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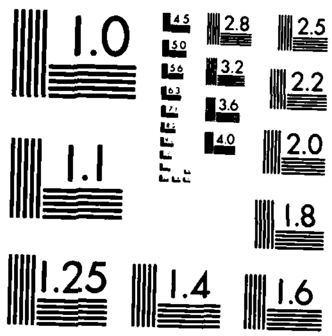

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GRAVITY GRADIOMETER SURVEY DATA PROCESSING

James V. White
Jacob D. Goldstein

AD-A156 165

The Analytic Sciences Corporation
One Jacob Way
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30 July 1984

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

This Technical Report describes a practical methodology for processing airborne gravity gradiometer survey data to estimate gravity disturbance vectors at the surface of the earth. The methodology also provides an error analysis of the disturbance estimates.

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20. Abstract (Continued)

Special consideration is given to the data processing and data documentation required for testing of the airborne Gravity Gradiometer Survey System (GGSS). As an example of the data processing methodology, a prototype data template is defined for the simplified problem of estimating the z (vertical) gravity disturbance from measurements of the zz gradient. This template is an interim example of the types of templates that are being developed in this study for processing real survey data. Gravity and measurement error covariance matrices (for the AWN worldwide gravity disturbance model) were computed for this prototype template. These matrices are sufficient for designing the optimal gravity disturbance estimator for the template and for computing its rms accuracy.

FOREWORD

This report along with the previously delivered data diskette and Technical Information Memorandum (Ref. 1) have been prepared in response to needs identified at the Gravity Gradiometer Survey System (GGSS) Data Analysis Working Group meeting of 4 April 1984. The work described herein was performed for the Air Force Geophysics Laboratory and the Defense Mapping Agency under Contract No. F19628-83-C-0053, "Independent Analysis of Gravity Gradiometer Survey System Development and Deployment."

Acknowledgements are made to several Staff Members of The Analytic Sciences Corporation. W.G. Heller and K.S. Tait offered important technical contributions to the development of the template data processing approach. A.R. LeSchack was responsible for verifying the numerical accuracy of the derived covariance formulas. J.H. Rotondo provided programming support, and S.J. Brzezowski assisted in the management of the study.

TABLE OF CONTENTS

LIST OF FIGURES

1. INTRODUCTION

1.1 OVERVIEW

This report addresses the problem of processing airborne gravity gradiometer measurements to estimate the gravity disturbance vector at the surface of the earth. The approach is based on a minimum-variance estimation technique, which is optimal with respect to probabalistic models for the uncertainties associated with the measurement errors and the *a priori* statistical information about the gravity disturbance vector.

A primary result of the work reported herein is a practical methodology for processing the gradiometer measurements to meet specified accuracy requirements. The methodology consists of two stages. The *first* stage involves averaging the individual gradiometer measurements over appropriate subregions within the survey area using a *data template*. During the *second* stage, these averaged gradiometer measurements are optimally weighted and summed to estimate the three components of the gravity disturbance vector at the surface of the earth. The weights are chosen optimally with respect to probabilistic models for (1) the measurement errors and (2) the *a priori* uncertainty about the true gravity disturbance vector. The rms accuracy of the estimated disturbance vector is characterized by an error covariance matrix.

In preparation for testing of the Gravity Gradiometer Survey System (GGSS), a prototype example of an averaging template is documented in this report. The gravity and measurement error covariance matrices (valid for this template and the Attenuated White-Noise (AWN) (Ref. 2) worldwide gravity model) are provided in Ref. 1. This template provides a simple, but modestly accurate example of the measurement averaging portion of the data processing methodology. Other templates, having more averaging zones for meeting the required rms accuracy and optimized for data processing, are being developed at TASC specifically for GGSS testing. The prototype template described here is provided as an interim example of the type of processing that will be required.

1.2 TECHNICAL APPROACH

The technical approach of this study is based on minimum-variance estimation theory. This approach has the flexibility needed to treat mixtures of different types of gravity quantities acquired at different survey heights. The approach applies to both point and mean gravity field data.

The central aspect of this study is the design of the data averaging templates. The templates are being developed to achieve required rms accuracy levels. This is accomplished by using error covariance analyses to study the tradeoffs between estimation accuracy and the template parameters for different blends of gradient data. The covariance analyses can be performed before any survey data are available because the error covariances are computed using gravity and measurement error models.

1.3 ORGANIZATION OF REPORT

The theoretical basis for the estimation algorithm is presented in Chapter 2. First, the quantities to be estimated and the quantities to be measured are defined. The practical need for the data processing to reduce the dimensionality of the computations is explained. Based on these preliminaries, the estimation problem is stated and a recursive algorithm for solving it is presented.

The remainder of Chapter 2 describes some recommended conventions for documenting the data analysis. A prototype data processing template is defined and the derivation of the covariances which are needed to design an optimal estimator for this template is explained. The report ends with Chapter 3, which presents a summary and conclusions.

2. TECHNICAL DISCUSSION

The purpose of this chapter is to formulate and discuss the problem of processing data from an airborne gravity gradiometer survey. The principal result of this analysis is a practical algorithm for (1) estimating the point gravity disturbance vector at the surface of the earth (or other appropriately defined downward continuation point) from the survey data and (2) computing mean-square errors of those estimates.

2.1 STATEMENT OF THE PROBLEM

Processing survey data to estimate the gravity disturbance vector is formulated as a minimum-variance estimation problem. This formulation involves three important elements:

- Definitions of the gravity quantities that are being estimated
- Definitions of the gravity quantities that are being measured
- Probabilistic models for the uncertainties associated with the survey measurement data and all associated gravity quantities (both those being measured and those being estimated).

2.1.1 Estimated Gravity Quantities

The gravity quantities to be estimated depend on the purpose of the data processing. For data processing that will follow routine GGSS survey operations, the gravity disturbance vector at the surface of the earth will typically be estimated. In contrast, for the verification tests of the GGSS, a specially defined gravity disturbance vector should be estimated, as explained in the following discussion.

The gravity disturbance vector at position $\underline{r}$ is defined as follows:

$$\underline{\delta}(\underline{r}) \triangleq \underline{g}(\underline{r}) - \underline{g}_R(\underline{r}) \tag{2.1-1}$$

In Eq. 2.1-1, $g(\underline{r})$ is the whole value gravity vector at $\underline{r}$, and $\underline{g}_R(\underline{r})$ is the reference gravity vector. The reference field and the coordinate systems for expressing gravity vectors and position vectors must be specified to complete the definition of the disturbance vector. Appropriate candidate reference systems are WGS 72 or WGS 84.

Although $\underline{\delta}(\underline{r})$ is the quantity that is usually sought, it is inappropriate to estimate this gravity disturbance vector during airborne testing of the GGSS. The reason is that initial gradiometer survey testing will be confined to a limited geographic area (probably on the order of 500 km by 500 km). This limited extent makes long-wavelength testing of the survey accuracy imprecise. However, the medium- and short-wavelength accuracy of survey data can be tested by estimating a redefined gravity disturbance vector, which in this report is called the residual gravity disturbance. This residual vector is defined so that its variance at long wavelengths can be controlled to suit the accuracy objectives of GGSS testing. The residual gravity disturbance vector, denoted $\underline{d}(\underline{r})$, is defined as the departure of the disturbance $\underline{\delta}(r)$ from its local mean $\underline{\delta}_m(\underline{r})$:

$$\underline{d}(\underline{r}) \triangleq \underline{\delta}(\underline{r}) - \underline{\delta}_m(\underline{r}) \tag{2.1-2}$$

For this style of template, which is comprised of concentric circuits, the documentation matrix D is $n \times 8$. The k^{th} gravity quantity, whose value is given by $\underline{x}(k)$, is described by the following eight numbers in the k^{th} row of D:

D(k,1) = ID Number of the k^{th} gravity quantity

D(k,2) = x coordinate (x_c) of the upper-left corner of the template circuit (km)

D(k,3) = y coordinate (y_c) of the upper-left corner of the template circuit (km)

D(k,4) = width (w) of the rectangular subregions (km)

D(k,5) = mean altitude (A_1) of rectangular subregion No. 1 (km)

D(k,6) = mean altitude (A_2) of rectangular subregion No. 2 (km)

D(k,7) = mean altitude (A_3) of rectangular subregion No. 3 (km)

D(k,8) = mean altitude (A_4) of rectangular subregion No. 4 (km)

For a <u>point</u> gravity quantity at the template's center, as distinguished from a mean quantity, the template circuit reduces to a point at the origin, which yields D(k,2) = D(k,3) = D(k,4) = 0 and D(k,5) = D(k,6) = D(k,7) = D(k,8). For the mean z gravity disturbance (i.e., when ID No. = 2), the mean refers to an average over a <u>single square area</u> centered at the origin. Therefore, if ID = 2, the parameter w denotes the length of each side of this square area.

Other forms of templates, having different symmetries, are appropriate for estimating x (east) and y (north) gravity disturbance components (i.e., the deflections of the vertical). Such templates are being developed and will be provided in subsequent reports.

discussion, each of the picture-frame averaging zones will be referred to as a "circuit." For mathematical tractability, each of these circuits is composed of simple rectangular subregions. The recommended way of defining a single circuit using four rectangular subregions is illustrated in Fig. 2.4-2. The geometry of each circuit is then completely specified by seven parameters: the coordinates (x_c, y_c) of the upper-left corner, the width w of each rectangular subregion, and the altitude of each rectangular subregion. The width w represents the crosstrack resolution of the gradient measurements, given that the actual aircraft paths are expected to have minor deviations from straight paths.

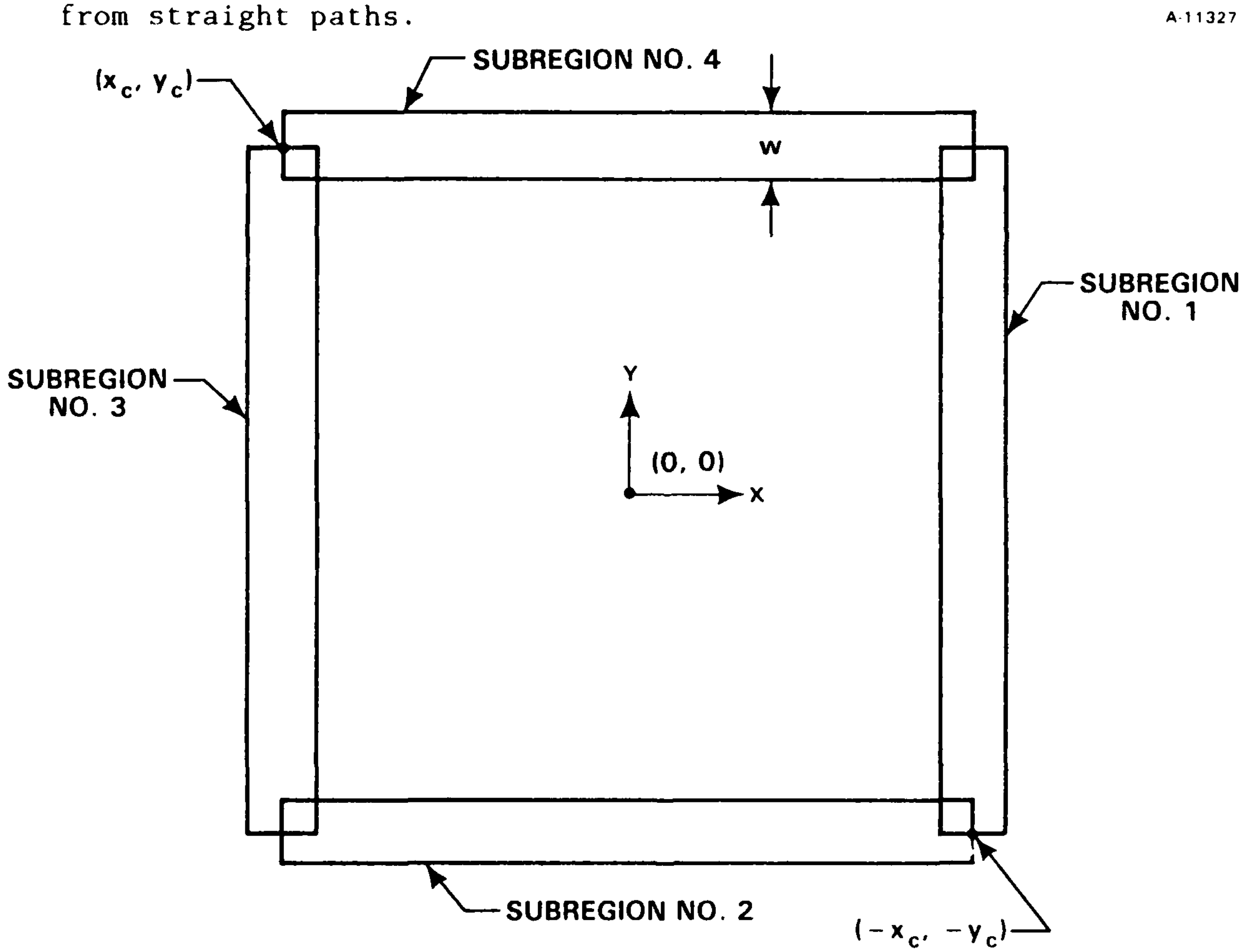

Figure 2.4-2 A Typical Template Circuit, Specified by the Three Parameters x_c, y_c, w and Altitude of Each Subregion (Not to Scale)

data consists of concentric "picture-frame" averaging zones, which are depicted (not to scale) in Fig. 2.4-1.* The geometry illustrated in Fig. 2.4-1 reflects the recommended test flight pattern characterized by variable track spacing which increases with distance from the survey pattern center. In the following

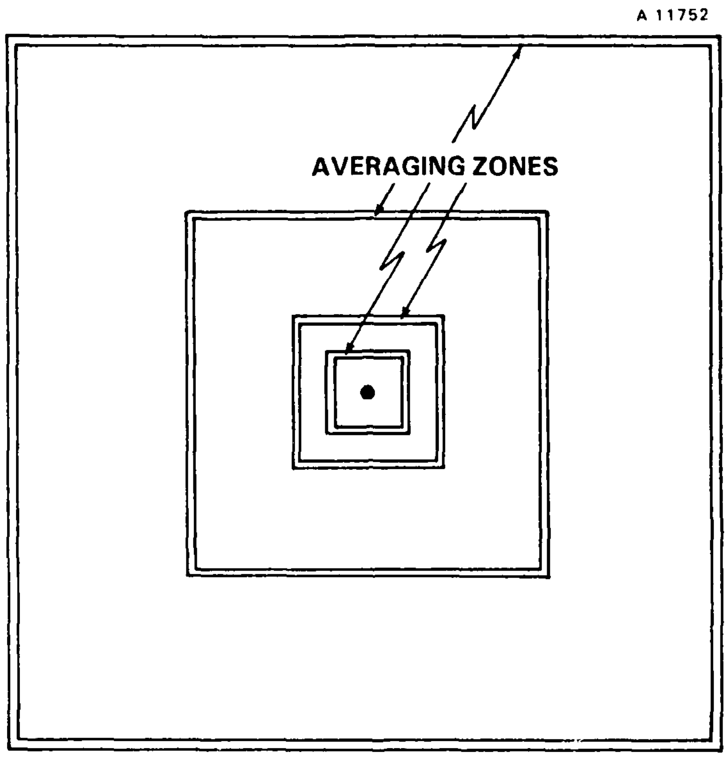

Figure 2.4-1 Data Processing Template with Rectangular Geometry and Azimuthal Symmetry (Not to Scale)

*The choice of the template involves a compromise between the number of zones (the size of the covariance matrix that has to be inverted) and the loss of accuracy with respect to the optimal template in which each measurement would be considered an individual zone. Thus, during GGSS testing, data collected between template zones, although used during the track-crossing adjustment preprocessing, do not appear explicitly in the final estimation equations. For operational data processing, all data will eventually be used directly in the estimation. This is because template shifts are needed to obtain estimates at different locations, and each shift of the template uses a different subset of the data in the estimation.

As discussed in Section 2.1, the survey data are processed by averaging them over subregions of the surveyed area. (It is expected that the data processing will include track-crossing adjustments to reduce low-frequency measurement errors.) The resulting mean gradient data are listed in the gravity measurement vector $\underline{z}_1$. According to Eq. 2.1-7, these averaged data are a linear combination of components of the truth vector $\underline{x}$ and the measurement error vector $\underline{v}_1$. To document the gravity-related quantities in $\underline{x}$, an n×p <u>documentation matrix</u> D is defined, where n is the number of quantities listed in $\underline{x}$, and p is the number of parameters used to describe each of these quantities. The parameters depend on the particular template that is used for the data averaging, as discussed in the following section.

2.4 DATA TEMPLATES

For concreteness, the discussion of data templates focuses on a simplified example of practical importance in testing the GGSS: estimating the z (up) component of the residual disturbance vector using gradiometer measurements of the zz element of the gravity gradient. An appropriate template for averaging these gradiometer measurements would have azimuthal symmetry. This symmetry applies to data processing that is optimal with respect to gravity models having an isotropic crosscorrelation function relating the vertical disturbance with the measurements of the zz gradient element. Thus, a reasonable template with complete azimuthal symmetry would have concentric annular averaging zones like a bulls-eye target. However, airborne gradiometer surveys will yield gradient measurements taken along nearly straight tracks that form a rectangular grid. To be consistent with this survey geometry, the template should also have a rectangular geometry. Therefore, a reasonable template geometry for reduction of test

2.3 GRAVITY DATA DOCUMENTATION

The analysis of gravity gradiometer survey data involves several kinds of gravity quantities. These include point values of gravity disturbance vectors and disturbance gradients, expressed with respect to a reference field. Other important gravity disturbance quantities are mean values of gravity disturbance vectors and disturbance gradients, averaged over template zones. For convenience in designing future surveys and for storing, processing, and documenting actual survey data, each gravity quantity can be assigned an identification (ID) number. These numbers are selected sequentially. The beginning of such a list follows; additional quantities can be added to the list as appropriate.

ID NUMBER	GRAVITY QUANTITY	UNITS
1	Point z (Up) Disturbance	mgal
2	Mean z (Up) Disturbance	mgal
3	Point x (East) Disturbance	mgal
4	Mean x (East) Disturbance	mgal
5	Point y (North) Disturbance	mgal
6	Mean y (North) Disturbance	mgal
7	Point zz Disturbance Gradient	mgal/km
8	Mean zz Disturbance Gradient	mgal/km
9	Point xx Disturbance Gradient	mgal/km
10	Mean xx Disturbance Gradient	mgal/km
11	Point xz Disturbance Gradient	mgal/km
12	Mean xz Disturbance Gradient	mgal/km
13	Point yy Disturbance Gradient	mgal/km
14	Mean yy Disturbance Gradient	mgal/km
15	Point xy Disturbance Gradient	mgal/km
16	Mean xy Disturbance Gradient	mgal/km
17	Point yz Disturbance Gradient	mgal/km
18	Mean yz Disturbance Gradient	mgal/km
.	.	.
.	. (entries to be added as	.
.	. future needs arise).	.

2.2 ESTIMATION ALGORITHM

The solution to the estimation problem of the previous section is given by the following recursive equations (Ref. 3), which update the prior estimates and their error covariances. The optimal estimate of the truth vector $\underline{x}$ and the error covariance matrix for this estimate are

$$\hat{\underline{x}}_1 = \hat{\underline{x}}_0 + K_1(\underline{z}_1 - H_1\hat{\underline{x}}_0) \qquad (2.2\text{-}1)$$

$$P_1 = P_0 - K_1H_1P_0 \qquad (2.2\text{-}2)$$

In Eqs. 2.2-1 and 2.2-2, the n×m gain matrix K_1 is computed as follows:

$$K_1 = P_0H_1^T[H_1P_0H_1^T + R_1]^{-1} \qquad (2.2\text{-}3)$$

The estimate of the residual disturbance vector and the covariance matrix P_{dd} of the errors in this estimate are given by the following two formulas:

$$\hat{\underline{d}}(\underline{r}) = B\hat{\underline{x}}_1 \qquad (2.2\text{-}4)$$

$$P_{dd} = BP_1B^T \qquad (2.2\text{-}5)$$

For the case where the prior estimate $\hat{\underline{x}}_0 = \underline{0}$, the optimal estimate can be written directly in terms of the averaged data $\underline{z}_1$ as follows:

$$\hat{\underline{d}}(\underline{r}) = G\underline{z}_1 \qquad (2.2\text{-}6)$$

$$G = BK_1 \qquad (2.2\text{-}7)$$

2.1.4 Estimation Problem

The problem of estimating the residual gravity disturbance vector $\underline{d}(\underline{r})$ from averaged gradiometer measurements can be stated as follows:

Given:

1. The *a priori* estimate $\hat{\underline{x}}_0$ of the truth vector $\underline{x}$.
2. The covariance matrix P_0 of the errors in this prior estimate.
3. The measurement vector $\underline{z}_1$ of averaged gradiometer data.
4. The covariance matrix R_1 of the errors in these averaged data.

Find:

1. An estimate $\hat{\underline{d}}(\underline{r})$ of the residual gravity disturbance vector $\underline{d}(\underline{r})$ at the template's center.
2. The covariance matrix P_{dd} of the errors $\tilde{\underline{d}}(\underline{r}) = \hat{\underline{d}}(\underline{r}) - \underline{d}(\underline{r})$.

Optimality: The estimates are to be optimal in the sense that the weighted mean-square errors, J_x and J_d, are minimized for any positive-definite symmetric weighting matrices W_x and W_d:

$$J_x \triangleq E[\tilde{\underline{x}}_1^T W_x \tilde{\underline{x}}_1] \tag{2.1-20}$$

$$J_d \triangleq E[\tilde{\underline{d}}(\underline{r})^T W_d \tilde{\underline{d}}(\underline{r})] \tag{2.1-21}$$

$$B = [1 \quad -1 \quad 0_{1\times 12}] \tag{2.1-12}$$

$$H_1 = [0_{12\times 2} \quad I_{12\times 12}] \tag{2.1-13}$$

An important conclusion to be drawn from this discussion is that the definitions of the truth vector $\underline{x}$, the observation matrix H_1, and the output matrix B are each selected to suit the needs of the particular estimation problem that is being formulated.

The prior knowledge of the truth vector $\underline{x}$ is modeled by an unbiased estimate denoted $\hat{\underline{x}}_0$:

$$\hat{\underline{x}}_0 = E[\underline{x}] \tag{2.1-14}$$

Usually $\hat{\underline{x}}_0 = \underline{0}$ because a reference field has been subtracted from all gravity-related quantities. The uncertainty of this prior estimate is modeled by its error covariance. The error vector $\tilde{\underline{x}}_0$ and its n×n covariance matrix P_0 are defined as follows:

$$\tilde{\underline{x}}_0 \triangleq \hat{\underline{x}}_0 - \underline{x} \tag{2.1-15}$$

$$P_0 \triangleq E[\tilde{\underline{x}}_0 \tilde{\underline{x}}_0^T] \tag{2.1-16}$$

The prior knowledge of the measurement error vector $\underline{v}_1$ is modeled by stating that its mean is zero, its covariance matrix is R_1 (a known matrix), and its crosscovariance with the error vector $\tilde{\underline{x}}_0$ is zero:

$$E[\underline{v}_1] = \underline{0} \tag{2.1-17}$$

$$E[\underline{v}_1 \underline{v}_1^T] = R_1 \tag{2.1-18}$$

$$E[\underline{v}_1 \tilde{\underline{x}}_0^T] = 0 \tag{2.1-19}$$

The truth vector in Eq. 2.1-9 serves only as an illustrative example of how $\underline{x}$ can be organized; this particular blend of gravity quantities would not actually be used in practice because additional gradient quantities (besides T_{zz}) must be included to estimate the residual gravity disturbance vector.

For the example case under discussion, the number of measurements in $\underline{z}_1$ is m = 12 and the number of gravity-related scalars in $\underline{x}$ is n = 18. The B matrix in Eq. 2.1-9 is

$$B = [I_{3\times 3} \quad -I_{3\times 3} \quad 0_{3\times 12}] \tag{2.1-10}$$

where $I_{3\times 3}$ is the 3×3 identity matrix, and $0_{3\times 12}$ is the 3×12 zero matrix. This zero matrix accounts for the fact that the 12 gradient quantities do not enter the definition of the point residual disturbance.

To simplify the bookkeeping, the values of the mean gradients in $\underline{x}$ are listed in the same order as their measured values in $\underline{z}_1$. With this convention, the observation matrix has the following form:

$$H_1 = [0_{12\times 6} \quad I_{12\times 12}] \tag{2.1-11}$$

In Eq. 2.1-11, the 12×6 zero matrix indicates (1) that there are 12 scalar quantities in the measurement vector $\underline{z}_1$, and (2) that the first six elements of $\underline{x}$ are not being measured.

Later in this report, a prototype template is defined for estimating the z (up) component of the residual gravity disturbance $d_z(\underline{r})$, rather than all three components of $\underline{d}(\underline{r})$. For this simplified case, the truth vector $\underline{x}$ is 14×1. The first two elements of $\underline{x}$ are the scalars, $\delta_z(\underline{r})$ and its local mean, and the B and H_1 matrices are

$$\underline{d}(\underline{r}) = B\underline{x} \tag{2.1-8}$$

In Eq. 2.1-8, the 3×n output matrix B selects those linear combinations of the elements of $\underline{x}$ that are being estimated.

The following discussion provides an example of how this formulation is used for gradiometer test data reduction. For convenience, the quantities to be estimated are separated from those being measured when defining the elements of $\underline{x}$. (In many estimation problems, the quantities to be estimated are included among those being measured because it is often appropriate to treat measured and estimated quantities in a unified manner.) For GGSS testing, the residual gravity disturbance $[\underline{d}(\underline{r}) = \underline{\delta}(\underline{r}) - \underline{\delta}_m(\underline{r})]$ is to be estimated. Therefore, the two conventional gravity quantities needed to define $d(\underline{r})$ [the point disturbance $\underline{\delta}(\underline{r})$ and its local mean $\underline{\delta}_m(\underline{r})$] are listed first in $\underline{x}$. The remaining elements of $\underline{x}$ are used to represent the actual average values of the measured quantities. For gradiometer data, these average values are the mean gradients that are defined by the appropriate data template zones. For example, the particular prototype template used in this report has 12 zones, over which the zz component of the gravity gradient is averaged. Therefore, when using this template, the average values of T_{zz} for these zones are the last 12 elements of $\underline{x}$ (these mean values are denoted as $T_{zz}(i)$ for i = 1 to 12):

$$\underline{x} = \begin{bmatrix} \underline{\delta}(\underline{r}) \\ \underline{\delta}_m(\underline{r}) \\ T_{zz}(1) \\ T_{zz}(2) \\ \vdots \\ T_{zz}(12) \end{bmatrix} \tag{2.1-9}$$

number of computations. The process of template design can be performed systematically by using error covariance analyses. Therefore, the template geometry is selected to be not only appropriate for estimating the residual gravity disturbance vector, but also amenable to an efficient and accurate covariance analysis.

In the following discussion, the measurement vector $\underline{z}_1$ is a list of mean gravity gradient elements, which are computed during the data processing. To define these quantities completely, the location (including altitude) and size of each averaging window must be specified.

2.1.3 Error Models

The last part of formulating the estimation problem is to model the errors in the observed data and the *a priori* uncertainty in the values of the gravity disturbance and gravity gradient field. The observed data are represented by the measurement vector $\underline{z}_1$ of appropriately averaged gradiometer measurements. A model for the averaged data is given as follows:

$$\underline{z}_1 = H_1\underline{x} + \underline{v}_1 \tag{2.1-7}$$

In Eq. 2.1-7, $\underline{v}_1$ is an $m\times 1$ vector of measurement errors. The $n\times 1$ truth vector $\underline{x}$ represents the actual values of the gravity-related quantities. These quantities consist of those that are being estimated as well as those that are being measured. The $m\times n$ observation matrix H_1 selects from $\underline{x}$ those linear combinations of its elements that are being measured.

The residual gravity disturbance vector $\underline{d}(\underline{r})$, which is to be estimated, is also expressed in terms of the truth vector:

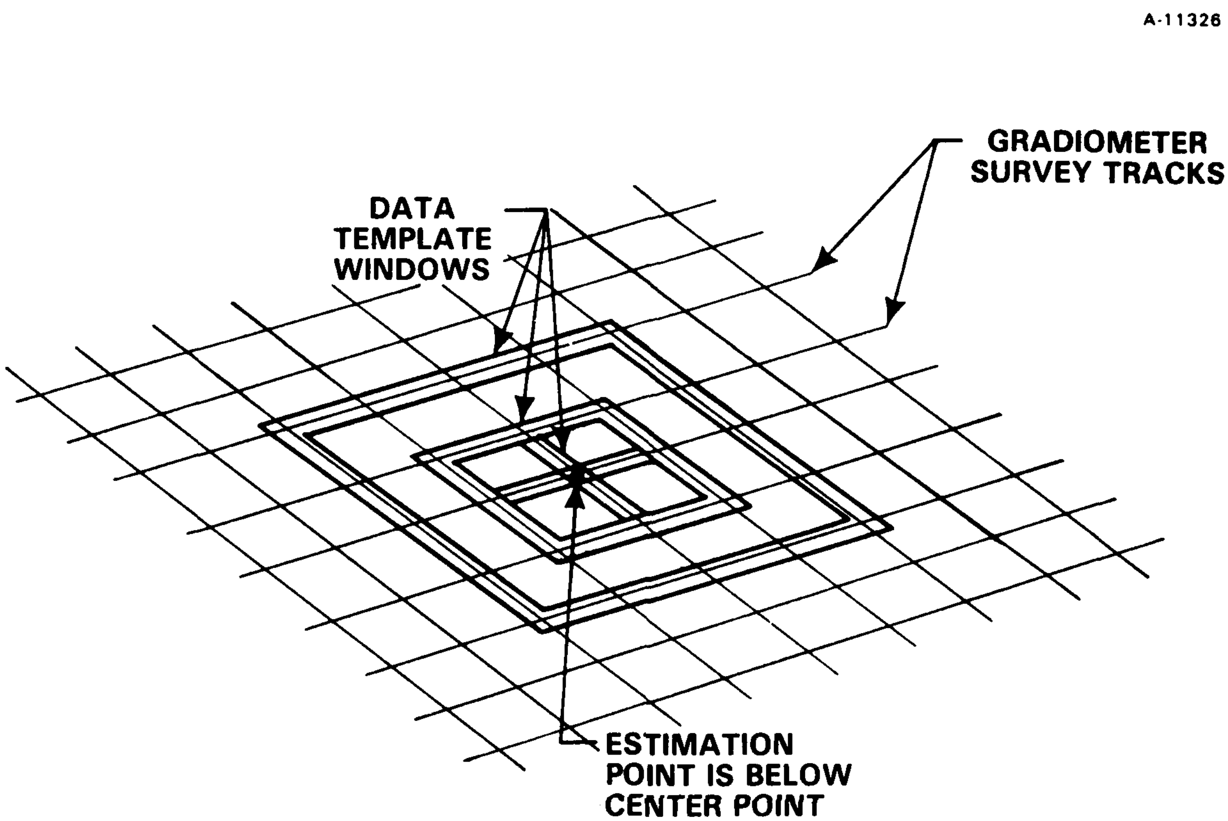

Figure 2.1-1 Data Template Windows Superimposed on Survey Tracks (Not to Scale)

are then listed in a vector denoted $\underline{z}_1$ and used optimally to estimate the residual disturbance vector. The locations and sizes of these subregions are selected to achieve specified rms accuracies, while minimizing the computational effort.

A convenient way of visualizing this processing is to imagine a template, as depicted in Fig. 2.1-1, that is superimposed on a map of the survey tracks, with the center of the template located over the position where the gravity departure is to be estimated. The "windows" in this template define the subregions within which the gradient measurements are averaged. The shapes and locations of these windows are selected to achieve specified rms accuracies with the smallest possible

2.1.2 Measured Gravity Quantities

During testing of the GGSS, the measured gravity quantities will include the six distinct elements of the gravity gradient tensor.* These gradient measurements will be made along nearly straight flight paths (within ±100 m of the nominal flight path) that form a grid over the survey region. For analysis, all of the measurements in the survey can be listed in a measurement vector $\underline{z}$. To estimate the number of these measurements, consider, for illustrative purposes,† flight paths that are 5 km apart and form a square grid 300 km on a side. Such a survey consists of 122 paths, each 300 km long, for a total track length of 36,600 km. If the six gradient elements are measured at 1 km intervals (e.g., one set of measurements every 12 seconds provided by a plane flying 300 km/hr), then the measurement vector $\underline{z}$ contains 219,600 numbers. To process $\underline{z}$ optimally (to estimate the residual gravity disturbance vector $\underline{d}(\underline{r})$ at the center of the survey region), 219,600 simultaneous equations must be solved. These computations are impractical because of the size of $\underline{z}$. By using a data averaging technique to reduce the dimension of the measurement vector $\underline{z}$, a practical estimation algorithm is developed.

To estimate the residual gravity disturbance vector $\underline{d}(\underline{r})$, the gradient measurements can be averaged over selected subregions of the survey area. The resulting average measurements

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*It is recognized that only five of these are independent. However, the redundancy of the sixth should be used to improve overall accuracy.

†The actual test flight pattern may entail a greater extent and variable track spacing which increases with distance from the center.

In Eq. 2.1-2, the local-mean disturbance vector is defined as the following average centered on position $\underline{r}$:

$$\underline{\delta}_m(\underline{r}) \triangleq \iint w(\underline{\rho})\ \underline{\delta}(\underline{r}-\underline{\rho})\ d\rho_1 d\rho_2 \qquad (2.1\text{-}3)$$

where

$$\underline{\rho} \triangleq \begin{bmatrix} \rho_1 \\ \rho_2 \end{bmatrix} \qquad (2.1\text{-}4)$$

In Eq. 2.1-3, $w(\underline{\rho})$ is the weighting function at shift $\underline{\rho}$. The weighting function for a uniform local average over a square geographic region centered on position $\underline{r}$ is given by the following equations in which L is the length of each side of the averaging area:

$$w(\underline{\rho}) \triangleq L^{-2}, \quad -L/2 \leq \rho_1 \leq L/2, \quad -L/2 \leq \rho_2 \leq L/2 \qquad (2.1\text{-}5)$$

$$w(\underline{\rho}) \triangleq 0, \quad \text{otherwise} \qquad (2.1\text{-}6)$$

<u>The weighting function defined by Eqs. 2.1-5 and 2.1-6 is recommended for use in testing because (1) it provides the needed control over the long-wavelength variance of the residual disturbance vector, and (2) it yields lagged covariance functions for the residual disturbance that can be evaluated in closed form</u>. The length parameter L controls the long-wavelength variance of the residual disturbance $\underline{d}(\underline{r})$. The variance at wavelengths longer than L is significantly attenuated, while the variance at wavelengths shorter than L is only slightly affected. Because the lagged covariances of $\underline{d}(\underline{r})$ can be evaluated in closed form, the weighting function defined by Eqs. 2.1-5 and 2.1-6 is especially suited for covariance analyses of survey accuracy. On the basis of such covariance studies, algorithms are being developed by TASC for processing the survey data to meet specified rms accuracies.

A prototype template for estimating the z residual disturbance component from airborne measurements of the zz gradient element is defined by the 14 × 8 documentation matrix presented in Table 2.4-1. This template is a prototype example of the templates that will be used for processing the full set of gradiometer test data. Later templates will contain more circuits and be applied to additional gravity gradients. The purpose of the prototype template is to provide an interim example of the type of data averaging and data documentation that is recommended for GGSS testing, as well as to offer a rudimentary data processing capability.

TABLE 2.4-1

DOCUMENTATION MATRIX FOR PROTOTYPE TEMPLATE

ROW NO.	ID	x_c (km)	y_c (km)	w (km)	A_1 (km)	A_2 (km)	A_3 (km)	A_4 (km)
1)	1	0	0	0	0	0	0	0
2)	2	-125	125	250	0	0	0	0
3)	7	0	0	0	0.6	0.6	0.6	0.6
4)	8	-5	5	1	0.6	0.6	0.6	0.6
5)	8	-10	10	1	0.6	0.6	0.6	0.6
6)	8	-15	15	1	0.6	0.6	0.6	0.6
7)	8	-20	20	1	0.6	0.6	0.6	0.6
8)	8	-30	30	1	0.6	0.6	0.6	0.6
9)	8	-40	40	1	0.6	0.6	0.6	0.6
10)	8	-60	60	1	0.6	0.6	0.6	0.6
11)	8	-80	80	1	0.6	0.6	0.6	0.6
12)	8	-120	120	1	0.6	0.6	0.6	0.6
13)	8	-160	160	1	0.6	0.6	0.6	0.6
14)	8	-320	320	1	0.6	0.6	0.6	0.6

The first two elements of the truth vector $\underline{x}$ described by Table 2.4-1 are the point z gravity disturbance and its mean value averaged over a 250-km × 250-km region. Therefore, the z component of the residual disturbance is computed from $\underline{x}$ as follows:

$$d_z(\underline{r}) = B\underline{x} \tag{2.4-1}$$

$$B = [1 \quad -1 \quad 0_{1\times 12}] \tag{2.4-2}$$

2.5 COVARIANCE MATRICES

As explained in Sections 2.1 and 2.2, to implement an optimal estimator for a particular data template, or to compute the rms accuracy of such an estimator, two covariance matrices are needed. These are (1) the error covariance matrix P_0 of the initial estimate $\hat{\underline{x}}_0$ of the truth vector $\underline{x}$ (Eqs. 2.1-15 and 2.1-16), and (2) the error covariance matrix R_1 of the errors $\underline{v}_1$ in the measurement vector $\underline{z}_1$ (Eqs. 2.1-7 and 2.1-18). In this section, these covariance matrices are defined for the prototype template defined in Table 2.4-1. Numerical values for these covariances are computed using the flat-earth version of the AWN worldwide gravity disturbance model (Ref. 2) and a white-noise error model for the gradiometer measurement errors.

2.5.1 Gravity Error Covariance Matrix

With reference to Table 2.4-1, the first gravity quantity has ID No. = 1, and is the point value of the z gravity disturbance. This is the vertical gravity disturbance at the point with coordinates x = 0 and y = 0. Therefore,

$$\underline{x}(1) \triangleq \delta_z(\underline{0}) \qquad \text{(altitude = 0 km)} \tag{2.5-1}$$

According to Table 2.4-1, the second gravity quantity has ID No. = 2. The table indicates that $\underline{x}(2)$ is the mean value of $\delta_z(\underline{r})$, averaged over a 250 km × 250 km square area with center at x = 0 and y = 0. Therefore, with $\underline{r} \equiv [x \ y]^T$,

$$\underline{x}(2) \triangleq \frac{1}{(250)^2} \iint_{-125}^{125} \delta_z(\underline{r})\ dx\ dy \qquad \text{(altitude = 0 km)}$$

(2.5-2)

The third quantity documented in Table 2.4-1 has ID No. = 7, which denotes the point value of the zz disturbance gradient T_{zz}. The point value is at location x = 0 and y = 0 and an altitude of 0.6 km. Therefore,

$$\underline{x}(3) \triangleq T_{zz}(\underline{0}) \qquad \text{(altitude = 0.6 km)} \qquad (2.5\text{-}3)$$

The remaining gravity quantities documented in Table 2.4-1 have ID No. = 8, which denotes mean zz disturbance gradients, with their averages taken over four rectangular subregions that are uniquely identified from the upper-left corner coordinates (x_c, y_c), the width w, and the altitudes A_1, A_2, A_3 and A_4. For example, the fourth quantity is defined as follows:

$$\underline{x}(4) \triangleq (I_1 + I_2 + I_3 + I_4)/4 \qquad (2.5\text{-}4)$$

$$I_1 \triangleq 1/10 \left\{ \int_{-5}^{5} \int_{4.5}^{5.5} T_{zz}(\underline{r})\ dx\ dy \right\} \qquad \text{(altitude = 0.6 km)}$$

(2.5-5)

$$I_2 \triangleq 1/10 \left\{ \int_{-5.5}^{-4.5} \int_{-5}^{5} T_{zz}(\underline{r})\ dx\ dy \right\} \qquad \text{(altitude = 0.6 km)}$$

(2.5-6)

$$I_3 \triangleq 1/10 \left\{ \int_{-5}^{5} \int_{-5.5}^{-4.5} T_{zz}(\underline{r})\, dx\, dy \right\} \text{ (altitude = 0.6 km)} \tag{2.5-7}$$

$$I_4 \triangleq 1/10 \left\{ \int_{4.5}^{5.5} \int_{-5}^{5} T_{zz}(\underline{r})\, dx\, dy \right\} \text{ (altitude = 0.6 km)} \tag{2.5-8}$$

The error covariance matrix P_0 (for an initial estimate $\hat{\underline{x}}_0 = \underline{0}$) has been computed for the AWN gravity model using the above definitions and their extensions to the other gravity quantities documented in Table 2.4-1. The resulting covariance matrix has been provided in Ref. 1.

2.5.2 Measurement Error Covariance Matrix

Previous analyses of gravity gradiometer test data (e.g., Refs. 4 and 5) provide the basis for the gradiometer measurement error model used in this report. The noise-like measurement errors are typically modeled by a zero-mean, white-noise signal component added to a random-walk signal component that models instrument drift. During the first stage of processing the raw gradiometer survey data, track-crossing adjustments will be applied to the measured gradients to suppress the long-wavelength errors caused by random instrument drift. The residual errors in the adjusted data will then consist primarily of random errors that are accurately modeled as zero-mean white noise.

The white noise in the individual gradient measurements will not be affected by the track-crossing adjustments. Therefore, the mean-square error σ^2 (E^2) caused by white noise in a single gradient measurement can be computed from the spectral density S (E^2/Hz) (two-sided spectrum) of the noise and the effective averaging time t_{ave} (sec) of the gradiometer output filter:

$$\sigma^2 = S/t_{ave} \qquad (2.5\text{-}9)$$

For example, if S = 80 E^2/Hz and t_{ave} = 12 sec (one measurement every kilometer at a speed at 300 km/hr), then the rms error of the individual measurements is σ = 2.6 E ≡ 0.26 mgal/km. When these measurements are averaged using the data template, the mean-square error σ_m^2 of the resulting mean gradient is determined by the number N of measurements in the average:

$$\sigma_m^2 = \sigma^2/N \qquad (2.5\text{-}10)$$

In Eq. 2.5-10, the number of measurements N is determined by the length L (km) of the averaging zone in the data template, the speed V (km/hr) of the survey aircraft, and the time t_{ave} (sec) between consecutive measurements:

$$N = L/(Vt_{ave}) \qquad (2.5\text{-}11)$$

It is recommended that the organization of the gravity-related quantities in the truth vector $\underline{x}$, which is defined by the documentation matrix D, should also be used to organize the measurement vector $\underline{z}_1$. To achieve the same organization in both vectors, the m×n matrix H_1 in Eq. 2.1-7 is defined for the template under discussion as follows:

$$H_1 \triangleq [0_{m\times p} \;\; I_{m\times m}] \quad , \quad p = n - m \qquad (2.5\text{-}12)$$

In Eq. 2.5-12, $0_{m\times p}$ is the m×p zero matrix and $I_{m\times m}$ is the m×m identity matrix. This definition of H_1, with m = 12 and p = 2, is appropriate for the example template defined by Table 2.4-1, in which the first <u>two</u> gravity quantities are used to characterize the residual gravity disturbance component which is being <u>estimated</u>. The remaining m quantities listed in Table 2.4-1 and the truth vector $\underline{x}$ are the <u>measured</u> quantities in $\underline{z}_1$. The order of listing these quantities is preserved because of the m×m identity matrix in Eq. 2.5-12.

The error covariance matrix R_1 for the prototype template documented in Table 2.4-1 has been computed using a gradiometer white-noise model with two-sided spectral density $S = 80\ E^2/Hz$, an rms error per gradient measurement of $\sigma = 0.26$ mgal/km, an aircraft speed of $V = 300$ km/hr, and an averaging time of $t_{ave} = 12$ seconds per measurement. The covariance matrix is diagonal because the gradiometer measurement errors are modeled as uncorrelated from measurement to measurement. The 12 diagonal elements of R_1, expressed in $(\text{mgal/km})^2$, are listed below:

$$
\begin{aligned}
R_1(1,1) &= 0.26^2 &&\equiv 6.76 \times 10^{-1} \\
R_1(2,2) &= 0.26^2/40 &&\equiv 1.69 \times 10^{-3} \\
R_1(3,3) &= 0.26^2/80 &&\equiv 8.45 \times 10^{-4} \\
R_1(4,4) &= 0.26^2/120 &&\equiv 5.63 \times 10^{-4} \\
R_1(5,5) &= 0.26^2/160 &&\equiv 4.23 \times 10^{-4} \\
R_1(6,6) &= 0.26^2/240 &&\equiv 2.82 \times 10^{-4} \\
R_1(7,7) &= 0.26^2/320 &&\equiv 2.11 \times 10^{-4} \\
R_1(8,8) &= 0.26^2/480 &&\equiv 1.41 \times 10^{-4} \\
R_1(9,9) &= 0.26^2/640 &&\equiv 1.06 \times 10^{-4} \\
R_1(10,10) &= 0.26^2/960 &&\equiv 7.04 \times 10^{-5} \\
R_1(11,11) &= 0.26^2/1280 &&\equiv 5.28 \times 10^{-5} \\
R_1(12,12) &= 0.26^2/2560 &&\equiv 2.64 \times 10^{-5}
\end{aligned}
$$

2.6 ESTIMATION AND ERROR COVARIANCE EQUATIONS

The purpose of this section is to summarize the calculation for estimating the z component of the residual disturbance vector using Eqs. 2.2-1 to 2.2-7 and the data template and covariances defined in Sections 2.4 and 2.5.

For an initial estimate $\hat{\underline{x}}_0 = \underline{0}$ of the truth vector, the optimal estimate of the residual vertical disturbance $d_z(\underline{r})$ (located at the point beneath the center of the data template) is given as follows:

$$\hat{d}_z(\underline{r}) = G\,\underline{z}_1 \qquad (2.6\text{-}1)$$

$$G = BK_1 \qquad (2.6\text{-}2)$$

$$B = [1 \quad -1 \quad 0_{1\times 12}] \qquad (2.6\text{-}3)$$

$$K_1 = P_0H_1^T[H_1P_0H_1^T + R_1]^{-1} \qquad (2.6\text{-}4)$$

$$H_1 = [0_{12\times 2} \quad I_{12\times 12}] \qquad (2.6\text{-}5)$$

In Eq. 2.6-1, $\underline{z}_1$ is the 12×1 vector of averaged gradiometer data, averaged using the data template documented in Table 2.4-1. The measurement error covariance matrix R_1 is given in Section 2.5.2 and the initial gravity covariance matrix P_0 has been provided (Ref. 1).

The error variance of the estimate in Eq. 2.6-1 is

$$P_{dd} = E[(\hat{d}_z(\underline{r}) - d_z(\underline{r}))^2] = BP_1B^T \qquad (2.6\text{-}6)$$

$$P_1 = P_0 - K_1H_1P_0 \qquad (2.6\text{-}7)$$

For the numerical values provided, the rms error ($P_{dd}^{1/2}$) of the estimate of the residual vertical disturbance is 3.6 mgal, as computed using Eq. 2.6-6. Other data templates are being developed at TASC to meet the accuracy requirements for processing of the GGSS test data. These templates will be described in future reports.

3. SUMMARY AND CONCLUSIONS

3.1 SUMMARY

This report has presented a practical methodology for processing airborne gravity gradiometer data to estimate gravity disturbance vectors at the surface of the earth. The methodology consists of two stages. During the first stage, the gradiometer measurements are averaged over subregions of the surveyed area using data templates. The purpose of this first stage is to reduce the dimensionality of the computations. During the second stage, the averaged gradiometer measurements are weighted optimally and summed to estimate the gravity disturbance vector (or in the case of GGSS testing, to estimate the residual gravity disturbance vector). The data templates are selected to meet specified rms accuracy requirements, while avoiding unnecessary computations.

As an interim example of the type of data processing and documentation that are required for handling real survey data, a prototype data template is defined. Associated with this template are gravity and measurement error covariance matrices, which are needed to design optimal estimators for this template and analyze their rms errors.

Special attention is given to the unique requirements for testing of the GGSS. Because the limited geographic extent of the survey area makes long-wavelength accuracy verification imprecise, a specially defined gravity disturbance is introduced, called the residual gravity disturbance vector (the departure of the disturbance vector from its local mean value).

The residual gravity disturbance vector should be estimated during testing of the GGSS, rather than the gravity disturbance vector, because the long-wavelength content of the residual disturbance vector can be more easily controlled.

3.2 CONCLUSIONS

Based on the results of this study, the following principal conclusions are reached.

- A practical methodology is available for processing airborne gradiometer data to estimate gravity disturbance vectors (or residual gravity disturbance vectors for testing of the GGSS) at the surface of the earth
- Data processing templates can be designed, using error covariance analyses, to reduce the dimensionality of the data processing while satisfying accuracy requirements
- The technical approach (minimum-variance estimation based on averaged measurements) is flexible and can handle mixtures of gravity quantities appropriate to future survey needs.

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