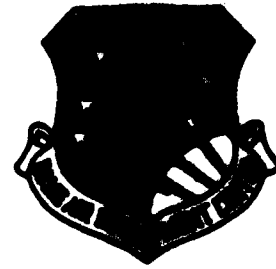


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RADC-TR-88-40
In-House Report
March 1988



SIDELOBE SECTOR NULLING WITH MINIMIZED WEIGHT PERTURBATIONS

Robert A. Shere

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<p>A new method is presented to lower the sidelobes of a given array pattern in a specified sidelobe sector while maintaining the integrity of the original pattern. The method consists of minimizing a performance measure defined to be equal to the weighted sum of the power in the sidelobe sector and the squared weight perturbations. By varying the weights in the performance measure, it is possible to shift the relative emphasis placed on the objectives of lowered sidelobes and small pattern distortion. An analytic solution is obtained for the perturbed weights that minimize the performance measure, and for the perturbed weights when a constant look direction gain constraint is imposed. Curves are plotted showing pattern parameters and examples of perturbed patterns. <i>Keywords:</i></p>					
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Sidelobe Sector Nulling With Minimized Weight Perturbations

1. INTRODUCTION

There is currently considerable interest¹⁻¹⁰ in modifying the patterns of linear array antennas, adaptively and synthetically, to lower the sidelobes at prescribed locations or in pattern sectors. Together with this objective of reduced sidelobes in specified portions of the pattern, it is generally desirable to preserve such pattern characteristics as gain and beam width, or an already low average sidelobe level of a design antenna pattern. Preservation of pattern integrity demands that the perturbations of the complex element weights, required to achieve lowered sidelobe levels in specified regions, be minimized. A trade-off exists, of course, between the two goals of lowered sidelobes and minimization of the weight perturbations. This suggests that a useful performance measure in sidelobe sector nulling is the weighted sum of the average power in a specified sidelobe region and the squared weight perturbations. By varying the weights assigned to the average power in the sidelobe region and the weight perturbations, and minimizing the performance measure, it is then possible to shift the relative emphasis placed on the two principal objectives.

(Received for Publication 21 March 1985)

Because of the large number of references cited above, they will not be listed here. See References, page 13.

The purpose of this report is to present a synthetic sidelobe sector nulling method based on the minimization of the performance measure described above. It is shown that the element weights that minimize the performance measure can be obtained analytically. Curves are plotted showing the variation of the average sector sidelobe power, sum of the squared weight perturbations, gain in the look direction, and magnitude of the perturbed weights, with the ratio of the weights in the performance measure. Examples are shown of the patterns with reduced sidelobes that are obtained using this method. An analytic solution is also obtained to the related problem of minimizing the same performance measure subject to the constraint that the gain in the look direction be constant. Weights and pattern parameters obtained with this solution are compared to those corresponding to the solution without the constant gain constraint.

2. ANALYSIS

We consider a linear array of N equispaced, isotropic elements with inter-element spacing d and phase reference at the array center. Let

$$\underline{w}_0 = [w_{01}, w_{02}, \dots, w_{0N}]^T$$

and

$$\underline{w} = [w_1, w_2, \dots, w_N]^T$$

denote the vectors of the original and perturbed complex weights respectively. The sum of the squared weight perturbations is then given by

$$(\underline{w} - \underline{w}_0)^\dagger (\underline{w} - \underline{w}_0) \quad .$$

The perturbed field pattern is given by

$$p(u) = \sum_{n=1}^N w_n \exp(jd_n u)$$

with

$$d_n = (N - 1)/2 - (n - 1), \quad n = 1, 2, \dots, N$$

and

$$u = (2\pi/\lambda)d \sin(\theta) \quad ,$$

where λ is the wavelength, and θ the pattern angle measured from broadside to the array. Let the sidelobe sector in which the pattern is to be minimized be specified by the interval $u = [u_0 - \varepsilon, u_0 + \varepsilon]$. Then the average power in the sidelobe sector is given by

$$\begin{aligned} (1/2\varepsilon) \int_{u_0 - \varepsilon}^{u_0 + \varepsilon} |p(u)|^2 du &= \sum_{n=1}^N \sum_{m=1}^N w_n w_m^* \exp [j(d_n - d_m)u_0] \\ &\quad \cdot \text{sinc} [(d_n - d_m)\varepsilon] \\ &= \underline{w}^\dagger H \underline{w} \end{aligned}$$

where H is the Hermitian Toeplitz matrix whose elements are

$$H_{mn} = \exp [j(d_n - d_m)u_0] \text{sinc} [(d_n - d_m)\varepsilon] \quad (1)$$

and $\text{sinc}(x) = \sin(x)/x$. We now define the performance measure

$$P = \mu_1 (\underline{w} - \underline{w}_0)^\dagger (\underline{w} - \underline{w}_0) + \mu_2 \underline{w}^\dagger H \underline{w} \quad (2)$$

where μ_1 and μ_2 are the respective weights assigned to the sum of the squared weight perturbations and the average power in the sidelobe sector. The derivative of P with respect to the weight vector \underline{w} is

$$2\mu_1 (\underline{w} - \underline{w}_0) + 2\mu_2 H \underline{w}$$

Equating the derivative to zero and solving for \underline{w} gives the desired perturbed weight vector

$$\begin{aligned} \underline{w} &= \mu_1 (\mu_1 I + \mu_2 H)^{-1} \underline{w}_0 \\ &= (I + \mu_2/\mu_1 H)^{-1} \underline{w}_0 \end{aligned} \quad (3)$$

In the limit as $\mu_2/\mu_1 \rightarrow 0$ and no weight is placed on lowering the sidelobes in the specified pattern sector relative to preserving the original pattern, $\underline{w} = \underline{w}_0$, the initial weight vector. To investigate the limit as $\mu_2/\mu_1 \rightarrow \infty$, we write

$$\underline{w} = (\mu_1/\mu_2) [(\mu_1/\mu_2) I + H]^{-1} \underline{w}_0$$

so that $\underline{w} \rightarrow 0$ as $\mu_2/\mu_1 \rightarrow \infty$. This suggests the possibility that the lowering of the sidelobes in the specified sector may be a trivial matter of shrinking the

magnitudes of the perturbed weights rather than redistributing the power in the array pattern. However, while it is true that the perturbed weights become zero in the limit, there is only a very gradual reduction in the magnitude of the perturbed weights until very large values of μ_2/μ_1 are attained, far larger than is necessary to reduce the sector sidelobe power to low levels. The reason for this is that the matrix H is strongly ill-conditioned and the magnitude of the elements of the matrix $[(\mu_1/\mu_2) I + H]^{-1}$ increase rapidly, approximately of the order of μ_2/μ_1 , as μ_1/μ_2 decreases until μ_1/μ_2 is very close to zero. This is discussed further in the following section and in Appendix A.

It is also possible to obtain an analytic solution for the perturbed weight vector that minimizes the performance measure P and satisfies the constant look direction gain condition

$$\underline{s}^\dagger \underline{w} = g \quad , \quad (4)$$

where \underline{s} is the look direction vector defined by

$$\underline{s} = [\exp(-jd_1 u_0), \exp(-jd_2 u_0), \dots, \exp(-jd_N u_0)]^T \quad ,$$

$u_0 = (2\pi/\lambda)d \sin(\theta_0)$, θ_0 is the look direction angle, and g is the desired pattern value in the look direction. The solution is found by using the method of Lagrangian multipliers and is given by¹¹

$$\underline{w} = A \left[\underline{w}_0 - \frac{\underline{s}^\dagger A \underline{w}_0 - g}{\underline{s}^\dagger A \underline{s}} \underline{s} \right] \quad \# \quad (5)$$

with the matrix A defined by

$$A = (I + \mu_2/\mu_1 H)^{-1} \quad . \quad (6)$$

As before, in the limit as $\mu_2/\mu_1 \rightarrow 0$, $\underline{w} = \underline{w}_0$, since $\underline{s}^\dagger \underline{w}_0 = g$ and the matrix A becomes the identity matrix. The limit of \underline{w} as $\mu_2/\mu_1 \rightarrow \infty$ is the solution to the problem of minimizing the power in a sidelobe sector subject to a constant look direction gain constraint. This problem is discussed in Reference 5. Letting $\mu_1 \rightarrow 0$ in Eq. (5) we obtain

[#]The factor $(\underline{s}^\dagger A \underline{w}_0 - g)/(\underline{s}^\dagger A \underline{s})$ in Eq. (5) is the Lagrangian multiplier and is shown in Appendix B to be real.

11. Brandwood, D. H. (1983) A complex gradient operator and its application in adaptive array theory, Proc. IEEE 130(F and H):11-16.

$$\underline{w} = \frac{H^{-1} \underline{s}}{\underline{s}^\dagger H^{-1} \underline{s}} g$$

Although in the above analysis we have considered a single sidelobe sector only, the nulling method presented generalizes immediately to any number of sidelobe sectors. The matrix H in Eq. (3) or (6) is simply replaced by a sum of matrices of the same form as given in Eq. (1) with different values of μ_0 and ϵ . Different weights can be assigned to the various sidelobe sectors if desired. It is also possible to let the width ϵ of a sidelobe sector become zero in which case the method synthesizes a null at a point location.

3. RESULTS

The weights required to minimize the performance measure defined by Eq. (2) were calculated from Eq. (3) for uniform arrays of 11, 21, and 41 elements with interelement spacing $\lambda/2$. The pattern sector for reduced sidelobe power was taken to be the interval $\theta = [20^\circ, 30^\circ]$. The ratio, μ_2/μ_1 , of the weights assigned to the average sector sidelobe power and the sum of the squared weight perturbations respectively was varied from 0.0001 to 100,000. In Figures 1, 2, and 3, respectively, the average sector sidelobe power, the sum of the squared weight perturbations, and the gain in the look direction (0°), are plotted as functions of $\log_{10}(\mu_2/\mu_1)$. It is apparent from the figures that the behavior of all three functions is quite complicated, although the general trends are clear. The ability to lower sidelobes in a wide pattern sector without significantly affecting the power density in the look direction increases rapidly with the number of array elements. For an 11 element array, a degradation of 1.1 dB in the look direction is associated with a 30-dB reduction in average sector sidelobe power, while for a 41-element array, only a 0.1-dB reduction in look direction gain is required to achieve a 30-dB reduction in average sector sidelobe power. No significant shift in the direction of the mainbeam was found with arrays of 41 and 21 elements over the entire range of μ_2/μ_1 . With the 11-element array, a shift of 0.1° was obtained with $\mu_2/\mu_1 = 100$, increasing in magnitude to a shift of -0.7° with $\mu_2/\mu_1 = 100,000$.

In Figure 4 the locations of the pattern nulls for a 41-element array closest to the sector $\theta = [20^\circ, 30^\circ]$ are plotted as a function of $\log_{10}(\mu_2/\mu_1)$. It is clearly seen how an increasing number of nulls are moved into, and close to, the sector $[20^\circ, 30^\circ]$ as μ_2/μ_1 increases, and hence as increasing weight is placed on lowering the sidelobes in the sector as compared with preserving the original pattern.

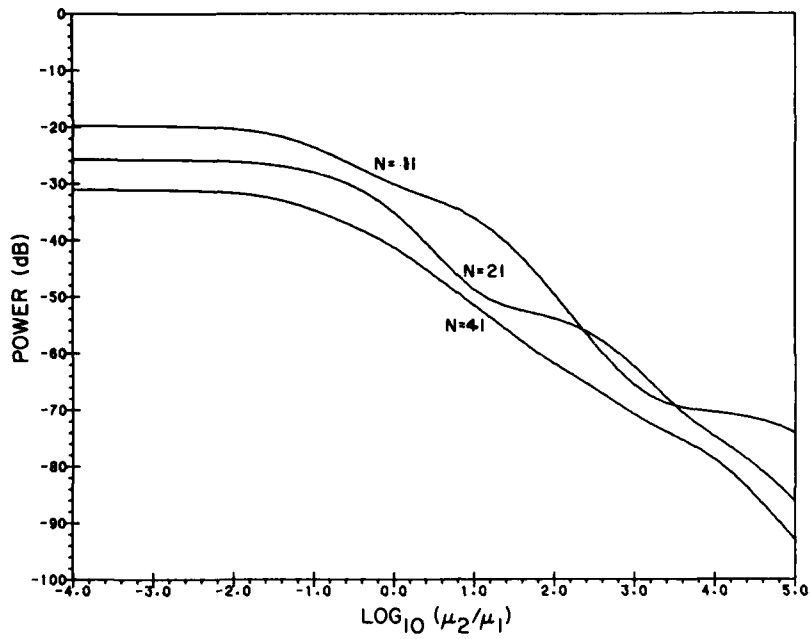


Figure 1. Average Sidelobe Power in the Sector [20°, 30°] for Arrays of 11, 21, and 41 Elements

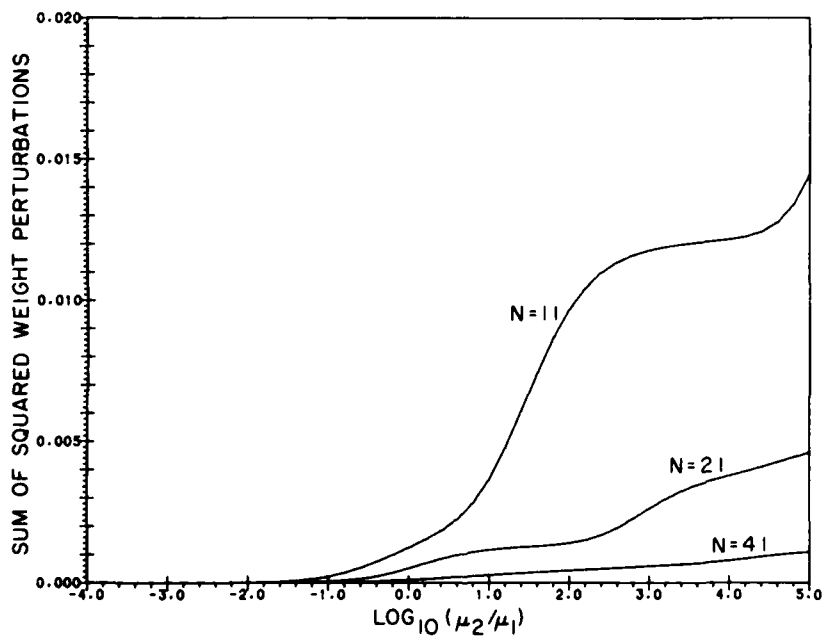


Figure 2. Sum of Squared Weight Perturbations for Arrays of 11, 21, and 41 Elements

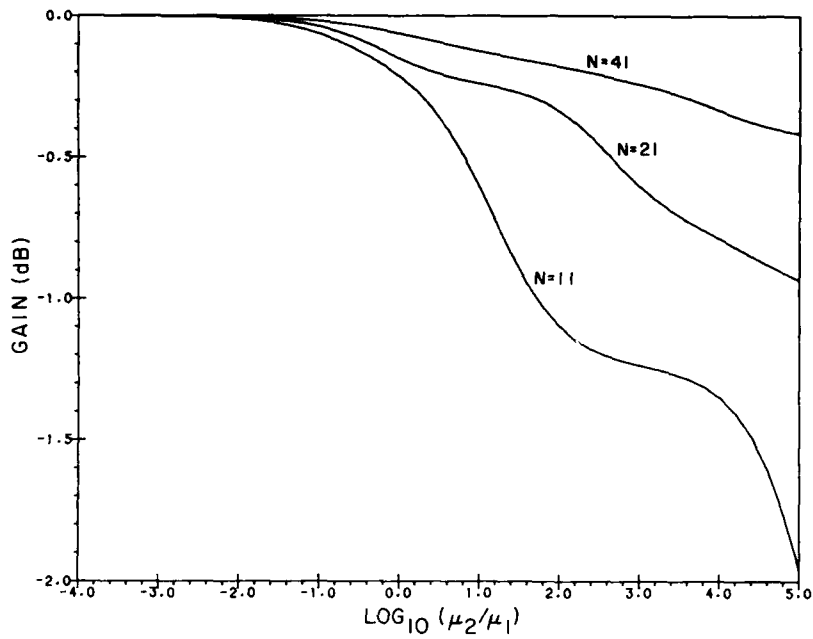


Figure 3. Look Direction Gain of Perturbed Pattern for Arrays of 11, 21, and 41 Elements

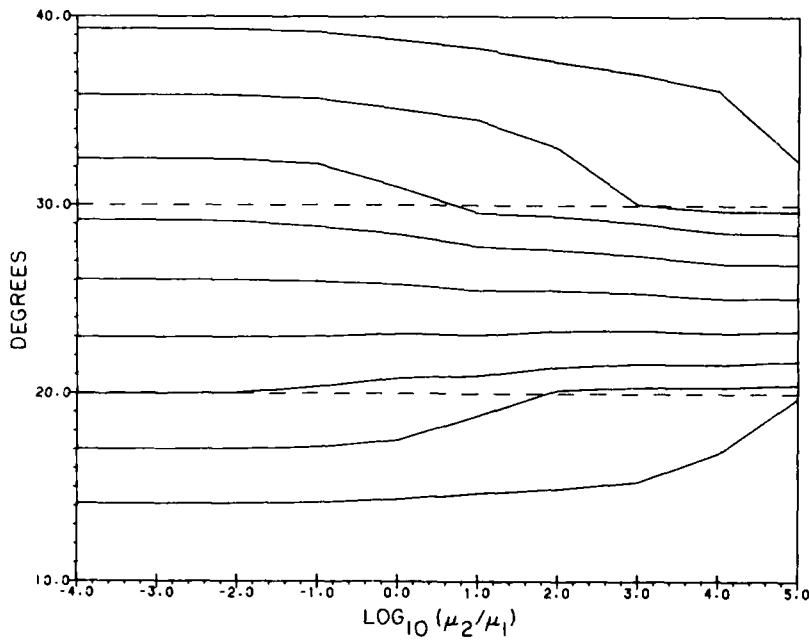


Figure 4. Location of Nulls in Perturbed Pattern of 41-Element Array in the Vicinity of the Sector [20°, 30°]

In the previous section it was stated that although in the limit as $\mu_2/\mu_1 \rightarrow \infty$ the perturbed weight vector $\underline{w} \rightarrow 0$, this decrease in $|\underline{w}|$ is very gradual. In Figure 5 we have plotted $|\underline{w}|/|\underline{w}_0|$ in dB for μ_2/μ_1 from 0.0001 to 100,000 and the sector for reduced sidelobes taken to be $[20^\circ, 30^\circ]$, for arrays of 11, 21, and 41 elements. (This is also the ratio of the total power in perturbed pattern to the total power in the original pattern since the interelement spacing is assumed to be $\lambda/2$.) It is seen that for the 11-element array and μ_2/μ_1 as high as 100,000, $|\underline{w}|/|\underline{w}_0|$ is only -1.2 dB, while for the 21- and 41-element arrays, the decrease is even less pronounced.

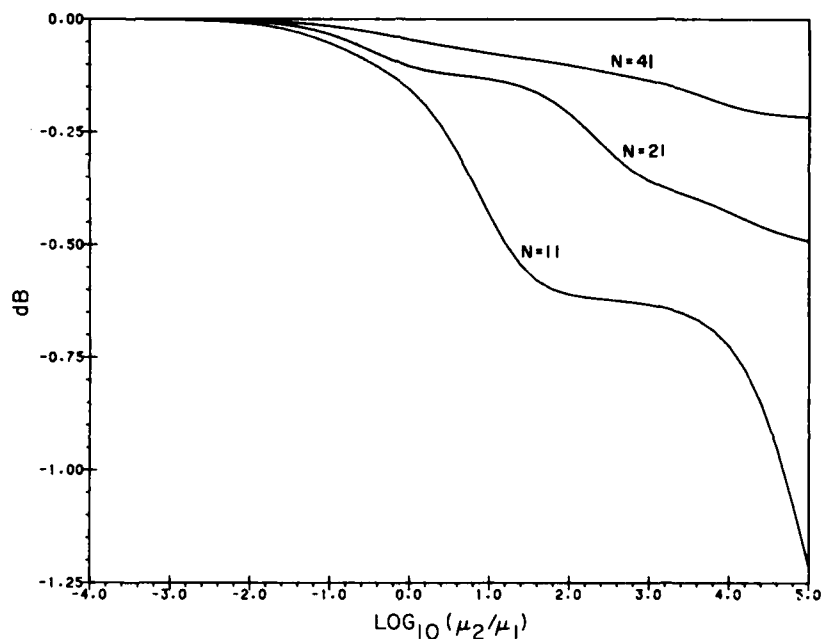


Figure 5. $|\underline{w}|/|\underline{w}_0|$ (dB) for Arrays of 11, 21, and 41 Elements. No look direction constraint

As examples of the patterns obtained with the null synthesis method described in this report, Figures 6, 7, and 8 show the unperturbed pattern and perturbed pattern obtained with $\mu_2/\mu_1 = 100$, and the sector for reduced sidelobes taken to be $[20^\circ, 30^\circ]$, for arrays of 11, 21, and 41 elements respectively. As the number of elements increases, the power in the sector $[20^\circ, 30^\circ]$ decreases and the perturbed pattern follows the original pattern more and more closely except in the immediate vicinity of the nulling sector and at $\theta = \pm 90^\circ$. The distortion of the

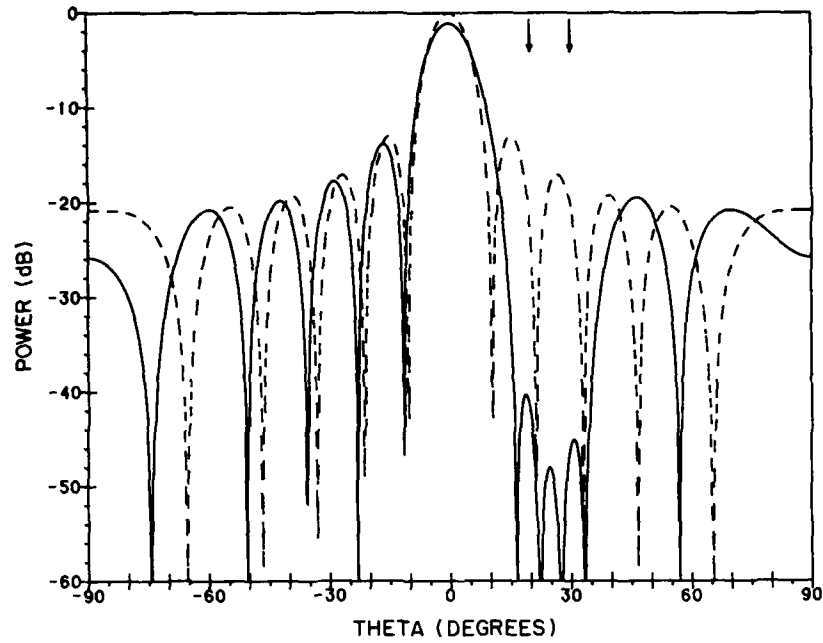


Figure 6. Unperturbed Uniform 11-Element Array Pattern (----) and Perturbed Pattern (—) With Lowered Sidelobes in the Sector [20°, 30°]. $\mu_2/\mu_1 = 100$

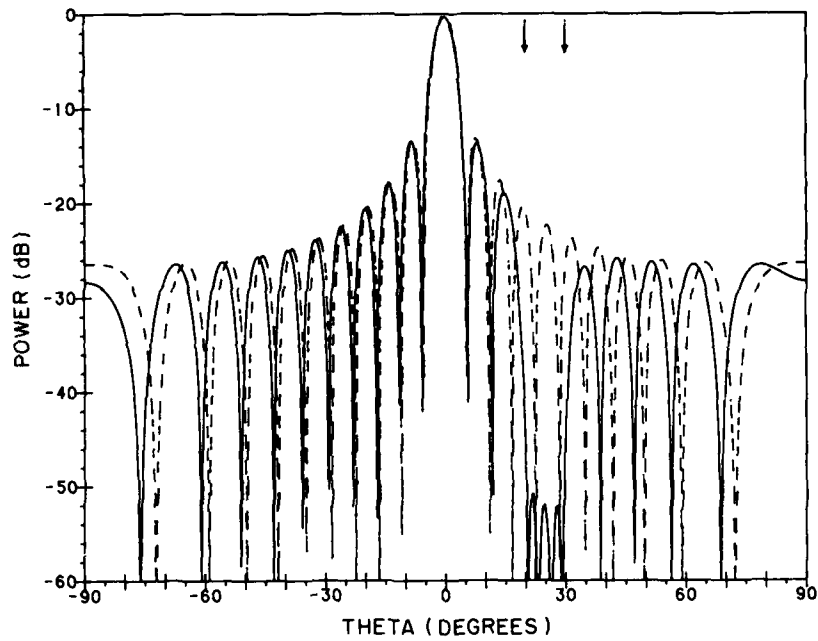


Figure 7. Unperturbed Uniform 21-Element Array Pattern (----) and Perturbed Pattern (—) With Lowered Sidelobes in the Sector [20°, 30°]. $\mu_2/\mu_1 = 100$

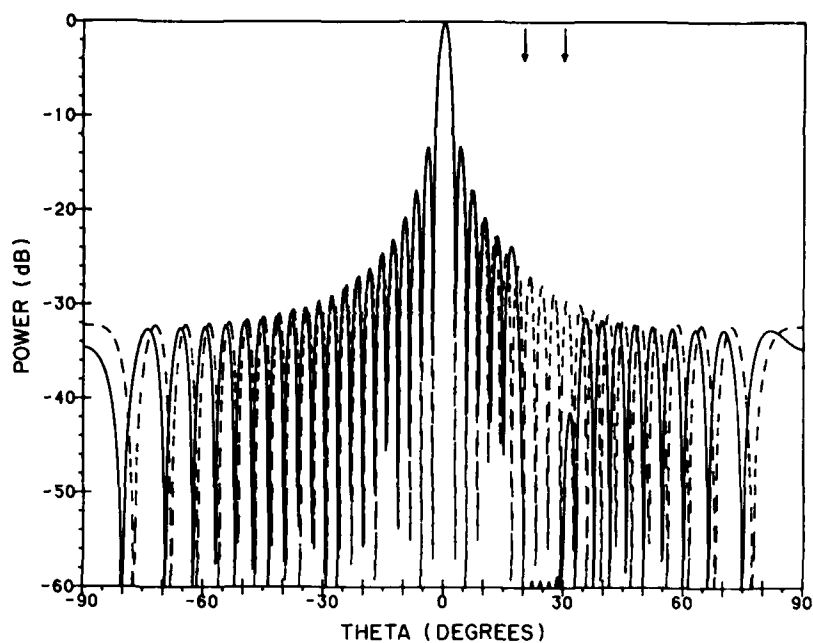


Figure 8. Unperturbed Uniform 41-Element Array Pattern (----) and Perturbed Pattern (—) With Lowered Sidelobes in the Sector [20°, 30°]. $\mu_2/\mu_1 = 100$

perturbed patterns produced by this nulling method is far less than that produced by the wideband nulling method described in Reference 5, in which the ratio of the sidelobe sector power to the mainbeam power is minimized with no attempt made to minimize weight perturbations.

Parallel calculations were also performed for the constant look direction gain solution of Eq. (5). The curves for average sector sidelobe power and the sum of the squared weight perturbations are not significantly different from Figures 1 and 2 and hence are not included here. Figure 9 shows the ratio of $|\underline{w}|/|\underline{w}_0|$ in dB. Here the perturbed weights gradually increase as μ_2/μ_1 increases. The ratio for the 11-element array increases to 0.8 dB for $\mu_2/\mu_1 = 100,000$, with smaller increases for the larger arrays.

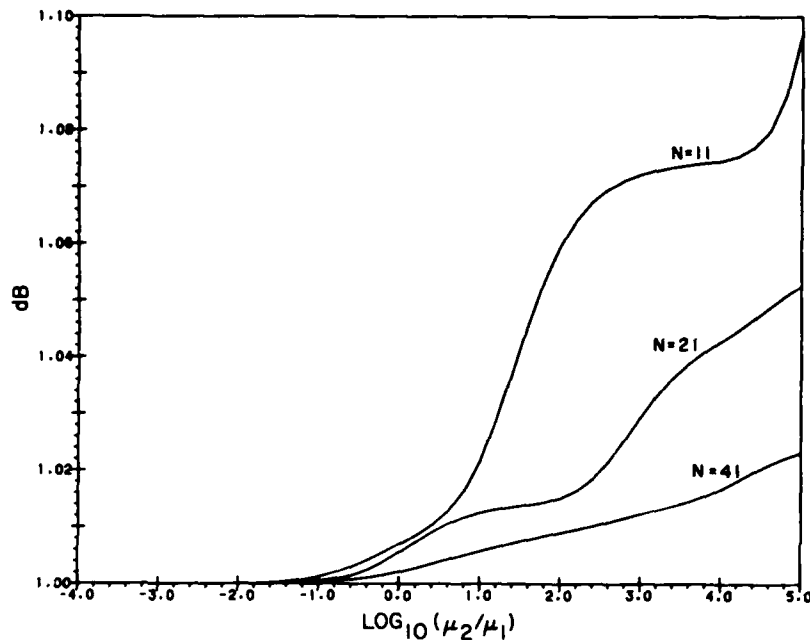


Figure 9. $|w|/|w_0|$ (dB) for Arrays of 11, 21, and 41 Elements. Constant look direction gain constraint

4. CONCLUSIONS

An analytic solution has been obtained for the set of array weights that minimize a performance measure consisting of a weighted sum of the power in a sidelobe sector and the squared weight perturbations. Minimization of this performance measure is effective in reducing the sidelobes in a specified array pattern sector while maintaining pattern integrity. Pattern distortion in other regions of the pattern is greatly reduced from what it is if no weight is placed on minimizing the weight perturbations. The relative emphasis placed on sidelobe reduction and on pattern integrity can be shifted by varying the ratio of the weights in the performance measure. As the number of elements in the array increases, the power in the specified sector decreases as does the pattern distortion. An analytic solution is also obtained for the set of array weights that minimize the performance measure subject to the constraint that the gain in the look direction be fixed. Numerical results obtained with the two solutions do not differ significantly except for extremely large weighting of the sidelobe sector power relative to the weight perturbations in which case the unconstrained solution tends to zero.

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Appendix A

On the Ill-Conditioned Nature of the Matrix H of Eq. (1)

The matrix H defined by Eq. (1) is ill-conditioned; that is, it is extremely sensitive to small changes in its elements. To demonstrate this we consider the matrix $(\mu_1/\mu_2) I + H$ with I the identity matrix, and examine the elements of the inverse matrix $A = [(\mu_1/\mu_2) I + H]^{-1}$ for large values of μ_2/μ_1 . In Figure A1 we have plotted $A_{11}/(\mu_2/\mu_1)$ for $N = 41$ and $u_0 = 1.3226$, $\epsilon = 0.2482$ corresponding to the sidelobe sector $[20^\circ, 30^\circ]$. It is seen, for example, that as μ_2/μ_1 increases from 10^6 to 10^8 , and hence as μ_1/μ_2 decreases from 10^{-6} to 10^{-8} , A_{11} increases from 200,000 to 12,600,000. Hence, merely by adding 10^{-8} to the unit diagonal terms of the matrix H instead of 10^{-6} , we obtain a more than sixty-fold increase in the terms of the resulting inverse matrix!

For sufficiently large values of μ_2/μ_1 , there is no further increase in the elements of A and hence $A_{11}/(\mu_2/\mu_1)$ goes to zero as $1/(\mu_2/\mu_1)$. This can be seen in Figure A2 where we have plotted $A_{11}/(\mu_2/\mu_1)$ for $N = 7$ and $u_0 = 1.3226$, $\epsilon = 0.2482$. Note the sudden change in the slope of the curve at around $\mu_2/\mu_1 = 10^{14}$, the point at which A becomes essentially equal to H^{-1} .

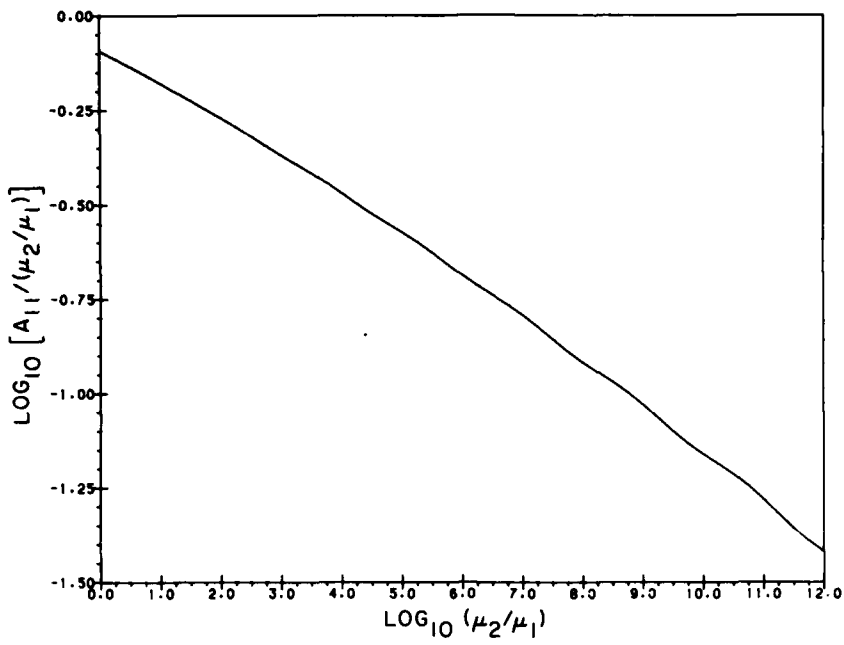


Figure A1. Plot of $A_{11}/(\mu_2/\mu_1)$ for $N = 41$, $A = [(\mu_2/\mu_1) I + H]^{-1}$

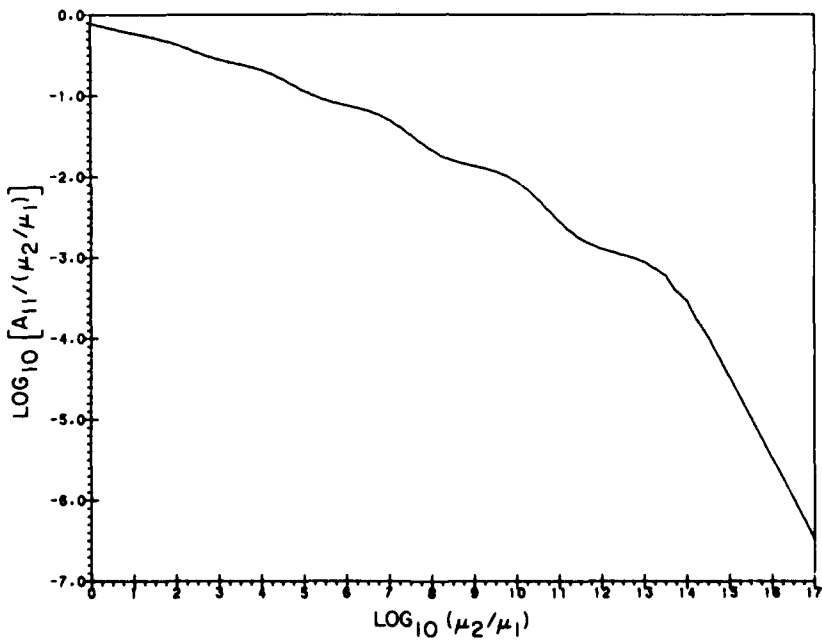


Figure A2. Plot of $A_{11}/(\mu_2/\mu_1)$ for $N = 7$, $A = [(\mu_2/\mu_1) I + H]^{-1}$

Appendix B

Proof That the Factor $(\underline{s}^\dagger A \underline{w}_0 - g)/(\underline{s}^\dagger A \underline{s})$ in Eq. (5) is Real

The matrix A in Eq. (5) is given by $A = [I + (\mu_2/\mu_1) H]^{-1}$ with I the identity matrix and H defined by Eq. (1). Since A is the inverse of an Hermitian matrix, A is likewise Hermitian so that $\underline{s}^\dagger A \underline{s}$ is real. It then remains to show that $\underline{s}^\dagger A \underline{w}_0$ is real. The elements of $A^{-1} = I + (\mu_2/\mu_1) H$ satisfy the symmetry relation

$$A^{-1}_{N+1-m, N+1-n} = A^{-1}_{mn}^* .$$

We will show below that the elements of A satisfy the same symmetry relation. Since the components of the look direction vector \underline{s} and the initial weight vector \underline{w}_0 satisfy the symmetry relations

$$s_{N+1-n} = s_n^*$$

$$w_{0, N+1-n} = w_{0n}^* ,$$

it then follows that

$$\begin{aligned}
 \underline{s}^\dagger A \underline{w}_0 &= \sum_{m=1}^N \sum_{n=1}^N s_m^* w_{on} A_{mn} \\
 &= 1/2 \sum_{m=1}^N \sum_{n=1}^N \left[s_m^* w_{on} A_{mn} + s_{N+1-m}^* w_{o, N+1-n} A_{N+1-m, N+1-n} \right] \\
 &= 1/2 \sum_{m=1}^N \sum_{n=1}^N \left[s_m^* w_{on} A_{mn} + (s_m^* w_{on} A_{mn})^* \right]
 \end{aligned}$$

is real.

To show that $A_{N+1-m, N+1-n} = A_{mn}^*$, we define in general the matrix $C^\#$ in terms of the $N \times N$ complex valued matrix C by

$$C_{mn}^\# = C_{N+1-m, N+1-n}^*$$

and show that

$$(CD)^\# = C^\# D^\# \quad . \quad (B1)$$

It then follows from

$$A A^{-1} = I$$

that

$$(A A^{-1})^\# = A^\# (A^{-1})^\# = I$$

and so if $(A^{-1})^\# = A^{-1*}$, then $A^\# = A^*$.

To prove Eq. (B1), we note that

$$\text{the } (m, p)\text{th element of } CD = \sum_{n=1}^N C_{mn} D_{np} \quad .$$

$$\text{the } (m, p)\text{th element of } (CD)^{\#} = (CD)_{N+1-m, N+1-p}^*$$

$$= \sum_{n=1}^N C_{N+1-m, n}^* D_{n, N+1-p}^* \quad .$$

$$\text{the } (m, n)\text{th element of } C^{\#} = C_{N+1-m, N+1-n}^* \quad .$$

$$\text{the } (n, p)\text{th element of } D^{\#} = D_{N+1-n, N+1-p}^* \quad .$$

$$\text{and the } (m, p)\text{th element of } C^{\#} D^{\#} = \sum_{n=1}^N C_{N+1-m, N+1-n}^* D_{N+1-n, N+1-p}^*$$

so that $(CD)^{\#} = C^{\#} D^{\#}$ as claimed.

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