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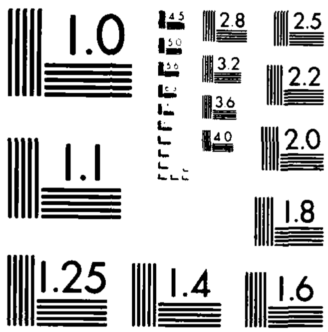

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R. W. Copplestone

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SUMMARY

An experimental technique is described to determine the disturbance to the response of one microphone due to the presence of a second identical microphone. The measurements were carried out in a free-field environment using plane waves incident normally onto the microphones and protection grids were used. The microphone diaphragms were coplanar and the spacing between the microphone centres was varied from one to sixteen microphone diameters. Amplitude deviations up to 1 dB and phase deviations up to 7 degrees have been measured. The magnitudes of the disturbances have been found to be in good agreement with theory.

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1 INTRODUCTION

Previous work at the RAE[1] (formerly NGTE) demonstrated that acoustic interference problems can arise due to sound reflected from microphone supports. That particular problem was solved by using a more compact mounting system.

Developments of that mounting have been adopted as standard in the RAE noise test facilities. In order to improve the reliability of the acoustic data acquisition system in the Anechoic Chamber Facility, two microphones are mounted at each measuring station so that if one should fail the other may be used without interrupting the tests. The distance between the two microphones should be small enough for there to be no significant difference between the microphone signals, but large enough for interference effects due to scattering to be insignificant. Drawing on experience, it seemed reasonable that the separation adopted, of the order of twenty microphone diameters, would be a good compromise, but no experimental verification had been carried out. Another example of microphones in close proximity occurs in a source location system developed by Rolls Royce[2] which requires the close mounting of several microphones in an approximately linear array. In this case the disturbance of a sound field around a microphone due to sound scattered by its neighbour(s) should be determined so that the effects of any phase distortion can be included in any subsequent data processing or so that the design of the array can be modified.

This Report describes an experimental determination of the change in the amplitude and phase response of a microphone due to the presence of a second identical microphone at frequencies from 1 kHz to 20 kHz. The measurements were carried out in a free-field environment using plane waves incident normally onto the microphones using the arrangement illustrated in Fig 1. The microphone diaphragms were coplanar and the spacing between microphone centres was varied from one to sixteen microphone diameters.

An attempt is made to interpret the results in the light of a theoretical analysis.

2 THEORETICAL CONSIDERATIONS

2.1 The scattering of sound

If an obstacle is placed in a sound field, the sound waves that would have been found in the volume occupied by the obstacle will have been displaced or scattered. This effect can be reproduced by a suitable distribution of sources which, when combined with the incident wave satisfies the boundary conditions at the surface of the obstacle. It would be useful to be able to calculate the intensity and directivity functions of such a distribution of sources so that the effect of the obstacle may be predicted, but such calculations are usually difficult. However, the theoretical analysis of the arrangement under investigation is comparatively straightforward since the microphones, which cause the scattering, may be considered as semi-infinite rigid cylinders. Furthermore, the microphone diaphragms are coplanar and parallel to the incident plane wavefront, a condition that further simplifies the analysis.

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2.2 The amplitude of the scattered wave

It will be convenient to refer to one microphone as the reference microphone and to the other as the scattering microphone although there can be no distinction in practice because either microphone will disturb the sound field experienced by the other.

Consider now the scattering microphone in isolation with plane waves incident normally onto the diaphragm. Following the introduction of the microphone, considered to be a rigid body, into the sound field, the particle velocity in the region now occupied by the diaphragm will be reduced from its free-field value of $P_0/\rho c$ to zero. This effect can be reproduced by replacing the d phragm by a disc source of equal diameter and vibrating with a velocity equal in magnitude to the incident particle velocity but in the opposite direction. The strength of the source of the scattered sound is thereby quantified. The incident wave and that radiated by the equivalent source now combine at the diaphragm to produce a particle velocity of zero, the required boundary conditions, and the sound pressure at any point in the field may then be found by adding the incident wave to the wave radiated by this source. This allows the resultant sound pressure at the reference microphone to be calculated. The conditions in the region now occupied by the curved cylindrical surface of the microphone body are unchanged because the particle velocity normal to the surface is zero whether or not the microphone is introduced.

It should be noted that all points on the equivalent source vibrate with the same phase since the plane of the diaphragm is parallel to the incident plane wavefront. For other angles of incidence the phase will vary across the source, and this will complicate the analysis considerably. Furthermore, the boundary conditions at the curved surface of the microphone body will no longer be trivial.

It will be clear from the above that the source of the scattered sound is equivalent to a disc vibrating at the end of a right cylinder. Such a source has been the subject of detailed theoretical analyses[3,4], and using data derived from these analyses the far-field sound pressure P_s, in a plane containing the source and at a distance r from it has been calculated. The results are shown in Fig 2, and in order that the information contained therein may have wider application, the independent variable is ka (the wave number multiplied by the radius of the cylinder) and not the frequency. A value of 2.4 for ka corresponds to a frequency of 20 kHz for the microphone used in this investigation.

In view of the tedious calculations necessary in the case examined above but not reproduced here, it would be useful to discover whether approximate methods could have given acceptable results and, to this end, two approximations are examined. The first approximation assumes that the scattered wave is generated by a simple source and this is represented in Fig 2 by the straight line. There is excellent agreement at low frequencies, but there is also divergence at higher frequencies, and this is due to the directivity of the disc source which increases with frequency while the simple source is non-directional. The second approximation assumes the scattered wave to be generated by the same disc source used to establish the correct result, but in this case set in an infinite baffle rather than at the end of a right cylinder. When the calculated sound pressures are halved, as indicated in Fig 2, agreement could be considered acceptable for all values of ka . This is not an arbitrary reduction since the baffle, which is absent in the experimental arrangement, causes pressure doubling at low frequencies. In view of the approximation made here, the agreement with the exact solution is remarkable. However both approximations discussed above are of interest only and the exact solution will be used in the subsequent analysis. The convergence of the results demonstrates that when ka is small, all compact monopole sources of the same strength are equivalent.

The configuration used in this investigation caused the scattered sound to arrive at the reference microphone at grazing incidence and manufacturer's data[5] show that under these conditions, the sensitivity of the microphone relative to its sensitivity at normal incidence decreases with increasing frequency, leading to an effective reduction of the amplitude of the scattered wave at high frequencies. The effective amplitude is shown in Fig 3, and since this time this is specific to a particular microphone the independent variable is frequency and not ka .

It has been assumed hitherto that the microphone behaves as a semi-infinite rigid cylinder and hence only one source of scattered sound need be considered. But to satisfy the boundary conditions for a real, and therefore finite, microphone, a second source must be established at the rear of the microphone body. However consideration of the geometry of this section of the microphone body, and of its position relative to the diaphragm of the reference microphone will lead to the conclusion that, in the present context, the effective amplitude of the wave scattered from this source will be negligible.

2.3 The phase of the scattered wave

The plane containing the microphone diaphragms is parallel to the incident plane wavefront, consequently there can be no phase difference between the incident sound pressures at each microphone. To calculate the phase of the scattered wave at the reference microphone, it is necessary to determine the phase of the equivalent source relative to the incident sound pressure, to which must be added the phase lag due to the propagation delay. It is a property of acoustic plane waves that the pressure and particle velocity are in phase; a positive pressure being associated with a positive particle velocity directed away from the source of the plane waves. Hence to achieve the necessary boundary conditions of zero particle velocity, a positive incident pressure at the diaphragm must cause the velocity of the source of the scattered wave to be directed outwards. Now, from Ref 6, the phase of the far-field sound pressure generated by the equivalent source leads the phase of the surface velocity, considered positive when directed outwards, by $\pi/2$, in addition to the phase lag ϕ' due to the propagation delay. Hence the phase of the scattered wave at the reference microphone relative to the incident wave is ϕ , where $\phi = \pi/2 - \phi'$.

2.4 The acoustic interference between adjacent microphones

The effective amplitude and phase of the scattered wave at the reference microphone have now been established and will be used in the following analysis. However to avoid the complexities that consideration of near field conditions will introduce, it will be assumed that the scattered sound radiates from a point located at the centre of the diaphragm of the scattering microphone and further it will be assumed that the sound is received at a point located at the centre of the reference microphone. Using this simplification expressions which describe the disturbances to the amplitude and phase responses of the reference microphone due to the proximity of the scattering microphone have been derived in Appendix B, and the relevant equations are listed here. The change in the amplitude response due to the disturbance is given by

$$A = 8.7 \frac{P_s}{P_0} \sin \phi' \quad \text{dB} \ . \tag{1}$$

The change in the phase response due to the disturbance is given by

$$\theta = 57 \frac{P_s}{P_0} \cos \phi' \quad \text{degrees} \ . \tag{2}$$

Since ϕ', the phase lag due to the propagation delay, is directly proportional to frequency, the response changes are seen to be periodic with frequency. It is shown in Appendix B that if the distance between the microphones is greater than about three microphone diameters, the maximum amplitude and phase deviations are given by

$$A_{max} R = 0.7 \quad \text{dB} \tag{3}$$

$$\theta_{max} R = 4.6 \quad \text{degrees} \tag{4}$$

and hence

$$\frac{\theta_{max}}{A_{max}} = 6.6 \quad \text{degrees/dB} \ . \tag{5}$$

Finally, the response deviation 'wavelength' is given by the relationship

$$\frac{Lr}{c} = 1 \ . \tag{6}$$

3 MEASUREMENT PROCEDURES

3.1 Test geometry

The measurements were carried out in a small anechoic chamber at an ambient temperature of 15°C; Fig 1 illustrates the arrangement of the essential components. The fixed reference microphone was mounted 1.565 m from, and co-axially with a fixed loudspeaker. This separation, the maximum available within the confines of the chamber, was used to obtain the best approximation to plane wave conditions at the microphones. The scattering microphone was suspended by cotton threads above the reference microphone so that the diaphragms were coplanar. An additional thread attached to the scattering microphone allowed it to be removed to a position where acoustic interference was insignificant without the operator opening the chamber door. This arrangement, by avoiding the possibility of disturbing the reference microphone and loudspeaker, assured the constancy of the distance between these components; a necessity when small phase changes are to be measured.

3.2 Generation and analysis of test signals

A block diagram of the equipment used for the generation and analysis of the test signals is given in Fig 4. A signal derived from a pseudo-random noise generator was applied to the loudspeaker, and the resulting signal generated by the reference microphone was analysed using a Fast Fourier Transform (FFT) analyser, thereby producing an amplitude and phase spectrum. Phase reference was obtained from the pseudo-random noise generator. The results of the spectral analysis were stored in the analyser memory and a second set of spectra obtained by repeating the above process, but this time with the scattering microphone removed. The change in the amplitude and phase response of the reference microphone due to the presence of the interfering microphone was obtained by subtracting the second set of spectra from the first. The use of a FFT analyser and a pseudo-random noise generator enabled the required information to be obtained quickly and accurately.

The form of the spectra prior to subtraction is of secondary importance since it is a spectral difference that is sought. The loudspeaker used in these tests provided an adequate signal over the frequency range of interest.

Further details concerned with the generation and analysis of the test signals will be found in Appendix C.

4 EXPERIMENTAL RESULTS

The effects on the amplitude response of the reference microphone due to the proximity of the scattering microphone for a range of separations are given in Fig 5, and the effects on phase response are given in Fig 6. The periodic nature of the response changes is quite evident.

For those cases where the microphone separation is small and hence the disturbance is relatively large, response changes have been calculated and are shown in Fig 5 and Fig 6 by the broken lines. It would be confusing to continue this process since the magnitude of the changes decrease with increasing separation. If the experimental data had been available in numerical form this would not be a problem. However the data were available in graphical form only to the scale given. To assess all of the experimental data, a different method of presentation has been used as shown in Fig 7, where the product of peak amplitude deviation and microphone separation and the product of peak phase deviation and microphone separation have been plotted against microphone separation. The calculated and experimental values differ by less than 10% except for the case when the microphones are in contact, and here the effective amplitude appears to be too high. The theory presented makes the assumption that the microphones as source and receiver of the scattered wave are operating under far-field conditions, and clearly in this instance such an assumption is not valid. However the effort required to extend the theory to cope with this case cannot be justified.

The measured ratios of peak phase deviation to peak amplitude deviations are seen in Fig 8 to agree with the calculated value of 6.6 degrees/dB to within 10%.

Finally, the products of deviation 'wavelength' and microphone separation divided by the velocity of sound are seen in Fig 9 to agree with the calculated value of unity to within 6%, with the exception of the phase 'wavelength' for the case where the microphones are in contact and here a value of 1.13 was obtained.

5 CONCLUDING REMARKS

A technique to determine the disturbance to the free-field response of a microphone due to the presence of a second identical microphone has been described for one configuration. The results show that the character of the disturbance is in good agreement with that predicted by theory.

The results can be summarised by stating that for B&K microphones (type 4133/2619) in the configuration described, the product of peak amplitude deviation in dB and microphone separation, in units of one microphone diameter, is 0.7 dB and the product of peak phase deviation and microphone separation is 4.6 degrees. These results apply for separations of two diameters or more. At one diameter, that is, with the microphones in contact, the peak deviations are 1.0 dB and 7 degrees.

The double microphone installations in the Anechoic Chamber utilise microphone separations exceeding sixteen diameters, giving a peak amplitude deviation of less than 0.04 dB, and since the resolution of the third-octave analyser currently in use is 0.25 dB, interference effects are not a problem. It would seem that the initial educated guess is vindicated.

The results obtained here also indicate that the spacing of the microphones in the source-location array, chosen on the basis of the effectiveness of the array, and which can be as close as twelve diameters, will not produce significant interference.

Appendix A

LIST OF SYMBOLS

Symbol	Definition	Unit
a	radius of cylinder/microphone	m
A	change in amplitude response due to interference	dB
c	velocity of sound	m/s
f_0	a frequency at which the response disturbance is zero	Hz
L	wavelength of the changes to the amplitude and phase responses	Hz
k	wave number = ω/c	rad/m
n	an integer	
P	instantaneous resultant sound pressure at the reference microphone	Pa
P_0	pressure amplitude of the incident plane wave	Pa
P_e	effective pressure amplitude of the scattered wave at the reference microphone	Pa
P_m	resultant pressure amplitude at the reference microphone	Pa
P_s	pressure amplitude of the scattered wave at a distance r from the scattering microphone	Pa
r	distance between microphone centres	m
R	distance between microphone centres	one microphone diameter
ϕ	phase of scattered wave at the reference microphone relative to the incident wave	radian
ϕ'	that part of ϕ due to propagation delay	radian
θ	change in phase response due to interference	degrees
ρ	density of air	kg/m^3
ω	frequency	rad/s

Appendix B

DERIVATION OF INTERFERENCE FORMULAE

If the phase of the direct wave at the microphones is zero, then for a sinusoidal signal, of frequency ω, the sound pressure at the reference microphone will be P, where

$$P = P_0 \sin \omega t + P_e \sin(\omega t + \phi) \quad .$$

The first term on the RHS is due to the direct wave, the second is due to the scattered wave. Since the phase of the source of the scattered wave leads the phase of the incident wave by $\pi/2$, then

$$\phi = \frac{\pi}{2} - \phi'$$

and

$$\phi' = \frac{\omega r}{c}$$

therefore

$$P = P_0 \sin \omega t + P_e \cos(\omega t - \phi')$$

$$= P_0 \left(\sin \omega t + \frac{P_e}{P_0} \cos(\omega t - \phi')\right)$$

$$= P_0 \left(1 + \left(\frac{P_e}{P_0}\right)^2 + 2 \frac{P_e}{P_0} \sin \phi'\right)^{\frac{1}{2}} \sin\left(\omega t + \tan^{-1} \frac{P_e/P_0 \cos \phi'}{1 + P_e/P_0 \sin \phi'}\right)$$

$$= P_m \sin(\omega t + \theta) \quad .$$

A value for P_e/P_0 can be obtained from Fig 3.

The change in the signal amplitude due the interference is given by

$$A = 20 \log_{10} \frac{P_m}{P_0} \quad \text{dB}$$

$$= 10 \log_{10} \left(1 + \left(\frac{P_e}{P_0}\right)^2 + 2 \frac{P_e}{P_0} \sin \phi'\right) \quad \text{dB} \quad .$$

Since

$$\frac{P_e}{P_0} \ll 1$$

we can write

Appendix B

$$A = 10 \log_{10}\left(1 + 2\frac{P_e}{P_0}\sin\phi'\right) \quad \text{dB}$$

$$\approx 4.343 \log_e\left(1 + 2\frac{P_e}{P_0}\sin\phi'\right) \quad \text{dB}$$

$$\approx 8.7\frac{P_e}{P_0}\sin\phi' \quad \text{dB} \quad . \qquad (1)$$

The change in the phase response due to the interference is given by

$$\theta = \tan^{-1}\frac{P_e/P_0 \ \cos\phi'}{1 + P_e/P_0 \ \sin\phi'} \quad \text{radians} \quad .$$

Since θ is small,

$$\theta = \frac{P_e/P_0 \ \cos\phi'}{1 + P_e/P_0 \ \sin\phi'} \quad \text{radians} \quad .$$

and, furthermore, since $P_e/P_0 \ll 1$

$$\theta = 57\frac{P_e}{P_0}\cos\phi' \quad \text{degrees} \quad . \qquad (2)$$

A particularly useful relationship, the maximum disturbance at a given separation, can be derived from equations (1) and (2) if further approximations are made. We can see that each disturbance equation consists essentially of two parts. Firstly, the P_e/P_0 term which varies slowly with frequency as shown in Fig 3, and secondly the $\sin\phi'$ or the $\cos\phi'$ terms which vary with frequency at a rate proportional to r. If r is large enough, the disturbances undergo a sufficient number of cycles for a maximum in the $\sin\phi'$ or $\cos\phi'$ terms to occur when $P_e = P_{e_{max}}$, and hence the maximum disturbance may be easily calculated. This condition is satisfied when R is of the order of three or more.

From Fig 3

$$\left(\frac{P_e}{P_0}\frac{r}{2a}\right)_{max} = 0.09 \quad .$$

For given conditions, P_e is the only variable

therefore

$$P_{e_{max}} = 0.09P_0\frac{2a}{r} \quad .$$

If

$$\sin\phi' = 1$$

REPORT DOCUMENTATION PAGE

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17. Abstract

An experimental technique is described to determine the disturbance to the response of one microphone due to the presence of a second identical microphone. The measurements were carried out in a free-field environment using plane waves incident normally onto the microphones and protection grids were used. The microphone diaphragms were coplanar and the spacing between the microphone centres was varied from one to sixteen microphone diameters. Amplitude deviations up to 1 dB and phase deviations up to 7 degrees have been measured. The magnitudes of the disturbances have been found to be in good agreement with theory.

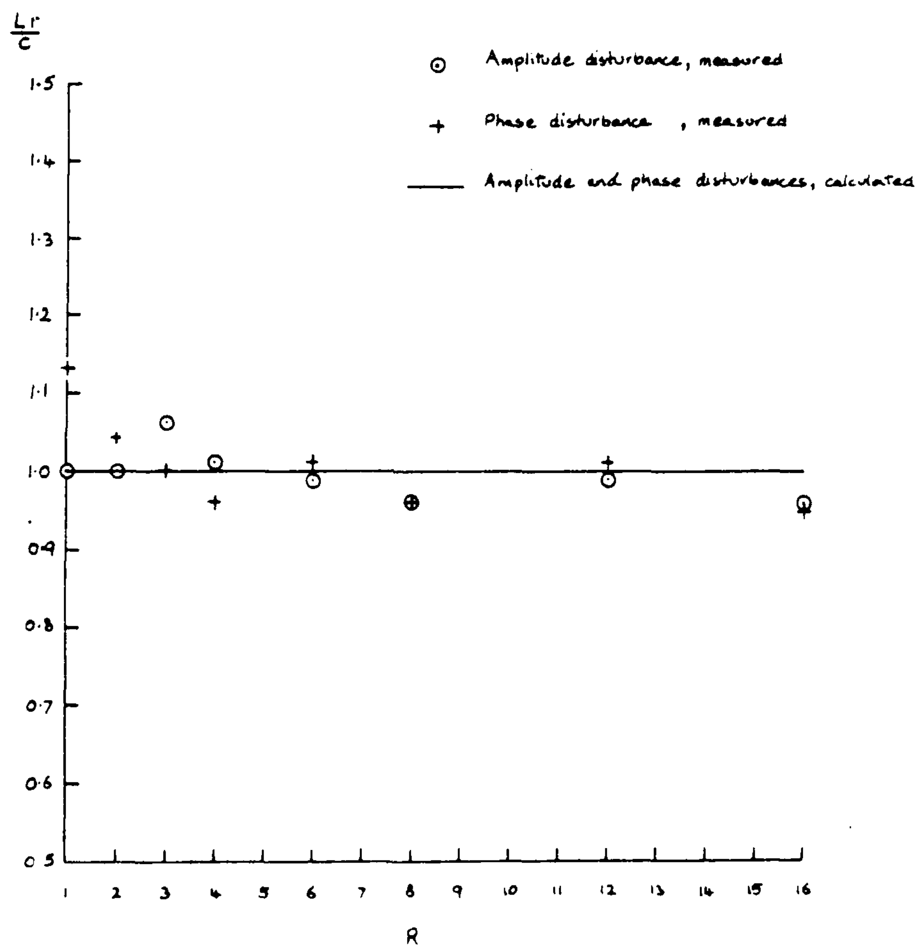

Fig 9 'Wavelengths' of amplitude and phase disturbances

Fig 8

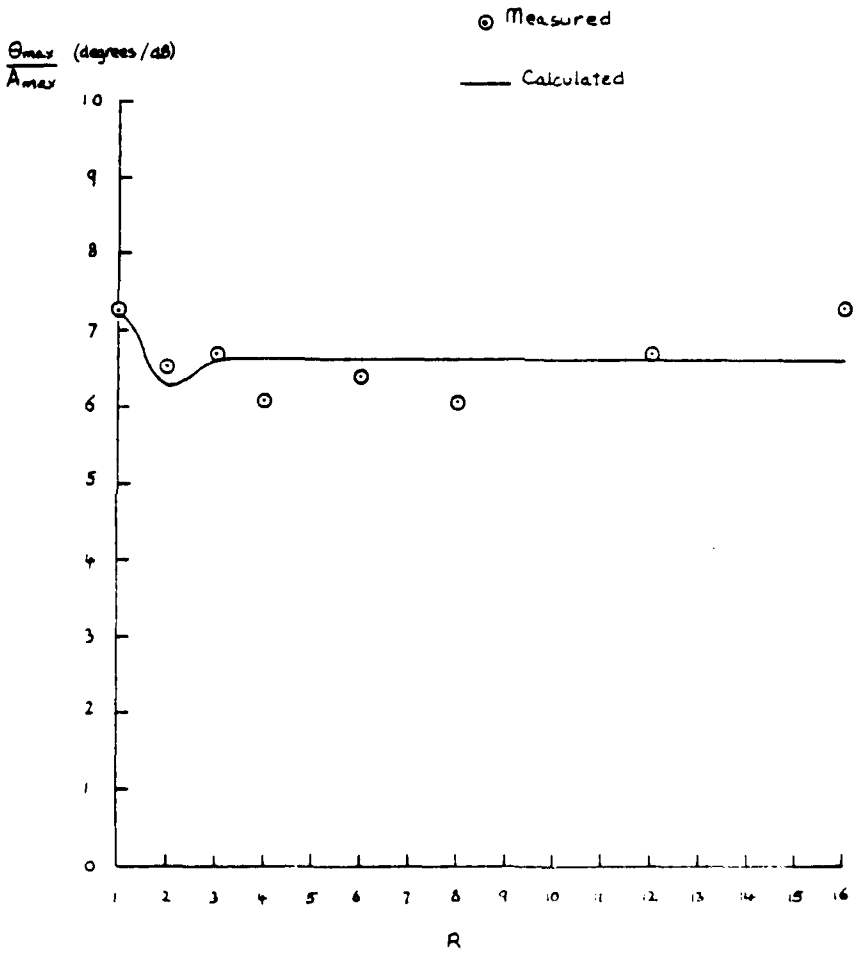

Fig 8 Maximum phase change per unit amplitude change

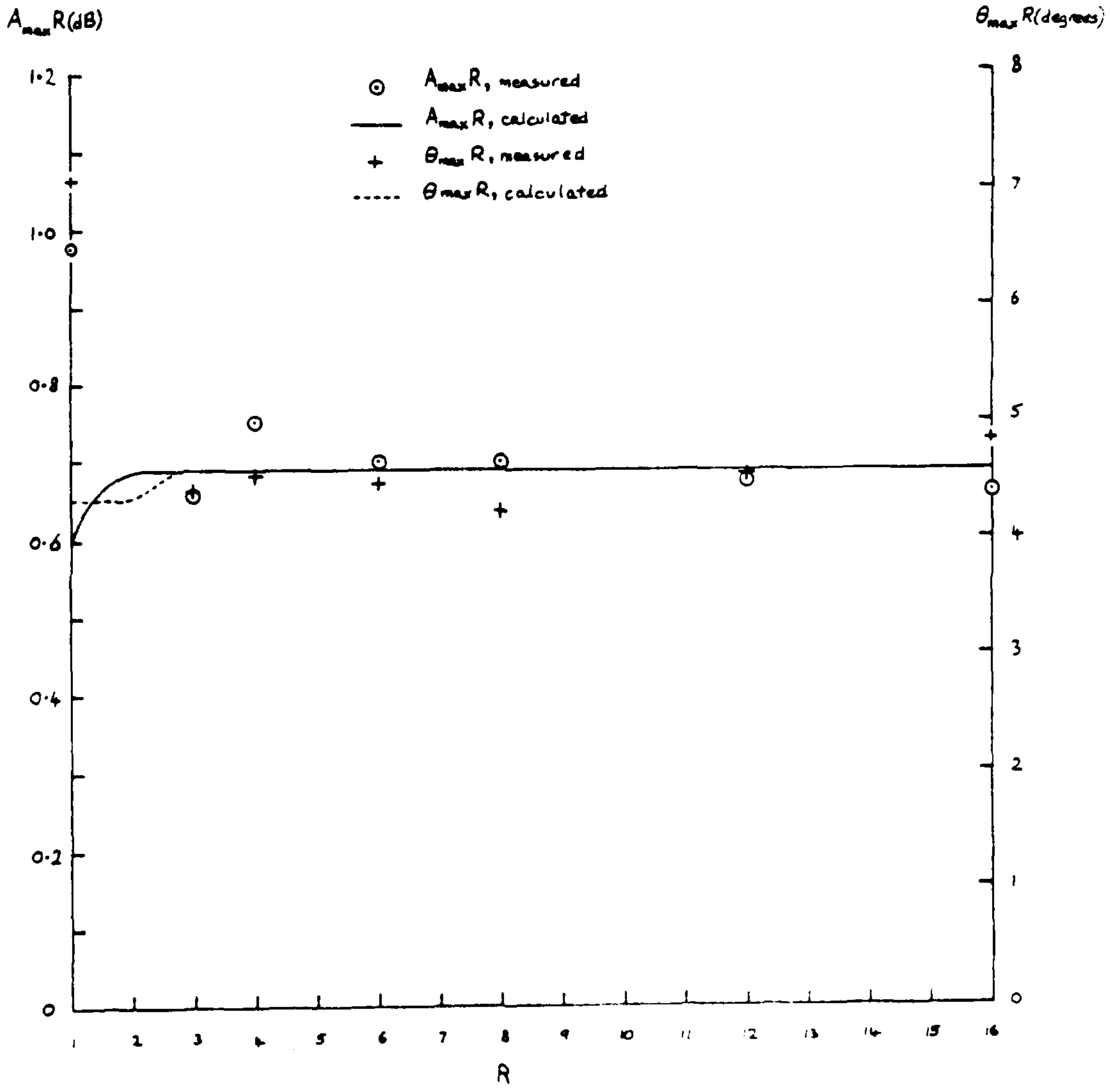

Fig 7 Maximum response changes

Fig 6

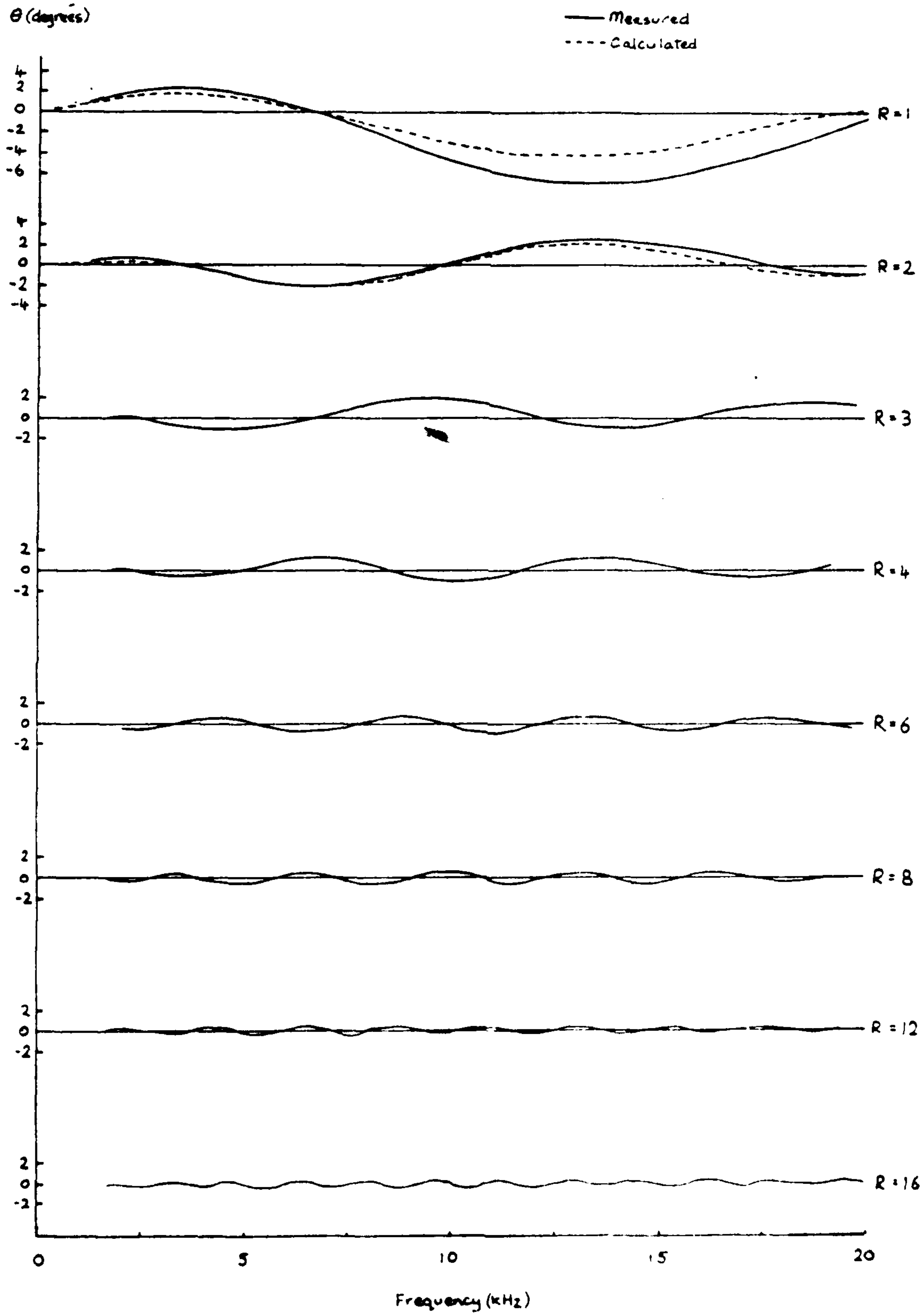

Fig 6 Phase response deviations

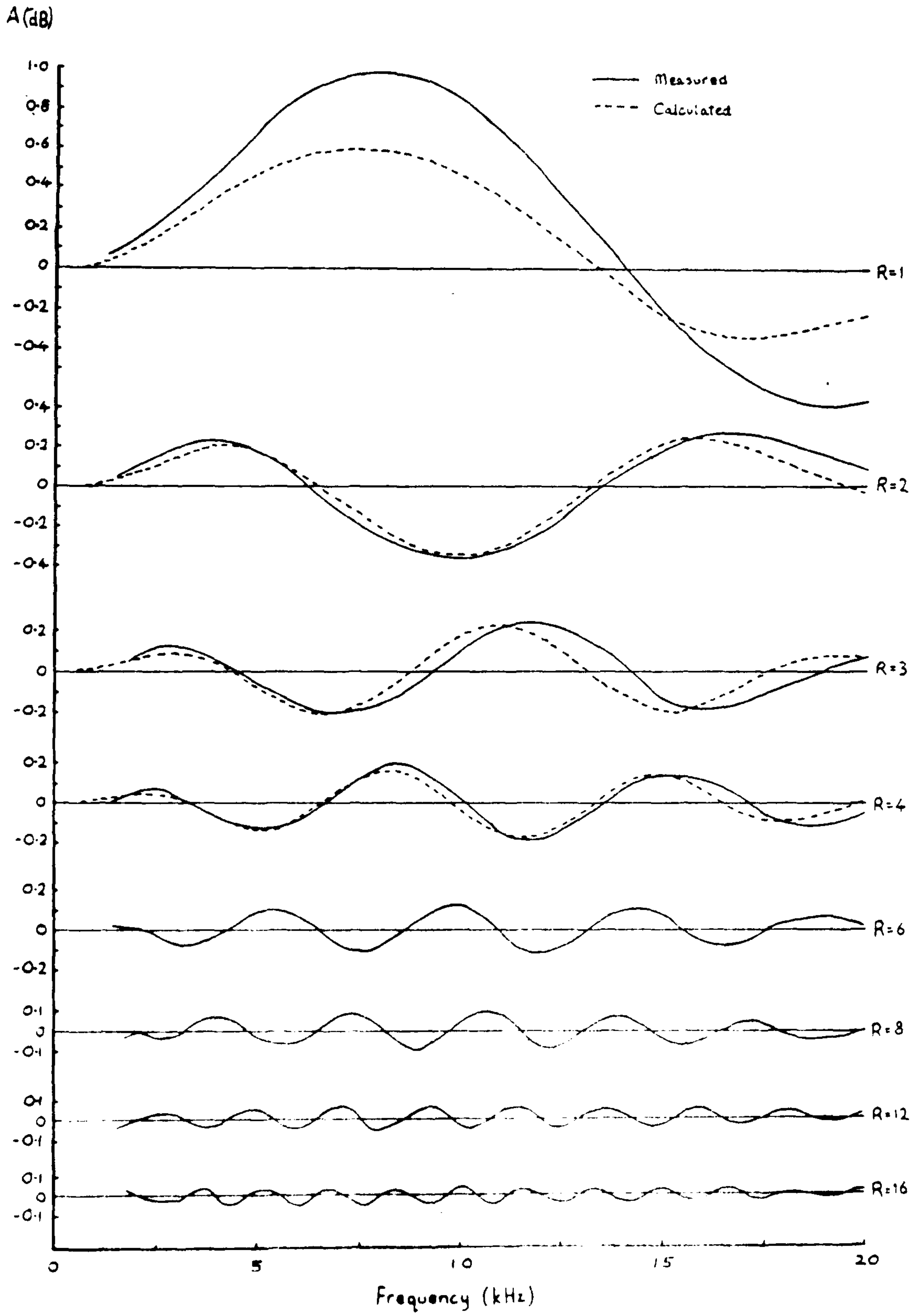

Fig 5 Amplitude response deviations

Fig 4

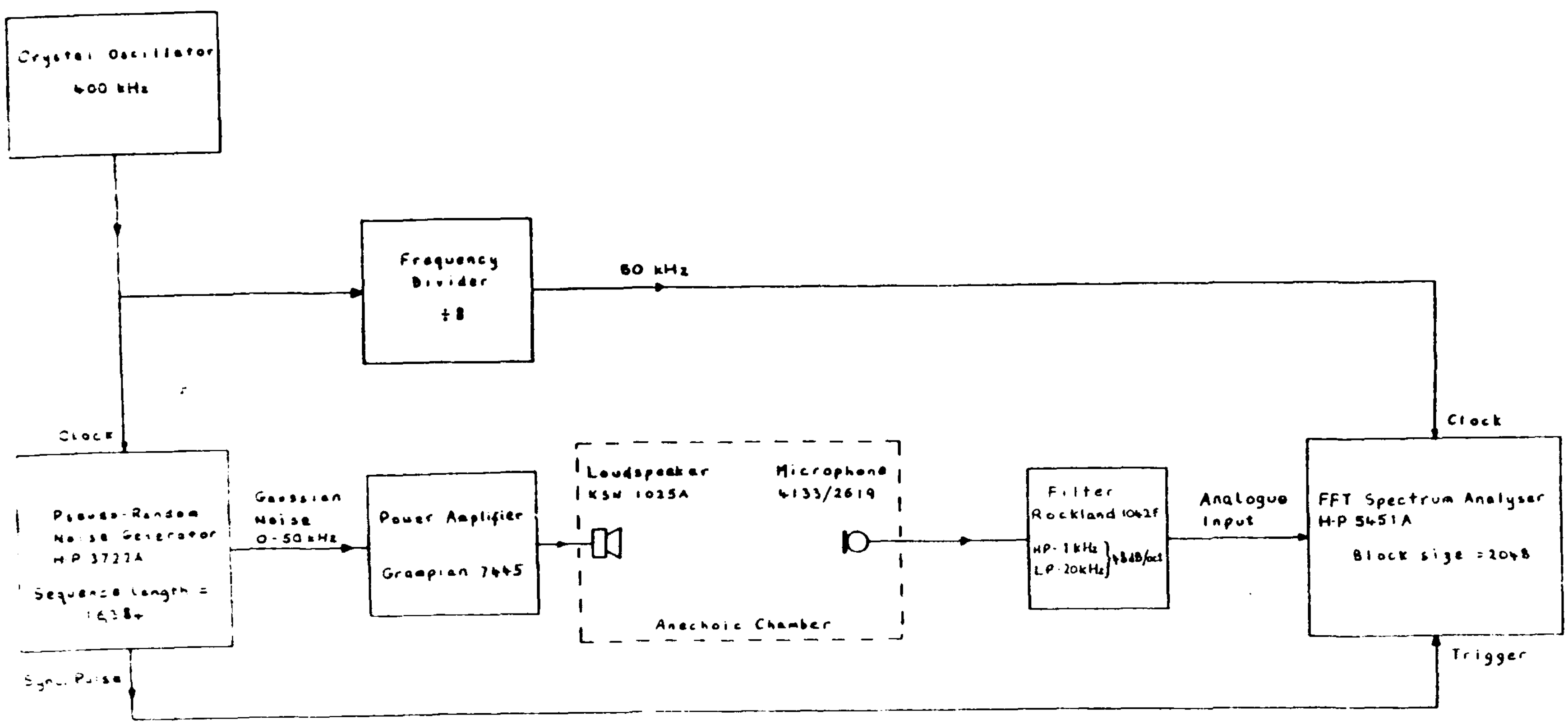

Fig 4 Block diagram of measuring system

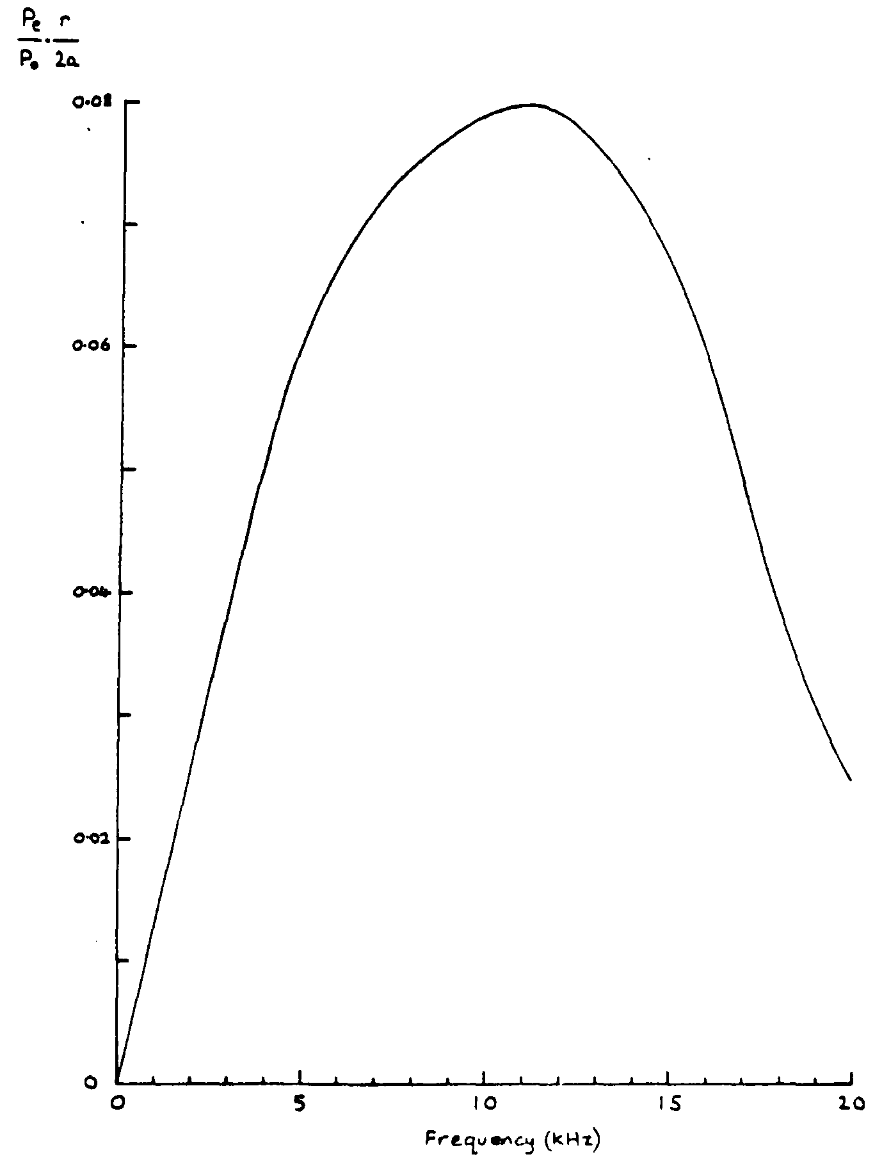

Fig 3 Effective amplitude of the scattered wave at the reference microphone

Fig 2

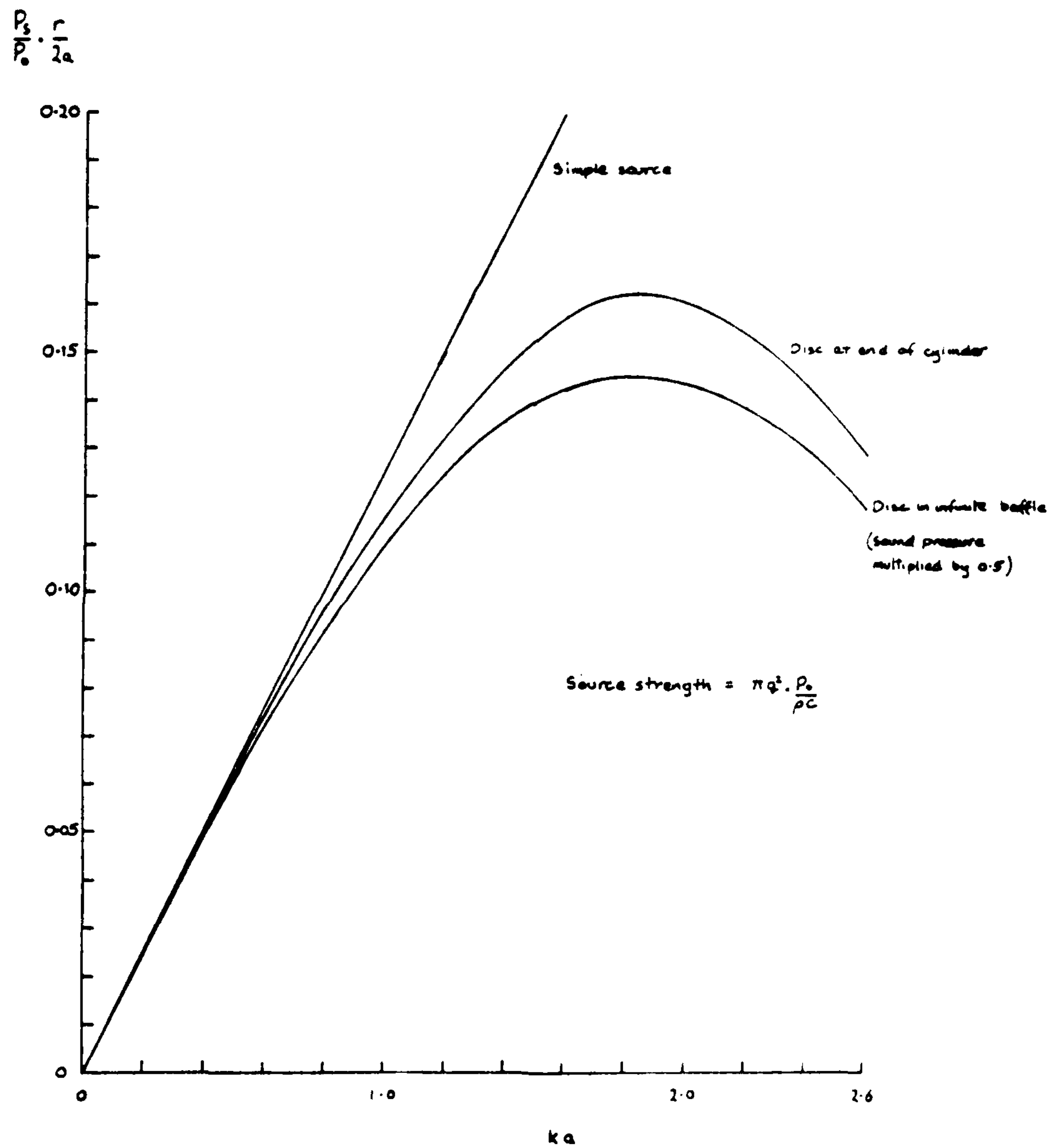

Fig 2 Far-field sound pressures generated by sources of the same strength, in the plane containing the source

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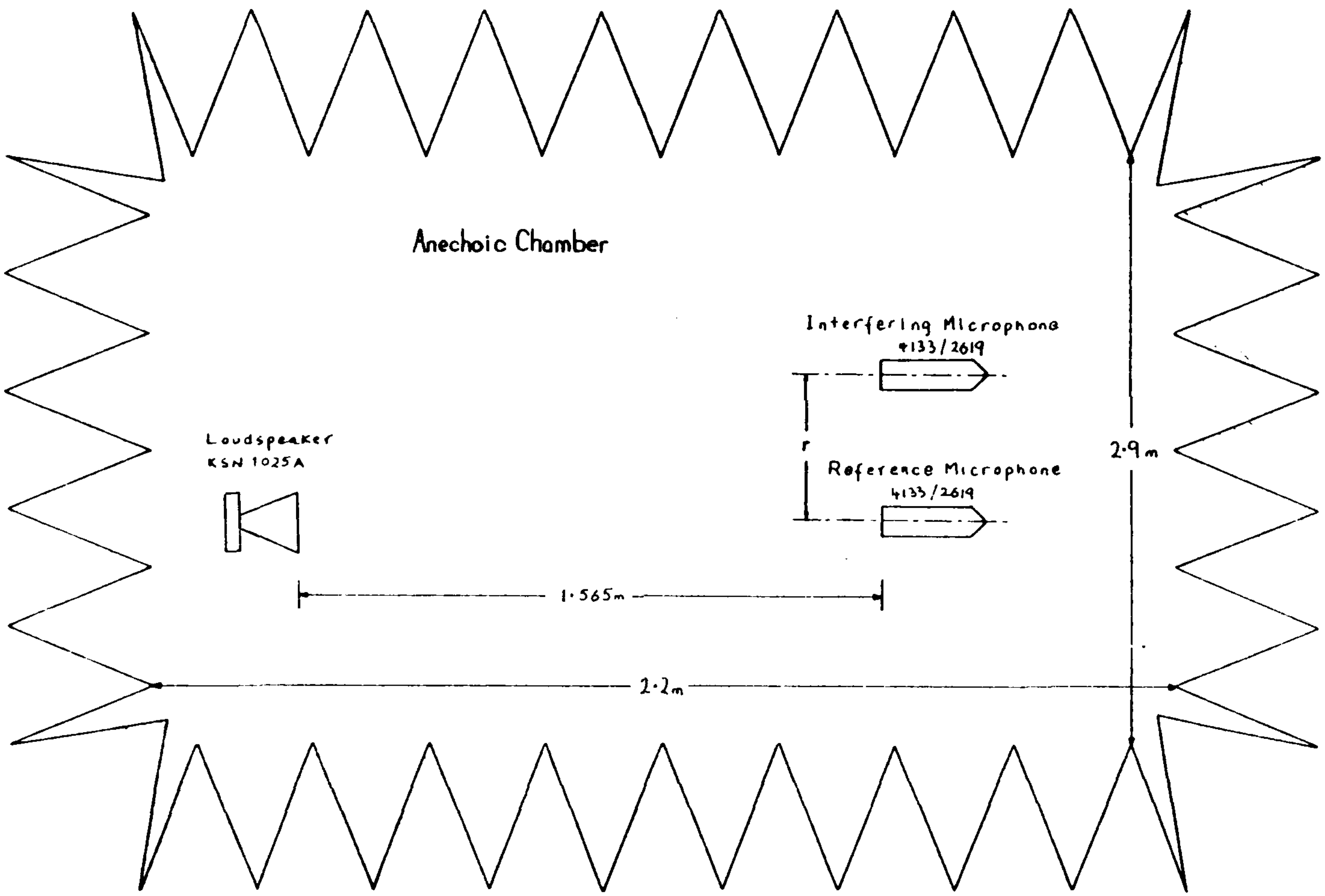

Fig 1 Arrangement of acoustic components

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Appendix C

GENERATION AND ANALYSIS OF TEST SIGNALS

The test signal, broadband Gaussian noise of nominal bandwidth 0-50 kHz, is produced by a pseudo-random noise generator. The bandwidth of the signal that reaches the microphone will be less than 50 kHz due to limitations within the power amplifier and loudspeaker; this limitation presents no problems. The clock pulses for the noise generator are derived from a 400 kHz crystal oscillator. This frequency combined with a sequence length of 2^{14} produces a repetitive complex waveform with a period of $2^{14} \div 400$ kHz = 40.96 ms . A synchronisation pulse is produced at the start of each sequence. The 400 kHz clock frequency is divided by 8 to produce a clock signal of 50 kHz, phase locked to the noise generator clock, to drive the analogue-to-digital converter (ADC) contained within the analyser. The ADC converts the reference microphone signal at a rate of 50000 samples per second in blocks of 2^{11} samples; the conversion being initiated by the noise generator synchronisation pulse which also provides a phase reference. Since $2^{14} \div 400$ kHz = $2^{11} \div 50$ kHz we have a data block consisting of a complete noise generator sequence and hence special window functions are not required. During the measurements, one hundred consecutive data blocks were averaged giving a signal-to-noise ratio improvement of 20 dB.

Prior to conversion the microphone signal is band limited by means of a high-pass filter set at 1 kHz, 48 dB/octave, to minimise acoustic background noise and a low-pass filter set at 20 kHz, -48 dB/octave, to minimise the effects of aliasing. Thus the analysis yields a spectrum with 2^{10} frequency points from 0-25 kHz subject to the limitation imposed by the filters.

therefore $$L = \left(\left(2(n + 2) - 1\right) - (2n - 1)\right) \frac{c}{4r}$$

therefore $$\frac{L_r}{c} = 1 .$$

when

$$P_e = P_{e_{max}}$$

then

$$A_{max} = 8.7 \frac{P_{e_{max}}}{P_0} \quad db \ .$$

$$= 8.7 \times 0.08 \times \frac{2a}{r} \quad dB \ .$$

But

$$R = \frac{r}{2a}$$

therefore

$$A_{max}R = 0.7 \quad dB \ . \qquad (9)$$

Similarly

$$\theta_{max}R = 4.6 \quad \text{degrees} \qquad (10)$$

and hence

$$\frac{\theta_{max}}{A_{max}} = 6.6 \quad \text{degrees/dB} \ . \qquad (11)$$

From equation (1), the amplitude disturbance will be zero when

$$\phi' = n\pi$$

but

$$\phi' = \frac{2\pi r f_0}{c}$$

therefore

$$f_0 = \frac{nc}{2r}$$

therefore

$$L = (n + 2)\frac{c}{2r} - \frac{nc}{2r}$$

therefore

$$\frac{L_r}{c} = 1 \ .$$

An identical result can be obtained for the phase disturbance 'wavelength'.

From equation (2), the phase disturbance will be zero when

$$\phi' = (2n - 1)\frac{\pi}{2}$$

therefore

$$(2n - 1)\frac{\pi}{2} = \frac{2\pi r f_0}{c}$$

therefore

$$f_0 = (2n - 1)\frac{c}{4r}$$

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