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**ANALYTIC SIMULATION OF THE
PERFORMANCE OF MOBILE
MAINTENANCE CONTACT TEAMS**

GEORGE J. SCHLENKER

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George J. Schlenker

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This report presents an approach to describing the performance of mobile, main- tenance contact teams (C-teams). These teams support a tactical military sys- tem within a specific service area, such as along a Division Front. The teams are dispatched to a failed unit, diagnose and repair the fault, and move to the next customer without leaving the service area. The mathematical model of this system describes the stochastic steady-state using Markov process theory. Solu- tions are obtained by solving the linear, steady-state equations (continued)		

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20. ABSTRACT (continued):

using an efficient numerical procedure. The implementing computer program source code is included in the report.

A parametric analysis was performed using values characteristic of an air defense system. Parameters examined include: (a) intercustomer speed and variation in speed, (b) density of the C-teams, (c) dedicated versus nondedicated teams, (d) mean time between service requests, (e) mean diagnostic and repair time, and (f) form of the probability distribution for total service time.

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EXECUTIVE SUMMARY

This report presents an approach to describing the performance of mobile, maintenance contact teams (C-teams). These teams support an operational system within a specific service area. In the interest of generality, the supported system is left undefined. However, values selected for a parametric analysis were based on a tactical air defense system. The ultimate objective of C-team service is to keep the customer system in a high state of operational readiness. Thus, C-team performance is measured by variables such as average number of nonoperational customer units and average time a failed unit stays nonoperational. Although the mathematical model of the system in steady-state is fully analytic, it is not an expected-value model. The probability distribution for number of nonoperational customer units is an output of the implementing computer program. Statistics summarizing the performance of- and utilization of C-teams are also outputs.

A parametric analysis was performed to examine the sensitivity of C-team performance to parameters such as: (a) intercustomer speed and variation in speed, (b) density of C-teams, (c) dedicated versus nondedicated C-teams, (d) mean time per unit between service requests, (e) mean diagnostic and repair time, and (f) form of the probability distribution for total service time. Some conclusions are drawn from these results, which may be applicable to an operational system of interest. For example, when each of 3 C-teams is dedicated to 1/3 of the customer population, a substantial performance penalty is paid relative to an arrangement of nondedicated C-teams, which serve the entire population.

For the interested analyst, the report sketches the method of derivation of formulas. Computer source code for the implementing programs is included with extensive comments.

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MEMORANDUM REPORT

SUBJECT: Analytic Simulation of the Performance of Mobile
Maintenance Contact Teams

1. References

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- a. Memorandum Report No DRSMC/SA/MR-4, (AD A136580), HQ, AMCCOM, Nov 83, title: Models of a Service System for Production Machine Maintenance.
 - b. Technical Report, Society for Industrial and Applied Math, 1979, title: LINPAC Users' Guide.
 - c. Textbook by Donald Gross and Carl Harris, J. Wiley, c. 1974, title: Fundamentals of Queueing Theory.

2. Background

A maintenance contact team (C-team) is an adjunct to Organizational- and/or DS- maintenance. Each C-team is considered a mobile service system which operates within a prescribed service area. The C-team concept treated here is a derivative of the radar/electronics van supporting the Vulcan Air Defense System, which was developed in the 1960's. A C-team is dispatched so that it can move to the next customer immediately upon completing service on the current customer. In addition to the time to diagnose and repair a fault, the total time a C-team "serves" a customer includes the time to travel to the customer, following a commitment to that customer. Intercustomer distance and speed are variables which affect the service performance. Variations in these random variables as well as the conventional variation in maintenance service time are factors considered in modeling this service system. Performance of this service system is measured by the operational availability of the system being supported. To be general, the latter system is not specified here.

3. Objectives

An objective of this MFR is to describe an analytic -- as opposed to Monte-Carlo -- model which simulates the stochastic steady state of a service system consisting of (possibly) multiple C-teams serving a finite population of customers within a service area. A secondary objective is to present some parametric results which may be helpful in describing the adequacy of C-team performance. Parameter values used in this analysis are based on a range of values considered applicable to the SGT York air defense system. The relative importance of certain parameters and insensitivity of others are displayed.

4. Model Assumptions

The customers are located with uniform probability density in a rectangular service area. Contact teams are dispatched from one customer to the next without leaving the service area. When a C-team completes service on one customer, it either (a) immediately travels to the next customer, if a customer is waiting, or (b) waits at current position until dispatched, if the queue of customers is empty. The path traveled by C-teams between customers follows a series of segments, each of which is parallel to one of the sides of the service area. (The last assumption represents use of road networks and the need to avoid obstructions.) The intercustomer speed varies randomly over occasions with a uniform probability density between prescribed limits. The next customer to be served is selected by means of one of two service disciplines -- FIFO or SDST. Using FIFO, the next customer is the one whose request came first. Using SDST, the next customer is the one with the shortest distance from the team's current position. All parametric results shown here use FIFO service discipline. (When the average queue length is small, there is no practical difference in results from the two disciplines.) The requests for C-team service are assumed to occur at random in a conditionally Poisson manner. The rate parameter per operational unit is a constant. Thus, arrival rate of requests is proportional to the number of operational units, i.e., units not down for C-team maintenance. Disabling faults other than those repairable by C-teams are not treated explicitly in this model. The diagnostic and repair function is always assumed to restore a failed unit to an operational status. The average time to do this is the MTTR. The nominal values of the variables used in this study are given in Table 1. Parametric excursions were made for selected variables.

TABLE 1

VARIABLES USED IN THE CONTACT-TEAM PARAMETRIC ANALYSIS

Standard Parameter Values

Frontal Width of Service Area	_____	20 km *
Cross-front Dimension of Area	_____	4 km
Intercustomer Average Speed	_____	20 km/hr
Range of Uniformly Dist'd Speed	+/-	5 km/hr
Avg Time Between Service Requests		100 hr/unit
Avg Diagnostic and Repair Time	_____	3 hr
Population Dens of Fire Units	_____	12 to 36
Number of Supporting C-Teams	_____	1 to 3

* Front of service area represents a Division Front.

5. Methodology

Every aspect of this problem has been approached analytically, and has been verified by Monte-Carlo methods. This includes the model of C-team movement between customers as well as the stochastic service or queueing model. One advantage of a purely analytic approach is that parametric analyses can be performed with assurance that the observed differences in output statistics are in no way affected by Monte-Carlo sampling variation. This aspect is particularly important when the natural variation in subject variables is relatively large or when differential parametric effects are small. Altho some of the model assumptions may seem somewhat restrictive -- such as a uniform distribution of customers within the service area, these have, in fact, been found not to be so. Some alternatives, which are not analytically tractable, have been examined via Monte Carlo. These auxiliary studies are not reported here.

6. The service performed by a C-team has two stages -- travel to a customer and repair of a fault. Therefore, it is appropriate to consider as a model of the service system a Markov model having two stages of service and serving a finite customer population. The total time to "serve" a customer is considered a gamma(2) random variable. This model is equivalent to two identical stages of exponential service in tandem. Alternative models of service were also used. These are discussed under "Results". A multi-server analytic model of this sort was used earlier (Ref a.) to represent the service system for production machine maintenance. Numerical methods are used to solve the steady-state equations, which derive from the Kolmogorov differential-difference equations for the state probability vector. In the present application, it was considered necessary to change the implementing computer program in order to reduce execution time for cases having a large customer population. Program changes responsible for a great reduction in CPU time are: (a) use of a better method for solving the state equations and (b) reduction in the number of Markov states. In the computer program, found in Annex B, the reduced state-transition matrix, derived from the Kolmogorov equations, is, first, factored (into upper and lower triangular matrices), and then inverted in place. Two LINPAC routines (Ref b.) are used for this purpose. This approach is about 2.5 times faster than the gaussian-reduction method of Ref a. for large (g.t. 100 X 100) matrices. The dimension of the state probability vector is reduced by eliminating states with negligible probability of being occupied. The choice of states to eliminate is based on the closed-form solution to a similar queueing problem. The similar problem involves a finite customer population with conditionally Poisson arrivals and exponential services, whose steady-state solution is found in Reference c. The deleted states are those associated with a number in the service system greater than N, where the probability that the system number exceeds N is smaller than some small value (acceptable error). An error probability of 1/10,000 generally reduces the number of states by more than a factor of 1/2 for the cases treated here. This reduction is quite significant, since CPU time is nearly proportional to the number of states squared.

7. Survey of Results

Several formulas of interest were derived for this model. The system performance requires inputs to the queueing model such as: (a) average time a C-team spends in travel between customers and (b) average maintenance service time per customer. The first variable depends upon the mean and variance of the intercustomer distance and the mean and variance of the intercustomer speed. Formulas were derived for the probability distribution of intercustomer distance and for the mean and variance of this distribution. Since the intercustomer speed is considered a uniform random variable over the range (s1, s2), the mean and variance of the speed are the standard results: (s1 + s2)/2 and (s2 - s1)**2/12, respectively. An approximation for the mean and variance of the ratio of two random variables can be used to obtain the mean and variance of the intercustomer travel time -- = distance/speed -- in terms of the means and variances of distance and speed. The accuracy of the approximation depends somewhat upon the skewness of the distributions of distance and speed. The approximation was checked against an exact method for a practical range of values of the distance and speed parameters and found to be quite adequate. The approximation can be given, simply, as follows. Let E(d) and V(d) be the mean and variance of the intercustomer distance. Similarly, let E(s) and V(s) be the mean and variance of the speed. Then, the mean and variance of the travel time are given, respectively, by

$$E(t) = E(d)/E(s) + E(d)V(s)/(E(s))^3$$

and

$$V(t) = V(d)/(E(s))^2 + V(s)(E(d))^2/(E(s))^4 + (E(d)V(s))/(E(s))^6 + V(d)V(s)/(E(s))^4 .$$

Equations for the mean and variance of the intercustomer distance for a FIFO service discipline, are given in terms of the dimensions of the rectangular service area, say, a by b:

$$E(d) = (a + b)/3$$

and

$$V(d) = (a^2 + b^2)/18 .$$

Equations for the cumulative distribution function (C.D.F.) of intercustomer distance are presented in Annex A. The method of derivation is outlined in this annex. Annex B contains a computer source listing of the program which implements all the auxiliary equations, such as those above, as well as the equations of the Markov queueing model. An example of the C.D.F. of intercustomer distance is given in Figure 1 using the nominal dimensions of the service area. The C.D.F. of intercustomer distance using a SDST service discipline, given n customers are queued (n g.t. 0), is also shown in Figure 1. Notationally, let the latter C.D.F.

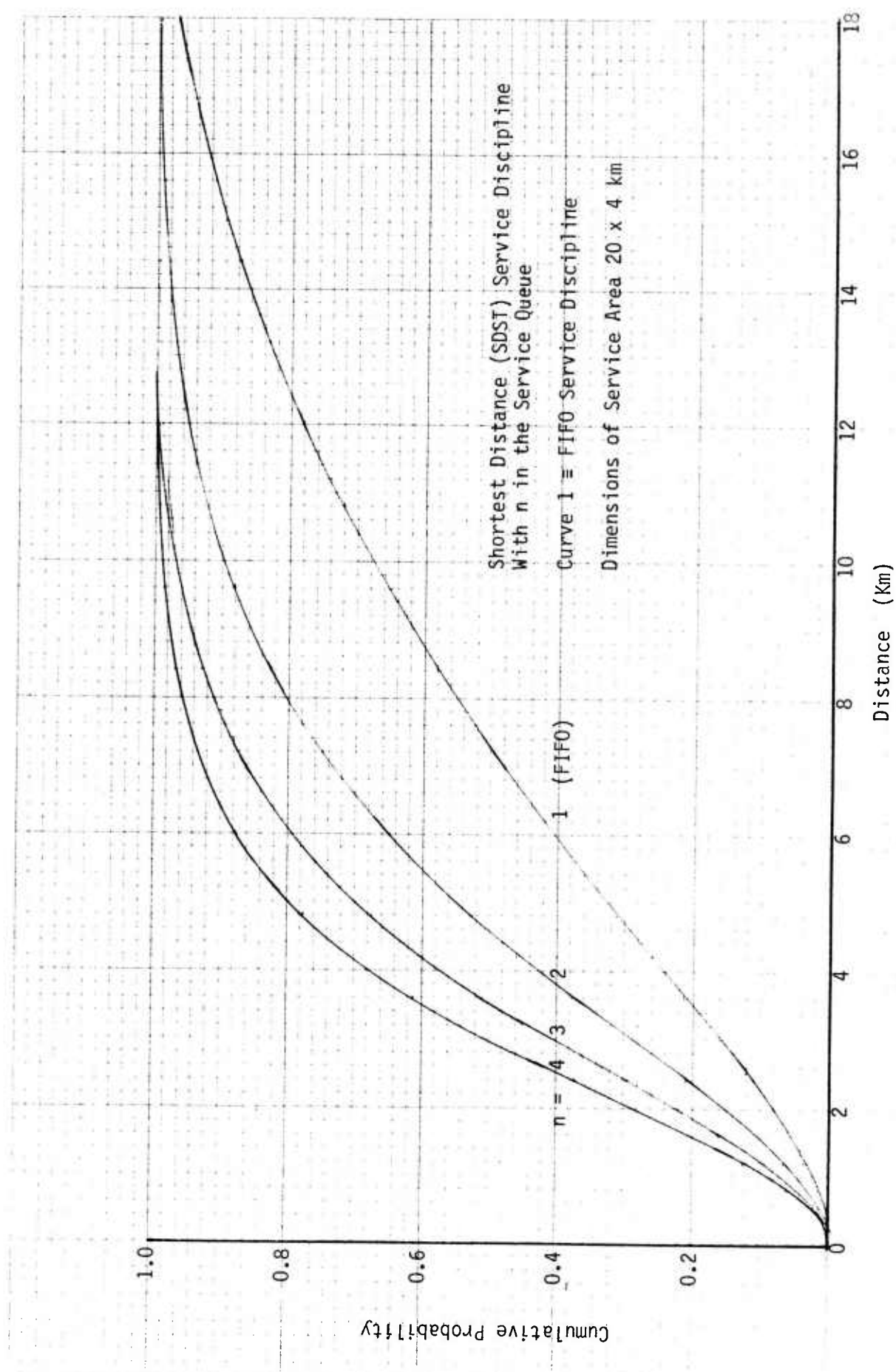


Figure 1. Conditional Cum Probability Distributions of Intercustomer Distance for Several Situations Under SDST Service Discipline

be $G(n,d)$ and the corresponding C.D.F. using FIFO be $F(d)$, for intercustomer distance d . Then, these probability distributions are related as follows:

$$G(n,d) = 1 - (1 - F(d))^n .$$

8. The auxiliary equations yield the mean total time in which a C-team is involved with a customer. This parameter and the mean time between failures of operational units (MTBF) are inputs to the Markov queueing model (Annex B). The outputs of this model are typical of stochastic service models: (a) Average and standard deviation of number of individuals in the service system, i.e., either waiting (in a service queue) for a C-team to be assigned or being "served". In this context "being served" means that a C-team is either in transit or conducting repairs, whereas the expression "in the service system", used in queueing theory, means that an individual operational unit has experienced a failure (of the type which is repairable by a C-team), which has not been repaired. In this context, "in the service system" just means "nonoperational". (b) Avg and S.D. of number of individuals in the service queue. In this situation there is only one service queue, since failed units request service from a single source, the dispatcher. (c) Avg and S.D. of number of busy service channels (or busy C-teams). (d) Avg and S.D. of waiting time in the service system, i.e., the time a customer is "down" with a failure, repairable by a C-team. (e) Avg and S.D. of waiting time in the service queue. This time interval starts with a request for service and ends when a C-team is committed to serve the requesting unit.

9. Model runs were made under two contingencies (or options) for a battalion population of 30 operational units. In the first option the 30 units are divided into batteries of 12 units, each of which has a single C-team dedicated to serving that battery. In the second option any of the 3 C-teams can serve any customer. The latter option does not "fence off" requests by the artifice of restricting the population which a C-team can serve.

10. Using a (battery) population of 12 units, the effect of intercustomer speed on maintenance performance is shown in Table 2. The average speed is increased from 10 to 40 km/hr, in the first three runs -- columns 1 thru 3. In the fourth run the range of speed is increased while preserving the average at the nominal value, 20 km/hr. This may be compared with the results in column 2 to determine the effect of variability in the speed. In the fifth run, the density of C-teams is tripled to shown density effects.

TABLE 2

PARAMETRIC EFFECTS OF CONTACT-TEAM MOBILITY AND DENSITY ON SERVICE

Attribute of the Performance	Service System Parameters: Teams per Btry, Range of Speed				
	1, 10+-5	1, 20+-5	1, 40+-5	1, 20+-10	3, 20+-5 km/hr
Avg Num in Sys*	0.648	0.546	0.503	0.552	0.399
Avg Units in Q	0.209	0.156	0.153	0.159	0.001
Avg Wait in Sys	5.712	4.770	4.375	4.819	3.439 Hrs
Avg Wait in Q	1.845	1.362	1.174	1.385	0.005 Hrs
Pr(Q Wait[8 hr)	0.933	0.961	0.970	0.959	0.999

* "Average Customers in the Service System" is equivalent to the number of customer units "down" for service by a contact team.

11. Using the second option, a series of runs were made with MTBF and MTTR as parameters. The excursions in these parameters were chosen to develop the locus of points, in the space of these parameters, on which the average number of nonoperational customer units has the constant value -- 0.5, 1.0, 1.5, or 2.0. This family of curves is displayed in Figure 2. These curves are isoavailability curves having availabilities of 98.6%, 97.2%, 95.8%, and 94.4% for the supported system, respectively. An alternative way of displaying these data is shown in Figure 3. In this figure the average number of customer units "down" is shown as a function of MTBF, with MTTR as a parameter. Note that the curve for each particular MTTR shows a "knee" at which the rate of change of average nonoperational units changes remarkably.

TABLE 3

SERVICE PERFORMANCE FOR A SYSTEM OF 3 C-TEAMS SERVING 36 CUSTOMERS

MTBF (Hrs)	! Avg Customers in Service Sys and Avg Wait (Hrs) in Service Sys! for an MTTR (Hrs):							
	! 2 !		! 3 !		! 4 !		! 5 !	
40	! 2.558	! 3.059 !						
60	! 1.496	! 2.602 !	! 2.342	! 4.175 !				
75			! 1.736	! 3.800 !	! 2.465	! 5.512 !		
80	! 1.088	! 2.494 !						
100	! 0.862	! 2.454 !	! 1.244	! 3.580 !	! 1.672	! 4.872 !	! 2.183	! 6.454 !
120	! 0.716	! 2.436 !						
125			! 0.981	! 3.500 !	! 1.292	! 4.653 !		
150			! 0.812	! 3.464 !	! 1.061	! 4.555 !	! 1.324	! 5.729 !
160	! 0.536	! 2.421 !						
200	! 0.430	! 2.415 !	! 0.608	! 3.433 !	! 0.788	! 4.474 !	! 0.972	! 5.551 !
250			! 0.486	! 3.422 !	! 0.629	! 4.444 !	! 0.773	! 5.485 !
300			! 0.405	! 3.416 !	! 0.524	! 4.429 !	! 0.645	! 5.454 !
350					! 0.449	! 4.422 !	! 0.551	! 5.438 !
400							! 0.482	! 5.429 !

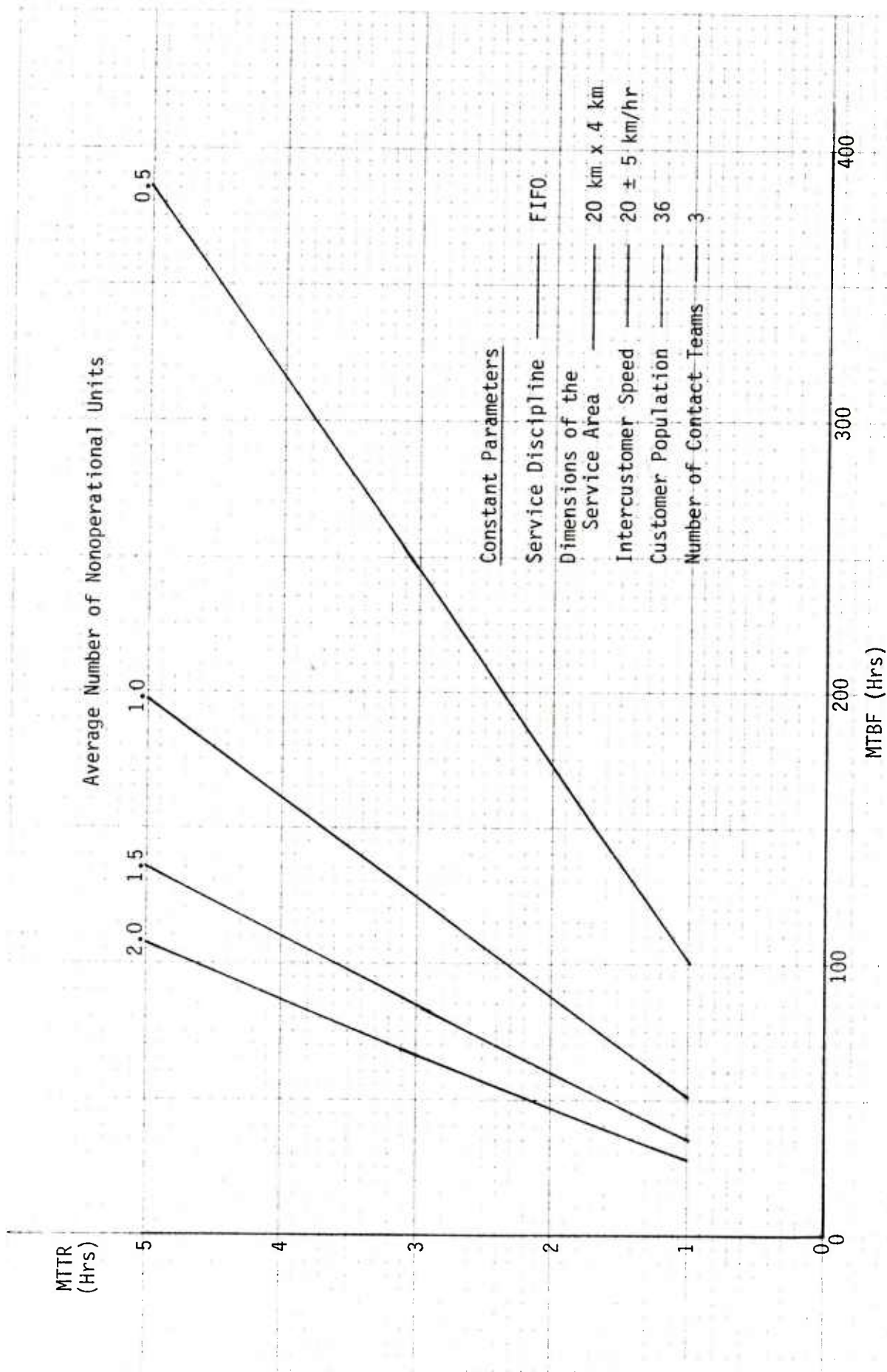


Figure 2. Customer Isoavailability Curves in the Space of MTBF and MTTR

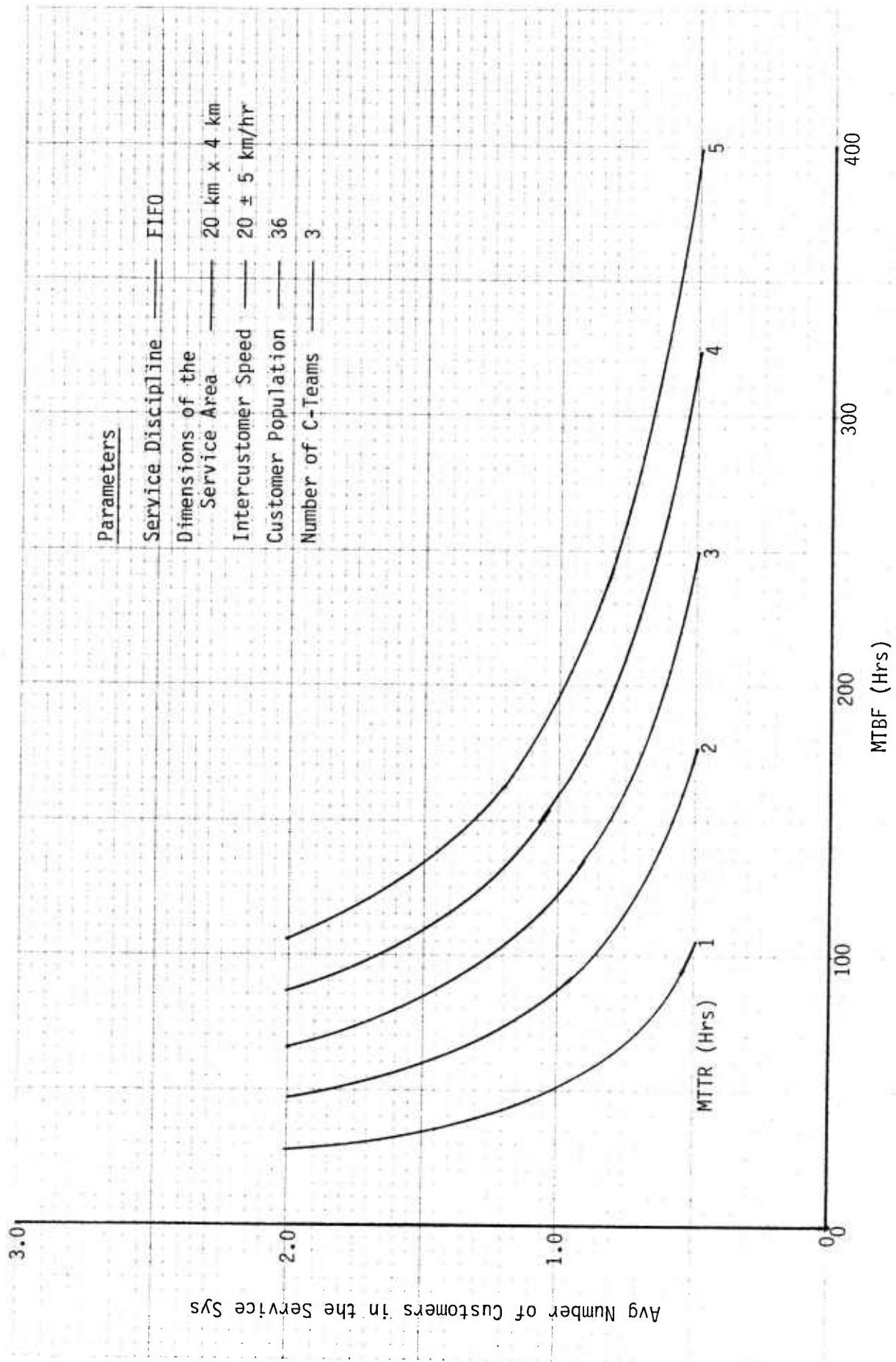


Figure 3. Average Nonoperational Customer Units as a Function of MTBF with MTTR as a Parameter

12. The results presented above were obtained for one statistical model of the time for a C-team to "serve" a customer, i.e., to reach a customer and to diagnose and repair the fault. This model uses a gamma probability distribution, with shape parameter of 2. This is denoted as the gamma(2) model. Among the alternative models one might reasonably take are: (a) the service activity time is exponentially distributed, or (b) the travel time is exponentially distributed (as a 1st stage of service), and the actual repair time is exponentially distributed, as a 2nd stage of service. To examine the sensitivity of the queueing results to the distributional assumption, comparison runs were made for the gamma(2) and exponential models. A gamma(2) model is actually equivalent to two tandem exponential models having identical rate parameters. Therefore, the gamma(2) and exponential models produce results which bound results of model (b), when all models are constrained to have the same mean activity time. The results of the paired comparisons are shown in Table 4. Note that when the C-teams are relatively inactive, the model alternatives produce results which are quite close, particularly with respect to the average number of units in the service system. The values of parameters which are treated as constants in Table 4 are:

Dimensions of the Service Area	20 km X 4 km
Intercustomer Speed	20 +- 5 km/hr
Customer Population Density	36
Number of Supporting C-Teams	3

The two parameters shown in Table 4 are MTBF, the mean time (hours) between service requests per unit, and MTTR, the mean diagnostic and repair time (hours).

TABLE 4

COMPARISON OF QUEUEING STATISTICS FOR ALTERNATIVE
DISTRIBUTIONS OF C-TEAM TRAVEL-AND-SERVICE TIME

C.D.F. Total Time	Statistical Attribute of C-Team Performance						
	Avg No in Sys	S.D. No in Sys	Avg No in Queue	Avg Busy Servers	S.D. Busy Servers	Avg-Sys Wait Tm	Avg-Queue Wait Time
MTBF = 60							
MTTR = 1							
Gamma(2)	0.8400	0.9213	0.0147	0.8253	0.8737	1.433	0.025
Exponen	0.8433	0.9347	0.0181	0.8252	0.8748	1.439	0.031
MTBF = 75							
MTTR = 1							
Gamma(2)	0.6697	0.8183	0.0063	0.6634	0.7946	1.422	0.013
Exponen	0.6711	0.8247	0.0078	0.6634	0.7953	1.425	0.016
MTBF = 100							
MTTR = 1							
Gamma(2)	0.5020	0.7066	0.0021	0.4999	0.6971	1.414	0.006
Exponen	0.5025	0.7090	0.0026	0.4999	0.6974	1.416	0.007
MTBF = 125							
MTTR = 1							
Gamma(2)	0.4020	0.6319	0.0009	0.4011	0.6273	1.411	0.003
Exponen	0.4022	0.6331	0.0011	0.4011	0.6275	1.412	0.004
MTBF = 80							
MTTR = 2							
Gamma(2)	1.0885	1.0619	0.0375	1.0510	0.9593	2.494	0.086
Exponen	1.0973	1.0916	0.0466	1.0507	0.9612	2.515	0.107
MTBF = 100							
MTTR = 2							
Gamma(2)	0.8624	0.9343	0.0162	0.8462	0.8828	2.454	0.046
Exponen	0.8661	0.9489	0.0200	0.8461	0.8840	2.465	0.057
MTBF = 100							
MTTR = 3							
Gamma(2)	1.2444	1.1474	0.0598	1.1846	0.9977	3.580	0.172
Exponen	1.2588	1.1914	0.0747	1.1841	1.0001	3.623	0.215

13. Conclusions

Several conclusions are derived from the parametric analysis:

- (a) Restricting the C-teams so that one team serves only a particular battery has the effect of significantly reducing the readiness of Battalion units. If three teams can serve any of 36 customer units on a dispatched, FIFO basis, the average number of nonoperational units is 1.24, versus 1.63, if the 36 units are divided into 3 exclusive service groups, served by dedicated teams. A comparison of average waiting time in the service area is 3.58 hours, in the first case, versus 4.77 hours, in the second -- a 33 % increase.
- (b) Little performance benefit is obtained from increasing the average C-team speed from 20 to 40 km/hr. For such a change, with one team serving 12 customers, the average number of customers down for this sort of maintenance changes from 0.546 to 0.503. The average time spent "in the maintenance system" goes from 4.77 hrs to 4.38 hrs with this speed doubling.
- (c) Maintenance performance of the C-teams suffers a serious degradation with a reduction of the average speed from 20 km/hr (nominal) to 10 km/hr. This halving of the speed increases the average number of customers in this service system from 0.546 to 0.648 units (19 % increase). The average time spent in the service system, i.e., "down", goes from 4.77 hrs to 5.71 hrs, a 20 % increase. These and other parametric effects are displayed in Tables 1, 2, 3, and 4 and Figures 2 and 3.
- (d) A knee occurs at an MTBF of 100 hrs in the functional relation of average customers in the service system versus MTBF, for an MTTR of 3 hrs. The location of the knee is somewhat subjective. In this case, a quite rapid increase occurs in the average number of customers down for maintenance with a small decrease in the MTBF below the location of the knee. The location of the knee decreases with decreasing MTTR. Decreasing the MTTR also makes the knee more pronounced.
- (e) Several alternatives were examined to describe the probability distribution of the time for a C-team to travel to a customer and to complete maintenance service on that customer. In general, the statistics which characterize service performance are relatively insensitive to the form of this distribution, given a mean value of the total "service" time.

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 1 ATTN: SARAC-DO

COMMANDER
 WATERVLIET ARSENAL
 WATERVLIET, NY 12189-5000
 1 ATTN: AMXMC-LCB-TL

COMMANDER
 CHEMICAL R AND D CENTER
 ABERDEEN PROVING GROUND
 (EDGEWOOD AREA), MD 21010-5423
 1 ATTN: AMSMC-CLJ-IA (A)

DIRECTOR US AMETA
 ROCK IS, IL 61299-6000
 1 ATTN: AMXOM-QA

DIRECTOR
 NAVAL POSTGRADUATE SCHOOL
 MONTEREY, CA 93940
 1 ATTN: DEPT OF OPERATIONS ANAL.

1 DIRECTOR
 ADVANCED RESEARCH PROJECTS AGENCY
 1400 WILSON BLVD
 ARLINGTON, VA 22209

DIRECTOR
 USA TRASANA
 WHITE SANDS MISSILE RANGE
 WHITE SANDS, NM 88002-5502
 1 ATTN: ATAA-SL

COMMANDER
 USA COMBINED ARMS COMBAT
 DEVELOPMENT ACTIVITY
 FT LEAVENWORTH, KS 66207
 1 ATTN: ATZL-CAM-M

ANNEX A

DERIVATION OF EQUATIONS

Probability Distribution for Intercustomer Distance

 The cumulative distribution function (c.d.f.) for the distance a C-team travels between customers is derived in a series of steps. We start by observing that this distance consists of the sum of two independent random variables, each of which is the projection of intercustomer distance along a line. This fact is a consequence of the requirement that the path followed by a C-team consists of series of orthogonal segments, each of which is parallel to one of the sides of the rectangular service area. One may equivalently view this type of path as an x-displacement (from the customer just served) followed by a y-displacement. Let the dimensions of the service area be: a by b, so that $0 \leq x \leq a$, $0 \leq y \leq b$. (Note that "le" denotes "less than or equal to". Similarly, "ge" is the abbreviation for "greater than or equal to", etc.). Suppose that the c.d.f. of the x-component of displacement is denoted by $F_1(d_1)$ and that for the y-component is $F_2(d_2)$, where the intercustomer distance, d, is

$$d = d_1 + d_2, \quad 0 \leq d \leq a+b. \quad (1)$$

Because the components of d are independent, the c.d.f. of d, $F_d(d)$, is the convolution of F_1 and F_2 . The method just outlined is the approach used to obtain the distribution of intercustomer distance, using a FIFO service discipline.

As a first step, consider the distance between two randomly selected points along a line segment of unit length (0,1), where the points are uniformly distributed. Let the random location of the first point be X_1 and that for the second be X_2 . Note that upper case letters are used for realizations of a random variable. The random variable denoting the distance is Z:

$$Z = \text{abs}(X_1 - X_2). \quad (2)$$

Equivalently,

$$Z = X_1 - X_2, \text{ for } X_1 \geq X_2,$$

or

$$Z = X_2 - X_1, \text{ for } X_2 \geq X_1. \quad (3)$$

Notationally, let the c.d.f. of z be

$$F(z) = \text{Pr}(Z \leq z). \quad (4)$$

Because of symmetry, $\text{Pr}(X_1 \geq X_2)$ is equal to $\text{Pr}(X_2 \geq X_1)$.

Therefore, consider only the first of these two contingencies, in which $X_1 \geq X_2$.

Notationally, let I_u represent the integration operator over the feasible space of the variable of integration u . To be specific the limits of integration -- u_1, u_2 -- may be denoted as arguments in this way: $I(u_1, u_2)$. To abbreviate, the expression $(X_i = u)$ means that the random variable X_i lies between u and $u+du$, ($i = 1, 2$).

Then,

$$F(z) = I_u \Pr(X_2 = u) \Pr(X_1 - u \leq z). \quad (5)$$

In performing the integration, one must choose limits so that $0 \leq u \leq 1$ and that $X_1 \geq X_2 = u$, as assumed.

From (4) and (5), remembering that $\Pr(X_i = u) = du$,

$$F(z) = I_u du \Pr(u \leq X_1 \leq z+u)$$

$$F(z) = I_u (0, 1-z) du z + \Pr(u \leq z). \quad (6)$$

After integrating, evaluating at the limits, and combining terms,

$$F(z) = 2z - z^2. \quad (7)$$

By a change of variable with $z = d/a$,

$$F(d) = 2(d/a) - (d/a)^2, \quad d \leq a. \quad (8)$$

The probability density function (p.d.f.) for d can be obtained from (8) by differentiation, yielding

$$f(d) = (2/a)(1 - d/a), \quad d \leq a. \quad (9)$$

Taking the first two origin moments of f -- $E(d)$ and $E(d^2)$, where

$$E(d^k) = I_u (0, a) u^k f(u) du, \quad k = 1, 2, \quad (10)$$

$$E(d_1) = a/3 \quad (11a)$$

and

$$E(d_1^2) = a^2/6 \quad (11b)$$

The variance of d_1 , $V(d_1)$, is obtained from (11) via the

difference between second origin moment and mean squared:

$$V(d_1) = a^2/18. \quad (12)$$

The c.d.f., p.d.f., and associated moments of d_2 can be developed using the above results for d_1 by merely substituting b for a in equations (8) thru (12). Note that the random variable for the x-component of d differs statistically from that of the y-component only in its range. Thus,

$$F(d_2) = 2(d_2/b) - (d_2/b)^2, \quad d_2 \leq b. \quad (13)$$

$$f(d_2) = (2/b)(1 - d_2/b), \quad d_2 \leq b. \quad (14)$$

The mean and variance of f_2 are

$$E(d_2) = b/3 \quad (15a)$$

and

$$V(d_2) = b^2/18. \quad (15b)$$

Since the mean of the sum of independent random variables is just the sum of the means, from (1), (11a), and (15a),

$$E(d) = (a + b)/3. \quad (16)$$

The same additive law applies to the variance of the sum of independent random variables. Thus,

$$V(d) = (a^2 + b^2)/18. \quad (17)$$

These results also appear on page 4 of this report. By contrast to the other equations on page 4, these results are exact.

Notationally, let the p.d.f. of d be denoted by $f(d)$.

As noted above, the probability distribution of d is the convolution of the distributions of its components. This statement also applies to the densities. Both $f_1(h)$ and $f_2(v)$ are defined on finite domains: $(0,a)$ and $(0,b)$, respectively. Therefore, the domain of $f(d)$, with $d = h+v$, is $(0,a+b)$.

It will be seen that the functional form changes over subdomains such as $(0, \min(a,b))$ and $(\min(a,b), \max(a,b))$. This results from the finite limits of integration in the convolution operation. The specification of these limits in the space of h and v is tricky. A straightforward way of handling this problem is to use Laplace transforms. The convolution theorem of integral transforms applies here: the transform of the convolution is the product of the transforms of the functions being convolved. Denoting the Laplace transform of f_i as $f_i^*(s)$, where i is 1 or 2,

$$f_d^*(s) = \int_0^{\text{liu}(i)} \exp(-su) f_i(u) du, \quad (18)$$

with $\text{liu}(1) = a$ and $\text{liu}(2) = b$.
Then,

$$f_d^*(s) = f_1^*(s) f_2^*(s). \quad (19)$$

In performing the inverse transform, the "problem" of subdomains is handled automatically by the mechanics of the operation. This attractive aspect of integral transforms motivated this approach.

After some algebraic manipulation of the results from (18), the transform for f_d becomes

$$f_d^*(s) = (2/a)/s - (2/a^2)/s^2 + (2/a^2) \exp(-sa)/s. \quad (20)$$

A comparable expression for $f_d^*(s)$ is obtained by substituting

b for a in equation (20). Note that terms involving $\exp(-sa)$ and $\exp(-sb)$ appear in $f_1^*(s)$ and $f_2^*(s)$, respectively.

Using (19), one notes that terms involving these factors and the factor $\exp(-s(a+b))$ appear in $f_d^*(s)$. During inversion, terms

which involve a factor of the form $\exp(-cs)$, with c constant, are converted to terms in the inverse which have a unit step function as a factor. The unit step function in the variable d is defined as

$$\begin{aligned} u(d - c) &= 0, & d < c \\ &= 1/2, & d = c \\ &= 1, & d > c. \end{aligned} \quad (21)$$

Thus, terms in the inverse which have a unit step factor contribute only to the result for that portion of the domain for which d is greater than or equal to the parameter c .

From (19) and (20),

$$f^*(s) = \frac{2}{d} A + \frac{1}{s} \left(\frac{3}{a} B + B \exp(-sa) + B \exp(-sb) \right) +$$

$$\frac{4}{(1/s)} \left(C - C \exp(-sa) - C \exp(-sb) + C \exp(-s(a+b)) \right),$$

with

$$A = 4/(ab),$$

$$B = -4(a + b)/(ab)^2, \quad B = 4/a^2 b \quad \text{and} \quad B = 4/ab^2,$$

$$C = 4/(ab)^2. \tag{22}$$

Terms in this transform involving $\exp(-s(a+b))$ are needed to insure that the inverse transform (p.d.f.) is zero for $d \geq a+b$. By restricting the domain of the p.d.f. to $(0, a+b)$, these terms can be neglected. With this restriction, the inverse transform becomes.

$$f(d) = Ad + (B/2)d^2 + (B/2)(d-a)^2 u(d-a) + (B/2)(d-b)^2 u(d-b) + (C/6)(d^3 - (d-a)^3 u(d-a) - (d-b)^3 u(d-b)), \quad d \leq a+b. \tag{23}$$

From (21) and (23), it can be seen that the polynomial in d for the p.d.f. assumes different forms over the subdomains: $(0, \min(a,b))$, $(\min(a,b), \max(a,b))$, and $(\max(a,b), a+b)$. The c.d.f. of intercustomer distance is obtained from (23) by integration:

$$F(d) = \int_0^d f(t) dt, \quad d \leq a+b. \tag{24}$$

The implementing computer code for the result of (24) is in Annex B on page B-7.

The c.d.f. of intercustomer distance from (24) was derived assuming the coordinates of the positions of the two customers were statistically independent and uniformly distributed. This situation exists under both FIFO service discipline and when less than 2 customers are queued with SDST discipline. Suppose the service discipline is shortest distance (SDST). Denote the conditional c.d.f. of d , given that n customers are waiting, by $G(n,d)$. By definition,

$$\Pr(D \geq d, n) = 1 - G(n,d). \tag{25}$$

But,

$$\Pr(D \geq d, n) = \Pr(D_1 \geq d) \Pr(D_2 \geq d) \dots \Pr(D_n \geq d),$$

where D_n is the random value of the n th distance from $F(d)$.

Thus,

$$\Pr(D \geq d, n) = (1 - F(d))^n,$$

and

$$G(n, d) = 1 - (1 - F(d))^n. \quad (26)$$

The unconditional c.d.f. of d under SDST discipline, $G(d)$, is obtained by summing, over n , products of the form:

$$\Pr(\text{number queued} = n)G(d, n).$$

The probability weights in the resulting expression can be obtained from the analytic queueing model. An iterative evaluation of these probabilities is required, since the service rate, being a model input, depends upon the mean and variance of d . The first two origin moments of $G(d)$ can be obtained by numerical integration of the following equations using Simpson's rule.

$$E(d) = \int_0^{\infty} (1 - G(t)) dt \quad (27a)$$

$$E(d^2) = \int_0^{\infty} t^2 (1 - G(t)) dt. \quad (27b)$$

The computer program which implements these operations for any c.d.f., $G(d)$, is found in Annex B, starting on page B-24.

Approximations for Mean and Variance of Travel Time

Approximations are given on page 4 for the mean and variance of intercustomer travel time. The derivation of these results is sketched here. The intercustomer time, t , is the ratio of the distance, d , to the (average) speed, s , over that distance. This (avg) speed is considered a random variable, varying from one customer to the next. The mean and variance of s are considered as given. Since t is a function of d and s , it can be expanded in a Maclaurin series about the mean values of d and of s . The mathematical expectation of both sides of this equation is taken. Cubic- and higher -terms of $d - E(d)$ and of $s - E(s)$ are discarded. This procedure leads to the approximation for $E(t)$, given on p. 4. The Maclaurin expansion for t is squared, and mathematical expectations are taken on both sides of this equation. After discarding the higher-order terms, one obtains an approximation for the second origin moment of t . The approximation for the variance of t is this second origin moment minus the square of the above mean. The neglect of the 3rd and higher central moments in the above derivations may cause one to be concerned about the accuracy of these approximations when the distributions of d and of s are rather skewed. To evaluate the error of approximation one can compare these results with exact results for a special case. The special case, discussed next, involves distributional assumptions for intercustomer distance and speed. The c.d.f. of t for the special case is given in closed form. Using this expression, the mean and variance of t are calculated via the procedure in (27). The approximation has nearly 3-digit accuracy for an example with $a = 15$ km and with s uniform over the interval 15 to 25 km/nr.

Derivation of the Distribution of Travel Time for a Special Case

To obtain a probability distribution of intercustomer distance, let the population be distributed uniformly along the x-axis, from zero to a. In this case the FIFO c.d.f. of intercustomer distance is given by (8), with the p.d.f. given by (9). Note that the distance distribution is positively skewed, and, thus has a non-zero 3rd central moment -- contrary to the above implicit assumption. For the present purpose, denote the p.d.f. and c.d.f. of the intercustomer distance, x, as f(x) and F(x), respectively.

To derive the c.d.f. of the intercustomer travel time, assume that the speed has a uniform distribution between u and v. Thus,

$$f(s) = 1/(v-u), \quad u \leq s \leq v. \quad (28)$$

Notationally, let the c.d.f. of time, t, be F(t), where

$$F(t) = \Pr(T \leq t), \text{ for a random time } T. \quad (29)$$

Observe that T can take any value from zero to (a/u). For a particular realization, T = X/S, with X being the particular distance and S being the particular value of the intercustomer speed. Then, the conditions for T ≤ t are :

$$X = x \text{ and } v \geq S \geq x/t, \text{ with } 0 \leq x \leq a.$$

Applying these conditions to (29),

$$F(t) = \int_0^t \int_x^a f(x) \Pr(S \geq x/t) dx, \quad (30a)$$

where

$$\Pr(S \geq x/t) = \begin{cases} 1, & x \leq ut, \\ 1 - (x/t - u)/(v-u), & u \leq x/t \leq v, \\ 0, & x > vt. \end{cases} \quad (30b)$$

Thus,

$$F(t) = \int_0^{ut} \int_x^a f(x) dx + \int_{ut}^a \int_x^a f(x) (1 - (x/t - u)/(v-u)) dx, \quad (31a)$$

for vt ≥ t a,

$$F(t) = F(ut) + \int_{ut}^a \int_x^a f(x) (1 - (x/t - u)/(v-u)) dx, \quad (31b)$$

for vt ≤ a, or for t ≤ a/v.

Carrying out the indicated integrations yields the following results.

For va/u > vt > a, (31a) becomes

$$F(t) = 1 - a/(3t(v-u)) + u^2 t/(a(v-u)) - (2/3)u^2 t^2/(a(v-u)) + (u/(v-u))(1 - F(ut)), \quad a/u \geq t \geq a/v. \quad (32a)$$

$$F(t) = F(vt)(1 + u/(v-u)) - F(ut) u/(v-u) - (v+u)/a t^2 + 2/(3a(v-u))(v-u)^2 t^3, \quad t \leq a/v. \quad (32b)$$

ANNEX B

COMPUTER SOURCE PROGRAMS

The source programs listed here are written in SIMSCRIPT 2.5 for the PRIME 750 minicomputer. However, the code does not employ features unique to this computer. The MAIN and subprograms calculate statistics for the stochastic steady state of a maintenance service system. This system consists of several mobile contact teams (C-teams) which are centrally dispatched. The teams serve a fixed population of customers within a service area. The customers are assumed to be uniformly distributed within a rectangular service area whose dimensions are input parameters. All program inputs are read interactively, with prompting messages displayed at the terminal. At the beginning of each program listing is found: (a) a functional description of the program, (b) a list of the program inputs -- or input arguments, for subprograms, (c) a list of the program outputs, and, in some instances, (d) a list of key endogenous variables. All utility functions and routines are included in this listing. No external files are used. Since the output is lengthy, it is necessary to set up a COMO file to display all of it and to obtain a permanent copy.

The programs found in this annex are located as follows:

	Page

MAIN 'CONTACT.Q	B-2
CDT.FIFO	B-7
FINITE.ME2.Q	B-8
SGEFA	B-20
SGEDI	B-22
MINRV	B-24
LIMSTATE	B-26

```

1  PREAMBLE ''CONTACT.Q
2  NORMALLY MODE IS REAL
3  DEFINE PDT.FIFO AS A REAL FUNCTION WITH 3 ARGUMENTS
4  DEFINE CDT.FIFO AS A REAL FUNCTION WITH 3 ARGUMENTS
5  DEFINE PDT.SDST AS A REAL FUNCTION WITH 3 ARGUMENTS
6  DEFINE CDT.SDST AS A REAL FUNCTION WITH 3 ARGUMENTS
7  DEFINE FUN.CDF TO MEAN CDT.FIFO
8  END ''PREAMBLE

1  MAIN ''CONTACT.Q
2  ''
3  ''PROGRAM SOLVES THE STEADY-STATE QUEUEING PROBLEM ASSOCIATED WITH A
4  ''MOBILE CONTACT TEAM WHICH VISITS AND SERVES CUSTOMERS IN A RECTANGULAR
5  ''SERVICE AREA. THE CUSTOMERS, REQUIRING MAINT SERVICE, ARE ASSUMED
6  ''TO BE UNIFORMLY DISTRIBUTED WITHIN A RECTANGLE WITH DIMENSIONS ARANGE
7  ''BY BRANGE. THE POPULATION OF CUSTOMERS IS FINITE (= MPOP). THE NUMBER
8  ''OF C-TEAMS IS EQUIVALENT TO THE NUMBER OF SERVICE CHANNELS (NSERVE).
9  ''TWO TYPES OF SERVICE DISCIPLINE ARE PERMITTED: FIFO AND SDST. USING
10 ''FIFO, THE CONTACT TEAM TRAVELS TO THE CUSTOMER HAVING THE FIRST REQUEST.
11 ''USING SDST, THE CONTACT TEAM TRAVELS TO THE CUSTOMER HAVING THE SHORTEST
12 ''DISTANCE FROM THE CURRENT LOCATION. AN ERLANG(2) DISTRIBUTION IS USED
13 ''TO APPROXIMATE THE TOTAL TIME TO TRAVEL TO AND SERVE A CUSTOMER.
14 ''
15 ''INPUTS:
16 ''FLAG.DIST _____ AN INTEGER FLAG TO INDICATE PRINTING OF THE PROBABILITY
17 ''                      DISTRIBUTION OF INTERCUSTOMER DISTANCE.
18 ''FLAG.FIFO _____ AN INTEGER FLAG TO INDICATE A FIFO SERVICE DISCIPLINE
19 ''                      (= 1). THE ALTERNATIVE DISCIPLINE IS TO SERVE THE CUST-
20 ''                      OMER HAVING THE SHORTEST DISTANCE (SDST) NEXT.
21 ''IPRINT _____ FLAG TO PRINT QUEUEING STATISTICS FROM A SUBROUTINE.
22 ''ARANGE _____ THE FRONTAL DIMENSION (KM) OF THE RECTANGULAR CUSTOMER
23 ''                      TERRITORY SERVED BY A SET OF CONTACT TEAMS.
24 ''BRANGE _____ THE CROSS-FRONTAL (ORTHOGONAL) DIMENSION (KM) OF THE
25 ''                      RECTANGULAR CUSTOMER TERRITORY.
26 ''SPEED _____ THE AVERAGE SPEED (KM/HR) OF MOVEMENT OF A CONTACT TEAM
27 ''                      BETWEEN CUSTOMERS.
28 ''MPOP _____ SIZE OF THE CUSTOMER POPULATION, I.E., THE NUMBER OF UNITS
29 ''                      SERVED BY CONTACT TEAMS.
30 ''NSERVE _____ THE NUMBER OF CONTACT TEAMS (OR SERVICE CHANNELS).
31 ''MTBF _____ MEAN TIME (HOURS) BETWEEN FAILURES OF THE AGGEGATE OF PARTS
32 ''                      REQUIRING SERVICE BY A CONTACT TEAM.
33 ''MTTR _____ MEAN TIME TO DIAGNOSE AND REPAIR A FAILURE WHICH REQUIRES
34 ''                      A CONTACT-TEAM SERVICE REQUEST.
35 ''
36 ''ENDOGENOUS VARIABLES:
37 ''P.NSTATE(*) _____ A REAL VECTOR OF MARKOV-STATE PROBABILITIES FOR THE SERVICE
38 ''                      SYSTEM. P.NSTATE(N) IS THE PROB THAT N UNITS ARE IN THE
39 ''                      "SERVICE SYSTEM", I.E., EITHER BEING SERVED OR AWAITING
40 ''                      SERVICE.
41 ''LAMBDA _____ AVERAGE ARRIVAL RATE PER CUSTOMER (PER HOUR).
42 ''MU1 _____ AVERAGE UNIT SERVICE RATE, INCLUDING TRAVEL, (PER HOUR).
43 ''MLIM _____ LIMITING NUMBER OF CUSTOMER PERMITTED IN THE SERVICE SYS.
44 ''PLIM _____ PROB ACCURACY IN C.D.F., TO LIMIT THE MARKOV STATE SPACE.

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45 ''
46 ''OUTPUTS:
47 ''P.NULL _____ STEADY-STATE PROBABILITY THAT THE SERVICE SYSTEM IS EMPTY.
48 ''P.SYS.FULL _____ PROBABILITY THAT ALL SERVICE CHANNELS ARE BUSY.
49 ''P.CUST.WAIT _____ PROB THAT AN "ARRIVING" CUSTOMER, I.E., A JUST FAILED
50 '' UNIT, MUST QUEUE.
51 ''ENQ _____ EXPECTED NUMBER OF CUSTOMER UNITS DOWN WITH A FAILURE,
52 '' WHICH REQUIRES THE USE OF A CONTACT TEAM.
53 ''SDNSYS _____ STANDARD DEVIATION OF CUSTOMER UNITS IN THE SERVICE SYS.
54 ''ENQ _____ EXPECTED NUMBER OF CUSTOMER UNITS WAITING FOR A SERVICE
55 '' CHANNEL.
56 ''ESYS.WAIT _____ EXPECTED TIME THAT A FAILED CUSTOMER UNIT MUST WAIT UNTIL
57 '' SERVICE BY A CONTACT TEAM IS COMPLETE.
58 ''EQ.WAIT _____ EXPECTED WAITING TIME IN THE SERVICE QUEUE.
59 ''EQ.WAIT.GQ _____ CONDITIONAL AVERAGE WAITING TIME IN THE SERVICE QUEUE,
60 '' GIVEN THAT THE CUSTOMER MUST QUEUE.
61 ''SDQ.WAIT.GQ _____ CONDITIONAL STANDARD DEVIATION OF THE WAITING TIME IN THE
62 '' SERVICE QUEUE, GIVEN THAT THE CUSTOMER MUST QUEUE.
63 ''E.BUSY.SERVERS _____ AVERAGE NUMBER OF CONTACT TEAMS WHICH ARE BUSY, I.E.,
64 '' THAT ARE DISPATCHED TO A CUSTOMER OR PERFORMING SERVICE.
65 ''SD.BUSY.SERVERS _____ STANDARD DEVIATION OF THE NUMBER OF BUSY SERVERS (TEAMS).
66 ''
67 DEFINE FLAG.DIST,FLAG.FIFO,I,IPRINT,J,K,L,M,MLIM,MPOP,N,NSERVE AS INTEGER
68 VARIABLES
69 DEFINE ANSWER AS A TEXT VARIABLE
70 DEFINE P.NSTATE AS A REAL, 1-DIMENSIONAL ARRAY
71 PRINT 1 LINE THUS
DO YOU WANT FIFO SERVICE? (Y OR N). ALTERNATIVE IS SHORTEST DISTANCE.
73 READ ANSWER
74 IF SUBSTR.F(ANSWER,1,1) = "Y"
75 LET FLAG.FIFO=1
76 OTHERWISE
77 LET FLAG.FIFO=0
78 ALWAYS
79 PRINT 1 LINE THUS
INPUT THE FRONTAL DIMENSION (KM) OF THE AREA OF THE CONTACT TEAM(S).
81 READ ARANGE
82 PRINT 1 LINE THUS
INPUT THE DEPTH (CROSS-FRONTAL) DIMENSION (KM) OF THE AREA OF THE TEAM(S).
84 READ BRANGE
85 PRINT 1 LINE THUS
INPUT THE AVG SPEED (KM/HR) OF THE CONTACT TEAM BETWEEN CUSTOMERS.
87 READ SPEED
88 PRINT 1 LINE THUS
INPUT THE PLUS OR MINUS MAX DEVIATION IN SPEED FROM AVERAGE (KM/HR).
90 READ DELTAS
91 LET DELTAS=MIN.F(SPEED/2.0,DELTAS)
92 LET VS=DELTAS**2/3.0
93 PRINT 1 LINE THUS
INPUT THE NUMBER OF CUSTOMERS (FIRE UNITS) SERVED BY THE CONTACT TEAMS.
95 READ MPOP
96 RESERVE P.NSTATE(*) AS MPOP
97 PRINT 1 LINE THUS
INPUT THE NUMBER OF CONTACT TEAMS (SERVICE CHANNELS) CONSIDERED, LE 3.
99 READ NSERVE
100 LET NSERVE=MIN.F(3,NSERVE)

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101     PRINT 1 LINE THUS
      INPUT THE MEAN TIME (HRS) BETWEEN PERTINENT FAILURES OF A FIRE UNIT.
103     READ MTBF
104     PRINT 1 LINE THUS
      INPUT THE AVERAGE DIAGNOSTIC AND REPAIR TIME (HRS) FOR THESE FAILURES.
106     READ MTTR
107     PRINT 1 LINE THUS
      DO YOU WANT TO PRINT THE PROB DIST OF INTERCUSTOMER DISTANCE? (Y OR N).
109     READ ANSWER
110     IF SUBSTR.F(ANSWER,1,1) = "Y"
111         LET FLAG.DIST=1
112     OTHERWISE
113         LET FLAG.DIST=2
114     ALWAYS
115     PRINT 1 LINE THUS
      DO YOU WANT TO PRINT FROM THE QUEUEING SUBROUTINE? (Y OR N).
117     READ ANSWER
118     IF SUBSTR.F(ANSWER,1,1) = "Y"
119         LET IPRINT=1
120     OTHERWISE
121         LET IPRINT=0
122     ALWAYS
123     SKIP 3 LINES
124     PRINT 2 LINES THUS
      INPUT DATA FOR PERFORMANCE OF CONTACT TEAMS

127  ''
128  ''CALCULATE AVG AND STD DEV OF INTERCUSTOMER DISTANCE AND TRAVEL TIME.
129  ''
130     IF FLAG.FIFO=1
131         PRINT 1 LINE THUS
      SERVICE DISCIPLINE USED IS "FIFO".
133         LET ER=ARANGE/3.0+BRANGE/3.0
134         LET VR=(ARANGE**2+BRANGE**2)/18.0
135     OTHERWISE
136         PRINT 1 LINE THUS
      SERVICE DISCIPLINE USED IS "SHORTEST-DISTANCE".
138  ''     LET ER=(ARANGE+BRANGE)/5.0
139  ''     LET VR=(ARANGE**2+BRANGE**2)*(1.0/15.0-1.0/25.0)
140         CALL MINRV (0,2,ARANGE+BRANGE,ARANGE,BRANGE) YIELDING ER,VR
141     ALWAYS
142     LET ETM.TRVL=ER/SPEED+ER*VS/SPEED**3 '' APPROXIMATELY
143     LET VTM.TRVL=VR/SPEED**2+ER**2*VS/SPEED**4+ER**2*VS**2/SPEED**6
144     +VR*VS/SPEED**4 '' APPROXIMATELY FROM MACLAUREN EXPANSION ABOUT AVG'S
145     PRINT 7 LINES WITH ARANGE,BRANGE,SPEED,DELTAS,MPOP,NSERVE,MTBF,MTTR
146     THUS
      FRONTAL DIMENSION (KM) OF SERVICE AREA     ___ **.*
      DEPTH DIMENSION (KM) OF SERVICE AREA        ___ **.*
      RANGE OF SPEED (KM/HR) OF CONTACT TEAM     ___ **.* +/ - **.*
      NUMBER OF CUST UNITS IN SERVICE AREA        ___ **
      NUMBER OF CONTACT TEAMS IN SERVICE AREA     ___ **
      AVG TIME (HR) BET SERVICE REQUESTS/CUST    ___ ****.*
      AVERAGE DIAGNOSTIC AND REPAIR TIME (HR)    ___ **.*
154     SKIP 1 LINE

```

```

155     PRINT 7 LINES WITH ER,SQRT.F(VR),SPEED,SQRT.F(VS),ETM.TRVL,
156     SQRT.F(VTM.TRVL),ETM.TRVL/(ETM.TRVL+MTTR)
157     THUS

```

```

AVERAGE DISTANCE (KM) BETWEEN CUSTOMERS ---- **.*
STD DEV DISTANCE (KM) BETWEEN CUSTOMERS ---- **.*
AVERAGE SPEED (KM/HR) BETWEEN CUSTOMERS ---- **.*
STD DEV SPEED (KM/HR) BETWEEN CUSTOMERS ---- **.*
AVG TRAVEL TIME (HR) BETWEEN CUSTOMERS ---- **.*
S.D. TRAVEL TIME (HR) BETWEEN CUSTOMERS ---- **.*
RATIO: TRAVEL TIME/(TRAVEL+REPAIR TIMES) -- **.*

```

```

165     SKIP 2 LINES
166     IF FLAG.DIST NE 1
167         GO TO L0
168     OTHERWISE
169     PRINT 0 LINES THUS

```

PROBABILITY DISTRIBUTION OF INTERCUSTOMER DISTANCE

```

DIST   C.D.F OF MIN DIST FOR N CUST.  N:
(KM)   FIFO      2          3          4

```

```

176     LET DR=(ARANGE+BRANGE)/20.0
177     FOR I=1 TO 20 DO
178         LET R=DR*I
179         LET CDF1=CDT.FIFO(ARANGE,BRANGE,R)
180         LET CDF2=1.0-(1.0-CDF1)**2
181         LET CDF3=1.0-(1.0-CDF1)**3
182         LET CDF4=1.0-(1.0-CDF1)**4
183         PRINT 1 LINE WITH R,CDF1,CDF2,CDF3,CDF4
184     THUS

```

```

**.* **.* **.* **.* **.*

```

```

186     IF CDF1 GE 0.9999
187         GO TO K0
188     OTHERWISE
189     LOOP 'OVER I
190 'K0'PRINT 2 LINES THUS

```

```

193 'LO'LET LAMBDA=1.0/MTBF
194     LET MU1=1.0/(ETM.TRVL+MTTR)
195     SKIP 2 LINES
196     PRINT 8 LINES WITH LAMBDA,MPOP*LAMBDA,MU1,NSERVE*MU1,MPOP,NSERVE
197     THUS

```

INPUT PARAMETERS FOR A STEADY-STATE, FINITE QUEUEING SYSTEM

```

ARRIVAL RATE (LAMBDA) _____ **.* PER HR PER OPERATING CUSTOMER UNIT
MAX ARRIVAL RATE _____ **.* UNITS PER HR
SERVICE RATE (MU1) _____ **.* PER HR PER CUSTOMER
MAX SERVICE RATE _____ **.* UNITS PER HR
CUSTOMER POPULATION _____ ** CUSTOMERS
SERVICE CHANNELS _____ ** SERVERS

```

```

206 ''
207 ''GET A LIMIT ON THE SYSTEM-STATE INDEX (MLIM) WHICH SATISFIES ACCURACY
208 ''REQUIREMENTS.
209 ''
210 LET PLIM=0.0001
211 CALL LIMSTATE (LAMBDA,MU1,MPOP,NSERVE,PLIM,0) YIELDING MLIM,P.NULL,
212 P.NSTATE(*)
213 PRINT 1 LINE WITH MLIM,PLIM
214 THUS
SYSTEM INDEX = ** FOR ____ *.***** ACCURACY IN CUM PROB
216 SKIP 4 LINES
217 CALL FINITE.ME2.Q(LAMBDA,MU1,MPOP,MLIM,NSERVE,IPRINT) YIELDING P.NULL,
218 P.SYS.FULL,P.CUST.WAIT,ENSY,SDNSYS,ENQ,ESYS.WAIT,EQ.WAIT,EQ.WAIT.GQ,
219 SDQ.WAIT.GQ,E.BUSY.SERVERS,SD.BUSY.SERVERS,P.NSTATE(*)
220 SKIP 2 LINES
221 IF IPRINT NE 1
222 PRINT 7 LINES WITH P.NULL,P.SYS.FULL,ENSY,SDNSYS,ESYS.WAIT,
223 E.BUSY.SERVERS,SD.BUSY.SERVERS
224 THUS
PROBABILITY THAT SERVICE SYSTEM IS EMPTY (NO UNITS DOWN) ____ *.*****
PROBABILITY THAT ALL SERVICE CHANNELS ARE BUSY _____ *.*****
AVERAGE NUMBER OF CUSTOMERS IN THE SERVICE SYSTEM _____ **.***
STD DEV NUMBER OF CUSTOMERS IN THE SERVICE SYSTEM _____ **.***
UNCOND AVG WAITING TIME (HR) IN THE SERVICE SYSTEM _____ **.***
AVERAGE NUMBER OF BUSY SERVICE CHANNELS _____ **.***
STD DEV NUMBER OF BUSY SERVICE CHANNELS _____ **.***
232 ALWAYS
233 SKIP 2 LINES
234 STOP
235 END ''CONTACT.Q

```

```

1 FUNCTION CDT.FIFO (A,B,T)
2 ''
3 ''FUNCTION EVALUATES THE CUM DISTRIBUTION FUNCTION (C.D.F) FOR THE
4 ''INTERCUSTOMER DISTANCE OF A DISPATCHED MOBILE SERVICE SYSTEM, UNDER A
5 ''FIFO SERVICE DISCIPLINE. THE CUSTOMER AREA (OR TERRITORY) IS A RECTANGLE
6 ''WITH DIMENSIONS A BY B. CUSTOMERS ARE ASSUMED TO BE UNIFORMLY DISTRI-
7 ''BUTED IN THIS AREA. INDEPENDENCE OF THE X- AND Y- COORDINATE POSITIONS
8 ''OF THE CUSTOMERS IS ASSUMED. THE PATH THAT THE SERVICE SYSTEM IS
9 ''ASSUMED TO FOLLOW IS A SERIES OF ORTHOGONAL SEGMENTS PARALLEL TO THE
10 ''SIDES OF THE RECTANGULAR SERVICE AREA.
11 ''
12     IF T GT A+B
13         RETURN WITH 1.0
14     OTHERWISE
15         LET AB=A*B
16         LET ABS=AB**2
17         LET CDF=2.0*T**2/AB*(1.0-T*(A+B)/AB/3.0)+T**4/ABS/6.0
18         IF T GT B
19             ADD (T-B)**3/6.0/AB*(4.0/B-(T-B)/AB) TO CDF
20         ALWAYS
21         IF T GT A
22             ADD (T-A)**3/6.0/AB*(4.0/A-(T-A)/AB) TO CDF
23         ALWAYS
24         RETURN WITH CDF
25 END ''CDT.FIFO

```

```

1 ROUTINE FINITE.ME2.Q (LAMBDA,MU1,MPOP,MLIM,NSERVE,MODE) YIELDING P.NULL,
2   P.SYS.FULL, P.CUST.WAIT, ENSYS, SDNSYS, ENQ, ESYS.WAIT, EQ.WAIT,
3   EQ.WAIT.GQ, SDQ.WAIT.GQ, E.BUSY.SERVERS, SD.BUSY.SERVERS, P.NSTATE
4   ''
5   ''THIS ROUTINE CALCULATES THE STEADY-STATE, SYSTEM STATE-PROBABILITY
6   ''VECTOR FOR A SERVICE SYSTEM HAVING EXPONENTIAL INTERARRIVAL TIMES FOR
7   ''EACH MEMBER OF A FINITE CUSTOMER POPULATION, AND HAVING NSERVE SERVERS,
8   ''WITH SERVICE TIMES DISTRIBUTED AS ERLANG WITH SHAPE PARAMETER 2.
9   ''
10  ''INPUT:
11  ''LAMBDA          ARRIVAL RATE PER INDIVIDUAL IN THE POPULATION
12  ''MU1            SERVICE RATE OR RECIPROCAL OF MEAN SERVICE TIME
13  ''MPOP           CUSTOMER POPULATION SIZE
14  ''MLIM           MAX ALLOWED CUSTOMERS IN THE SERVICE SYSTEM
15  ''NSERVE         NUMBER OF SERVERS (N LE 3)
16  ''MODE           INTEGER FLAG FOR PRINTING FROM ROUTINE (FOR MODE=1)
17  ''
18  ''OUTPUT:
19  ''P.NSTATE       VECTOR OF PROBABILITY ELEMENTS IN WHICH THE N TH
20  ''              ELEMENT IS THE PROBABILITY THAT N INDIVIDUALS ARE
21  ''              IN THE SERVICE SYSTEM IN THE STEADY STATE.
22  ''P.NULL         PROBABILITY THAT THE SYSTEM IS EMPTY
23  ''P.SYS.FULL     PROBABILITY THAT ALL SERVICE CHANNELS ARE FULL
24  ''P.CUST.WAIT    PROBABILITY THAT AN ARRIVING CUSTOMER MUST QUEUE
25  ''ENSYS         EXPECTED NUMBER OF CUSTOMERS IN THE SYSTEM
26  ''SDNSYS        STD DEVIATION OF THE NUMBER IN THE SYSTEM
27  ''ENQ           EXPECTED NUMBER OF CUSTOMERS IN THE QUEUE
28  ''ESYS.WAIT     EXPECTED WAIT IN THE SERVICE SYSTEM
29  ''EQ.WAIT       EXPECTED WAIT IN THE SERVICE QUEUE
30  ''EQ.WAIT.GQ    EXPECTED QUEUE WAITING TIME, GIVEN THAT A
31  ''              CUSTOMER MUST QUEUE.
32  ''SDQ.WAIT.GQ   STANDARD DEVIATION OF QUEUE WAITING TIME, GIVEN
33  ''              THAT A CUSTOMER MUST QUEUE.
34  ''E.BUSY.SERVERS EXPECTED VALUE OF BUSY SERVERS
35  ''SD.BUSY.SERVERS STANDARD DEVIATION OF THE NUMBER OF BUSY SERVERS
36  ''OPTIONAL PRINTING OF THE QUEUE WAITING TIME DISTRIBUTION IS PROVIDED.
37  ''
38  ''ENDOGENOUS VARIABLES:
39  ''AM            KOLMOGOROV STATE-TRANSITION MATRIX
40  ''BV            VECTOR OF PROBABILITIES IN WHICH THE N TH
41  ''              ELEMENT IS THE PROBABILITY OF FINDING THE
42  ''              SYSTEM IN THE N TH STATE, WHERE
43  ''              N=2*(NO CUSTOMERS-1)+STAGE OF SERVICE, NO GT 0.
44  ''QV           VECTOR OF PROBABILITIES IN WHICH THE N TH
45  ''              ELEMENT IS THE PROBABILITY THAT AN ARRIVING CUSTOMER
46  ''              FINDS THE SYSTEM IN THE N TH STATE. THIS VECTOR
47  ''              IS USED TO CALCULATE THE DISTRIBUTION OF
48  ''              WAIT IN QUEUE.
49  ''IPVT(*)      INTEGER VECTOR USED IN FACTORING THE STATE-TRANSITION
50  ''              MATRIX.
51  ''DET(*)       TWO-ELEMENT VECTOR USED TO STORE THE DETERMINANT OF
52  ''              THE STATE-TRANSITION MATRIX.
53  ''
54  DEFINE I,INFO,J,MLIM,MPOP,TWOM,MAX,MAXM,MODE,N,NSERVE AS INTEGER VARIABLES

```

```

55 DEFINE IPVT AS AN INTEGER, 1-DIMENSIONAL ARRAY
56 DEFINE BV,DET,PV,QV,P.NSTATE,AND Q.NSTATE AS REAL,1-DIMENSIONAL ARRAYS
57 DEFINE AM AS A REAL, 2-DIMENSIONAL ARRAY
58     IF MLIM LE 0 OR MLIM GT MPOP
59         PRINT 1 LINE WITH MPOP,MLIM
60         THUS
        ERROR IN INPUT TO FINITE.ME2.Q.  MPOP = ***  MLIM = ***.
62     STOP
63     OTHERWISE
64     LET TWOM=2*MLIM
65     RESERVE DET(*) AS 2
66     LET MU=2.0*MU1 ''SERVICE RATE FOR EACH OF THE 2 STAGES
67     LET ML=REAL.F(MPOP)*LAMBDA ''MAX ARRIVAL RATE
68     RESERVE P.NSTATE(*) AS MPOP
69     RESERVE Q.NSTATE(*) AS MPOP
70     IF NSERVE LE 0
71         RETURN
72     OTHERWISE
73     IF NSERVE GT 1
74         GO TO L0
75     OTHERWISE ''NSERVE=1
76     RESERVE PV(*) AS TWOM
77     RESERVE QV(*) AS TWOM
78     RESERVE AM(*,*) AS TWOM BY TWOM
79     RESERVE IPVT(*) AS TWOM
80     FOR I=1 TO TWOM DO
81     LET QV(I)=0.0
82     FOR J=1 TO TWOM DO
83     LET AM(I,J)=0.0
84     LOOP ''OVER J
85     LOOP ''OVER I TO INITIALIZE
86     ''
87     ''FILL NON-ZERO ELEMENTS OF THE REDUCED STATE-TRANSITION MATRIX (AM)
88     ''AND THE CONSTANT VECTOR (BV).
89     ''
90     FOR J=1 TO TWOM DO
91     LET AM(1,J)=-ML
92     LOOP ''OVER COLUMNS
93     SUBTRACT MU+REAL.F(MPOP-1)*LAMBDA FROM AM(1,1)
94     ADD MU TO AM(1,4)
95     LET AM(2,1)=MU
96     LET AM(2,2)=-MU-ML+LAMBDA
97     FOR N=2 TO MLIM-1 DO
98     LET I=2*N-1
99     LET AM(I,I-2)=(MPOP-N+1)*LAMBDA
100    LET AM(I,I)=-MU-(MPOP-N)*LAMBDA
101    LET AM(I,I+3)=MU
102    LET AM(I+1,I-1)=AM(I,I-2)
103    LET AM(I+1,I)=MU
104    LET AM(I+1,I+1)=AM(I,I)
105    LOOP ''OVER N
106    ''FINALLY,
107    LET AM(TWOM-1,TWOM-3)=LAMBDA*(MPOP-MLIM+1)
108    LET AM(TWOM-1,TWOM-1)=-MU
109    LET AM(TWOM,TWOM-2)=LAMBDA*(MPOP-MLIM+1)
110    LET AM(TWOM,TWOM-1)=MU
111    LET AM(TWOM,TWOM)=-MU

```

```

113 ''
114 ''OBTAIN THE INVERSE OF AM(*,*) IN PLACE.
115 ''
116 ''
117 ''FACTOR AM(*,*) FIRST TO OBTAIN AN INVERSE.
118 ''
119     CALL SGEFA (AM(*,*),IPVT(*),INFO)
120     IF INFO NE 0
121         PRINT 1 LINE WITH INFO THUS
TROUBLE FACTORING AM.     INFO = *.
123         STOP
124     OTHERWISE
125         CALL SGEDI (AM(*,*),IPVT(*),DET(*),11)
126 ''
127 ''OBTAIN THE SOLUTION VECTOR OF THE MATRIX EQUATION.
128 ''
129     FOR I=1 TO TWOM, LET PV(I)=-ML*AM(I,1)
130 ''
131 ''SOLVE FOR THE EMPTY-SYSTEM PROBABILITY (P.NULL) AND CALCULATE
132 ''THE STATE-PROBABILITY VECTOR (P.NSTATE).
133 ''
134     LET ENSYS=0.0
135     LET VNSYS=0.0
136     LET ENQ=0.0
137     LET ESQ=0.0
138     LET SUM=0.0
139     FOR N=1 TO MLIM DO
140         LET I=2*N-1
141         LET P.NSTATE(N)=PV(I)+PV(I+1)
142         ADD P.NSTATE(N) TO SUM
143         ADD N*P.NSTATE(N) TO ENSYS
144         ADD N**2*P.NSTATE(N) TO VNSYS
145         ADD (N-1)*P.NSTATE(N) TO ENQ
146         ADD (N-1)**2*P.NSTATE(N) TO ESQ
147     LOOP ''OVER NON-ZERO SYSTEM STATES
148 ''
149 ''CHECK VALIDITY OF PROBABILITY SUM.
150 ''
151     IF SUM LT 0.0 OR SUM GT 1.0
152         PRINT 1 LINE WITH SUM THUS
ERROR IN ROUTINE FINITE.ME2.Q.  PARTIAL SUM OF STATE PROBS = **.***.
154         STOP
155     OTHERWISE
156         LET P.NULL=1.0-SUM
157         LET P.SYS.FULL=SUM ''FOR A SINGLE SERVER
158 ''
159 ''CALCULATE THE VARIANCE AND STD DEV OF THE NUMBER IN THE SYSTEM.
160 ''
161     LET VNSYS=VNSYS-ENSYS**2
162     LET SDNSYS=SQRT.F(VNSYS)
163     LET VNQ=ESQ-ENQ**2
164     LET SDNQ=SQRT.F(VNQ)
165 ''
166 ''CALCULATE THE MEAN AND STANDARD DEVIATION OF BUSY SERVERS.
167 ''
168     LET E.BUSY.SERVERS=1.0-P.NULL ''FOR A SINGLE SERVER
169     LET VAR.BUSY.SERVERS=1.0-P.NULL-E.BUSY.SERVERS**2
170     LET SD.BUSY.SERVERS=SQRT.F(VAR.BUSY.SERVERS)

```

```

171 ''
172 ''CALCULATE MEAN WAITING TIMES USING LITTLE'S FORMULA.
173 ''
174     LET ESYS.WAIT=ENSY/ LAMBDA/(REAL.F(MPOP)-ENSY)
175     LET EQ.WAIT=ESYS.WAIT-1.0/MU1
176     IF MPOP=1
177         RETURN
178     OTHERWISE ''CALCULATE AND PRINT WAITING-TIME DISTRIBUTIONS
179 ''
180 ''CALCULATE THE PROBABILITY THAT AN ARRIVAL FINDS THE SYSTEM
181 ''IN THE N TH STATE (QV(N)).
182 ''
183     LET NORM.CONST=MPOP*P.NULL
184     FOR N=1 TO MLIM-1 DO
185         ADD (MPOP-N)*P.NSTATE(N) TO NORM.CONST
186     LOOP ''TO CALCULATE THE NORMALIZATION CONSTANT
187     LET Q.NULL=MPOP*P.NULL/NORM.CONST
188     LET P.CUST.WAIT=1.0-Q.NULL
189     LET EQ.WAIT.GQ=0.0
190     LET VQ.WAIT.GQ=0.0
191     FOR N=1 TO MLIM-1 DO
192         LET I=2*N-1
193         LET QV(I)=(MPOP-N)*PV(I)/NORM.CONST
194         LET QV(I+1)=(MPOP-N)*PV(I+1)/NORM.CONST
195         LET Q.NSTATE(N)=QV(I)+QV(I+1)
196         ADD (I+1)*QV(I)+I*QV(I+1) TO EQ.WAIT.GQ
197         ADD (I+1)*(I+2)*QV(I)+I*(I+1)*QV(I+1) TO VQ.WAIT.GQ
198     LOOP ''OVER THE NUMBER OF ARRIVING CUSTOMERS
199     LET EQ.WAIT.CHECK=EQ.WAIT.GQ./MU
200     LET ESYS.WAIT.CHECK=EQ.WAIT.CHECK +1.0/MU1
201     LET EQ.WAIT.GQ=EQ.WAIT.GQ/MU/(1.0-Q.NULL)
202     LET VQ.WAIT.GQ=VQ.WAIT.GQ/MU/MU/(1.0-Q.NULL)-EQ.WAIT.GQ**2
203     LET SDQ.WAIT.GQ=SQRT.F(VQ.WAIT.GQ)
204     IF MODE NE 1
205         RETURN
206     OTHERWISE
207 'L1'SKIP 4 LINES
208     PRINT 21 LINES WITH NSERVE,P.SYS.FULL,ENSY,SDNSYS,ENQ,SDNQ,E.BUSY.SERVERS,
209     SD.BUSY.SERVERS,ESYS.WAIT,EQ.WAIT,EQ.WAIT.GQ,P.NULL,P.NULL,Q.NULL,Q.NULL
210     THUS

```

STATE OF THE SYSTEM IN STEADY STATE WITH GAMMA(2) SERVICE FOR ** SERVERS

```

PROBABILITY: SYSTEM IS FULL          *.****
EXPECTED NUMBER IN THE SYSTEM        **.****
STD DEV NUMBER IN THE SYSTEM         **.****
EXPECTED NUMBER IN THE QUEUE         **.****
STD DEV NUMBER IN THE QUEUE          **.****
EXPECTED NUMBER BUSY SERVERS         **.****
STD DEV NUMBER BUSY SERVERS          **.****
MEAN WAITING TIME (HRS) IN SYSTEM    ***.***
MEAN WAITING TIME (HRS) IN QUEUE     ***.***
COND'AL MEAN TIME (HRS) IN QUEUE     ***.***

```

PROBABILITY DISTRIBUTIONS FOR NUMBER OF CUSTOMERS IN THE SYSTEM

NO IN SYSTEM	UNCONDITIONAL		GIVEN ARRIVAL	
	PROB	CUML PROB	PROB	CUML PROB
	DENS		DENS	

```

0          *.****  *.****  *.****  *.****
232      LET CUM.DIST=P.NULL
233      LET CUM.DIST.COND=Q.NULL
234  FOR N=1 TO MLIM DO
235      ADD P.NSTATE(N) TO CUM.DIST
236      ADD Q.NSTATE(N) TO CUM.DIST.COND
237      PRINT 1 LINE WITH N, P.NSTATE(N), CUM.DIST, Q.NSTATE(N), CUM.DIST.COND
238      THUS
**        *.****  *.****  *.****  *.****
240      IF CUM.DIST GE 0.9999
241          GO TO K1
242      OTHERWISE
243  LOOP 'OVER THE SYSTEM STATES
244  'K1'PRINT 2 LINES THUS

```

```

247      IF NSERVE GT 1
248          RETURN
249      OTHERWISE
250      LET DELT=MAX.F(1.0,TRUNC.F(EQ.WAIT.GQ/10.0))
251      LET MAX=60
252      SKIP 2 LINES
253  ''
254  ''PRINT HEADINGS FOR THE QUEUE WAITING TIME DISTRIBUTION.
255  ''
256      PRINT 8 LINES WITH MPOP AND Q.NULL THUS
      CUM AND CONDITIONAL CUM PROB DISTRIBUTIONS OF WAITING TIME IN QUEUE
      EXPONENTIAL INTERARRIVALS FOR EACH OF ** CUSTOMERS.ERLANG(2) SERVICE.

```

TIME	CUML	COND
(HRS)	PROB	PROB

```

0          *.****  0
265  ''
266  ''START TIME LOOP.
267  ''
268      FOR I=1 TO MAX 'TIME STEPS' DO
269          LET TM=I*DELT
270          LET ARG=TM*MU
271          LET CUM.DIST=1.0-EXP.F(-ARG)*(QV(1)*(1.0+ARG)+QV(2))
272          LET SUMN=0.0
273          FOR N=2 TO MLIM-1 DO
274              LET SUMEJ=1.0
275              LET ARGJ=1.0
276              LET JFACTORIAL=1.0
277              FOR J=1 TO 2*(N-1) DO
278                  LET ARGJ=ARGJ*ARG
279                  LET JFACTORIAL=JFACTORIAL*J
280                  ADD ARGJ/JFACTORIAL TO SUMEJ
281              LOOP 'OVER J
282              LET ARGJ=ARGJ*ARG
283              LET JFACTORIAL=JFACTORIAL*(2*N-1)
284              LET SUMOJ=SUMEJ+ARGJ/JFACTORIAL
285              ADD QV(2*N-1)*SUMOJ+QV(2*N)*SUMEJ TO SUMN
286          LOOP 'OVER N
287      LET CUM.DIST=CUM.DIST-EXP.F(-ARG)*SUMN

```

```

288         LET CON.DIST=(CUM.DIST-Q.NULL)/(1.0-Q.NULL)
289         PRINT 1 LINE WITH TM, CUM.DIST, AND CON.DIST THUS
***.* *.**** *.****
291         IF CUM.DIST GE 0.9999
292             GO TO K2
293         OTHERWISE
294         LOOP ''OVER I TIME STEPS
295     'K2'PRINT 10 LINES WITH EQ.WAIT.GQ, SDQ.WAIT.GQ, EQ.WAIT.CHECK,
296         ESYS.WAIT.CHECK, Q.NULL, E.BUSY.SERVERS, SD.BUSY.SERVERS
297         THUS

```

```

MEAN WAITING TIME IN QUEUE, GIVEN A WAIT     ***.***
STD DEV WAITING TIME IN QUEUE, GIVEN A WAIT ***.***
MEAN WAITING TIME IN QUEUE (UNCONDITIONAL) ***.***
MEAN WAITING TIME IN SYS (UNCONDITIONAL) ***.***
PROBABILITY THAT AN ARRIVAL FINDS SYS EMPTY *.****
MEAN NUMBER OF BUSY SERVERS                 **.****
STD DEV NUMBER OF BUSY SERVERS _____ **.****

```

```

308         RETURN
309     'LO'IF NSERVE GT 2
310         GO TO L3
311     OTHERWISE
312     ''
313     ''RESERVE ARRAYS FOR THE CASE: NSERVE=2
314     ''
315         LET MAXM=3*MLIM-1
316     '' RESERVE BV(*) AS MAXM
317         RESERVE PV(*) AS MAXM
318         RESERVE QV(*) AS MAXM
319         RESERVE AM(*,*) AS MAXM BY MAXM
320         RESERVE IPVT(*) AS MAXM
321     ''
322     ''FILL THE REDUCED STATE-TRANSITION MATRIX (AM).
323     ''
324     FOR I=1 TO MAXM DO
325         FOR J=1 TO MAXM DO
326             LET AM(I,J)=0.0
327         LOOP ''OVER J
328     LOOP ''OVER I
329     FOR J=1 TO MAXM DO
330         LET AM(1,J)=ML
331     LOOP ''OVER J
332         LET AM(1,2)=AM(1,2)+MU
333         LET AM(2,1)=MU
334         LET AM(2,2)=-(MPOP-1)*LAMBDA-MU
335         LET AM(2,5)=2.0*MU
336         LET AM(3,1)=(MPOP-1)*LAMBDA
337         LET AM(3,3)=-(MPOP-2)*LAMBDA-2.0*MU
338         LET AM(3,7)=MU
339         LET AM(4,2)=(MPOP-1)*LAMBDA
340         LET AM(4,3)=2.0*MU
341         LET AM(4,4)=AM(3,3)
342         LET AM(4,8)=2.0*MU
343         LET AM(5,4)=MU
344         LET AM(5,5)=AM(4,4)

```

```

345 FOR N=3 TO MLIM-1 DO
346   LET I=3*(N-1)
347   LET AM(I,3*N-6)=(MPOP-N+1)*LAMBDA
348   LET AM(I,I)=-((MPOP-N)*LAMBDA+2.0*MU)
349   LET AM(I,3*N+1)=MU
350   ADD 1 TO I
351   LET AM(I,3*N-5)=(MPOP-N+1)*LAMBDA
352   LET AM(I,3*N-3)=2.0*MU
353   LET AM(I,I)=AM(I-1,I-1)
354   LET AM(I,3*N+2)=2.0*MU
355   ADD 1 TO I
356   LET AM(I,3*N-4)=(MPOP-N+1)*LAMBDA
357   LET AM(I,3*N-2)=MU
358   LET AM(I,I)=-((MPOP-N)*LAMBDA+2.0*MU)
359 LOOP ''OVER NUMBER OF CUSTOMERS IN THE SYSTEM
360   LET I=3*(MLIM-1)
361   LET AM(I,3*MLIM-6)=LAMBDA*(MPOP-MLIM+1)
362   LET AM(I,I)=-2.0*MU
363   ADD 1 TO I
364   LET AM(I,3*MLIM-5)=LAMBDA*(MPOP-MLIM+1)
365   LET AM(I,3*MLIM-3)=2.0*MU
366   LET AM(I,I)=-2.0*MU
367   ADD 1 TO I
368   LET AM(I,3*MLIM-4)=LAMBDA*(MPOP-MLIM+1)
369   LET AM(I,3*MLIM-2)=MU
370   LET AM(I,I)=-2.0*MU
371 ''
372 ''OBTAIN THE INVERSE OF AM(*,*) IN PLACE.
373 ''
374 ''
375 ''FACTOR AM(*,*).
376 ''
377   CALL SGEFA (AM(*,*),IPVT(*),INFO)
378   IF INFO NE 0
379     PRINT 1 LINE WITH INFO THUS
TROUBLE FACTORING AM.  INFO = *.
381     STOP
382   OTHERWISE
383     CALL SGEDI (AM(*,*),IPVT(*),DET(*),11)
384 ''
385 ''OBTAIN THE SOLUTION VECTOR OF THE MATRIX EQUATION.
386 ''
387   FOR I=1 TO MAXM, LET PV(I)=ML*AM(I,1)
388 ''
389 ''SOLVE FOR THE EMPTY-SYSTEM PROBABILITY (P.NULL) AND CALCULATE THE
390 ''CUSTOMER STATE PROBABILITY VECTOR (P.NSTATE).
391 ''
392   LET ENSYS=PV(1)+PV(2)
393   LET P.NSTATE(1)=ENSYS
394   LET VNSYS=ENSYS
395   LET ENQ=0.0
396   LET ESQ=0.0
397   LET SUM=ENSYS
398 FOR N=2 TO MLIM DO
399   LET I=3*(N-1)

```

```

400     LET P.NSTATE(N)=PV(I)+PV(I+1)+PV(I+2)
401     ADD P.NSTATE(N) TO SUM
402     ADD N*P.NSTATE(N) TO ENSYS
403     ADD (N-2)*P.NSTATE(N) TO ENQ
404     ADD (N-2)**2*P.NSTATE(N) TO ESQ
405     ADD N**2*P.NSTATE(N) TO VNSYS
406     LOOP ''OVER THE OTHER NON-ZERO SYSTEM STATES
407     ''
408     ''CHECK VALIDITY OF THE PROBABILITY SUM.
409     ''
410     IF SUM LT 0.0 OR SUM GT 1.0
411     PRINT 1 LINE WITH SUM THUS
ERROR IN ROUTINE FINITE.ME2.Q. PARTIAL SUM OF STATE PROBS = **.****.
413     STOP
414     OTHERWISE
415     LET P.NULL=1.0-SUM
416     LET P.SYS.FULL=1.0-P.NULL-P.NSTATE(1) ''FOR 2 SERVERS
417     ''
418     ''CALCULATE THE VARIANCE AND STANDARD DEV OF THE NUMBER IN THE SYSTEM.
419     ''
420     LET VNSYS=VNSYS-ENSYS**2
421     LET SDNSYS=SQRT.F(VNSYS)
422     LET VNQ=ESQ-ENQ**2
423     LET SDNQ=SQRT.F(VNQ)
424     ''
425     ''CALCULATE THE MEAN AND STANDARD DEVIATION OF BUSY SERVERS.
426     ''
427     LET E.BUSY.SERVERS=P.NSTATE(1)+2.0*(1.0-P.NULL-P.NSTATE(1))
428     LET VAR.BUSY.SERVERS=P.NSTATE(1)+4.0*(1.0-P.NULL-P.NSTATE(1))
429     -E.BUSY.SERVERS**2
430     LET SD.BUSY.SERVERS=SQRT.F(VAR.BUSY.SERVERS)
431     ''
432     ''CALCULATE MEAN WAITING TIME USING LITTLE'S FORMULA.
433     ''
434     LET ESYS.WAIT=ENSYS/LAMBDA/(REAL.F(MPOP)-ENSYS)
435     LET EQ.WAIT=ENQ/LAMBDA/(REAL.F(MPOP)-ENSYS)
436     IF MPOP LE 2
437     RETURN
438     OTHERWISE ''CALCULATE AND PRINT CONDITIONAL PROB DISTRIBUTIONS
439     ''
440     ''CALCULATE THE PROBABILITY THAT AN ARRIVAL FINDS THE SYSTEM IN STATE N.
441     ''
442     LET NORM.CONST=MPOP*P.NULL
443     FOR N=1 TO MLIM-1 DO
444     ADD (MPOP-N)*P.NSTATE(N) TO NORM.CONST
445     LOOP ''TO CALCULATE THE NORMALIZATION CONSTANT
446     LET Q.NULL=MPOP*P.NULL/NORM.CONST
447     LET QV(1)=(MPOP-1)*PV(1)/NORM.CONST
448     LET QV(2)=(MPOP-1)*PV(2)/NORM.CONST
449     LET Q.NSTATE(1)=QV(1)+QV(2)
450     FOR N=2 TO MLIM-1 DO
451     LET I=3*(N-1)
452     LET QV(I)=(MPOP-N)*PV(I)/NORM.CONST
453     LET QV(I+1)=(MPOP-N)*PV(I+1)/NORM.CONST
454     LET QV(I+2)=(MPOP-N)*PV(I+2)/NORM.CONST
455     LET Q.NSTATE(N)=QV(I)+QV(I+1)+QV(I+2)
456     LOOP ''OVER NUMBER OF ARRIVING CUSTOMERS

```

```

457     LET P.CUST.WAIT=1.0-Q.NULL-Q.NSTATE(1)
458 'L2' LET EQ.WAIT.GQ=EQ.WAIT/P.CUST.WAIT
459     LET SDQ.WAIT.GQ=EQ.WAIT.GQ ''APPROXIMATELY
460     IF MODE NE 1 RETURN
461     OTHERWISE
462     GO TO L1
463 'L3' IF MPOP LE 3
464     PRINT 1 LINE WITH MPOP AND NSERVE THUS
ERROR IN FINITE.ME2.Q. MPOP = **. NSERVE = **.
465     STOP
466     OTHERWISE
467 ''
468 ''
469 ''RESERVE ARRAYS FOR THE CASE: NSERVE = 3.
470 ''
471     LET MAXM=4*MLIM-3
472     RESERVE PV(*) AS MAXM
473     RESERVE QV(*) AS MAXM
474     RESERVE AM(*,*) AS MAXM BY MAXM
475     RESERVE IPVT(*) AS MAXM
476 ''
477 ''
478 ''FILL THE REDUCED STATE TRANSITION MATRIX (AM) .
479 ''
480 FOR I=1 TO MAXM DO
481     FOR J=1 TO MAXM DO
482         LET AM(I,J)=0.0
483     LOOP ''OVER J
484 LOOP ''OVER I
485 FOR J=1 TO MAXM DO
486     LET AM(1,J)=ML
487 LOOP ''OVER J
488     LET AM(1,2)=AM(1,2)+MU
489     LET AM(2,1)=MU
490     LET AM(2,2)=-((MPOP-1)*LAMBDA+MU)
491     LET AM(2,5)=2.0*MU
492     LET AM(3,1)=(MPOP-1)*LAMBDA
493     LET AM(3,3)=-((MPOP-2)*LAMBDA+2.0*MU)
494     LET AM(3,7)=MU
495     LET AM(4,2)=(MPOP-1)*LAMBDA
496     LET AM(4,3)=2.0*MU
497     LET AM(4,4)=AM(3,3)
498     LET AM(4,8)=2.0*MU
499     LET AM(5,4)=MU
500     LET AM(5,5)=AM(4,4)
501     LET AM(5,9)=3.0*MU
502     LET AM(6,5)=(MPOP-2)*LAMBDA
503     LET AM(6,6)=-((MPOP-3)*LAMBDA+3.0*MU)
504     LET AM(6,11)=MU
505     LET AM(7,4)=AM(6,5)
506     LET AM(7,6)=3.0*MU
507     LET AM(7,7)=AM(6,6)
508     LET AM(7,12)=2.0*MU
509     LET AM(8,5)=AM(7,4)
510     LET AM(8,7)=2.0*MU
511     LET AM(8,8)=AM(7,7)
512     LET AM(8,13)=3.0*MU
513     LET AM(9,8)=MU
514     LET AM(9,9)=AM(8,8)

```

```

516 FOR N=4 TO MLIM-1 DO
517   LET I=4*N-6 ''AS THE ROW INDEX
518   LET AM(I,4*N-10)=(MPOP-N+1)*LAMBDA
519   LET AM(I,I)=-((MPOP-N)*LAMBDA+3.0*MU)
520   LET AM(I,4*N-1)=MU
521   ADD 1 TO I ''I=4*N-5
522   LET AM(I,4*N-9)=(MPOP-N+1)*LAMBDA
523   LET AM(I,I-1)=3.0*MU
524   LET AM(I,I)=AM(I-1,I-1)
525   LET AM(I,4*N)=2.0*MU
526   ADD 1 TO I ''I=4*N-4
527   LET AM(I,4*N-8)=(MPOP-N+1)*LAMBDA
528   LET AM(I,I-1)=2.0*MU
529   LET AM(I,I)=AM(I-1,I-1)
530   LET AM(I,4*N+1)=3.0*MU
531   ADD 1 TO I ''I=4*N-3
532   LET AM(I,4*N-7)=(MPOP-N+1)*LAMBDA
533   LET AM(I,I-1)=MU
534   LET AM(I,I)=AM(I-1,I-1)
535 LOOP ''OVER THE NUMBER OF CUSTOMERS IN THE SYSTEM
536 ''
537 ''EQUATIONS FOR THE LAST FOUR STATES.
538 ''
539   LET I=4*MLIM-6
540   LET AM(I,4*MLIM-10)=LAMBDA*(MPOP-MLIM+1)
541   LET AM(I,I)=-3.0*MU
542   ADD 1 TO I ''I=4*MLIM-5
543   LET AM(I,4*MLIM-9)=LAMBDA*(MPOP-MLIM+1)
544   LET AM(I,I-1)=3.0*MU
545   LET AM(I,I)=AM(I-1,I-1)
546   ADD 1 TO I ''I=4*MLIM-4
547   LET AM(I,4*MLIM-8)=LAMBDA*(MPOP-MLIM+1)
548   LET AM(I,I-1)=2.0*MU
549   LET AM(I,I)=AM(I-1,I-1)
550   ADD 1 TO I ''I=4*MLIM-3
551   LET AM(I,4*MLIM-7)=LAMBDA*(MPOP-MLIM+1)
552   LET AM(I,I-1)=MU
553   LET AM(I,I)=AM(I-1,I-1)
554 ''
555 ''OBTAIN THE INVERSE OF AM(*,*) IN PLACE.
556 ''
557 ''
558 ''FACTOR AM(*,*).
559 ''
560   CALL SGEFA (AM(*,*),IPVT(*),INFO)
561   IF INFO NE 0
562     PRINT 1 LINE WITH INFO THUS
TROUBLE FACTORING AM. INFO = *.
564   STOP
565   OTHERWISE
566     CALL SGEDI (AM(*,*),IPVT(*),DET(*),11)
567 ''
568 ''OBTAIN THE SOLUTION VECTOR OF THE MATRIX EQUATION.
569 ''
570   FOR I=1 TO MAXM, LET PV(I)=ML*AM(I,1)

```

```

571 ''
572 ''SOLVE FOR THE EMPTY SYSTEM STATE PROBABILITY (P.NULL) AND CALCULATE
573 ''THE CUSTOMER STATE PROBABILITY VECTOR (P.NSTATE(*)).
574 ''
575     LET P.NSTATE(1)=PV(1)+PV(2) ''FOR 1 CUSTOMER
576     LET P.NSTATE(2)=PV(3)+PV(4)+PV(5)
577     LET ENSYS=P.NSTATE(1)+2.0*P.NSTATE(2)
578     LET VNSYS=P.NSTATE(1)+4.0*P.NSTATE(2)
579     LET ENQ=0.0
580     LET ESQ=0.0
581     LET SUM=P.NSTATE(1)+P.NSTATE(2)
582 FOR N=3 TO MLIM DO
583     LET I=4*N-6
584     LET P.NSTATE(N)=PV(I)+PV(I+1)+PV(I+2)+PV(I+3)
585     ADD P.NSTATE(N) TO SUM
586     ADD N*P.NSTATE(N) TO ENSYS
587     ADD (N-3)*P.NSTATE(N) TO ENQ
588     ADD (N-3)**2*P.NSTATE(N) TO ESQ
589     ADD N**2*P.NSTATE(N) TO VNSYS
590 LOOP ''OVER THE OTHER NON-ZERO STATES
591 ''
592 ''CHECK VALIDITY OF THE PROBABILITY SUM.
593 ''
594     IF SUM LT 0.0 OR SUM GT 1.0
595         PRINT 1 LINE WITH SUM THUS
596     ERROR IN ROUTINE FINITE.ME2.Q. PARTIAL SUM OF STATE PROBS = **.*****.
597     STOP
598     OTHERWISE
599     LET P.NULL=1.0-SUM
600     LET P.SYS.FULL=1.0-P.NULL-P.NSTATE(1)-P.NSTATE(2) ''FOR 3 SERVERS
601 ''
602 ''CALCULATE THE VARIANCE AND STANDARD DEVIATION OF NUMBER IN THE SYSTEM.
603 ''
604     LET VNSYS=VNSYS-ENSYS**2
605     LET SDNSYS=SQRT.F(VNSYS)
606     LET VNQ=ESQ-ENQ**2
607     LET SDNQ=SQRT.F(VNQ)
608 ''
609 ''CALCULATE THE MEAN AND STANDARD DEVIATION OF BUSY SERVERS.
610 ''
611     LET E.BUSY.SERVERS=P.NSTATE(1)+2.0*P.NSTATE(2)+3.0*(1.0-P.NULL
612     -P.NSTATE(1)-P.NSTATE(2))
613     LET VAR.BUSY.SERVERS=P.NSTATE(1)+4.0*P.NSTATE(2)+9.0*(1.0-P.NULL
614     -P.NSTATE(1)-P.NSTATE(2))-E.BUSY.SERVERS**2
615     LET SD.BUSY.SERVERS=SQRT.F(VAR.BUSY.SERVERS)
616 ''
617 ''CALCULATE MEAN WAITING TIME USING LITTLE'S FORMULA.
618 ''
619     LET ESYS.WAIT=ENSYS/LAMBDA/(REAL.F(MPOP)-ENSYS)
620     LET EQ.WAIT=ENQ/LAMBDA/(REAL.F(MPOP)-ENSYS)
621     IF MPOP=3
622         RETURN
623     OTHERWISE ''CALCULATE AND PRINT CONDITIONAL PROB DISTRIBUTIONS

```

```

624 ''
625 ''CALCULATE THE PROBABILITY THAT AN ARRIVAL FINDS THE SYSTEM IN STATE N.
626 ''
627     LET NORM.CONST=MPOP*P.NULL
628 FOR N=1 TO MLIM-1 DO
629     ADD (MPOP-N)*P.NSTATE(N) TO NORM.CONST
630 LOOP ''TO CALCULATE THE NORMALIZATION CONSTANT
631     LET Q.NULL=MPOP*P.NULL/NORM.CONST
632     LET QV(1)=(MPOP-1)*PV(1)/NORM.CONST
633     LET QV(2)=(MPOP-1)*PV(2)/NORM.CONST
634     LET QV(3)=(MPOP-2)*PV(3)/NORM.CONST
635     LET QV(4)=(MPOP-2)*PV(4)/NORM.CONST
636     LET QV(5)=(MPOP-2)*PV(5)/NORM.CONST
637     LET Q.NSTATE(1)=QV(1)+QV(2)
638     LET Q.NSTATE(2)=QV(3)+QV(4)+QV(5)
639 FOR N=3 TO MLIM-1 DO
640     LET I=4*N-6
641     LET QV(I)=(MPOP-N)*PV(I)/NORM.CONST
642     LET QV(I+1)=(MPOP-N)*PV(I+1)/NORM.CONST
643     LET QV(I+2)=(MPOP-N)*PV(I+2)/NORM.CONST
644     LET QV(I+3)=(MPOP-N)*PV(I+3)/NORM.CONST
645     LET Q.NSTATE(N)=QV(I)+QV(I+1)+QV(I+2)+QV(I+3)
646 LOOP ''OVER NUMBER OF ARRIVING CUSTOMERS
647     LET P.CUST.WAIT=1.0-Q.NULL-Q.NSTATE(1)-Q.NSTATE(2)
648     GO TO L2
649 END ''ROUTINE FINITE.ME2.Q

```

```

1 ROUTINE SGEFA ( A, IPVT, INFO)
2 ''
3 ''ROUTINE FACTORS THE MATRIX A(*,*) INTO UPPER (U) AND STRICTLY LOWER (L)
4 ''TRIANGULAR MATRICES SUCH THAT A(*,*) = U(*,*)L(*,*).
5 ''ROUTINE IS INTENDED FOR USE WITH OTHER ROUTINES OF THE LINEAR OPERATIONS
6 ''PACKAGE--LINPACK. THIS VERSION IS A CONVERSION OF THE FORTRAN ROUTINE
7 ''WRITTEN BY CLEVE MOLER, U. OF N.M. AND ARGONNE NAT LAB.
8 ''
9 ''ARGUMENTS:
10 ''NAME          MODE          ENTRY VALUE          RETURN VALUE
11 ''-----
12 ''A              REAL(N, N) SQUARE MATRIX.      AN UPPER TRIANGULAR MATRIX AND
13 ''              THE MULTIPLIERS WHICH WERE USED TO
14 ''              GET IT. THESE ARE STORED IN L.
15 ''N              INTEGER ORDER OF THE MATRIX A.  DIMENSION OF A(*,*).
16 ''IPVT           INTEGER(N).          VECTOR OF PIVOT INDICES.
17 ''INFO           INTEGER INDICATOR.          = 0 FOR NORMAL VALUE.
18 ''              = K IF U(K,K) EQ 0.0. THIS
19 ''              INDICATES THAT SGESL OR SGEDI
20 ''              WILL DIVIDE BY ZERO IF CALLED.
21 ''
22     DEFINE I,INFO,J,K,KP1,L,N,NM1 AS INTEGER VARIABLES
23     DEFINE IPVT AS AN INTEGER, 1-DIMENSIONAL ARRAY
24     DEFINE A AS A REAL, 2-DIMENSIONAL ARRAY
25 ''
26 ''GAUSSIAN ELIMINATION WITH PARTIAL PIVOTING.
27 ''
28     LET N=DIM.F(IPVT(*))
29     LET INFO=0
30     LET NM1=N-1
31     IF NM1 LT 1
32         GO TO L7
33     OTHERWISE
34     FOR K=1 TO NM1 DO
35         LET KP1=K+1
36 ''
37 ''FIND L = PIVOT INDEX IN THIS COLUMN.
38 ''
39     LET SMAX=ABS.F(A(K,K))
40     LET L=K
41     FOR I=K+1 TO N DO
42         IF ABS.F(A(I,K)) GT SMAX
43             LET L=I
44             LET SMAX=ABS.F(A(I,K))
45     ALWAYS
46     LOOP ''FOR MAX ELEMENT
47     LET IPVT(K)=L
48 ''
49 ''ZERO PIVOT IMPLIES THIS COLUMN ALREADY TRIANGULARIZED.
50 ''
51     IF A(L,K) = 0.0
52         GO TO L4
53     OTHERWISE
54 ''
55 ''INTERCHANGE IF NECESSARY.
56 ''

```

```

57         IF L = K
58             GO TO L1
59         OTHERWISE
60             LET T=A(L,K)
61             LET A(L,K)=A(K,K)
62             LET A(K,K)=T
63 'L1'     LET T=-1.0/A(K,K)
64             FOR I=K+1 TO N, LET A(I,K)=T*A(I,K)
65         ''
66         ''ROW ELIMINATION WITH COLUMN INDEXING.
67         ''
68             FOR J=K+1 TO N DO
69                 LET T=A(L,J)
70                 IF L=K
71                     GO TO L2
72                 OTHERWISE
73                     LET A(L,J)=A(K,J)
74                     LET A(K,J)=T
75 'L2'     FOR I=K+1 TO N, LET A(I,J)=T*A(I,K)+A(I,J)
76             LOOP ''OVER (J) COLUMNS
77             GO TO L5
78 'L4'     LET INFO=K
79 'L5' LOOP ''OVER K
80 'L7' LET IPVT(N)=N
81         IF A(N,N)=0.0
82             LET INFO=N
83         ALWAYS
84         RETURN
85 END ''SGEFA

```

```

1 ROUTINE SGEDI (A, IPVT, DET, JOB)
2 ''
3 ''
4 ''SGEDI COMPUTES THE DETERMINANT AND INVERSE OF A MATRIX USING THE RESULTS
5 ''PRODUCED BY SGEFA.
6 ''
7 ''ARGUMENTS:
8 ''A(*,*)  _ THE REAL FACTORED MATRIX FROM SGEFA ON INPUT.  ON OUTPUT THE
9 ''          ARRAY CONTAINS THE MATRIX INV, IF REQUESTED. OTHERWISE UNCHANGED.
10 ''IPVT(*)  THE INTEGER PIVOT VECTOR FROM SGEFA.
11 ''JOB  _ _ _ AN INTEGER SWITCH.
12 ''          = 11 FOR BOTH DETERMINANT AND INVERSE.
13 ''          = 01 FOR INVERSE ONLY.
14 ''          = 10 FOR DETERMINANT ONLY.
15 ''DET(*)  _ CONTAINS THE DETERMINANT OF THE MATRIX, IF REQUESTED. OTHERWISE
16 ''          IS NOT REFERENCED. THE DETERMINANT = DET(1)*10.0**DET(2), WITH
17 ''          DET(1) BETWEEN 0 AND 10, AND WITH DET(2) A FLOATED INTEGER.
18 ''
19 ''NOTE: A DIVISION BY ZERO WILL OCCUR IF THE INPUT FACTOR CONTAINS A
20 ''ZERO ON THE DIAGONAL AND THE INVERSE IS REQUESTED.
21 ''
22     DEFINE I,J,JOB,K,KB,KP1,L,N,NM1 AS INTEGER VARIABLES
23     DEFINE IPVT AS AN INTEGER, 1-DIMENSIONAL ARRAY
24     DEFINE DET, WORK AS REAL, 1-DIMENSIONAL ARRAYS
25     DEFINE A AS A REAL, 2-DIMENSIONAL ARRAY
26     LET N=DIM.F(IPVT(*))
27     RESERVE WORK(*) AS N ''LOCALLY
28 ''
29 ''CALCULATE THE DETERMINANT IF REQUESTED.
30 ''
31     IF DIV.F(JOB,10) = 0
32         GO TO L6
33     OTHERWISE
34         LET DET(1)=1.0
35         LET DET(2)=0.0
36         LET TEN=10.0
37         FOR I=1 TO N DO
38             IF IPVT(I) NE I
39                 LET DET(1) = -DET(1)
40             ALWAYS
41             LET DET(1)=A(I,1)*DET(1)
42             IF DET(1)=0.0
43                 GO TO L6
44             OTHERWISE
45 'L1'     IF ABS.F(DET(1)) GE 1.0
46                 GO TO L2
47             OTHERWISE
48                 LET DET(1)=TEN*DET(1)
49                 SUBTRACT 1.0 FROM DET(2)
50                 GO TO L1
51 'L2'     IF ABS.F(DET(1)) LT TEN
52                 GO TO L4
53             OTHERWISE
54                 LET DET(1)=DET(1)/TEN
55                 ADD 1.0 TO DET(2)
56                 GO TO L2
57 'L4'LOOP ''OVER I

```

```

58 ''
59 ''GET INVERSE OF UPPER TRIANGULAR MATRIX U(*,*).
60 ''
61 'L0' IF MOD.F(JOB,10)=0
62     RELEASE WORK(*)
63     RETURN
64     OTHERWISE
65     FOR K=1 TO N DO
66         LET A(K,K)=1.0/A(K,K)
67         LET T=-A(K,K)
68         FOR I=1 TO K-1, LET A(I,K)=T*A(I,K)
69         LET KP1=K+1
70         IF N LT KP1
71             GO TO L9
72         OTHERWISE
73         FOR J=KP1 TO N DO
74             LET T=A(K,J)
75             LET A(K,J)=0.0
76             FOR I=1 TO K, LET A(I,J)=T*A(I,K)+A(I,J)
77         LOOP ''OVER J
78 'L9' LOOP ''OVER K
79 ''
80 ''FORM INVERSE(U)*INVERSE(L)
81 ''
82     LET NM1=N-1
83     IF NM1 LT 1
84         RELEASE WORK(*)
85         RETURN
86     OTHERWISE
87     FOR KB=1 TO NM1 DO
88         LET K=N-KB
89         LET KP1=K+1
90         FOR I=KP1 TO N DO
91             LET WORK(I)=A(I,K)
92             LET A(I,K)=0.0
93         LOOP ''OVER I
94         FOR J=KP1 TO N DO
95             LET T=WORK(J)
96             FOR I=1 TO N, LET A(I,K)=T*A(I,J)+A(I,K)
97         LOOP ''OVER J
98         LET L=IPVT(K)
99         IF L NE K ''SWAP ELEMENTS OF VECTORS K AND L
100             FOR I=1 TO N DO
101                 LET T=A(I,K)
102                 LET A(I,K)=A(I,L)
103                 LET A(I,L)=T
104             LOOP ''OVER I TO SWAP
105         ALWAYS
106     LOOP ''OVER KB
107     RELEASE WORK(*)
108     RETURN
109 END ''SGEDI

```

```

1 ROUTINE MINRV (IPRINT,N,XRANGE,APARM,BPARM) YIELDING EMINX,VMINX
2 ''
3 ''ROUTINE CALCULATES THE MEAN AND VARIANCE OF THE MINIMUM OF N CONTINUOUS,
4 ''POSITIVE, I.I.D. RANDOM VARIABLES. THE C.D.F. FOR THESE R.V.'S IS
5 ''CALLED "FUN.CDF" HERE, AND IS SPECIFIED BY A DEFINE-TO-MEAN STATEMENT
6 ''IN THE PREAMBLE. IF THE PRINT FLAG IPRINT =1, THE BASIS C.D.F. AND
7 ''C.D.F. OF THE MINIMUM ARE PRINTED FROM THIS ROUTINE.
8 ''
9 ''INPUTS:
10 ''IPRINT ___ INTEGER FLAG TO PRINT FROM THE ROUTINE ( = 1).
11 ''N _____ NUMBER OF I.I.D. RANDOM (X- ) VARIABLES IN THE SET.
12 ''XRANGE _____ UPPER LIMIT ON RANGE OF X, USED TO CALCULATE INTEGRATION STEP.
13 ''APARM _____ FIRST (REAL-VALUED) PARAMETER OF THE C.D.F. OF X.
14 ''BPARM _____ 2 ND (REAL-VALUED) PARAMETER OF THE C.D.F. OF X.
15 ''
16 ''OUTPUTS:
17 ''EMINX _____ EXPECTED VALUE OF THE MIN OF N RANDOM X-VARIABLES.
18 ''VMINX _____ VARIANCE OF THE MIN X.
19 ''
20 DEFINE I,IPRINT,J,K,M,N AS INTEGER VARIABLES
21 DEFINE FV,SUMV AS REAL, 1-DIMENSIONAL ARRAYS
22 RESERVE FV(*),SUMV(*) AS 2
23 LET M=2000 ''STEPS TO INTEGRATE ALLOWED
24 LET DELX=XRANGE/M ''INTEGRATION STEP SIZE (SIMPSON'S RULE)
25 IF IPRINT NE 1
26 GO TO L0
27 OTHERWISE
28 SKIP 2 LINES
29 PRINT 7 LINES WITH N,APARM,BPARM
30 THUS
PROB DISTRIBUTION FOR THE SMALLEST OF ** CONTINUOUS RANDOM VARIABLES
PARAMETERS OF BASIS DISTRIBUTION: 1ST = ..... 2ND = .....
-----
ARGU-      BASIS      CDF OF
MENT       C.D.F.     MIN SET
-----
38 'L0'LET FV(1)=1.0
39 LET FV(2)=0.0
40 FOR J=1 TO 2, LET SUMV(J)=FV(J)
41 FOR I=1 TO M DO
42 LET X=I*DELX
43 IF MOD.F(I,2)=0
44 LET COEF=2.0
45 OTHERWISE
46 LET COEF=4.0
47 ALWAYS
48 LET FX=FUN.CDF(APARM,BPARM,X)
49 LET GN.COMPL=(1.0-FX)**N
50 LET FV(1)=GN.COMPL
51 LET FV(2)=X*GN.COMPL
52 FOR J=1 TO 2, ADD COEF*FV(J) TO SUMV(J)
53 IF MOD.F(I,20)=0 AND IPRINT=1 ''PRINT RESULTS
54 PRINT 1 LINE WITH X,FX,1.0-GN.COMPL
55 THUS
.....*.***** *.*****

```

```

57         ALWAYS
58         IF GN.COMPL LT 0.0001 AND COEF = 2.0
59             FOR J=1 TO 2, SUBTRACT FV(J) FROM SUMV(J)
60             GO TO L1
61         OTHERWISE
62     LOOP 'OVER I
63 'L1' IF IPRINT=1
64     PRINT 2 LINES THUS

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```

67     ALWAYS
68     LET EMINX=SUMV(1)*DELX/3.0
69     LET VMINX=2.0*SUMV(2)*DELX/3.0-EMINX**2
70     IF IPRINT=1
71         PRINT 2 LINES WITH EMINX,SQRT.F(VMINX)
72     THUS
    MEAN VALUE OF MIN R.V. IN THE SET _ .....
    STANDARD DEV MIN R.V. IN THE SET _ .....
75     ALWAYS
76     RELEASE FV(*)
77     RELEASE SUMV(*)
78     RETURN
79 END 'MINRV

```

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1 ROUTINE TO LIMSTATE 'OF A FINITE, MULTI-SERVER QUEUEING SYSTEM' GIVEN
2 LAMBDA, MU, MPOP, NSERVE, PLIM, IPRINT YIELDING LSTATE, P.NULL, P.NSTATE
3 ''
4 ''ROUTINE SOLVES FOR THE PROBABILITY VECTOR WHICH CHARACTERIZES THE STEADY-
5 ''STATE OF A QUEUEING SYSTEM WITH A FINITE POPULATION (MPOP) OF CUSTOMERS
6 ''SERVED BY NSERVE CHANNELS OF EXPONENTIAL SERVICE. THE ARRIVAL RATE FOR
7 ''EACH INDIVIDUAL NOT IN THE SYSTEM IS A CONSTANT (LAMBDA). THE SERVICE
8 ''RATE IS THE CONSTANT MU. THE ROUTINE RETURNS THE (INTEGER) STATE INDEX
9 ''WHICH IS EXCEEDED WITH PROBABILITY PLIM. THE ROUTINE ALSO RETURNS
10 ''THE PROBABILITY OF AN EMPTY SERVICE SYSTEM (P.NULL) AND THE VECTOR OF
11 ''STATE PROBABILITIES (P.NSTATE(*)). REF: GROSS AND HARRIS, FUND. QUEUE.
12 ''
13 ''INPUT:
14 ''LAMBDA ARRIVAL RATE FOR SERVICE PER INDIVIDUAL.
15 ''MU SERVICE RATE PER SERVER.
16 ''MPOP CUSTOMER POPULATION SIZE.
17 ''NSERVE NUMBER OF SERVICE CHANNELS.
18 ''PLIM LIMIT ON THE UPPER-TAIL PROBABILITY OF THE STATE
19 '' PROBABILITY DISTRIBUTION.
20 ''IPRINT INTEGER SWITCH TO PRINT FROM THE SUBROUTINE. PRINTING
21 '' OCCURS WHEN IPRINT=1.
22 ''OUTPUT:
23 ''LSTATE INDEX OF THE MARKOV STATE WHICH IS EXCEEDED WITH
24 '' PROBABILITY PLIM.
25 ''P.NULL PROBABILITY THE SERVICE SYSTEM IS EMPTY.
26 ''P.NSTATE VECTOR IN WHICH THE N TH ELEMENT IS THE PROBABILITY
27 '' THAT N CUSTOMERS ARE IN THE SERVICE SYSTEM.
28 ''
29 ''ENDOGENOUS VARIABLES:
30 ''ENO.DOWN AVERAGE NUMBER OF INDIVIDUALS IN THE SERVICE SYSTEM.
31 ''SDNO.DOWN STD DEV NUMBER OF INDIVIDUALS IN THE SERVICE SYSTEM.
32 ''P.SYS.FULL PROBABILITY THAT ALL SERVICE CHANNELS ARE BUSY.
33 ''E.BUSY.SERVERS AVERAGE NUMBER OF BUSY SERVICE CHANNELS.
34 ''SD.BUSY.SERVERS STD DEV NUMBER OF BUSY SERVICE CHANNELS.
35 ''ENO.QUEUED AVERAGE NUMBER OF UNITS WAITING IN THE SERVICE QUEUE.
36 ''ESYS.WAIT AVERAGE WAITING TIME IN THE SERVICE SYSTEM.
37 ''EQ.WAIT AVERAGE WAITING TIME IN THE SERVICE QUEUE, INCLUDING
38 '' THE INSTANCES OF ZERO QUEUE TIME.
39 ''
40 DEFINE I,IPRINT,J,LSTATE,M,MAX,MPOP,N,NSERVE AS INTEGER VARIABLES
41 DEFINE P.NSTATE AS A REAL, 1-DIMENSIONAL ARRAY
42 RESERVE P.NSTATE(*) AS MPOP
43 LET R=LAMBDA/MU
44 LET C=NSERVE
45 IF NSERVE GE MPOP
46 PRINT 1 LINE WITH NSERVE,MPOP
47 THUS
INPUT ERROR TO ROUTINE LIMSTATE. NSERVE = ***** MPOP = *****
49 RETURN
50 OTHERWISE
51 LET CDF.LIM=1.0-PLIM
52 LET RHO=R/C
53 LET ENO.DOWN=0.0
54 LET E.BUSY.SERVERS=0.0
55 LET VAR.BUSY.SERVERS=0.0
56 LET SUM=1.0

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57     LET QSUM=C
58     LET NFACTORIAL=1.0
59     LET RATIO.FACTORIALS=1.0
60     IF NSERVE=1
61         GO TO PASS
62     OTHERWISE
63     FOR N=1 TO NSERVE-1 DO
64         LET NFACTORIAL=NFACTORIAL*N
65         LET RATIO.FACTORIALS=RATIO.FACTORIALS*(MPOP-N+1)
66         LET P.N=R**N/NFACTORIAL*RATIO.FACTORIALS ''OMITTING *P.NULL
67         LET P.NSTATE(N)=P.N ''WITH THE NTH STATE STORED IN THE NTH ELEMENT
68         ADD N*P.N TO ENO.DOWN
69         ADD (NSERVE-N)*P.N TO QSUM
70         ADD P.N TO SUM
71         ADD N*P.N TO E.BUSY.SERVERS
72         ADD N**2*P.N TO VAR.BUSY.SERVERS
73     LOOP ''TO DEVELOP SUMS
74 ''
75 ''CALCULATE THE LAST TERM IN THE SUM FOR P.NULL
76 ''
77 ''PASS'LET CFACTORIAL=C*NFACTORIAL
78     LET CEXPOC=C**NSERVE
79     LET COEF=CEXPOC/CFACTORIAL
80     LET SUM.LAST=0.0 ''SUM FOR PROB THAT NO IN SYSTEM GT OR = NSERVE
81     LET VNO.DOWN=VAR.BUSY.SERVERS
82     FOR N=NSERVE TO MPOP DO
83         LET RATIO.FACTORIALS=RATIO.FACTORIALS*(MPOP-N+1)
84         LET P.N=COEF*RATIO.FACTORIALS*RHO**N
85         ADD P.N TO SUM.LAST ''OMITTING *P.NULL
86         ADD N*P.N TO ENO.DOWN ''OMITTING *P.NULL
87         ADD N**2*P.N TO VNO.DOWN ''OMITTING *P.NULL
88         LET P.NSTATE(N)=P.N ''OMITTING *P.NULL
89     LOOP ''TO OBTAIN THE LAST TERM
90     ADD SUM.LAST TO SUM
91     LET P.NULL=1.0/SUM ''PROB OF SYSTEM NULL STATE
92 ''
93 ''CHECK OF CALCULATED SERVICE SYSTEM STATE-PROBABILITIES.
94 ''
95     IF P.NULL GE 1.0 OR P.NULL LE 0.0
96         PRINT 1 LINE WITH P.NULL THUS
ERROR IN CALCULATING STATE PROBABILITIES.  P.NULL = .....
98     STOP
99     OTHERWISE ''CALCULATE EXPECTED VALUES
100     LET ENO.DOWN=ENO.DOWN*P.NULL
101     LET VNO.DOWN=VNO.DOWN*P.NULL-ENO.DOWN**2
102     LET SDNO.DOWN=SQRT.F(VNO.DOWN)
103     LET P.SYS.FULL=P.NULL*SUM.LAST
104     LET E.BUSY.SERVERS=P.NULL*E.BUSY.SERVERS+C*P.SYS.FULL
105     LET VAR.BUSY.SERVERS=P.NULL*VAR.BUSY.SERVERS+C**2*P.SYS.FULL
106     -E.BUSY.SERVERS**2
107     LET SD.BUSY.SERVERS=SQRT.F(VAR.BUSY.SERVERS)
108     LET ENO.QUEUED=ENO.DOWN-NSERVE+P.NULL*QSUM
109     LET ESYS.WAIT=ENO.DOWN/LAMBDA/(REAL.F(MPOP)-ENO.DOWN)
110 ''ABOVE EXPRESSION IS KNOWN AS LITTLE'S FORMULA.
111     LET EQ.WAIT=ESYS.WAIT-1.0/MU

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112 ''
113 ''CALCULATE SERVICE SYSTEM STATE PROBABILITIES.
114 ''
115     LET PSUM=P.NULL
116     FOR N=1 TO MPOP DO
117         LET P.NSTATE(N)=P.NSTATE(N)*P.NULL
118         ADD P.NSTATE(N) TO PSUM
119         IF PSUM LE CDF.LIM
120             LET LSTATE=N
121         ALWAYS
122     LOOP ''OVER ALL STATES
123     IF IPRINT NE 1
124         RETURN
125     OTHERWISE
126         SKIP 2 LINES
127         PRINT 2 LINES THUS
        STATISTICS FOR THE STEADY STATE OF THE SERVICE SYSTEM

130     PRINT 8 LINES WITH ENO.DOWN,SDNO.DOWN,P.SYS.FULL,E.BUSY.SERVERS,
131     SD.BUSY.SERVERS,ENO.QUEUED,ESYS.WAIT,EQ.WAIT
132     THUS
        AVERAGE NUMBER OF INDIVIDUALS IN THE SERVICE SYSTEM _____ ***.****
        STD DEV NUMBER OF INDIVIDUALS IN THE SERVICE SYSTEM _____ ***.****
        PROBABILITY THAT ALL SERVICE CHANNELS ARE BUSY _____ *.****
        AVERAGE NUMBER OF BUSY SERVICE CHANNELS _____ *.****
        STD DEV NUMBER OF BUSY SERVICE CHANNELS _____ *.****
        AVERAGE NUMBER OF INDIVIDUALS QUEUED FOR SERVICE _____ ***.****
        AVERAGE WAITING TIME IN THE SERVICE SYSTEM _____ ****.***
        AVERAGE WAITING TIME IN THE SERVICE QUEUE _____ ****.***

141     SKIP 2 LINES
142     RETURN
143 END ''LIMSTATE

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