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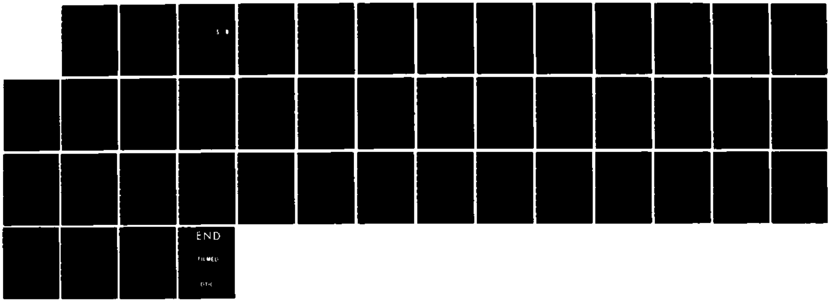

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OPTIMAL LOW THRUST TRANSFER
USING A FIRST-ORDER PERTURBATION MODEL

THESIS

AFIT/GA/AA/84D-1

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THESIS

Presented to the Faculty of the School of Engineering
of the Air Force Institute of Technology
Air University
in Partial Fulfillment of the
Requirements for the Degree of
Master of Science

by
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March 1985

Acknowledgements

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List of Symbols

N - number of revolutions of the central body in an orbital transfer

a - semimajor axis

t - time

E - energy

μ - gravitational parameter

$\mathbf{F}$ - vehicle thrust vector

$\mathbf{V}$ - vehicle velocity vector

V_{circ} - circular orbit velocity

m - vehicle mass

ν - true anomaly

R - radial acceleration component

T - tangential acceleration component

N - normal acceleration component

e - eccentricity

n - mean motion

r - distance from center of mass of the central body

i - inclination

ω - arguement of periapsis

Ω - longitude of ascending node

u - arguement of latitude

p - semi-latus rectum

h - alternate orbital element

k - alternate orbital element

H - magnitude of angular momentum

$\mathbf{A}$ - magnitude of vehicle acceleration

θ - pitch angle

Ψ - yaw angle

ϵ - dimensionless ratio of vehicle acceleration to local gravitational acceleration

$\mathbf{H}$ - Hamiltonian

λ_1 - lagrange multiplier associated with a

λ_2 - lagrange multiplier associated with $\mathbf{h}$

λ_3 - lagrange multiplier associated with $\mathbf{k}$

U_1 - quantity combining λ_2 and λ_1

U_2 - quantity combining λ_3 and λ_1

a_0 - initial value of a

a_f - final value of a

u_0 - initial value of u

u_f - final value of u

Abstract

An analytic solution to the optimal low thrust transfer problem is developed using a first-order perturbation approach for planar orbit transfers. This solution is verified by comparison with the solution obtained by integrating the equations of motion. For low vehicle accelerations the analytic solution yields results quite close to those obtained by the integrated solution. At higher acceleration levels, however, the analytic solution diverges from the integrated solution. For very low acceleration levels the analytic solution approachs the limiting case of the infinitesimally low thrust spiral solution.

OPTIMAL LOW THRUST ORBIT TRANSFER USING A FIRST-ORDER PERTURBATION MODEL

I. Introduction

In performing any orbital transfer it is essential to determine the optimum thrust vector profile. This problem is easily solved for two extreme cases: the continuous thrust spiral transfer for a vehicle having infinitesimally small acceleration, and the two burn impulsive transfer for very high vehicle acceleration. Between these two extremes, the optimal transfer problem has been addressed by a variety of individuals. Several authors[1,2,3] have considered continuous thrust transfers requiring one revolution or less to complete, while others[4] have investigated transfers requiring many revolutions using some version of the method of averaging. This latter approach usually leads to a series of small, nearly impulsive, burns and is suboptimal.

It is always possible, in principle, to solve the transfer problem through integration of the equations of motion. Unfortunately, the amount of computation required for transfers involving many revolutions is prohibitive, and numerical errors rapidly become unacceptably large. Wiesel and Alfano[5,6] have developed an analytical approach which leads to a more tractable solution of the optimal transfer problem for many revolutions. Their approach involves splitting the control problem into a "fast" timescale problem over one or a few orbits, and a "slow" timescale problem over an entire transfer. The slow timescale solution incorporates a series of solutions to the fast timescale problem. This approach leads to an optimal solution for an infinite number of revolutions. Wiesel and Alfano found that the errors in obtaining desired final conditions were proportional to $1/N$, where N was the number of revolutions in the transfer. This result suggests that the solution of the optimal control problem for finite revolution transfers should be amenable to a perturbation theory approach.

This paper will develop an analytic solution to the fast timescale problem using a first-order perturbation approach. This analytic solution is compared with the solution obtained by integrating the equations of motion to determine its range of validity. Based on these results, some conclusions are drawn regarding the application of perturbation theory to the low thrust transfer problem.

II. Low Thrust Spiral Solution

As a limiting case, the optimal control solution for a vehicle having infinitesimally small acceleration is developed for a planar transfer beginning in a circular orbit. The solution is optimal in the sense of minimizing the time required to perform the transfer and, hence, maximizing the time rate of change of the semimajor axis, a. The rate of change of the semimajor axis is given by[7]

$$\frac{da}{dt} = \frac{da}{dE}\frac{dE}{dt} = \frac{\mu}{2E^2}\frac{dE}{dt} \tag{1}$$

where the energy per unit mass is

$$E = -\frac{\mu}{2a} \tag{2}$$

and the time rate of change of the energy is

$$\frac{dE}{dt} = \frac{\mathbf{F}\cdot\mathbf{V}}{m} = \dot{\nu}\left(\frac{dr}{d\nu}R + rT\right) \tag{3}$$

In the above equations, the gravitational parameter is represented by μ, m is the vehicle mass, ν is the true anomaly, and $\mathbf{F}$ and $\mathbf{V}$ are the vehicle thrust and velocity vectors, respectively. The orbital radius, r, is the same as the semimajor axis for a circular orbit. R and T are, respectively, the radial and tangential components of the vehicle acceleration. The derivative of r with respect to ν is

$$\frac{dr}{d\nu} = \frac{r\,e\sin\nu}{1 + e\cos\nu} \tag{4}$$

where e is the orbital eccentricity.

For a circular orbit eccentricity is zero, so the right hand side of Equation (4) is zero. Hence, the radial term in Equation (3) is initially zero and the vehicle directs its thrust tangentially. Since the vehicle has infinitesimally small acceleration, orbital eccentricity must remain infinitesimally small and the right hand side of Equation (4) will always be nearly zero. As a consequence, in accordance with Equation (3), a radial acceleration component will not result in any change in energy. Since energy is affected only by the tangential acceleration component, the optimal control solution is to direct the thrust vector tangent to the orbit. This corresponds to a yaw angle of zero.

For zero yaw angle, the dot product of the vehicle thrust vector and the vehicle velocity vector is

$$\mathbf{F}\cdot\mathbf{V} = FV \tag{5}$$

so the rate of change of the semimajor axis is given by

$$\frac{da}{dt} = \frac{2a^2}{\mu}\frac{F}{m}V_{circ} = \frac{2F}{m}\sqrt{\frac{a^3}{\mu}} \tag{6}$$

where V_{circ} is the circular orbit velocity. Separating variables and integrating:

$$\int \frac{da}{a^{3/2}} = \frac{2F}{m\mu^{1/2}} \int dt \tag{7}$$

yields the semimajor axis as a function of time:

$$a(t) = \frac{1}{a_0^{-1/2} - \dfrac{2F}{m\mu^{1/2}}(t - t_0)} \tag{8}$$

III. Equations of Motion

Selection of Form

It is desirable to select a form of the equations of motion that will make analysis of the low thrust transfer problem as easy as possible. A useful form is the Lagrange planetary equations expressed in terms of force components.[7,8]

$$\frac{da}{dt} = \frac{2e\sin\nu}{n\sqrt{1-e^2}}R + \frac{2a\sqrt{1-e^2}}{nr}T \tag{9}$$

$$\frac{de}{dt} = \frac{\sqrt{1-e^2}\sin\nu}{na}R + \frac{\sqrt{1-e^2}}{na^2e}\left[\frac{a^2(1-e^2)}{r} - r\right]T$$

$$\frac{di}{dt} = \frac{r\cos u}{na^2\sqrt{1-e^2}}N$$

$$\frac{d\omega}{dt} = \frac{\sqrt{1-e^2}\cos\nu}{nae}R + \frac{p}{eH}\left[\sin\nu\left(1 + \frac{1}{1+e\cos\nu}\right)\right]T - \frac{r\cot i\sin u}{na^2\sqrt{1-e^2}}N$$

$$\frac{d\Omega}{dt} = \frac{r\sin u}{na^2\sqrt{1-e^2}\sin i}N$$

Unfortunately, this form involves the argument of perigee, ω, and the eccentricity, e, so it is singular for small eccentricity. This singularity problem can be eliminated by making the change of variables:

$$h = e\cos\omega \tag{10}$$

$$k = e\sin\omega$$

Another problem is that the true anomaly, ν, is not well defined for nearly circular orbits. The true anomaly can be replaced by the argument of latitude, u, where:

$$u = \omega + \nu \tag{11}$$

This change of variables transforms the independent variable from time (implicit in ν) to u.

Applying these variable changes to the Lagrange planetary equations, expanding for small eccentricity, and retaining only the lowest order terms yields:[9]

(12)

$$\frac{da}{du} \approx \frac{2}{n^2} T$$

$$\frac{dh}{du} \approx \frac{\sin u}{n^2 a} R + \frac{2}{n}\left[\frac{a}{\mu}\right]^{1/2} \cos u \, T$$

$$\frac{dk}{du} \approx -\frac{\cos u}{n^2 a} R + \frac{2}{n}\left[\frac{a}{\mu}\right]^{1/2} \sin u \, T$$

$$\frac{di}{du} \approx \frac{\cos u}{n^2 a} N$$

$$\frac{d\Omega}{du} \approx \frac{\sin u}{n^2 a \sin i} N$$

In these equations R, T, and N are the acceleration components in the radial, tangential, and normal directions, respectively, i is the inclination, and Ω is the right ascension of the ascending node. The quantity n is the mean motion.

(13)

$$n = \sqrt{\frac{\mu}{a^3}}$$

The mass and acceleration of a low thrust vehicle making small orbit changes can be treated as constant. This assumption can not be made when considering the long timescale problem. For a vehicle with constant acceleration A, pitch angle θ, and yaw angle Ψ (see Figure 1), the acceleration components are:

(14)

$$T = A \cos\theta \cos\Psi$$

$$R = -A \cos\theta \sin\Psi$$

$$N = A \sin\theta$$

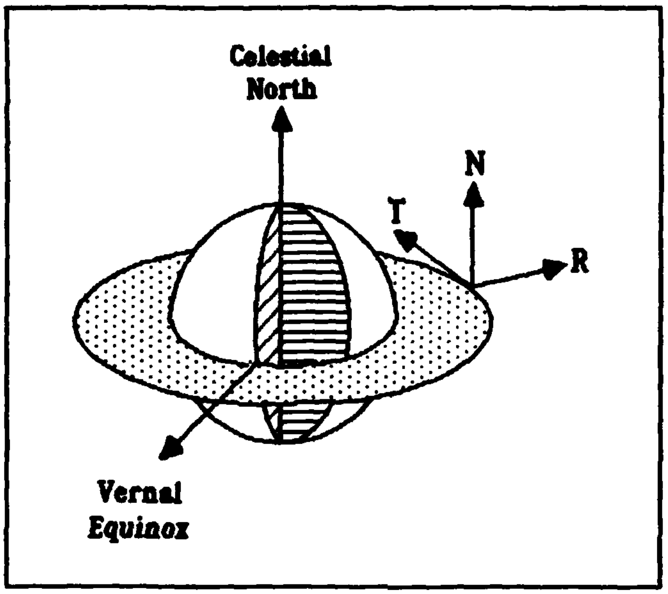

Figure 1. Vehicle Acceleration Components

Another useful relationship is

$$\epsilon = \frac{A}{n^2 a} \tag{15}$$

where ϵ is the dimensionless ratio of vehicle acceleration to local gravitational acceleration. Substituting the expressions for the acceleration components and ϵ into the equations of motion yields:

$$\frac{da}{du} = 2a\epsilon \cos\theta \cos\Psi$$

$$\frac{dh}{du} = \epsilon[-\sin u \cos\theta \sin\Psi + 2\cos u \cos\theta \cos\Psi]$$

$$\frac{dk}{du} = \epsilon[\cos u \cos\theta \sin\Psi + 2 \sin u \cos\theta \cos\Psi]$$

$$\frac{di}{du} = \epsilon \cos u \sin\theta$$

$$\frac{d\Omega}{du} = \frac{\epsilon}{\sin i} \sin u \sin\theta \tag{16}$$

Simplification to Planar Case

It was decided to develop only the more limited case of orbital changes within a single plane. This is equivalent to setting the pitch angle θ equal to zero in the equations above. This assumption results in the following set of three equations:

$$\frac{da}{du} = 2a\epsilon \cos\Psi$$

$$\frac{dh}{du} = \epsilon[-\sin u \sin\Psi + 2\cos u \cos\Psi]$$

$$\frac{dk}{du} = \epsilon[\cos u \sin\Psi + 2 \sin u \cos\Psi] \tag{17}$$

This is the final form of the equations of motion used to develop the analytical solution to the optimal low thrust transfer problem.

IV. Analytic Solution

Control Optimization

To solve the low thrust transfer problem, it is necessary to determine the optimal yaw angle program to perform an orbital transfer from a given set of initial conditions (a_0, h_0, k_0) to a given set of final conditions (a_f, h_f, k_f). For a vehicle under constant thrust, an optimal trajectory is equivalent to a minimum time trajectory.[9] If the vehicle's orbital eccentricity is very small at all points in the trajectory then time is simply proportional to u. Based on this assumption, the Hamiltonian for the optimal transfer is

$$H = 1 + \lambda_1 \frac{da}{du} + \lambda_2 \frac{dh}{du} + \lambda_3 \frac{dk}{du} \tag{18}$$

Since only small orbital changes are being considered, the dependent variables on the right sides of the equations of motion can be treated as constants. Using this assumption, the canonical equations for the lagrange multipliers are

$$\frac{d\lambda_1}{du} = -\frac{\partial H}{\partial a} = 0$$
$$\frac{d\lambda_2}{du} = -\frac{\partial H}{\partial h} = 0 \tag{19}$$
$$\frac{d\lambda_3}{du} = -\frac{\partial H}{\partial k} = 0$$

so all three of the lagrange multipliers are constants. The solution to the long timescale problem would incorporate a series of such small changes. Inserting the equations of motion into the Hamiltonian and computing the yaw optimality condition

$$\frac{\partial H}{\partial \Psi} = 0 \tag{20}$$

yields the control law:

$$\tan\Psi = \frac{-\lambda_2 \sin u + \lambda_3 \cos u}{2(a\lambda_1 + \lambda_2 \cos u + \lambda_3 \sin u)} \tag{21}$$

Note that, even though the λ's are constant, Ψ will vary because of its dependence on u. The direction of the transfer, inward or outward, is used to resolve the ambiguity in the solution for Ψ. For outward transfers Ψ will take on values in the range $-\pi/2 \leq \Psi \leq \pi/2$, while for inward transfers Ψ will take on values in the other two quadrants.

Since the lagrange multipliers are constant throughout a given orbital transfer so long as the change in a is small, and Ψ is dependent on these multipliers, the multipliers are effectively the control variables for this problem. The yaw control law can be simplified by letting

$$U_1 = \frac{\lambda_2}{\lambda_1} \qquad U_2 = \frac{\lambda_3}{\lambda_1} \tag{22}$$

The yaw control law then becomes:

$$\tan\Psi = \frac{-U_1 \sin u + U_2 \cos u}{2(a + U_1 \cos u + U_2 \sin u)} \tag{23}$$

<u>Small Angle Assumption</u>

To further simplify the analytical solution to the low thrust transfer problem, it is assumed that the yaw angle Ψ is small. Since the optimal yaw angle for a vehicle having infinitesimal thrust is $\Psi=0$, this is a reasonable assumption for a low thrust vehicle. Note that if Ψ is small, U_1 and U_2 must be small. Using this small angle assumption, the equations of motion become:

(24)

$$\frac{da}{du} = 2a\epsilon$$

$$\frac{dh}{du} = \epsilon[-\Psi \sin u + 2\cos u]$$

$$\frac{dk}{du} = \epsilon[\Psi \cos u + 2\sin u]$$

and the yaw control law becomes

(25)

$$\Psi = \frac{-U_1 \sin u + U_2 \cos u}{2a}$$

Note that the equation for the semimajor axis does not involve Ψ. At this level of approximation the analytical solution is identical to the spiral solution for a. The differential equation for a can be solved directly in terms of u to yield

(26)

$$a = a_0 \sqrt{\frac{\mu}{\mu - 4\Lambda a_0^2(u - u_0)}}$$

Orbital Element Changes

The changes in orbital elements over an entire orbit transfer are computed by integrating the equations of motion over u. For the semimajor axis this is

(27)

$$\Delta a = a_f - a_0 = \int_{u_0}^{u_f} \frac{da}{du}\, du$$

and similarly for the other two orbital elements. Carrying out these integrations for the equations of motion resulting from the small Ψ assumption yields

$$\Delta a = 2a_0 \epsilon u \tag{28}$$

$$\Delta h = \frac{2\epsilon}{a_0}\left[a_0 \sin u + \frac{U_1}{8}(-\sin u \cos u + u) - \frac{U_2}{8}\sin^2 u\right]$$

$$\Delta k = \frac{2\epsilon}{a_0}\left[a_0(1 - \cos u) + \frac{U_1}{8}\sin^2 u + \frac{U_2}{8}(\sin u \cos u + u)\right]$$

Note that Equation (26) is valid for large changes in a while the first of Equations (28) is valid only for small changes.

Control Solution

Given the initial and final conditions for an orbital transfer it is necessary to compute the controls (i.e. the λ's) required to achieve that transfer. It is assumed that each transfer begins and ends in a circular orbit. The total changes in h and k over an entire transfer of this type must therefore be zero, since these two elements are directly proportional to the eccentricity. Using this fact, the variables U_1 and U_2 can be determined by simultaneously solving the following two equations.

$$\Delta h = 0 = a_0 \sin u_f + \frac{U_1}{8}(-\sin u_f \cos u_f + u_f) - \frac{U_2}{8}\sin^2 u_f \tag{29}$$

$$\Delta k = 0 = a_0(1 - \cos u_f) + \frac{U_1}{8}\sin^2 u_f + \frac{U_2}{8}(\sin u_f \cos u_f + u_f)$$

Solution of these equations yields:

$$U_1 = \frac{(U_2 \sin u_f - 8a_0)\sin u_f}{u_f - \sin u_f \cos u_f} \tag{30}$$

$$U_2 = \frac{8a_0(\sin^3 u_f - (1 - \cos u_f)(u_f - \sin u_f \cos u_f))}{u_f^2 - \sin^2 u_f \cos^2 u_f + \sin^4 u_f}$$

The lagrange multiplier associated with the semimajor axis is computed using the transversality condition

$$H(u_f) = 0 = 1 + \lambda_1\left(\frac{da}{du} + U_1\frac{dh}{du} + U_2\frac{dk}{du}\right)\Bigg|_{u_f} \tag{31}$$

Solving this equation for λ_1 yields

$$\lambda_1 = \frac{-2\mu}{Aa[4a(a + \cos u + \sin u) + (U_1 \sin u - U_2 \cos u)(\sin u - \cos u)]} \tag{32}$$

where a as a function of u is given by Equation 26. The values of λ_2 and λ_3 are computed from λ_1, U_1, and U_2. With the λ's in hand, the yaw angle, Ψ, can be computed for any value of u using Equation (25).

V. Verification

The analytical solution to the low thrust transfer problem developed in Section IV was verified by comparison with the integrated equations of motion (Eqns. 13) for a set of orbital transfers. All computations were carried out in canonical units, for which the distance unit is one Earth radius and the gravitational parameter, μ, is numerically equal to one. All of the verification trajectories began at $u = 0$. The initial and final conditions for all of these trajectories were

$$(a_0, h_0, k_0) = (1, 0, 0)$$
$$(a_f, h_f, k_f) = (1.01, 0, 0) \tag{33}$$

Note the small change in the semimajor axis. Solutions for these conditions can be extended to other values of a_0 and a_f by scaling ϵ.

Using the above conditions and specifying u_f, the λ's and the required acceleration were computed. In order to make a valid comparison, these computed values were used in both the analytical solution and the solution via direct integration of the equations of motion. For the analytical solution, Equation (26) was used instead of the first of Equations (28) to provide higher accuracy for changes in the semimajor axis. In the integrated solution, however, the analytical result of Equation (26) was not used; all three of Equations (17) were integrated. Four hundred integration steps per revolution were used. The small angle assumption was <u>not</u> made for the integrated solution.

Orbital transfers computed using both the analytical solution and the integrated solution are compared in Figures 2-9. Figures 2-5 show the orbital elements and yaw angle as a function of u for a transfer to $u_f = 1.8$ revolutions. The correspondence between the analytic and integrated solutions is poor. Note that for small changes in a the yaw angle computed using the two solution methods is essentially identical, so Figure 5 shows only one curve. Figures 6-9 show a transfer to $u_f = 4.9$ revolutions, and in this case the correspondence between the two solutions is very good. Figures 2-9 indicate that the analytical solution is valid for large u_f, but breaks down at small values of u_f (i.e. higher accelerations).

Using the analytical solution, the maximum yaw angles for transfers involving large values of u_f (near 100 revolutions) were computed. Since a_0 and a_f are fixed (Eqns. 33), large u_f corresponds to a very low acceleration level. As shown in Figure 10, the maximum yaw angles for these transfers were very small. Note the continuing decrease in the maximum value of the yaw angle as u_f increases. Hence, for very low vehicle accelerations the analytical solution gives results close to those of the spiral solution.

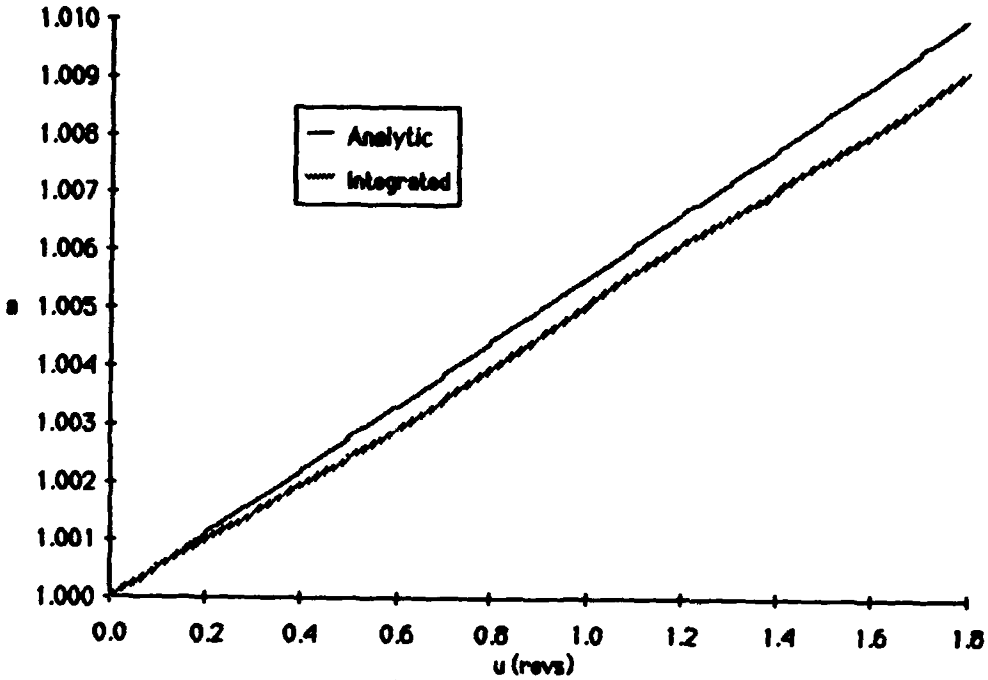

Figure 2. a vs. u (u_f = 1.8 rev.)

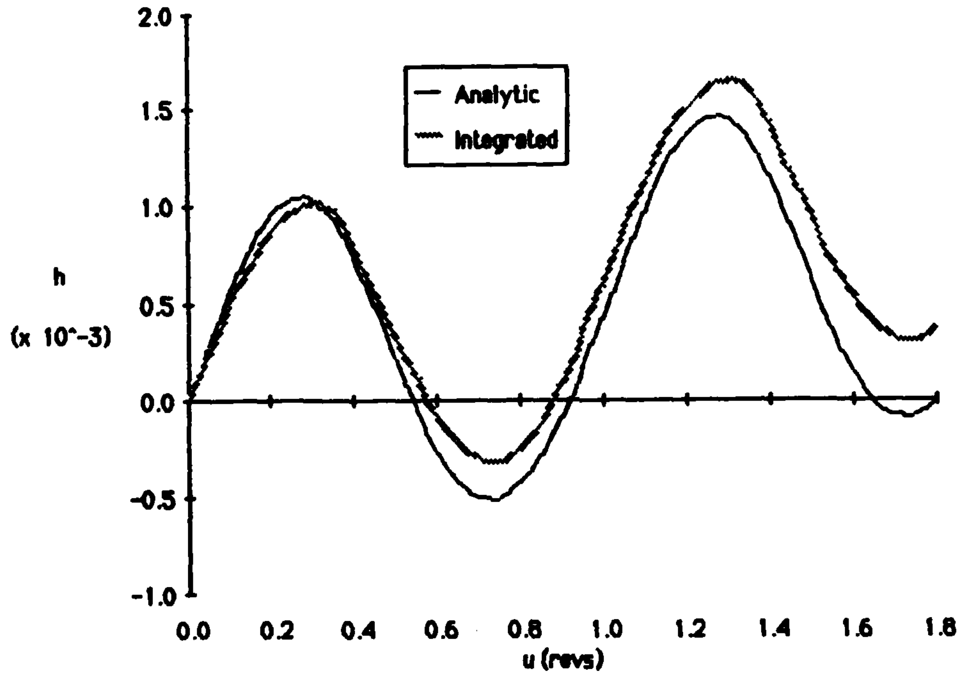

Figure 3. h vs. u (u_f = 1.8 rev.)

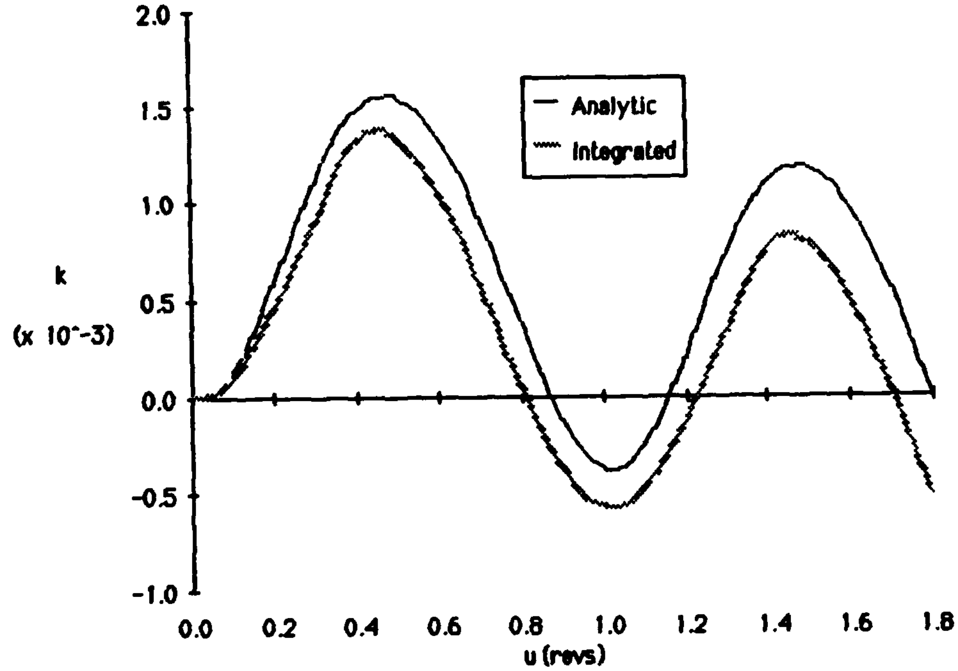

Figure 4. k vs. u (u_f = 1.8 rev.)

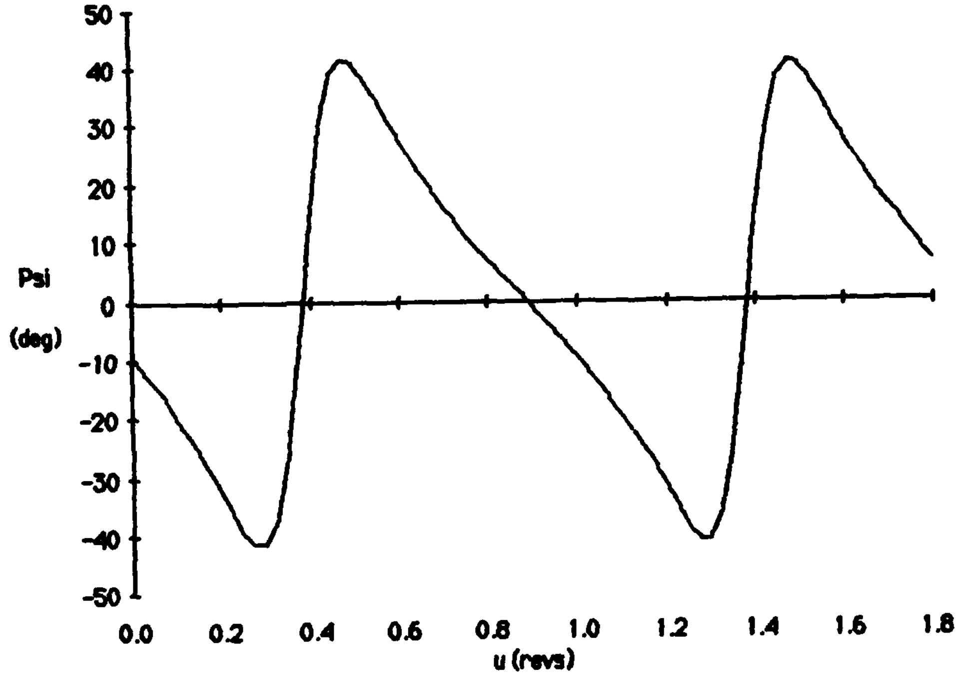

Figure 5. Ψ vs. u (u_f = 1.8 rev.)

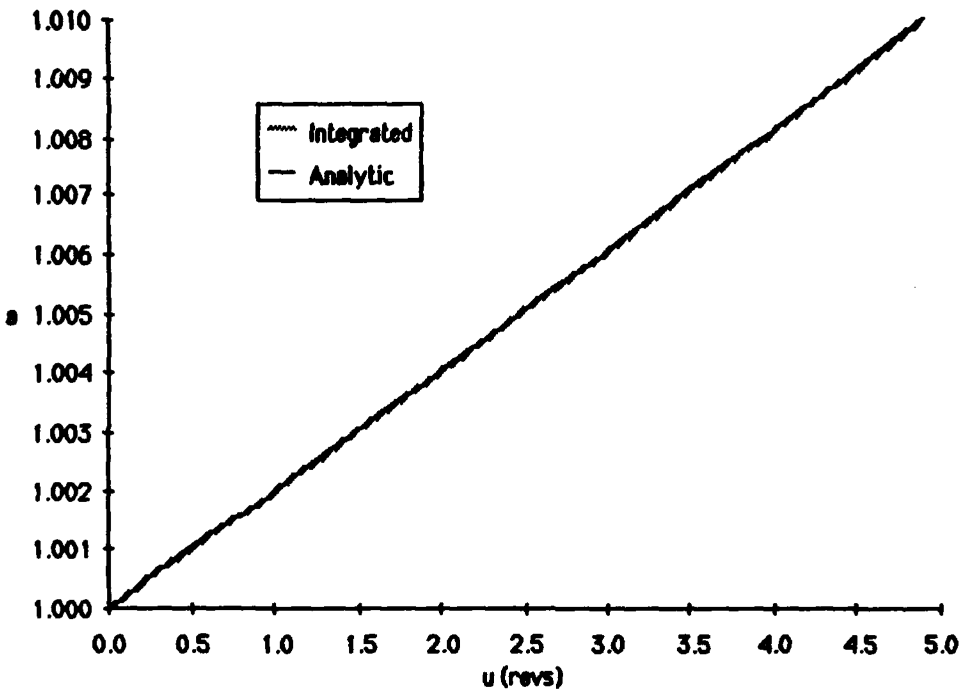

Figure 6. a vs. u (u_f = 4.9 rev.)

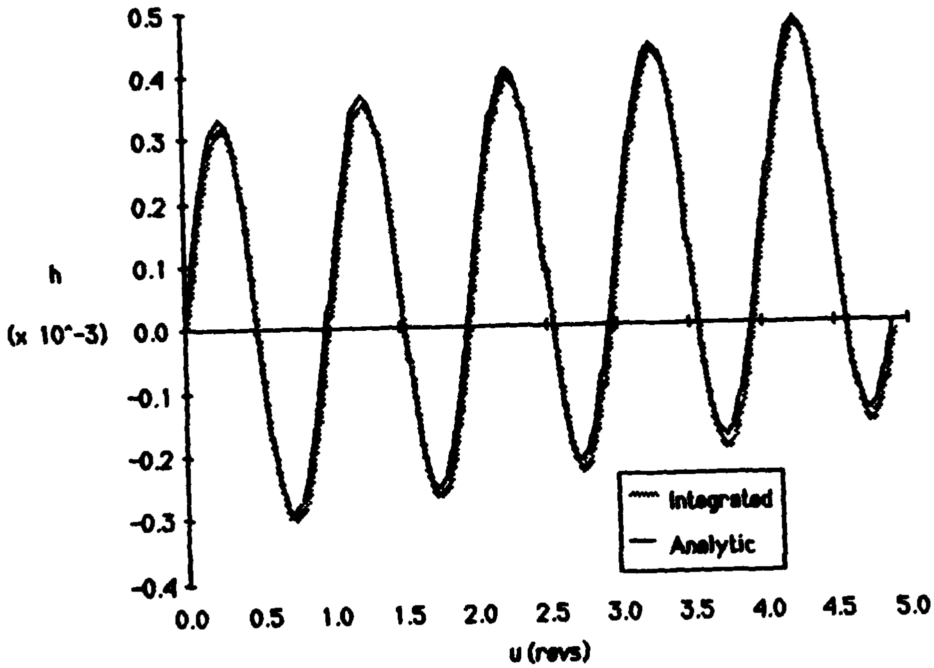

Figure 7. h vs. u (u_f = 4.9 rev.)

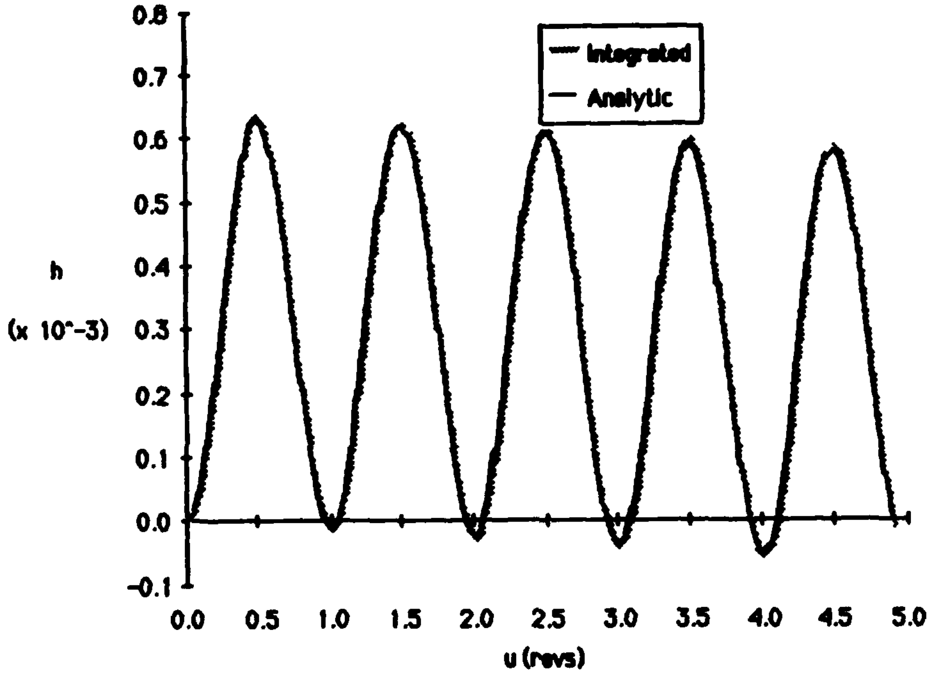

Figure 8. h vs. u (u_f = 4.9 rev.)

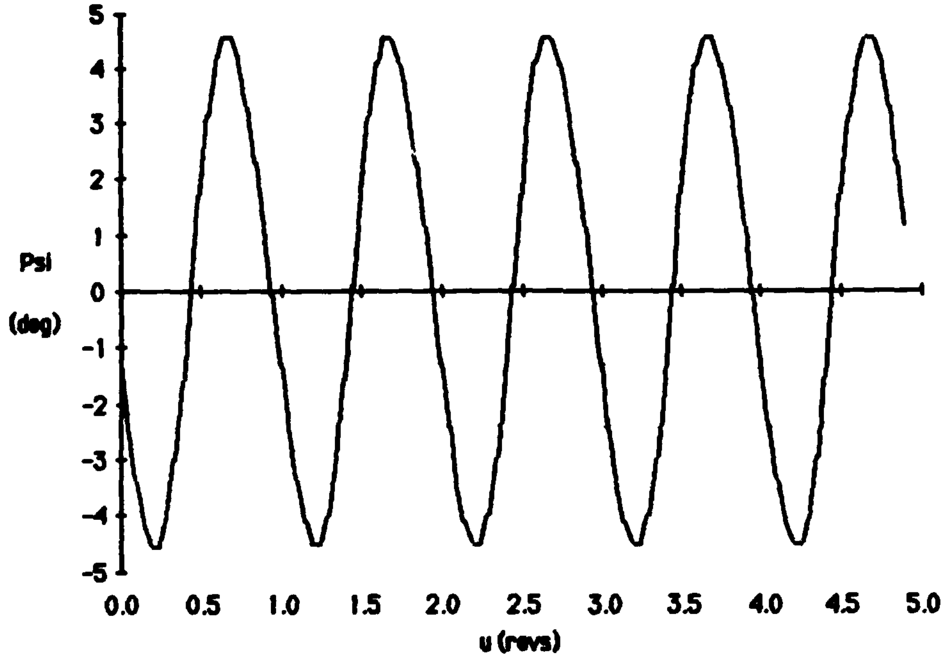

Figure 9 Ψ vs. u (u_f = 4.9 rev.)

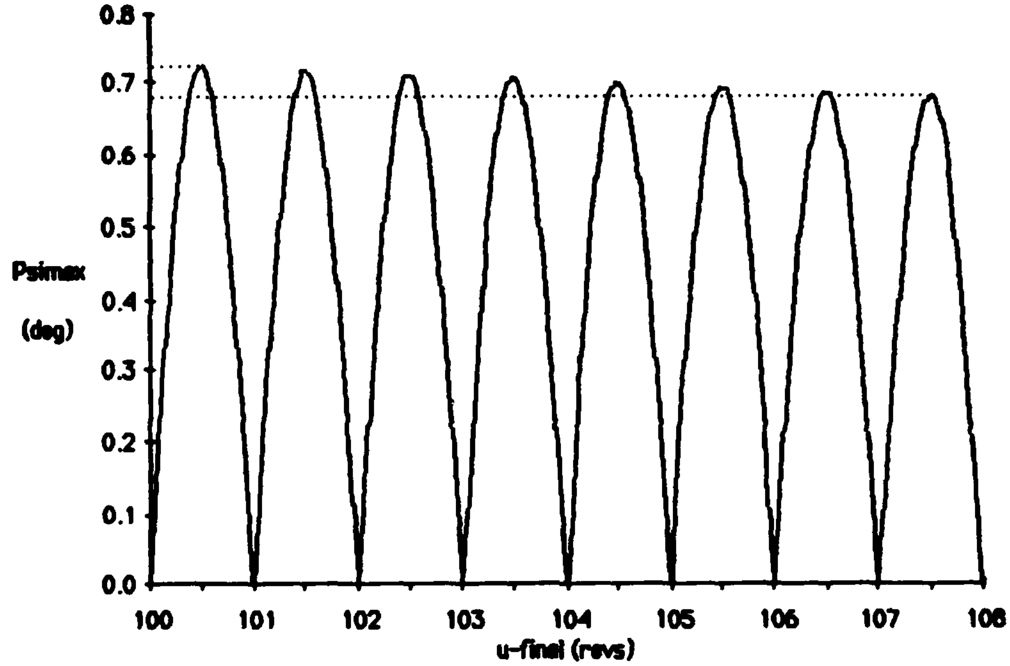

Figure 10. Ψ_{max} vs. u_f

VI. Results

A solution to the low thrust boundary value problem, for a given transfer, occurs when the value of the Hamiltonian first reaches zero. A typical plot of the Hamiltonian as a function of u is shown in Figure 11. The Hamiltonian oscilates up and down, but its mean value decreases as the semimajor axis increases. If the Hamiltonian is extended beyond the point at which it first equals zero, it will dip below zero with each subsequent revolution. This indicates that a minimum fuel solution to the low thrust transfer problem, as opposed to minimum time solution, can be developed. For a minimum fuel solution, the vehicle's engine would be on when the Hamiltonian was positive and off when it was negative. The turn on and turn off points for an inward transfer from u_f to u_o would simply be the mirror image of the turn on and turn off points for an outward transfer from u_o to u_f.

The quantities ϵ, λ_1, λ_2, λ_3, and the absolute value of Ψ_{max} are shown as a function of u_f in Figures 12-16. Note that solutions do not exist for some values of u_f below 2.5 revolutions. All three λ's are periodic with a period of 2π (one revolution) and, in each revolution, they are all singular at the 0.5 and 0.75 revolution points. Because of dependence on the inverse rate of change of the semimajor axis, the mean magnitude of λ_1 increases as u_f increases. The mean values of λ_2 and λ_3 do not change. Ψ_{max} also has a period of 2π, but its maximum value per revolution asymptotically approaches zero as u_f increases and vehicle acceleration decreases. The decrease in Ψ_{max} as u_f increases is shown in both Figure 16 and Figure 10.

One major advantage of the analytical solution over the integrated solution is that it requires much less computation. For any transfer, using the analytical solution, the values of the orbital elements for any given value of u can be computed directly from Equations (28). In comparison, obtaining the orbital elements for the same value of u using the integrated solution requires integrating over the entire range from $u = 0$ to u. Computing and outputting the orbital elements forty times per revolution, the analytical solution was able to complete one revolution about ninety times as quickly as the integrated solution. This gives a rough indication of the computational advantage of the analytical solution.

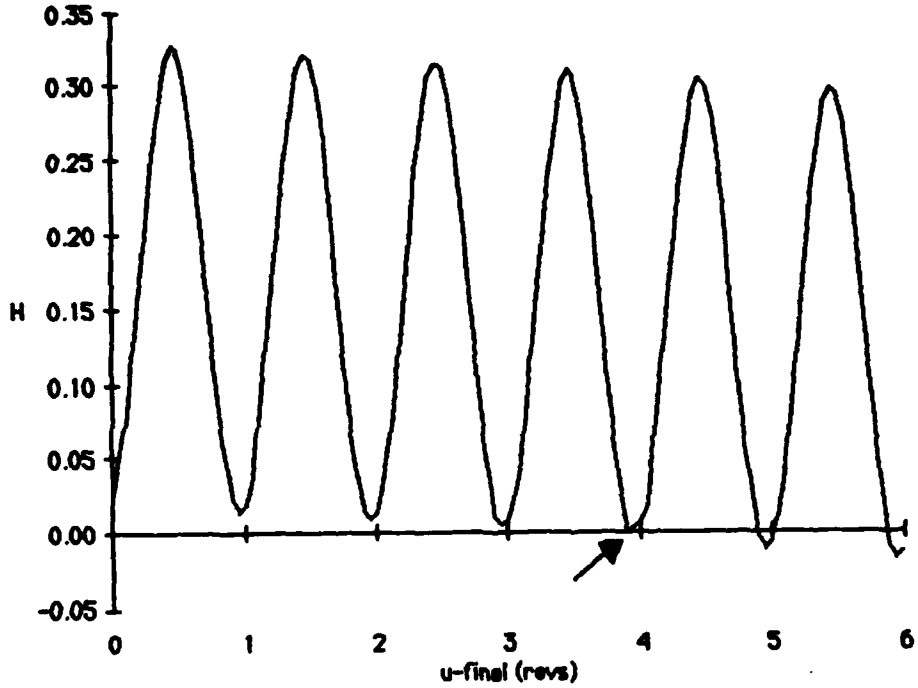

Figure 11. H vs. u

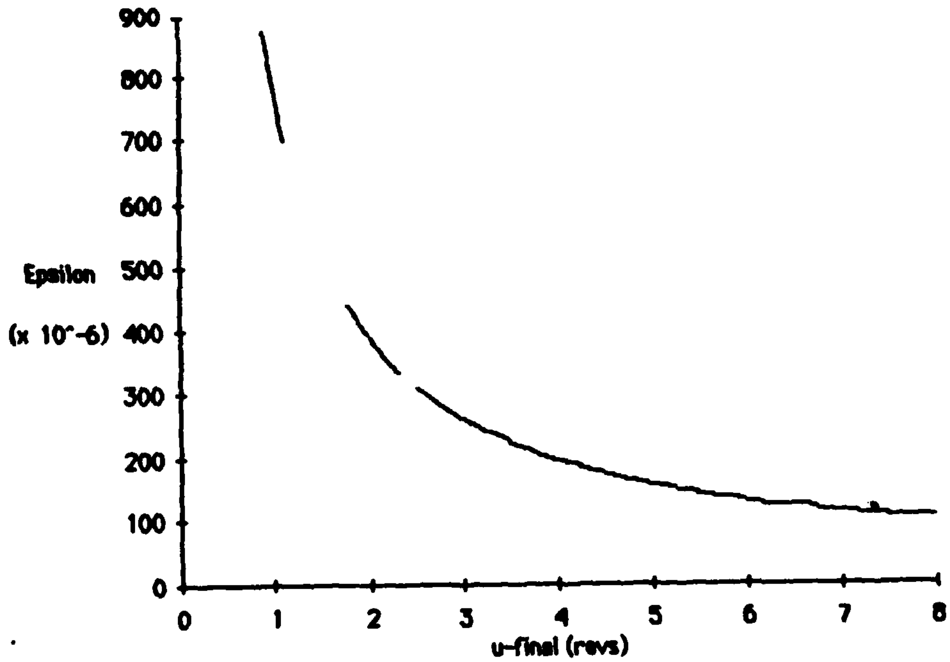

Figure 12 ϵ vs. u_f

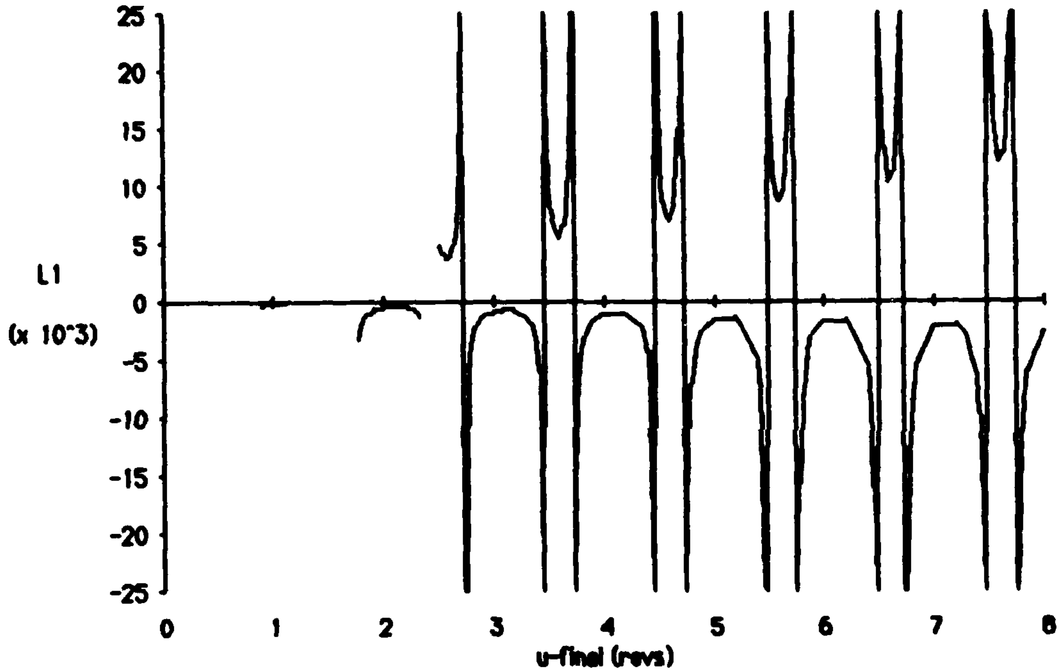

Figure 13. λ_1 vs. u_f

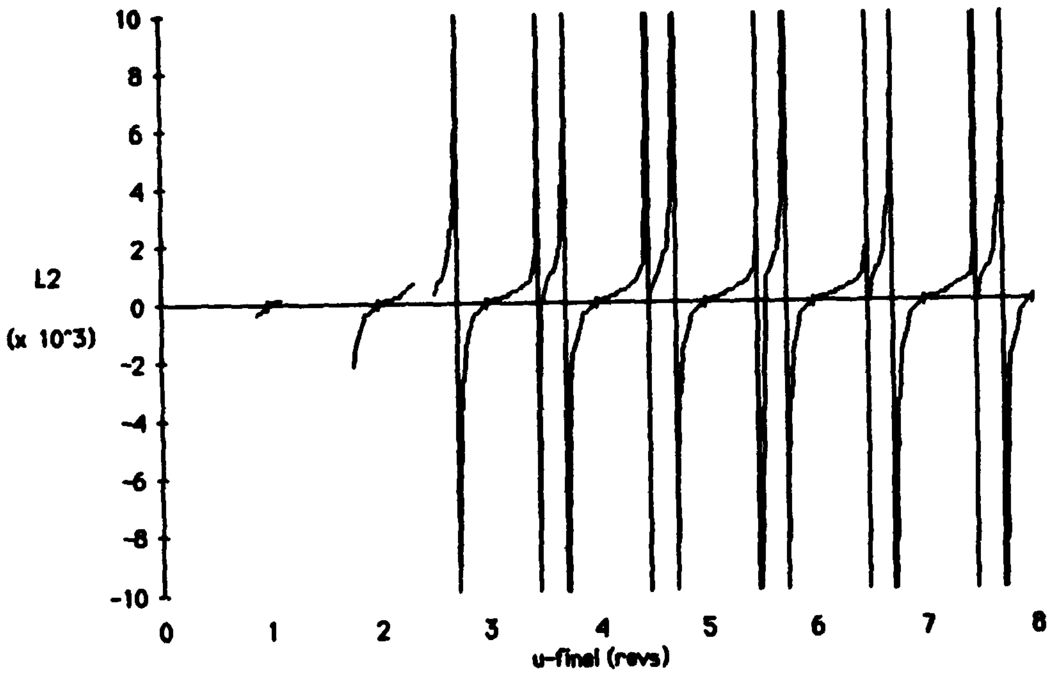

Figure 14. λ_2 vs. u_f

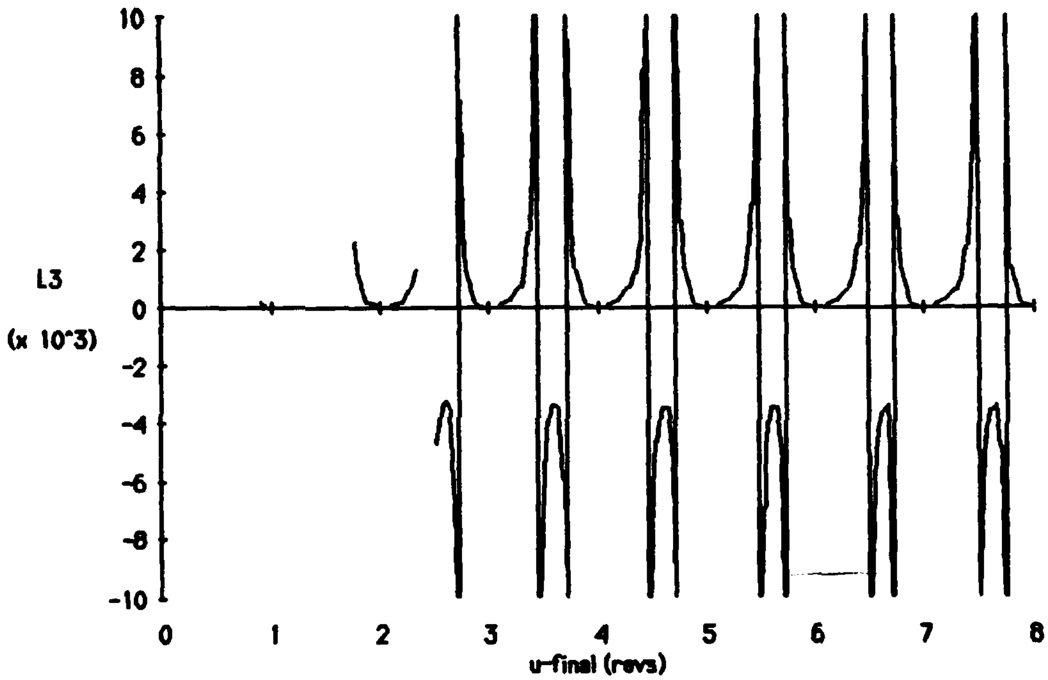

Figure 15. λ_3 vs. u_f

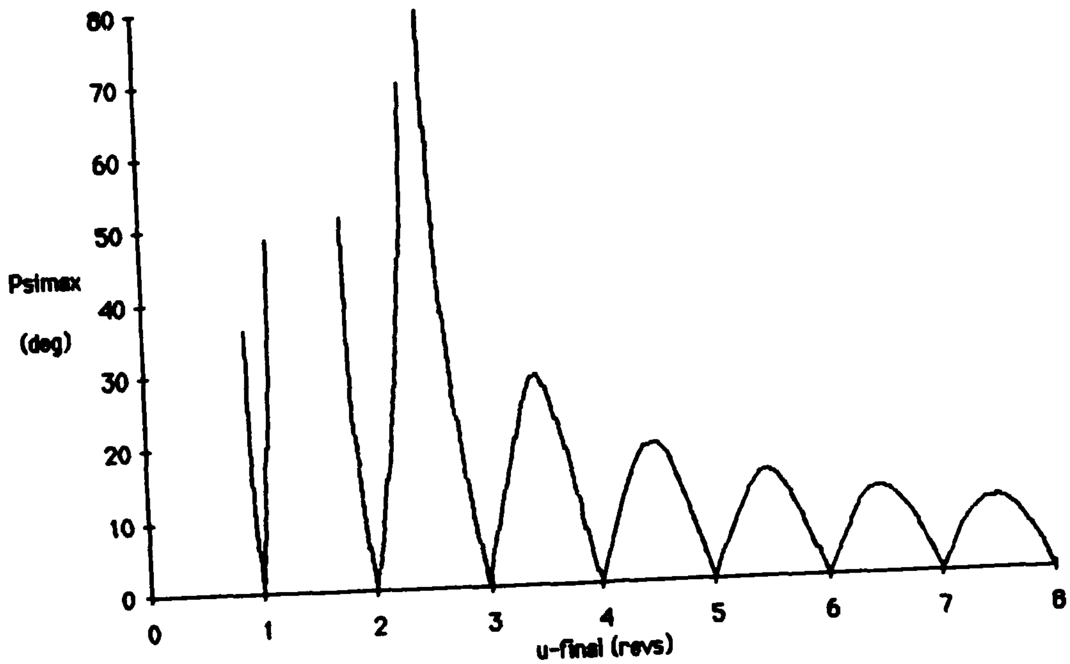

Figure 16. Ψ_{max} vs. u_f

VII. Conclusions and Recommendations

In this paper an analytic solution to the optimal low thrust transfer problem was developed using a first-order perturbation approach. For low vehicle accelerations this solution yielded results quite close to those obtained by integrating the vehicle equations of motion. At higher acceleration levels, however, the analytic solution diverged from the integrated solution. For very low acceleration levels the analytic solution was found to approach the limiting case of the infinitesimally low thrust spiral solution.

The results described in this paper could be further developed by incorporating a higher order perturbation model. Such a higher order model would extend the validity of the analytic solution to higher levels of vehicle acceleration. Another useful activity would be to develop an analytical solution for the full, three dimensional orbital transfer problem, eliminating the limitation to planar transfers. Using the minimum time transfer solution as a basis, the minimum fuel solution should also be developed. By incorporating results from all of these recommended areas of study, there is potential for the development of powerful and efficient tools for the analysis of low thrust transfers.

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Vita

Thomas Black was born on 31 January 1958 in Euclid, Ohio. He graduated from high school in Canandaigua, New York in 1976 and attended the University of Virginia from which he received the degree of Bachelor of Science in Aerospace Engineering in May 1980. While at the University of Virginia he entered Air Force ROTC, and was commissioned upon graduation. His first Air Force assignment was at the Rocket Propulsion Laboratory, Edwards AFB, CA, where he served as a project manager. He entered the School of Engineering, Air Force Institute of Technology, in May 1983. Since May 1984 he has worked in the Technology Assessment Office of the Flight Dynamics Laboratory.

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An analytic solution to the optimal low thrust transfer problem is developed using a first-order perturbation approach for planar orbit transfers. This solution is verified by comparison with the solution obtained by integrating the equations of motion. For low vehicle accelerations the analytic solution yields results quite close to those obtained by the integrated solution. At higher acceleration levels, however, the analytic solution diverges from the integrated solution. For very low acceleration levels the analytic solution approachs the limiting case of the infinitesimally low thrust spiral solution. cont keywords include:

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