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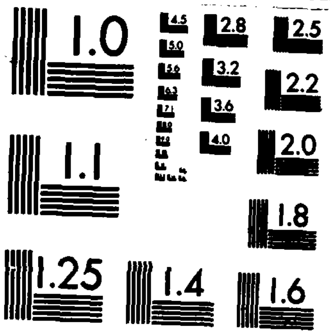

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LARGE-TIME BEHAVIOR OF VISCOUS SURFACE WAVES

J. Thomas Beale and Takaaki Nishida

Mathematics Research Center
University of Wisconsin—Madison
610 Walnut Street
Madison, Wisconsin 53705

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ABSTRACT

We consider the mathematical behavior of a viscous, incompressible fluid bounded above by an atmosphere of constant pressure and below by a horizontal bottom. After reviewing the existence and regularity theory for the equations governing the motion, we establish rates of decay for solutions near equilibrium. The function describing the height of the free surface decays like $t^{-1/2}$; the velocity field decays like t^{-1}. These estimates are shown first for the linearization about equilibrium and then for the full nonlinear problem. Complete details will be given elsewhere.

AMS (MOS) Subject Classifications: 68D05, 35Q10, 35R35

Key Words: viscous incompressible flow, surface waves, free surface

Work Unit Number 1 - Applied Analysis

*Department of Mathematics, Duke University, Durham, NC 27706
**Department of Mathematics, Kyoto University, Kyoto, 606 Japan

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SIGNIFICANCE AND EXPLANATION

One primary purpose of mathematical analysis is to describe the behavior of solutions of equations representing physical phenomena. The results shed light on the validity of the mathematical model as a representation of the physical process and may in turn offer new information about the phenomena. For the problem considered here, the motion near equilibrium of a viscous fluid bounded by a free surface, fundamental properties of existence and regularity were established by the first author in earlier work. This paper continues the study by describing further the behavior of solutions over a horizontal bottom. Solutions decay at a fixed rate because of the effect of viscosity in the interior of the fluid. The slow rate of decay is determined by the behavior of long wavelengths in the linearization about equilibrium.

The responsibility for the wording and views expressed in this descriptive summary lies with MRC, and not with the authors of this report.

LARGE-TIME BEHAVIOR OF VISCOUS SURFACE WAVES

J. Thomas Beale* and Takaaki Nishida**

§1. Introduction.

We are concerned with solutions global in time to a free surface problem of a viscous incompressible fluid, which is formulated as follows: The motion of the fluid is governed by the Navier-Stokes equations

$$(1.1)\qquad \left.\begin{aligned} u_t + (u\cdot\nabla)u - \nu\Delta u + \nabla p &= 0 \\ \nabla\cdot u &= 0 \end{aligned}\right\} \quad \text{in } \Omega(t),$$

where $\Omega(t) = \{x \in R^2, -b < y < \eta(t,x)\}$ is the domain occupied by the fluid. The free surface S_F: $y = \eta(t,x)$ satisfies the kinematic boundary condition

$$(1.2)\qquad \eta_t + u_1\eta_{x_1} + u_2\eta_{x_2} - u_3 = 0 \quad \text{on } S_F.$$

The stress tensor satisfies the free boundary condition:

*Department of Mathematics, Duke University, Durham, NC 27706
**Department of Mathematics, Kyoto University, Kyoto, 606 Japan

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$$(1.3)\quad pn_i - \nu(u_{i,x_j} + u_{j,x_i})n_j =$$

$$[g\eta - \beta\nabla\{(1 + |\nabla\eta|^2)^{-1/2}\nabla\eta\}]n_i \quad \text{on} \quad S_F,$$

where n is the outward normal to S_F, g is the acceleration of gravity and β is the nondimensionalized coefficient of surface tension. On the bottom S_B: $y = -b$ we have the fixed boundary condition

$$(1.4)\qquad u = 0 \quad \text{on} \quad S_B.$$

We assume here that b is constant, although Theorem 1.1 below holds for $b = b(x)$.

We consider the initial value problem (1.1)-(1.4) with the data at $t = 0$

$$(1.5)\qquad \begin{cases} \eta = \eta_0(x) & x \in R^2, \\ u = u_0(x,y) & \text{in} \quad \Omega_0, \end{cases}$$

where $\Omega_0 = \Omega(0)$. Local existence theorems for (1.1)-(1.5) are proved for both cases with or without considering surface tension ([1],[2]). The problem of existence global in time for (1.1)-(1.5) neglecting the surface tension ($\beta = 0$) has a difficulty which was pointed out in [1]. However if the surface tension is taken into account, the following global existence and regularity theorem is proved.

Theorem 1.1 ([2])

Let $3 < r < 7/2$. *Assume the compatibility conditions on the initial data:*

$$(1.6) \quad \begin{cases} \nabla \cdot u_0 = 0 & \text{in } \Omega_0, \\ \{((u_0)_{i,x_j} + (u_0)_{j,x_i})n_j\}_{\tan} = 0 & \text{on } y = \eta_0(x), \\ u_0 = 0 & \text{on } y = \eta_0(x), \end{cases}$$

There exists $\delta_0 > 0$ *such that if the initial data satisfy*

$$(1.7) \qquad E_0 = |\eta_0|_{H^r(R^2)} + |u_0|_{H^{r-1/2}(\Omega_0)} < \delta_0 ,$$

then there exists a unique global solution η, u, p *of* (1.1)-(1.5), *which satisfies*

$$(1.8) \quad \eta \in \tilde{K}^{r+1/2}(R^+ \times R^2),\ u \in K^r(R^+ \times \Omega(t)),\ \nabla p \in K^{r-2}(R^+ \times \Omega(t)).$$

Further, given any $T_1 > 0$ *and any* $k > 0$, *there exists* $\delta_1 > 0$ *such that if*

$$(1.9) \qquad E_0 < \delta_1$$

then the solution becomes smooth for $t > T_1$, *i.e.,*

$$(1.10) \quad \eta \in \tilde{K}^{r+k+1/2}((T_1,\infty) \times R^2),\ u \in K^{r+k}((T_1,\infty) \times \Omega(t)),$$
$$\nabla p \in K^{r+k-2}((T_1,\infty) \times \Omega(t)).$$

In particular the solution with $k \geq 2$ is classical. Here $H^r(\cdot)$ is the usual Sobolev space with norm $|\ |_r$ on the domain $\cdot$. $K^r((T_1,T_2) \times \Omega(t))$ is composed of the restriction to the fluid domain $\Omega(t)$ of the functions belonging to

$$(1.11) \qquad K^r((T_1,T_2)\times R^3) =$$

$$H^0((T_1,T_2);H^r(R^3)) \cap H^{r/2}((T_1,T_2),H^0(R^3)).$$

$\eta \in \tilde{K}^r(R^+ \times R^2)$ is defined as follows: $\eta \in K^r((0,T) \times R^2)$ for any $T > 0$ and $\eta = \eta_1 + \eta_2$ such that $\eta_1 \in K^r(R^+ \times R^2)$ and η_2 is the Fourier transform in space-time of L^1 function of bounded support. See [2] for the details of the function spaces.

In this summary we give an asymptotic decay rate for the solution of the above theorem.

Theorem 1.2. *If* $\eta_0 \in L^1(R^2)$, *then there exists* $\delta_2 > 0$ *such that if*

$$(1.12) \qquad E_1 \equiv E_0 + |\eta_0|_{L^1} < \delta_2 ,$$

then the solution has the decay rate:

$$|\partial^\alpha \eta(t)|_0 < CE_1(1 + t)^{-(1+\alpha)/2}, \quad \alpha = 0,1,2$$
$$|u(t)|_2, \ |\nabla p(t)|_0 < CE_1(1 + t)^{-1}.$$

In §2 we transform the free boundary problem (1.1)-(1.5) to an equivalent one on a fixed domain and reduce the components of the stress tensor to zero. The linear decay estimate is discussed in §3 and the nonlinear one in §4.

§2. Reduction of the Problem.

We recall some of the main ideas for the reduction of the free surface problem in [2]. First we use the transformation of the free boundary problem (1.1)-(1.5) to that on the fixed (equilibrium) domain: $\Omega = \{x \in R^2, -b < y < 0\}$. Given $\eta(t,x)$ we extend it for $y < 0$ as follows:

$$(2.1) \qquad \tilde{\eta}(t,x,y) = F^{-1}(e^{|\xi|y}\,\hat{\eta}(t,\xi)),$$

where $\hat{\eta}(t,\xi)$ is the Fourier transform with respect to x and F^{-1} is the inverse. If $\eta(t,\cdot)$ belongs to $H^s(S_F)$, then $\tilde{\eta}(t,\cdot,\cdot)$ belongs to $H^{s+1/2}(\Omega)$, where S_F now denotes the upper surface $y = 0$ of Ω. For each $t > 0$ we define the transformation θ on Ω onto $\Omega(t) = \{x \in R^2, -b < y < \eta(t,x)\}$ by

$$(2.2) \qquad \theta(x_1,x_2,y;t) = (x_1,x_2,\tilde{\eta} + y(1 + \tilde{\eta}/b)).$$

The vector field u on $\Omega(t) = \theta(\Omega)$ is defined from the vector field v on Ω by

$$u_i = \theta_{i,x_j} v_j / J \equiv \alpha_{ij} v_j, \tag{2.3}$$

where J is the Jacobian determinant of $d\theta = (\theta_{i,x_j})$ $= 1 + \tilde{\eta}/b + \tilde{\eta}_y(1 + y/b)$. This map preserves the property of being divergence free, that is, $\nabla \cdot v = 0$ in Ω iff $\nabla \cdot u = 0$ in $\Omega(t)$.

Using the transformation (2.2), (2.3) and $u_{i,x_j} = \zeta_{lj}\, \partial_l(\alpha_{ik} v_k)$, where $\zeta = (d\theta)^{-1}$ and so on, we can rewrite the free surface problem (1.1)-(1.5) as one on the equilibrium domain Ω as follows:

$$\eta_t - v_3 = 0 \quad \text{on} \quad S_F, \tag{2.4}$$

$$\left.\begin{aligned} &v_t - \nu\, \Delta v + \nabla q = F(\eta, v, \nabla q) \qquad (2.5)\\ &\nabla \cdot v = 0 \qquad (2.6)\end{aligned}\right\} \quad \text{in} \quad \Omega,$$

$$v = 0 \quad \text{on} \quad S_B\,, \tag{2.7}$$

$$\left.\begin{aligned} &v_{i,x_3} + v_{3,x_i} = F_i(\eta, v), \quad i = 1,2 \qquad (2.8)\\ &q - 2\nu\, v_{3,x_3} - (g - \beta\Delta)\eta = F_3(\eta, v) \qquad (2.9)\end{aligned}\right\} \quad \text{on} \quad S_F.$$

Here we have gathered the linear terms on the left hand side and all the nonlinear terms on the right hand side of the equation.

Next we reduce the tangential component of the stress tensor F_i, $i = 1,2$ to zero: Given $F_i \in H^{r-3/2}(S_F)$, $i = 1,2$, choose the vector $z \in H^{r+1}(\Omega)$ satisfying the condition

$$\begin{cases} z = 0, \ \partial_y z = 0, \ \partial_y^2 z = (F_2, -F_1, 0) & \text{on } S_F, \\ z = 0, \ \partial_y z = 0 & \text{on } S_B. \end{cases}$$

Then $w = \nabla \times z$ satisfies

$$\begin{aligned} w_3 = 0, \quad w_{i,x_3} + w_{3,x_i} = F_i, \quad i = 1,2 \quad & \text{on } S_F, \\ \nabla \cdot w = 0 \quad & \text{in } \Omega, \\ w_3 = 0 \quad & \text{on } S_F. \end{aligned}$$

Therefore η, $v' = v - w$, q satisfy the system (2.4)-(2.9) with the replacements F by $F_4 = F - w_t + \nu\Delta w$ and F_i, $i = 1,2$ by 0. The prime in v' is omitted hereafter.

Finally we rewrite the system (2.4)-(2.9) with $F = F_4$, $F_i = 0$, $i = 1,2$, for η, v, q in the operator form. Let P be the projection on the subspace of solenoidal vectors orthogonal to the subspace $Gr^0 = \{\nabla P : P \in H^1(\Omega), P = 0 \text{ on } S_F\}$ of $H^0(\Omega)$, i.e.,

(2.10) $$H^0 = PH^0 \oplus Gr^0.$$

Applying P to (2.5) we have

(2.11) $$v_t - \nu P\Delta v + P\nabla q = PF_4.$$

Here $P\nabla q$ can be decomposed to three parts as follows:

$$p\nabla q = \nabla\pi^{(1)} + \nabla\pi^{(2)} + \nabla\pi^{(3)},$$

where $\pi^{(i)}$, $i = 1,2,3$, are defined by

$$\pi^{(1)} = 2\nu v_{3,x_3}, \quad \pi^{(2)} = g\eta - \beta\Delta\eta, \quad \pi^{(3)} = F_3 \quad \text{on} \quad S_F,$$

(2.12)
$$\Delta\,\pi^{(i)} = 0 \quad \text{in} \quad \Omega\ ,$$
$$\partial_y\pi^{(i)} = 0 \quad \text{on} \quad S_B\ .$$

We define

$$A\ v = -P\Delta v + \nabla\pi^{(1)}\ ,$$

(2.13)
$$\begin{cases} R\ v = v_3|_{S_F}\ , \\ \\ R^*((g - \beta\Delta)\eta) = \nabla\pi^{(2)}\ . \end{cases}$$

Using these notations the system (2.4), (2.11) has the form

(2.14)
$$\eta_t = R\ v\ ,$$

(2.15)
$$v_t + Av + R^*((g - \beta\Delta)\eta) = f\ ,$$

where $f(\eta,v,\nabla q) = PF_4 - \nabla\pi^{(3)}$. We regard (2.6), (2.7), (2.8) with $F_i = 0$ as domain conditions on A.

3. Rates of Decay for the Linear Problem

We investigate the decay rate of the solution of the inearized equations

(3.1) $$\eta_t = Ru ,$$

(3.2) $$u_t + Au + R^*((g - \beta\Delta)\eta) = 0 ,$$

(3.3) $$\eta(0) = \eta_0, \quad u(0) = u_0 \quad \text{at} \quad t = 0.$$

These are supplied with the conditions:

(3.4) $$\nabla \cdot u = 0 \quad \text{in} \quad \Omega ,$$

(3.5) $$u_{i,x_3} + u_{3,x_i} = 0, \quad i = 1,2 \quad \text{on} \quad S_F,$$

(3.6) $$u = 0 \quad \text{on} \quad S_B.$$

Theorem 3.1. *Let* $E_2 = |\eta_0|_{L^1} + |\eta_0|_{5/2} + |u_0|_0$. *Then the solution of* (3.1)-(3.6) *has the decay rate* $(t \geq 1)$:

(3.7)
$$|\partial^\alpha\eta(t)|_0 \leq C_0E_2\, t^{-(1+\alpha)/2}, \quad 0 \leq \alpha \leq 5/2.$$
$$|u(t)|_2 \leq C_0\, E_2\, t^{-1}.$$

The theorem is proved in several steps.

Let $\mathcal{H} = \{v = (\eta,u) : \eta \in H^1(S_F), \ u \in PH^0(\Omega)\}$, where $(\rho,\eta)_1 = g(\rho,\eta)_0 + \beta(\nabla\rho,\nabla\eta)_0$ is the inner product of $H^1(S_F)$

and set $W = \{v: \eta \in H^{5/2}(S_F),\ \ u \in PH^2(\Omega)$ and u satisfies (3.4), (3.5), (3.6)$\}$. Let us define the operator

$$(3.9)\qquad G\, v = \begin{bmatrix} 0 & R \\ -R^*(g-\beta\Delta)\eta & -A \end{bmatrix} \begin{bmatrix} n \\ u \end{bmatrix} \quad \text{on} \quad D(G) \subseteq \mathscr{H},$$

and consider its closed extension which will be denoted by G again.

Lemma 3.2. *The operator* G *generates a contraction semigroup* e^{tG} *on* $\mathscr{H}$, *and* $W \subset D(G)$.

Consider the resolvent equation:

$$(3.10)\qquad (\lambda - G)\begin{bmatrix} \eta \\ u \end{bmatrix} = \begin{bmatrix} h \\ f \end{bmatrix} .$$

The resolvent of G can be extended in to the left half-plane, as shown in lemmas 3.3-3.5.

Lemma 3.3. *For any* $\tau_0 > 0$ *there exists* $c_0 > 0$ *such that if*

$$\lambda \in \{\lambda = \sigma + i\tau,\ -c_0|\tau| < \sigma < \tau_0,\ |\tau| > \tau_0\},$$

then the solution of (3.10) has the estimate

$$(3.11)\quad |u|_2 + |\lambda||u|_0 + |\lambda^{-1}Ru|_{5/2} + |\eta|_{5/2} + |\lambda||\eta|_{3/2} \le C(|f|_0 + |h|_{5/2}) .$$

We treat the resolvent near $\lambda = 0$ in two cases separately: (i) the supports of $\hat{h}(\xi)$, $\hat{f}(\xi,y)$ belong to $\{|\xi| \ge \xi_0\}$; (ii) the supports belong to $\{|\xi| \le \xi_0\}$. Here $\hat{}$ means the Fourier transform with respect to x.

Lemma 3.4. *For any* $\xi_0 > 0$ *there exists* $r_0 > 0$ *such that if* $\lambda \in \{|\lambda| < r_0\}$ *and the supports of* $h(\xi)$, $f(\xi,y)$ *belong to* $\{|\xi| \ge \xi_0\}$, *then the resolvent equation* (3.10) *has the solution* (η,u) *satisfying*

$$(3.12)\qquad |u|_2, |\eta|_{5/2} \le C(|h|_{5/2} + |f|_0) .$$

Let $\hat{G}(\xi)$ be the Fourier transform of G with respect to x.

Lemma 3.5. *There exist* $\xi_1 > 0$ *and* r_1, r_2, with $\nu(\pi/2b)^2 > r_2 > r_1 > 0$, *such that if* $r_1 < |\lambda| < r_2$, *then* $(\lambda - \hat{G}(\xi))^{-1}$ *exists for* $|\xi| < \xi_1$. *For small* ξ *there is a one-dimensional eigenspace which is analytic with respect to* ξ. *The eigenvalue and eigenvector have the following expansions.*

$$
(3.13)\quad \begin{cases} \lambda = -(gb^3/3\nu)|\xi|^2 + O(|\xi|^4)\ , \\ \eta = 1 + O(|\xi|^2), \\ u_j = i(g\xi_j/2\nu)(y^2-b^2) + O(|\xi|^3), \quad j = 1,2, \\ u_3 = (g|\xi|^2/2\nu)(y^3/3-b^2y-2b^3/3) + O(|\xi|^4). \end{cases}
$$

By using lemmas 3.2-3.5 the decay estimate (3.7) can be proved by the transformation of the integral path of the representation

$$
(3.14)\quad v(t) = e^{tG}v_0 = \lim_{\tau\to\infty} \int_{\sigma-i\tau}^{\sigma+i\tau} e^{\lambda t}(\lambda-G)^{-1}v_0\, d\lambda, \quad \sigma > 0
$$

to the left half-plane.

§4. Nonlinear Decay Estimates.

The free surface problem (1.1)-(1.5) was reduced to the following system in §2.

$$
(4.1)\quad \eta_t - R\, u = 0,
$$

$$
(4.2)\quad u_t + A\, u + R^*((g - \beta\Delta)\eta) = f\ ,
$$

$$
(4.3)\quad \eta(0) = \eta_0, \quad u(0) = u_0,
$$

where f consists of nonlinear terms depending on η, u, ∇p and their derivatives.

The initial value problem (4.1)-(4.3) has a unique solution (Theorem 1.1) which becomes smooth for $t \geq T_1 > 0$. Namely we know that if $E_0 < \delta_1$, then

$$(4.4) \qquad \begin{cases} \eta(t) \in C(0,\infty;H^{5/2}), \text{ i.e., } \tilde{\eta}(t) \in C(0,\infty;H^3), \\ u(t) \in C(0,\infty;H^2), \text{ and} \end{cases}$$

$$(4.5) \qquad \tilde{\eta} \in \tilde{K}^7((1,\infty) \times \Omega), \quad u(t) \in K^6((1,\infty) \times \Omega).$$

Let us define

$$M(\eta,u;t) = \max\Big(\max_{\alpha=0,1,2} \sup_{0\leq s\leq t} (1+s)^{(1+\alpha)/2} |D^\alpha \tilde{\eta}(s)|_0 ,$$

$$(4.6) \qquad \sup_{0\leq s\leq t} (1+s)^{3/2} |D^3 \tilde{\eta}(s)|_0, \ \sup_{0\leq s\leq t} (1+s)|u(s)|_2 \Big).$$

$$E(\eta,u) = |\tilde{\eta}|_{\tilde{K}^7((1,\infty)\times\Omega)} + |u|_{K^6((1,\infty)\times\Omega)} .$$

Since the linearized equation is solved by the semigroup e^{tG} on $\mathcal{H}$ in §3, this solution satisfies the variation of constants formula:

$$(4.7) \qquad v(t) = e^{tG} v_0 + \int_0^t e^{(t-s)G} f(s)ds,$$

where $v = \binom{\eta}{u}$ and $f = \binom{0}{f(\eta,u,\nabla p)}$.

The first term of (4.7) was already estimated in §3, i.e.,

$$(4.8)\qquad \begin{bmatrix}\eta_0(t)\\ u_0(t)\end{bmatrix} = e^{tG}\begin{bmatrix}\eta_0\\ u_0\end{bmatrix} \text{ has the decay rate:}$$

$$|\partial^\alpha \eta_0(t)|_0 \le C_0 E_2\, t^{-(1+\alpha)/2}, \quad 0 \le \alpha \le 5/2, \text{ i.e.,}$$

$$(4.9)\qquad \begin{cases} |D^\alpha \tilde{\eta}_0(t)|_0 \le C_0\, E_2\, t^{-(1+\alpha)/2}, \quad \alpha = 0,1,2, \text{ and} \\ |D^3 \tilde{\eta}_0(t)|_0 \le C_0\, E_2\, t^{-3/2}, \\ |u_0(t)|_2 \le C_0\, E_2\, t^{-1}. \end{cases}$$

It is sufficient by (4.4) to prove the decay rate for $t \ge 2$. Let us decompose the second term of (4.7) into three parts

$$(4.10)\qquad \int_0^t \left(e^{(t-s)G}\begin{bmatrix}0\\ f(s)\end{bmatrix}\right)ds = \int_0^{t/2}(\quad)ds + \int_{t/2}^{t-1}(\quad)ds + \int_{t-1}^{t}(\quad)ds$$

$$\equiv \begin{bmatrix}\eta_1\\ u_1\end{bmatrix}(t) + \begin{bmatrix}\eta_2\\ u_2\end{bmatrix}(t) + \begin{bmatrix}\eta_3\\ u_3\end{bmatrix}(t).$$

Lemma 4.1. *Let* $E_3 = |\tilde{\eta}|_3 + |u|_2 + |Dp|_0$. *We have the estimates:*

$$(4.11)\qquad \begin{aligned} |F|_0 &\le C(E_3)\{(|D\tilde{\eta}|_2 + |u|_2)(|D\tilde{\eta}|_2 + |u|_2 + |Dp|_0) \\ &\qquad + |\tilde{\eta}|_{L^\infty}(|D^2u|_0 + |Dp|_0)\}. \\ |DF|_0 &\le C(E_3)\{(|D\tilde{\eta}|_2 + |u|_2)(|D\tilde{\eta}|_3 + |u|_3 + |Dp|_1) \\ &\qquad + |\tilde{\eta}|_{L^\infty}(|D^3u|_0 + |D^2p|_0)\}. \\ |D^2F|_0 &\le C(E_3)\{(|D\tilde{\eta}|_2 + |u|_2)(|D\tilde{\eta}|_4 + |u|_3 + |Dp|_1) \\ &\qquad + (|\tilde{\eta}|_{L^\infty} + |D\tilde{\eta}|_{L^\infty} + |D^2\tilde{\eta}|_1)(|D^4u|_0 + |D^3p|_0)\}. \end{aligned}$$

Let $\tilde{F}_i$, $i = 1,2,3$ on Ω be the extension of F_i on S_F by using $\tilde{\eta}$ for η. We have

$$(4.12) \qquad |\tilde{F}_i|_0 \le C(E_3)(|D\tilde{\eta}|_2 + |\tilde{\eta}|_{L^\infty})|u|_2$$

$D^\alpha \tilde{F}_i$, $\alpha = 1,2,3$, have the same estimate as $D^{\alpha-1}F$ as above. Therefore f has the same estimate as (4.11).

Lemma 4.2. *For* $t \ge 2$, *we have for* $i = 1,2$

$$(4.13) \qquad M(\eta_i,u_i;t) \le C_1 \sup_{0\le s\le t} (1+s)^{3/2}|f(s)|_0$$

$$\le C_1C(E_3)M(\eta,u;t)^2.$$

Also we have for $i = 3$

$$(4.14) \qquad M(\eta_3,u_3;t) \le C_2 \sup_{t-1\le s\le t} (1+s)^{3/2}|f(s)|_2$$

$$\le C_2C(E_3)M(\eta,u;t)(M(\eta,u;t) + E(\eta,u)) .$$

This is proved by Theorem 3.1 and by Lemma 4.1 and Sobolev-Nirenberg's inequality [4]. In particular, we know that since by (4.5) $\tilde{\eta}(t)$ and $u(t)$ are bounded in H^6 and H^5 respectively for any $t \ge 1$, we have by (4.6) for any $t \ge 1$

$$|D^{3+\alpha}\,\tilde{\eta}(t)|_0 \le C\,t^{(\alpha-3)/2}, \quad \alpha = 1,2,$$

$$|D^{2+\alpha}\,u(t)|_0 \le C\,t^{(\alpha/3-1)}, \quad \alpha = 1,2.$$

Proof of Theorem 1.2. It follows from (4.9) and Lemma 4.2 that

$$M(\eta,u;t) \le C_0E_2 + C_1CM(\eta,u;t)^2 + C_2CM(\eta,u;t)(M(\eta,u;t) + E(\eta,u)).$$

Therefore there exists $\delta_2 > 0$ such that if $E_2 < \delta_2$, $E_0 < \delta_1$, then

$$M(\eta,u;t) \le C\,E_2\ .$$

This proves Theorem 1.2.

Remark. If the fluid has an infinite depth, the eigenvalue of $G(\xi)$ has the following expansion:

$$\lambda(\xi) = \pm\sqrt{g|\xi|} - 2\nu|\xi|^2 - O(|\xi|^{11/4})\ , \tag{4.15}$$

which is quite different from (3.13). The details will be published elsewhere.

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We consider the mathematical behavior of a viscous, incompressible fluid bounded above by an atmosphere of constant pressure and below by a horizontal bottom. After reviewing the existence and regularity theory for the equations governing the motion, we establish rates of decay for solutions near equilibrium. The function describing the height of the free surface decays like $t^{-1/2}$; the velocity field decays like t^{-1}. These estimates are shown first for the linearization about equilibrium and then for the full nonlinear problem. Complete details will be given elsewhere. cont Keywords include:

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