.0 seconds at integer multiples of 200 microseconds.

The following figures provide the raw velocities as taken directly from the taped records from 0 to 1.6682 seconds with no corrections at all. Records which were of acceleration have been converted to velocity by a simple rectangular rule integration. The velocities have been converted to units of meters/sec to be consistent with that used in the main text as well as that used by Perret. A few of the cases recorded on tape had two versions. In this case, we have produced the one which corresponds to that given by Perret.

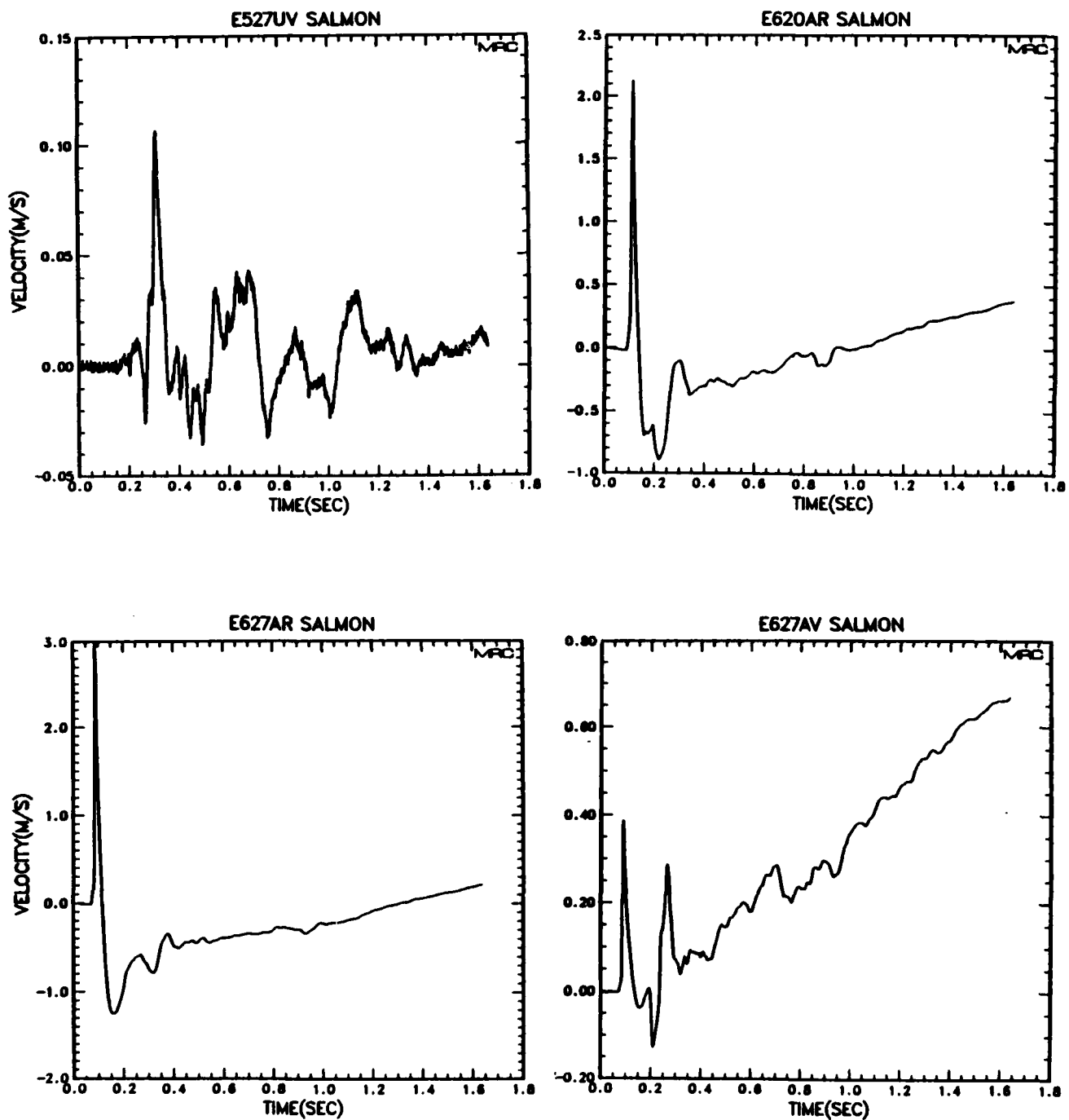


Figure B-2. Raw SALMON velocity records.

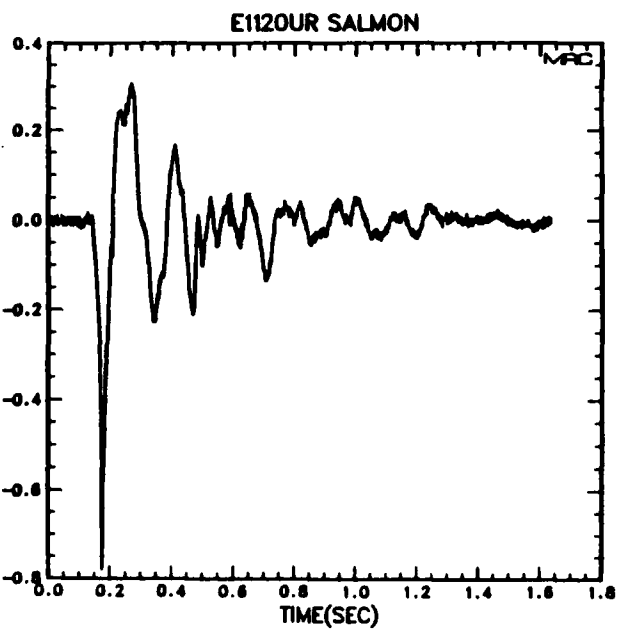
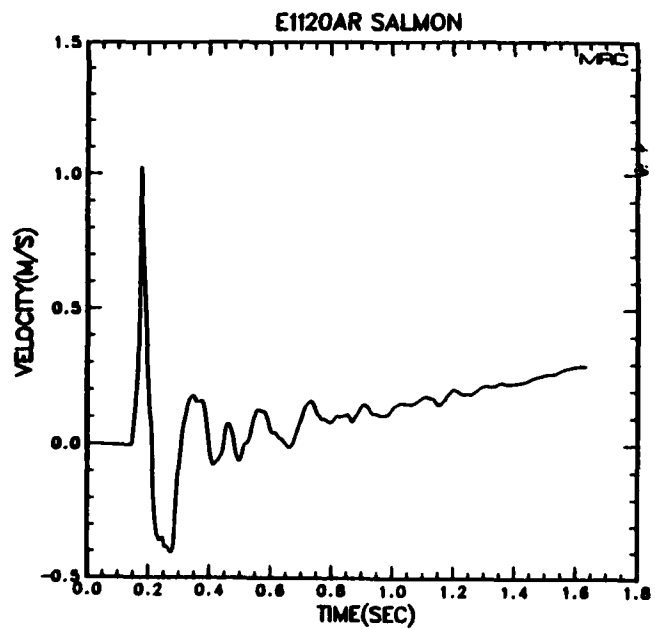
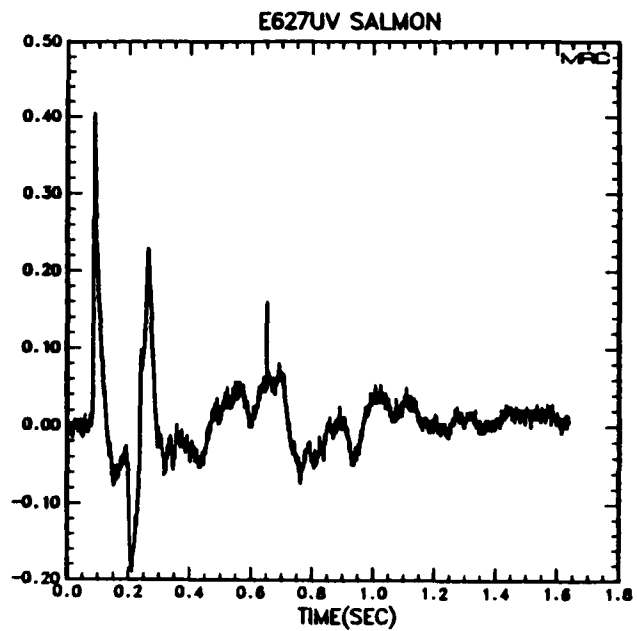
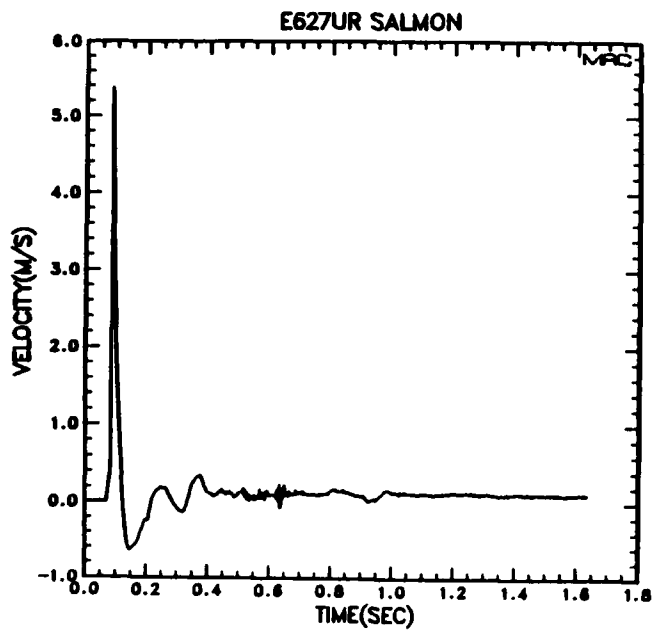


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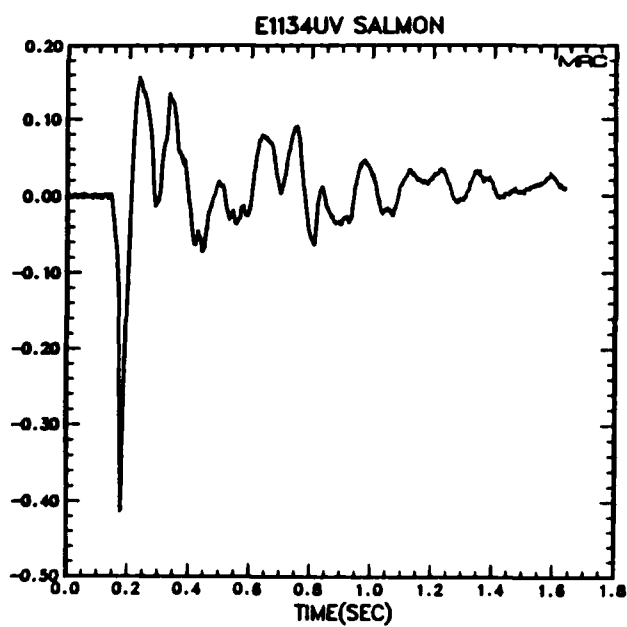
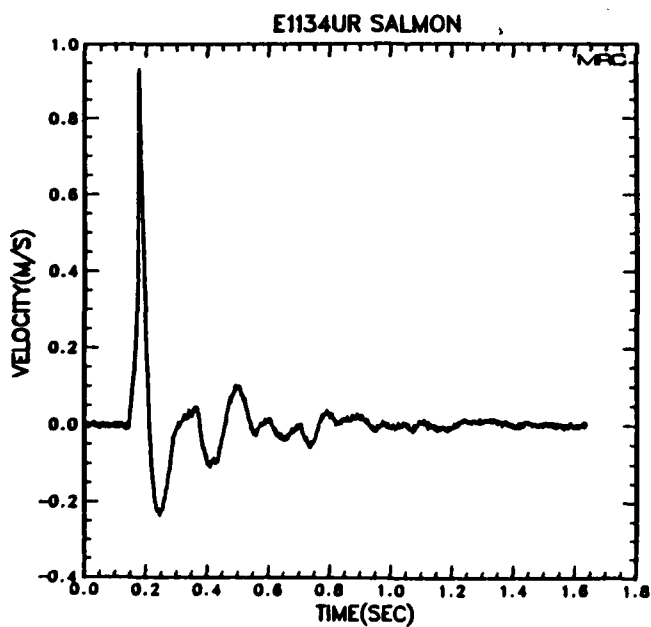
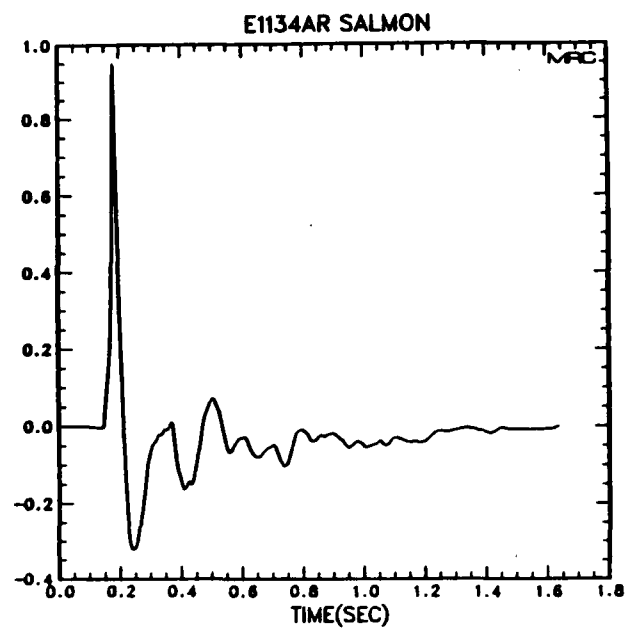
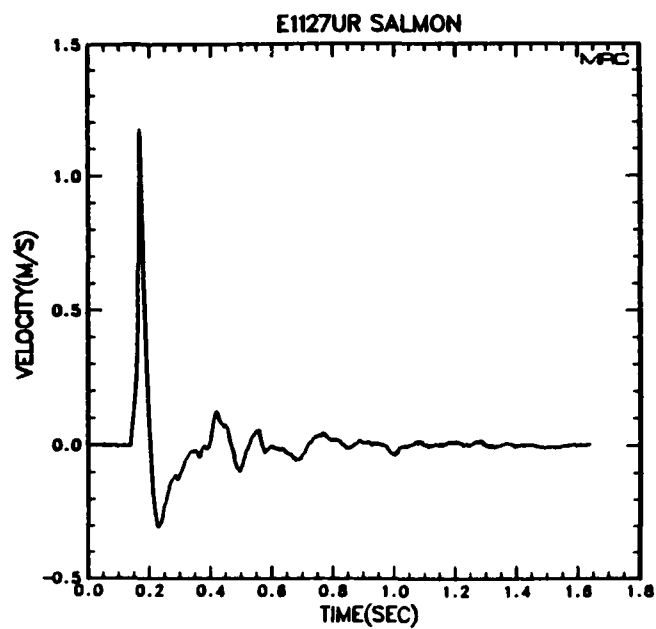


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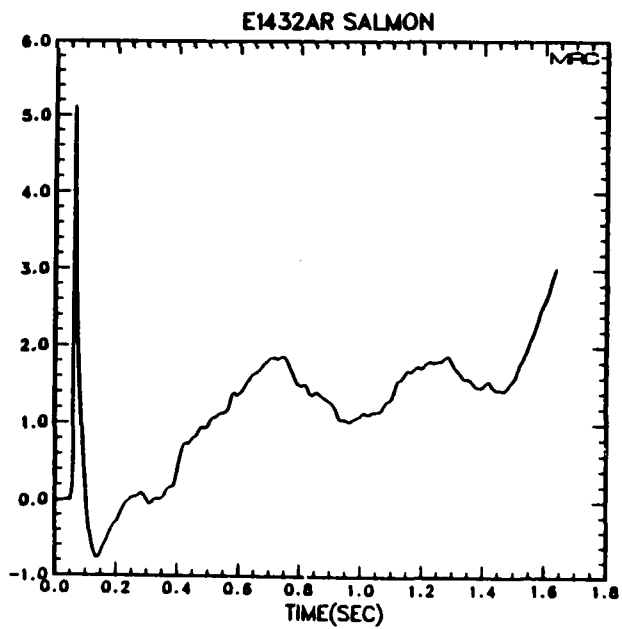
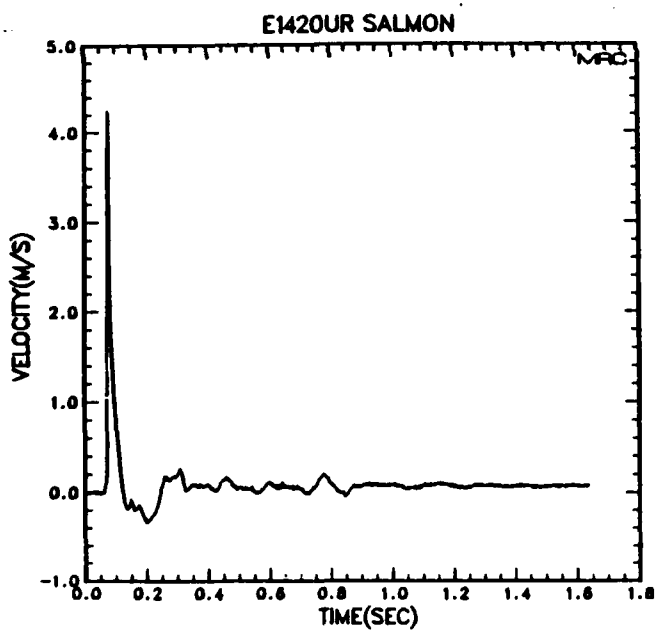
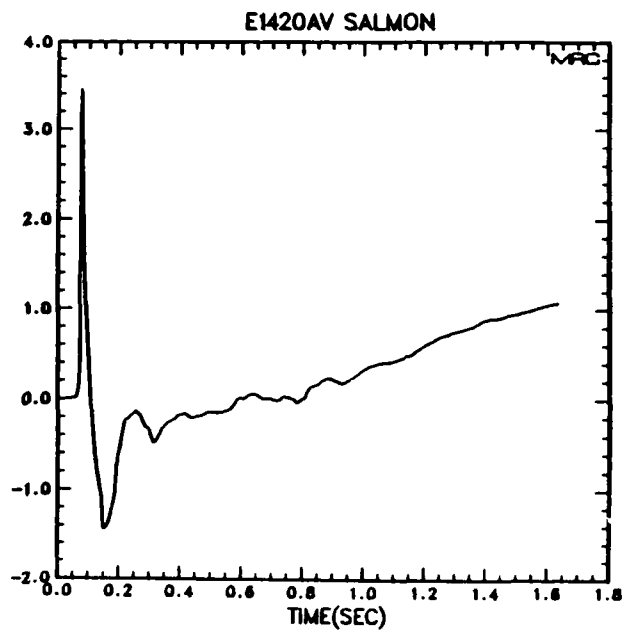
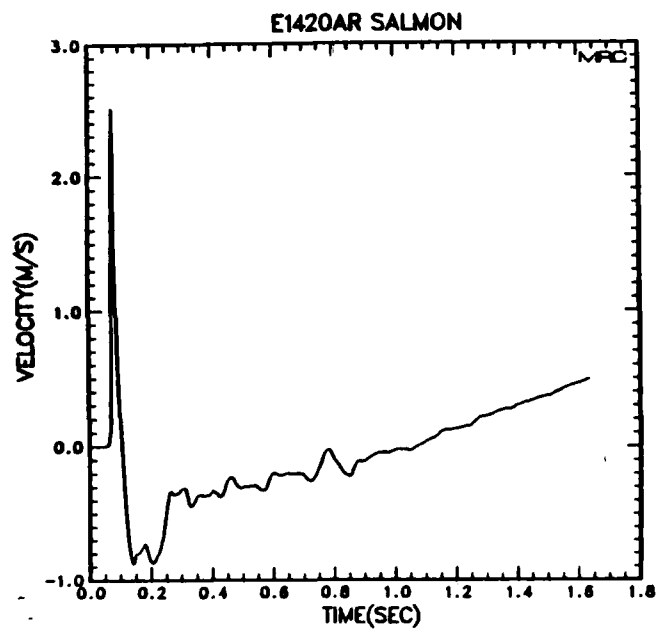


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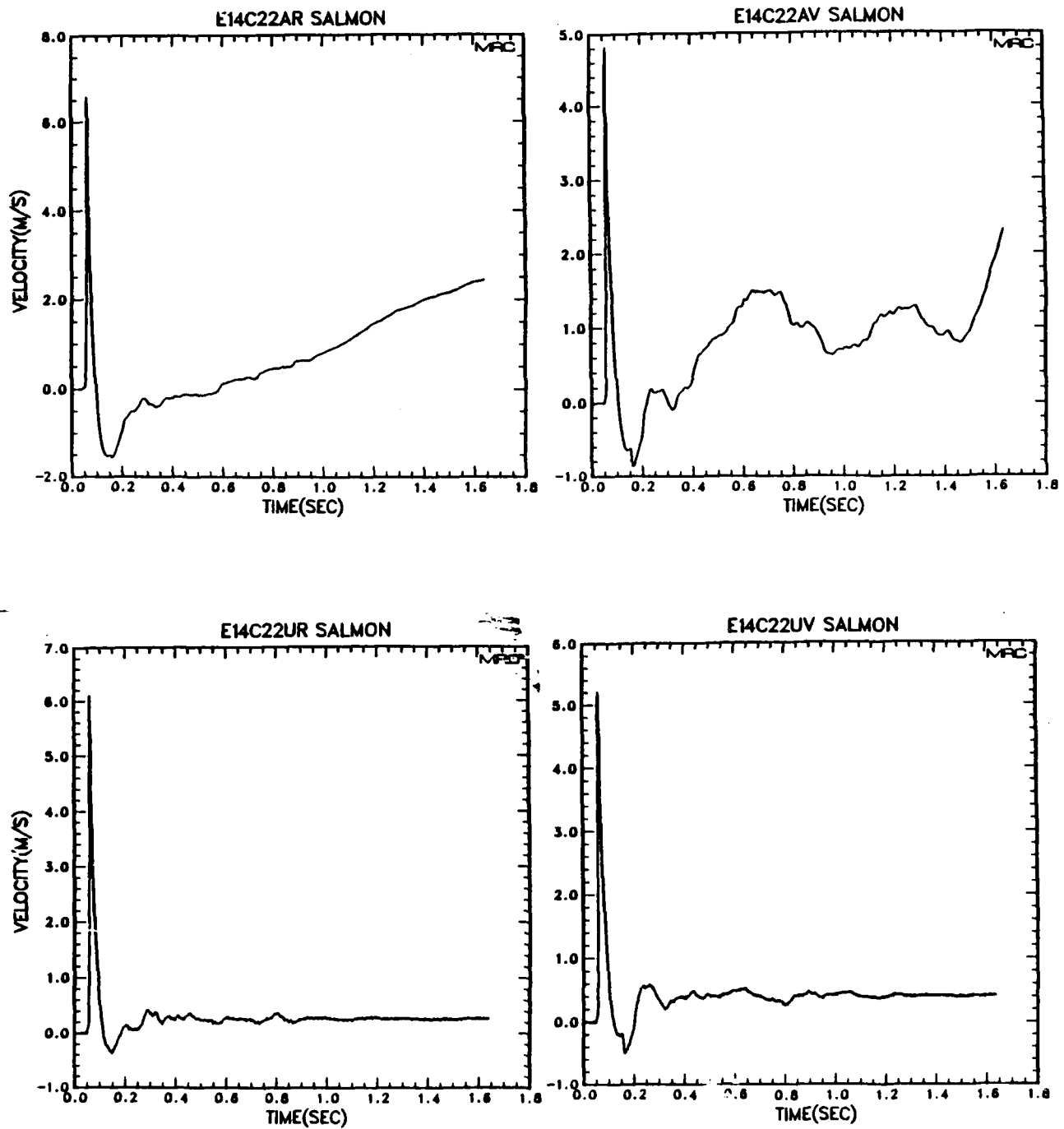


Figure B-2. (Continued).

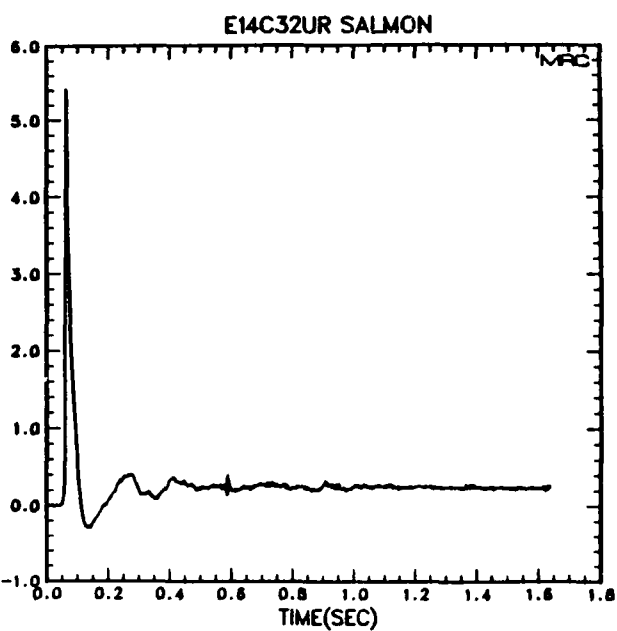
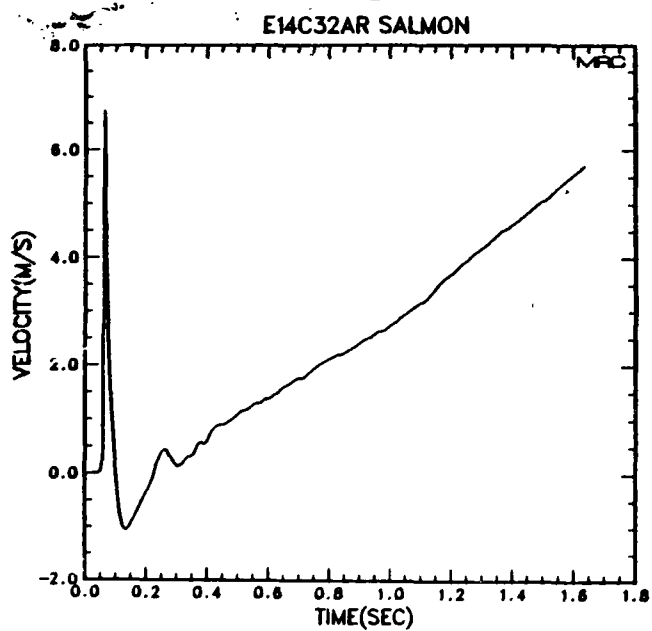
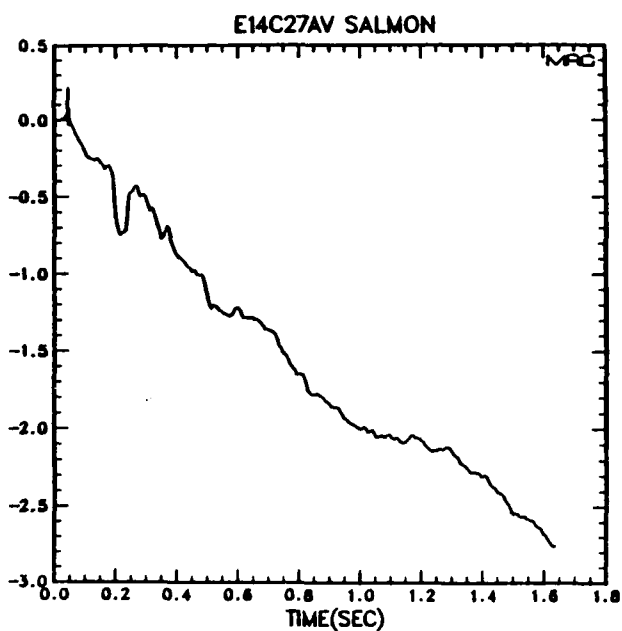
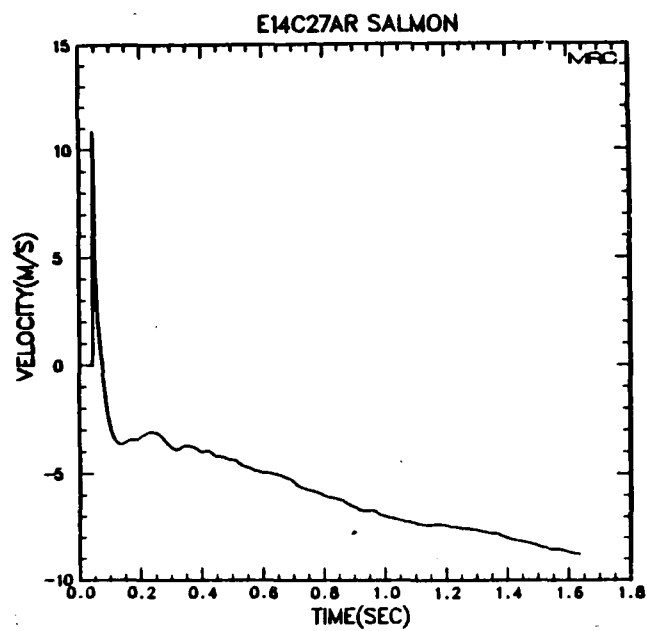


Figure B-2. (Continued).

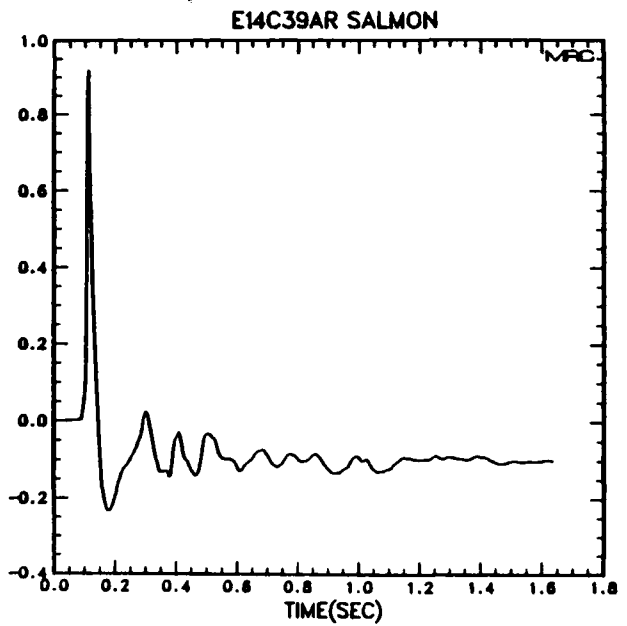
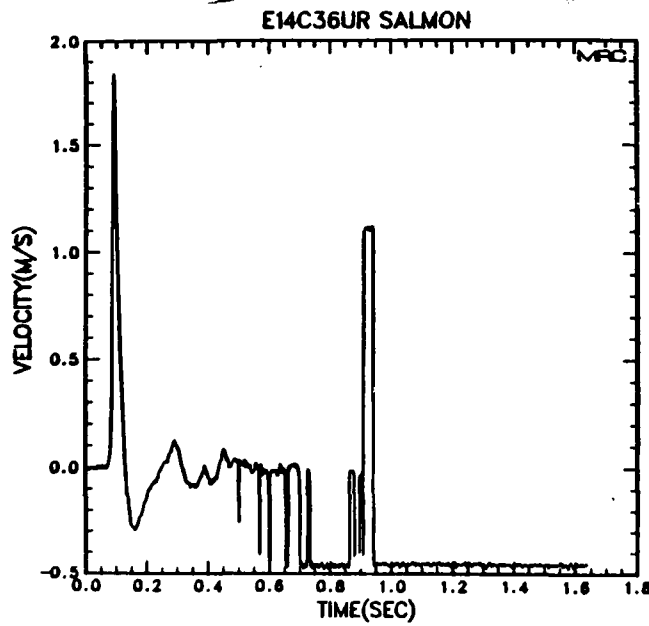
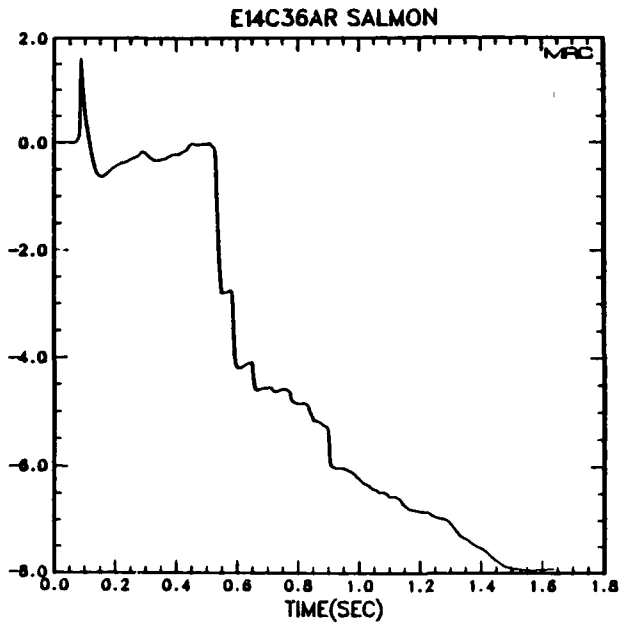
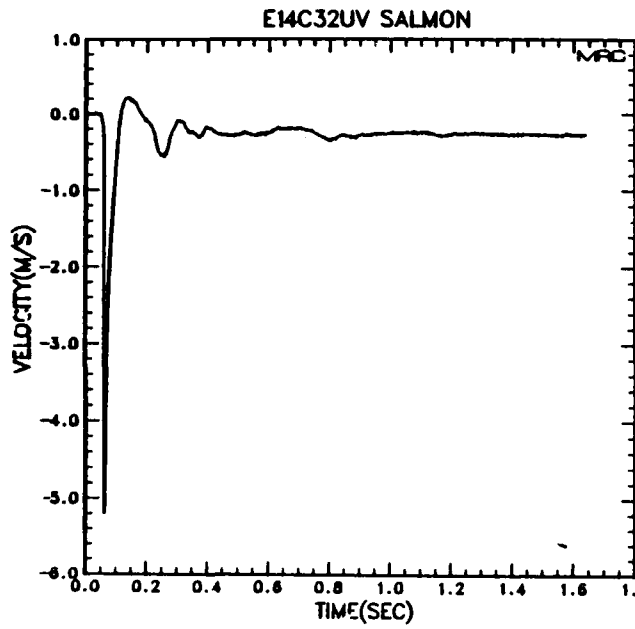


Figure B-2. (Continued).

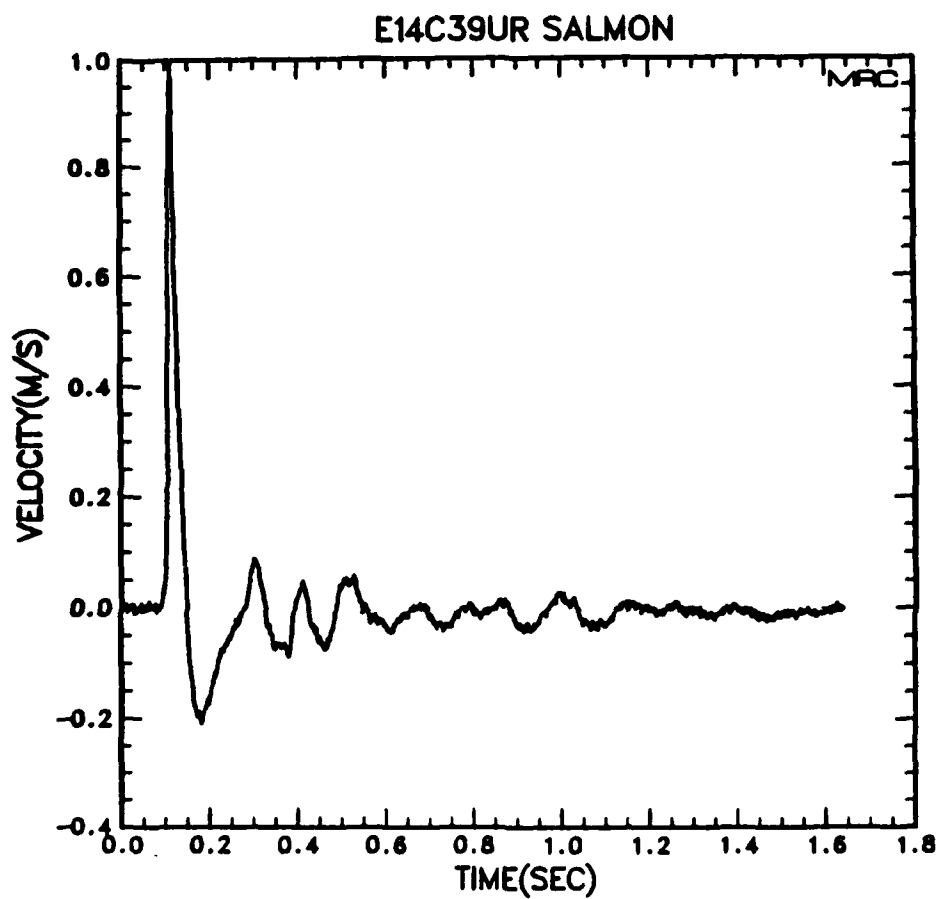


Figure B-2. (Continued).

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