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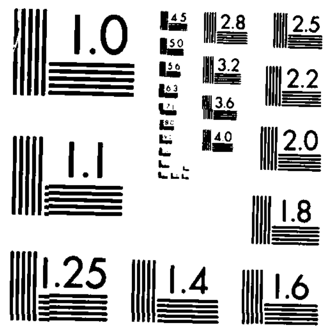

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Aerodynamics Technical Memorandum 369

GENERATION BY CONFORMAL MAPPINGS OF AEROFOIL SECTIONS AND OF CERTAIN OTHER SIMPLE SHAPES SUITABLE FOR BOTH AERODYNAMIC AND STRESS-CONCENTRATION PROBLEMS

by

Ton Tran-Cong

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SUMMARY

Aerofoil shapes as well as other simple shapes are produced using conformal mappings. These mappings are applicable to the two-dimensional aerofoil problems of aerodynamics and also to the stress concentration problems of linear elasticity.

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1. INTRODUCTION

In flight dynamic studies of aircraft, there is frequently a requirement for the estimation of aerodynamic derivatives. Theoretical approaches to this estimation often require a conformal mapping from a unit circle to the aerofoil shape concerned as one of the steps involved. Therefore an investigation has been carried out into the transformation from circles into practical shapes. Some results of this investigation are presented in this Technical Memorandum, which is essentially a compendium of shapes (aerodynamic and non-aerodynamic) in the z-plane obtainable by conformal mappings from the unit circle in a ζ-plane. Shapes obtained by varying the position and size of this circle are too numerous to be catalogued and are considered to be essentially just variants of the main ones considered here.

The method of conformal mapping from the exterior of the unit circle to the exterior of a desired closed curve has widespread applications, for example in aerodynamics and in solid mechanics. However, comparatively few shapes have been specifically given in applications. The best known are the ellipse and Joukowski aerofoils, which are variants of the narrow slit, and the Karman-Trefftz aerofoils which are variants of areas bounded by two circular arcs with equal curvature (see Kober [1] and Glauert [2]) and hypocyloids (see the recent paper by Clement [3]). Most other shapes are generated only by multistep and ad-hoc mappings (see Clement [4] and Wu [5], [6]) or by infinite series (see Abbott and Doenhoff's book [7] on the method by Theodorsen and the technique by Naiman). The present work aims to bring to light another range of curves, besides the well-known ones, which can be produced by simple analytic formulae or by simple numerical methods. These curves are:

1. Double-cusped aerofoils,
2. Biconvex-aerofoils with different leading and trailing edge angles,
3. Biconvex-aerofoils, by integration,
4. Triangles with arbitrary vertex angles,

5. Regular polygons,
6. Rhombs,
7. Rectangles,
8. Aerofoils by finite product formulae.

The first two types of aerofoil curve are derived from synthesised formulae. The next five types of curve are obtained from the integration of $dz/d\zeta$. The last type of aerofoil curve is obtained from a finite product formula subjected to some constraint as developed previously in Ref. 9.

The following sections will treat those types of curves and their generating methods in the above order. Of particular concern is the closure condition for the integration method. This is automatically satisfied for Schwarz-Christoffel transformations but not necessarily for the type of transformations used to generate the shapes from 3. to 7. A discussion of this condition has not been found in literature available to the author. Section 6 describes the new integral method used to plot the curves in the transformed region and the Fourier integral method used to evaluate the complex coefficients of the descending series, which are given in the corresponding figure, for each transformation. A novel way to generate an approximate square is also mentioned there.

By the use of these transformations, two-dimensional problems of pure circulation, inviscid, incompressible flow about these shapes can be easily solved. Similarly, they can be used to solve the problem of incompressible Stokes' flow about these shapes. The two-dimensional problem of stress-concentration for an infinite plate can also be solved for these shapes by the use of the well known complex-variable method.

2. DOUBLE-CUSPED AEROFOILS

The formula for this aerofoil was synthesised by closely following Joukowski's formula. We guess a formula of the form

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$$z = \zeta + k\,\frac{\zeta+b}{\zeta^2-a}$$

and select a, b and k such that $dz/d\zeta$ vanishes only at $\zeta = \pm 1$ and to zeros of order one. The resulting formula is

$$z(\zeta) = \zeta + \frac{(1-a)^2}{(1+a)}\;\frac{\zeta}{\zeta^2-a}\;, \qquad (1)$$

which has its derivative as

$$\frac{dz}{d\zeta} = 1 - \frac{(1-a)^2}{1+a}\;\frac{\zeta^2+a}{(\zeta^2-a)^2}\;. \qquad (2)$$

This is clearly a function with the desired properties. The constant a determines the maximum thickness of the aerofoil Figure 1 gives such an aerofoil with the constant a being 0.07. The image of a nest of concentric circles is plotted using a graphic plotter. The small irregularities on these curves are due to the limitation on the resolution of the plotter as well as of the integration subroutine used. The small table in the same figure gives the values of the coefficients a_n for the descending series

$$z(\zeta) = a_1\zeta + \sum_{n=-1}^{-\infty} a_n\zeta^n$$

which is a Laurent expansion of the transformation function $z(\zeta)$. Transformations for aerofoils in subsequent figures also have their series given in this way.

The double-cusped aerofoils with different leading and trailing halves are given by the formula

$$z = \zeta - \frac{b}{1+a}\,\frac{1}{\zeta^2} + \frac{(1-a)^2}{(1+a)} \times \frac{\zeta+b}{\zeta^2-a}\;. \qquad (3)$$

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Its derivative is

$$\frac{dz}{d\zeta} = 1 + \frac{2b}{1+a}\,\frac{1}{\zeta^3} - \frac{(1-a)^2}{(1+a)} \times \frac{\zeta^2+2b\zeta+a}{(\zeta^2-a)^2} \quad , \tag{4}$$

which vanishes only at $\zeta = \pm 1$ to zeros of order one. The constant a determines the maximum thickness of the aerofoil and the constant b determines the extent of asymmetry between the leading and trailing edge. Figure 2 is a plot of an aerofoil so generated with a = 0.07, b = -0.3.

3. AEROFOILS WITH FINITE AND DIFFERENT LEADING AND TRAILING EDGE ANGLES (GENERALISED KARMAN-TREFFTZ AEROFOILS)

From the Karman-Trefftz formula

$$z = \mu \, \frac{\left(\frac{\zeta+1}{\zeta-1}\right)^{\mu} + 1}{\left(\frac{\zeta+1}{\zeta-1}\right)^{\mu} - 1} \quad , \; 0<\mu\leq 2,$$

we construct the new formula

$$z = \frac{\mu(1+\kappa\lambda)}{(1+\kappa)} \times \frac{(1+\kappa)^{-\mu}\left[\left(\frac{\zeta+1}{\zeta-1}\right)^{\frac{2\eta}{1-\lambda}} + \kappa\left(\frac{\zeta+1}{\zeta-1}\right)^{\frac{2\eta\lambda}{1-\lambda}}\right]^{\frac{\mu(1-\lambda)}{2\eta}} + 1}{(1+\kappa)^{-\mu}\left[\left(\frac{\zeta+1}{\zeta-1}\right)^{\frac{2\eta}{1-\lambda}} + \kappa\left(\frac{\zeta+1}{\zeta-1}\right)^{\frac{2\eta\lambda}{1-\lambda}}\right]^{\frac{\mu(1-\lambda)}{2\eta}} - 1} \tag{5}$$

for which the derivative vanishes only at $\zeta = \pm 1$ to zeros of order $(\mu-1)$ and $(\lambda\mu-1)$ respectively.

In the above formula, μ is the external angle for the trailing edge, $\lambda\mu$ is the external angle for the leading edge $(0\leq\lambda\leq 1)$, κ affects the ratio between the influence by the leading edge and that by the trailing edge $(0<\kappa<\infty)$, and η affects the propagation of the

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circular arcs from the leading and trailing edges ($0<\eta<1$). For the special case where λ is equal to one, the above formula reduces to the standard Karman-Trefftz formula.

Figure 3 gives such an aerofoil with $\mu = 1.9$, $\lambda = 0.7$, $\kappa = 0.4$ and $\eta = 0.4$. Effects of each individual parameter on the shape of the aerofoil are demonstrated in figures 4, 5 and 6. In these latter figures, individual parameters κ, η, λ, are set to 1.0, 0.2, 0.6 respectively while the remaining three parameters are left the same as in figure 3.

Analysis easily shows that the curve given by equation (5) asymptotes circular arcs of the Karman-Trefftz type at the leading and trailing edges of the aerofoil. By using all four parameters at our disposal, the basic NACA 0015 aerofoil with no camber is approximated with the choice $\mu = 1.9$, $\lambda = 0.5263$, $\kappa = 0.22$ and $\eta = 0.53$. The resulting aerofoil is given in figure 7. Note that this aerofoil is the image of the unit circle in the ζ-plane. The images of other circles in the ζ plane can give even better approximation of the NACA aerofoil.

4. SHAPES DERIVED FROM THE INTEGRATION OF $dz/d\zeta$ IN THE FORM OF FINITE PRODUCTS

It has been known (see Carrier, Krook and Pearson [8] that the integration of

$$\frac{dz}{d\zeta} = \frac{1}{\zeta^2} \prod_{k=1}^{n} (\zeta-\phi_k)^{2\gamma_k}, \quad |\phi_k| = 1, \quad \sum_{k=1}^{n} \gamma_k = 1$$

gives a transformation from the outside of a unit circle to the outside of a polygon with <u>some</u> choice of $\{\phi_k\}$. Here we point out that transformation by integration of $dz/d\zeta$ need not be a haphazard process. Indeed, if we have

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$$\frac{dz}{d\zeta} = \frac{1}{\zeta^2} \prod_{k=1}^{n} (\zeta-\phi_k)^{2\gamma_k}, \quad |\phi_k| \le 1, \quad \sum_{k=1}^{n} \gamma_k = 1 \tag{6}$$

and

$$\sum_{k=1}^{n} \gamma_k \phi_k = 0 \tag{7}$$

then the integration of $dz/d\zeta$ will give a single-valued function $z(\zeta)$ for $|\zeta| > 1$, such that its derivative is given by the first member of equation (6). The closure equation (7) has not been found in available literature. This condition here is arrived at by requiring that $z(\zeta)$ is single-valued and by considering the contour integral of the first member of equation (6) along a circle of very large radius. The vanishing of this integral has (7) as it necessary and sufficient condition. (Another result from the condition of closure is that if $\underset{\sim}{\gamma} = (\gamma_1, \gamma_2, \ldots, \gamma_n)$ is a vector of real values of external angles of a polygonal then equation

$$\sum_{k=1}^{n} \gamma_k e^{ix_k} = 0$$

will have at least a real vector solution $\underset{\sim}{x} = (x_1, x_2, \ldots, x_n)$. Each shape of the polygon has a unique solution $\underset{\sim}{x} = (x_1, x_2, \ldots, x_n)$, all x_i within $[0, 2\pi]$, determined up to a constant common to all components. Proof is by the Rienann mapping theorem for the exterior of the polygon from the region $|\zeta| > 1$).

Our application of the transformation (6) subject to the condition (7) gives the biconvex aerofoil of figure 8. The derivative $dz/d\zeta$ is taken as

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$$\frac{dz}{d\zeta} = \frac{(\zeta-1)^{\alpha}(\zeta+1)^{\beta}}{(\zeta-c)^{\alpha+\beta}} ,$$

with $\alpha = 0.8$, $\beta = 0.6$, $c = 0.142$. The condition (7) is satisfied with this choice of parameters. Note that α determines the leading edge angle, β the trailing edge angle and c is a function of α and β by equation (7). This way of generating aerofoils gives more flexibility than the use of formula (3).

The generation of a triangle with given vertex angles is also a special case of equation (6) subject to the closure condition (7). The derivative for this case is

$$\frac{dz}{d\zeta} = \frac{(\zeta-e^{i0})^{2-\alpha-\beta}(\zeta-e^{ia})^{\alpha}(\zeta-e^{ib})^{\beta}}{\zeta^{2}} .$$

Figure 9 is a triangle corresponding to $\alpha = 0.5$, $\beta = 0.7$, $a = 2.094$ and $b = 3.807$. Note that a and b are chosen such that equation (7) is satisfied with the values of α and β given.

Shapes with some kind of symmetry usually have condition (7) automatically satisfied, and are given in text books (such as [8]) without any mention of the latter condition. The following curves belong to this category.

By choosing

$$\frac{dz}{d\zeta} = \frac{1}{\zeta^{2}} (\zeta^{n}-1)^{2/n}$$

we have an n-sided polygon. The examples are given in figures 10, 11, 12 and 13. As n tends to infinity, the polygon becomes a circle.

Putting n = 4 we have a square as in figure 11. By giving different powers to (ζ^2-1) and (ζ^2+1) we have

$$\frac{dz}{d\zeta} = \frac{1}{\zeta^2}(\zeta^2-1)^{\alpha}(\zeta^2+1)^{1-\alpha} , \quad 0<\alpha<1 ,$$

which gives a rhomb with external angles α and (1-α), as in figure 14.

Another variation of the formula for the square into

$$\frac{dz}{d\zeta} = \frac{1}{\zeta^2}(\zeta^2-e^{i\alpha\pi})^{\frac{1}{2}}(\zeta^2-e^{-i\alpha\pi})^{\frac{1}{2}} , \quad 0<\alpha<1 ,$$

gives rectangles with different aspect ratio, such as in figure 15.

5. AEROFOILS BY FINITE PRODUCT FORMULAE

Aerofoil shapes can also be generated by the use of finite product formulae as considered in a previous study [9]. This method consists of using formulae such as

$$z(\zeta)-z_c = (\zeta-1)^{1.9}(\zeta+0.1+0.2i)^{0.1}(\zeta-0.1+0.2i)^{-1} ,$$

with the constant z_c chosen such that $z-\zeta$ is of order ζ^{-1} or smaller as $|\zeta|$ tends to infinity. A computer program is used to ensure that all singularities of $dz/d\zeta$ are contained in the unit disc $|\zeta|\leq 1$ with one at $\zeta=1$ to generate a finite acute angle at the trailing edge. (The computer programs for checking the singularities of $dz/d\zeta$ and the initial shapes of these aerofoils were written by Mr. C. A. Martin. The author acknowledges his help to sustantiate this method). An aerofoil generated by this method is given in figure 16. The constant z_c has been calculated in the computer program and the value of $z(\zeta)$ is plotted in the figure.

6. PLOTTING OF CURVES AND COMPUTATION OF DESCENDING SERIES

To plot curves in the transformed region we use the formula developed in Ref. [9]

$$z(\zeta) = \int_0^1 e^{i2\pi t} d\alpha(t) + \exp[\int_0^1 \log(\zeta - e^{i2\pi t})[d\alpha(t)+dt]]$$

where $\alpha(t)+t$ is given by

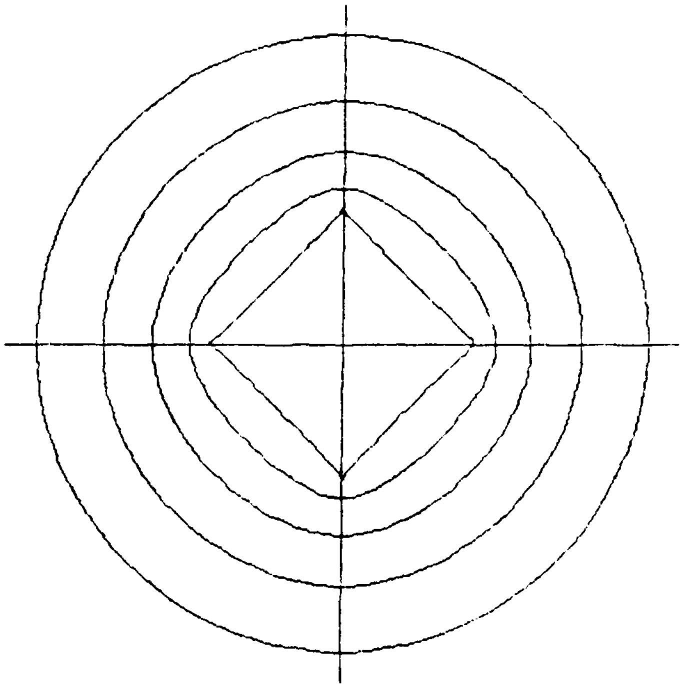
$$2\pi i[\alpha(t)+t] = \log[z(e^{i2\pi t}) - z_D], \quad 0 \leq t \leq 1 ,$$

and z_D is an arbitrarily chosen point interior to the closed curve $z(e^{i2\pi t})$. This method of plotting was used to generate the curves in all figures of this Memo. The advantage of this method is that infinitesimal features close to the unit circle are preserved, in contrast to the descending power method where a very large number of terms are required close to the points of singularity of $dz/d\zeta$ and this number becomes infinite as any singularity is approached.

Far away from the aerofoil it is quite economical to use decending series, as demonstrated by the shapes of the images of concentric circles in Figures 1 to 17. The descending series for each aerofoil is given in the table in its corresponding figure as described previously in section 2. The complex coefficients of these series are computed using Fourier integrals on $z(e^{i2\pi t})$. This method is applicable here as the image $z(\zeta)$ is known for each point $\zeta = e^{i2\pi t}$ of the unit circle in the ζ-plane.

As a point of curiosity, we note that the descending series for the function

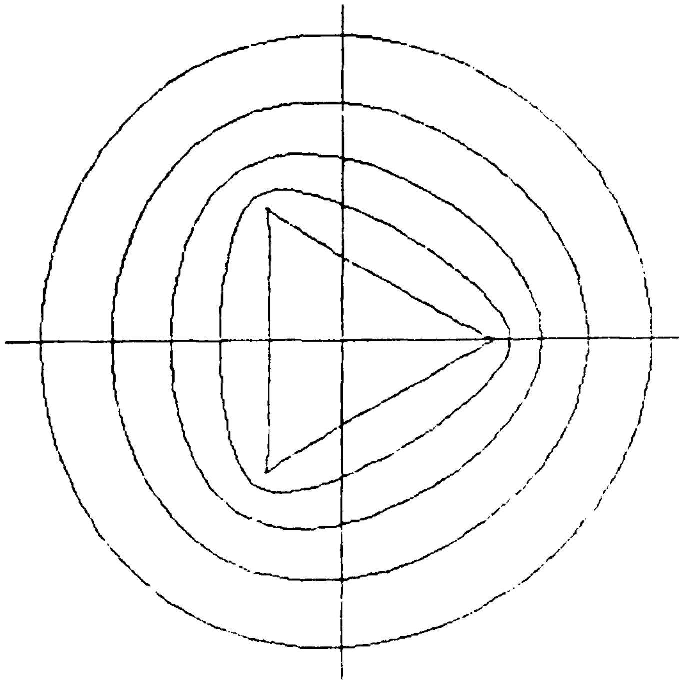
$$z = x + iy$$



Coefficients of the descending series:

```
        A1R,A1I=   1.          1.
 -N=   1,AR,AI=-0.00187-0.00251
 -N=   2,AR,AI=-0.00000  0.00031
 -N=   3,AR,AI= 0.00042  0.10676
 -N=   4,AR,AI=-0.00000  0.00016
 -N=   5,AR,AI=-0.00050-0.00038
 -N=   6,AR,AI=-0.00000  0.00010
 -N=   7,AR,AI= 0.00031  0.01799
 -N=   8,AR,AI=-0.00000  0.00008
 -N=   9,AR,AI=-0.00028-0.00021
 -N=  10,AR,AI=-0.00000  0.00006
 -N=  11,AR,AI= 0.00021  0.0056[illegible]
 -N=  12,AR,AI=-0.00000  0.00005
 -N=  13,AR,AI=-0.00019-0.00015
 -N=  14,AR,AI=-0.00000  0.00004
 -N=  15,AR,AI= 0.00016  0.00261
 -N=  16,AR,AI=-0.00000  0.00004
 -N=  17,AR,AI=-0.00015-0.00011
 -N=  18,AR,AI=-0.00000  0.00003
 -N=  19,AR,AI= 0.00013  0.00144
```

FIG. 11 A [illegible] APE BY INTEGRATION METHOD



Coefficients of the descending series:

```
        A1R,A1I=   1.         1.
 -N=    1,AR,AI=-0.00014-0.00000
 -N=    2,AR,AI=-0.00000 0.33313
 -N=    3,AR,AI= 0.00000 0.00001
 -N=    4,AR,AI=-0.00000-0.00004
 -N=    5,AR,AI= 0.00000 0.02218
 -N=    6,AR,AI=-0.00000 0.00000
 -N=    7,AR,AI=-0.00000-0.00002
 -N=    8,AR,AI= 0.00000 0.00615
 -N=    9,AR,AI= 0.00000 0.00000
 -N=   10,AR,AI=-0.00000-0.00001
 -N=   11,AR,AI=-0.00000 0.00260
 -N=   12,AR,AI=-0.00000 0.00000
 -N=   13,AR,AI=-0.00000-0.00001
 -N=   14,AR,AI= 0.00000 0.00136
 -N=   15,AR,AI= 0.00000-0.00000
 -N=   16,AR,AI= 0.00000-0.00001
 -N=   17,AR,AI= 0.00000 0.00030
 -N=   18,AR,AI= 0.00000-0.00000
 -N=   19,AR,AI= 0.00000-0.00001
```

FIG. 10 A RE[illegible]AR TRIANGLE BY [illegible] METHOD,

[illegible]

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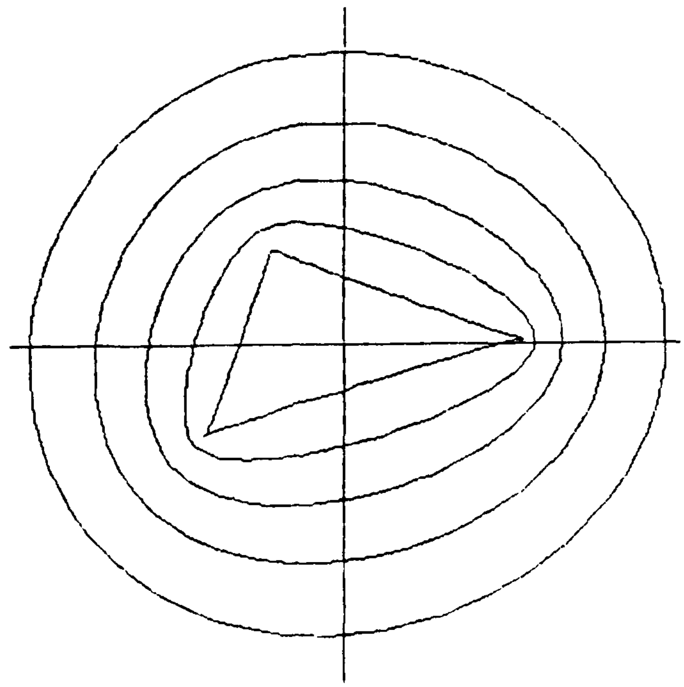
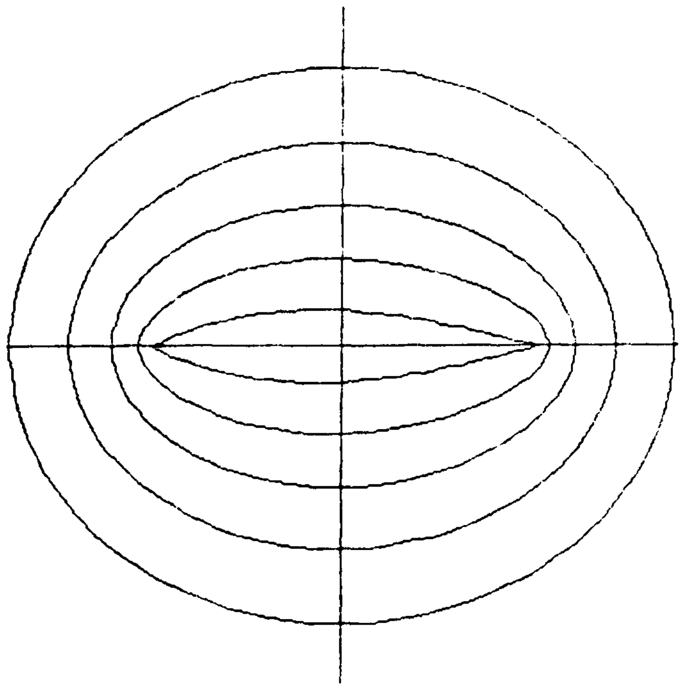


Coefficients of the descending series:

```
       A1R,A1I=  1.         1.
-N=   1,AR,AI= 0.35381  0.12254
-N=   2,AR,AI= 0.00090-0.28273
-N=   3,AR,AI=-0.03461  0.04024
-N=   4,AR,AI= 0.00862-0.01132
-N=   5,AR,AI= 0.00229-0.01251
-N=   6,AR,AI=-0.00863  0.00418
-N=   7,AR,AI= 0.00498-0.00234
-N=   8,AR,AI=-0.00064-0.00220
-N=   9,AR,AI=-0.00262  0.00022
-N=  10,AR,AI= 0.00246  0.00007
-N=  11,AR,AI=-0.00089-0.00118
-N=  12,AR,AI=-0.00061-0.00000
-N=  13,AR,AI= 0.00096  0.00045
-N=  14,AR,AI=-0.00042-0.00095
-N=  15,AR,AI=-0.00012  0.00025
-N=  16,AR,AI= 0.00032  0.00022
-N=  17,AR,AI=-0.00004-0.00061
-N=  18,AR,AI=-0.00013  0.00030
-N=  19,AR,AI= 0.00017  0.00001
```

FIG. 9 AN ARBIT[illegible] TRIANGLE BY [illegible] METHOD

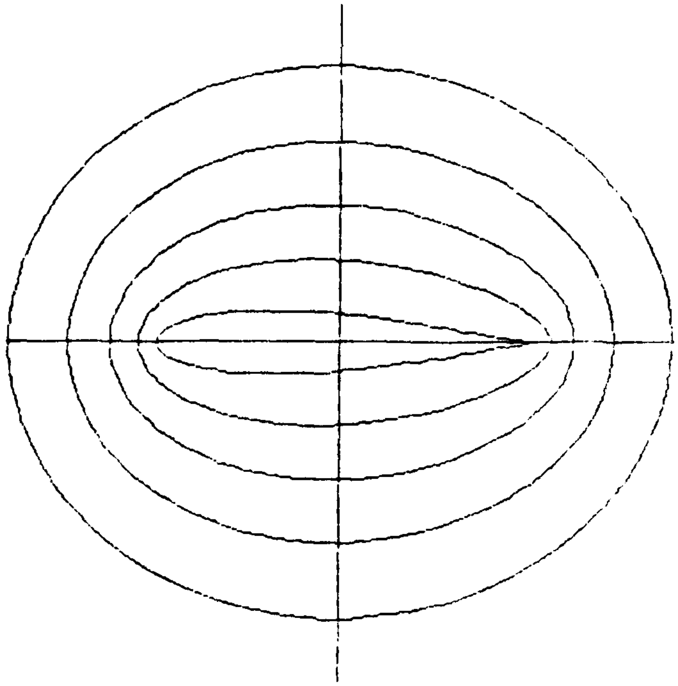
[illegible]



Coefficients of the descending series:

	AR	AI
A1R,A1I=	1.	1.
-N= 1,AR,AI=	0.68014	0.00000
-N= 2,AR,AI=	0.00223	0.02998
-N= 3,AR,AI=	0.00288	0.03664
-N= 4,AR,AI=	-0.00013	-0.00229
-N= 5,AR,AI=	0.00065	0.0082[illegible]
-N= 6,AR,AI=	-0.00017	-0.0016[illegible]
-N= 7,AR,AI=	0.0002[illegible]	0.0031[illegible]
-N= 8,AR,AI=	-0.0000[illegible]	[illegible]
-N= 9,AR,AI=	0.00012	0.001[illegible]
-N= 10,AR,AI=	-0.0000[illegible]	-0.00[illegible]
-N= 11,AR,AI=	0.0000[illegible]	0.00[illegible]
-N= 12,AR,AI=	-0.0000[illegible]	[illegible]
-N= 13,AR,AI=	0.0000[illegible]	[illegible]
-N= 14,AR,AI=	-0.0000[illegible]	[illegible]
-N= 15,AR,AI=	0.000[illegible]	0.000[illegible]
-N= 16,AR,AI=	-0.0000[illegible]	[illegible]
-N= 17,AR,AI=	[illegible]	[illegible]
-N= 18,AR,AI=	-0.00002	[illegible]
-N= 19,AR,AI=	0.00001	[illegible]

FIG. [illegible]



Coefficients of the descending series:

```
          A1R,A1I=   1.          1.
  -N=   1,AR,AI= 0.71645  0.00000
  -N=   2,AR,AI= 0.00000  0.07176
  -N=   3,AR,AI=-0.00000  0.01781
  -N=   4,AR,AI= 0.00000  0.00598
  -N=   5,AR,AI=-0.00000  0.00213
  -N=   6,AR,AI=-0.00000  0.00164
  -N=   7,AR,AI= 0.00000  0.00059
  -N=   8,AR,AI= 0.00000  0.00069
  -N=   9,AR,AI=-0.00000  0.00025
  -N=  10,AR,AI= 0.00000  0.00036
  -N=  11,AR,AI=-0.00000  0.00014
  -N=  12,AR,AI=-0.00000  0.00021
  -N=  13,AR,AI= 0.00000  0.00008
  -N=  14,AR,AI= 0.00000  0.00013
  -N=  15,AR,AI=-0.00000  0.00005
  -N=  16,AR,AI=-0.00000  0.00009
  -N=  17,AR,AI=-0.00000  0.00004
  -N=  18,AR,AI= 0.00000  0.00006
  -N=  19,AR,AI= 0.00000  0.00003
```

FIG. 7 APPROXIMATION OF THE AEROFOIL [illegible] THICKNESS, BY AN ABEL [illegible] EQUATION (5),
[illegible]

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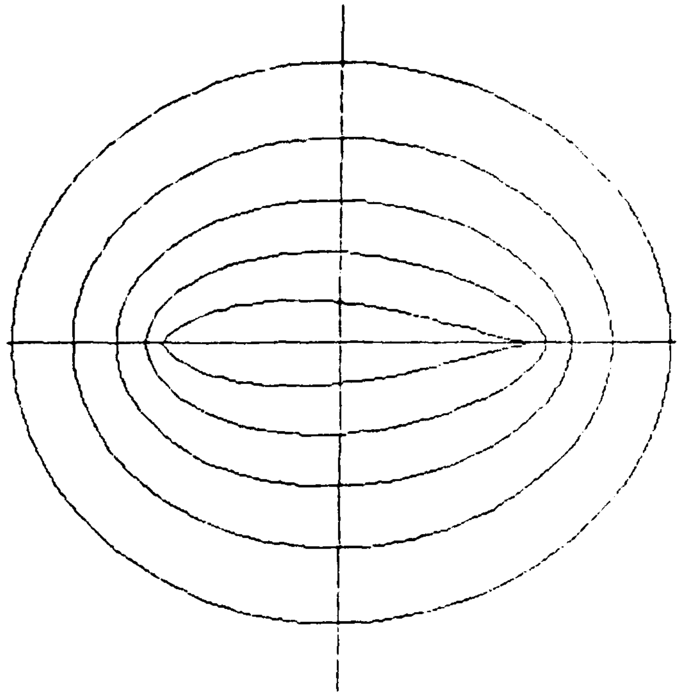


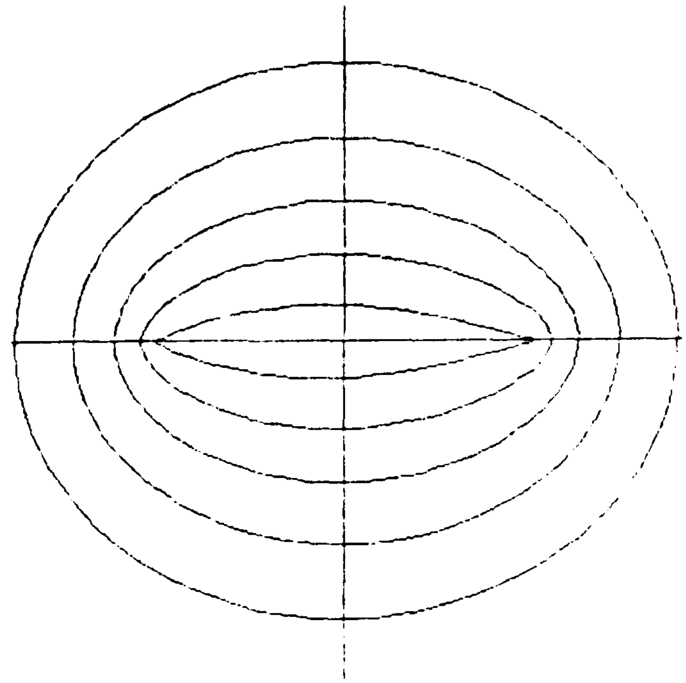
Coefficients of the descending series:

		A1R.A1I=	1.	1.
-N=	1	.AR.AI=	0.63296	0.00000
-N=	2	.AR.AI=	0.00000	0.07024
-N=	3	.AR.AI=	0.00000	0.03602
-N=	4	.AR.AI=	0.00000	0.00479
-N=	5	.AR.AI=	0.00000	0.00695
-N=	6	.AR.AI=	0.00000	0.00034
-N=	7	.AR.AI=	-0.00000	0.0025[illegible]
-N=	8	.AR.AI=	-0.00000	-0.00015
-N=	9	.AR.AI=	-0.00000	0.00125
-N=	10	.AR.AI=	-0.00000	-0.00019
-N=	11	.AR.AI=	0.00000	0.00074
-N=	12	.AR.AI=	0.00000	-0.00017
-N=	13	.AR.AI=	0.00000	0.00048
-N=	14	.AR.AI=	0.00000	-0.00014
-N=	15	.AR.AI=	-0.00000	0.00034
-N=	16	.AR.AI=	-0.00000	-0.00012
-N=	17	.AR.AI=	-0.00000	0.00025
-N=	18	.AR.AI=	0.00000	-0.00010
-N=	19	.AR.AI=	0.00000	0.00019

FIG. 6 FINI[illegible]ED AIRFOIL [illegible]

[illegible]

REPRODUCED AT GOVERNMENT EXPENSE

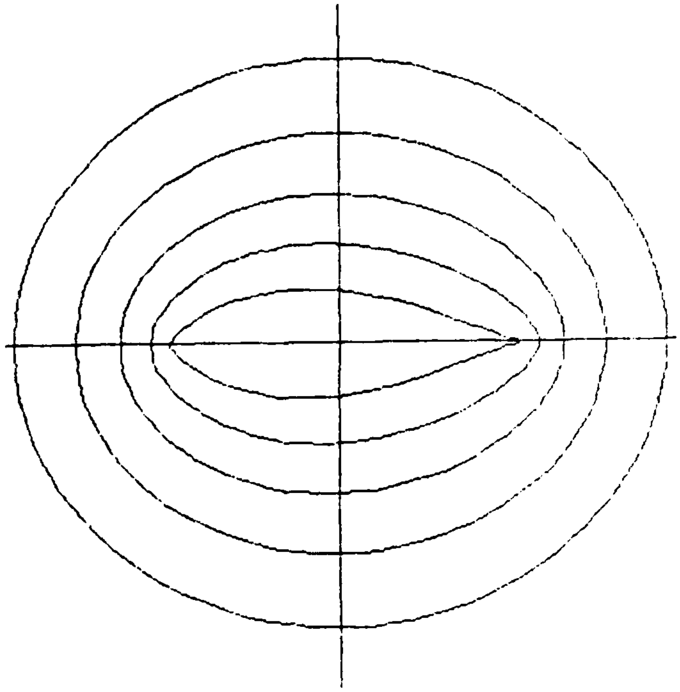


Coefficients of the descending series:

```
            A1R,A1I=  1.        1.
  -N=   1,AR,AI= 0.67634  0.00000
  -N=   2,AR,AI= 0.00000  0.02708
  -N=   3,AR,AI=-0.00000  0.04209
  -N=   4,AR,AI=-0.00000  0.00148
  -N=   5,AR,AI= 0.00000  0.00952
  -N=   6,AR,AI=-0.00000-0.00013
  -N=   7,AR,AI= 0.00000  0.00361
  -N=   8,AR,AI= 0.00000-0.00023
  -N=   9,AR,AI=-0.00000  0.00178
  -N=  10,AR,AI=-0.00000-0.00023
  -N=  11,AR,AI= 0.00000  0.00102
  -N=  12,AR,AI=-0.00000-0.00018
  -N=  13,AR,AI= 0.00000  0.00064
  -N=  14,AR,AI= 0.00000-0.00014
  -N=  15,AR,AI=-0.00000  0.00044
  -N=  16,AR,AI=-0.00000-0.00011
  -N=  17,AR,AI= 0.00000  0.00031
  -N=  18,AR,AI= 0.00000-0.00009
  -N=  19,AR,AI= 0.00000  0.00027
```

FIG. [illegible] FINI[illegible]
[illegible] = 1.[illegible], [illegible] = [illegible], [illegible] .4,

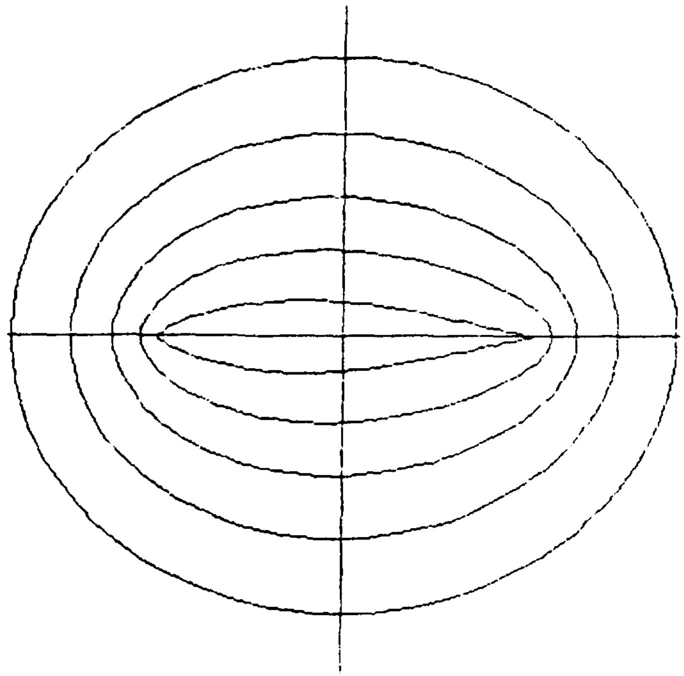
REPRODUCED AT GOVERNMENT EXPENSE [illegible]



Coefficients of the descending series:

```
        A1R,A1I=  1.        1.
 -N=   1,AR,AI= 0.54106  0.00000
 -N=   2,AR,AI= 0.00000  0.06854
 -N=   3,AR,AI=-0.00000  0.05037
 -N=   4,AR,AI= 0.00000  0.00595
 -N=   5,AR,AI= 0.00000  0.01164
 -N=   6,AR,AI=-0.00000-0.00000
 -N=   7,AR,AI=-0.00000  0.00437
 -N=   8,AR,AI= 0.00000-0.00061
 -N=   9,AR,AI=-0.00000  0.00218
 -N=  10,AR,AI= 0.00000-0.00055
 -N=  11,AR,AI= 0.00000  0.00129
 -N=  12,AR,AI=-0.00000-0.00044
 -N=  13,AR,AI= 0.00000  0.00084
 -N=  14,AR,AI= 0.00000-0.00034
 -N=  15,AR,AI=-0.00000  0.00059
 -N=  16,AR,AI=-0.00000-0.00027
 -N=  17,AR,AI=-0.00000  0.00043
 -N=  18,AR,AI= 0.00000-0.00022
 -N=  19,AR,AI= 0.00000  0.00033
```

FIG. 4 [illegible]
[illegible] = 1.[illegible], [illegible] = 0.7, [illegible] = 1.[illegible], [illegible]

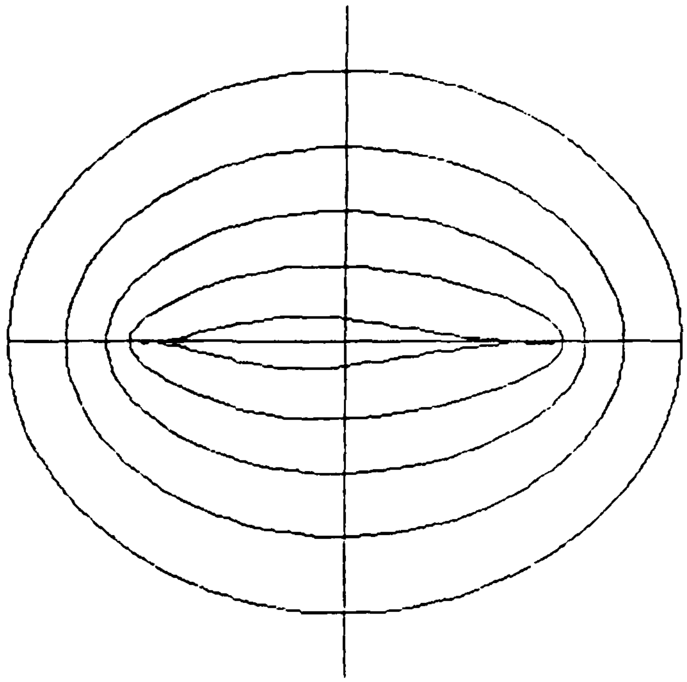


Coefficients of the desc[illegible]ing series:

```
          A1R,A1I=   1.        1.
 -N=   1,AR,AI= 0.68767 0.00000
 -N=   2,AR,AI= 0.00000 0.05498
 -N=   3,AR,AI= 0.00000 0.03449
 -N=   4,AR,AI=-0.00000 0.00196
 -N=   5,AR,AI= 0.00000 0.00709
 -N=   6,AR,AI=-0.00000-0.00051
 -N=   7,AR,AI=-0.00000 0.00275
 -N=   8,AR,AI=-0.00000-0.00054
 -N=   9,AR,AI=-0.00000 0.00143
 -N=  10,AR,AI= 0.00000-0.00041
 -N=  11,AR,AI= 0.00000 0.00036
 -N=  12,AR,AI= 0.00000-0.00031
 -N=  13,AR,AI= 0.00000 0.00057
 -N=  14,AR,AI= 0.00000-0.00024
 -N=  15,AR,AI=-0.00000 0.00040
 -N=  16,AR,AI=-0.00000-0.00019
 -N=  17,AR,AI=-0.00000 0.00030
 -N=  18,AR,AI= 0.00000-0.00015
 -N=  19,AR,AI= 0.00000 0.00023
```

FIG. [illegible] FINIT[illegible] APPROXI[illegible]ATION (5),
[illegible] = 1.[illegible], [illegible] = 0.7, [illegible] [illegible].4, [illegible] [illegible].4

REPRODUCED AT [illegible] NMENT EXPENSE



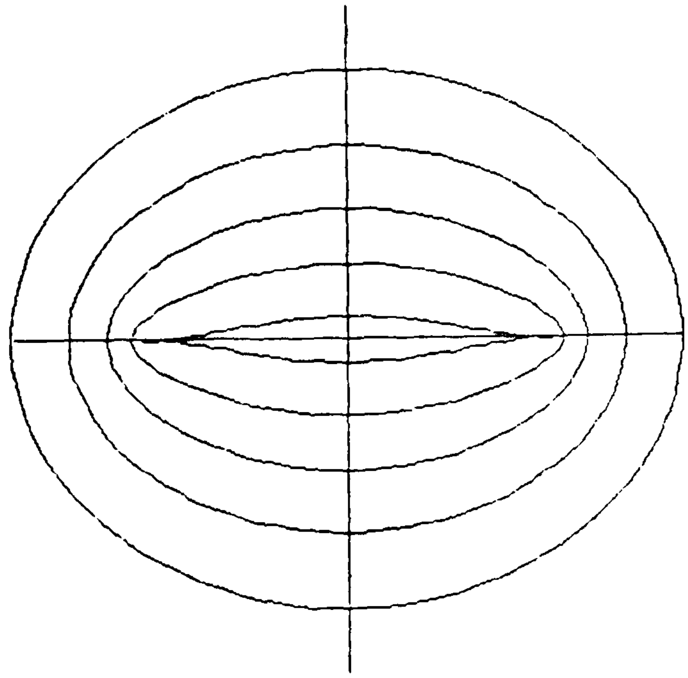
Coefficients of the descending series:

```
        A1R,A1I=  1.        1.
-N=   1,AR,AI=  0.80832  0.00000
-N=   2,AR,AI=  0.00000  0.03788
-N=   3,AR,AI=  0.00000  0.05658
-N=   4,AR,AI=  0.00000-0.01697
-N=   5,AR,AI=  0.00000  0.00396
-N=   6,AR,AI=-0.00000-0.00119
-N=   7,AR,AI=-0.00000  0.00028
-N=   8,AR,AI=  0.00000-0.00008
-N=   9,AR,AI=-0.00000  0.00002
-N=  10,AR,AI=-0.00000-0.00001
-N=  11,AR,AI=  0.00000  0.00000
-N=  12,AR,AI=  0.00000-0.00000
-N=  13,AR,AI=  0.00000  0.00000
-N=  14,AR,AI=  0.00000-0.00000
-N=  15,AR,AI=  0.00000  0.00000
-N=  16,AR,AI=-0.00000  0.00000
-N=  17,AR,AI=-0.00000-0.00000
-N=  18,AR,AI=  0.00000  0.00000
-N=  19,AR,AI=-0.00000-0.00000
```

FIG. 2 DOUBLE-CUSP AIRFOIL

z = [illegible] − b / (1 + [illegible]) [illegible] + (1 − [illegible]) (· + b) / ([illegible]) ([illegible] − [illegible])

[illegible] .[illegible], b = [illegible]



Coefficients of the descending series:

```
        A1R,A1I=   1.        1.
-N=   1,AR,AI= 0.80832 0.00000
-N=   2,AR,AI=-0.00000 0.00000
-N=   3,AR,AI=-0.00000 0.05658
-N=   4,AR,AI=-0.00000 0.00000
-N=   5,AR,AI=-0.00000 0.00396
-N=   6,AR,AI=-0.00000-0.00000
-N=   7,AR,AI= 0.00000 0.00028
-N=   8,AR,AI= 0.00000-0.00000
-N=   9,AR,AI=-0.00000 0.00002
-N=  10,AR,AI=-0.00000-0.00000
-N=  11,AR,AI= 0.00000 0.00000
-N=  12,AR,AI= 0.00000 0.00000
-N=  13,AR,AI= 0.00000 0.00000
-N=  14,AR,AI= 0.00000 0.00000
-N=  15,AR,AI=-0.00000 0.00000
-N=  16,AR,AI=-0.00000-0.00000
-N=  17,AR,AI= 0.00000-0.00000
-N=  18,AR,AI= 0.00000-0.00000
-N=  19,AR,AI= 0.00000 0.00000
```

FIG. 1 [illegible]UBLE-CUSPED [illegible],

$$z = \zeta + \frac{(1-a)^{2}\,\zeta\ [illegible]}{(1+a)\ ([illegible])}$$

REPRODUCED AT GOVERNMENT EXPENSE

ACKNOWLEDGEMENTS

Valuable discussions have been held with Mr. D. A. Secomb, and Mr. C. A. Martin during the course of this work. References [3] to [6] were kindly provided by Dr. L. R. F. Rose. The author wishes to thank them for their assistance and comments.

Two-dimensional problems of incompressible Stokes' flow about these cross-sections and of stress-concentration for an infinite elastic plate with a hole of these shapes can also be solved using the conformal mappings given here.

where

$$x = \begin{cases} \cos(2\pi t) \text{ if } |\cos(2\pi t)| \le \dfrac{1}{\sqrt{2}} \\[2ex] \dfrac{\cos(2\pi t)}{|\cos(2\pi t)|\sqrt{2}} \text{ if } |\cos(2\pi t)| > \dfrac{1}{\sqrt{2}} \end{cases}$$

$$y = \begin{cases} \sin(2\pi t) \text{ if } |\sin(2\pi t)| \le \dfrac{1}{\sqrt{2}} \\[2ex] \dfrac{\sin(2\pi t)}{|\sin(2\pi t)|\sqrt{2}} \text{ if } |\sin(2\pi t)| > \dfrac{1}{\sqrt{2}} \end{cases}$$

gives an approximate square, as in figure 17. The reason is that the coefficients of its ascending series are mostly zero or very small and decrease in magnitude very rapidly with increasing power of z. It is also convenient to note in this Memo that the n-corner starred shape used in some stress concentration problems is generated by the formula

$$z = (\zeta^n + \frac{m}{\zeta^n} + 1 + m)^{1/n} ,$$

where n is positive integer and m is a positive constant less than unity.

7. CONCLUSIONS

It has been shown here that there are many practical shapes obtainable from conformal mapping using simple techniques. Employing the results given here, it is only a matter of straightforward application to calculate the pure circulation, inviscid, incompressible flow about the aerofoils of this Memo. The pressure distribution then follows directly.

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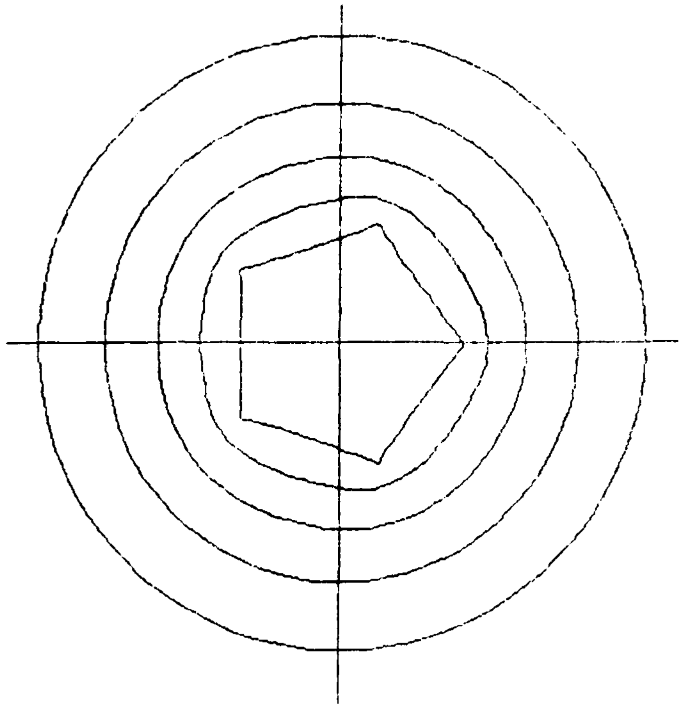
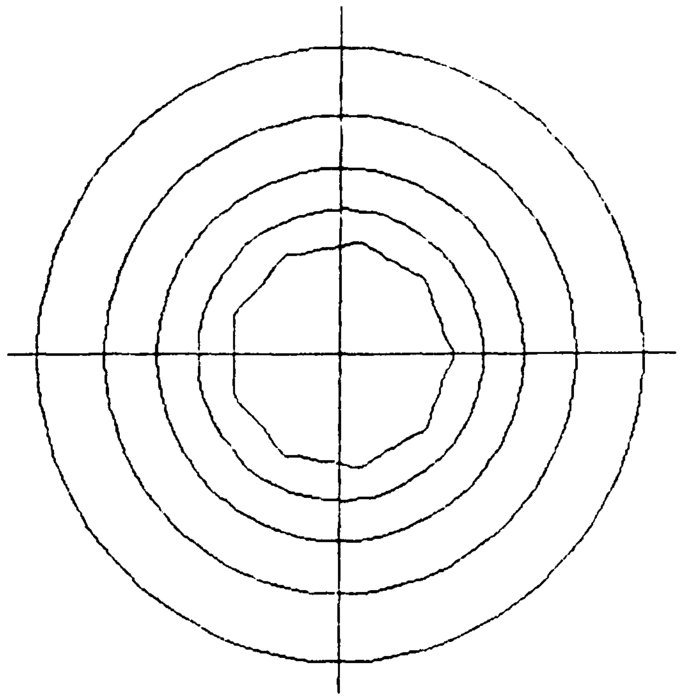


Coefficients of the descending series:

```
             A1R,A1I=  1.          1.
     -N=   1,AR,AI=-0.00385-0.00352
     -N=   2,AR,AI=-0.00000 0.00048
     -N=   3,AR,AI=-0.00000 0.00032
     -N=   4,AR,AI= 0.00052 0.00031
     -N=   5,AR,AI=-0.00000 0.00019
     -N=   6,AR,AI=-0.00058-0.00065
     -N=   7,AR,AI=-0.00000 0.00014
     -N=   8,AR,AI=-0.00000 0.00012
     -N=   9,AR,AI= 0.00034 0.01343
     -N=  10,AR,AI=-0.00000 0.00010
     -N=  11,AR,AI=-0.00032-0.00035
     -N=  12,AR,AI=-0.00000 0.00009
     -N=  13,AR,AI=-0.00000 0.00007
     -N=  14,AR,AI= 0.00023 0.00463
     -N=  15,AR,AI=-0.00000 0.00006
     -N=  16,AR,AI=-0.00022-0.00024
     -N=  17,AR,AI=-0.00000 0.00006
     -N=  18,AR,AI=-0.00000 0.00005
     -N=  19,AR,AI= 0.00018 0.00223
```

FI[illegible] [illegible] A [illegible]TAIN BY INT[illegible] METHOD

REPRODUCED AT GOVERNMENT EXPENSE

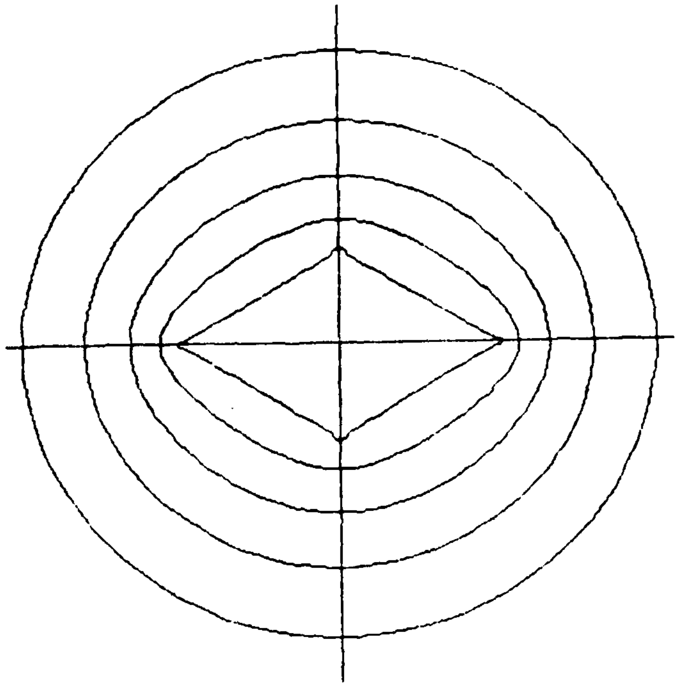


Coefficients of the descending series:

```
          A1R,A1I=   1.         1.
 -N=   1,AR,AI=-0.00024-0.00000
 -N=   2,AR,AI= 0.00000 0.00033
 -N=   3,AR,AI=-0.00000-0.00006
 -N=   4,AR,AI= 0.00000 0.00034
 -N=   5,AR,AI=-0.00000-0.00001
 -N=   6,AR,AI= 0.00000 0.00062
 -N=   7,AR,AI=-0.00000 0.00001
 -N=   8,AR,AI= 0.00000 0.02773
 -N=   9,AR,AI= 0.00000 0.00003
 -N=  10,AR,AI=-0.00000-0.00002
 -N=  11,AR,AI= 0.00000 0.00006
 -N=  12,AR,AI=-0.00000-0.00001
 -N=  13,AR,AI=-0.00000 0.00010
 -N=  14,AR,AI=-0.00000-0.00000
 -N=  15,AR,AI= 0.00000 0.00023
 -N=  16,AR,AI=-0.00000 0.00000
 -N=  17,AR,AI= 0.00000 0.00506
 -N=  18,AR,AI= 0.00000 0.00002
 -N=  19,AR,AI=-0.00000-0.00001
```

FIG. 13 A NINE-SIDED RE[illegible] BY
[illegible]GATION METH[illegible]
[illegible]

REPRODUCED AT GOVERNMENT EXPENSE

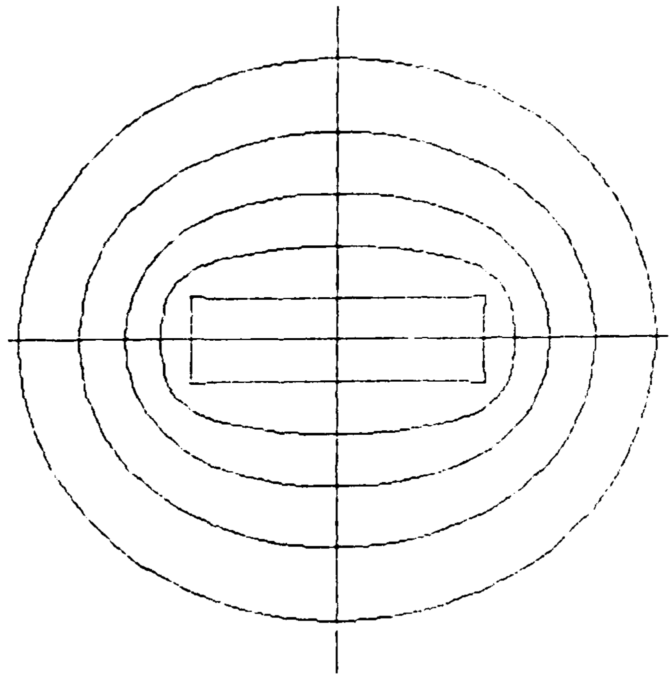


Coefficients of the desc[illegible] series:

	N	AR	AI
AIR,AII=		1.	1.
-N=	1	0.33173	-0.00000
-N=	2	-0.00000	0.00000
-N=	3	-0.00000	0.14975
-N=	4	0.00000	-0.00000
-N=	5	-0.00000	-0.01045
-N=	6	-0.00000	0.00000
-N=	7	-0.00000	0.01699
-N=	8	0.00000	0.00000
-N=	9	-0.00000	-0.00336
-N=	10	-0.00000	-0.00000
-N=	11	0.00000	0.00569
-N=	12	-0.00000	0.00000
-N=	13	-0.00000	-0.00163
-N=	14	-0.00000	-0.00000
-N=	15	0.00000	0.00274
-N=	16	-0.00000	0.00000
-N=	17	-0.00000	-0.00096
-N=	18	-0.00000	-0.00000
-N=	19	0.00000	0.00159

FIG. 14 A [illegible] BY INTE[illegible] METHOD

[illegible]



Coefficients of the Descending Series:

```
           AIR.AII=   1.        1.
  -N=   1,AR,AI=  0.50019  0.00000
  -N=   2,AR,AI=  0.00000-0.00000
  -N=   3,AR,AI=-0.00000-0.12477
  -N=   4,AR,AI=  0.00000  0.00000
  -N=   5,AR,AI=-0.00000-0.03744
  -N=   6,AR,AI=  0.00000-0.00000
  -N=   7,AR,AI=-0.00000-0.0033[illegible]
  -N=   8,AR,AI=-0.00000-0.00000
  -N=   9,AR,AI=  0.00000  0.00645
  -N=  10,AR,AI=-0.00000  0.00000
  -N=  11,AR,AI=  0.00000  0.00504
  -N=  12,AR,AI=-0.00000  0.00000
  -N=  13,AR,AI=  0.00000  0.00091
  -N=  14,AR,AI=  0.00000-0.00000
  -N=  15,AR,AI=-0.00000-0.00173
  -N=  16,AR,AI=  0.00000-0.00000
  -N=  17,AR,AI=-0.00000-0.00168
  -N=  18,AR,AI=  0.00000  0.00000
  -N=  19,AR,AI=-0.00000-0.00033
```

FIG. 15 A R[illegible]TANGLE BY [illegible] METHOD

[illegible]

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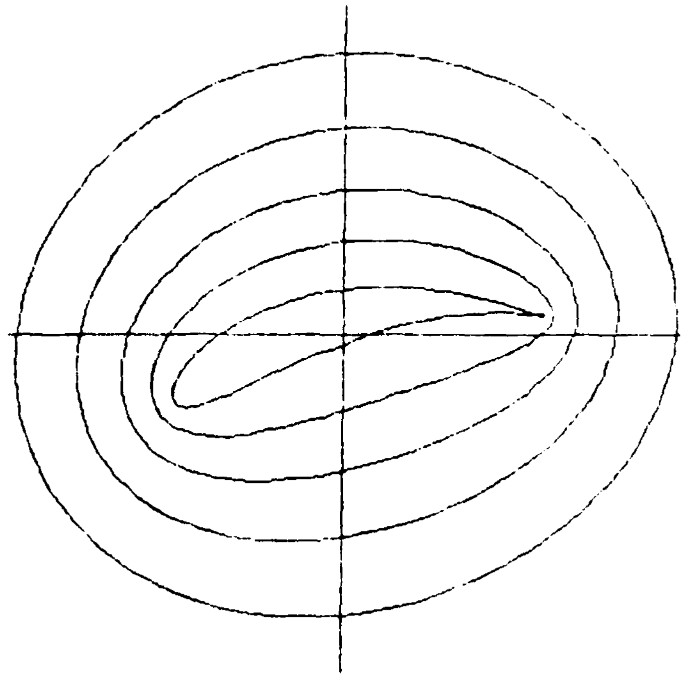
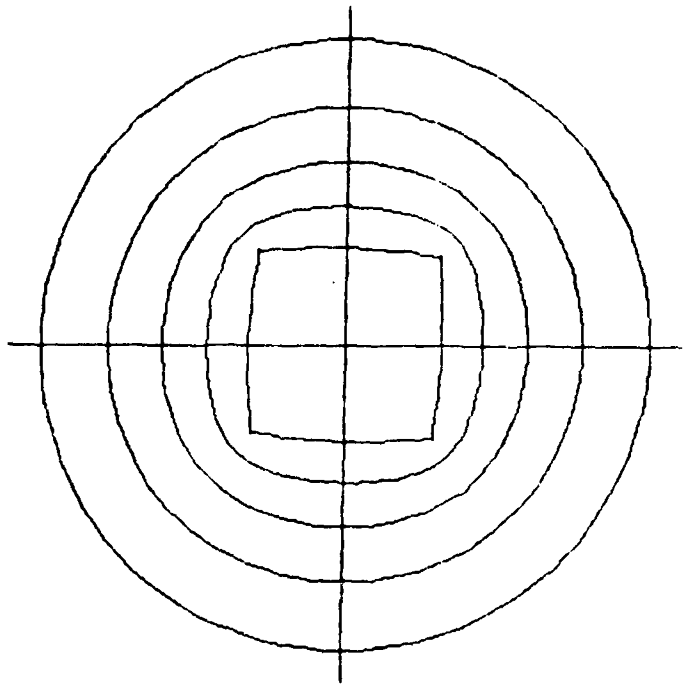


Coefficients of the descending series:

	A1R,A1I=	1.	1.
-N=	1,AR,AI=	0.62256	0.30022
-N=	2,AR,AI=	-0.02165	0.17175
-N=	3,AR,AI=	-0.03395	0.02257
-N=	4,AR,AI=	-0.00672	-0.00144
-N=	5,AR,AI=	0.00050	0.00028
-N=	6,AR,AI=	0.00056	0.00127
-N=	7,AR,AI=	0.00021	0.00106
-N=	8,AR,AI=	0.00012	0.00079
-N=	9,AR,AI=	0.00008	0.00065
-N=	10,AR,AI=	0.00004	0.00056
-N=	11,AR,AI=	0.00000	0.00050
-N=	12,AR,AI=	-0.00004	0.00045
-N=	13,AR,AI=	-0.00009	0.00040
-N=	14,AR,AI=	-0.00013	0.00036
-N=	15,AR,AI=	-0.00016	0.00032
-N=	16,AR,AI=	-0.00019	0.00028
-N=	17,AR,AI=	-0.00022	0.00023
-N=	18,AR,AI=	-0.00024	0.00018
-N=	19,AR,AI=	-0.00025	0.00014

FIG. 16 AEROFOIL BY A FINITE PRODUCT FORMULA

$$z-z_c = ([illegible] -1)^{1.9} (\zeta+0.1+0.2i)^{0.1} ([illegible] 0.1+0.2i)^{-1}$$



Coefficients of the descending series:

```
           A1R,A1I=   1.          1.
    -N=   1,AR,AI=-0.00000  0.00000
    -N=   2,AR,AI=-0.00000  0.00000
    -N=   3,AR,AI=-0.00000-0.13006
    -N=   4,AR,AI= 0.00000-0.00000
    -N=   5,AR,AI= 0.00000-0.00000
    -N=   6,AR,AI=-0.00000-0.00000
    -N=   7,AR,AI= 0.00000  0.01893
    -N=   8,AR,AI= 0.00000  0.00000
    -N=   9,AR,AI=-0.00000-0.00000
    -N=  10,AR,AI=-0.00000-0.00000
    -N=  11,AR,AI=-0.00000-0.00749
    -N=  12,AR,AI=-0.00000-0.00000
    -N=  13,AR,AI= 0.00000  0.00000
    -N=  14,AR,AI= 0.00000-0.00000
    -N=  15,AR,AI= 0.00000  0.00413
    -N=  16,AR,AI=-0.00000  0.00000
    -N=  17,AR,AI= 0.00000-0.00000
    -N=  18,AR,AI= 0.00000  0.00000
    -N=  19,AR,AI=-0.00000-0.00272
```

FIG. 17 AN APPROXIMATE SQUARE BY DESCENDING POWERS OF CHOPPED SINE-WAVE

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14. Descriptors	15. COSATI Group
Conformal mapping	12010
Grids	01010
Airfoils	20110
Holes (openings)	20110

16. Abstract

Aerofoil shapes as well as other simple shapes are produced using conformal mappings. These mappings are applicable to the two-dimensional aerofoil problems of aerodynamics and also to the stress concentration problems of linear elasticity.

This page is to be used to record information which is required by the Establishment for its own use but which will not be added to the DISTIS data base unless specifically requested.

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